Teskari matritsa va ularga doir misollar.

Ushbu A kvadrat matritsani qaraylik:

$$A = \begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \cdots \cdots \cdots & \cdots \cdots \cdots & \cdots \cdots \\ a_{n1} & a_{n2} \cdots & a_{nn} \end{pmatrix} \qquad (1)$$

Agar

$$AB = BA = E$$

bo’lsa, B matritsa A matritsa uchun *teskari matritsa* deb ataladi.

A matritsaga teskari matrisani A^{-1} kabi belgilanadi.

1-teorema. Agar A matritsa xos, ya'ni $detA = 0$ bo'lsa, u holda A^{-1}teskari matritsa mavjud emas.

Isbot. A matritsa uchun $AB = E$ bo'ladigan B matritsa mavjud deb faraz qilaylik. U holda $\det(AB) = detE$. 7-xossaga asosan: $det(AB) = detA \cdot detB = detE$. Biroq,

$det\,A = 0$, $detE$ ≠0 ekanligini hisobga olsak, 0=1 ni hosil qilamiz. Bu ziddiyat teoremani isbot qiladi.

2-teorema. Agar A matritsa xosmas, ya'ni $detA$ ≠0 bo'lsa, u holda uning uchun A^{-1}teskari matritsa mavjud.

Isbot. (1) matritsaning determinantini Δ orqali ifodalaymiz:

$$\Delta = detA = \begin{vmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \cdots & \cdots \cdots \cdots \cdots & \cdots \cdots \\ a_{n1} & a_{n2} \cdots & a_{nn} \end{vmatrix} \qquad (3)$$

Bu determinantni a_{ij} elementining algebraik to'ldiruvchisi A_{ij} orqali ifodalaymiz. A_{ij} algebraik to'ldiruvchilardan yangi $\tilde{A}$ matritsa tuzamiz.

$$\tilde{A} = \begin{pmatrix} A_{11} & A_{21} \cdots & A_{n1} \\ A_{12} & A_{22} \cdots & A_{n2} \\ \cdots & \cdots \cdots \cdots \cdots & \cdots \cdots \\ A_{1n} & A_{2n} \cdots & A_{nn} \end{pmatrix} \qquad (4)$$

Bu matritsa *A* matritsaga *biriktirilgan matritsa* deb ataladi.

Bu matritsaning barcha elementlarini $detA = \Delta$ ga bo'lamiz. U holda B matritsa hosil bo'ladi:

$$B = \begin{pmatrix} \frac{A_{11}}{\Delta} & \frac{A_{21}}{\Delta} \cdots & \frac{A_{n1}}{\Delta} \\ \frac{A_{12}}{\Delta} & \frac{A_{22}}{\Delta} \cdots & \frac{A_{n2}}{\Delta} \\ \cdots & \cdots & \cdots \\ \frac{A_{1n}}{\Delta} & \frac{A_{2n}}{\Delta} \cdots & \frac{A_{nn}}{\Delta} \end{pmatrix} \qquad (5)$$

Bu matritsa A matritsaga teskari matritsa bo'ladi. B matritsa A matritsa uchun teskari matritsadir, ya'ni $B = A^{-1}$.

3-teorema. *Agar A matritsa xosmas bo'lsa*, *u holda* A^{-1} *matritsa yagonadir.*

Shunday qilib, berilgan A matritsaga teskari A^{-1} matritsani hosil qilish uchun quyidagi ishlarni amalga oshirish zarur:

1. $detA = \Delta$ ni hisoblash.
2. Agar $\Delta \neq 0$ bo'lsa $detA$ ning barcha elementlari uchun algebraic to'ldiruvchilardan tuzilgan $\tilde{A}$ biriktirilgan matritsani (4) formulada ko'rsatilgandek tuzish.
3. Bu matritsaning barcha elementlarini $\Delta = detA$ ga bo'lish.

1-misol. Ushbu

$$\begin{pmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ -2 & 1 & 1 \end{pmatrix}$$

matritsa uchun teskari matritsa tuzing.

Yechish. Avval $detA$ ni topamiz:

$$\Delta = detA = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ -2 & 1 & 1 \end{vmatrix} = 3 \neq 0$$

Matritsa barcha elementlarining algebraic to’ldiruvchilarini hisoblaymiz:

$$A_{11}=\begin{vmatrix}0 & -1\\ 1 & 1\end{vmatrix}=1,\ A_{21}=-\begin{vmatrix}-2 & 1\\ 1 & 1\end{vmatrix}=3,\ A_{31}=\begin{vmatrix}-2 & 1\\ 0 & -1\end{vmatrix}=2,$$

$$A_{12}=\begin{vmatrix}2 & -1\\ -2 & 1\end{vmatrix}=0,\ A_{22}=\begin{vmatrix}1 & 1\\ -2 & 1\end{vmatrix}=3,\ A_{32}=-\begin{vmatrix}1 & 1\\ 2 & -1\end{vmatrix}=3,$$

$$A_{13}=\begin{vmatrix}2 & 0\\ -2 & 1\end{vmatrix}=2,\ A_{23}=-\begin{vmatrix}1 & -2\\ -2 & 1\end{vmatrix}=3,\ A_{33}=\begin{vmatrix}1 & -2\\ 2 & 0\end{vmatrix}=4.$$

A matritsaga biriktirilgan $\tilde{A}$ matritsa bunday bo'ladi.

$$\tilde{A}=\begin{pmatrix}1 & 3 & 2\\ 0 & 3 & 3\\ 2 & 3 & 4\end{pmatrix}$$

$\tilde{A}$ matritsaning hamma elementlarini $\Delta = 3$ ga bo'lsak, teskari A^{-1} matritsani hosil qilamiz.

$$A^{-1} = \begin{pmatrix} \frac{1}{3} & 1 & \frac{2}{3} \\ 0 & 1 & 1 \\ \frac{2}{3} & 1 & \frac{4}{3} \end{pmatrix}$$

Teskari matritsaning ikkita xossasini keltiramiz:

1. $detA^{-1} = \frac{1}{detA}$

2. $(AB)^{-1} = B^{-1}A^{-1}$.