Mavzu:**Determinantlar. Kramer formulasi.**

Determinantlar, xossalari, hisoblash.

nxn ta elementdan tuzilgan, kvadrat jadval ko'rinishidagi, ikki vertikal kesma orasiga olingan

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \dots a_{1n} \\ a_{21} & a_{22} & a_{23} \dots a_{2n} \\ a_{31} & a_{32} & a_{33} \dots a_{33} \\ . & . & . \dots . \\ a_{n1} & a_{n2} & a_{n3} \dots a_{nn} \end{vmatrix}$$

ifoda n −tartibli determinant, $a_{ij} \epsilon R \ (i, j = \overline{1, n})$ sonlari esa determinant elementlari deyiladi.

Gorizantal qatorlar yo'llar (satrlar), vertikal qatorlar esa ustunlar deyiladi.

Birinchi indeksi *i* bo'lgan elementlar *i*-yo'l (satr) elementlari, ikkichi indeksi *j* bo'lgan elementlar esa *j*-ustun elementlari deyiladi.

Masalan, a_{34} element 3-yo'l (satr), 4-ustunda joylashgan. a_{11}, a_{22},…,a_{nn} joylashgan dioganal determinant bosh dioganali, ikkinchi dioganal esa yordamchi dioganal deyiladi.

$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ ifoda 2-tartibli determinant deyilib, qiymati $a_{11} a_{22} - a_{12} a_{21}$ ayirmaga teng hisoblanadi .

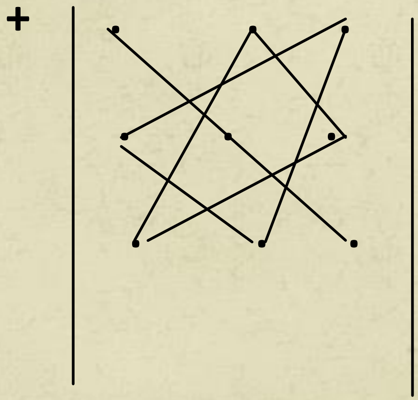
$$\begin{vmatrix} a_{11} a_{12} a_{13} \\ a_{21} a_{22} a_{23} \\ a_{31} a_{32} a_{33} \end{vmatrix}$$

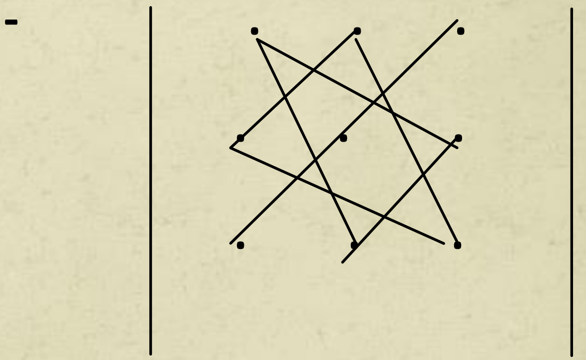
ifoda 3-tartibli determinant, uning qiymati

$$a_{11}\ a_{22}\ a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{13}\ a_{22} a_{31} - a_{12} a_{21} a_{33} - \ a_{11} a_{23} a_{32}$$

songa teng deyiladi.

3-tartibli determinant 6 ta had yig'indisidan iborat, uchtasi musbat, qolgan uchtasi manfiy ishoralidir. Hadlar yozilish tartibi, ishoralarni eslab qolish uchun "uchburchak qoidasi" deb ataluvchi sxemadan foydalaniladi.





1) $\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix} = 4 \cdot 1 - (-2) \cdot 3 = 10,$

2) $\begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{vmatrix} = 4 \cdot (-2)(-5) + (-3) \cdot 8 \cdot 1 + 5 \cdot 3 \cdot (-7) - 5 \cdot (-2) \cdot 1 - 3 \cdot 3 \cdot (-5) - 4 \cdot 8 \cdot (-7) = 40 - 24 - 105 + 10 - 45 + +224 = 100$

n −tartibli Δ determinantda a_{ij} element joylashgan yo'l va ustun o'chirilsa, ($n-1$) tartibli determinant hosil bo'lib, uni a_{ij} element minori deyiladi va M_{ij} harfi bilan belgilanadi.

$A_{ij} = (-1)^{i+j} \cdot M_{ij}$ soni esa a_{ij} element algebraik to'ldiruvchisi deyiladi.

Masalan , $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ bo'lsa,

$A_{11} = (-1)^2 \cdot \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = -3$, $A_{21} = (-1)^3 \cdot \begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} = 6$, $A_{32} = (-1)^5 \cdot \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = 6, ..$

Kelgusida yo'l satr uchun o'rinli munosabatlarni ixtiyoriy qator uchun deb ataymiz.

Teorema. 1). Ixtiyoriy qator elemenlarini o'z algebraik to'ldiruvchilariga ko'paytmalari yig'indisi determinant qiymatiga teng.

2). Ixtiyoriy qator elementlari parallel qator elementlari algebraik to'ldiruvchilariga ko'paytmalari yig'indisi nolga teng.

$\Delta = \sum_{k=1}^{n} a_{i,k} A_{i,k}$, $0 = \sum_{k=1}^{n} a_{i,k} A_{i,s}$, bunda S=1...n, ≠k

Isbot. Soddalik uchun isbotni 3-tartibli determinantlar uchun keltiramiz(3-yo’l elementlarini tanladik).

$$\Delta= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{31}\mathrm{A}_{31} + a_{32}\mathrm{A}_{32} + a_{33}\mathrm{A}_{33} =$$

$$= a_{31} \cdot (-1)^4 \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} + a_{32} \cdot (-1)^5 \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{22} \end{vmatrix} a_{33} \cdot (-1)^6 \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} =$$

$$= a_{12}\ a_{23}\ a_{31} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} + a_{13}\ a_{21}a_{32} + a_{11}a_{22}a_{33} - a_{12}\ a_{21}a_{33}$$

Masalan, $a_{31}\mathrm{A}_{11} + \mathrm{a}_{32}\mathrm{A}_{12} + a_{33}\mathrm{A}_{13} = 0, a_{31}\mathrm{A}_{21} + \mathrm{a}_{32}\mathrm{A}_{22} + a_{33}\mathrm{A}_{33} = 0$ tengliklarni ham shunday tekshirish mumkin.

Misol. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 2 \\ -1 & 4 & 3 & 1 \\ 0 & 0 & 1 & -1 \end{vmatrix} = 2 \cdot (-1)^6 \begin{vmatrix} 1 & 2 & 3 \\ -1 & 4 & 3 \\ 0 & 0 & 1 \end{vmatrix} = 2 \cdot 1 \cdot (-1)^6 \begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix} = 2 \cdot (4+2) = 12$

Natija 1. Determinant biror qatori barcha elementlari nol bo'lsa, determinant qiymati nolga teng.

Natija 2. Agar determinantda bosh dioganal bir tarafida turgan elementlar nol bo'lsa, determinant qiymati bosh dioganal elementlari ko'paytmasiga teng.

Isboti yoyish teoremasidan kelib chiqadi.

$$\begin{vmatrix} d_{11} & d_{12} & d_{13} & \dots & d_{1n} \\ 0 & d_{22} & d_{23} & \dots & d_{2n} \\ 0 & 0 & d_{33} & \dots & d_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & d_{nn} \end{vmatrix} = d_{11} \begin{vmatrix} d_{22} & d_{23} & \dots & d_{2n} \\ 0 & d_{33} & \dots & d_{3n} \\ 0 & 0 & \dots & d_{4n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & d_{nn} \end{vmatrix} = \dots = d_{11}\, d_{22} \dots d_{nn}$$

Determinant xossalarini 3-tartibli determinantlarda tushunishga harakat qilamiz.

$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ berilgan bo'lsin.

1°. *Determinant biror yo'li unga mos ustun bilan almashtirilsa determinant qiymati o'zgarmaydi, umuman, barcha yo'llari mos ustunlar bilan almashtirilsa (trasponirlansa) ham determinant qiymati o'zgarmaydi.*

2°. Determinant ikki parallel qatori o'rinlari almashtirilsa, determinant qiymati ishorasi o'zgaradi.

Natija. Determinant ikki parallel qatori bir xil bo'lsa, determinant qiymati nolga teng.

3°. Determinant biror qatori o'zgarmas k songa ko'paytirilsa, uning qiymati ham k ga ko'payadi.

Isboti. K songa ko'paygan qator bo'yicha yoyib, xossa o'rinliligiga amin bo'lamiz.

Natija. Determinant ikki parallel qatori o'zaro proporsional bo'lsa, determinant qiymati nolga teng.

$$4^{\circ}.\ \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21}+\alpha_1 & a_{22}+\alpha_2 & a_{23}+\alpha_3 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ \alpha_1 & \alpha_2 & \alpha_3 \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Natija. Determinant biror qatori o'zgarmas k songa ko'paytirilib, o'ziga parallel qator elementlariga qo'shilsa va natijasi ular o'rniga yozilsa, determinant qiymati o'zgarmaydi.

Bu natijadan yuqori tartibli determinantlarni diogonal determinantga keltirishda foydalaniladi.

$$\Delta = \begin{vmatrix} 1 & a_1 & a_2 & \dots & a_n \\ 1 & a_1+b_1 & a_2 & \dots & a_n \\ 1 & a_1 & a_2+b_2 & \dots & a_n \\ \dots & \dots & \dots & \dots & \dots \\ 1 & a_1 & a_2 & \dots & a_n+b_n \end{vmatrix} =$$

1-yo'lni (-1)ga ko'paytirib, qolgan barcha yo'llarga qo'shamiz

$$= \begin{vmatrix} 1 & a_1 & a_2 & \dots & a_n \\ 0 & b_1 & 0 & \dots & \\ 0 & 0 & b_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & b_n \end{vmatrix}$$

$= b_1\, b_2\, b_3 \dots b_n.$

Kramer formulasi.

Noma"lumlar koeffisientlaridan tuzilgan $\Delta=\begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \dots & \cdot \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{vmatrix}$ determinant tenglamalar sisremasining asosiy determinanti, undagi j-ustun o'rniga ozod b_i hadlardan iborat ustun qo'yilgan determinant esa *j*-yordamchi determinant deyiladi va Δ_j ko'rinishida belgilanadi.

$$\Delta_j=\begin{vmatrix} a_{11}\dots a_{1j-1} b_1 \; a_{1j+1}\dots a_{1n} \\ a_{21}\dots a_{2j-1} b_2 \; a_{2j+1}\dots a_{2n} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}\dots a_{nj-1} b_n \; a_{nj+1}\dots a_{nn} \end{vmatrix}$$

Dastlab, berilgan tenglamalar sistemasidan har bir i-tenglamani A_{i1} *ga* ko'paytiramiz va hosil bo'lgan tenglamalarni qo'shamiz:

$$(a_{11}A_{11}+a_{21}A_{21}+\cdots+a_{n1}A_{n1})x_1+((a_{12}A_{11}+a_{22}A_{21}+\cdots+a_{n2}A_{n1})x_2+\cdots+(a_{1n}A_{11} + a_{2n}A_{21}+\cdots+a_{nn}A_{n1})x_n=b_1A_{11}+b_2A_{21}+\cdots+b_nA_{n1}$$

Determinantni yoyish haqidagi teoremaga ko’ra: $\Delta \cdot x_1 = \Delta_1$.Endi sistamadagi har bir i-tenglama A_{i2} ga ko’paytirilib qo’shilsa, $\Delta \cdot x_2 = \Delta_2$,..., A_{in} ga ko'paytirib qo’shilsa, $\Delta x_n = \Delta_n$ tenglik hosil bo’ladi.

Demak, sistemadagi noma”lumlar $x_j = \frac{\Delta_j}{\Delta}$ formula yordamida xisoblanar ekan. Bu Kramer formulasidir.

$\Delta \cdot x_j = \Delta_j$ tenglikdan quyidagilar kelib chiqadi:

1) $\Delta \neq 0$ da sistema yagona echimga ega, uni birgalikda deyiladi.

2) $\Delta = 0$, $\Delta_j = 0$ bo’lsa, sistema cheksiz ko’p echimga ega.

3) $\Delta = 0$, Δ_j lardan birortasi noldan farqli bo’lsa, sistema echimga ega emas.

Misol. Kramer formulasi yordamida yeching:

$$\begin{cases} x_1 + x_2 + x_3 - x_4 = 2 \\ x_1 + 2x_2 + 3x_3 - 4x_4 = -2 \\ 2x_1 + x_2 - x_3 + x_4 = 5 \\ 4x_1 + 3x_2 + 2x_3 - 4x_4 = 0 \end{cases} \qquad \Delta = \begin{vmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & -4 \\ 2 & 1 & -1 & 1 \\ 4 & 3 & 2 & -4 \end{vmatrix} = 3,$$

$$\Delta_1 = \begin{vmatrix} 2 & 1 & 1 & -1 \\ -2 & 2 & 3 & -4 \\ 5 & 1 & -1 & 1 \\ 0 & 3 & 2 & -4 \end{vmatrix} = 3, \quad \Delta_2 = \begin{vmatrix} 1 & 2 & 1 & -1 \\ 1 & -2 & 3 & -4 \\ 2 & 2 & -1 & 1 \\ 4 & 0 & 2 & -4 \end{vmatrix} = 6,$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 & -1 \\ 1 & 2 & -2 & -4 \\ 2 & 1 & 5 & 1 \\ 4 & 3 & 0 & -4 \end{vmatrix} = 9, \quad \Delta_4 = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & -2 \\ 2 & 1 & -1 & 5 \\ 4 & 3 & 2 & 0 \end{vmatrix} = 12$$ bo'lganligi uchun

$x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4.$