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River channel deformations in the area of damless water intake

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Abstract. A mathematical model of the unsteady uneven movement of the river flow in the area of the damless water intake, flowing in deformable channels, was developed and numerically implemented. In the developed model of the deformation of the river channel, flowing on easily eroded beds, adapted formulas of the transporting ability of the flow are used, the hydrodynamic equations of motion and continuity of the flow, deformation of the channel, and sediment balance. To close the system of differential equations, we use the adapted formula of the conveying ability of the flow–Benold. In the calculations, the non-erosion speed was taken according to the formula C.E. Mirtschulava, taking into account standard values of adaptive coefficients. It is suitable for describing deformation processes, taking into account the drift of dredging slots. The model contains the minimum number of equations and empirical constants. In the present work, a numerical model is proposed for predicting channel processes in river beds in the area of damless water intakes and is verified on the results of experimental studies. The figures show sequential deformations of the channel, arising due to natural causes, as well as due to channel adjustment and treatment works (discharge of soil from the supply channel into the river). As can be seen from these figures, the channel from multi-sleeve is gradually turning into one-sleeve. It has been established that after 1 year of observation at low water, as a result of discharge of sediment from the sewage treatment works, the river ducts die off. These ducts are observed only in the flood. Note that below the Karshi main channel (KMCh), the left bank eroded by about 150 m. After a 10-year course, the low-water course turned into a one-arm riverbed. As a result of the erosion of the left-bank islands, it turned into the river itself. During the flood, there are also no pronounced sleeves, a significant erosion of the left bank is noted, up to 700 m. After that, this bank was strengthened. In the future, the channel process did not undergo significant changes. Towards the end of the 10th year of observation, the channel broadened somewhat, and the flood depth decreased in the flood. Comparison of the results of field and numerical studies shows that the developed model gives good results. For low-water conditions, coincidence is better than for flood conditions. During the flood, under natural conditions, somewhat greater depths are observed on the floodplain than in the calculations. It should be noted that both in calculations and in kind:
- there was a significant erosion of the left bank below the KMCh;
- the channel of the Amudarya river in the considered section from multi-sleeve turned into practically one-sleeve.

1. Introduction

With damless water intake from rivers, the channel of which passes on easily eroded soils, many operational problems arise. One of them is the lack of the possibility of guaranteed selection of the volume of water and ensuring the flow of water into the head structure with the lowest turbidity. The problems of protecting the head structures of the supply channels of pumping stations during damless water intake from the entrainment of channel sediments, mainly bottom ones from rivers, the channel of which passes on easily eroded soils transporting a large amount of sediment, is not always successfully solved [1–3]. For a rational solution of the above problem, the multifactor and multidirectional channel processes that are occurring in the main are impeded. Recently, for predictive calculations of channel processes, numerical modeling is increasingly used. Numerous studies performed have shown that the use of numerical modeling is not only many times cheaper and faster than the so-called physical modeling, but it allows you to simulate processes that could not be modeled using traditional methods. Obviously, it is necessary that the mathematical model adequately reflect the real phenomenon and describe all the main effects inherent in this phenomenon [4 – 6].In hydraulic engineering research practice for modeling channel flows, approaches based on the numerical solution of the two-dimensional Saint-Venant equations have shown high efficiency and sufficient accuracy. The derivation of these equations, numerical integration algorithms and calculation examples are given. Many researchers in computational hydraulics have pioneered the development and use of numerical modeling in river hydraulics. The Novosibirsk School of Hydrotechnicians, led by Academician O. F. Vasiliev, of the Hydroproject Institute, - B., has great merit in this area. L. The historian, V. M. Lyakhter, A. N. Militeev, V. V. Belikov, A. M. Prudovsky, S.Y. Shkolnikov [7–9].

However, for deformable channels, the Saint-Venant model turns out to be open, and needs to be supplemented, for example, with an equation or a system of equations for finding variables in time and space of bottom marks.

Often, the so-called two-dimensional equation of sediment balance is used for this purpose. In [8–10], the necessity of introducing diffusion terms into the sediment balance equation was theoretically substantiated, and an expression was obtained for the diffusion coefficient of the bottom mark (which is accepted by the spherical tensor) proportional to the longitudinal (in direction of the velocity vector) specific sediment discharge:

$$(1-\mathrm{p})\frac{\partial \mathrm{Z_b}}{\partial \mathrm{t}}+\frac{\partial US_{*}h}{\partial x}+\frac{\partial VS_{*}h}{\partial y}=\frac{\partial}{\partial x}D\frac{\partial Z_b}{\partial x}+\frac{\partial}{\partial y}D\frac{\partial Z_b}{\partial y}\ ; \qquad (1)$$

here $D = ahS_{*}\left|\vec{U}\right|$, $\left|\vec{U}\right| = \sqrt{U^2+V^2}$, a - coefficient of proportionality.

U, V - components of the vector of averaged over depth water velocity along the x, y axes;

Model (3) was verified on experimental data. A different model was proposed in [10–12]. Its main differences from the previous one are that the diffusion term is written only for the directional flow velocity orthogonal to the vector; the additional term takes into account the influence of the curvature of the streamlines on the change in the bottom marks. In this case, the diffusion coefficient is assumed to be proportional not to the flow velocity, but to the value of the non-moving flow velocity. Using this model, interesting results have been obtained on the calculation of channel reformation.

In all the deformation models mentioned above, it is assumed that the concentration of sediment particles in the flow is close to equilibrium. At the same time, there are a number of problems for the solution of which this assumption is not applicable. For example, a clarified (without sediment) water stream usually enters the lower downstream water bodies of a hydrothermal stream, a reservoir saturated with sediment flows into reservoirs, on the contrary, excessive turbidity occurs during the operation of dredgers, and the concentration of sediment in a stream also differs from equilibrium in sharply unsteady and wave flows. In, a model of sediment transport in an arbitrary flow (in a one-dimensional formulation) was proposed and experimentally substantiated, in which the concentration of particles in the general case does not coincide with the equilibrium one. Moreover, it was shown that the specific sediment consumption from the bottom into the thickness of the flow is proportional to *(S-S_*)*. In view of the above, the authors of the present work improved the method for calculating the deformation of the river channel taking into account the selection of part of the water flow to the head structures and using the developed model to conduct numerical studies of the deformation of the river channel in the area of the damless water intake was determined as the main goal of this work and an attempt was made to take into account the advantages of a number of the above models and to propose a mathematical model of sediment transport in uneven and unsteady river flows.

2. Method

An analysis of existing methods for calculating deformation, determining the merits of the developed models, and proposing a mathematical model of sediment transport with uneven unsteady movement of river flows in the area of damless water intake is accepted as a research method for this work.

3. Result and Discussion

For numerical calculations of river flows with a deformable bottom, it is proposed to use the mathematical model described by the system of two-dimensional Saint-Venant equations [12–14]:

$$\frac{\partial Q_x}{\partial t}+\frac{\partial Q_x U_y}{\partial x_y}+\frac{1}{2}g\frac{\partial h^2}{\partial x_y}=-\lambda Q_x\frac{|U|}{2h}-gh\frac{\partial z_b}{\partial x_y};$$

$$\frac{\partial z}{\partial t}+\frac{\partial Q_x}{\partial x_x}=0$$

and closing ratios:

$$\frac{\partial hS}{\partial t}+\frac{\partial USh}{\partial x}+\frac{\partial VSh}{\partial y}=-K(S-S_*)\,; \tag{2}$$

$$(1-\mathrm{p})\frac{\partial \mathrm{Z_b}}{\partial \mathrm{t}}=K(S-S_*)+\frac{\partial}{\partial x}D\frac{\partial Z_b}{\partial x}+\frac{\partial}{\partial y}D\frac{\partial Z_b}{\partial y}\,; \tag{3}$$

$$K=\begin{cases}\alpha U_*+(1-\alpha)W, & U_*\ge W\\ W, & U_*\le W\end{cases}\quad 0\le\alpha<1; \tag{4}$$

$$D=\beta\tilde{S}hW\,; \tag{5}$$

$$S_*=\alpha\frac{\lambda\rho}{2\rho_s}\frac{(|\vec{U}|-U_N)^2}{gh}\left(\frac{0.13}{tg\varphi}+0.01\frac{|\vec{U}|}{W}\right),\qquad \lambda=2gn^2h^{-1/3} \tag{6}$$

IOP Conf. Series: Materials Science and Engineering **869** (2020) 072014 doi:10.1088/1757-899X/869/7/072014

here t - time; h- flow depth; U, V – components of the flow velocity along the axis X and Y respectively; $\left|\vec{U}\right| = \sqrt{U^2 + V^2}$; S - volumetric concentration of sediment particles in the stream; S_* - equilibrium volume concentration of particles (saturation concentration), adopted by the modified Reynolds formula; K - intensity factor of vertical sediment exchange between the bottom and the stream; p - soil porosity (ratio of pore volume to total soil volume with pores); ρ_s, ρ - density of soil and water, respectively; φ - soil internal friction angle; W – hydraulic coarseness; U_* - dynamic speed; $\left|\vec{U}\right|, U_N$ modulus of the average vertical flow velocity and non-shifting velocity, respectively; λ - hydraulic friction coefficient calculated by the Mannig formula; n– roughness coefficient.

In the calculations, the non-shifting speed was taken according to the formula of C.E. Mirtskhulava, which, taking into account the standard values of the coefficients, has two formally equivalent types of records:

$$U_N = \lg\frac{8.8h}{d_{90}}\sqrt{\frac{2}{3.5}\left[(\rho_s - \rho)gd_{50} + 1.25C_y^H\right]} \qquad (7)$$

$$U_N = 0.18\sqrt{\frac{2}{3.5\lambda}\left[(\rho_s - \rho)gd_{50} + 1.25C_y^H\right]}, \qquad (8)$$

where C_y^H - traction in t/m^2; d_{50} - average diameter of soil particles, d_{90} - diameter of 90% of soil particles. When calculating the diffusion coefficient (7), three versions of the formulas were used for:

a) according to the total equilibrium concentration of sediment and suspended sediment

$$\tilde{S} = S_* \qquad (9)$$

b) by bottom equilibrium concentration

$$\tilde{S} = \alpha_1\frac{\lambda\rho}{2\rho_s}\frac{(\left|\vec{U}\right| - U_N)^2 \cdot 0.13}{gh\ \ tg\varphi}, \qquad (10)$$

v) by bottom concentration “without square”

$$\tilde{S} = \alpha_1\frac{\lambda\rho}{2\rho_s}\frac{(\left|\vec{U}\right| - U_N)\left|\vec{U}\right| \cdot 0.13}{gh\ \ tg\varphi}, \qquad (11)$$

The initial bottom surface was specified as the initial conditions $Z(x,y,0)$, the corresponding instantaneous velocity fields $\vec{U}$ $(x,y,0)$, depths $h(x,y,0)$ and concentration S $(x,y,0)$.

The condition for non-leakage is set as the boundary conditions on the “solid” boundaries, the flow rates or water levels are usually set on the “liquid” boundaries. At the boundaries through which the flow flows into the computational domain, the sediment concentration was set. Sometimes more complex types of boundary conditions were applied (the connection of expenditures with levels, “non-reflecting” boundary conditions, etc.).

The solution of the system of equations (2), (3) regarding the concentration of sediments and bottom marks was carried out by the finite volume method on mixed triangular-quadrangular grids in conjunction with the Saint-Venant equations. The developed numerical scheme is consistent with the scheme for the equation of continuity of the liquid phase, which eliminates the occurrence of the so-called "dipoles" - sources and sinks of mass. The scheme of the type of “directed differences” excludes unphysical oscillations of the bottom topography, provides the “transportability” property and the implementation of the difference analog of mass conservation for the solid phase.

The system of equations (2), (3) is, in a sense, “minimal” for modeling the processes of bottom deformations in uneven and unsteady river flows. Indeed, the rejection of equation (2) leads, as already noted above, to the postulation of the proximity of the concentration in the flow to the equilibrium one and does not allow one to set the boundary conditions for the concentrations. The refusal in the equation (3) of the terms describing the diffusion of the bottom marks makes it impossible to take into account the transformation processes (flattening) of underwater slopes. It is also clear that to describe three different physical processes (blast-sedimentation, longitudinal sediment transport, transverse diffusion of bottom marks) at present (before reaching more general regularities of sediment transport), at least three empirical coefficients are necessary (in our model it α, β, α_1). At the same time, the question of choosing the closing relations of the model (4-6) - (11) remains open and requires further research.

In the test problem considered below, the adjustment of the underwater slope was supposed $\alpha = 0.5$, kind of closing relations (5) – (11) and parameters α_1 and β varied during the calculation process. To test the model, we used laboratory experiments on erosion of a slope by a stream, the velocity vector of which is directed along the generatrix of this slope, conducted in the Hydroproject Research Laboratory [15–20].

Numerical calculations for identical conditions were performed according to model (2), (3). For a 18.0 m×2.0 m tray, a rectangular grid of 1800 cells with a size of 0.1 m × 0.2 m was constructed. At the input boundary for the first row of cells, the bottom was assumed to be indelible, which corresponded to the input section of the tray reinforced with a cement crust in the experiment [15].

At the inlet boundary, the water flow rate *Q=112 l/s* and the clarified flow *(S=0)* were set. Parameters of the numerical model α_1 and β were selected in the calculation process from the condition of the best coincidence in the average diameter of the tray of the calculated profile of the diffuse slope with the experimental data. The calculation results are presented in figure 1-3.

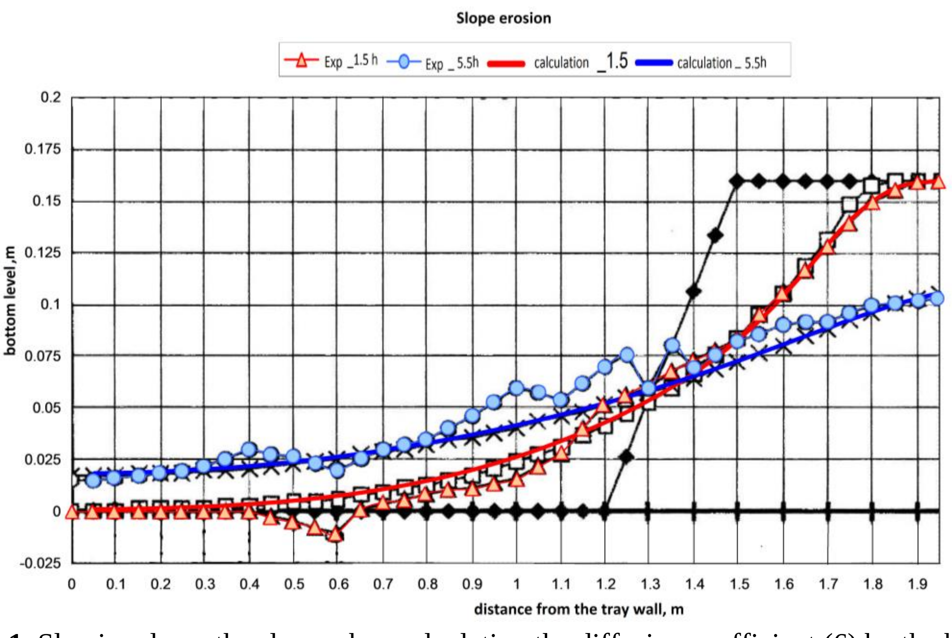

Figure 1. Slowing down the slope when calculating the diffusion coefficient (6) by the bottom concentration (12) and non-shifting velocity according to the formula (9): β =40, α_1=0.25.

IOP Conf. Series: Materials Science and Engineering **869** (2020) 072014 doi:10.1088/1757-899X/869/7/072014

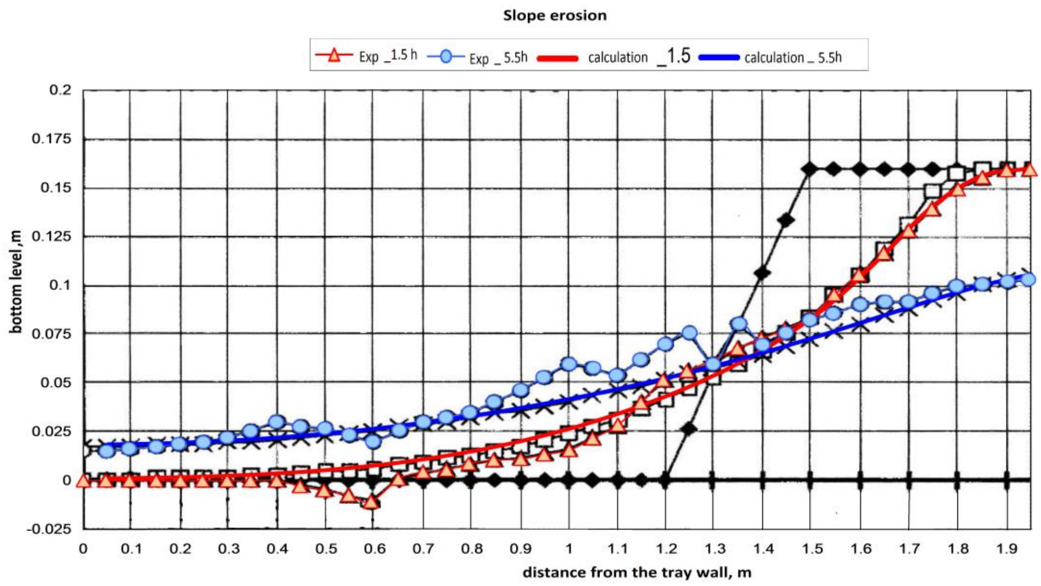

Figure 2. Slowing down the slope when calculating the diffusion coefficient according to bottom concentration (12) and non-shifting velocity according to the formula (10): β =15, α_1 =0.25.

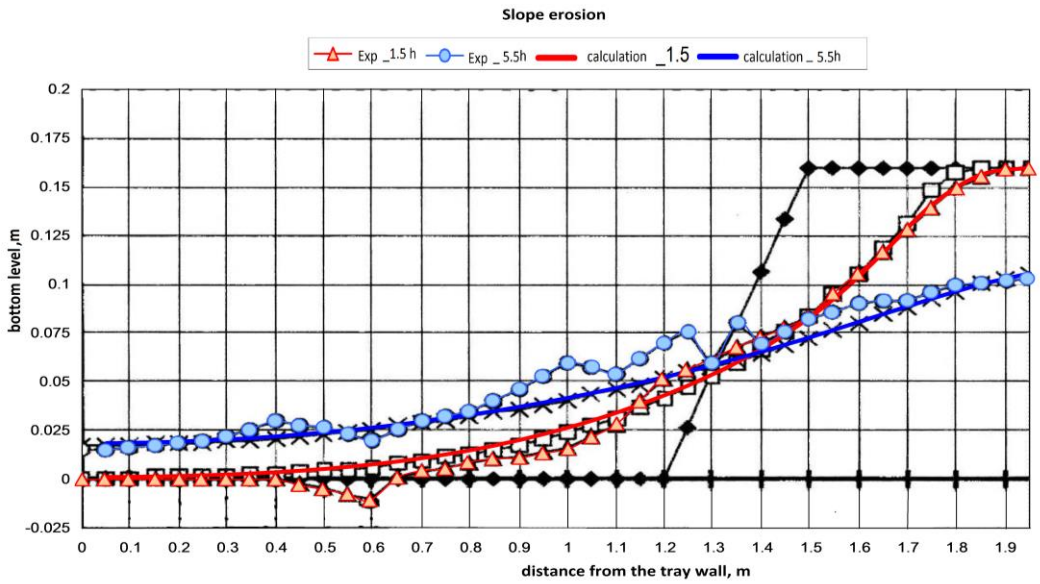

Figure 3. Slowing down the slope when calculating diffusion by the total concentration (8), (11) and non-shifting velocity according to the formula (10): β =30, α_1 =0.25

The best results, as can be seen from the presented figures, are given by the first option (according to the bottom concentration (10) and non-shifting velocity according to the formula (7): β =40, α_1 =0.25).

Thus, it can be argued that the proposed mathematical model of unsteady uneven motion of the river flow is suitable for describing the processes of deformation, including the insertion of dredging slots. The model contains the minimum number of equations and empirical constants.

A comparison with the data of experimental studies showed good convergence of the results, however, the optimal values of empirical coefficients for different experiments were slightly different. Therefore, further work is required to obtain more universal closing relations of the model.

The model was validated using full-scale data for the section of the river of the damless water intake of the Amudarya into the Karshi Main channel. The Amudarya River flows on hydraulically heterogeneous soils. This was possible to take into account, since the concentration transfer equation (2) allows independent calculation for fractions of various sizes (provided that the total concentration of particles in the stream is not very high). The channel process in this area, as in almost the entire Amu Darya, can be attributed to the type of channel multi-arm. This type is characterized by the fact that the geological structure of the floodplain and the channel are slightly different, therefore, only channel-forming sediments, which can be either suspended or bottom, will be considered below. As is known, the channel-forming fraction is determined by the composition of bottom sediments. Since the bulk of the bottom sediment has a size of 0.25 to 0.05 mm, their average particle size in the calculations was set to 0.12 mm.

For calculations according to the foregoing, we used the two-dimensional model described in this paper. The use of the Bagnold formula as a formula for conveying ability is justified by the fact that, unlike other formulas, individual terms in it are caused by bottom and suspended sediments. Therefore, having field material on the flow rate and fractional composition of bottom and suspended sediments, it is easier to adapt. Note that in this formula, the first term is caused by entrained (bottom) sediments, and the second term is weighted. As noted above, the ratio of sediment to suspended sediment consumption is. Then for the coefficient we get the formula:

$$\alpha_W = \frac{W}{U}\frac{0{,}24}{\alpha_p} \tag{7.2}$$

Based on data from field studies of the Amudarya River, in the area of the damless water intake into the Karshi main channel, the share of the channel-forming fraction in the composition of suspended sediments α_p is 0.31 and the ratio of the flow of bottom sediment to the flow of all (and not just channel-forming) sediment $\sum a$ during the flood is about 0.1 ... 0.15. Then we have that during the flood is $\alpha = \frac{a_\Sigma}{a_r} \approx 0{,}032 \div 0{,}049.$ During summer floods, the characteristic flow rate 1,5 – 2 м/с. Для принятой средней крупности гидравлическую крупность можно принять равной 0,01 м/с.

Moreover, it should be accepted that a higher number corresponds to higher speeds α Then: $\alpha_w = 0{,}036 \div 0{,}037.$ Such a value α_w и принималось в расчетах. As initial data on the levels and discharges of water, measurements at the Kerky waterfall for the period under consideration were taken. Kerki is located 15 km below the KMCh. Since there are no sources and drains except KMCh in this section of the Amu Darya, data on expenditures are quite representative.

The results of numerical studies of channel processes are shown in the following figures.

IOP Conf. Series: Materials Science and Engineering **869** (2020) 072014 doi:10.1088/1757-899X/869/7/072014

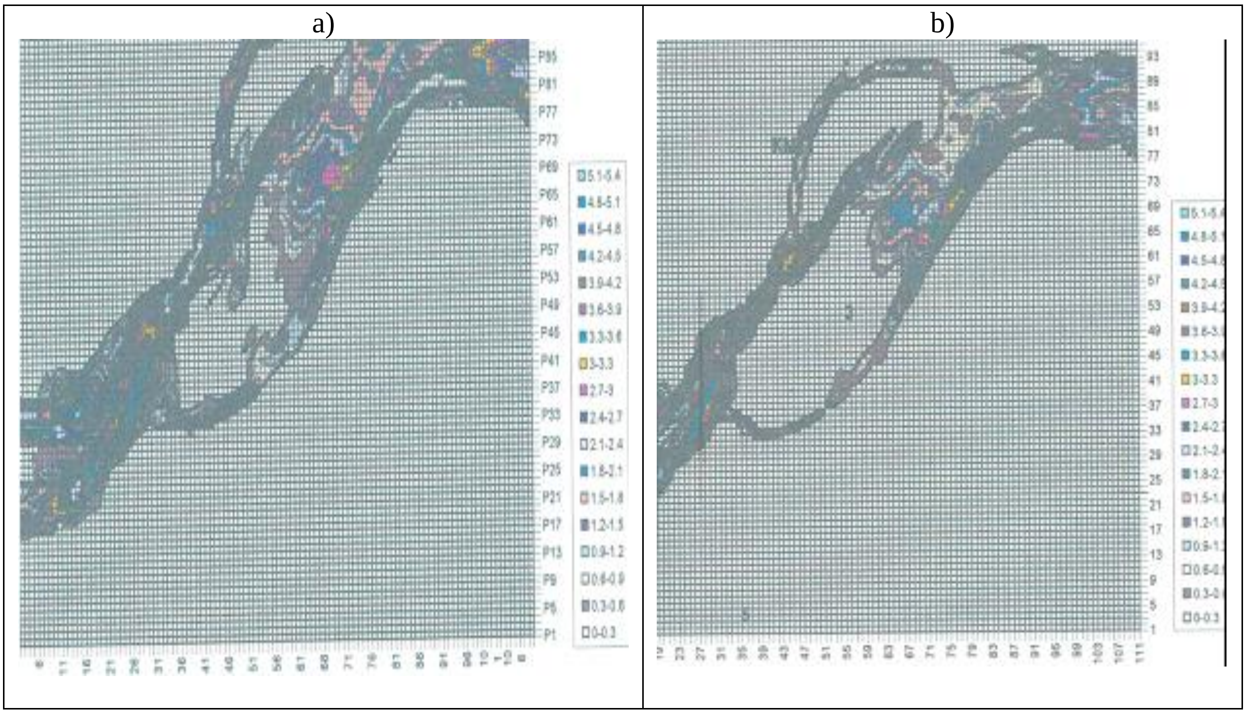

Figure 4. a) Distribution of the depths of the water flow in the area of the damless intake into the Karshi Main Canal at low water on the initial bottom of the bottom; b) Distribution of water flow depths in the area of damless water intake into the Karshi Main Chanel in the flood at the initial bottom of the bottom

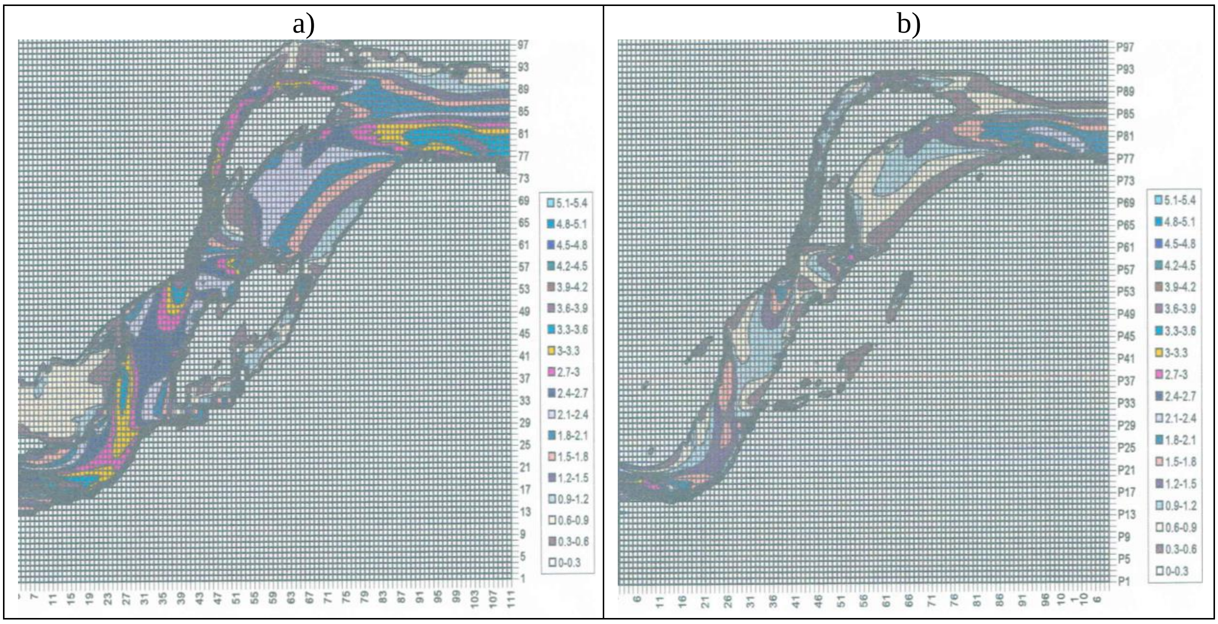

Figure 5. a) distribution of the depths of the water flow in the area of the damless intake in the Karshi Main Chanel at the low water calculated after 1 year of observation b) distribution of the depths of the water flow in the region of the damless intake in the Karshi Main Chanel in the flood calculated after 1 year

IOP Conf. Series: Materials Science and Engineering **869** (2020) 072014 doi:10.1088/1757-899X/869/7/072014

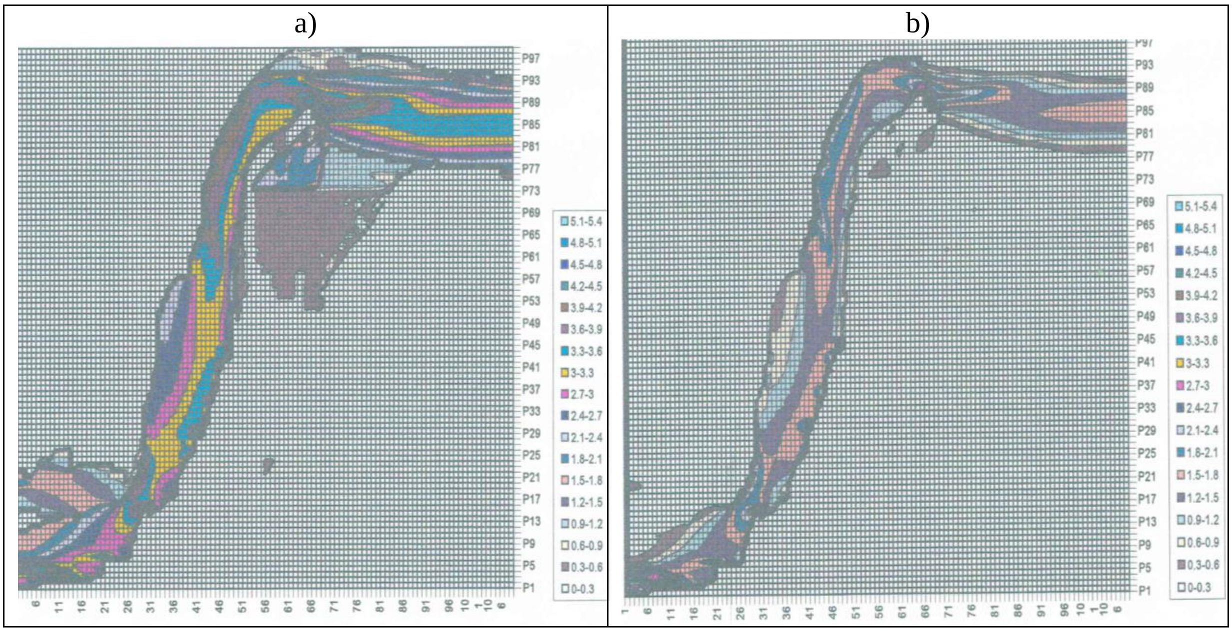

Figure 6. a) The distribution of the depths of the water flow in the area of the damless intake into the Karshi Main Channel at the low water calculated after 10 years of observation; b) distribution of water flow depths in the area of damless water intake into the Karshi Highway Canal into the flood calculated after 10 years of observation

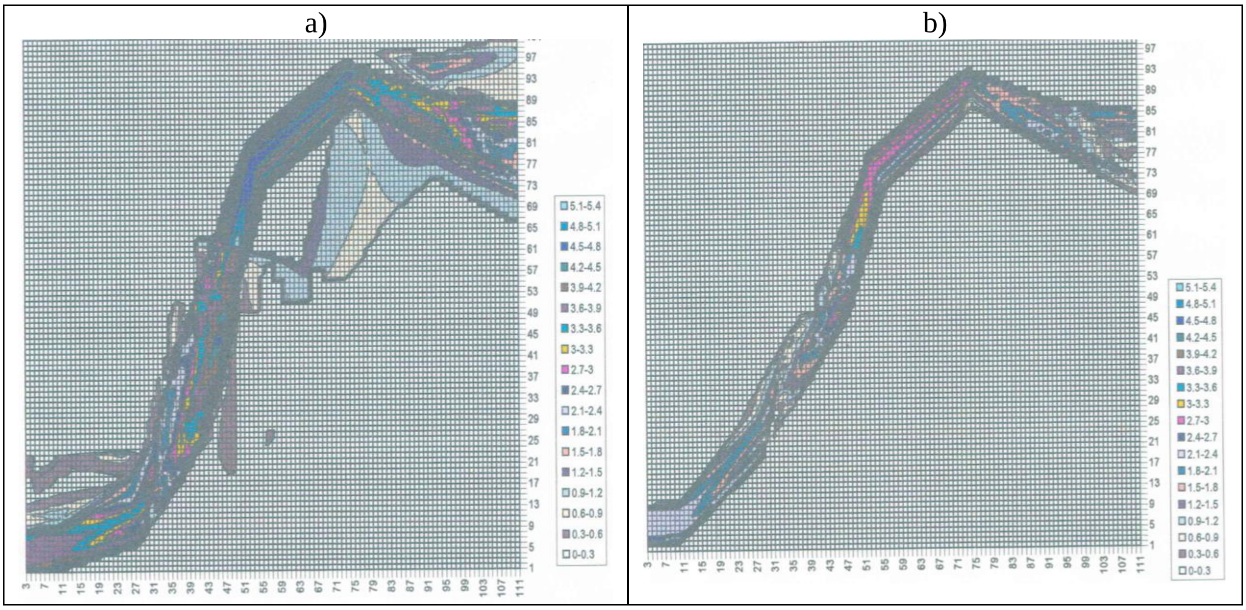

Figure 7. a) distribution of water flow depths in the area of damless water intake in the Karshi Main Canal at low water measured after 10 years of observation b) distribution of water flow depths in the area of damless water intake in the Karshi Main Channel in the flood measured after 10 years of observation

In figure 5 b, 7 b, successive deformations of the channel are presented that arise due to natural causes, as well as due to the channel of adjustment and treatment works (discharge of soil from the supply channel into the river). As can be seen from these figures, the channel from multi-sleeve is gradually turning into one-sleeve.

After 1 year of observation at low water, as a result of discharge of sediment from the sewage treatment works into the river, the river ducts die off (figure 5 a). These ducts are observed only in the flood (figure 6 b). Note that below the KMCh the left bank eroded by about 150 m. After a 10-year course, the low-water course turned into a one-arm channel (figure 6 a). As a result of the erosion of the left-bank islands, it turned into the river itself. No pronounced sleeves are also observed in the flood (figure 7 b). There is a significant erosion of the left bank, up to 700 m. After that, this bank was strengthened.

In the future, the channel process did not undergo significant changes. Towards the end of the 10th year of observation, the channel broadened somewhat, and the flood depth decreased during the flood (figure 6 a,b).

Figures 7 a, b show the depth distribution at the bottom marks measured at the end of 1995 for the same hydrological conditions. Comparison of fig. 6 a, b and 7 a, b shows that the developed model gives good results. For low-water conditions, coincidence is better than for flood conditions. During the flood, under natural conditions, somewhat greater depths are observed on the floodplain than in the calculations. It should be noted that both in calculations and in kind:

- there was a significant erosion of the left coast below the KMCh;
- the channel of the Amudarya river in the considered section from multi-sleeve turned into practically one-sleeve.

4. Conclusions

According to the results of the research, the following conclusions and recommendations can be made:

- The developed mathematical model of unsteady uneven movement of the river flow is suitable for describing the processes of deformation, including the insertion of dredging slots. The model contains the minimum number of equations and empirical constants;
- Comparison with the data of experimental studies showed good convergence of the results, however, the optimal values of empirical coefficients for different experiments were slightly different;
- A comparison of the results of field and numerical studies shows that the developed model gives good results. For low-water conditions, coincidence is better than for flood conditions. During the flood, under natural conditions, somewhat greater depths are observed on the floodplain than in the calculations. It should be noted that both in calculations and in kind;
- According to the results of calculations and field studies, it was found that there was a significant erosion of the left bank below the KMCh;
- Установлено, что русло реки Амударьи на рассматриваемом участке из многорукавного превратилось практически в однорукавное.

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