Natural Vibrations Of Structurally Inhomogeneous Multi-Connected Shell Structures With Viscoelastic Elements

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Abstract. The methods for solving the problems of natural vibrations of structurally inhomogeneous, multi-connected, axisymmetric shell structures based on the hereditary theory of viscoelasticity are given in the paper. Using the basic relations of the hereditary theory of viscoelasticity and asymptotic methods, the problem of natural vibrations of structurally inhomogeneous, multi-connected, axisymmetric shell structures is reduced to the effectively solvable mathematical problem of complex eigenvalues, in which approximate engineering methods are proposed.

1. Introduction

In the study of the dynamics of multi-connected problems of structurally inhomogeneous shell structures, a special place is occupied by eigenfrequencies and the modes of vibration. An extensive class of thin-walled spatial structures can be attributed to shell structures; their design scheme can be represented in the form of some compositions of thin shells and plates.

Dissipative processes in the materials are taken into account by a model of the Boltzmann-Volterra type. The algorithms for the dynamic calculation of multi- connected structurally inhomogeneous shell structures were developed in the framework of V.V. Novozhilov linear theory of shells and A.V. Aleksandrov physical relations, based on A.V. Aleksandrov method of displacements and S.K. Godunov method of orthogonal sweep. On the whole, using the basic relations of the hereditary theory of viscoelasticity and asymptotic methods the problem is reduced to an effectively solvable mathematical problem of complex eigenvalues, in the framework of which approximate engineering methods are proposed.

In [1-3], the basic equations were obtained that characterize the dynamics of structurally inhomogeneous, multi-connected shell structures considering viscoelastic characteristics of structure elements.

Eigenfrequencies were analyzed in [6], and, based on a mathematical model, modified methods for determining the eigenfrequencies and damping coefficients were proposed.

In [7-14], the dynamics of complex shell structure elements were studied. For example, the structure element strength was investigated in [7], the cylindrical shell vibrations - in [8,9], and the natural vibrations of thin conical shells - in [10]. The studies in [11] were devoted to the dynamics of axisymmetric shell structures. In [12-14], the methods for assessing the stress-strain state and steady-state vibrations of the elements of hydro-technical structures were proposed.

However, it should be noted that in literature there is an insignificant amount of information about studies of the dynamics of multiply connected visco-elastic deformable systems combining dissimilar

materials, bonds, and structural elements as a whole. The present paper is aimed to fill the gap in such studies, which determine the relevance of the work.

So, the improvement of the methods and algorithm for solving problems to determine dynamic characteristics of shell structures that take into account the dissipative properties of the material is an urgent task.

2. Methods and algorithms to determine dynamic characteristics of shell structures.

To obtain the basic equations of the dynamics of structurally inhomogeneous multi-connected shell structures, the Lagrange variation equation is used:

$$\sum_{p=1}^{N_S} \delta \Im_p + \sum_{i=1}^{N_r} \delta \Im_i + \sum_{e=1}^{N_b} \delta \Im_e - \sum_{p=1}^{N_S} \delta A_p - \sum_{i=1}^{N_r} \delta A_i = 0 \tag{1}$$

where $\delta \Im_p$ is the variation of potential strain energy of p-th shell element; $\delta \Im_i$ is the variation of potential strain energy of the i-th ring element; ; $\delta \Im_e$ is the variation of potential strain energy of the e-th viscoelastic connection; δA_p is the elementary work of external loads applied to the p-th shell element; δA_i is the elementary work of external loads applied to the i-th ring element.

After some simple mathematical transformations, the problem is reduced to solving the equations of the dynamics of shell structures with complex coefficients [1, 2]:

$$L_p + q_{p0} + \omega_R^2 [\bar{\rho}_p] U_p = 0 \ (p = 1, 2, ..., N_s), \tag{2}$$

$$L_r^i + \|\theta_i\| f_{i0} + \omega_R^2 [G_\omega] \Delta_i + \sum_j \sum_S (\xi_{ci}^{ijs} [\bar{\eta}_i^{ijs}] Q_s^{ijs} + \sum_j \sum_S (\xi_{ci}^{ijs} [\bar{\eta}_{ci}^{ijs}] N_s^{ijs} = 0, \tag{3}$$

$$(i = 1, 2, ..., N_r)$$

In the problem of structure natural vibrations, the solution to equations (2), (3) is sought in the form

$$U_p = U_p e^{-i\omega\tau}, \Delta_i = \Delta_i e^{-i\omega\tau} \tag{4}$$

Here ω - is the complex value of vibration frequency, the real part of which $\omega_I \, \omega_R$ represents the frequency of natural vibrations, ω_I is the damping coefficient. So, equations of structure natural vibrations will have the form:

$$L_p + q_{p0} + \omega^2 [\bar{\rho}_p] U_p = 0 \ (p = 1, 2, ..., N_S), \tag{5}$$

$$L_r^i + \|\theta\| f_{i0} + \tilde{\omega}^2 [G_\omega] \Delta_i + \sum_j \sum_S (\xi_{ci}^{ijs} [\bar{\eta}_i^{ijs}] Q_i^{ijs}) + \sum_j \sum_S (\xi_{ci}^{ijs} [\bar{\eta}_{ci}^{ijs}] N_{ci}^{ijs}) = 0 \tag{6}$$

$$(i = 1, 2..., N_r)$$

The values of $\omega*$ for which there is a nontrivial solution to the system with complex coefficients (6) are the complex values of natural frequencies of the structurally inhomogeneous shell structures under consideration. Each of these equations describes the behavior of an individual shell element of a thin-walled shell structure. In our case, the difference with the known equations is fundamental and consists in the fact that the solution of these equations is complex due to the complexity of the physical relationships describing the structural inhomogeneity and rheological properties of the individual layers and hereditary connections of the shell element. Each of these relations is an equation of vibrations in the complex form of an individual stringer or frame of multi-connected structurally

inhomogeneous shell structure, taking into account reactions from the adjacent shell elements and viscoelastic connections. The vector Q_i^{ijs} as can be seen from the expressions for Q_p and W_p is the vector of generalized reactions from the side of the *ijs*-th shell element adjacent to the *i*-th ring or stringer element in local coordinate system of the shell element under consideration. The vector N_{ci}^{ijs} is, in turn, the vector of generalized reactions on the part of the *ijs*-th and viscoelastic connections adjacent to the *i*-th ring or stringer element in local coordinate system.

The matrices $[\overline{\eta}_i^{ijs}],[\overline{\eta}_{ci}^{ijs}]$ and coefficients $\xi_i^{ijs},\xi_{ci}^{ijs}$ are the matrices and coefficients of transformation from the local coordinate system of the shell element or viscoelastic connection to the coordinate system of the shell structure under consideration.

In the case when at the connection node of two or more shell elements or viscoelastic connections there is no rod or ring element, the first and third terms of the corresponding equation () are identically equal to zero and this solution is interpreted as follows: the sum of the reactions from the shell elements and viscoelastic connections converging in this node, plus the sum of the external loads applied to the node under consideration, is zero.

Let us consider the solutions to a number of practical problems of calculating dynamic characteristics of a wide class of axisymmetric structurally inhomogeneous structures. To determine the frequencies and modes of natural vibrations, the problem-oriented procedure described above is used. It is known that the mathematical convergence of each of these methods has been widely and completely studied. However, the question of convergence of the proposed algorithm for the numerical implementation of the problems of structurally inhomogeneous viscoelastic shell structures, as a whole, has been studied insufficiently and represents an independent problem.

3. The study of obtained solutions accuracy on a test example

An assessment of practical convergence of the proposed algorithms is given based on a comparison of the results of solving test problems found in literature and the results of solutions obtained in the framework of the developed methods, with the results from complex arithmetic. Shell structures with exact or approximate solutions are chosen as the test cases.

To calculate the stiffness matrices of shell elements made of a viscoelastic material, the orthogonal sweep method is used. The mathematical convergence and stability of this method as applied to the theory of elastic shells has been comprehensively studied by Ya.M. Grigorienko, V.I. Myachenkov, A.N. Frolov, V.P. Maltsev, T. Mavlanov and others.

Here, the convergence and stability of the orthogonal sweep method is studied with complex arithmetic. Let us consider a part of the results of this study for structurally inhomogeneous viscoelastic shell elements whose rheological properties are described by the relaxation kernel $R(t) = Ae^{-\beta t}t^{\alpha-1}$. It should be noted that, unlike [1], we took into account the effects of viscoelastic properties of structure elements.

Figure 1. Four layer rectangular plate.

As a test example of using the developed algorithm, calculate the matrix of complex stiffness values for a four-layer rectangular plate (Figure 1.) with a different number of orthogonalization points m. The plate has the following geometric and mechanical parameters:

$$a = b = 100sm; j = 1,2,3,4; h_j = 0.25sm; E_j = 2*10^6 N / sm^2;$$

$$\gamma_j = 0.3; \rho_j = 7.8*10^{-6} Ns / sm^4; A_j = 0.03; \beta_j = 0.003; \alpha_j = 0.1; \omega_R = 300s^{-1}$$

The calculation results for m = 2, 4, 8, 16, 32, 64 and one step of integration between points (m- is the number of orthogonalization points in the interval length l) are given in Table 1. The top row of each element of the matrix is the real component, and the bottom row is the imaginary component of this element.

One of the results of the algorithm work is the relative accuracy in determining the roots achieved in the solution process. Some data from these protocols are given in Tables 1-2, the solution protocol contains the following data: the number of waves in the longitudinal direction (N); real (QR) and imaginary (QI) components of the complex frequency value ω^* the real (DR) and imaginary (DI) components of the mantissa of determinant $|P(\omega)|$; the order of the determinant $(IS | P(\omega)|)$.

As seen from the tables the practical convergence in calculating the matrix [K] is reached even at the number of orthogonalization points m = 4, and at m = 32 and m = 64 the elements of the matrix [K] coincide in all significant figures. The symmetry of the matrix is reached at m = 16.

Table 1. Values of eigenfrequencies of the four-layer plate

№	QR	QI	DR	DI	IS
I	5.4593E+00	0.0000E+00	9.7342E+00	-2.0188E+00	80
I	5.4675E+00	0.0000E+00	9.9396E-01	-2.0294E-0I	81
I	5.4757E+00	0.0000E+00	I.0I5IE+00	-2.0387E-0I	81
I	5.I447E+00	-3.0728E-0I	3.I727E+00	-3.9077B+00	80
I	4.9792E+00	-5.I567E-0I	I.0939E+00	-3.5966E+00	80
I	4.74I6E+00	-8.43I3E-0I	-7.6850E-0I	-2.8779E+00	80
I	4.480IE+00	-I.3494E-0I	8.5936E-0I	-6.3399E-0I	80
I	4.2435E+00	-3.9976E-02	3.5008E+00	-6.7106E-0I	79
I	4.0974E+00	-5.0428E-02	2.222IE+00	4.8778E+00	78
I	4.0977E+00	-7.5445E-02	-3.92I9E-0I	-I.2I54E+00	77
I	4.0976E+00	-7.4869E-02	-6.I493E-0I	-5.9784E-0I	74

Table 2. Values of eigenfrequencies of the four-layer plate

№	QR	QI	DR	DI	IS
2	2.8474E+0I	0.0000E+00	-7.0908E+00	4.3724E-0I	93
2	2.85I7E+0I	0.0000E+00	-7.I773E+00	4.5420E-0I	93
2	2.8560E+0I	0.0000E+00	-7.2633E+00	4.7I09E-0I	93

2	2.5678E+0I	-2.7600E-0I	-7.3030E+00	-6.4527E-0I	91
2	2.5654E+0I	-2.8I60E-0I	-7.2172E+00	-I.6035E+00	89
2	2.5654E+0I	-2.8I69E-0I	I.I3I8E-0I	I.5549E+00	86

The final values of the vibration frequency for the second example (Table 2) are $\omega_R^* = 25.6535; \omega_I^* = 0.281688;$; relative accuracy - $\varepsilon_r = 4.11986 \cdot 10^{-8}; \varepsilon_I = 5.24144 \cdot 10^{-6}$.

4. Study of natural vibrations of shell structures.

As an example, consider a structurally inhomogeneous axisymmetric structure - a two-cavity high-pressure reservoir, consisting of 6 shell and 6 nodal elements (Fig. 2). The nodes under numbers 3,5,6 are circular frames, the cross sections of which are rectangular in shape with a dimensions of 0.04 m x 0.06 m, other geometric dimensions are shown in Fig. 1.

Node No. 6 of the structure is fixed, shell elements No. 1,4,6 and the frames are elastic $(E = 2 \cdot 10^{11} N / m^2; v = 0.3; \rho = 7.8 \cdot 10^2 kg / sm^3)$. Shell elements forming the internal cavity (No. 2, 3, 5) are viscoelastic ones

$(\rho = 7.8 \cdot 10^2 kg / sm^3; v = 0.3)$ Their instantaneous modulus of elasticity - 1 was defined as a parameter of structural inhomogeneity and varied from $E = 2 \cdot 10^8 N / m^2$ to $E = 2 \cdot 10^{11} N / m^2$.
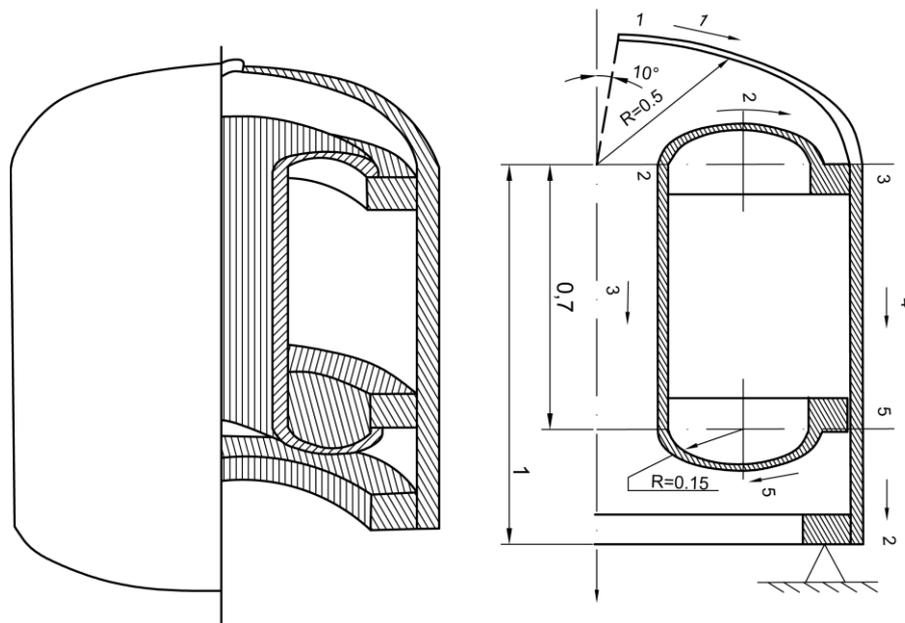

Figure 2. Design scheme of the high pressure tank

The parameters of the relaxation kernel of the shell elements material have values $A = 0.01; \alpha = 0.1; \beta = 0.05$. The thickness of shell elements is constant and equal to 0.01 m. In accordance with the design requirements, the behavior of the determinant damping coefficient of this structure was studied depending on the instantaneous modulus of elasticity of visco-elastic elements.

Figure 2 shows the calculated dependences of the damping coefficients $\omega_{I1}, \omega_{I2}, \omega_{I3}$ of the 3 lowest eigenmodes, determined by the operating conditions of the structure on the structural inhomogeneity parameter - E. As in the previous case, a synergism of dissipative properties was revealed for a multi-connected shell structure.
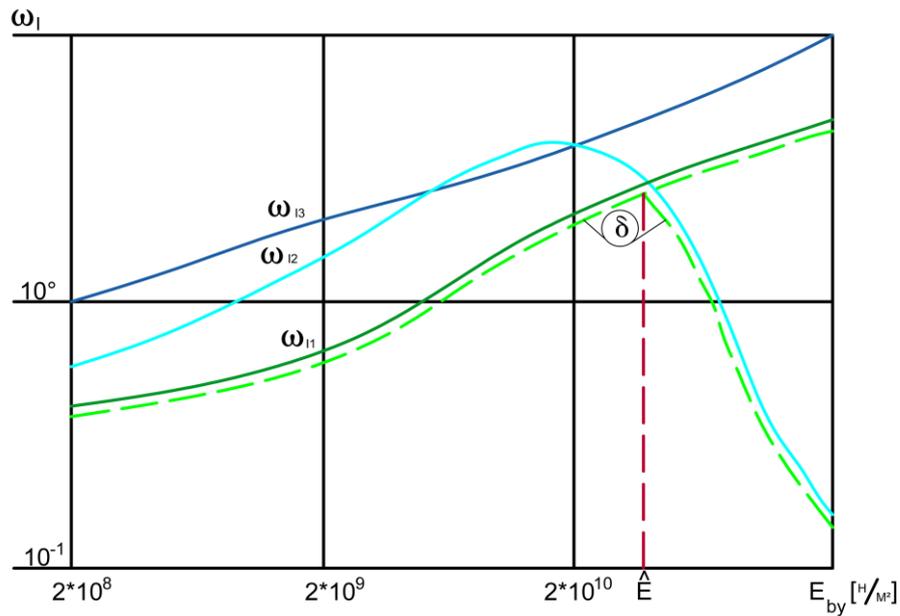
Figure 3. Change in the imaginary part of natural frequency depending on elastic modulus

Figure 3 shows the calculated dependences of the natural frequencies, two of which ω_{R1} and ω_{R2} in the vicinity of the intersection point of the corresponding damping coefficients become somewhat closer. The above calculation and analysis of the dynamics of this structure allows already at the stage of preliminary design to provide the necessary characteristics of dissipative properties required by the technical conditions and the normals, and the ability to damp the oscillations of the frequencies ω_{R1} and ω_{R2} specified by the operating conditions. The greatest energy dissipation is provided by the structure with an instantaneous elasticity modulus of viscoelastic elements $\hat{E}$ (Fig. 3).
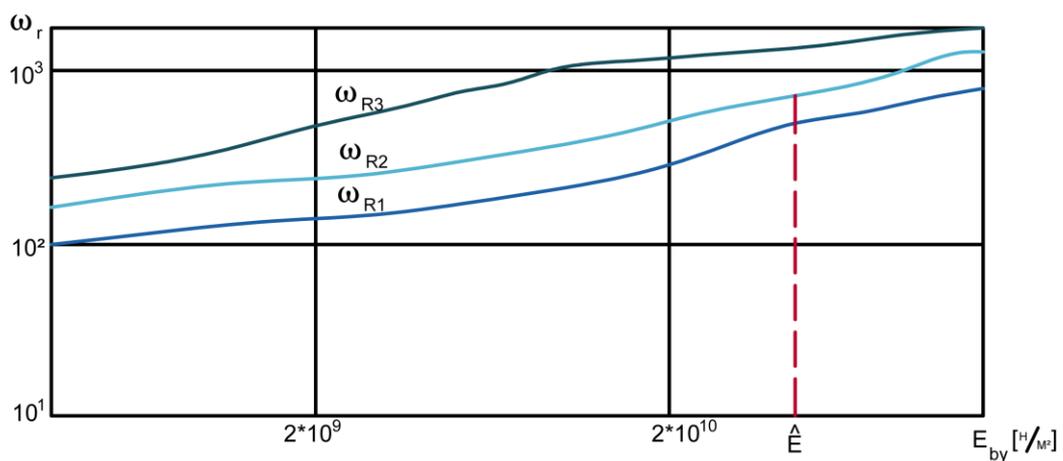

Figure 4. Change in the real part of natural frequency depending on elastic modulus.

To determine complex values of natural vibration frequencies of structurally inhomogeneous shell structures, i.e. to find the roots of a nonlinear functional equation in complex variables

$$D(\omega) = |P(\omega)| = 0 \qquad (7)$$

the Muller's method was used. The convergence of the iterative process proposed by Muller requires further research.

The algorithm for determining the frequencies and vibration modes of structurally inhomogeneous viscoelastic shell structures includes a protocol to find the roots of equation (6), i.e. the designer can always use this protocol to control the iterative process. Besides, one of the results of the algorithm work is the relative accuracy in determining the roots achieved in the solution process.

5. Conclusion

1. Based on the mathematical theory of viscoelasticity, variation principles of the dynamics, the problem of studying the dissipative properties of structurally inhomogeneous multi-connected shell systems with account for viscoelastic properties is reduced to an effective solvable mathematical problem for complex eigenvalues.
2. The hereditary Boltzmann-Volterra theory is proposed to describe the viscoelastic properties of the material of structure elements.
3. The engineering methods were developed to determine the eigenfrequencies of multiconnected, structurally inhomogeneous shell structures.
4. On the example of the reservoir, natural vibrations were investigated.
5. Numerical modeling was carried out, as a result of which the convergence of the developed methods and algorithms for solving dynamic problems of structurally inhomogeneous shell structures was established.

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