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Free oscillations of three-layered plates

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Abstract. The paper is devoted to improving the theory of bending and vibrations of three-layer plates with transverse compressible filler and thin outer bearing layers. For the outer layers, the Kirchhoff-Love hypothesis is accepted and the motion of their points is described by the equations of the theory of thin plates relative to forces and moments. Unlike bearing layers, a filler is considered as a three-dimensional body that does not obey any simplifying hypotheses. The equations of the bimoment theory of thick plates with respect to forces, moments and bimoments, created in the framework of the three-dimensional theory of elasticity, taking into account the nonlinearity of the distribution law of displacements and stresses over the thickness, are taken as the equations of motion of the filler. Expressions of forces, moments, and bimoments in the layers, as well as boundary conditions at the edges of a three-layer plate with respect to force factors, are given. In the conjugate zones of the layers, the complete contact conditions for the continuity of displacements and stresses are set. An example is considered and numerical results are obtained.

1. Introduction

The development of methods for calculating structural elements, buildings, and structures within the framework of the theory of plates and shells remains one of the urgent and complex problems of the mechanics of the deformable rigid body.

When constructing the theory of plates and shells, it is of great importance to take into account the rheological nonlinear properties of the structure material.

When modeling the dynamic behavior of the elements of buildings and structures, as well as three-layer structural elements, the theory of thick plates is successfully developed and used, taking into account forces, moments, and bimoments.

The calculation of thin plates consider in rheological and nonlinear properties of the structure material is of great important cheat the present stage of development of the theory of thin plates and shells.

In the field of the theory of thick plates, the methods for calculating thick plates taking into account forces, moments, and bimoments are successfully developed and used in modeling seismic vibrations of multi-story buildings and structures, in calculations of three-layer structural elements with a thick filler made of relatively soft material.

The study in [1] is devoted to the development of a method for calculating structures, which allows determining the lower boundaries of the frequencies of free vibrations based on the finite element method in displacements.

In [2], the problems of plate bending were solved using the finite element method based on approximations of the moment fields. Calculations of square fixed and hinged-supported plates under a uniformly distributed load were performed.

In [3], a method for calculating bending plates by the finite element method based on the Reissner theory was proposed. The method is based on fundamental principles of the minimum of additional energy and virtual displacements. Arbitrary quadrangular finite elements were used to sample the

The paper [4] is devoted to analytical and numerical studies of the operation of end elements of buildings in the form of axisymmetric thick plates for nuclear reactors under high pressure.

Numerical results were obtained [5], for a Reissner rectangular bent plate fixed along the contour under uniform loading by an iterative method of superposition of four types of trigonometric series.

In [6, 7], in a geometrically nonlinear statement, the problem of parametric vibrations of a viscoelastic orthotropic plate of variable thickness was considered using the classical Kirchhoff-Love hypothesis. A technique was proposed for solving a nonlinear problem using the Bubnov-Galerkin method in a polynomial approximation of displacements.

In [8, 9], the problem of oscillations of a viscoelastic cylindrical panel with concentrated masses was considered, based on the Kirchhoff-Love hypothesis in a geometrically nonlinear statement. The problem was solved using the Bubnov-Galerkin method.

The studies on the topic include the works [10, 11], devoted to dynamic calculations of structural elements under parametric vibrations under the action of a uniformly distributed dynamic load

The studies in [12] were devoted to solving the problem of dynamic stability of viscoelastic plates of variable stiffness. The equations of motion of a viscoelastic plate were described by particular integro-differential equations. Based on the Bubnov-Galerkin method, a numerical algorithm for solving problems has been developed

In [13], a flutter of a viscoelastic three-layer plate flowing around a gas flow was studied. The main direction of work is to consider the properties of a viscoelastic material at supersonic speeds. The equations of oscillations with respect to the deflection were described by integro-differential equations in partial derivatives. The critical flutter velocities of a three-layer plate were determined based on the Bubnov-Galerkin method using the quadrature formula.

A well-known Reissner–Mindlin theory on layered anisotropic plates was generalized in [14]. The most complete case-history review of publications was presented, which set out the basic provisions of existing theories and their elaboration.

The studies conducted in [15, 16] investigated the dynamic stability of circular three-layer plates of composite materials with viscoelastic properties. It was assumed that the three-layer plate consisted of a thin multilayer fiber-reinforced composite with a filler of foam. The plate is loaded in its plane with a rapidly increasing compressive dynamic force and loses its stability.

In [17, 18] using the theory of plates, the spatial model of the box structure was improved taking into account displacements and stresses in the butt joints of its beam and plate elements. The equations of motion of the elements of the box structure with the boundary conditions in the bases and the contact conditions between the elements of the box are constructed.

In [19, 20], forced vibrations of multi-story buildings were considered using the continuum plate model developed as part of the bimoment theory of thick plates [21, 22].

The current article is devoted to the development of the theory of three-layer plates with thin bearing layers and thick fillers of relatively soft material. For thin bearing layers, we apply the Kirchhoff-Love theory. Consider the thick filler as a three-dimensional body and apply the bimoment theory of thick plates.

Based on the general case, the layer materials are considered elastic and orthotropic ones. For orthotropic bearing layers of elastic materials, introduce the following notation:

$E_1^{(+)},\ E_2^{(+)},\ E_3^{(+)}$ are the elasticity moduli; $G_{12}^{(+)},\ G_{13}^{(+)}, G_{23}^{(+)}$ are the shear moduli; $\nu_{12}^{(+)},\ \nu_{13}^{(+)},\ \nu_{23}^{(+)}$ are the Poisson's ratios of the material of the lower bearing layer;
$E_1^{(-)},\ E_2^{(-)},\ E_3^{(-)}$ are the elasticity moduli; $G_{12}^{(-)},\ G_{13}^{(-)}, G_{23}^{(-)}$ are the shear moduli; $\nu_{12}^{(-)},\ \nu_{13}^{(-)},\ \nu_{23}^{(-)}$ are the Poisson's ratios of the material of the upper bearing layer;
E_1, E_2, E_3 are the elasticity moduli; G_{12}, G_{13}, G_{23} are the shear moduli; $\nu_{12}, \nu_{13}, \nu_{23}$ are the Poisson's ratios of the filler layer material;

$H = 2h$ is the filler thickness; $2h_u,\ 2h_l$ are the thickness of the upper and lower bearing layers; a and b are the dimensions of a three-layer plate in the plan.

2. Methods

The layer displacement field is described with respect to a rectangular Cartesian coordinate system (x_1, x_2, z) with an origin on the middle plane of the filler. The (oz) axis is directed normal down. Due to layers strain on the surfaces $z = -h$ and $z = +h$, distributed contact stresses $q_1^{(+)},\ q_1^{(+)},\ q_3^{(+)}$ and $q_1^{(-)},\ q_2^{(-)},\ q_3^{(-)}$ arise.

The displacements of the upper bearing layer are set according to the Kirchhoff – Love hypothesis in the form ($-h - 2h_u \le z =\le -h$):

$$U_{kz}^{(-)} = U_k^{(-)} - (z + h + h_u)\frac{\partial W^{(-)}}{\partial x_k}, \quad (k = 1{,}2). \qquad (1)$$

Similarly, the law of displacements distribution of the lower bearing layer can be represented as ($h \le z \le h + 2h_l$):

$$U_{kz}^{(+)} = U_k^{(+)} - (z - h - h_l)\frac{\partial W^{(+)}}{\partial x_k}, \quad (k = 1{,}2) \qquad (2)$$

Here $W^{(+)},\ W^{(-)}$ are the deflections of bearing layers, $U_1^{(+)},\ U_2^{(+)}$ and $U_1^{(-)},\ U_2^{(-)}$ are the displacements of the middle planes of the layers.

Introduce the notation for contact zones displacements between layers.

The displacement of the contact zones between the bearing layers $z = h$ and $z = -h$ and the filler is denoted by $u_1^{(+)},\ u_2^{(+)},\ u_3^{(+)}$ and $u_1^{(-)},\ u_2^{(-)},\ u_3^{(-)}$. The laws of distribution of the bearing layers (1) and (2) displacements must satisfy the continuity conditions on the joining points $z = h$ and $z = -h$:

$$u_1^{(+)} = U_{1z}^{(+)}, \quad u_2^{(+)} = U_{2z}^{(+)}, \ u_3^{(+)} = W^{(+)} \qquad (3)$$

$$u_1^{(-)} = U_{1z}^{(-)}, \quad u_2^{(-)} = U_{2z}^{(-)}, \ u_3^{(-)} = W^{(-)} \qquad (4)$$

Using the distribution law of displacements of bearing layers (1), (2) and the continuity conditions for displacements (3), (4) in the zone of layers contact $z = +h$ and $z = -h$, we obtain the expressions:

$$u_k^{(+)} = U_k^{(+)} + h_l\frac{\partial W^{(+)}}{\partial x_k}, \quad u_k^{(-)} = U_k^{(-)} - h_u\frac{\partial W^{(-)}}{\partial x_k}, \quad (k = 1.2) \qquad (5)$$

Due to layers strain on the surfaces $z = -h$ and $z = +h$, distributed contact stresses $q_1^{(+)},\ q_1^{(+)},\ q_3^{(+)}$ and $q_1^{(-)},\ q_2^{(-)},\ q_3^{(-)}$ arise.

Using the Cauchy relations, Hooke's law, and expressions (1) and (2), we obtain the strain and stress expressions of the bearing layers. Using the expressions of Hooke's law and the expressions for the bearing layers, we obtain the expressions of forces and the moments of the layers.

The longitudinal, tangential, and transverse forces of the lower bearing layer are determined through unknown functions of the displacements of the points of the middle surface of the lower bearing layer $U_1^{(+)},\ U_2^{(+)}$:

$$N_{11}^{(+)} = \int_{h}^{h+h_l} \sigma_{11} dz = B_{11}^{(+)} \frac{\partial U_1^{(+)}}{\partial x_1} + B_{12}^{(+)} \frac{\partial U_2^{(+)}}{\partial x_2},$$
$$N_{12}^{(+)} = \int_{h}^{h+h_l} \sigma_{12} dz = S_{12}^{(+)} \left(\frac{\partial U_1^{(+)}}{\partial x_2} + \frac{\partial U_2^{(+)}}{\partial x_1} \right), \quad N_{22}^{(+)} = \int_{h}^{h+h_l} \sigma_{22} dz = B_{12}^{(+)} \frac{\partial U_1^{(+)}}{\partial x_1} + B_{22}^{(+)} \frac{\partial U_2^{(+)}}{\partial x_2}, \tag{6}$$
$$Q_{13}^{(+)} = \frac{\partial M_{11}^{(+)}}{\partial x_1} + \frac{\partial M_{12}^{(+)}}{\partial x_2} - h_l q_1^{(+)}, \quad Q_{23}^{(+)} = \frac{\partial M_{21}^{(+)}}{\partial x_1} + \frac{\partial M_{22}^{(+)}}{\partial x_2} - h_l q_2^{(+)},$$

where $B_{11}^{(+)}, B_{22}^{(+)}, B_{12}^{(+)}, S_{12}^{(+)}$ are the cylindrical rigidity under tension and compression of the lower bearing layer of orthotropic material.

Bending and torsional moments of the lower bearing layer are determined through unknown functions of deflection of the points of the lower bearing layer $W^{(+)}$ in the form:

$$M_{11}^{(+)} = \int_{h}^{h+h_l} \sigma_{11} z dz = -D_{11}^{(+)} \frac{\partial^2 W^{(+)}}{\partial x_1^2} - D_{12}^{(+)} \frac{\partial^2 W^{(+)}}{\partial x_2^2}$$
$$M_{22}^{(+)} = \int_{h}^{h+h_l} \sigma_{22} z dz = -D_{12}^{(+)} \frac{\partial^2 W^{(+)}}{\partial x_1^2} - D_{22}^{(+)} \frac{\partial^2 W^{(+)}}{\partial x_2^2}, \quad M_{12}^{(+)} = \int_{h}^{h+h_l} \sigma_{12} z dz = -C_{12}^{(+)} \frac{\partial^2 W^{(+)}}{\partial x_1 \partial x_2} \tag{7}$$

where $D_{11}^{(+)}, D_{11}^{(+)}, D_{12}^{(+)}, C_{12}^{(+)}$ are the cylindrical rigidity of the lower supporting layer of orthotropic material under bending.

The longitudinal, tangential and transverse forces of the upper bearing layer $N_{kj}^{(-)}$ are determined through unknown functions of displacements of the middle surface points in the lower bearing layer $U_1^{(-)}, U_2^{(-)}$ in the form:

$$N_{11}^{(-)} = \int_{-h-h_u}^{-h} \sigma_{11} dz = B_{11}^{(-)} \frac{\partial U_1^{(-)}}{\partial x_1} + B_{12}^{(-)} \frac{\partial U_2^{(-)}}{\partial x_2},$$
$$N_{12}^{(-)} = \int_{-h-h_u}^{-h} \sigma_{12} dz = S_{12}^{(-)} \left(\frac{\partial U_1^{(-)}}{\partial x_2} + \frac{\partial U_2^{(-)}}{\partial x_1} \right),$$
$$N_{22}^{(-)} = \int_{-h-h_u}^{-h} \sigma_{22} dz = B_{12}^{(-)} \frac{\partial U_1^{(-)}}{\partial x_1} + B_{22}^{(-)} \frac{\partial U_2^{(-)}}{\partial x_2}, \tag{8}$$
$$Q_{13}^{(-)} = \frac{\partial M_{11}^{(-)}}{\partial x_1} + \frac{\partial M_{12}^{(-)}}{\partial x_2} - h_u q_1^{(-)},$$
$$Q_{23}^{(-)} = \frac{\partial M_{21}^{(-)}}{\partial x_1} + \frac{\partial M_{22}^{(-)}}{\partial x_2} - h_u q_2^{(-)}.$$

Here $B_{11}^{(-)}, B_{11}^{(-)}, B_{12}^{(-)}, S_{12}^{(-)}$ are the cylindrical rigidity of the upper bearing layer of orthotropic material under tension and compression.

Bending and torsion of the upper bearing layer have the expressions

$$M_{11}^{(-)} = \int_{-h-h_u}^{-h} \sigma_{11} z dz = -D_{11}^{(-)} \frac{\partial^2 W^{(-)}}{\partial x_1^2} - D_{12}^{(-)} \frac{\partial^2 W^{(-)}}{\partial x_2^2},$$
$$M_{22}^{(-)} = \int_{-h-h_u}^{-h} \sigma_{22} z dz = -D_{12}^{(-)} \frac{\partial^2 W^{(-)}}{\partial x_1^2} - D_{22}^{(-)} \frac{\partial^2 W^{(-)}}{\partial x_2^2}, \tag{9}$$
$$M_{12}^{(-)} = \int_{-h-h_u}^{-h} \sigma_{12} z dz = -C_{12}^{(-)} \frac{\partial^2 W^{(-)}}{\partial x_1 \partial x_2},$$

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where $D_{11}^{(-)}$, $D_{11}^{(-)}$, $D_{12}^{(-)}$, $C_{12}^{(-)}$ are the cylindrical rigidity of the upper bearing layer of orthotropic material under bending, $W^{(-)}$ are the unknown functions of the displacements and deflections of the points of the middle surface of the upper bearing layer.

Introduce the following notation for the displacements and internal force factors of the bearing layers of the plate.

The half-sums of internal longitudinal, tangential and transverse forces and moments are denoted in the form

$$\tilde{P}_{ij} = N_{ij}^{(+)} - N_{ij}^{(-)}, \quad \tilde{M}_{ij} = \tilde{M}_{ij}^{(+)} + \tilde{M}_{ij}^{(-)}, \quad \tilde{Q}_{i3} = Q_{i3}^{(+)} + Q_{i3}^{(-)}, \ (i, j = 1.2). \qquad (10)$$

Introduce the following generalized contact stresses using contact stresses defined as

$$\tilde{q}_k = \frac{q_k^{(+)} + q_k^{(-)}}{2}, \ (k = 1.2), \quad \tilde{q}_3 = \frac{q_3^{(+)} - q_3^{(-)}}{2} \qquad (11)$$

The equations of motion of the bearing layers with respect to shear moments have the form

$$\frac{\partial \tilde{P}_{k1}}{\partial x_1} + \frac{\partial \tilde{P}_{k2}}{\partial x_2} + 2\tilde{q}_k - 4(\rho_l h_l + \rho_u h_u)\ddot{\tilde{U}}_k = 0, \ (k = 1.2) \qquad (12)$$

Here ρ_l, ρ_u are the densities of the bearing layers.

The equation of motion of the bending vibrations of the bearing layers is obtained in the form:

$$2\frac{\partial^2 \tilde{M}_{11}}{\partial x_1^2} + 4\frac{\partial^2 \tilde{M}_{12}}{\partial x_1 \partial x_2} + 2\frac{\partial^2 \tilde{M}_{22}}{\partial x_2^2} - 4(\rho_l h_l + \rho_u h_u)\ddot{\tilde{W}} + (h_l + h_u)\frac{\partial \tilde{q}_1}{\partial x_1} + (h_l + h_u)\frac{\partial \tilde{q}_2}{\partial x_2} = 0 \qquad (13)$$

In contrast to the classical theory of plates, the components of the displacement vector are defined as functions of three spatial coordinates and time. The components of the strain tensor $u_1(x_1, x_2, z, t)$, $u_2(x_1, x_2, z, t)$, $u_3(x_1, x_2, z, t)$ are determined by the Cauchy relations.

Consider a filler as a three-dimensional body [21, 22], loaded with surface stresses $q_1^{(+)}, q_1^{(+)}, q_3^{(+)}$ and $q_1^{(-)}$, $q_1^{(-)}$, $q_3^{(-)}$ whose material obeys the generalized Hooke law:

$$\sigma_{11} = E_{11}\varepsilon_{11} + E_{12}\varepsilon_{22} + E_{13}\varepsilon_{33}, \quad \sigma_{22} = E_{21}\varepsilon_{11} + E_{22}\varepsilon_{22} + E_{23}\varepsilon_{33},$$
$$\sigma_{33} = E_{31}\varepsilon_{11} + E_{32}\varepsilon_{22} + E_{33}\varepsilon_{33}, \quad \sigma_{12} = 2G_{12}\varepsilon_{12}, \qquad (14)$$
$$\sigma_{13} = 2G_{13}\varepsilon_{13}, \quad \sigma_{23} = 2G_{23}\varepsilon_{23},$$

where E_{11}, $E_{12}, ..., E_{33}$ are the elastic constants determined through the Poisson's ratios and elastic moduli, G_{12}, G_{13}, G_{23} are the shear moduli of the filler material.

Note that the motion of the filler points in the framework of the bimoment theory of plates [22] is described by two problems.

To describe the second problem, introduce forces, moments, and bimoments using nine unknown functions $\tilde{\psi}_1$, $\tilde{\psi}_2$, $\tilde{u}_1$, $\tilde{u}_2$, $\tilde{\beta}_1$, $\tilde{\beta}_2$, $\tilde{r}$, $\tilde{\gamma}$, $\tilde{W}$, which are determined by the following formulas:

$$\tilde{u}_k = \frac{u_k^{(+)} - u_k^{(-)}}{2}, \quad (k = 1.2), \ \tilde{W} = \frac{u_3^{(+)} + u_3^{(-)}}{2} \qquad (15)$$

$$\tilde{\psi}_k = \frac{1}{2h^2}\int_{-h}^{h} u_k z dz, \quad \tilde{\beta}_k = \frac{1}{2h^4}\int_{-h}^{h} u_k z^3 dz \quad (k = 1.2), \ \tilde{r} = \frac{1}{2h}\int_{-h}^{h} u_3 dz, \ \tilde{\gamma} = \frac{1}{2h^3}\int_{-h}^{h} u_3 z^2 dz, \qquad (16)$$

and by the relations

$$M_{ij} = \int_{-h}^{h} \sigma_{ij} z dz, \quad P_{ij} = \frac{1}{h^2}\int_{-h}^{h} \sigma_{ij} z^3 dz \ (i, j = 1.2), \qquad (17)$$

$$Q_{i3} = \int_{-h}^{h} \sigma_{i3} dz, \quad \tilde{p}_{i3} = \frac{1}{2h^3}\int_{-h}^{h} \sigma_{i3} z^2 dz, \quad (i = 1.2), \quad \tilde{\tau}_{33} = \frac{1}{2h^4}\int_{-h}^{h} \sigma_{33} z^3 dz. \qquad (18)$$

The equations of motion of the second problem of a thick plate [22] with respect to bending, torques, shear forces, and relative to longitudinal, transverse bimoments (17), (18) are written in the form

$$\frac{\partial M_{k1}}{\partial x_1}+\frac{\partial M_{k2}}{\partial x_2}-Q_k+H\tilde{q}_k=\frac{H^2}{2}\rho\ddot{\tilde{\psi}}_k,$$
$$\frac{\partial P_{k1}}{\partial x_1}+\frac{\partial P_{k2}}{\partial x_2}-3\tilde{p}_{k3}+H\tilde{q}_k=\frac{H^2}{2}\rho\ddot{\tilde{\beta}}_k,\ \left(k=1.2\right), \tag{19}$$

$$\frac{\partial Q_{13}}{\partial x_1}+\frac{\partial Q_{23}}{\partial x_2}+2\tilde{q}_3=\rho H\ddot{\tilde{r}},$$
$$H\frac{\partial \tilde{p}_{13}}{\partial x_1}+H\frac{\partial \tilde{p}_{23}}{\partial x_2}-4\tilde{p}_{33}+2\tilde{q}_3=H\rho\ddot{\tilde{\gamma}} \tag{20}$$

The system of six differential equations of motion (19), (20) includes nine unknown functions $\tilde{\psi}_1,\ \tilde{\psi}_2,\ \tilde{u}_1,\ \tilde{u}_2,\ \tilde{\beta}_1,\ \tilde{\beta}_2,\ \tilde{r},\ \tilde{\gamma},\ \tilde{W}$.

Expand the components of the displacement vector in a Maclaurin series in the form [21, 22]:

$$u_k=B_0^{(k)}+B_1^{(k)}\frac{z}{h}+B_2^{(k)}\left(\frac{z}{h}\right)^2+B_3^{(k)}\left(\frac{z}{h}\right)^3+\ldots+B_m^{(k)}\left(\frac{z}{h}\right)^m,\ \ \left(k=1.2\right),$$
$$u_3=A_0+A_1\frac{z}{h}+A_2\left(\frac{z}{h}\right)^2+A_3\left(\frac{z}{h}\right)^3+\ldots+A_m\left(\frac{z}{h}\right)^m, \tag{21}$$

where $B_m^{(k)}, A_m$ are the unknown functions of two spatial coordinates and time.

And for the second problem, we have equations for contact stresses through unknown generalized functions in the form $\tilde{u}_1,\ \tilde{u}_2,\ \tilde{\psi}_1,\ \tilde{\psi}_2,\ \tilde{\beta}_1,\ \tilde{\beta}_2,\ \tilde{W}$

$$\tilde{q}_k=G_{k3}\left(105\tilde{\psi}_k+30\tilde{u}_k-315\tilde{\beta}_k+H\frac{\partial\tilde{W}}{\partial x_k}\right),\ \left(k=1.2\right),$$
$$\tilde{q}_3=E_{33}\left(15\tilde{r}+20\tilde{W}-105\tilde{\gamma}\right)+E_{31}H\frac{\partial\tilde{u}_1}{\partial x_1}+E_{32}H\frac{\partial\tilde{u}_2}{\partial x_2} \tag{22}$$

Taking into account the notation from (15), (16), from the conjugation conditions of the layers (5) we obtain equations that are the conjugation conditions between the bearing layers and the filler.

$$\tilde{u}_k=\tilde{U}_k+\frac{h_l+h_u}{2}\frac{\partial\tilde{W}}{\partial x_k},\ \left(k=1.2\right) \tag{23}$$

Here, generalized displacements of the bearing layers are introduced in the form

$$\tilde{U}_k=\frac{U_k^{(+)}-U_k^{(-)}}{2},\ \ \left(k=1.2\right) \tag{24}$$

Equations (23) determine the generalized contact displacements of the filler $\bar{u}_k$ and $\tilde{u}_k,\ \left(k=1.2\right)$ through the displacements of the median surfaces of the outer layers.

Thus, the expressions of contact displacements and stresses (22)-(24) of the symmetric and asymmetric displacement problems of the theory of three-layer plates are constructed.

The equations of motion of the bearing layers (12) and (13) the equations of motion of the filler (19), (20) taking into account expressions (22), (23) make up the joint system of equations of motion of the points of the layers of a three-layer plate with respect to unknown generalized displacements determined by formulas (15), (16).

The boundary conditions of the problem are set with respect to generalized displacements (15), (16), or relative to force factors (10), and (17), (18) depending on the conditions of the tasks posed [22].

It should be noted that for three-layer plates of a symmetric structure by material and thickness, the mechanical characteristics and geometric dimensions of the bearing layers will be the same:

$$E_1^{(+)} = E_1^{(-)},\ E_2^{(+)} = E_2^{(-)},\ G_{12}^{(+)} = G_{12}^{(-)},\ G_{13}^{(+)} = G_{13}^{(-)},\ G_{23}^{(+)} = G_{23}^{(-)},$$
$$\nu_{12}^{(+)} = \nu_{12}^{(-)},\ \nu_{13}^{(+)} = \nu_{13}^{(-)},\ \nu_{23}^{(+)} = \nu_{23}^{(-)},\ \rho_l = \rho_u,\ h_l = h_u.$$

Then the problem at hand is divided into two independent tasks. The first problem describes the symmetric deformation of a three-layer plate and the second asymmetric one.

The problem of intrinsic flexural-shear vibrations (asymmetric problem) of a three-layer plate of the symmetric structure with an orthotropic filler is posed. The equations of motion of the carrier layers and the filler of three-layer plates are written by the equations of motion (12), (13) and (19), (20) with respect to unknown functions $\tilde{U}_1,\ \tilde{U}_2,\ \tilde{\psi}_1,\ \tilde{\psi}_2,\ \tilde{\beta}_1,\ \tilde{\beta}_2,\ \tilde{W}$.

The boundary conditions are the conditions of free support along all edges of the plate for bending free vibrations, taking into account the transverse shift of the aggregate layer (asymmetric problem) of three-layer plates of symmetric structure. At two opposite edges of a three-layer plate, $x_1 = 0,\ x_1 = a$ have the conditions

$$M_{11} = 0,\ \ P_{11} = 0,\ \ \tilde{\psi}_2 = 0,\ \ \tilde{\beta}_2 = 0,\ \ \tilde{r} = 0,$$
$$\tilde{\gamma} = 0,\ \ \tilde{M}_{11} = 0,\ \ \tilde{P}_{11} = 0,\ \ \tilde{U}_2 = 0,\ \ \tilde{W} = 0. \qquad (25)$$

At the other two opposite edges of the three-layer plate, the boundary conditions must be satisfied

$$M_{22} = 0,\ \ P_{22} = 0,\ \ \tilde{\psi}_1 = 0,\ \ \tilde{\beta}_1 = 0,\ \ \tilde{r} = 0$$
$$\tilde{\gamma} = 0,\ \tilde{M}_{22} = 0,\ \tilde{P}_{22} = 0,\ \ \tilde{U}_1 = 0,\ \ \tilde{W} = 0 \qquad (26)$$

3. Results and discussion

Give an analytical solution to the problem of natural vibrations of a three-layer plate in trigonometric functions. Analytical solutions of the equations of natural vibrations of the bearing layers of the three-layer plate (12), (13), satisfying the boundary conditions (25) and (26) are written in the form:

$$\tilde{U}_1 = C_7 f_1(x_1,x_2)\cos(\omega t+\beta),\ \ \tilde{U}_2 = C_8 f_2(x_1,x_2)\cos(\omega t+\beta),$$
$$\tilde{W} = C_9 f_3(x_1,x_2)\cos(\omega t+\beta), \qquad (27)$$

where m, n is the number of half-waves according to the size of the plate in the plan, ω and β is the natural frequency and phase of the oscillation,

$$f_1(x_1,x_2) = \cos(\frac{n\pi x_1}{a})\sin(\frac{m\pi x_2}{b}),\ \ f_2(x_1,x_2) = \sin(\frac{n\pi x_1}{a})\cos(\frac{m\pi x_2}{b})$$
$$f_3(x_1,x_2) = \sin(\frac{n\pi x_1}{a})\sin(\frac{m\pi x_2}{b})$$

Analytical solutions of the equations of natural vibrations of the aggregate (12), (13) and (19), (20) are written in the form:

$$\tilde{\psi}_1 = C_1 f_1(x_1,x_2)\cos(\omega t+\beta),\ \ \tilde{\psi}_2 = C_2 f_2(x_1,x_2)\cos(\omega t+\beta),$$
$$\tilde{\beta}_1 = C_3 f_1(x_1,x_2)\cos(\omega t+\beta),\ \ \tilde{\beta}_2 = C_4 f_2(x_1,x_2)\cos(\omega t+\beta), \qquad (28)$$
$$\tilde{r} = C_5 f_3(x_1,x_2)\cos(\omega t+\beta),\ \ \tilde{\gamma} = C_6 f_3(x_1,x_2)\cos(\omega t+\beta).$$

Where in (27) and (28) $\tilde{C}_1, \tilde{C}_2, ..., \tilde{C}_9$ are the unknown constants.

Substituting the solution (27) and (28) into the system of equations (12), (13) and (19), (20), we obtain the frequency equation with respect to the dimensionless frequency $p_0 = \dfrac{\rho\omega^2 H^2}{G_{13}}$.

The dimensionless filler parameters are taken equal to the following: $E_{11}/G_{13}=4$, $E_{33}/G_{13}=3.2;\ \nu_{21}=0.4;\ \nu_{31}=0.1;\ h_l/h=0.05,\ H/a=0.333$, and the values $E_1^{(+)}/G_{13}, h\rho/h_l\rho_l$ vary. The calculations were performed for a square three-layer plate with a transversely isotropic filler.

In the Table1 and 2 shows the values of the dimensionless natural frequency p_0 for various values of the mechanical-geometric parameters of a three-layer plate.

The first column of the tables shows the values n at $m=1$. In the remaining columns of the table, the first 6 lowest values of the natural frequency are given for the five values of the ratio $E_1^{(+)}/G_{13}$.

An analysis of the calculations shows that in this range of values $E_1^{(+)}/G_{13}=250-1000$ the natural frequency increases significantly, and then approaches the asymptote for all values of the mechanical-geometric parameters of the three-layer plate.

Table 1. Values of dimensionless natural frequency p_0 at $h\rho/h_l\rho_l=1$

n	$E_1^{(+)}/G_{13}$		
	250.00	500.00	1000.0
1	0.274	0.294	0.308
2	0.679	0.732	0.759
3	1.331	1.456	1.531
4	2.219	2.499	2.643
5	3.302	3.763	4.102
6	4.611	5.352	6.005

Table 2. Values of dimensionless natural frequency p_0 at $h\rho/h_l\rho_l=2$

n	$E_1^{(+)}/G_{13}$		
	250.00	500.00	1000.0
1	0.237	0.258	0.266
2	0.579	0.633	0.661
3	1.132	1.252	1.374
4	1.861	2.124	2.354
5	2.749	3.283	3.6733
6	3.796	4.637	5.399

Note that the geometric dimensions of a three-layer plate also significantly affect the values of the natural frequency. With a decrease in the relative thickness of the filler H/a, the natural frequency decreases sharply. A change in the relative thickness of the carrier layers h/h_l results in a proportional change in the natural frequency. With an increase in relative density, the natural frequency decreases.

4. Conclusions

The article proposes a theory and methods for calculating with high accuracy the three-layer plates with compressive spatial filler. The basic equations of layer motion and boundary conditions are derived for various fixation options of edges of rectangular three-layer plates of the symmetric structure. Analytical solutions to the problem of free bending-shear vibrations of three-layer plates fixed at the edges are constructed. Eigen frequencies are found for various mechanical and geometrical parameters of three-layer plates. The theory and methods of calculating three-layer plates with compressive spatial filler are developed.

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