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Simulation of free vibrations of a thick plate without simplifying hypotheses

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Abstract. The paper is devoted to the simulation of free vibrations of orthotropic thick plates taking into account internal forces, moments and bimoments. To solve the problem, equations of motion of a plate are built, described by a six-equation system, constructed in the framework of a simplified version of the bimoment theory of thick plates. As an example, consider the problem of determining natural frequencies of thick isotropic and orthotropic fixed plates under transverse vibrations, taking into account the transverse shear and compression along the thickness.

1. Introduction

The theory of plates and shells occupies a special place in structure elements calculation. Refined plate theories have been constructed by many authors. All existing plate theories are developed based on a number of simplifying hypotheses.

Studies in the field of thick plates showed that in the spatial case of plate strains along its thickness, nonlinear laws of displacements distribution take place and the hypothesis of plane sections is violated. Moreover, in addition to tensile and shear forces, bending and torques, in the cross sections of the plate the additional force factors appear, the so-called bimoments. The authors in [1-4], solved the problems of bending and vibrations of thick plates on the basis of the bimoment theory of plates built in the framework of the three-dimensional theory of elasticity, using the method of displacement expansion on one of the spatial coordinates into an infinite Maclaurin series.

The studies in [5, 6] are devoted to the development of the methods of dynamic spatial calculation of a structure based on the finite difference method in the framework of the bimoment theory taking into account the spatial stress-strain state. Solutions to the problem of transverse and longitudinal vibrations of buildings and structures using the plate model developed in the framework of the bimoment theory of plates are given in [1-4].

In [7,8], vibrations of structure elements from isotropic viscoelastic plates of variable thickness were considered; the plates are under uniformly distributed vibration load applied on one of the parallel sides, leading (at certain combinations of natural vibration frequencies and excitation force) to a parametric resonance.

The stress-strain state and dynamic behavior of various thin-walled systems were studied in [9-12], taking into account their geometrical and physical features.

This paper poses the problem of free vibrations of a thick orthotropic plate. A brief description of the methods is given to construct the theory of plates taking into account the bimoments generated by nonlinear distribution of displacements in cross-section points. The equations of motion of moments and forces are constructed according to the traditional method. The bimoment equations are constructed using the equations of the three-dimensional dynamic theory of elasticity and boundary conditions described on the front surfaces of a plate. In the paper, we will use the notations and determinant relations of forces, moments, bimoments and equations of motion with respect to the force factors given in [1-4].

2. Statement of the problem

Consider an orthotropic thick plate of constant thickness $H = 2h$ and dimensions a, b in plan. Introduce the following notations: E_1, E_2, E_3 - moduli of elasticity; G_{12}, G_{13}, G_{23} - shear moduli; v_{12}, v_{13}, v_{23} - Poisson's ratios of the plate material. To describe the plate motion, introduce a Cartesian coordinate system with variables x_1, x_2 and z. The origin is located in the middle surface of the plate. The axis OZ is directed downward.

When constructing equation of motion, the plate is considered as a three-dimensional body, and all components of the stress and strain tensors are taken into account: σ_{ij}, ε_{ij}, $(i, j = 1,3)$. The components of the displacement vector are functions of three spatial coordinates and time $u_1(x_1, x_2, z, t)$, $u_2(x_1, x_2, z, t)$, $u_3(x_1, x_2, z, t)$.

The components of the strain tensor ε_{ij} are determined from the Cauchy relation:

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}, \ \ \varepsilon_{22} = \frac{\partial u_2}{\partial x_2}, \ \ \varepsilon_{33} = \frac{\partial u_3}{\partial z},$$

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right), \ \ \varepsilon_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x_1}\right), \ \ \varepsilon_{23} = \frac{1}{2}\left(\frac{\partial u_2}{\partial z} + \frac{\partial u_3}{\partial x_2}\right). \tag{1}$$

For an orthotropic plate, we write the Hooke'slaw in the general case:

$$\begin{aligned}
\sigma_{11} &= E_{11}\varepsilon_{11} + E_{12}\varepsilon_{22} + E_{13}\varepsilon_{33}, \\
\sigma_{22} &= E_{21}\varepsilon_{11} + E_{22}\varepsilon_{22} + E_{23}\varepsilon_{33}, \\
\sigma_{33} &= E_{31}\varepsilon_{11} + E_{32}\varepsilon_{22} + E_{33}\varepsilon_{33}, \\
\sigma_{12} &= 2G_{12}\varepsilon_{12}, \ \ \ \sigma_{13} = 2G_{13}\varepsilon_{13}, \ \ \ \sigma_{23} = 2G_{23}\varepsilon_{23}.
\end{aligned} \tag{2}$$

where E_{11}, $E_{12},...,E_{33}$ are the elastic constants determined by Poisson's ratios and elastic moduli.

Three-dimensional equations of dynamic theory of elasticityare usedas the equation of motion of a plate:

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial z} = \rho \ddot{\tilde{u}}_1,$$

$$\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial z} = \rho \ddot{\tilde{u}}_2,, \qquad (3)$$

$$\frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial z} = \rho \ddot{\tilde{u}}_3.$$

where ρ - is the plate material density.

Boundary conditions on the lower and upper face surfaces of the plate $z = h$ and $z = -h$ are:

$$\sigma_{33} = 0, \quad \sigma_{31} = 0, \; \sigma_{32} = 0, \qquad \text{at } z = h; \qquad (4,a)$$

$$\sigma_{33} = 0, \quad \sigma_{31} = 0, \; \sigma_{32} = 0, \qquad \text{at } z = -h. \qquad (4,b)$$

3. Method of the solution

Three-dimensional problem of elasticity theory of thick plate vibrations is solved using a simplified version of the bimoment theory of thick plates [1-4]. The method to construct the bimoment theory of plates is based on the Cauchy relation (1), on the generalized Hooke's law (2), three-dimensional equations of the theory of elasticity (3), boundary conditions on the front surfaces (4) and the method of displacement expansion into an infinite Maclaurin series. The proposed bimoment theory of plates is described by two unrelated problems, each of which is formulated on the basis of six two-dimensional equations of motion with corresponding boundary conditions.

Displacements and stresses of the points of the upper $z = -h$ and lower $z = h$ plate fibersare denotedby $u_i^{(-)}$, $u_i^{(+)}$, $(i = 1,3)$ and $\sigma_{ij}^{(-)}$, $\sigma_{ij}^{(+)}$, $(i = 1,3; \; j = 1,3)$.

The first task of the bimoment theory describes the tension-compression and transverse compression of a plate, and the second task describes the bending and transverse shear of a plate.

The second task of the bimoment theory consists of equations for bending moments, torque, shear forces with respect to six unknown kinematic functions $\tilde{\psi}_1$, $\tilde{\psi}_2$, $\tilde{u}_1$, $\tilde{u}_2$, $\tilde{r}$, $\tilde{W}$, defined by the formulas:

$$\tilde{u}_k = \frac{u_k^{(+)} - u_k^{(-)}}{2}, \qquad \tilde{\psi}_k = \frac{1}{2h^2} \int_{-h}^{h} u_k z\, dz, \quad (k = 1,2), \qquad (5)$$

$$\tilde{W} = \frac{u_3^{(+)} + u_3^{(-)}}{2}, \qquad \tilde{r} = \frac{1}{2h} \int_{-h}^{h} u_3\, dz. \qquad (6)$$

Introduce generalized external loads for the second problem

$$\tilde{q}_1 = \frac{q_1^{(+)} + q_1^{(-)}}{2}, \qquad \tilde{q}_2 = \frac{q_2^{(+)} + q_2^{(-)}}{2}, \qquad \tilde{q}_3 = \frac{q_3^{(+)} - q_3^{(-)}}{2}. \qquad (7)$$

Theexpressionsofbending, torquesandshearforcesareconstructedintheform:

$$M_{11} = \frac{H^2}{2}\left(E_{11}\frac{\partial \widetilde{\psi}_1}{\partial x_1} + E_{12}\frac{\partial \widetilde{\psi}_2}{\partial x_2} - E_{13}\frac{2(\widetilde{r}-\widetilde{W})}{H}\right),$$

$$M_{22} = \frac{H^2}{2}\left(E_{12}\frac{\partial \widetilde{\psi}_1}{\partial x_1} + E_{22}\frac{\partial \widetilde{\psi}_2}{\partial x_2} - E_{23}\frac{2(\widetilde{r}-\widetilde{W})}{H}\right), \tag{8}$$

$$M_{12} = \frac{H^2}{2}G_{12}\left(\frac{\partial \widetilde{\psi}_1}{\partial x_2} + \frac{\partial \widetilde{\psi}_2}{\partial x_1}\right),$$

$$Q_{13} = G_{13}\left(2\widetilde{u}_1 + H\frac{\partial \widetilde{r}}{\partial x_1}\right), \quad Q_{23} = G_{23}\left(\widetilde{u}_2 + H\frac{\partial \widetilde{r}}{\partial x_2}\right). \tag{9}$$

Thesystemofequationsofmotionofthesecondproblemconsistsoftwoequations, withrespecttobending, torques, oneequationwithrespecttoshearforces, andoneequationwithrespecttobimoment:

$$\frac{\partial M_{11}}{\partial x_1} + \frac{\partial M_{12}}{\partial x_2} - Q_{13} = \frac{H^2}{2}\rho\ddot{\widetilde{\psi}}_1 - H\widetilde{q}_1, \quad \frac{\partial M_{21}}{\partial x_1} + \frac{\partial M_{22}}{\partial x_2} - Q_{23} = \frac{H^2}{2}\rho\ddot{\widetilde{\psi}}_2 - H\widetilde{q}_2, \tag{10}$$

$$\frac{\partial Q_{13}}{\partial x_1} + \frac{\partial Q_{23}}{\partial x_2} = H\rho\ddot{\widetilde{r}} - 2\widetilde{q}_3. \tag{11}$$

Note that the relationships of forces and moments (8) (9), so, the equations of motion of system (10), (11) are constructedexactly. The system of equations of motion (10), (11) consists of three equations with respect to six unknown functions $\widetilde{\psi}_1$, $\widetilde{\psi}_2$, $\widetilde{u}_1$, $\widetilde{u}_2$, $\widetilde{r}$, $\widetilde{W}$. If in expressions of forces and moments, in (17) and (18), we conditionally assume $\widetilde{u}_1 = 3\widetilde{\psi}_1$, $\widetilde{u}_2 = 3\widetilde{\psi}_2$ and $E_{13} = E_{23} = 0$, and replace the shear modules G_{13}, G_{23} with $k_z G_{13}$, $k_z G_{23}$, (where k_z is the shear coefficient), then the equations of motion of the plates are obtained according to the Timoshenko theory.

To supplement these systems (10), (11) using the third equation of system (3) and boundary conditions (4), three more equations are constructed

$$\widetilde{u}_1 = \frac{5}{2}\widetilde{\psi}_1 + \frac{1}{12}\left(H\frac{\partial \widetilde{W}}{\partial x_1} - \frac{H\widetilde{q}_1}{G_{13}}\right), \quad \widetilde{u}_2 = \frac{5}{2}\widetilde{\psi}_2 + \frac{1}{12}\left(H\frac{\partial \widetilde{W}}{\partial x_2} - \frac{H\widetilde{q}_2}{G_{23}}\right). \tag{12}$$

$$\frac{\partial \widetilde{q}_1}{\partial x_1} + \frac{\partial \widetilde{q}_2}{\partial x_2} + \frac{\widetilde{\sigma}_{33}^*}{H} = \rho\ddot{\widetilde{W}}. \tag{13}$$

Here

$$\widetilde{\sigma}_{33}^* = 60E_{33}\frac{\widetilde{r}-\widetilde{W}}{H} - E_{31}H\frac{\partial}{\partial x_1}\left(\frac{\partial \widetilde{W}}{\partial x_1} - \frac{\widetilde{q}_1}{G_{31}}\right) - E_{32}H\frac{\partial}{\partial x_2}\left(\frac{\partial \widetilde{W}}{\partial x_2} - \frac{\widetilde{q}_2}{G_{32}}\right) -$$
$$- 12\left(E_{31}\frac{\partial \widetilde{u}_1}{\partial x_1} + E_{32}\frac{\partial \widetilde{u}_2}{\partial x_2} - \widetilde{q}_3\right). \tag{14}$$

It should be noted that equations (12), (13) and expression (14) are constructed with the accuracy up to the fourth order of a relatively small parameter $\delta = H/(10a)$.

Consider the boundary conditions of the problem for thick plates with fixed edges. At the edges of the plate $x_1 = const$ and $x_2 = const$ the following conditions must be met:

$$\widetilde{\psi}_1 = 0, \quad \widetilde{\psi}_2 = 0, \quad \widetilde{r} = 0, \quad \widetilde{u}_1 = 0; \quad \widetilde{u}_2 = 0, \quad \widetilde{W} = 0. \tag{15}$$

4. Solution to the test problem

Consider free transverse vibrations of a thick plate. The solution to the problem is presented in the form

$$\widetilde{\psi}_1 = \widetilde{\psi}_1(x_1, x_2)\cos(\omega t + \beta), \quad \widetilde{\psi}_2 = \widetilde{\psi}_2(x_1, x_2)\cos(\omega t + \beta),$$

$$\widetilde{u}_1 = \widetilde{u}_1(x_1, x_2)\cos(\omega t + \beta), \quad \widetilde{u}_2 = \widetilde{u}_2(x_1, x_2)\cos(\omega t + \beta), \tag{16}$$

$$\widetilde{r} = \widetilde{r}(x_1, x_2)\cos(\omega t + \beta), \quad \widetilde{W} = \widetilde{W}(x_1, x_2)\cos(\omega t + \beta).$$

where ω and β – are the natural frequency and phase of oscillations, respectively.

Substituting the solution (16) into equations (12) and (13), we obtain partial differential equations with respect to spatial coordinates, solved by the finite difference method. To approximate the derivatives of the displacements of internal points, the expressions of the central difference schemes are used.

The first and second derivatives are determined by formulas:

$$f_i^{'} = \frac{f_{i+1} - f_{i-1}}{2\Delta x_l}, \quad f_i^{''} = \frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta x_l^{2}}. \tag{17}$$

Based on expressions of the finite difference method (17) and solution (16), from the equation of motion (12), (13) and boundary conditions (14), we obtain the frequency equation for the square of the dimensionless frequency $p = H\omega\sqrt{\dfrac{\rho}{E}}$, solved numerically by dividing in half, from which the dimensionless eigenfrequencies p are found.

In calculations, the plates with isotropic and orthotropic materials are considered. Anisotropic plate has elastic moduli $E_1 = E_2 = E_3 = E_0$, shear moduli $G_{12} = G_{13} = G_{23} = E_0 / 2(1 + v)$, Poisson's ratios $v_{21} = v_{23} = v_{31} = v = 0.3$.

As an orthotropic material, SWAM 15:1 is taken with Poisson's ratios $v_{21} = 0.27$, $v_{23} = 0.3$, $v_{31} = 0.07$ and moduli of elasticity, shear (at $E_0 = 10^4$ MPa), and $E_1 = 4.6E_0$, $E_2 = 1.6E_0$, $E_2 = 1.12E_0$, $G_{12} = 0.56E_0$, $G_{13} = 0.33E_0$, $G_{23} = 0.43E_0$, For convenience, the following dimensionless variables are introduced

$$x = \frac{x_1}{a}, \quad y = \frac{x_2}{b}.$$

Calculation steps for dimensionless coordinates are taken as

$$\Delta x = \frac{1}{N}, \quad \Delta y = \frac{1}{M}, \quad M \le N.$$

Calculations show that satisfactory numerical results can be obtained at the number of divisions $N = M = 30$. Table 1 gives the first natural frequencies p for three sizes of square isotropic and orthotropic plates.

Table 1. Dimensionless values of the first natural frequencies for three sizes of square isotropic and orthotropic plates.

Plate material	$a = b = 3H$	$a = b = 5H$	$a = b = 8H$
Isotropic	0.695	0.332	0.156
Orthotropic	0.751	0.398	0.201

5. Conclusion

Based on the application of a simplified version of the bimoment theory of plates within the framework of the finite difference method, a mathematical model and a method for solving three-dimensional dynamic problem of the theory of elasticity of thick orthotropic plates natural vibrations are developed. The effectiveness of the methods is confirmed by solving a number of test problems. Numericalresultsobtained well agreewiththeresultsofstudies [4].

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