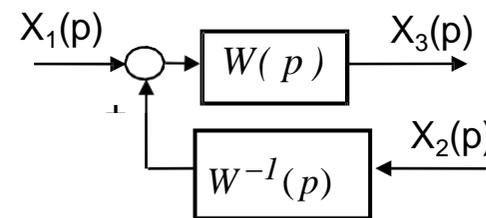
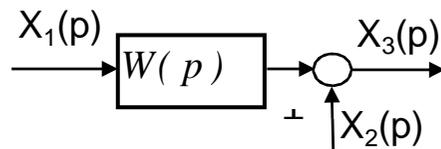
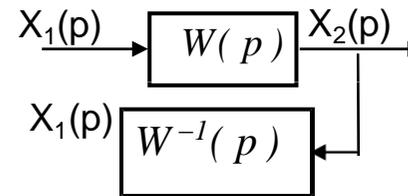
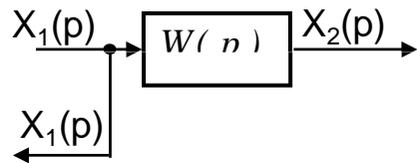
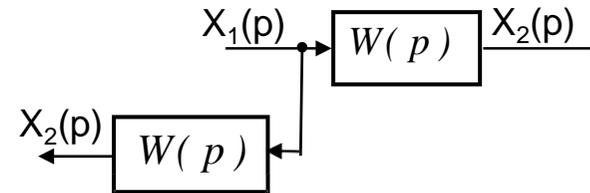
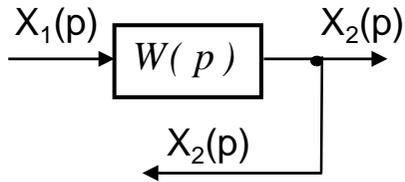
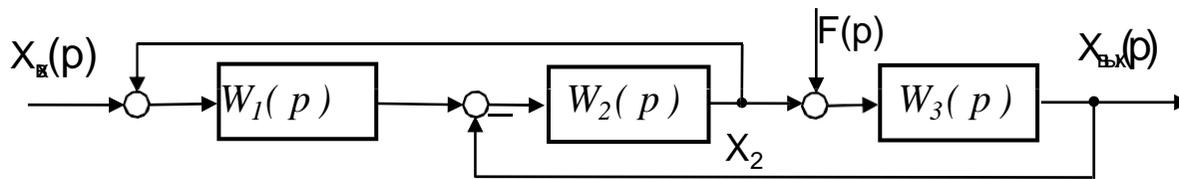
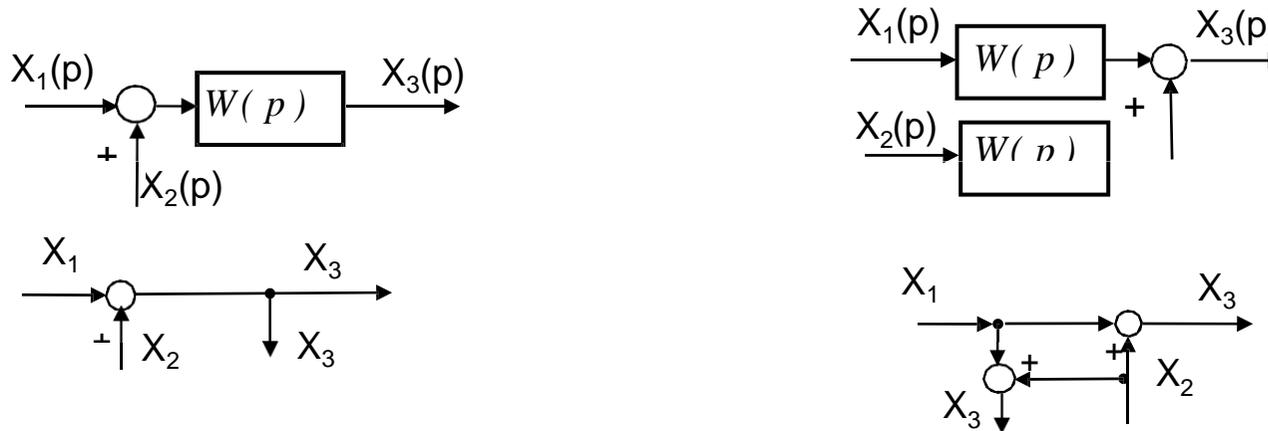


Правила преобразования структурных схем

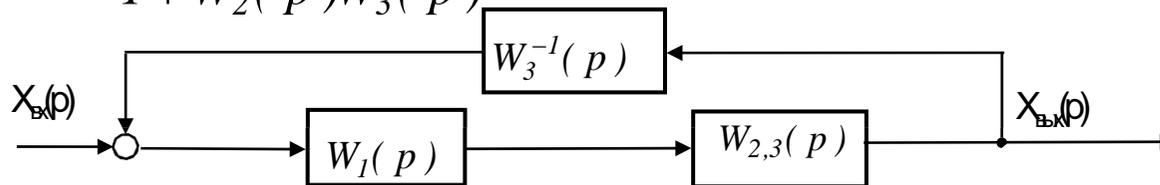
Правила переноса узлов разветвления



Правила преобразования структурных схем



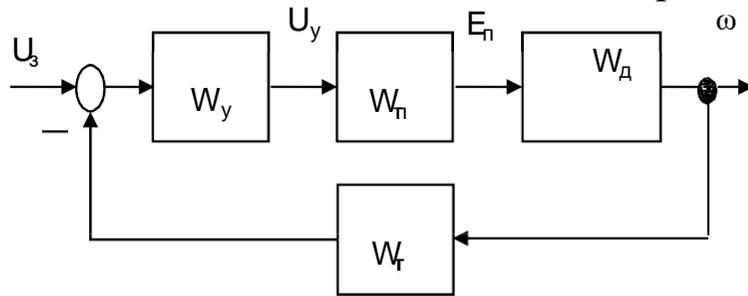
$$2,3 \quad - \frac{W_2(p)W_3(p)}{1 + W_2(p)W_3(p)}$$



$$W(p) = \frac{X_{vbx}(p)}{X_{ex}(p)} = \frac{W_1(p)W_{2,3}(p)}{1 + W_1(p)W_{2,3}(p)W_3^{-1}(p)} =$$

$$= \frac{W_1(p) \frac{W_2(p)W_3(p)}{1+W_2(p)W_3(p)}}{1+W_1(p) \frac{W_2(p)W_3(p)}{1+W_2(p)W_3(p)} W_3^{-1}(p)} = \frac{W_1(p)W_2(p)W_3(p)}{1+W_2(p)W_3(p)+W_1(p)W_2(p)}$$

$$W_p(p) = W_Y(p)W_{ТП}(p)W_{Д}(p)W_{ТГ}(p)$$

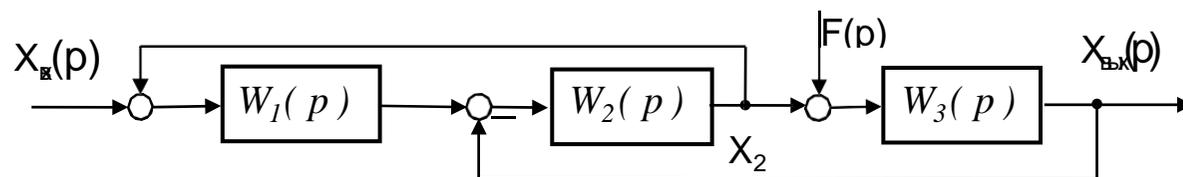


$$W_3(p) = \frac{\omega(p)}{U_3(p)} = \frac{W_p(p)/W_{ТГ}(p)}{1+W_p(p)} = \frac{K_y \frac{K_{\Pi}}{T_{\Pi}p+1} \frac{K_{Д}}{T_M T_{Я}p^2 + T_M p + 1}}{1 + K_y \frac{K_{\Pi}}{T_{\Pi}p+1} \frac{K_{Д}}{T_M T_{Я}p^2 + T_M p + 1} K_{ТГ}} =$$

$$= \frac{K_y K_{\Pi} K_{Д}}{(T_{\Pi}p+1)(T_M T_{Я}p^2 + T_M p + 1) + K_y K_{\Pi} K_{Д} K_{ТГ}}$$

Правило Мезона

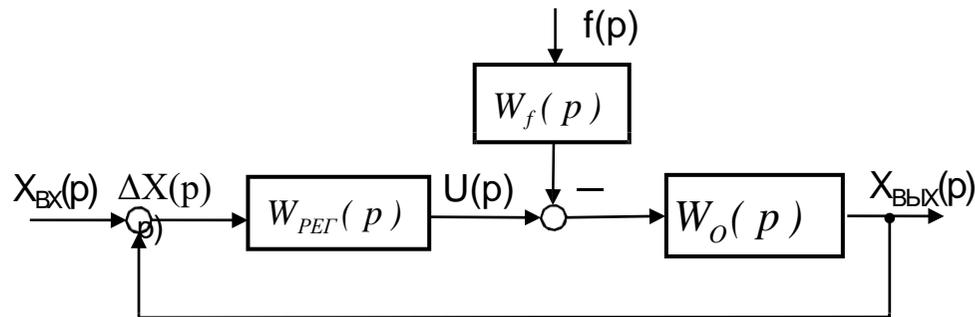
$$W(p) = \frac{X_{\text{вых}}(p)}{X_{\text{вх}}(p)} = \frac{\sum_i [W_{i \text{ вх}}(p) \Delta_i(p)]}{\Delta(p)}, \quad \Delta(p) = 1 - \sum W_{k1}(p) + \sum W_{k2}(p) - \sum W_{k3}(p)$$



$$W(p) = \frac{X_{\text{вых}}(p)}{X_{\text{вх}}(p)} = \frac{W_1(p)W_2(p)W_3(p)}{1 + W_1(p)W_2(p) + W_2(p)W_3(p)}$$

$$W(p) = \frac{X_2(p)}{F(p)} = \frac{W_2(p)W_3(p)}{1 + W_1(p)W_2(p) + W_2(p)W_3(p)}$$

Типовые передаточные функции САУ

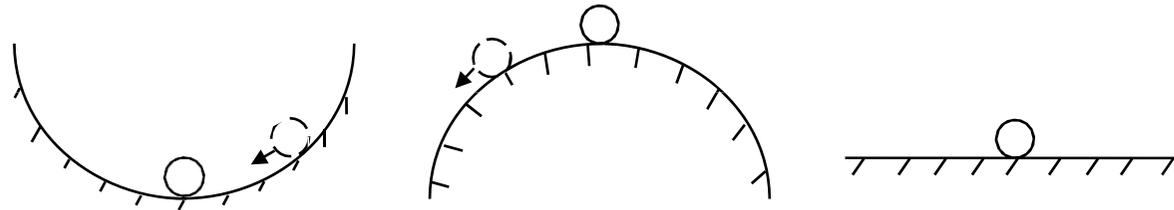


$$W_p(p) = \frac{X_{BbX}(p)}{X_{BX}(p)} = W_{PEΓ}(p)W_o(p) \quad W(p) = \frac{X_{BbX}(p)}{X_{BX}(p)} = \frac{W_p(p)}{1 + W_p(p)}$$

$$W_o(p) = \frac{X_{BbX}(p)}{U(p)} \quad W_{of}(p) = \frac{X_{BbX}(p)}{f(p)} = W_f(p)W_o(p)$$

$$\Phi(p) = \frac{X_{BbX}(p)}{f(p)} = \frac{W_f(p)W_o(p)}{1 + W_p(p)} \quad H(p) = \frac{\Delta X(p)}{X_{BX}(p)} = \frac{1}{1 + W_p(p)} = 1 - W(p).$$

УСТОЙЧИВОСТЬ ЛИНЕЙНЫХ САУ



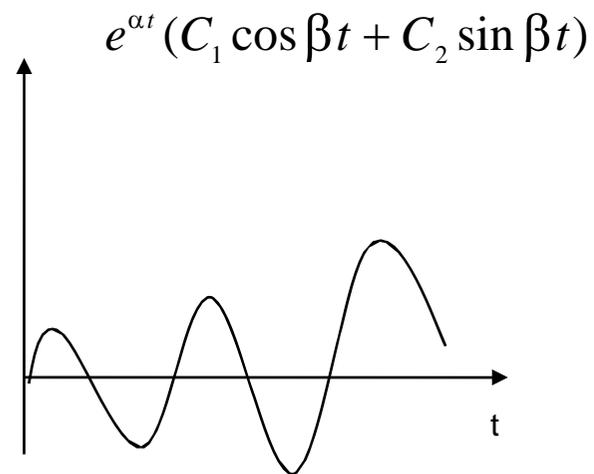
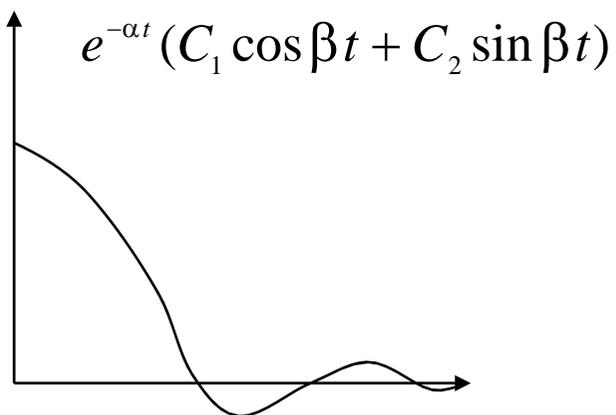
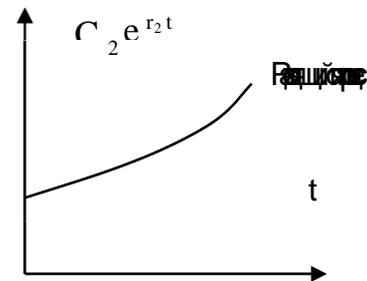
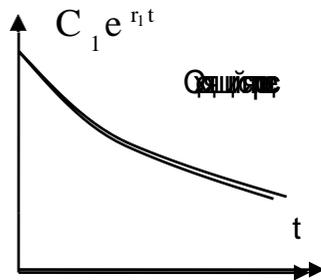
$$(a_0 p^n + a_1 p^{n-1} + \dots + a_n) X_{ввх} = (b_0 p^m + b_1 p^{m-1} + \dots + b_m) X_{вх}$$

$$X_{ввх}(t) = \bar{X}_{ввх}(t) + X_{ввх}^*(t)$$

$$a_0 r^n + a_1 r^{n-1} + \dots + a_n = 0$$

УСТОЙЧИВОСТЬ ЛИНЕЙНЫХ САУ

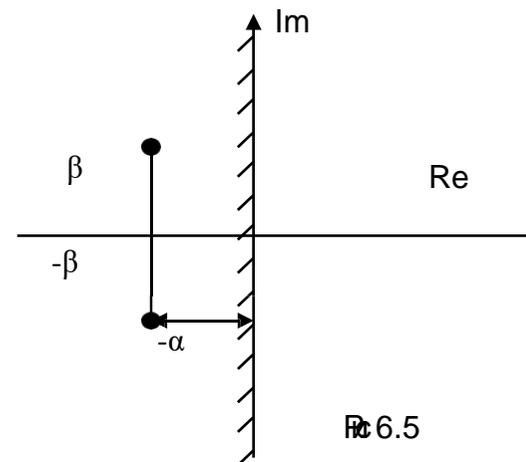
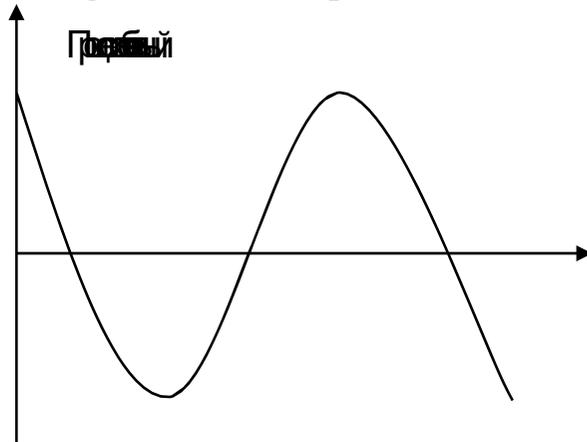
Выражения



УСТОЙЧИВОСТЬ ЛИНЕЙНЫХ САУ

Миллер

$$C_1 \cos \beta t + C_2 \sin \beta t$$



Критерии устойчивости

Критерий Гурвица

$$a_0 r^n + a_1 r^{n-1} + \dots + a_n = 0$$

$$\Delta_n = \begin{vmatrix} a_1 & a_3 & a_5 & \dots & 0 \\ a_0 & a_2 & a_4 & \dots & 0 \\ 0 & a_1 & a_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & a_n \end{vmatrix}$$

$$1 \quad n. \quad 2 \quad 0; \dots, \quad n-1 \quad 0.$$

$$\Delta_1 = a_1$$

$$\Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix};$$

$$\Delta_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix}$$

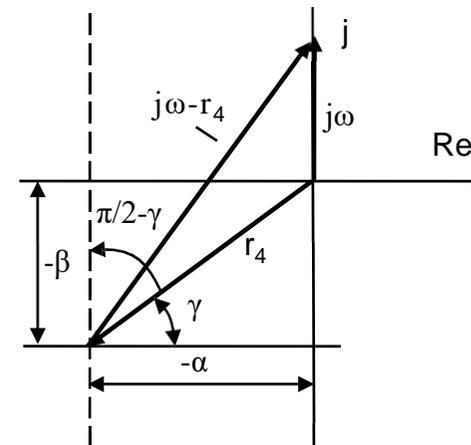
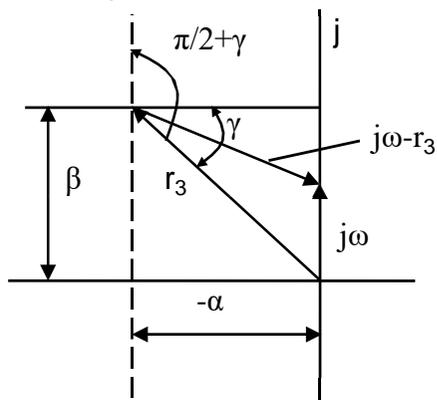
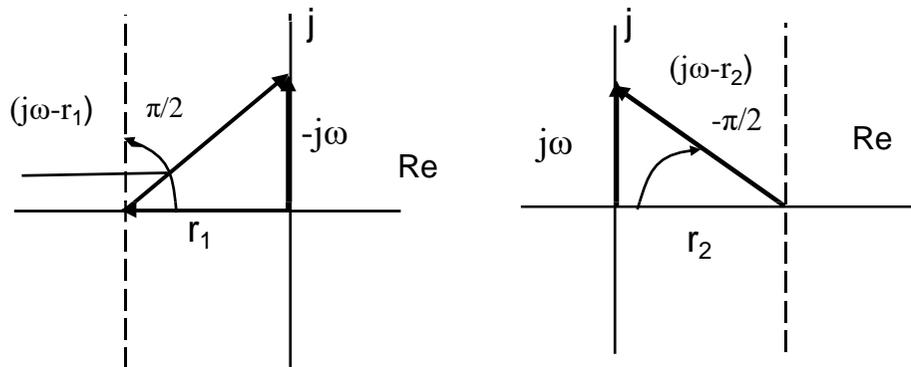
Критерии устойчивости

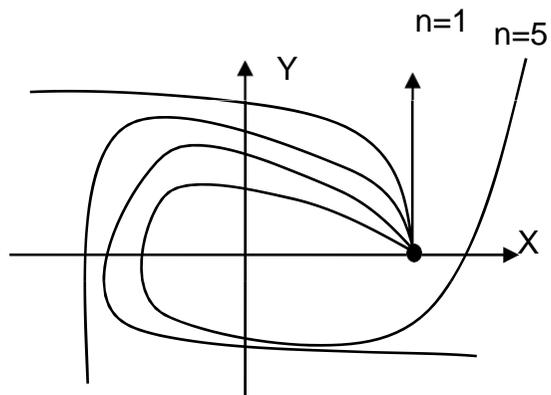
Критерий Михайлова

$$A(r) = a_0 r^n + a_1 r^{n-1} + \dots + a_n \quad A(j\omega) = a_0 (j\omega - r_1)(j\omega - r_2) \dots (j\omega - r_n)$$

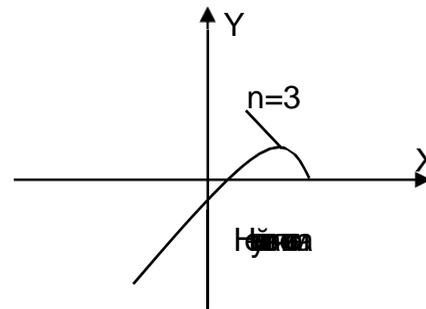
$$A(j\omega) = a_0 A_1(\omega) e^{j\varphi_1(\omega)} A_2(\omega) e^{j\varphi_2(\omega)} \dots A_n(\omega) e^{j\varphi_n(\omega)}$$

$$A(j\omega) = |A(j\omega)| e^{j\varphi(\omega)}$$





Функционал



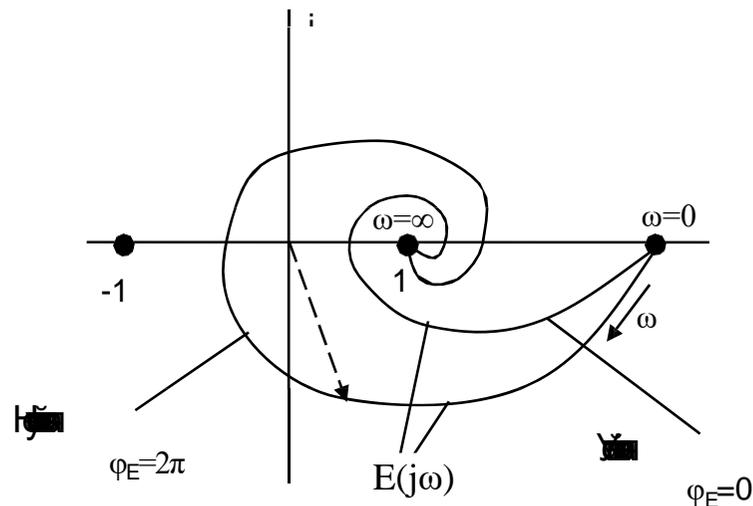
Функционал

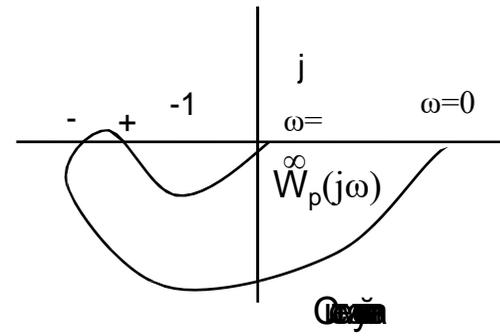
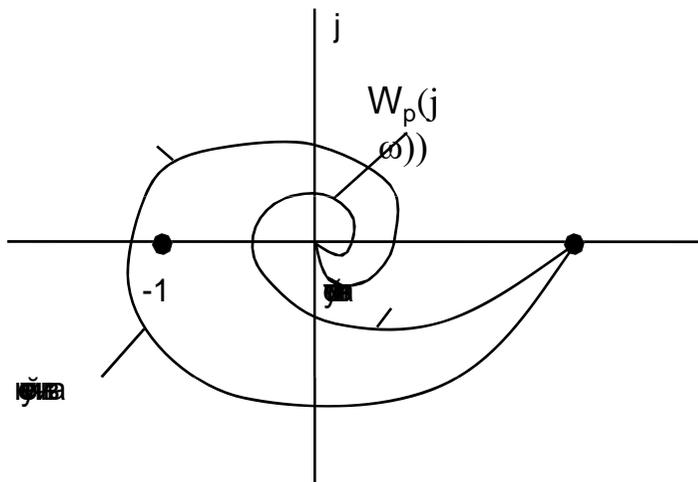
Критерии устойчивости

Критерий устойчивости Найквиста

$$W_p(p) = \frac{b_0 p^m + b_1 p^{m-1} + \dots + b_m}{c_0 p^n + c_1 p^{n-1} + \dots + c_n} = \frac{B(p)}{C(p)} \quad W_p(p) = \frac{\frac{B(p)}{C(p)}}{1 + \frac{B(p)}{C(p)}} = \frac{B(p)}{B(p) + C(p)}; \quad n > m$$

$$E(j\omega) = 1 + W_p(j\omega) = 1 + \frac{B(j\omega)}{C(j\omega)} = \frac{C(j\omega) + B(j\omega)}{C(j\omega)} = \frac{A(j\omega)}{C(j\omega)} = \frac{A(\omega)e^{j\varphi_A}}{C(\omega)e^{j\varphi_C}} = E(\omega)e^{j\varphi_E}$$





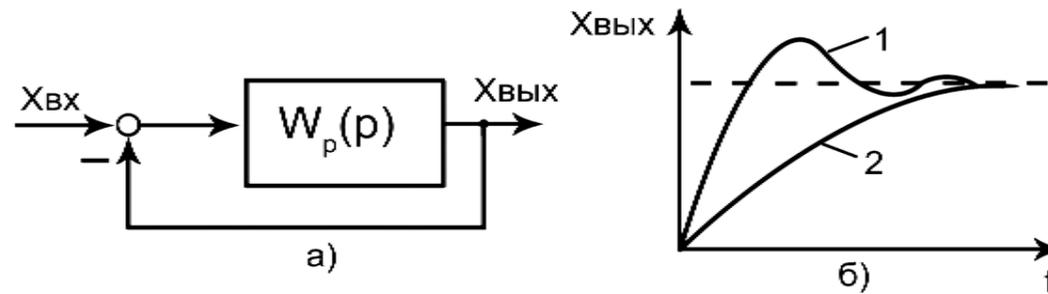
Анализ устойчивости САУ по логарифмическим частотным характеристикам

$$W_p(p) = W_1(p)W_2(p)W_3(p)$$

$$L_p(\omega) = L_1(\omega) + L_2(\omega) + L_3(\omega)$$

$$\varphi_p(\omega) = \varphi_1(\omega) + \varphi_2(\omega) + \varphi_3(\omega)$$

РАСЧЕТ ПЕРЕХОДНЫХ ПРОЦЕССОВ САУ



Операторный метод решения дифференциальных уравнений

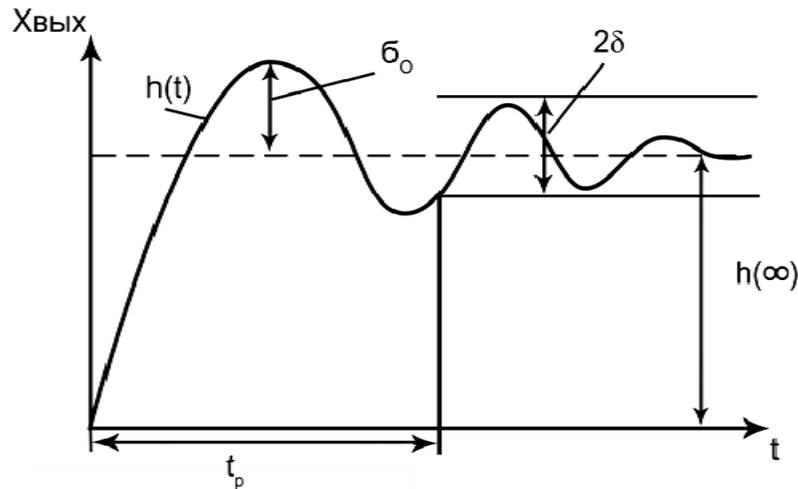
$$x_{вых}(t) = \frac{B(0)}{\Delta(0)} + \sum_{i=1}^n \frac{B(p_i)}{p_i \cdot A'(p_i)} \cdot e^{p_i \cdot t}$$

Частотный метод расчета переходных процессов

$$h(t) = \frac{2}{\pi} \int_0^{\infty} P(\omega) \frac{\sin \omega t}{\omega} d\omega$$

КАЧЕСТВО ПРОЦЕССОВ РЕГУЛИРОВАНИЯ

Оценка качества по переходной функции



Быстродействие $|x_{\text{вых}}(t) - x_{\text{вых}}(\infty)| <$

Перерегулирование σ_0

$$\sigma \% = \frac{\sigma_0}{x_{\text{вых}}(\infty)} \cdot 100\%$$

Статическая ошибка

$$\Delta x_{\text{уст}} = x_{\text{вх}} - x_{\text{вых}}(\infty)$$

Динамическая ошибка

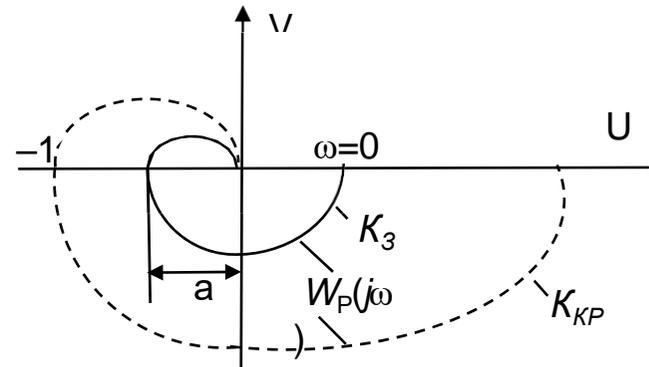
$$\Delta x(t) = x_{\text{вх}}(t) - x_{\text{вых}}(t)$$

Косвенные методы оценки качества

Цели урока

Запас устойчивости по модулю

$$\Delta K = \frac{K_{кр}}{K_3}$$



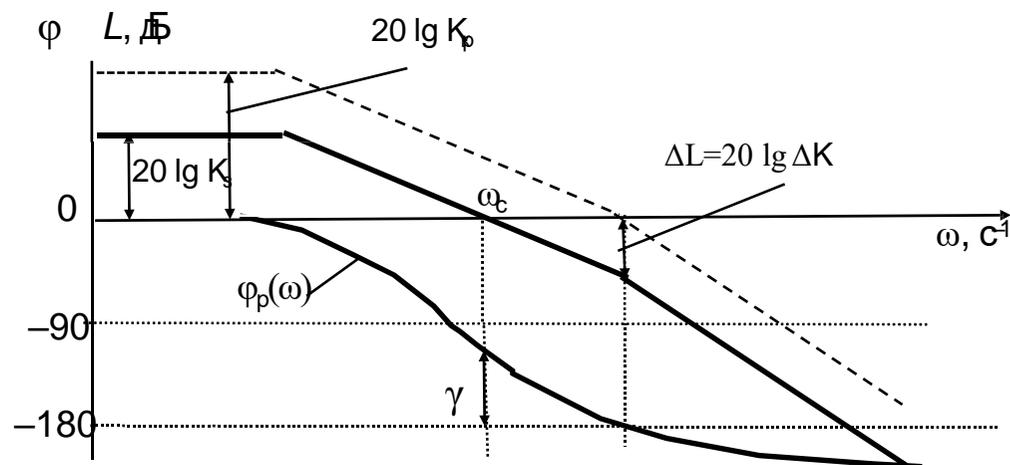
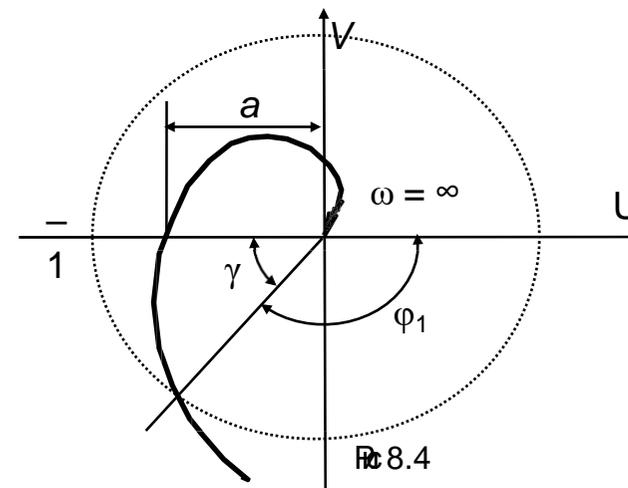
$$W_p(j\omega) = U(\omega) + jV(\omega) = K[U_1(\omega) + jV_2(\omega)] = KU_1(\omega)$$

$$K_3 \cdot U_1(\omega_1) = -a \quad \Delta K = \frac{K_{кр}}{K_3} = \frac{1}{a} > 1$$

$$K_{кр} \cdot U_1(\omega_1) = -1$$

Запас устойчивости по фазе

Приложение



Критерии

Степень устойчивости

$$\eta = |\alpha|_{\min}$$

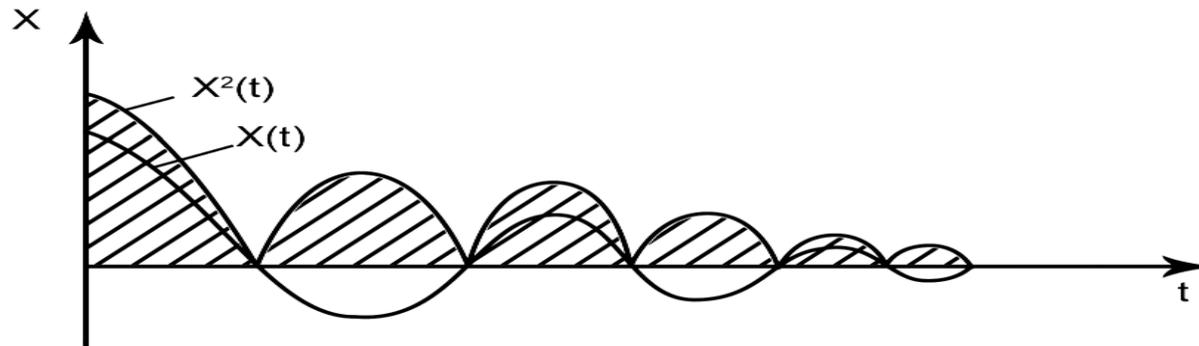
Колебательность

$$\lambda = \operatorname{tg} \theta = \frac{\beta_i}{\alpha_i}$$

Индикатор

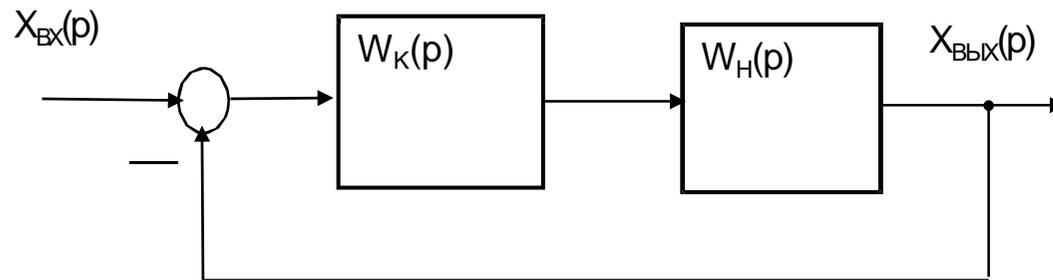
колебаний

$$I = \int_0^{\infty} x^2(t) dt$$



СИНТЕЗ СИСТЕМ РЕГУЛИРОВАНИЯ

- ~~ЦЕЛЬ~~
- ~~ЗАДАЧА~~
- ~~МЕТОДЫ~~
- ~~РЕЗУЛЬТАТЫ~~



$$W_K(p)$$

$$W_C(p) = W_H(p) \cdot W_K(p)$$

ЛЖ

$$20 \lg |W_C(j\omega)| = 20 \lg |W_H(j\omega)| + 20 \lg |W_K(j\omega)|$$

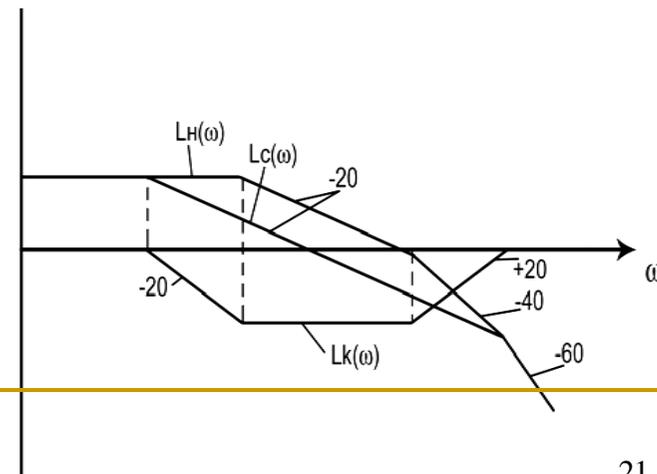
$$L_C(\omega) = L_H(\omega) + L_K(\omega)$$

$$L_K(\omega) = L_C(\omega) - L_H(\omega)$$

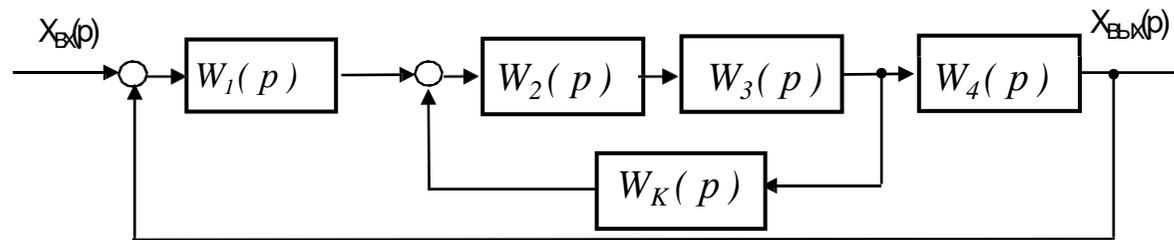
$$K(\quad) = C(\quad) - H(\quad)$$

Средства
формы
Средства
формы
Средства
формы

$L_C(\omega)$



Выбор параллельных корректирующих устройств в виде обратных связей



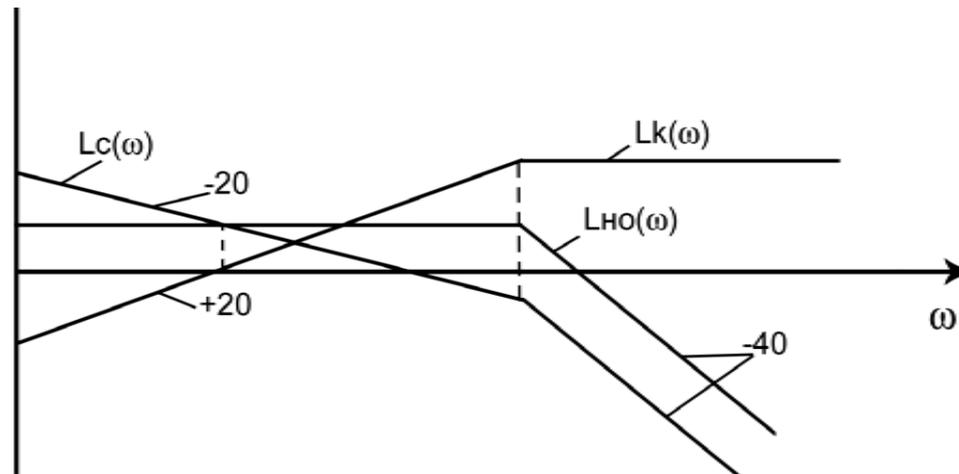
$$W_C(p) = \frac{W_{HO}(p) \cdot W_{OXB}(p)}{1 + W_K(p) W_{OXB}(p)}$$

$$W'_C(j\omega) = \frac{W_{HO}(j\omega)}{W_K(j\omega)}$$

$$20 \lg |W'_C(j\omega)| = 20 \lg |W_{HO}(j\omega)| - 20 \lg |W_K(j\omega)|$$

$$W_C(j\omega) = W_{HO}(j\omega) \cdot W_{OXB}(j\omega) = W_H(j\omega)$$

Порядок выбора параллельного корректирующего звена



$$W_C(j\omega) = \frac{W_{HO}(j\omega)}{W_K(j\omega)} \cdot \frac{W_K(j\omega) \cdot W_{OXB}(j\omega)}{1 + W_K(j\omega) \cdot W_{OXB}(j\omega)}$$

$$\left. \begin{aligned} L_C(\omega) &= L_{HO}(\omega) - L_K(\omega) + \Delta L(\omega) \\ \varphi_C(\omega) &= \varphi_{HO}(\omega) - \varphi_K(\omega) + \Delta\varphi(\omega) \end{aligned} \right\}$$

$$\Delta L(\omega) = 20 \lg \left| \frac{W_{OXB}(j\omega) \cdot W_K(j\omega)}{1 + W_{OXB}(j\omega) \cdot W_K(j\omega)} \right|$$

$$\Delta \varphi(\omega) = \arg \left| \frac{W_{OXB}(j\omega) \cdot W_K(j\omega)}{1 + W_{OXB}(j\omega) \cdot W_K(j\omega)} \right|$$

$$L_{BK}(\omega) = L_{OXB}(\omega) + L_K(\omega)$$

$$\varphi_{RK}(\omega) = \varphi_{OXR}(\omega) + \varphi_K(\omega)$$

Построение желаемой логарифмической амплитудно-частотной характеристики скорректированной системы

Задана

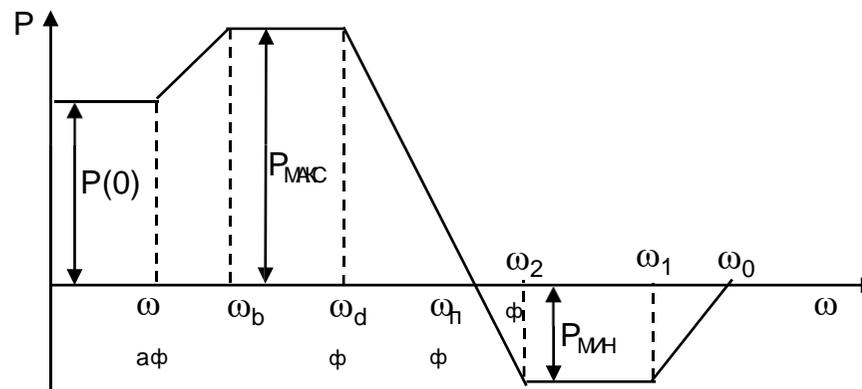
1. Величина



0%

2. Время t_p с

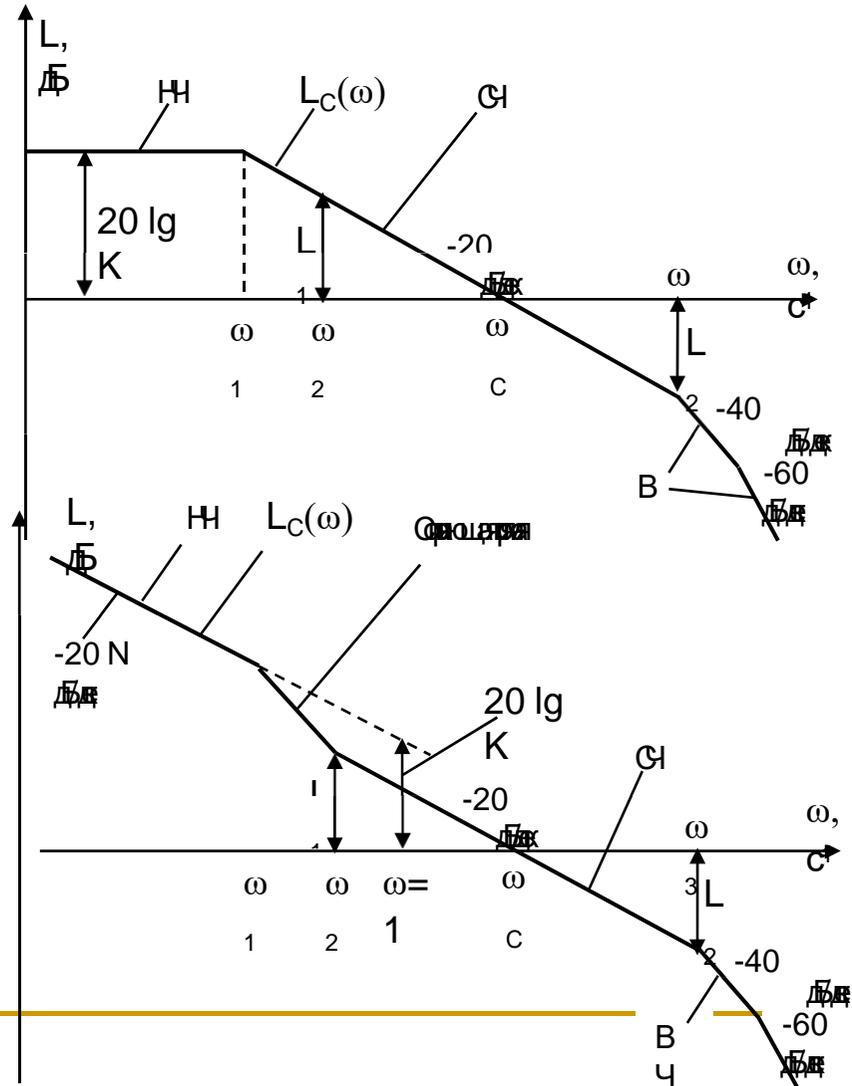
3. Коэффициент K



Типовые ЛАЧХ для статической астатической систем

$$W_D(p) = \frac{K \prod_{i=1}^m W_i(p)}{\prod_{j=1}^n W_j(p)}$$

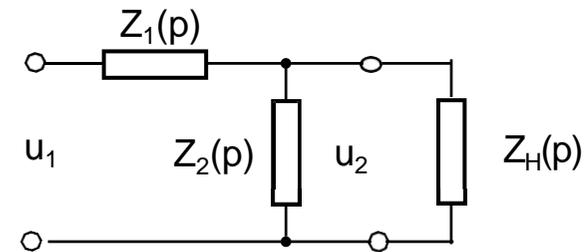
$$W_P(p) = \frac{K \prod_{i=1}^m W_i(p)}{P^N \prod_{j=1}^n W_j(p)}$$



Корректирующие звенья

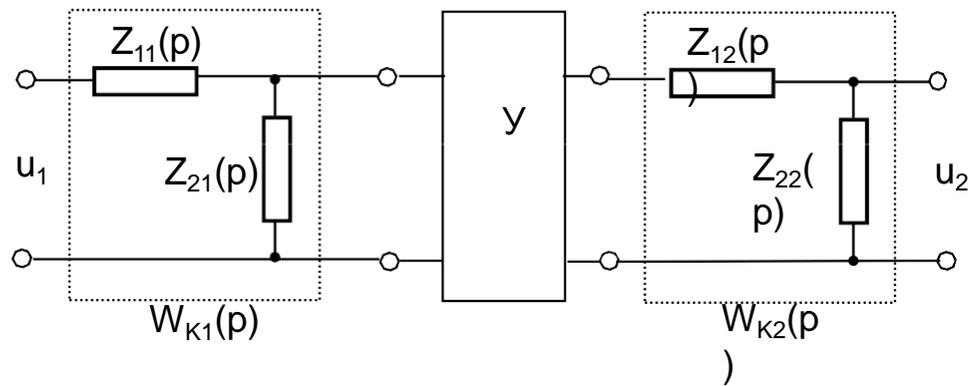
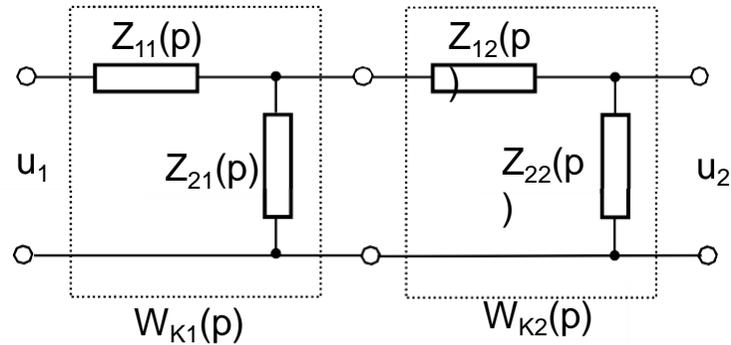
Пассивные четырехполюсники

$$W(p) = \frac{U_2(p)}{U_1(p)} = \frac{Z_2(p)}{Z_1(p) + Z_2(p)}$$



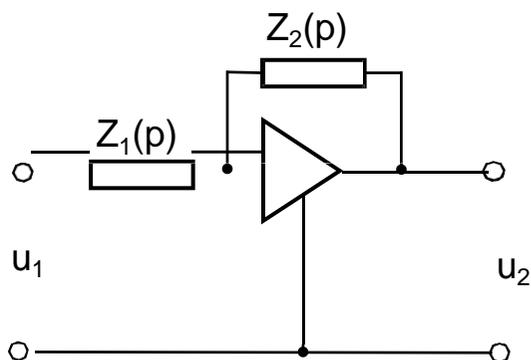
Дифференцирующие
интегрирующие
интегро-дифференцирующие

Реализация сложных корректирующих звеньев



$$W_{\Sigma}(p) = W_{K1}(p) \cdot K_Y \cdot W_{K2}(p)$$

Активные четырехполюсники постоянного тока



$$W(p) = \frac{U_2(p)}{U_1(p)} = -\frac{Z_2(p)}{Z_1(p)}$$

Активные двухполюсники постоянного тока
в операторной форме
в операторной форме

ПОВЫШЕНИЕ ТОЧНОСТИ САУ

Общие методы.

~~Устойчивость~~

~~Выход~~

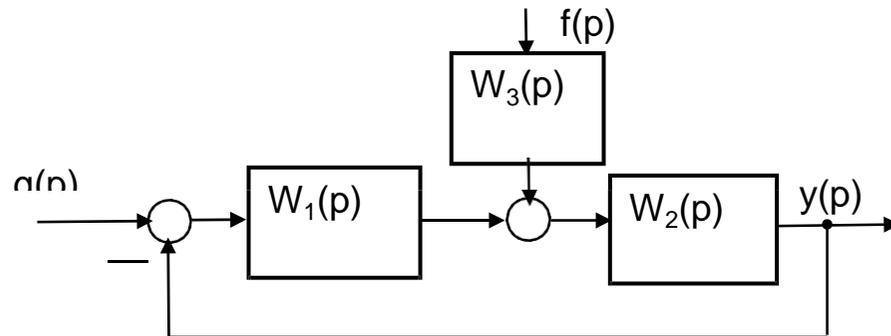
~~Приведение к нормальному виду~~

~~Приведение к 2-х тв~~

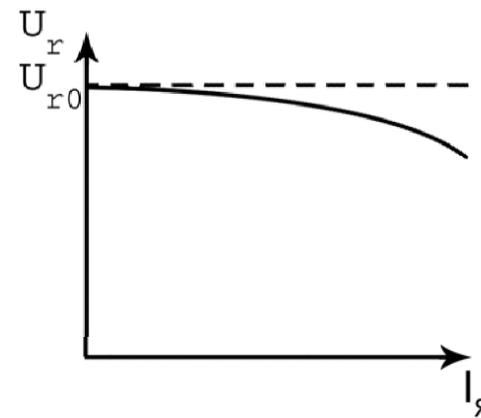
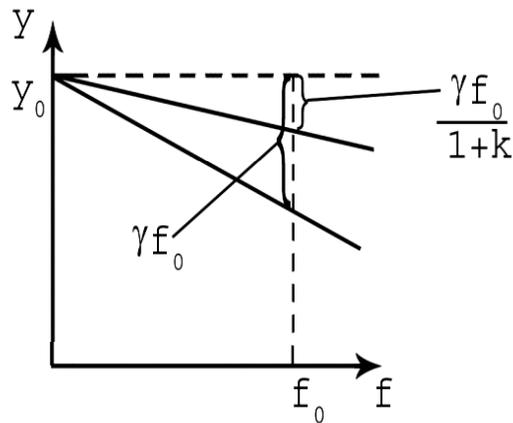
— ~~Применение метода Рунге-Кутты~~

— ~~Применение метода Эйлера~~

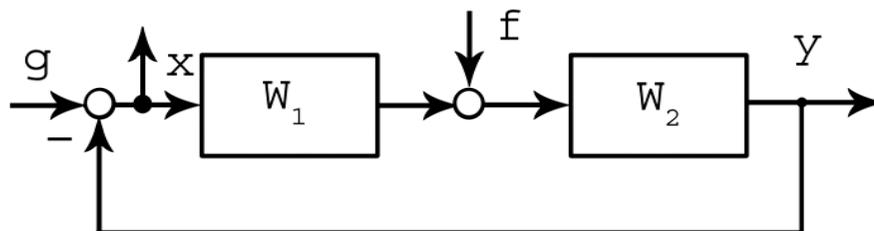
Увеличение коэффициента усиления разомкнутой системы



$$X(p) = \frac{1}{1 + W_P(p)} g(p) + \frac{W_{f0}(p)}{1 + W_P(p)} f(p)$$



Повышение степени астатизма



$$W_2(p) = \frac{W_0}{p} = \frac{1}{p} \cdot \frac{K}{T^2 p^2 + 2\xi T p + 1}$$

$$W_1(p) = \frac{K_u}{p}$$

МАТЕМАТИЧЕСКОЕ ОПИСАНИЕ САУ В ПРОСТРАНСТВЕ СОСТОЯНИЙ

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t), \end{cases}$$

$$x(t) = \begin{bmatrix} x_1(t) \\ \cdot \\ \cdot \\ x_n(t) \end{bmatrix} \quad u(t) = \begin{bmatrix} u_1(t) \\ \cdot \\ \cdot \\ u_k(t) \end{bmatrix} \quad y(t) = \begin{bmatrix} y_1(t) \\ \cdot \\ \cdot \\ y_m(t) \end{bmatrix}$$

Составление уравнений состояния по известной передаточной функции

$$W(p) = \frac{Y(p)}{U(p)} = \frac{b_0 p^m + b_1 p^{m-1} + \dots + b_m}{a_0 p^n + a_1 p^{n-1} + \dots + a_n}$$
$$\frac{Y(p)}{(b_0 p^m + b_1 p^{m-1} + \dots + b_m)} = \frac{U(p)}{(a_0 p^n + a_1 p^{n-1} + \dots + a_n)} = X(p).$$

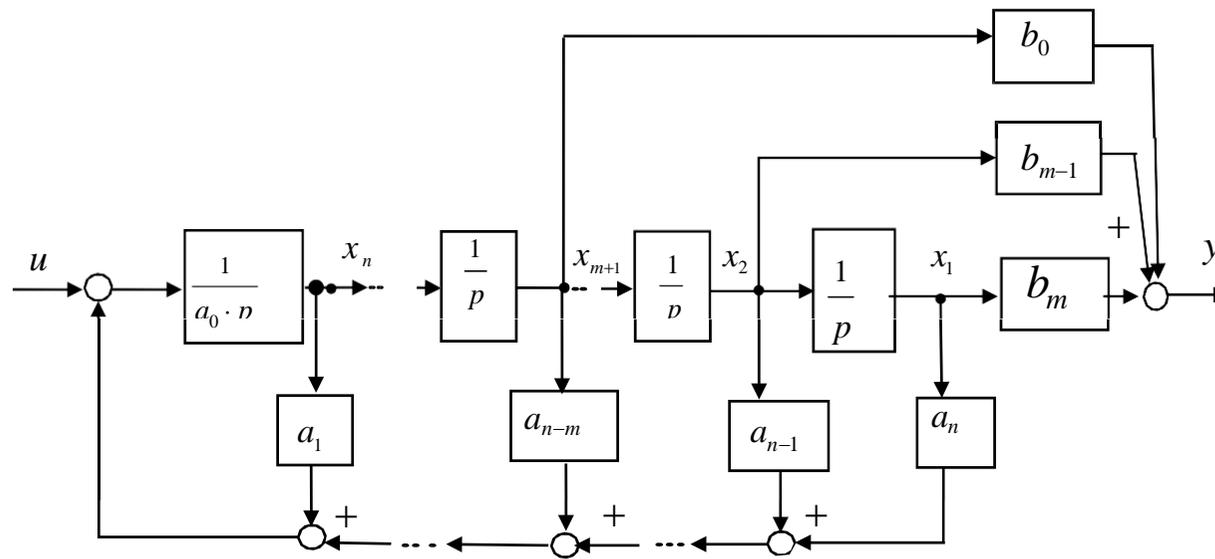
$$\begin{cases} (a_0 p^n + a_1 p^{n-1} + \dots + a_n) X(p) = U(p) \\ (b_0 p^m + b_1 p^{m-1} + \dots + b_m) X(p) = Y(p) \end{cases}$$

$$Y(p) = b_0 p^m X(p) + b_1 p^{m-1} X(p) + \dots + b_m X(p)$$

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{pmatrix} 0 & 1 & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & 1 \\ a & a & \dots & -a_1 \\ -\frac{n}{a_0} & -\frac{n-1}{a_0} & \dots & -\frac{1}{a_0} \end{pmatrix} \times \begin{bmatrix} x_1 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ \cdot \\ 0 \\ \frac{1}{a_0} \end{bmatrix} \times U$$

$$y = (b_m \quad \cdot \quad b_0 \quad 0 \quad \cdot \quad 0) \times \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ x_{n-1} \\ x_n \end{bmatrix}$$

$$\begin{cases} \dot{x}(t) = A x(t) + B u(t), \\ y(t) = C x(t). \end{cases}$$

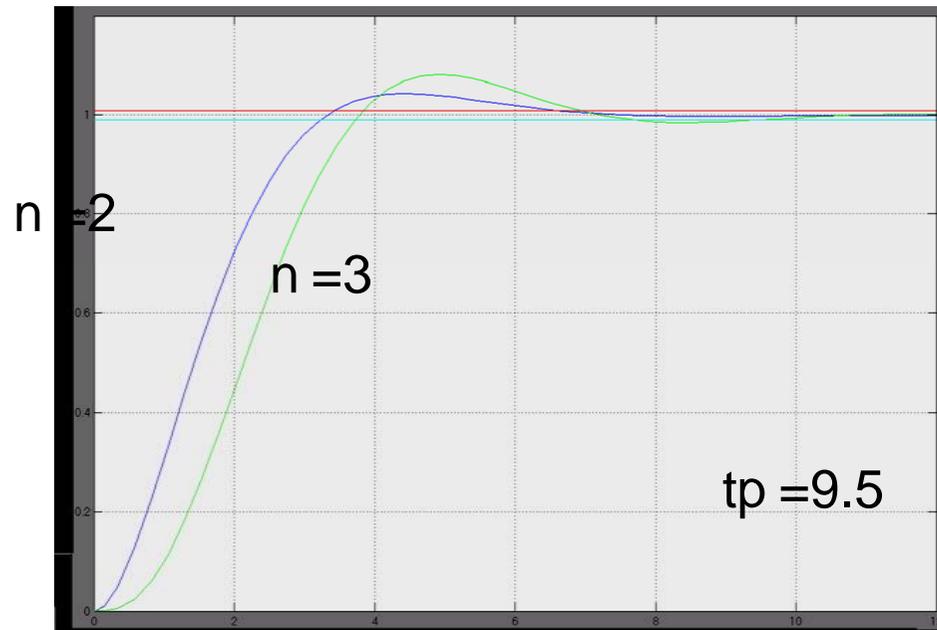


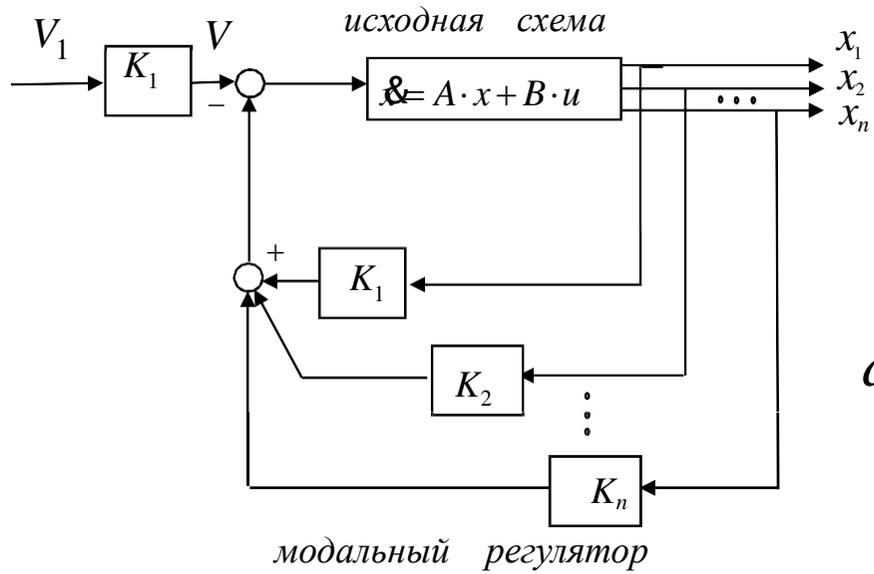
СИНТЕЗ МОДАЛЬНЫХ РЕГУЛЯТОРОВ

Фазовая

Билинейная

$$\begin{array}{l} n = 2 \\ n = 3 \end{array} \left| \begin{array}{l} p^2 + \sqrt{2}\omega_0 p + \omega_0^2 \\ p^3 + 2\omega_0 p^2 + 2\omega_0^2 p + \omega_0^3 \end{array} \right| \begin{array}{l} p^2 + 2\omega_0 p + \omega_0^2 \\ p^3 + 3\omega_0 p^2 + 3\omega_0^2 p + \omega_0^3 \end{array}$$





$$\dot{x} = Ax + B(v - Kx) = (A - BK)x + Bv$$

$$A = A - BK.$$

$$\det(pI - \overset{\circ}{A}) = D_{жс}(p)$$

УПРАВЛЯЕМОСТЬ И НАБЛЮДАЕМОСТЬ САУ

Управляемость САУ

$$P = (B, AB, A^2B, \dots, A^{n-1}B)$$

$$\text{rank } P = n$$

Наблюдаемость САУ

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t)$$

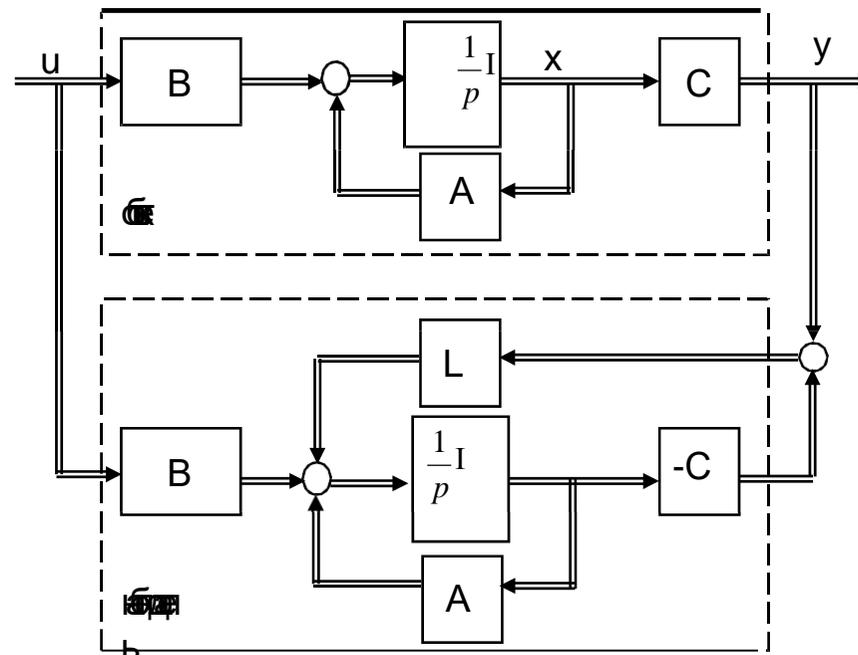
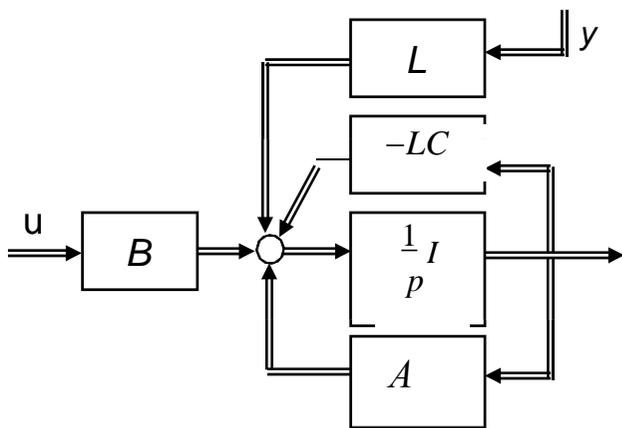
$$\text{rank } Q = n$$

$$Q = [C^T, A^T C^T, (A^T)^2 C^T, \dots, (A^T)^{n-1} C^T]$$

ИДЕНТИФИКАЦИЯ ВЕКТОРА СОСТОЯНИЯ СИСТЕМЫ

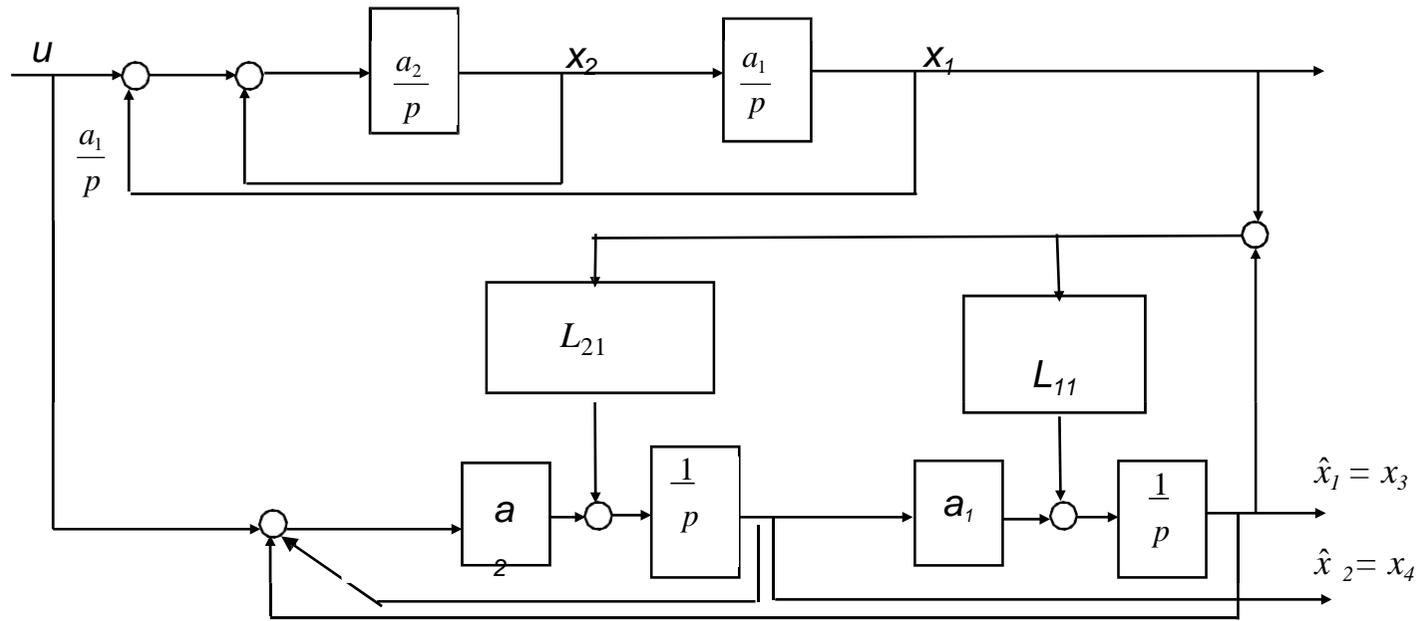
Принцип построения

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad \begin{cases} \dot{\hat{x}} = A\hat{x} + Bu \\ \hat{y} = C\hat{x} \end{cases}$$



$$\dot{\hat{x}} - \dot{x} = (A - LC)(\hat{x} - x)$$

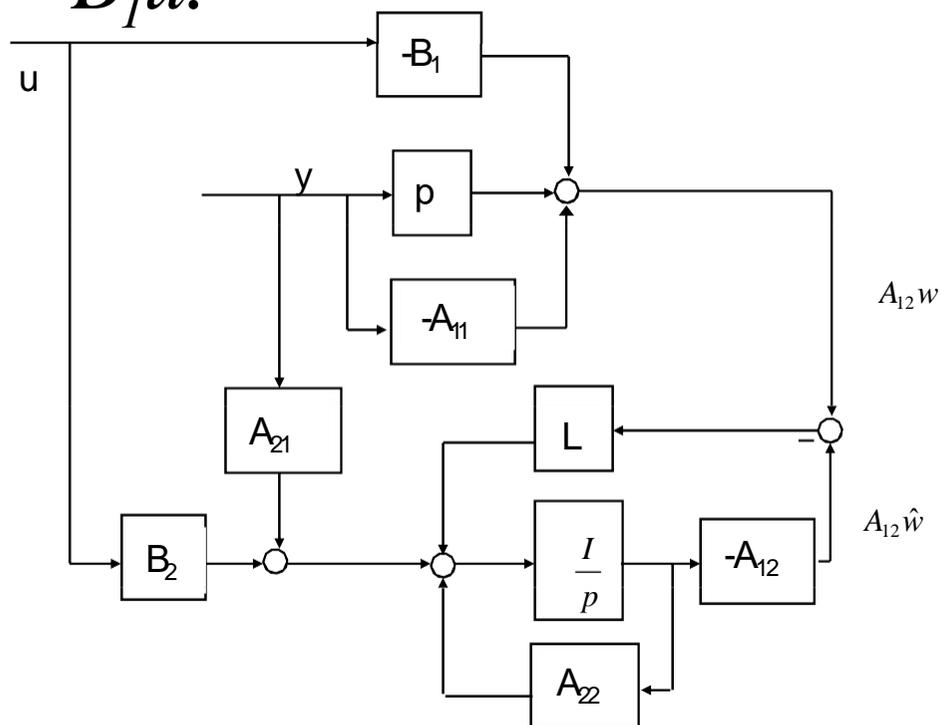
Объект

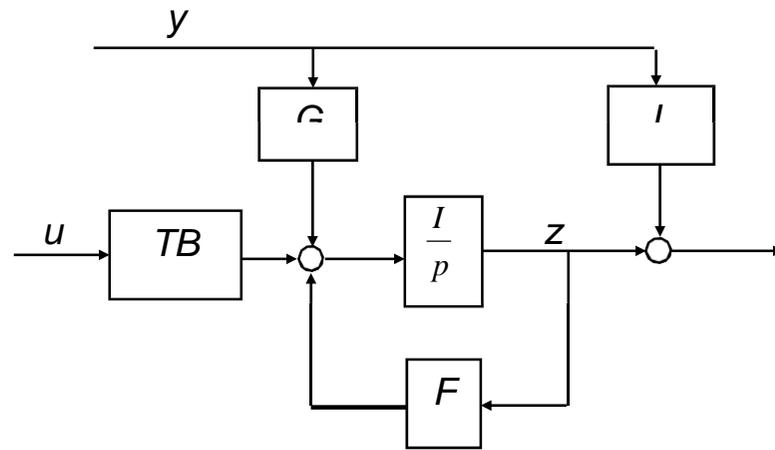
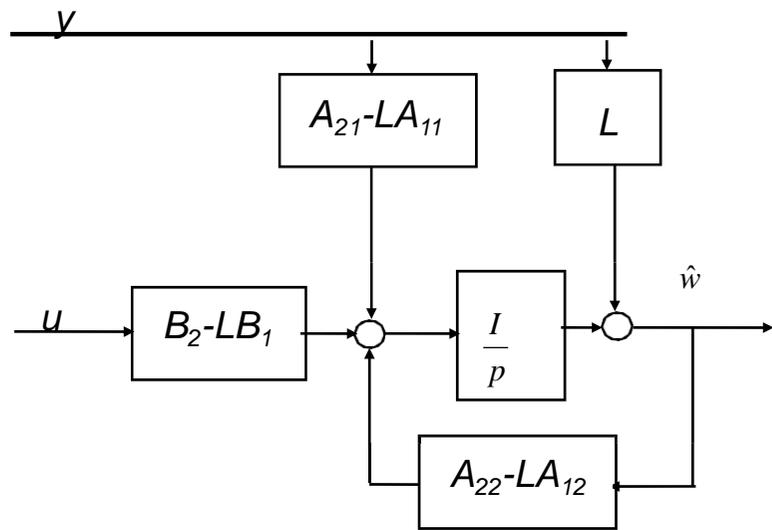


Редуцированный наблюдатель

$$x = \begin{cases} y & \} m \\ w & \} n - m \end{cases} \quad \left. \begin{aligned} \dot{y} &= A_{11}y + A_{12}w + B_1u \\ \dot{w} &= A_{21}y + A_{22}w + B_2u \end{aligned} \right\}$$

$$A_{12}w = \dot{y} - A_{11}y - B_1u.$$



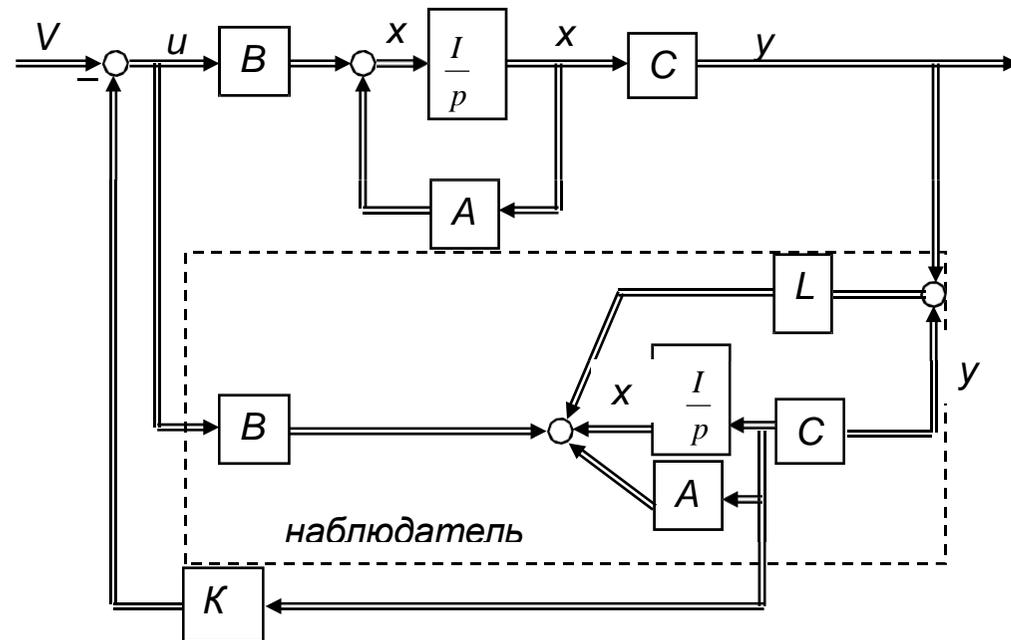


ОСОБЕННОСТИ ДИНАМИКИ СИСТЕМЫ, ЗАМКНУТОЙ ЧЕРЕЗ НАБЛЮДАТЕЛЬ

$$\dot{x} = Ax + Bu$$

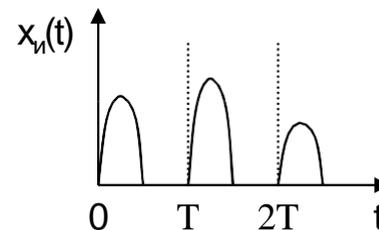
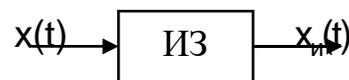
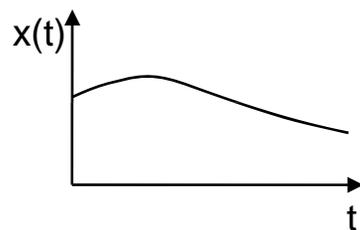
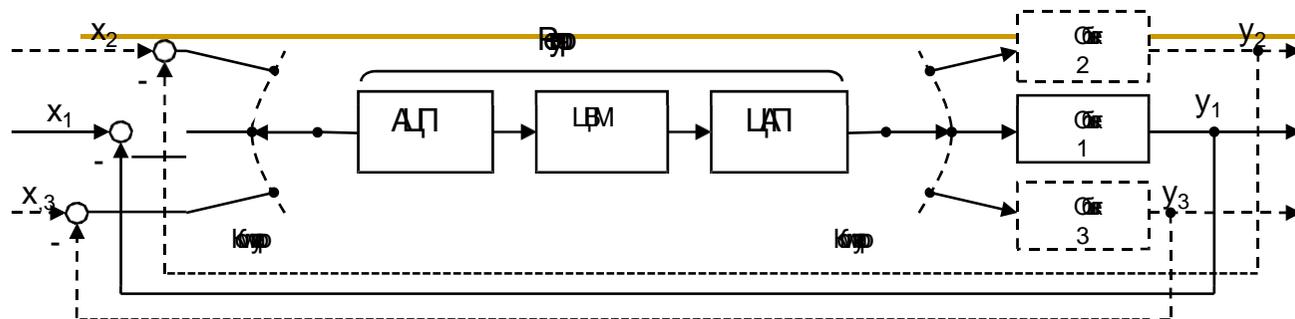
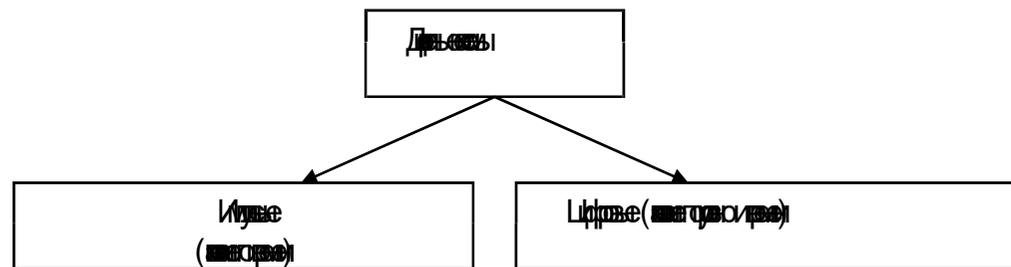
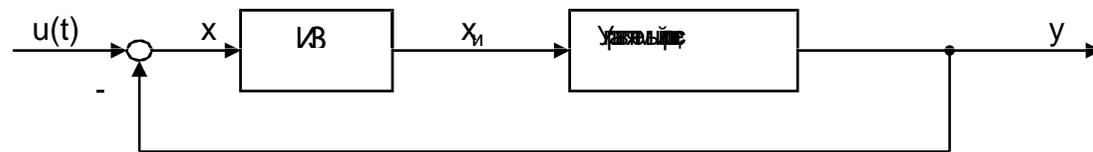
$$\dot{\hat{x}} = (A - LC)\hat{x} + Bu + Ly$$

$$u = V - K\hat{x}$$



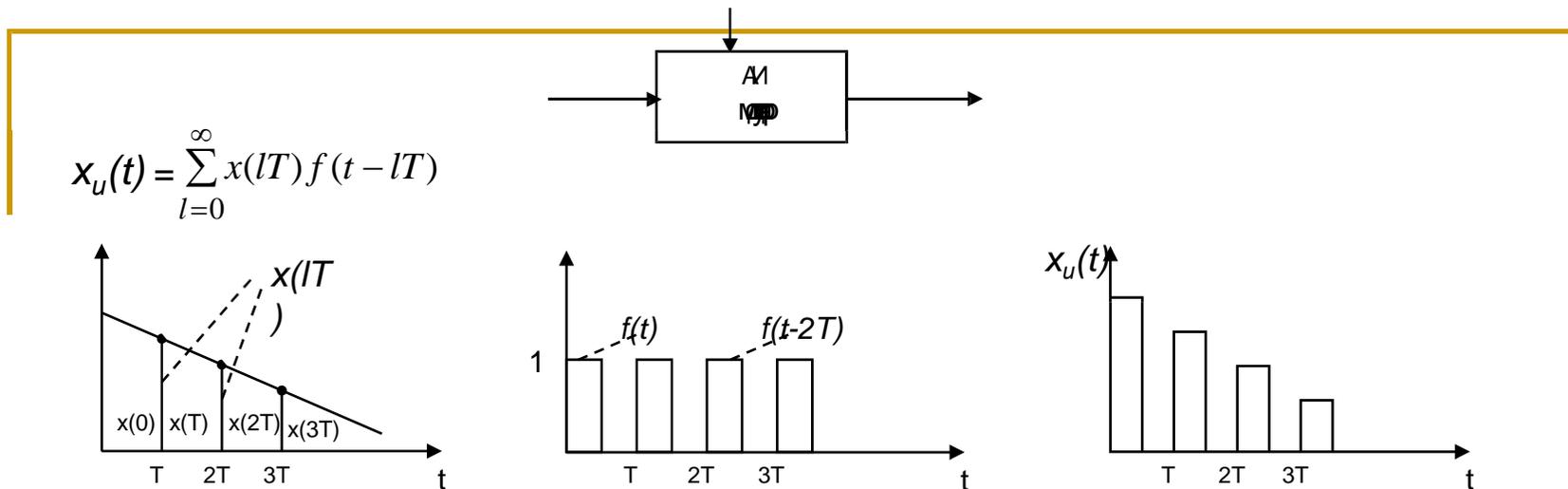
ДИСКРЕТНЫЕ СИСТЕМЫ АВТОМАТИЧЕСКОГО УПРАВЛЕНИЯ

ЛИНЕЙНЫЕ ИМПУЛЬСНЫЕ СИСТЕМЫ



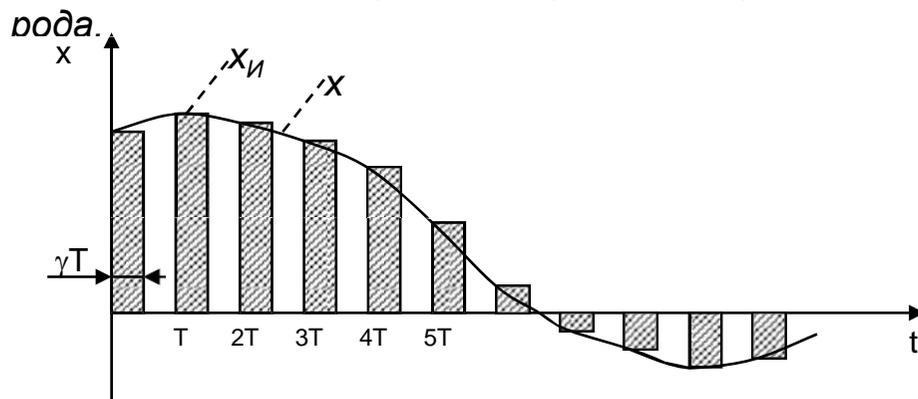
Виды модуляции сигналов

Общее понятие амплитудной модуляции сигналов



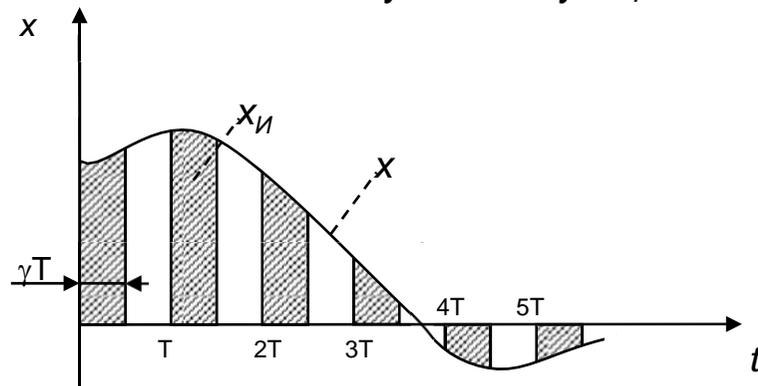
16.2.2. ИЗ с амплитудной и широтной модуляцией

1. ИЗ с амплитудной модуляцией первого



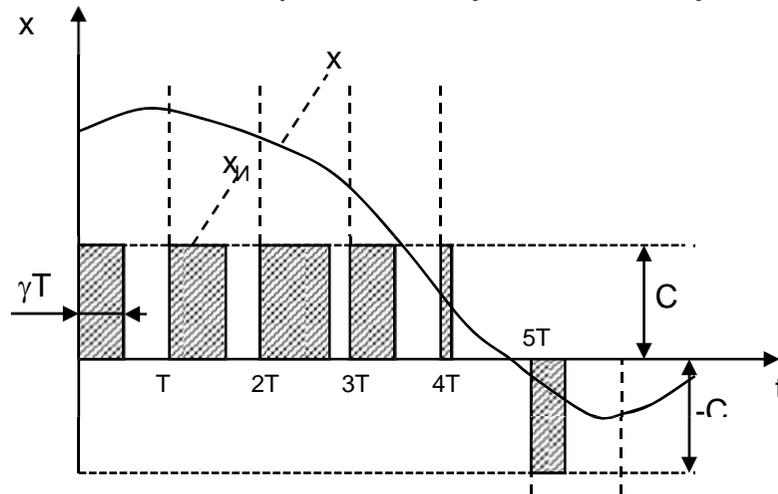
$$\begin{cases} x_{И} = kx(lT) & \text{при } lT \leq t \leq (lT + \gamma T) \\ x_{И} = 0 & \text{при } (lT + \gamma T) < t < (l + 1)T \end{cases}$$

2. ИЗ с амплитудной модуляцией второго рода.



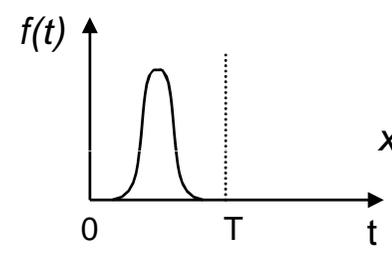
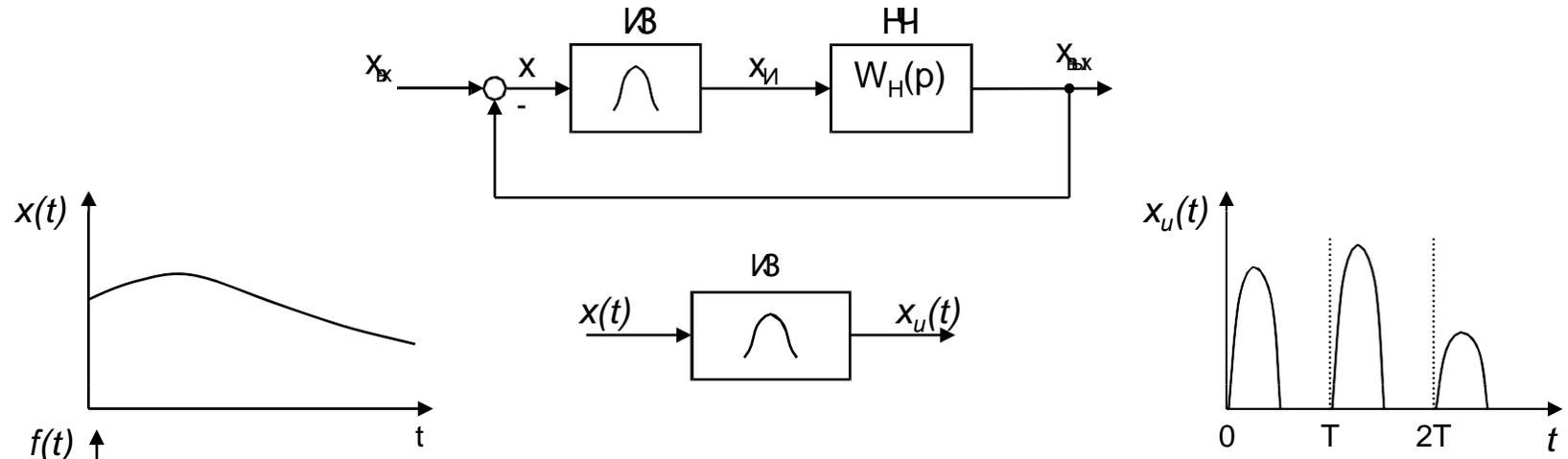
$$\begin{cases} x_{И} = kx(t) \text{ при } lT \leq t \leq (l+1)T \\ x_{И} = 0 \text{ при } (lT + \gamma T) < t < (l+1)T \end{cases}$$

3. ИЗ с широтно-импульсной модуляцией (ШИМ).

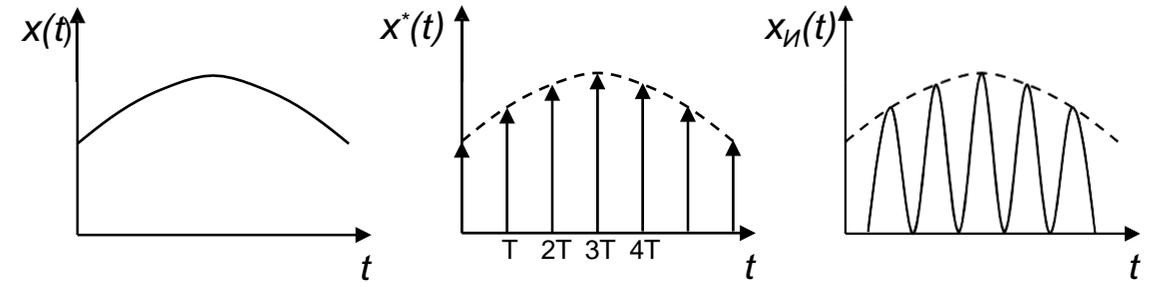
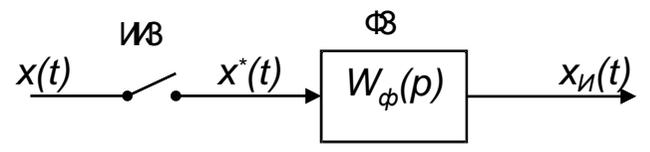


$$\begin{cases} x_{И} = C \text{ sign } x[lT] \text{ при } lT \leq t \leq (lT + \gamma T) \\ x_{И} = 0 \text{ при } (lT + \gamma T) < t < (l+1)T \end{cases}$$

16.3. Эквивалентная схема импульсной системы



$$x_u(t) = \sum_{l=0}^{\infty} x(lT) f(t - lT)$$



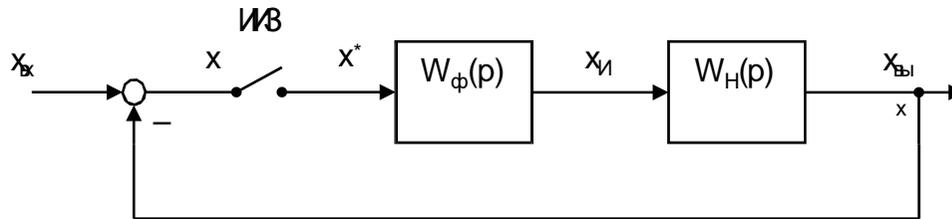
$$x^*(t) = \sum_{l=0}^{\infty} x(lT) \delta(t - lT)$$

$$w_{\phi}(t) = f(t)$$

$$W_{\phi}(p) = F(p)$$

$$W_{\phi}(p) = \int_0^{\infty} w_{\phi}(t) e^{-pt} dt = \int_0^{\infty} f(t) e^{-pt} dt$$

$$x(t) = \sum_{l=0}^{\infty} x(lT) w_{\phi}(t - lT)$$



$$W_{\phi H}(p) = W_{\phi}(p) W_H(p)$$

16.4. Применение преобразований Фурье и Лапласа для дискретных сигналов

$$\begin{array}{c} x \\ \bullet \\ \text{ИВ} \\ \bullet \\ x^* \end{array} \quad x^*(t) = \sum_{l=0}^{\infty} x(lT) \delta(t - lT)$$

$$X^*(p) = \int_0^{\infty} x^*(t) e^{-pt} dt$$

$$X^*(p) = \sum_{l=0}^{\infty} \int_0^{\infty} x(lT) \delta(t - lT) e^{-pt} dt$$

$$\int_0^{\infty} x(lT) \delta(t - lT) e^{-pt} dt = x(lT) e^{-plT}$$

$$X^*(p) = \sum_{l=0}^{\infty} x(lT) e^{-plT}$$

$$X(z) = \sum_{l=0}^{\infty} x(lT) z^{-l}$$

ОСОБЕННОСТИ СПЕКТРОВ ДИСКРЕТНЫХ СИГНАЛОВ

$$X^*(j\omega) = \int_0^{\infty} x^*(t) e^{-j\omega t} dt$$

$$X^*(j\omega) = \sum_{l=0}^{\infty} x(lT) e^{-j\omega lT}$$

$\underbrace{\quad}_x \quad \underbrace{\quad}_{x^*} \quad x^*(t) = \sum_{l=0}^{\infty} x(lT) \delta(t - lT)$

$$x^*(t) = x(t) \sum_{l=-\infty}^{\infty} \delta(t - lT)$$

$$\sum_{l=-\infty}^{\infty} \delta(t - lT) = \sum_{r=-\infty}^{\infty} C_r e^{jr\omega_u t} = \frac{1}{T} \sum_{r=-\infty}^{\infty} e^{jr\omega_u t}$$

$$C_r = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left[\sum_{l=-\infty}^{\infty} \delta(t - lT) \right] e^{-jr\omega_u t} dt = \frac{1}{T}$$

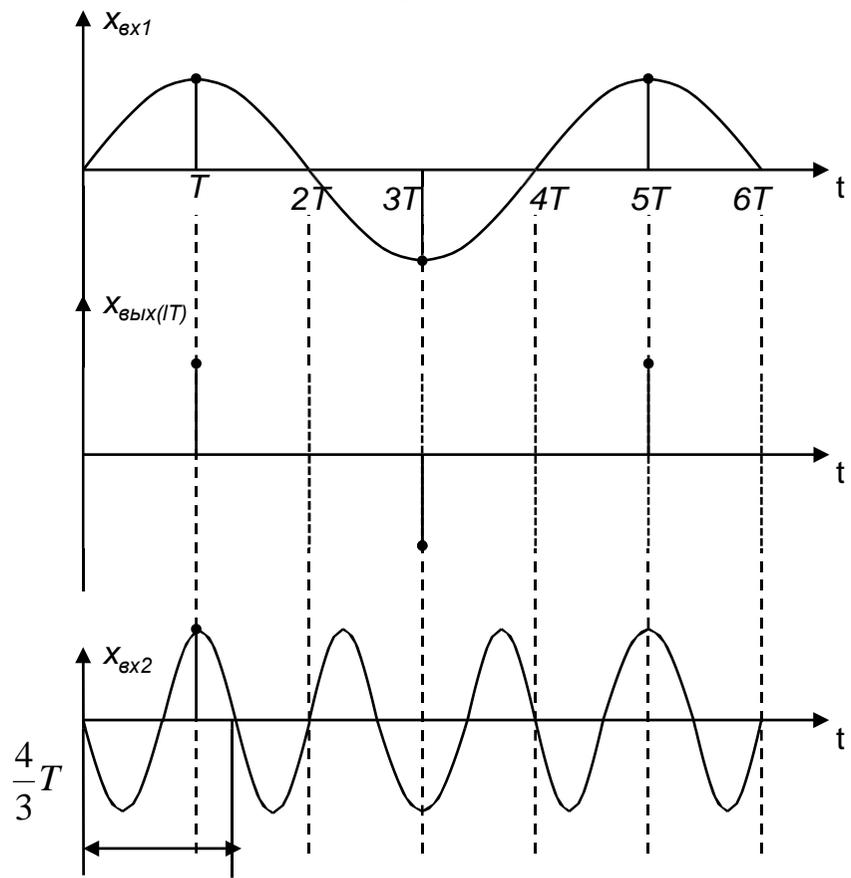
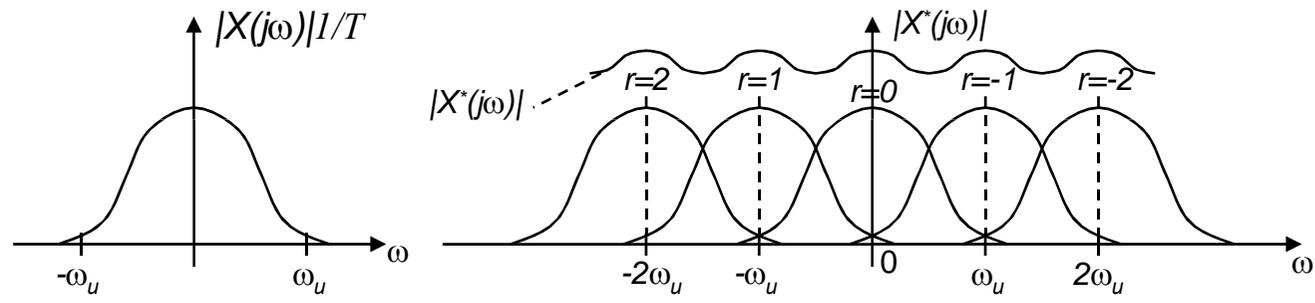
$$x^*(t) = x(t) \frac{1}{T} \sum_{r=-\infty}^{\infty} e^{jr\omega_u t}$$

$$X^*(j\omega) = \frac{1}{T} \int_0^{\infty} \left[x(t) \sum_{r=-\infty}^{\infty} e^{jr\omega_u t} \right] e^{-j\omega t} dt = \frac{1}{T} \sum_{r=-\infty}^{\infty} \int_0^{\infty} x(t) e^{-j(\omega - r\omega_u)t} dt$$

$$\int_0^{\infty} x(t) e^{-j(\omega - r\omega_u)t} dt = X[j(\omega - r\omega_u)]$$

$$X^*(j\omega) = \frac{1}{T} \sum_{r=-\infty}^{\infty} X[j(\omega - r\omega_u)]$$

$$X^*(j\omega) = \frac{1}{T} \sum_{r=-\infty}^{\infty} X[j(\omega + r\omega_u)]$$

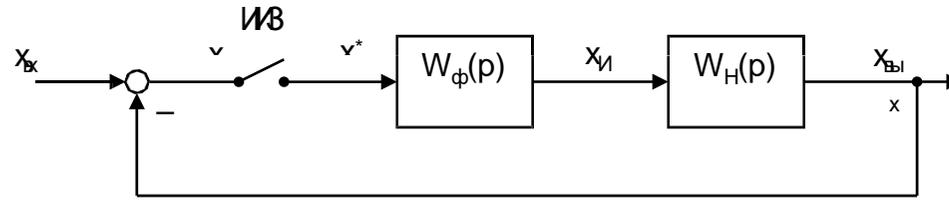


$$\omega_1 = \frac{\pi}{2T} = \frac{2\pi}{4T}$$

$$\omega_2 = \frac{3\pi}{2T} = \frac{2\pi}{\frac{4}{3}T}$$

$$X^*(p) = \frac{1}{T} \sum_{r=-\infty}^{\infty} X(p + jr\omega_u)$$

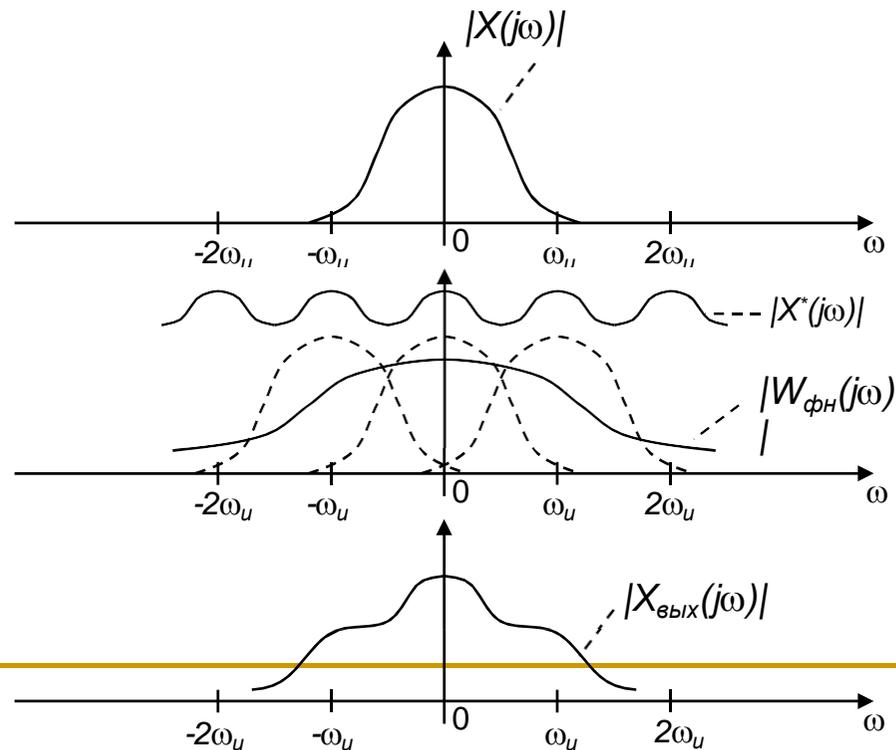
Прохождение сигналов через импульсную САУ



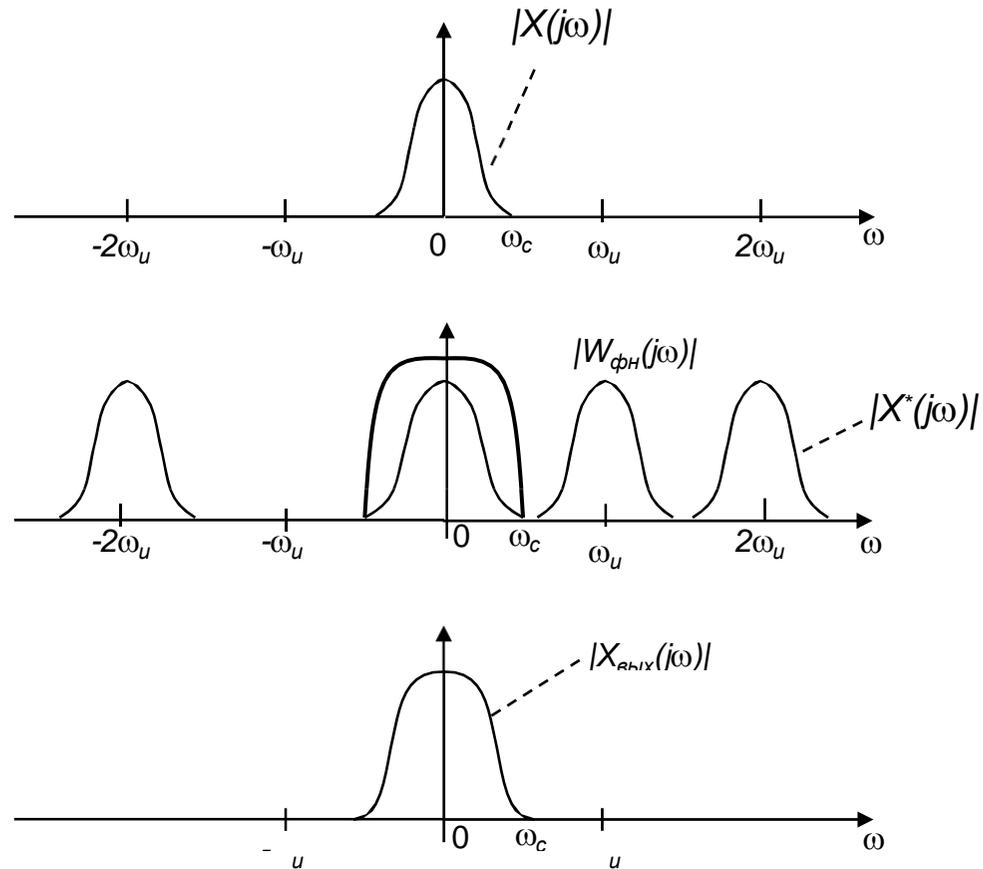
$$X_{\text{ВыХ}}(j) = W_{\text{ФН}}(j) \cdot X^*(j)$$

$$X_{\text{ВыХ}}(j\omega) = W_{\text{ФН}}(j\omega) \cdot \frac{1}{T} \sum_{r=-\infty}^{\infty} X[j(\omega + r\omega_u)]$$

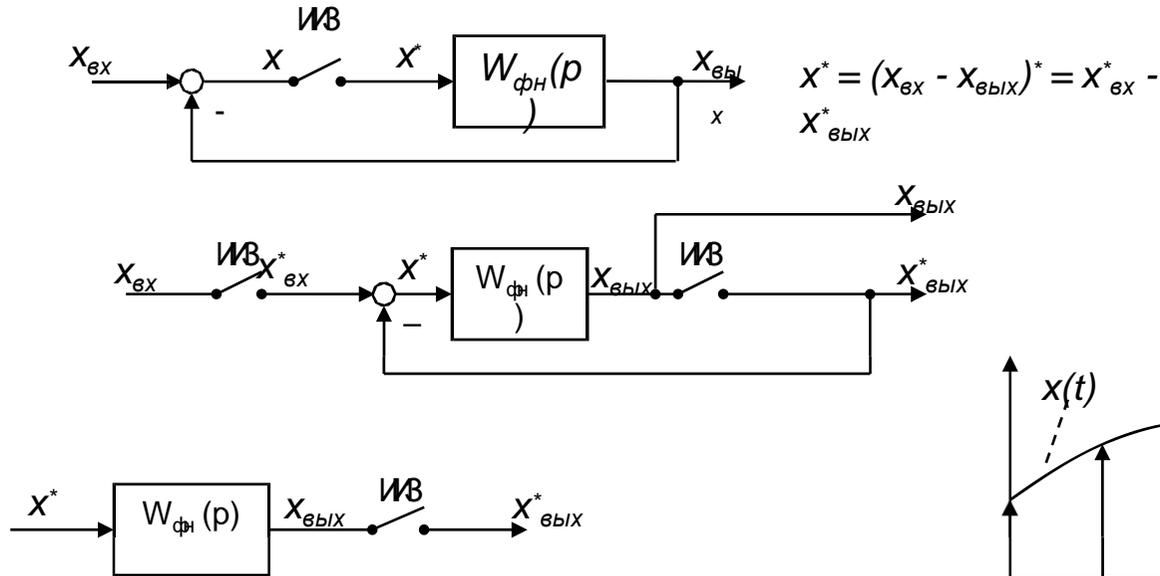
$$X_{\text{ВыХ}}(j) = \frac{1}{T} W_{\text{ФН}}(j\omega) X(j\omega) + \frac{1}{T} W_{\text{ФН}}(j\omega) \cdot \sum_{\substack{r=-\infty \\ r \neq 0}}^{r=\infty} X[j(\omega + r\omega_u)]$$



Прохождение сигналов через ИСАУ при выполнении условий теоремы Котельникова



Передаточная функция и частотная передаточная функция разомкнутой импульсной системы



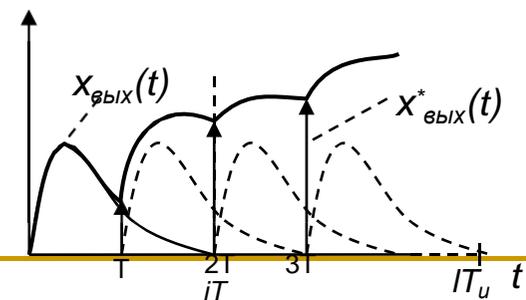
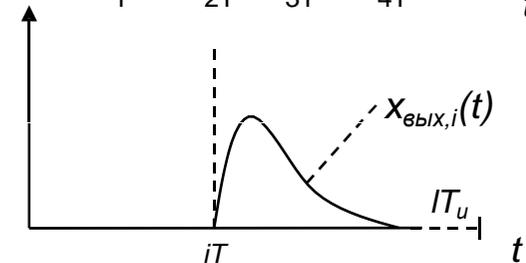
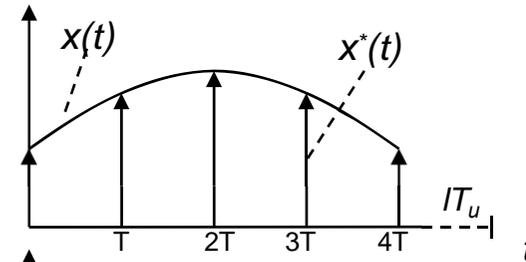
$$x_{вых,i}(t) = x(iT) w_{\phi H}(t - iT)$$

$$X_{вых}(t) = \sum_{i=0}^l x(iT) w_{\phi H}(t - iT)$$

$$X^*(t) = \sum_{i=0}^{\infty} x(iT) \delta(t - iT)$$

$$X_{вых}(t) = \sum_{i=0}^{\infty} x(iT) w_{\phi H}(t - iT)$$

$$X_{вых}(iT) = \sum_{i=0}^l x(iT) w_{\phi H}((l - i)T)$$



$$X_{\text{ввх}}^*(p) = \sum_{l=0}^{\infty} x_{\text{ввх}}(lT) e^{-plT} = \sum_{l=0}^{\infty} \sum_{i=0}^{\infty} x(iT) w_{\phi_n}((l-i)T) e^{-plT}$$

$$X_{\text{ввх}}^*(p) = \sum_{k=-i}^{\infty} \sum_{i=0}^{\infty} x(iT) w_{\phi_n}(kT) \cdot e^{-pkT} e^{-piT}$$

$$X_{\text{ввх}}^*(p) = \underbrace{\sum_{k=0}^{\infty} w_{\phi_n}(kT) e^{-pkT}}_{W_p^*(p)} \underbrace{\sum_{i=0}^{\infty} x(iT) e^{-piT}}_{X^*(p)}$$

$$W_p^*(p) = \frac{X_{\text{ввх}}^*(p)}{X^*(p)} = \sum_{k=0}^{\infty} w_{\phi_n}(kT) e^{-pkT}$$

$$X_{\text{ввх}}^*(j\omega) = W_p^*(j\omega) \cdot X^*(j\omega)$$

$$W_p^*(j\omega) = \frac{1}{T} \sum_{r=-\infty}^{\infty} W_{\phi_n} [j(\omega + r\omega_u)]$$

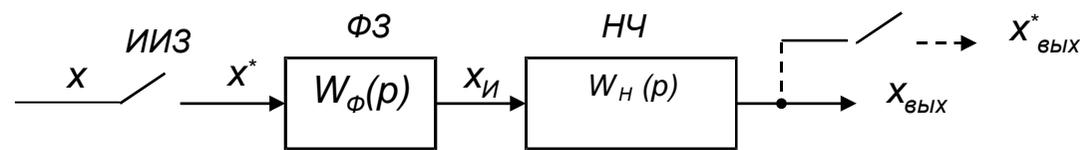
Определение передаточной функции разомкнутой системы по передаточной функции непрерывной части

$$W_p^*(p) = \sum_{l=0}^{\infty} w_{\phi_n}(lT) e^{-plT}$$

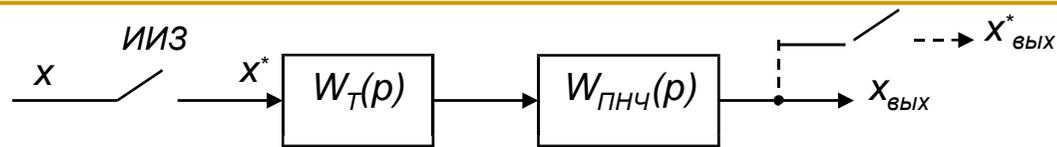
$$W_p^*(j\omega) = \sum_{l=0}^{\infty} w_{\phi_n}(lT) e^{-j\omega lT}$$

$$W_p^*(p) = \frac{1}{T} \sum_{r=-\infty}^{\infty} W_{\phi_n}(p + jr\omega_u)$$

$$W_p^*(j\omega) = \frac{1}{T} \sum_{r=-\infty}^{\infty} W_{\phi_n} [j(\omega + r\omega_u)]$$



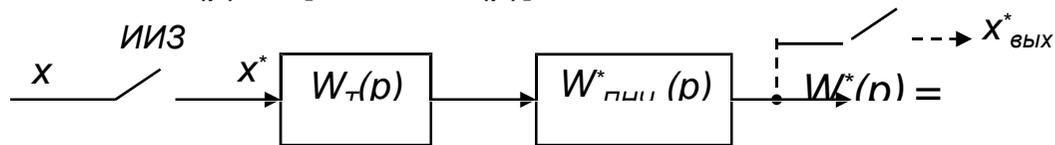
$$W\Phi(p) = W_T(p) \cdot W\Phi_{\text{НЧ}}(p)$$



$$W^*(p) = Z[W(p)]$$

$$W(p) = \frac{B_1(p)}{p(T_1 p + 1)(T_2^2 p^2 + 2\varepsilon T_2 p + 1)} = \frac{A}{p} + \frac{B}{T_1 p + 1} + \frac{C + Dp}{T_2^2 p^2 + 2\varepsilon T_2 p + 1}$$

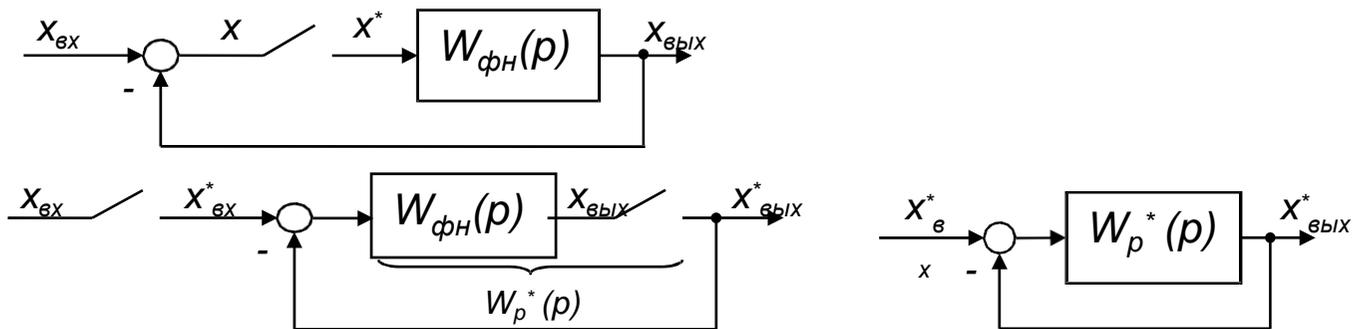
$$W^*_{П.Н.Ч.}(p) = Z[W_{П.Н.Ч.}(p)]$$



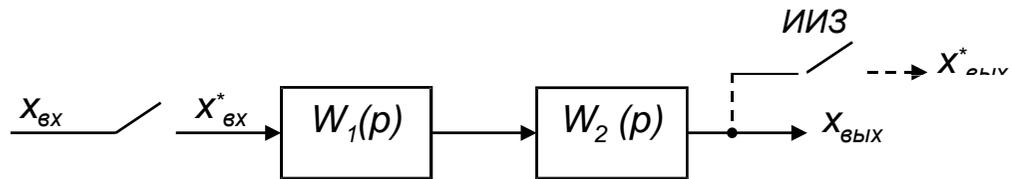
$$W_p^*(p) = W_T(p) \cdot W^*_{П.Н.Ч.}(p) = \frac{X^*_{вых}(p)}{X^*(p)}$$

$$W_p^*(p) = \frac{B^*(p)}{C^*(p)} = \frac{b_0 e^{mpT} + b_1 e^{(m-1)pT} + \dots + b_m}{c_0 e^{npT} + c_1 e^{(n-1)pT} + \dots + c_n}$$

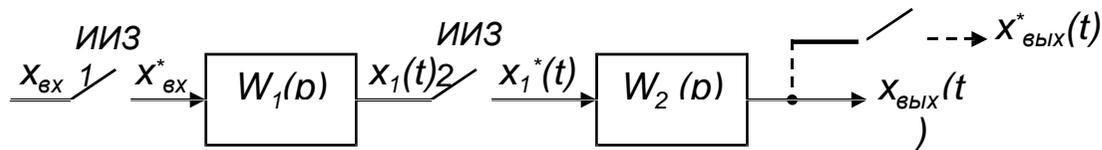
Передаточные функции замкнутых систем



$$\frac{X^*_{вых}(p)}{X^*(p)} = \frac{W_p^*(p)}{1 + W_p^*(p)}$$

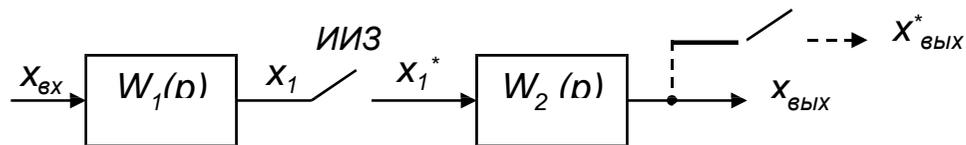


$$X^*_{вых}(p) = X^*_{вх}(p)W1W2^*(p)$$



$$X^*_{вых}(p) = W2^*(p) \cdot W1^*(p) \cdot X^*_{вх}(p) = W^*p(p) \cdot X^*_{вх}(p)$$

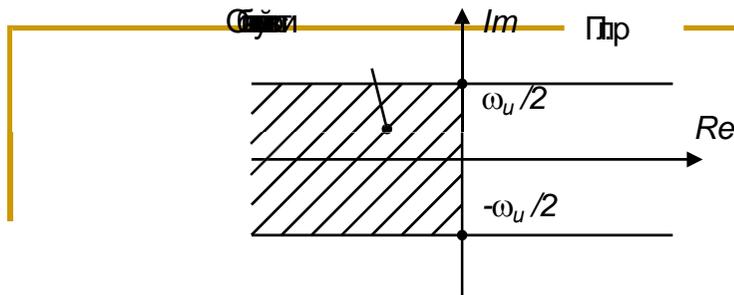
$$W1W2^*(p) \neq W1^*(p) \cdot W2^*(p)$$



$$X^*_{вых}(p) = X_{вх}W1^*(p)$$

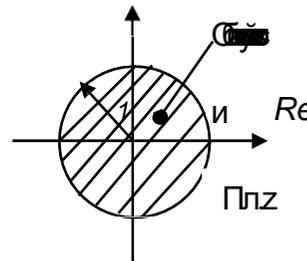
УСТОЙЧИВОСТЬ ИМПУЛЬСНЫХ СИСТЕМ АВТОМАТИЧЕСКОГО УПРАВЛЕНИЯ

Понятие об устойчивости



$$z_i = e^{p_i T} = e^{c_i T} \cdot e^{j\omega_i T} = \alpha_z + j\beta_z$$

$$e^{c_i T} = |z_i| = |z + jz|$$



Критерий устойчивости Гурвица для импульсных систем

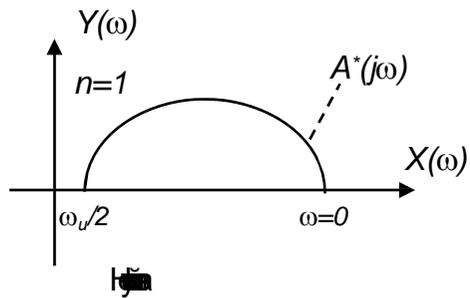
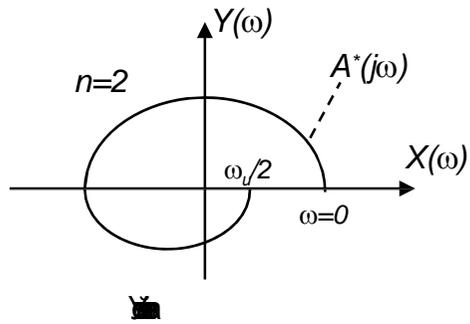
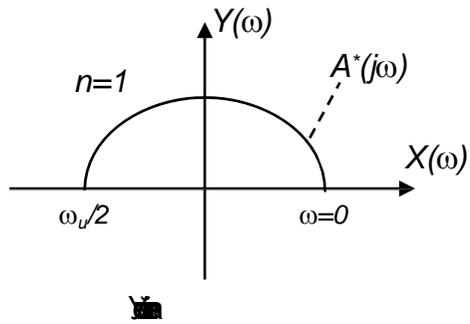
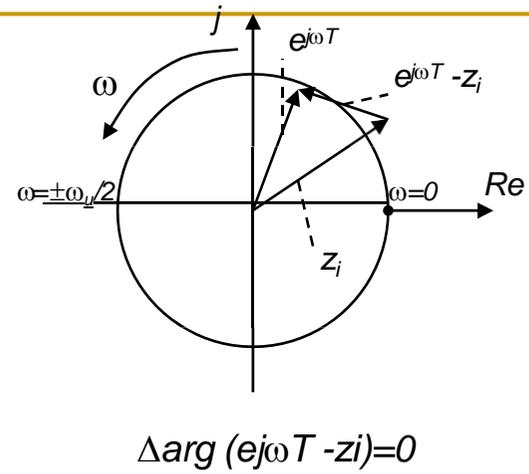
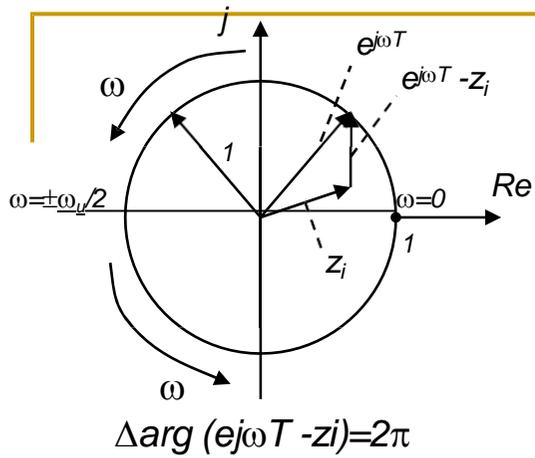
$$A(v) = a_0 \left(\frac{1+v}{1-v} \right)^n + a_1 \left(\frac{1+v}{1-v} \right)^{n-1} + \dots + a_n = 0$$

$$a_0(1+v)^n + a_1(1+v)^{n-1}(1-v) + \dots + a_n(1-v)^n = 0$$

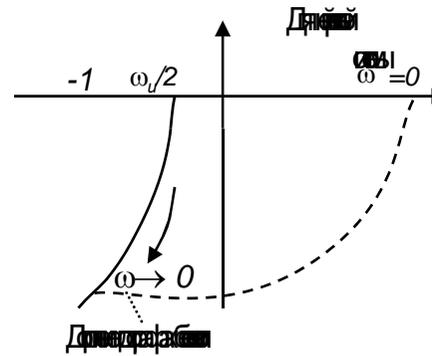
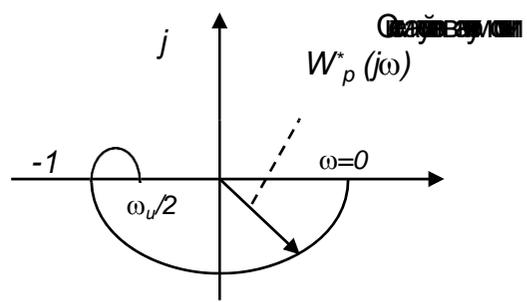
$$a_0' v^n + a_1' v^{n-1} + \dots + a_n' = 0$$

Критерий устойчивости Михайлова для импульсных систем

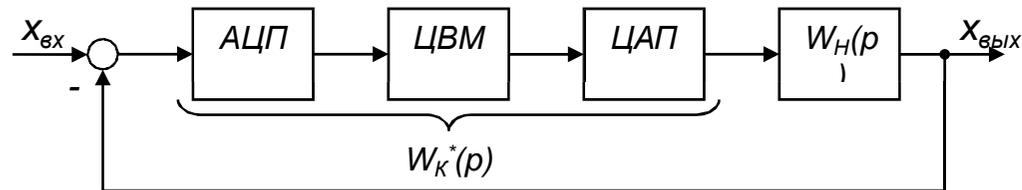
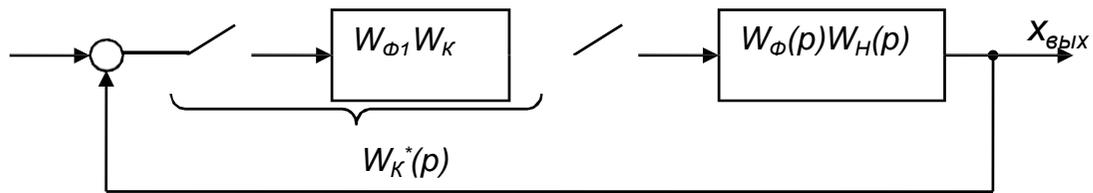
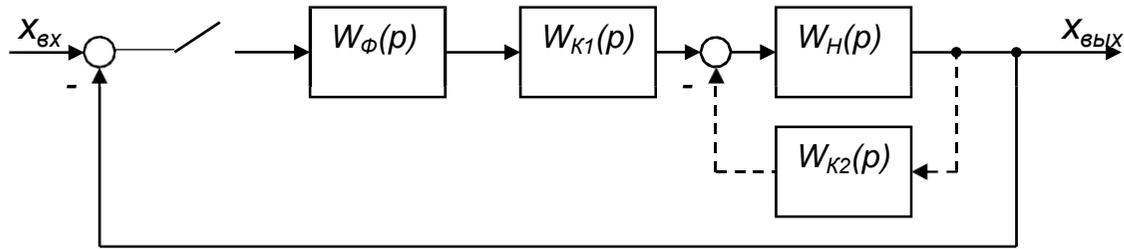
$$A(z) = a_0(z-z_1)(z-z_2)\dots(z-z_n)$$



Критерий устойчивости Найквиста для импульсных систем



КОРРЕКЦИЯ ИМПУЛЬСНЫХ СИСТЕМ



Условия конечной длительности переходного процесса

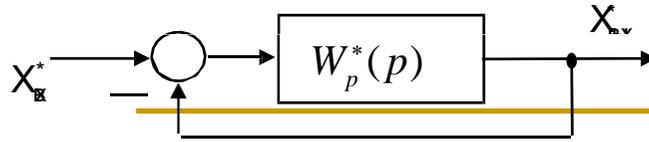
$$W^*(p) = \sum_{l=0}^{\infty} w(lT) e^{-plT}$$

$$W^*(p) = \frac{B^*(p)}{A^*(p)} = \frac{b_0 e^{mpT} + b_1 e^{(m-1)pT} + \dots + b_m}{a_0 e^{npT} + a_1 e^{(n-1)pT} + \dots + a_n}$$

$$W^*(p) = \frac{1}{a_0} \left[b_0 e^{-(n-m)pT} + b_1 e^{-(n-m+1)pT} + \dots + b_m e^{-npT} \right]$$

$$w[(n-m)T] e^{-(n-m)pT} + w[(n-m+1)T] e^{-(n-m+1)pT} + \dots$$

РАСЧЕТ ПЕРЕХОДНЫХ ПРОЦЕССОВ ИМПУЛЬСНЫХ САУ



$$W_p^*(p) = \frac{W_p^*(p)}{1 + W_p^*(p)}$$

$$X_{\text{вых}}^*(p) = W_p^*(p) \cdot X_{\text{вх}}^*(p)$$

$$x_{\text{вых}}(lT) = \frac{T}{2\pi j} \int_{c-j\frac{\omega_u}{T}}^{c+j\frac{\omega_u}{T}} X_{\text{вых}}^*(p) e^{plT} dp$$

$$X_{\text{вых}}^*(p) = \frac{B^*(p)}{A^*(p)} = \frac{b_0 e^{mpT} + b_1 e^{(m-1)pT} + \dots + b_m}{a_0 e^{npT} + a_1 e^{(n-1)pT} + \dots + a_n}$$

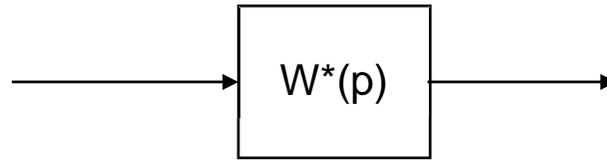
$$\dot{A}^*(p) = \frac{dA^*(p)}{d(e^{pT})}$$

$$x_{\text{вых}}(lT) = \sum_{k=1}^n \frac{B^*(p_k)}{\dot{A}^*(p_k)} e^{p_k lT}$$

$$W^*(p) = \frac{B^*(p)}{A^*(p)}$$

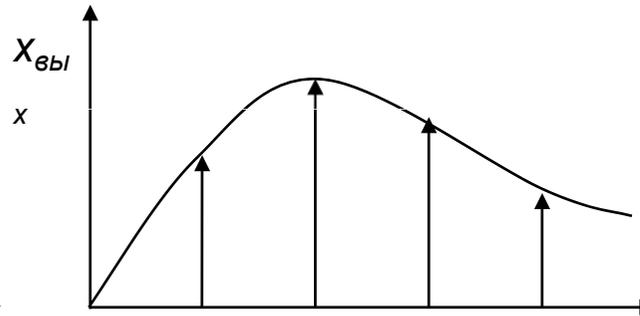
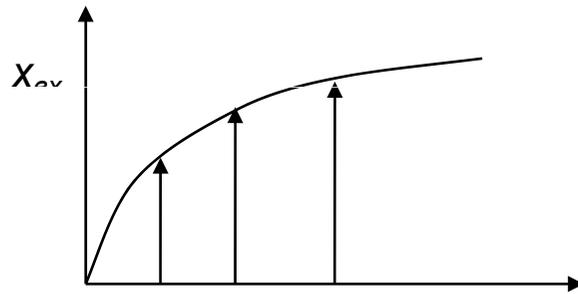
$$h(lT) = \frac{B^*(0)}{A^*(0)} + \sum_{k=1}^n \frac{B^*(p_k)}{(e^{p_k T} - 1) \dot{A}^*(p_k)} e^{p_k lT}$$

$$w(lT) = \sum_{k=1}^n \frac{B^*(p_k)}{e^{p_k T} \dot{A}^*(p_k)} e^{p_k lT}$$



$$x_{\text{вблх}}(t) = \sum_{i=0}^l x_{\text{вх}}(iT)w(t-iT) \quad \text{для } lT < t \leq (l+1)T$$

$$x_{\text{вблх}}(lT) = \sum_{i=0}^l x_{\text{вх}}(iT)w(lT-iT)$$



$$X_{\text{вблх}}^*(p) = \sum_{l=0}^{\infty} x_{\text{вблх}}(lT)e^{-pT} = x_{\text{вблх}}(0T)e^{-0pT} + x_{\text{вблх}}(T)e^{-pT} +$$

$$+ x_{\text{вблх}}(2T)e^{-2pT} + x_{\text{вблх}}(3T)e^{-3pT} + \dots$$

ЛЧХ импульсных САУ

$$v = j \frac{\sin \omega T}{1 + \cos \omega T} = j \operatorname{tg} \frac{\omega T}{2}$$

$$w = \frac{2}{T} v = \frac{2}{T} \cdot \frac{z-1}{z+1} = \frac{2}{T} \cdot j \cdot \operatorname{tg} \frac{\omega T}{2} = j\omega^*,$$

$$z = \frac{1 + wT/2}{1 - wT/2}$$

ПРИМЕНЕНИЕ ЛЧХ ДЛЯ ИМПУЛЬСНЫХ САУ

ЛЧХ импульсных САУ

$$z = \frac{1+v}{1-v} \quad v = \frac{z-1}{z+1} \quad \text{При } z = e^{j\omega T} = \cos T + j \sin T$$

$$v = j \frac{\sin T}{1 + \cos \omega T} = j \operatorname{tg} \frac{T}{2} \quad w = \frac{2}{T} v = \frac{2}{T} \cdot \frac{z-1}{z+1} = \frac{2}{T} \cdot j \cdot \operatorname{tg} \frac{\omega T}{2} = j\omega^*,$$

$$\text{в } \omega^* = \frac{2}{T} \operatorname{tg} \frac{T}{2} \quad \operatorname{tg} \frac{T}{2} \approx \frac{T}{2} \quad v = \frac{wT}{2}$$

$$z = \frac{1 + wT/2}{1 - wT/2}$$

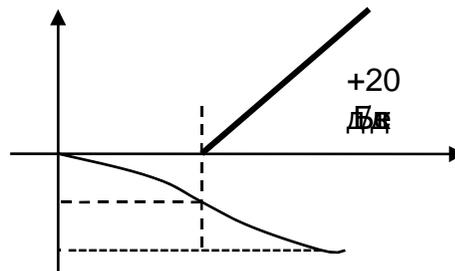
Методика построения ЛЧХ

$$W_p(z) = \frac{b_0 z^m + b_1 z^{m-1} + \dots + b_m}{a_0 z^n + a_1 z^{n-1} + \dots + a_n}$$

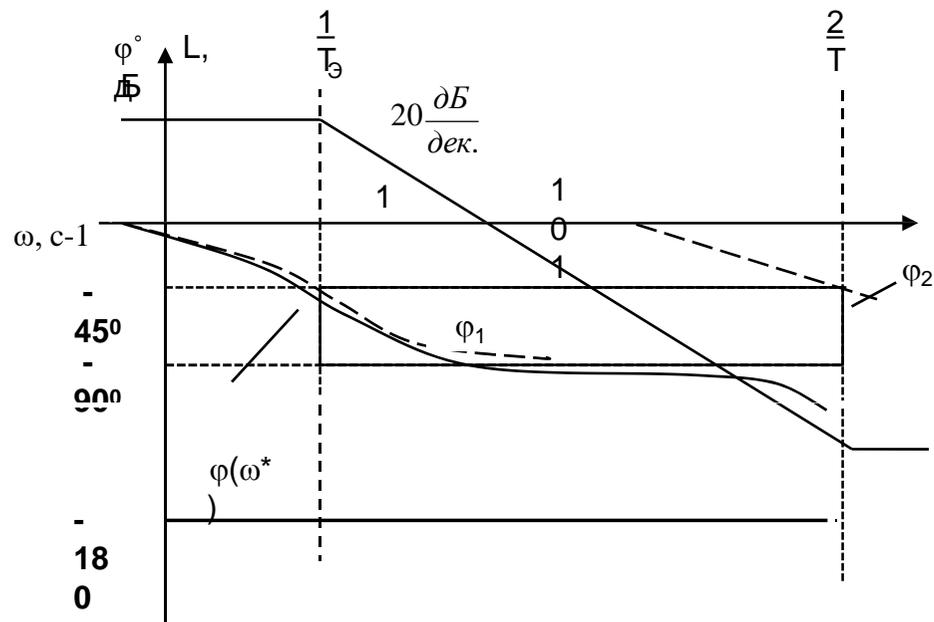
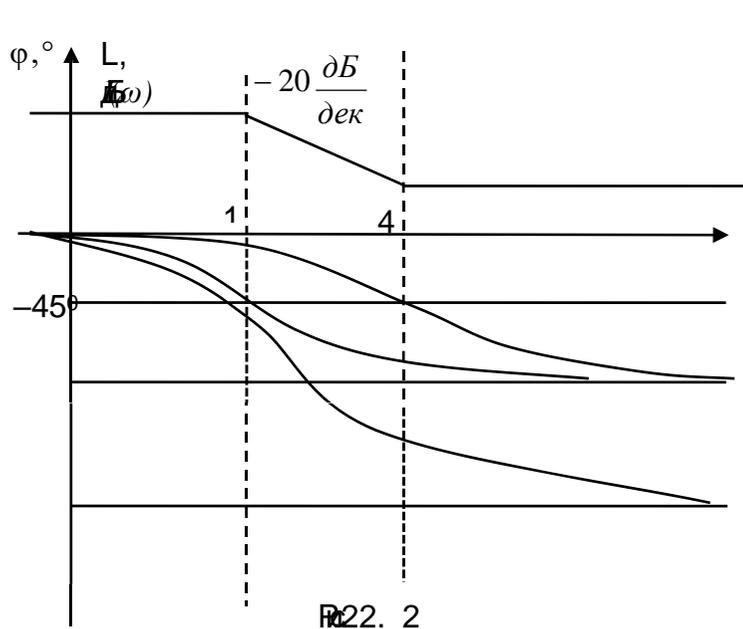
$$W_p(w) = \frac{b_0 \left(\frac{1+wT/2}{1-wT/2} \right)^m + b_1 \left(\frac{1+wT/2}{1-wT/2} \right)^{m-1} + \dots + b_m}{a_0 \left(\frac{1+wT/2}{1-wT/2} \right)^n + a_1 \left(\frac{1+wT/2}{1-wT/2} \right)^{n-1} + \dots + a_n} =$$

$$= \left(1 - \frac{wT}{2} \right)^{n-m} \cdot \frac{b'_0 w^m + b'_1 w^{m-1} + \dots + b'_m}{a'_0 w^n + a'_1 w^{n-1} + \dots + a'_n}$$

$$W_p(z) \xrightarrow{\frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}} W_p(w) \xrightarrow{w=j\omega^*} W_p(j\omega^*)$$



Примеры построения ЛЧХ



Синтез ИСАУ методом ЛЧХ

1. ~~Синтез ИСАУ~~ $LH(\omega^*)$

2. ~~Синтез ИСАУ~~

3. ~~Синтез ИСАУ~~

~~Синтез ИСАУ~~

$$20\lg|W_K(j\omega^*)| = 20\lg|W_C(j\omega^*)| - 20\lg|W_H(j\omega^*)|$$

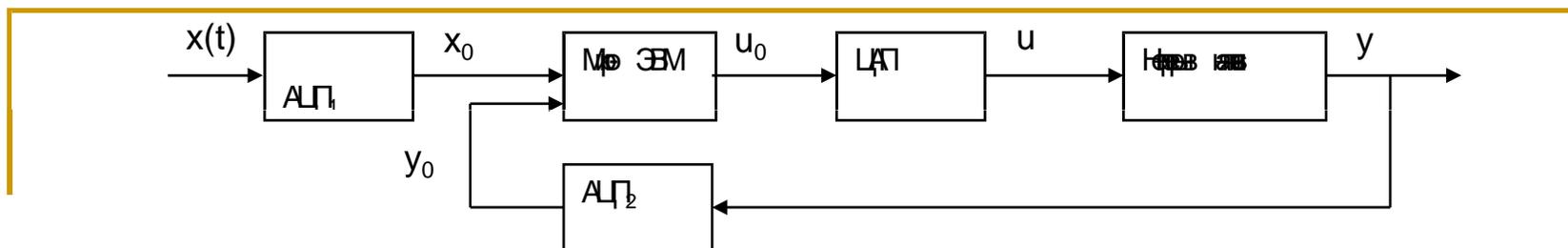
$$W_K(j\omega^*) = \frac{W_C(j\omega^*)}{W_H(j\omega^*)}$$

4. ~~Синтез ИСАУ~~ $W_K(z)$

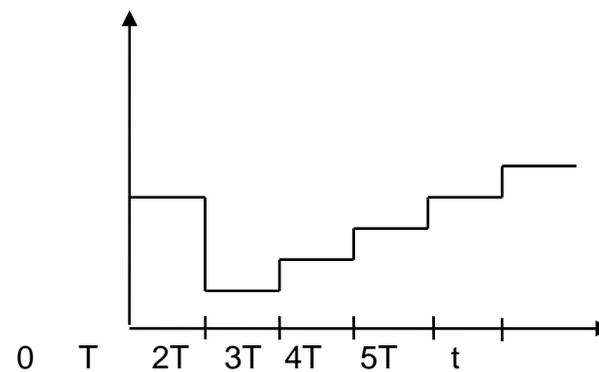
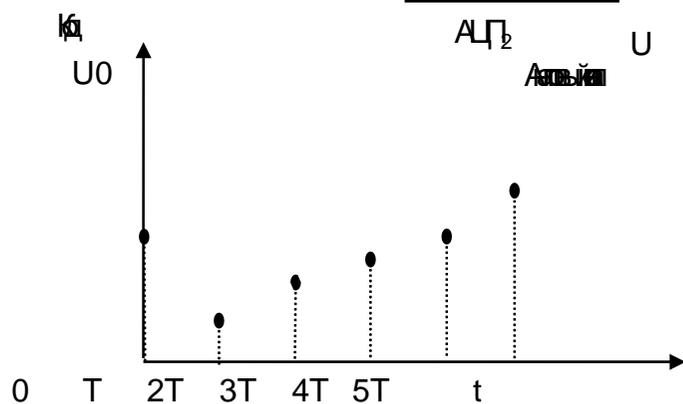
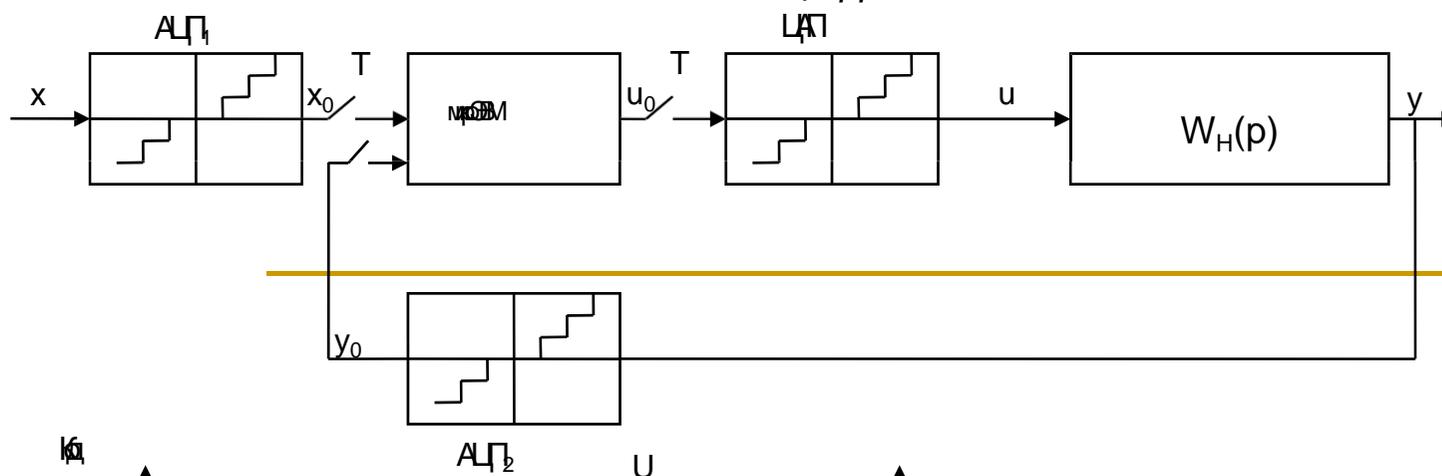
$$j\omega^* = \frac{2}{T} \cdot \frac{z-1}{z+1}$$

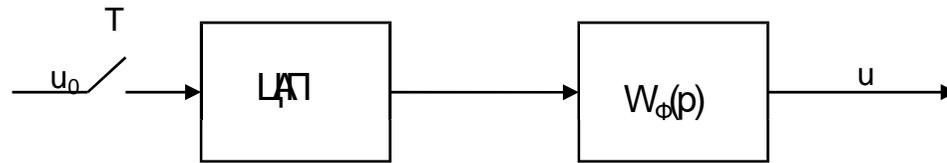
ЦИФРОВЫЕ САУ С МИКРО-ЭВМ

Функциональная схема цифровой САУ



23.2. Особенности цифровых САУ

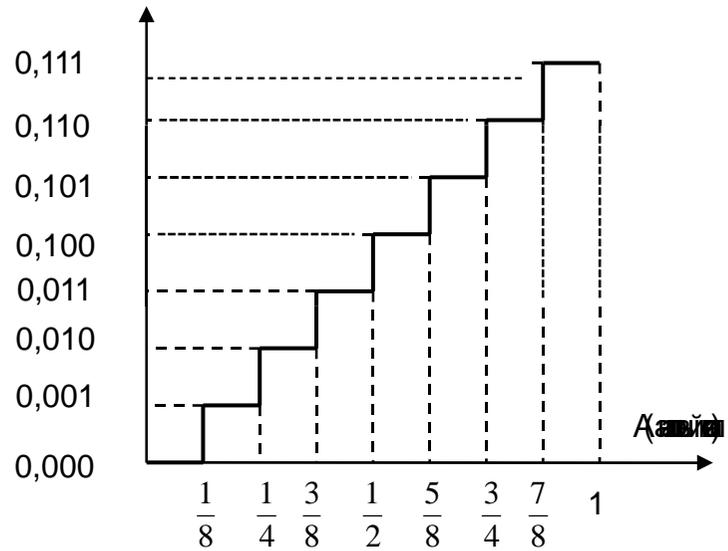


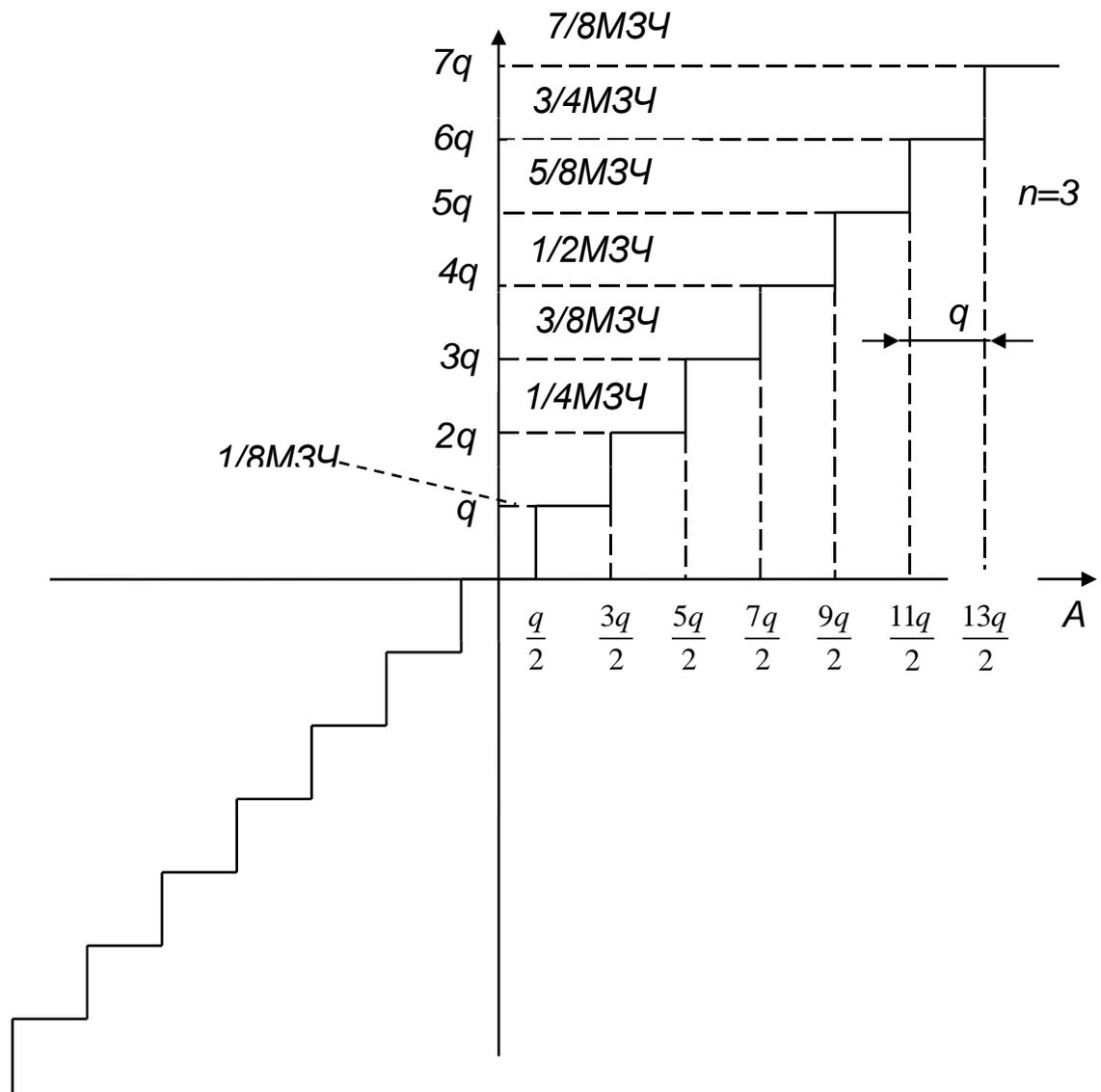


$$W_{\phi}(p) = \int_0^T 1 \cdot e^{-pt} dt = \frac{1 - e^{-pT}}{p} = \frac{z-1}{z} \cdot \frac{1}{p}$$

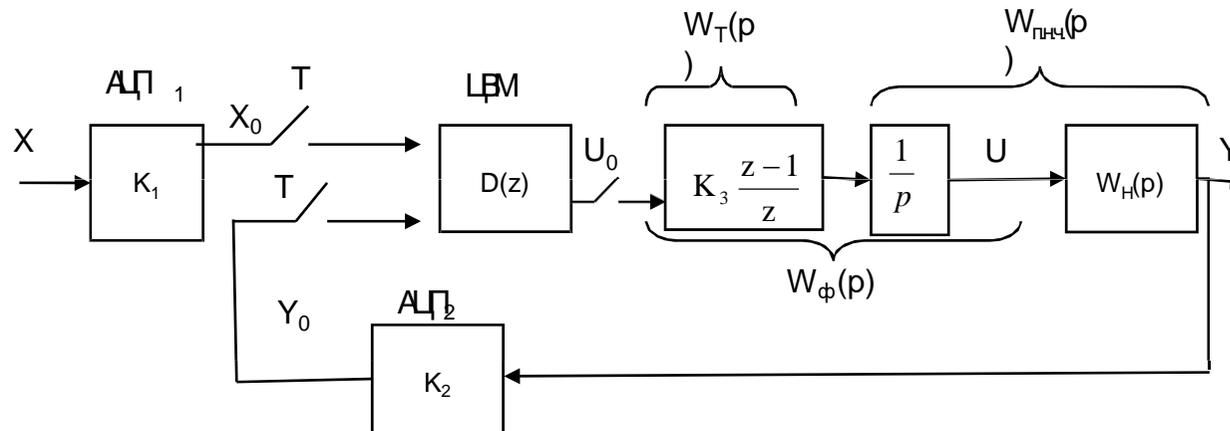
. Преобразование данных и квантование по уровню

Меню (д)





Структурная схема и передаточная функция цифровой САУ



$$W_H(z) = W_T(z) \cdot Z\{W_{П.Н.Ч.}(p)\} = W_T(z) \cdot Z\left\{\frac{W_H(p)}{p}\right\} = \frac{z-1}{z} \cdot Z\left\{\frac{W_H(p)}{p}\right\}$$

$$D(z) = \frac{U_0(z)}{E_0(z)} = \frac{b_0 z^s + b_1 z^{s-1} + \dots + b_s}{a_0 z^k + a_1 z^{k-1} + \dots + a_k}$$

$$D(z) = \frac{U_0(z)}{E_0(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_k z^{-k}}{a_0 + a_1 z^{-1} + \dots + a_k z^{-k}}$$

$$(a_0 + a_1 z^{-1} + \dots + a_k z^{-k}) U_0(z) = (b_0 + b_1 z^{-1} + \dots + b_k z^{-k}) E_0(z)$$

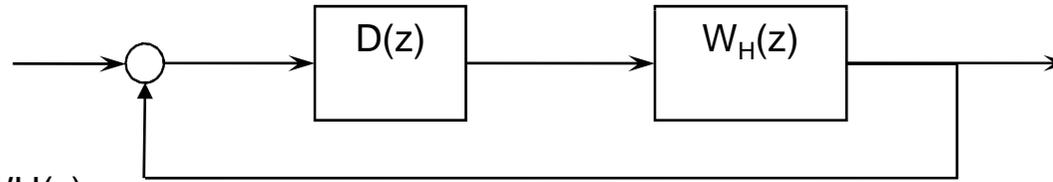
$$Z[f(n)] = F(z),$$

$$Z[f(n-m)T] = z^{-m} F(z).$$

$$\begin{aligned} a_0 \cdot u_0(nT) + a_1 u_0[(n-1)T] + \dots + a_k u_0[(n-k)T] = \\ = b_0 e_0(nT) + b_1 e_0[(n-1)T] + \dots + b_k e_0[(n-k)T] \end{aligned}$$

$$\begin{aligned} u_0(nT) = b_0 e_0(nT) + b_1 e_0[(n-1)T] + \dots + b_k e_0[(n-k)T] - \\ - \{a_1 u_0[(n-1)T] + a_2 u_0[(n-2)T] + \dots + a_k u_0[(n-k)T]\} \end{aligned}$$

Передаточные функции цифровой САУ



$$W_p(z) = D(z) \cdot W_H(z)$$

$$W(z) = \frac{W_p(z)}{1 + W_p(z)}$$

Приближенное определение дискретной передаточной функции по передаточной функции непрерывной части (подстановочный метод)

$$\frac{1}{p} = T \frac{1}{z-1};$$

$$p = \frac{z-1}{T};$$

$$\frac{1}{p} = T \frac{z}{z-1};$$

$$p = \frac{z-1}{T \cdot z};$$

$$\frac{1}{p} = \frac{T}{2} \cdot \frac{z+1}{z-1};$$

$$p = \frac{2(z-1)}{T(z+1)};$$

Дифференцирование цифровых последовательностей

$$p = \frac{1}{T} \left[(z-1) - \frac{1}{2}(z-1)^2 + \frac{1}{3}(z-1)^3 - \dots \right];$$

$$p = \frac{2}{T} \left[\frac{z-1}{z+1} + \frac{1}{3} \left(\frac{z-1}{z+1} \right)^3 + \frac{1}{5} \left(\frac{z-1}{z+1} \right)^5 + \dots \right];$$

$$p = \frac{1}{T} \left[(1-z^{-1}) + \frac{1}{2}(1-z^{-1})^2 + \frac{1}{3}(1-z^{-1})^3 + \dots \right];$$

$$\dot{f}(z) = \frac{1}{T} (1 - z^{-1}) \cdot f(z)$$

$$\dot{f}(n) = \frac{1}{T} [f(n) - f(n-1)]$$

$$D(z) = \frac{\dot{f}(z)}{f(z)} = \frac{1}{T} (1 - z^{-1})$$

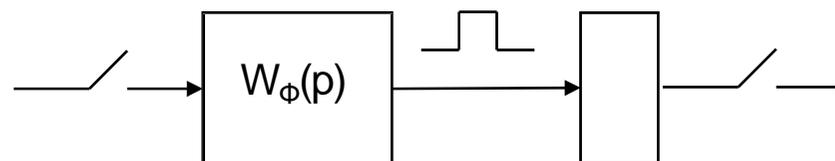


Цифровые интеграторы

$$\frac{1}{p} \approx \frac{T}{z-1} = \frac{T \cdot z^{-1}}{1-z^{-1}}$$

$$\frac{1}{p} \approx \frac{T}{2} \cdot \frac{z+1}{z-1} = \frac{T}{2} \cdot \frac{1+z^{-1}}{1-z^{-1}}$$

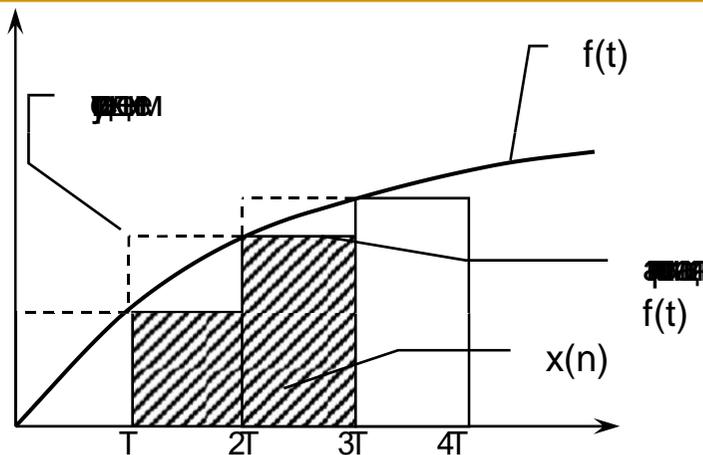
$$\frac{1}{p} \approx \frac{T}{1-z^{-1}}$$



$$\frac{X(z)}{F(z)} = \frac{z-1}{z} Z \left[\frac{1}{p^2} \right] = \frac{z-1}{z} \cdot \left[\frac{T \cdot z}{(z-1)^2} \right] = \frac{T}{z-1} = \frac{T \cdot z^{-1}}{1-z^{-1}}$$

$$X(z) \cdot (1 - z^{-1}) = F(z) \cdot T \cdot z^{-1}$$

$$x[n] = x[n-1] + T \cdot f[n-1]$$

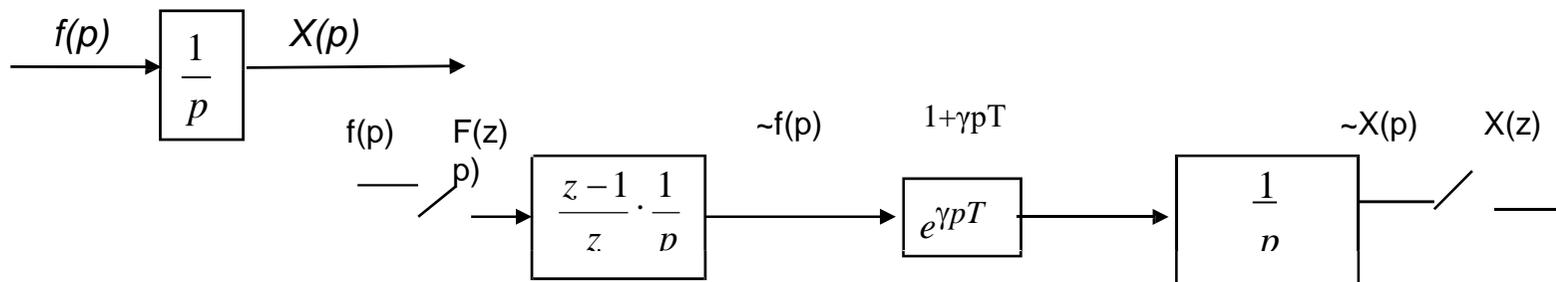


$$x(n) = x(n-1) + Tf(n)$$

$$X(z) \cdot (1 - z^{-1}) = \frac{T}{2} \cdot (1 + z^{-1}) \cdot F(z),$$

$$x(n) = x(n-1) + \frac{T}{2} [f(n) + f(n-1)].$$

Обобщенная формула численного интегрирования. Компенсация ошибок



$$\frac{X(z)}{F(z)} = \frac{z-1}{z} \cdot Z \left\{ \frac{1 + \gamma \cdot pT}{p^2} \right\} = \frac{z-1}{z} \cdot Z \left\{ \frac{1}{p^2} + \frac{\gamma T}{p} \right\} = \frac{z-1}{z} \cdot \left\{ \frac{Tz}{(z-1)^2} + \frac{\gamma Tz}{z-1} \right\} =$$

$$= T \left[\frac{\gamma z + (1-\gamma)}{z-1} \right] = T \left[\frac{\gamma + (1-\gamma) z^{-1}}{1 - z^{-1}} \right]$$

. Дискретные регуляторы

$$W(p) = k \left[1 + \frac{1}{T_I p} + T_D p \right] = \frac{U(p)}{X(p)} \quad W(z) = \frac{U(z)}{X(z)} = k \left[1 + \frac{T}{T_I} \cdot \frac{z^{-1}}{1 - z^{-1}} + \frac{T_D}{T} \cdot (1 - z^{-1}) \right] =$$

$$= \frac{k \left[(1 - z^{-1}) + \frac{T}{T_I} \cdot z^{-1} + \frac{T_D}{T} (1 - z^{-1})^2 \right]}{1 - z^{-1}}$$

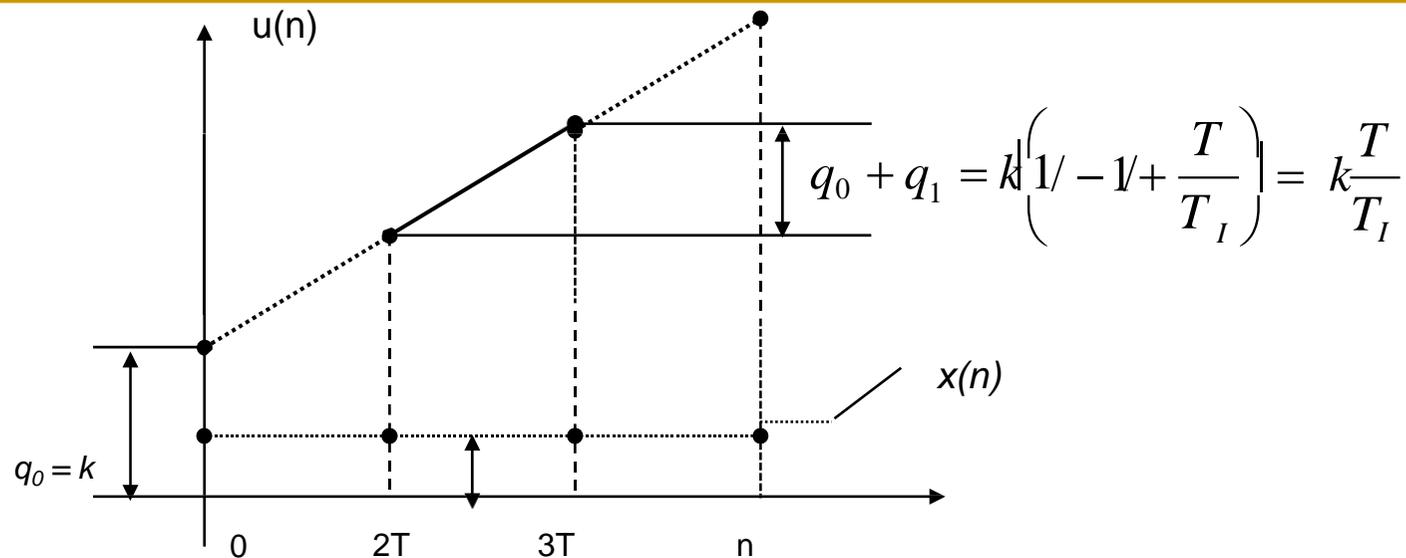
$$U(z) \cdot (1 - z^{-1}) = k \left[(1 - z^{-1}) + \frac{T}{T_I} \cdot z^{-1} + \frac{T_D}{T} (1 - z^{-1})^2 \right] \cdot X(z);$$

$$U(z) = U(z) \cdot z^{-1} + k \left[1 + \frac{T_D}{T} + \left(-1 + \frac{T}{T_I} - 2 \frac{T_D}{T} \right) \cdot z^{-1} + \frac{T_D}{T} \cdot z^{-2} \right] \cdot X(z)$$

$$u(n) = u(n-1) + k \left[\left(1 + \frac{T_D}{T} \right) \cdot x(n) + \left(-1 + \frac{T}{T_I} - 2 \frac{T_D}{T} \right) \cdot x(n-1) + \frac{T_D}{T} \cdot x(n-2) \right]$$

$$u(n) = u(n-1) + k \left[x(n) + \left(-1 + \frac{T}{T_I} \right) \cdot x(n-1) \right]$$

$$q_1 = \left(-1 + \frac{T}{T_I} \right) k$$



Выбор параметров цифрового регулятора

$$\frac{1}{p} \approx \frac{T}{z-1} \quad W(p) = k \left[1 + \frac{1}{T_I p} \right]$$

$$\text{тогда } W(z) = k \left[1 + \frac{T}{T_I \cdot (z-1)} \right]$$

$$u(n) = u(n-1) + k \left[x(n) + \left(-1 + \frac{T}{T_I} \right) \cdot x(n-1) \right]$$

$$q_0 = k; \quad q_1 = k \left(-1 + \frac{T}{T_I} \right).$$

$$\frac{1}{p} \approx T \frac{z}{z-1}.$$

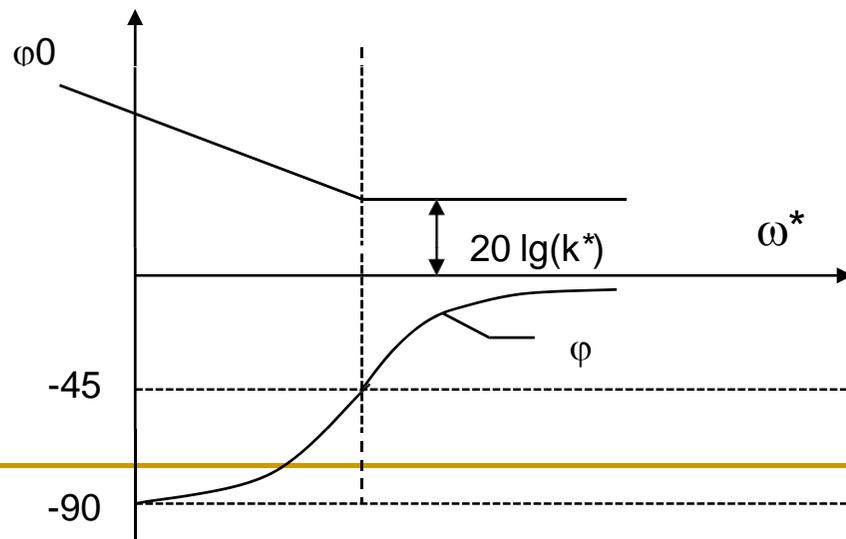
тогда $W(z) = k \left[1 + \frac{Tz}{T_I(z-1)} \right]$ - передаточная функция

$u(n) = u(n-1) + k \left(1 + \frac{T}{T_I} \right) \cdot x(n) - kx(n-1)$ - разностное уравнение

$$q_0 = k \left(1 + \frac{T}{T_I} \right); \quad q_1 = -k.$$

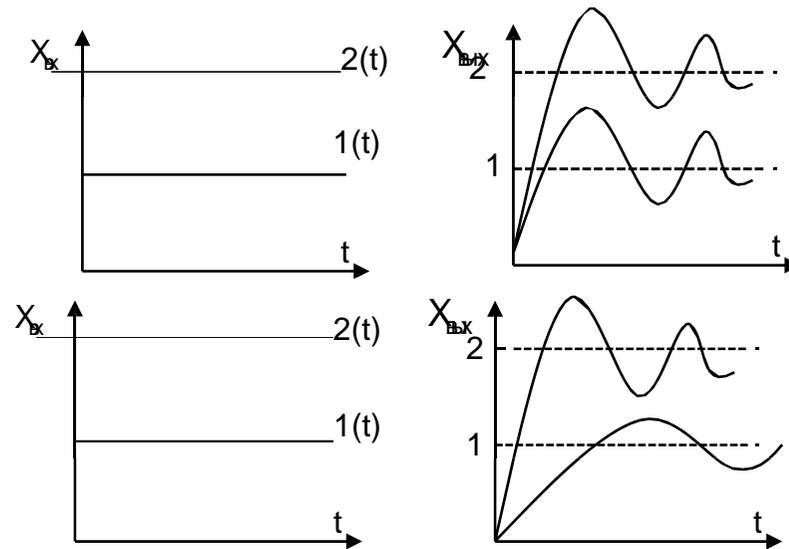
$$W(z) = k \left[1 + \frac{T}{T_I \cdot (z-1)} \right]$$

$$W(j\omega^*) = k^* \frac{\tau^* j\omega^* + 1}{\tau^* j\omega^*}$$

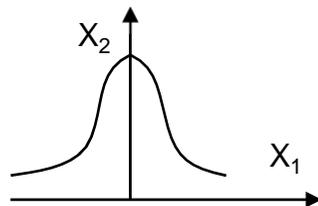


НЕЛИНЕЙНЫЕ СИСТЕМЫ АВТОМАТИЧЕСКОГО УПРАВЛЕНИЯ

Особенности нелинейных систем управления

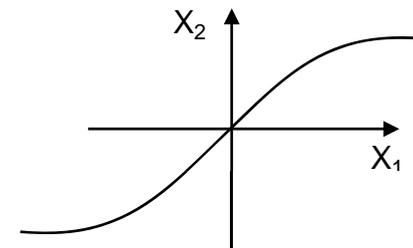


Общие характеристики НЗ

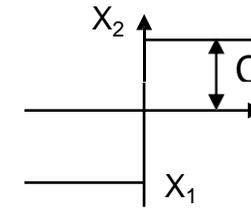
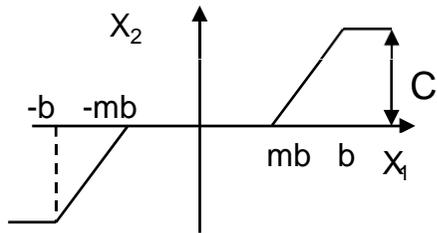
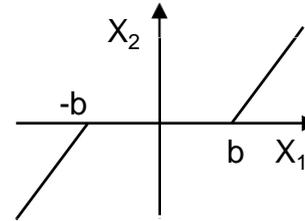
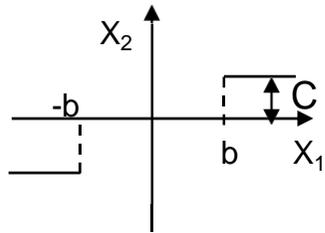


$$F(x_1) = F(-x_1)$$

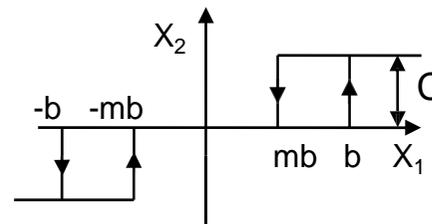
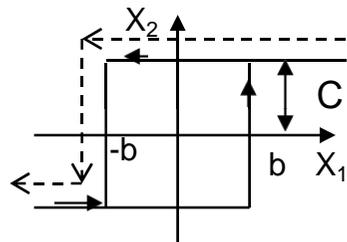
$$F(x_1) = -F(-x_1)$$

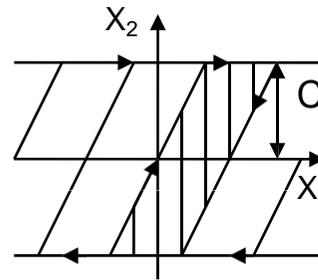
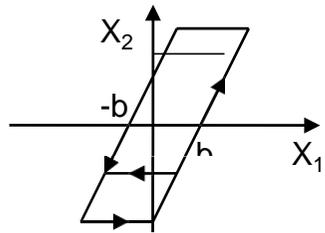


Типовые нелинейные звенья с однозначными характеристиками

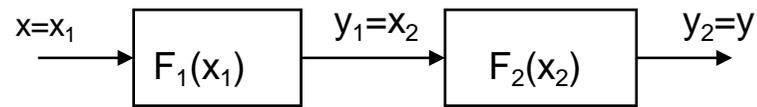


Звенья с неоднозначными характеристиками

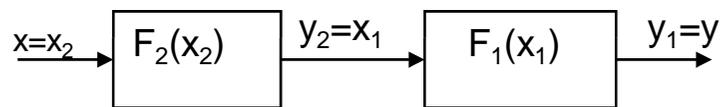




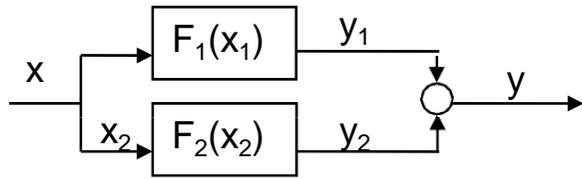
Основные виды соединений нелинейных звеньев



$$y = F_2(x_2) = F_2[F_1(x_1)] = F_2[F_1(x)]$$

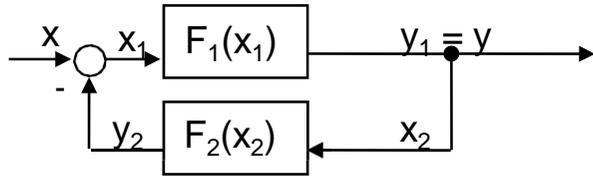


$$y = F_1(x_1) = F_1[F_2(x_2)] = F_1[F_2(x)]$$



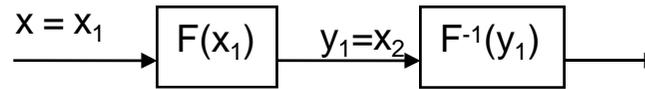
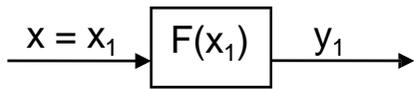
$$x = x_1 = x_2$$

$$y = y_1 + y_2 = F(x_1) + F(x_2)$$

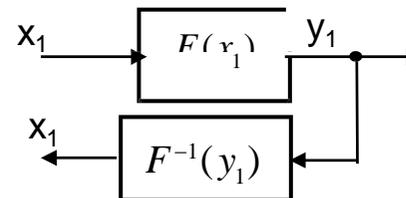
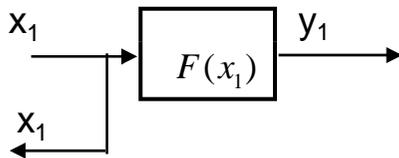


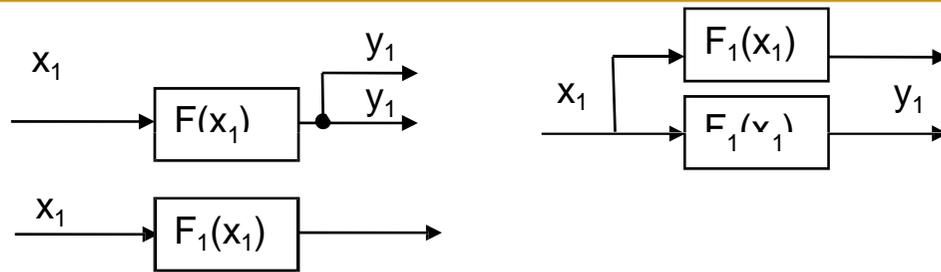
$$y = F_1(x_1) = F_1(x - y_2) = F_1[x - F_2(y)]$$

Способы нейтрализации нелинейных звеньев



$$y = F^{-1}(y_1) = F^{-1}[F(x_1)] = x$$





ПРИБЛИЖЕННЫЕ МЕТОДЫ ИССЛЕДОВАНИЯ НА ОСНОВЕ МЕТОДА ГАРМОНИЧЕСКОЙ ЛИНЕАРИЗАЦИИ НЕЛИНЕЙНОСТЕЙ

27.1. Метод гармонической линеаризации нелинейностей

$$x_2 = F(x_1, px_1)$$

$$x_2(t) = F(a \cdot \sin \psi) = a_0 + a_1 \cdot \sin \psi + b_1 \cdot \cos \psi + \text{высшие гармоники}$$

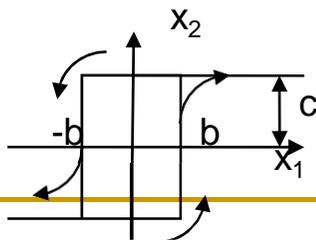
$$\sin \psi = \frac{x_1}{a}, \quad \cos \psi = \frac{px_1}{a\omega}$$

$$x_2 = q(a, \omega)x_1 + \frac{q'(a, \omega)}{\omega} px_1 + \text{в.г.}$$

$$q(a, \omega) = \frac{a_1}{a} = \frac{1}{\pi a} \int_0^{2\pi} F(a \cdot \sin \psi, a\omega \cos \psi) \sin \psi \, d\psi,$$

$$q'(a, \omega) = \frac{b_1}{a} = \frac{1}{\pi a} \int_0^{2\pi} F(a \cdot \sin \psi, a\omega \cos \psi) \cos \psi \, d\psi,$$

$$x_2 = F(x_1)$$

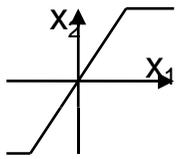


$$x_1 = a \cdot \sin \psi, \psi = \omega t$$

$$x_2(t) = q(a, \omega) x_1 + \frac{q'(a, \omega)}{\omega} \dot{x}_1$$

$$q(a, \omega) = \frac{1}{\pi a} \int_0^{2\pi} F(a \cdot \sin \psi) \sin \psi \, d\psi,$$

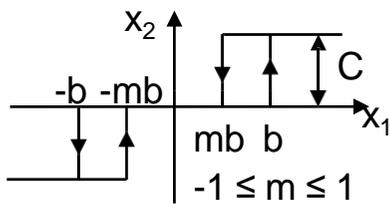
$$q'(a, \omega) = \frac{1}{\pi a} \int_0^{2\pi} F(a \cdot \sin \psi) \cos \psi \, d\psi,$$



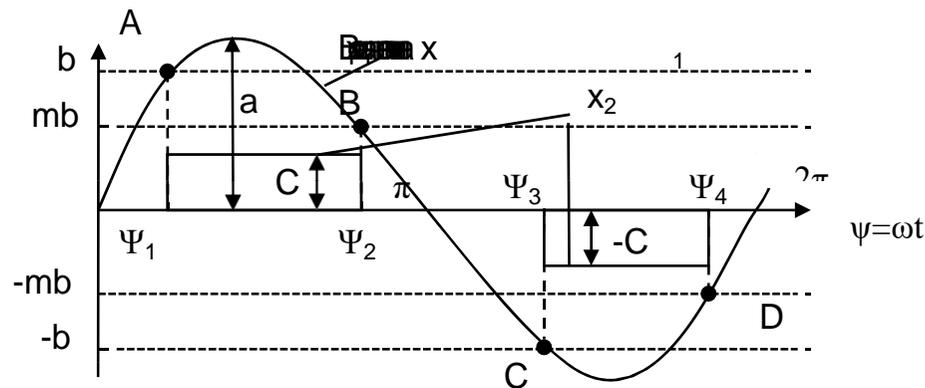
$$q'(a, \omega) = \frac{1}{\pi \cdot a} \cdot \int_0^{2\pi} F(a \cdot \sin(\psi)) \cdot \cos(\psi) \, d\psi = \frac{1}{\pi \cdot a^2} \cdot \int_0^0 F(x) \, dx = 0$$

$$x_2 = a(a) x_1$$

Коэффициенты гармонической линейзации релейных звеньев



$$x_1 = a \sin \omega t$$



$$\psi_1 = \arcsin(b/a)$$

$$\psi_2 = \pi - \arcsin(mb/a)$$

$$\psi_3 = \pi + \psi_1$$

$$\psi_4 = \pi + \psi_2$$

$$\int_0^{2\pi} = 2 \int_0^{\pi}; \quad \int_0^{\pi} = \int_0^{\psi_1} + \int_{\psi_1}^{\psi_2} + \int_{\psi_2}^{\pi} = \int_{\psi_1}^{\psi_2}$$

$$q(a) = \frac{2}{\pi a} \int_{\psi_1}^{\psi_2} C \cdot \sin(\psi) d\psi$$

$$q'(a) = \frac{2 \cdot C}{\pi a} \cdot (\cos \psi_1 - \cos \psi_2)$$

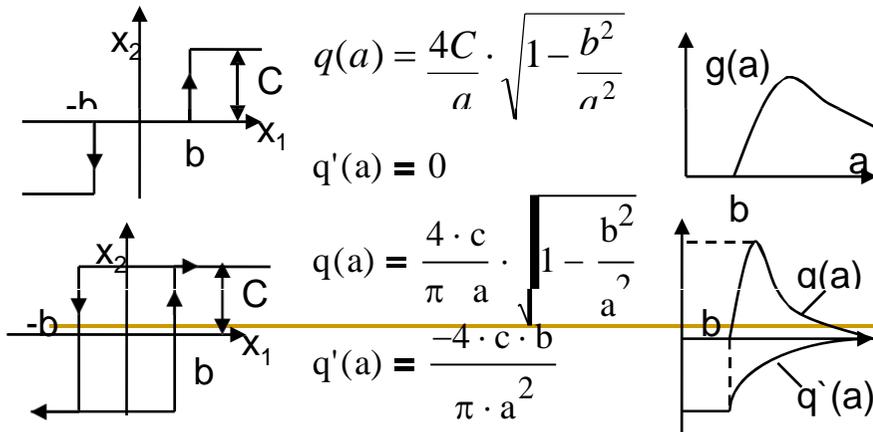
$$\cos \psi = \sqrt{1 - \sin^2 \psi} \quad \psi_2 = \arcsin\left(\frac{m \cdot b}{a}\right) < 90^\circ$$

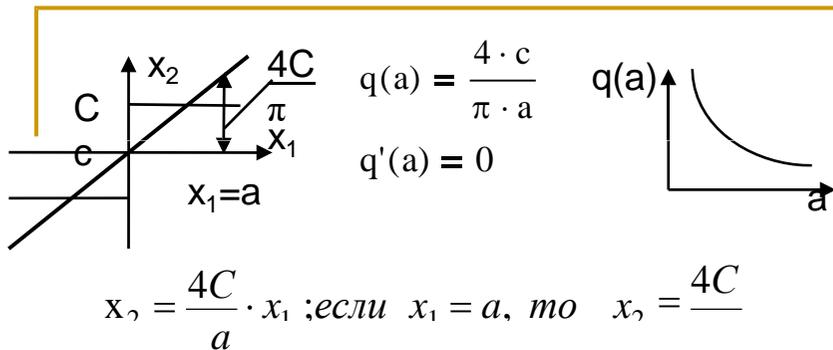
$$-\cos \psi_2 = -\cos(-\psi_2) = \sqrt{1 - \sin^2 \psi_2} = \sqrt{1 - \left(\frac{mb}{a}\right)^2}$$

$$q(a) = \frac{2C}{\pi a} \cdot \left[\sqrt{1 - \left(\frac{b}{a}\right)^2} + \sqrt{1 - \left(\frac{mb}{a}\right)^2} \right]$$

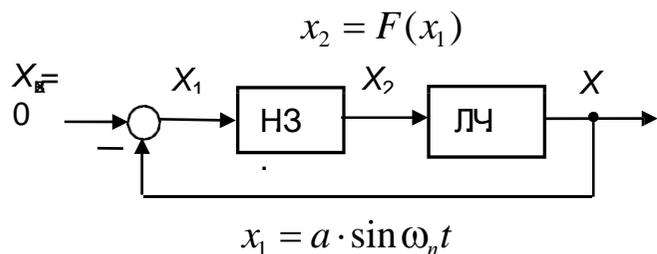
$$q'(a) = \frac{2}{\pi a} \cdot \int_{\psi_1}^{\psi_2} C \cdot \cos \psi d\psi = -\frac{2C}{\pi a} \cdot (\sin \psi_1 - \sin \psi_2) =$$

$$= \frac{-2C}{\pi a} \cdot \left(\frac{b}{a} - \frac{mb}{a} \right) = \frac{-2 \cdot C \cdot b}{\pi a^2} \cdot (1 - m)$$

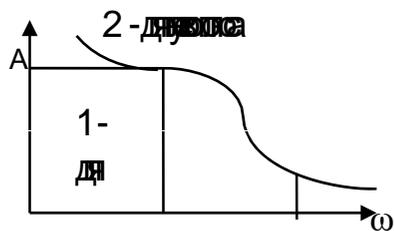




Метод гармонического баланса



$$x_2(t) = F(a \cdot \sin \omega_n t) = a_1 \cdot \sin \omega_n t + b_1 \cdot \cos \omega_n t + \text{высшие гармоники}$$



$$x_2(t) = a_1 \cdot \sin \omega_n t + b_1 \cdot \cos \omega_n t = b_m \sin(\omega_n t + \varphi_n)$$

$$W_H(a) = q(a) + jq'(a) = A_H(a) \cdot e^{j\varphi_H(a)}$$

Условие гармонического баланса

$$W_H(a) = \frac{\tilde{X}_2}{\tilde{X}_1} \quad W_{Л}(j\omega) = -\frac{\tilde{X}_1}{\tilde{X}_2}$$

$$W_H(a)W_{Л}(j\omega) = W_P(a, j\omega) = -1$$

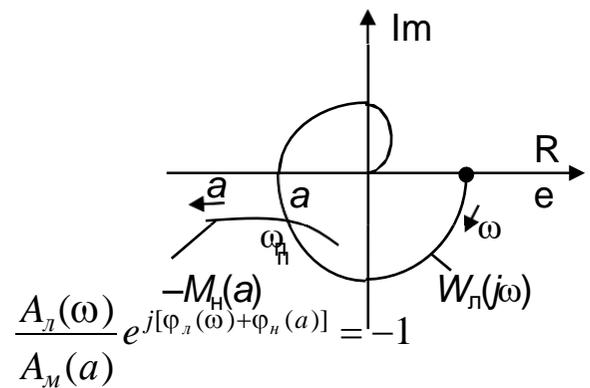
Графоаналитический метод определения параметров автоколебательного режима и его устойчивости

$$W_{\Pi}(j\omega) = -\frac{1}{W_H(a)} = -M_H(a),$$

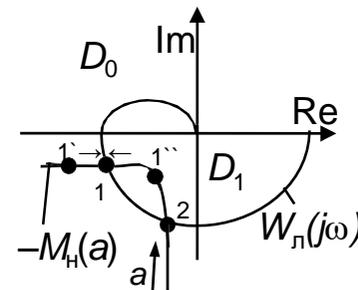
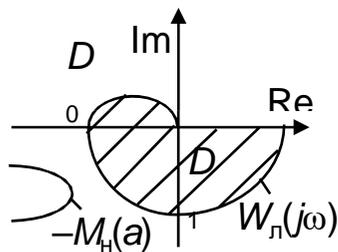
$$-M_H(a) = -\frac{1}{q(a) + jq'(a)}$$

$$W_{\Pi}(j\omega) = U(\omega) + jV(\omega)$$

$$-M_H(a) = U(a) + jV(a)$$



$$\begin{cases} \frac{A_L(\omega)}{A_M(a)} = 1 \\ \varphi_L(\omega) + \varphi_H(a) = -(2m+1)\pi; \text{ где } m = 0, \pm 1, \pm 2, \dots \end{cases}$$



МЕТОД ЛЯПУНОВА

Поиск функции Ляпунова

$$V = V(x_1, x_2, \dots, x_n)$$

$$V = x_1^2 + x_2^2$$

$$V = (x_1 + x_2)^2 + cx_3^2$$

$$V = x_1 + x_2$$

Функция Ляпунова и её производная

$$\left. \begin{aligned} \frac{dx_1}{dt} &= F_1(x_1, x_2, \dots, x_n) \\ \frac{dx_2}{dt} &= F_2(x_1, x_2, \dots, x_n) \\ &\dots \\ \frac{dx_n}{dt} &= F_n(x_1, x_2, \dots, x_n) \end{aligned} \right\}$$

$$\dots$$

$$\dots$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial x_1} \cdot F_1 + \frac{\partial V}{\partial x_2} \cdot F_2 + \dots + \frac{\partial V}{\partial x_n} \cdot F_n$$

$$\frac{dV}{dt} = W(x_1, x_2, \dots, x_n)$$

Формулировка теоремы Ляпунова в векторно-матричной форме

$$V(x_1, x_2, \dots, x_n) = \sum_{i=1}^n \sum_{j=1}^n p_{ij} x_i x_j,$$

$$V(x_1, x_2, \dots, x_n) = x^T P x$$

$$\dot{x} = Ax$$

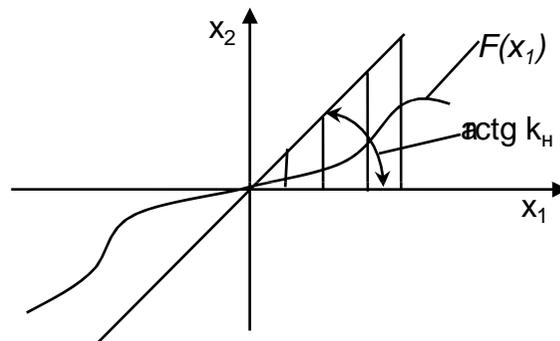
$$\dot{x}^T = x^T A^T$$

$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x} = x^T A^T P x + x^T P A x = x^T (A^T P + P A) x$$

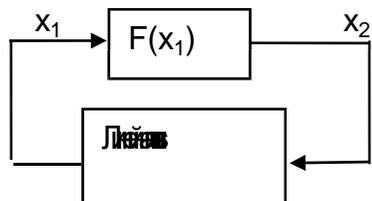
$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x} = x^T A^T P x + x^T P A x = x^T (A^T P + P A) x$$

УСЛОВИЯ АБСОЛЮТНОЙ УСТОЙЧИВОСТИ НЕЛИНЕЙНЫХ СИСТЕМ

$$x_2 = F(x_1) \quad 0 \leq \frac{F(x_1)}{x_1} < k_H$$



Частотный критерий абсолютной устойчивости В. М. Попова



$$W(p) = \frac{X_1(p)}{X_2(p)} = - \frac{R(p)}{Q(p)}$$

$$Q(p)X_1(p) = -R(p)X_2(p)$$

$$Q(p) = a_0 p^n + a_1 p^{n-1} + \dots + a_n$$

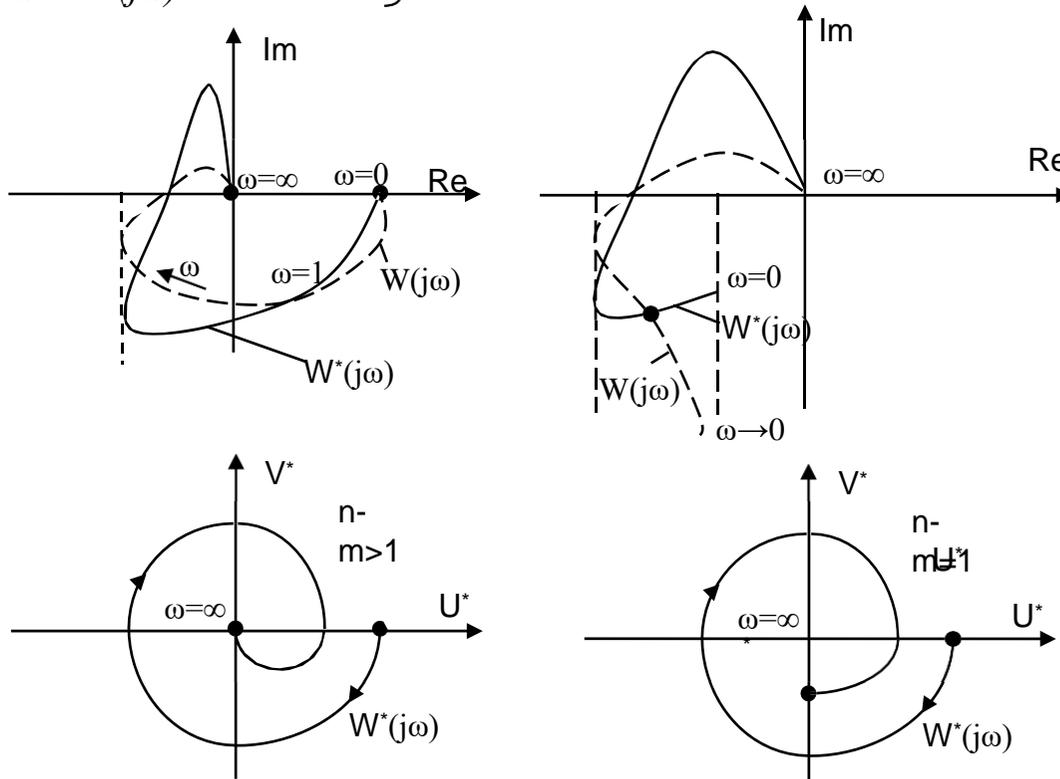
$$R(p) = b_0 p^m + b_1 p^{m-1} + \dots + b_m$$

$$W(p) = \frac{R(p)}{Q(p)}$$

$$\operatorname{Re}(1 + j\omega h)W(j\omega) + \frac{1}{k_h} > 0$$

$$W^*(j\omega) = U^*(\omega) + jV^*(\omega)$$

$$\left. \begin{aligned} U^*(\omega) &= \operatorname{Re} W^*(j\omega) = \operatorname{Re} W(j\omega) \\ V^*(\omega) &= \omega \operatorname{Im} W(j\omega) \end{aligned} \right\}$$



$$\operatorname{Re}(1 + j\omega h)W(j\omega) + \frac{1}{k_h} = \operatorname{Re} W(j\omega) - \omega h \operatorname{Im} W(j\omega) + \frac{1}{k_h}$$

$$U^*(\omega) - hV^*(\omega) + \frac{1}{k_h} \geq 0$$

$$V^* = \frac{1}{h} \left(U^* + \frac{1}{k_h} \right)$$