Algorithm for adaptive observation based on method of instrumental variables

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Abstract. When the input signal and the output value of the object of control cannot be measured accurately, the state vector is estimated. The instrumental variables (IVs) method is a commonly used parameter estimation method [1-10]. The task of adaptive observation is to create state observers containing parameter estimators. In adaptive observers, the matrices **A** and **b** or **c** (depending on the chosen canonical state-space representation form) are assumed to be unknown. In the monitoring process, parameter estimation is performed, the unknown matrices are determined, and then the state vector is calculated. The paper aims to present a non-recurrent adaptive observation algorithm for SISO linear time-invariant (LTI) discrete systems. The algorithm is based on the instrumental variables (IVs) method, and the adaptive state observer (ASO) estimates the parameters, the initial and the current state vectors of the discrete system. The algorithm's workability and effectiveness are proved by using simulation data in MATLAB/Simulink.

1 Introduction

To design control systems with state feedback, it is often necessary to recover the state vector by using measurements of the output and the input signals of the controlled object.

For the reconstruction of the state vector, an implementation of a state observer is necessary. The process of adaptive observation involves the creation of an observer with a parameter estimator [11,12]. The matrices A and b or c (depending on the canonical form for representing the object in the state space) are considered unknown.

During the observation process, the unknown parameters are estimated, the unknown matrices are determined, and the state vector is calculated.

This paper presents a non-recursive algorithm based on the IVs method [13] for adaptive observation of SISO LTI discrete systems.

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The parameters estimator in the adaptive observer is built based on a mathematical procedure with low computational complexity when inverting the information matrix presented in [14].

2 Problem Formulation

Consider a system which is mathematically described in the state space as follows:

$$
\begin{aligned}
\mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k), \quad \mathbf{x}(0) = \mathbf{x}_0, \\
y(k) &= \mathbf{c}^\mathsf{T}\mathbf{x}(k) + f(k), \quad k = 0, 1, 2, \cdots
\end{aligned}
\tag{1}
$$

where

$$
\mathbf{A} = \begin{bmatrix} \mathbf{0} & \vdots & \mathbf{I}_{n-1} \\ \cdots & \cdots & \cdots \\ & \mathbf{a}^\mathsf{T} & \end{bmatrix}
\tag{2}
$$

$$
\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, \quad
\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \quad
\mathbf{c} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\tag{3}
$$

The system order n is a priori known, $\mathbf{x}(0) \in R^n$ is the unknown initial vector of state, $\mathbf{x}(k) \in R^n$ is the unknown current vector of state, $u(k) \in R^1$ is a scalar input signal, $y(k) \in R^1$ is a scalar output signal, $f(k)$ is an additionally added noise signal, $\mathbf{a}$ and $\mathbf{b}$ are unknown vectors.

The state space description (1) corresponds to the following discrete transfer function:

$$
W(z) = \frac{h_1 z^{n-1} + h_2 z^{n-2} + \cdots + h_{n-1} z + h_n}{z^n - a_n z^{n-1} - \cdots - a_2 z - a_1}
\tag{4}
$$

The elements b_i of vector $\boldsymbol{b}$ in the chosen phase-coordinate canonical form are calculated by the coefficients h_i of the polynomial in the numerator and the coefficients a_i of the polynomial in the denominator of (4) as follows [15]:

$$
Tb = h,
\tag{5}
$$

where

$$
\mathbf{h}^\mathsf{T} = \begin{bmatrix} h_1 & h_2 & \cdots & h_n \end{bmatrix}, \quad
\mathbf{T} = \begin{bmatrix}
1 & 0 & \cdots & 0 & 0 \\
-a_n & 1 & \cdots & 0 & 0 \\
-a_{n-1} & -a_n & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
-a_2 & -a_3 & \cdots & -a_n & 1
\end{bmatrix}.
$$

The vector $\mathbf{a}$ elements a_i are the coefficients of the polynomial of the denominator of (4) and are presented in reverse order and have the opposite sign.

The purpose is to evaluate the elements of the unknown vectors **a** and **b**, the initial vector **x***(0),* and the current vector **x***(k), k=1, 2, …*

3 Solution

A stage algorithm of the adaptive observer based on the method of instrumental variables (IVs)

A computational procedure of the algorithm, which consists of 12 stages, is developed and is shown below:

Stage 1. Formation of matrices and vectors from input-output data [16,17]:

$$\mathbf{u}_1 = \begin{bmatrix} u(0) & u(1) & \cdots & u(N-2) \end{bmatrix},$$

$$\mathbf{y}_1 = \begin{bmatrix} y(0) & y(1) & \cdots & y(N-1) \end{bmatrix},$$

$$\mathbf{y}_2 = \begin{bmatrix} y(n) & y(n+1) & \cdots & y\left(\frac{N-n}{2}+n-1\right) \end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{y}_3 = \begin{bmatrix} y\left(\frac{N-n}{2}+n\right) & y\left(\frac{N-n}{2}+n+1\right) & \cdots & y(N-1) \end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{Y}_{11} = \begin{bmatrix} -y(n-1) & -y(n-2) & \cdots & -y(0) \\ -y(n) & -y(n-1) & \cdots & -y(1) \\ -y(n+1) & -y(n) & \cdots & -y(2) \\ \vdots & \vdots & \ddots & \vdots \\ -y\left(\frac{N-n}{2}+n-2\right) & -y\left(\frac{N-n}{2}+n-3\right) & \cdots & -y\left(\frac{N-n}{2}-1\right) \end{bmatrix},$$

$$\mathbf{Y}_{21} = \begin{bmatrix} -y\left(\frac{N-n}{2}+n-1\right) & -y\left(\frac{N-n}{2}+n-2\right) & \cdots & -y\left(\frac{N-n}{2}\right) \\ -y\left(\frac{N-n}{2}+n\right) & -y\left(\frac{N-n}{2}+n-1\right) & \cdots & -y\left(\frac{N-n}{2}+1\right) \\ -y\left(\frac{N-n}{2}+n+1\right) & -y\left(\frac{N-n}{2}+n\right) & \cdots & -y\left(\frac{N-n}{2}+2\right) \\ \vdots & \vdots & \ddots & \vdots \\ -y(N-2) & -y(N-3) & \cdots & -y(N-n-1) \end{bmatrix},$$

$$\mathbf{U}_{12} = \begin{bmatrix} u(n-1) & u(n-2) & \cdots & u(0) \\ u(n) & u(n-1) & \cdots & u(1) \\ u(n+1) & u(n) & \cdots & u(2) \\ \vdots & \vdots & \ddots & \vdots \\ u\left(\frac{N-n}{2}+n-2\right) & u\left(\frac{N-n}{2}+n-3\right) & \cdots & u\left(\frac{N-n}{2}-1\right) \end{bmatrix},$$

$$\mathbf{U}_{22} = \begin{bmatrix} u\left(\dfrac{N-n}{2}+n-1\right) & u\left(\dfrac{N-n}{2}+n-2\right) & \cdots & u\left(\dfrac{N-n}{2}\right) \\ u\left(\dfrac{N-n}{2}+n\right) & u\left(\dfrac{N-n}{2}+n-1\right) & \cdots & u\left(\dfrac{N-n}{2}+1\right) \\ u\left(\dfrac{N-n}{2}+n+1\right) & u\left(\dfrac{N-n}{2}+n\right) & \cdots & u\left(\dfrac{N-n}{2}+2\right) \\ \vdots & \vdots & \ddots & \vdots \\ u(N-2) & u(N-3) & \cdots & u(N-n-1) \end{bmatrix},$$

where $\mathbf{Y}_{11}$, $\mathbf{Y}_{21}$, $\mathbf{U}_{12}$ and $\mathbf{U}_{22}$ are Toeplitz matrices and $N=3n+2l$, $l=1, 2, 3, ...$

Stage 2. Calculate the sub-matrices $\mathbf{G}_{11}$, $\mathbf{G}_{12}$, $\mathbf{G}_{21}$, $\mathbf{G}_{22}$:

$$\mathbf{G}_{11} = \mathbf{Y}_{11}^{\mathsf{T}}\mathbf{Y}_{11} + \mathbf{Y}_{21}^{\mathsf{T}}\mathbf{Y}_{21}, \ \mathbf{G}_{12} = \mathbf{Y}_{11}^{\mathsf{T}}\mathbf{U}_{12} + \mathbf{Y}_{21}^{\mathsf{T}}\mathbf{U}_{22},$$

$$\mathbf{G}_{21} = \mathbf{U}_{12}^{\mathsf{T}}\mathbf{Y}_{11} + \mathbf{U}_{22}^{\mathsf{T}}\mathbf{Y}_{21}, \ \mathbf{G}_{22} = \mathbf{U}_{12}^{\mathsf{T}}\mathbf{U}_{12} + \mathbf{U}_{22}^{\mathsf{T}}\mathbf{U}_{22}.$$

Stage 3. Calculate the covariance matrix $\mathbf{C}$:

$$\mathbf{C} = \begin{bmatrix} \mathbf{M}_1 + \mathbf{M}_1\mathbf{G}_{12}\mathbf{M}_2\mathbf{G}_{21}\mathbf{M}_1 & \vdots & -\mathbf{M}_1\mathbf{G}_{12}\mathbf{M}_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots & \cdots & \cdots\cdots\cdots\cdots\cdots \\ -\mathbf{M}_2\mathbf{G}_{21}\mathbf{M}_1 & \vdots & \mathbf{M}_2 \end{bmatrix}$$

where

$$\mathbf{M}_1 = \mathbf{G}_{11}^{-1}, \ \mathbf{M}_2 = \left(\mathbf{G}_{22} - \mathbf{G}_{21}\mathbf{M}_1\mathbf{G}_{12}\right)^{-1}.$$

Stage 4. Calculate the vector $\hat{\mathbf{h}}$ and vector $\hat{\mathbf{a}}$ and form the estimated system matrix $\hat{A}$:

$$\hat{\mathbf{p}} = \mathbf{C}\begin{bmatrix} \mathbf{Y}_{11}^{\mathsf{T}}\mathbf{y}_2 + \mathbf{Y}_{21}^{\mathsf{T}}\mathbf{y}_3 \\ \mathbf{U}_{12}^{\mathsf{T}}\mathbf{y}_2 + \mathbf{U}_{22}^{\mathsf{T}}\mathbf{y}_3 \end{bmatrix},$$

$$\hat{\mathbf{h}} = \begin{bmatrix} \hat{h}_1 & \hat{h}_2 & \cdots & \hat{h}_n \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} \hat{p}_{n+1} & \hat{p}_{n+2} & \cdots & \hat{p}_{2n} \end{bmatrix}^{\mathsf{T}},$$

$$\hat{\mathbf{a}} = \begin{bmatrix} \hat{a}_1 & \hat{a}_2 & \cdots & \hat{a}_n \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} -\hat{p}_n & -\hat{p}_{n-1} & \cdots & -\hat{p}_1 \end{bmatrix}^{\mathsf{T}},$$

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{0} & \vdots & \mathbf{I}_{n-1} \\ \cdots & \cdots & \cdots \\ & \hat{\mathbf{a}}^{\mathsf{T}} & \end{bmatrix}.$$

Stage 5. Calculate vector $\mathbf{b}$ estimation by using the linear algebraic system of equations given below:

$$\mathbf{T}\hat{\mathbf{b}} = \hat{\mathbf{h}}$$

where $\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -\hat{a}_n & 1 & 0 & \cdots & 0 & 0 \\ -\hat{a}_{n-1} & -\hat{a}_n & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\hat{a}_2 & -\hat{a}_3 & -\hat{a}_4 & \cdots & -\hat{a}_n & 1 \end{bmatrix}$ is a Toeplitz matrix of lower-triangular type.

Stage 6. Estimate the initial vector of state $\mathbf{x_0}$:

$$\hat{\mathbf{x}}_0 = \left(\mathbf{D}^\mathsf{T}\mathbf{D}\right)^{-1}\mathbf{D}^\mathsf{T}\left(\mathbf{y_1} - \mathbf{Qu_1}\right) = \begin{bmatrix} \hat{x}_{01} & \hat{x}_{02} & \cdots & \hat{x}_{0n} \end{bmatrix}^\mathsf{T}$$

(it is required *det* $(\mathbf{D}^\mathsf{T}\mathbf{D}{\neq}0)$, where

$$\mathbf{D} = \begin{bmatrix} \mathbf{c}^\mathsf{T} \\ \mathbf{c}^\mathsf{T}\hat{\mathbf{A}} \\ \mathbf{c}^\mathsf{T}\hat{\mathbf{A}}^2 \\ \vdots \\ \mathbf{c}^\mathsf{T}\hat{\mathbf{A}}^{(N-1)} \end{bmatrix}_{(N \times n)}, \quad \mathbf{Q} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \mathbf{c}^\mathsf{T}\hat{\mathbf{b}} & 0 & \cdots & 0 \\ \mathbf{c}^\mathsf{T}\hat{\mathbf{A}}\hat{\mathbf{b}} & \mathbf{c}^\mathsf{T}\hat{\mathbf{b}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{c}^\mathsf{T}\hat{\mathbf{A}}^{(N-2)}\hat{\mathbf{b}} & \mathbf{c}^\mathsf{T}\hat{\mathbf{A}}^{(N-3)}\hat{\mathbf{b}} & \cdots & \mathbf{c}^\mathsf{T}\hat{\mathbf{b}} \end{bmatrix}_{(N \times (N-1))}.$$

Stage 7. Compute the output variable $y(k)$ estimation:

$$\left| \begin{aligned} \hat{\mathbf{x}}(k+1) &= \hat{\mathbf{A}}\hat{\mathbf{x}}(k) + \hat{\mathbf{b}}u(k), \quad \hat{\mathbf{x}}(0) = \hat{\mathbf{x}}_0 \\ \hat{y}(k) &= \mathbf{c}^\mathsf{T}\hat{\mathbf{x}}(k), \qquad k = 0,1,2,...,N-1 \end{aligned} \right.$$

$$\hat{\mathbf{F}} = \hat{\mathbf{A}} - \mathbf{g}\mathbf{c}^\mathsf{T}.$$

Stage 8. For the instrumental matrices $\mathbf{V}_{11}$, $\mathbf{V}_{21}$:

$$\mathbf{V}_{11} = \begin{bmatrix} -\hat{y}(n-1) & -\hat{y}(n-2) & \cdots & -\hat{y}(0) \\ -\hat{y}(n) & -\hat{y}(n-1) & \cdots & -\hat{y}(1) \\ -\hat{y}(n+1) & -\hat{y}(n) & \cdots & -\hat{y}(2) \\ \vdots & \vdots & \ddots & \vdots \\ -\hat{y}\left(\dfrac{N-n}{2}+n-2\right) & -\hat{y}\left(\dfrac{N-n}{2}+n-3\right) & \cdots & -\hat{y}\left(\dfrac{N-n}{2}-1\right) \end{bmatrix},$$

$$\mathbf{V_{21}} = \begin{bmatrix} -\hat{y}\left(\dfrac{N-n}{2}+n-1\right) & -\hat{y}\left(\dfrac{N-n}{2}+n-2\right) & \cdots & -\hat{y}\left(\dfrac{N-n}{2}\right) \\[2ex] -\hat{y}\left(\dfrac{N-n}{2}+n\right) & -\hat{y}\left(\dfrac{N-n}{2}+n-1\right) & \cdots & -\hat{y}\left(\dfrac{N-n}{2}+1\right) \\[2ex] -\hat{y}\left(\dfrac{N-n}{2}+n+1\right) & -\hat{y}\left(\dfrac{N-n}{2}+n\right) & \cdots & -\hat{y}\left(\dfrac{N-n}{2}+2\right) \\[2ex] \vdots & \vdots & \ddots & \vdots \\[1ex] -\hat{y}(N-2) & -\hat{y}(N-3) & \cdots & -\hat{y}(N-n-1) \end{bmatrix}$$

Stage 9. Recalculate the submatrices $\mathbf{G_{11}}$, and $\mathbf{G_{12}}$:

$$\mathbf{G_{11}} = \mathbf{V_{11}^{T}Y_{11}} + \mathbf{V_{21}^{T}Y_{21}}, \ \mathbf{G_{12}} = \mathbf{V_{11}^{T}U_{12}} + \mathbf{V_{21}^{T}U_{22}}.$$

Stage 10. Recalculate the vector of the parameters - **p**:

$$\mathbf{M_1} = \mathbf{G_{11}^{-1}}, \ \mathbf{M_2} = \left(\mathbf{G_{22}} - \mathbf{G_{21}M_1G_{12}}\right)^{-1},$$

$$\mathbf{C} = \begin{bmatrix} -\mathbf{M_2G_{22}M_1} & \vdots & \mathbf{M_2} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots & \cdots & \cdots\cdots\cdots\cdots\cdots\cdots \\ \mathbf{M_1} + \mathbf{M_1G_{11}M_2G_{22}M_1} & \vdots & -\mathbf{M_1G_{11}M_2} \end{bmatrix},$$

$$\hat{\mathbf{h}} = \begin{bmatrix} \hat{h}_1 & \hat{h}_2 & \cdots & \hat{h}_n \end{bmatrix}^{T} = \begin{bmatrix} \hat{p}_{n+1} & \hat{p}_{n+2} & \cdots & \hat{p}_{2n} \end{bmatrix}^{T},$$

$$\hat{\mathbf{a}} = \begin{bmatrix} \hat{a}_1 & \hat{a}_2 & \cdots & \hat{a}_n \end{bmatrix}^{T} = \begin{bmatrix} -\hat{p}_n & -\hat{p}_{n-1} & \cdots & -\hat{p}_1 \end{bmatrix}^{T},$$

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{0} & \vdots & \mathbf{I_{n-1}} \\ \cdots & \cdots & \cdots \\ & \hat{\mathbf{a}}^{T} & \end{bmatrix}.$$

Stage 11. Repeat stages 7 to 10 four times

Stage 12. Estimate the current vector of state $\mathbf{x}(k)$:

$$\hat{\mathbf{x}}(k+1) = \hat{\mathbf{F}}\hat{\mathbf{x}}(k) + \hat{\mathbf{b}}u(k) + \mathbf{g}y(k), \ \hat{\mathbf{x}}(0) = \hat{\mathbf{x}}_0, \quad \hat{\mathbf{F}} = \hat{\mathbf{A}} - \mathbf{gc}^{T}.$$

Vector **g** can be easily calculated by solving the so-called pole assignment problem (PAP), also known as a pole placement problem (PPP). The following options must be considered during the synthesis of vector **g**: the eigenvalues of the matrix $\hat{\mathbf{F}}$ must lay within the unit circle more inward than the eigenvalues of the matrix $\hat{\mathbf{A}}$ or must be zero. Implementing the options mentioned above guarantees good dynamic characteristics of the state observer.

4 Simulation Results

A computer experiment is performed in MATLAB by performing the following steps:
✓ The system under investigation is given by a transfer function, with input signal $u(k)$ and the respective output signal $y(k)$;

✓ To the system output is applied (added) colored noise signal *f(k)*;
✓ As input data for the observation algorithm are used: the input signal *u(k)* and the noise-corrupted output signal *y(k)*;
✓ The developed algorithm calculates the object parameters and the state vector estimates based on the input-output data sequences.

For the simulations is used the discrete transfer function of the system investigated and presented as follows:

$$W_{o\delta}(z) = \frac{0.6z^{-1} + 0.56z^{-2} + 0.2125z^{-3} + 0.3080z^{-4} + 0.5488z^{-5} + 0.7221z^{-6}}{1 - 1.4z^{-1} + 0.7875z^{-2} - 0.2275z^{-3} + 0.035525z^{-4} - 0.002835z^{-5} + 0.00009z^{-6}}$$

and its corresponding state space representation:

$$\mathbf{a} = \begin{bmatrix} -0.00009 \\ 0.002835 \\ -0.035525 \\ 0.2275 \\ -0.7875 \\ 1.4 \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} 0.6 \\ 0.2 \\ 0.1 \\ 0.3 \\ 0.4 \\ 0.5 \end{bmatrix}; \quad \mathbf{c} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

The MATLAB function *eig* (.) is implemented to obtain the matrix **A** eigenvalues:

$$eig(\mathbf{A}) = \begin{bmatrix} 0.4 & 0.3 & 0.25 & 0.2 & 0.15 & 0.1 \end{bmatrix}^{\mathsf{T}}.$$

As an input signal *u(k)* is used pseudo-random binary sequence (PRBS), which is generated by using the following MATLAB functions: u=(sign(randn(127,1)))*10.

By adding a color noise *f(k)*, the output signal *y(k)* is noise-corrupted. The following filter

$$W_{\phi}(z) = \frac{1}{1 - 1.4z^{-1} + 0.7875z^{-2} - 0.2275z^{-3} + 0.035525z^{-4} - 0.002835z^{-5} + 0.00009z^{-6}}$$

is used for filtering white noise to obtain the colored noise.

The noise level η is calculated by dividing the noise standard deviation σ_f to the output signal standard deviation σ_y following the following equation:

$$\eta = \frac{\sigma_f}{\sigma_y} 100 = 0 \div 10\% \tag{6}$$

Vector **a** estimation error e_a, vector **b** estimation error e_b, and the state vector **x**(k)

estimation error e_x are relative mean squared errors (RMSE) and could be determined by the equations given below:

$$e_a(k) = -\sqrt{\frac{\sum_{i=1}^{n}\left(a_i(k)-\hat{a}_i(k)\right)^2}{\sum_{i=1}^{n}a_i(k)}} \, , \, e_b(k) = -\sqrt{\frac{\sum_{i=1}^{n}\left(b_i(k)-\hat{b}_i(k)\right)^2}{\sum_{i=1}^{n}b_i(k)}} \, , \, e_x(k) = -\sqrt{\frac{\sum_{i=1}^{n}\left(x_i(k)-\hat{x}_i(k)\right)^2}{\sum_{i=1}^{n}x_i(k)}} \, .(7)$$

In Fig.1 are presented the results for the case of the noise-free output signal ($f(k)=0$), $l = 0$, $N=3n=18$). With these settings, the algorithm will start working at the 18th clock cycle, and in this case, in particular, the observation errors $e_a(k)$, $e_b(k)$, and $e_x(k)$ are equal to zero.
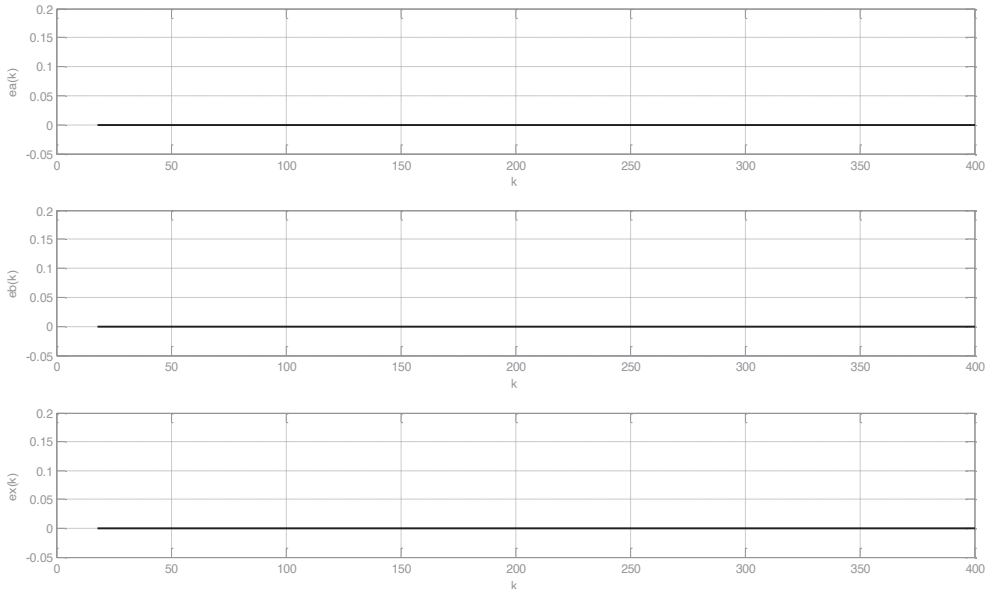

Fig. 1. RMSE for the noise-free output signal case

In the noise-corrupted output signal case, an experiment is held for noise level $\eta = 10.018\%$ and $N = 3n+2*40 = 98$ ($l = 40$). The results are presented in Fig.2. Under the described above initial settings, the algorithm will start working at the 98th calculations step, and the RMSE is as follows: $e_a(k)<0.033$, $e_b(k)<0.01$, $e_x(k)<0.065$.
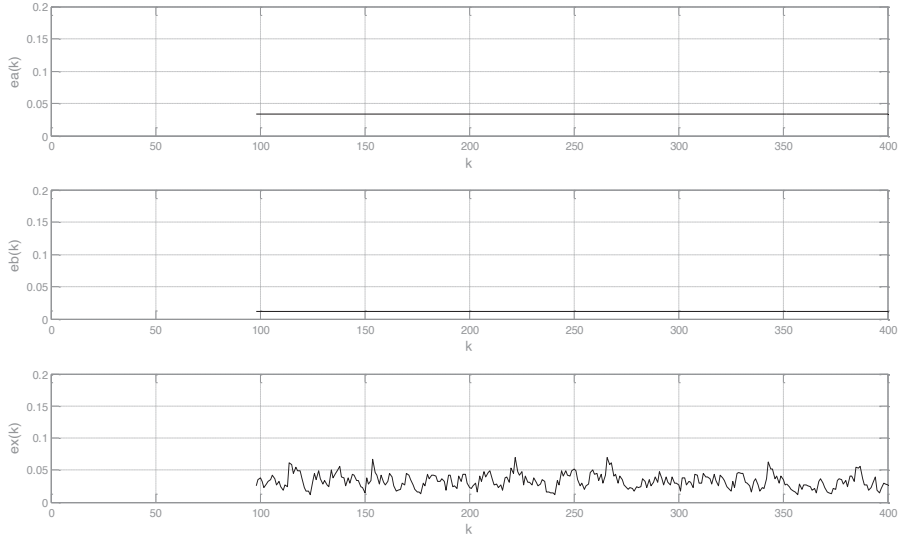
Fig. 2. RMSE for the noise-corrupted output signal case: an experiment is held for noise level $\eta = 10.018\%$, N=98

When the output signal is noise-corrupted with noise level $\eta = 10.014\%$ and $N = 3n + 2*100 = 218$ ($l = 100$). The results are: the algorithm starts at the 218th step of the calculations and the RMSE are: $e_a(k)<0.017$, $e_b(k)<0.0057$, $e_x(k)<0.024$, при $218<k>400$ $e_x(k)=0.01$ (shown in Fig.3).
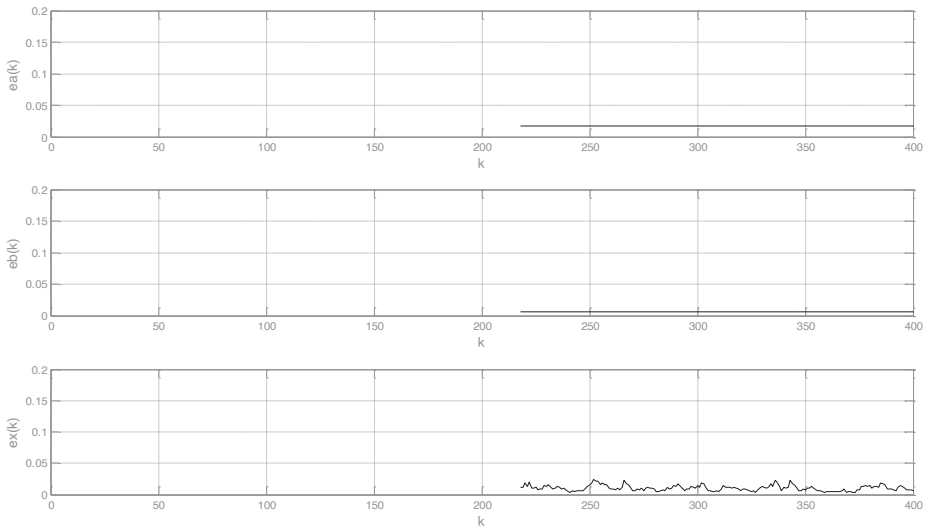

Fig. 3. RMSE for the noise-corrupted output signal case: an experiment is carried out for noise level $\eta = 10.014\%$, N=218

The results obtained by the computer experiment and the analysis of the graphs lead to the conclusion that as the number of input-output measurements (N) increases, the invariance of the algorithm against added noise increases proportionally, but the time required to collect the initial data set increases.

5 Conclusions

The proposed algorithm implements the IVs method to estimate system parameters, which are the basis for further reconstruction of the current state vector. Only at the zero iteration the Least-Squares Method (Stage 1 to Stage 4 of the suggested calculation procedure) is used.

The developed algorithm also estimates the initial state vector x_0, which allows the matrix of instrumental variables to be formed even under non-zero initial conditions.

The obtained results show that the number of input-output data measurements (N) is of great importance in terms of the accuracy of the estimations in the case of a noise-corrupted output. The highest accuracy should be expected for the highest number of N (Fig.2, Fig.3).

The method of the IVs guarantees the best results in the case of a priori collected data estimation [16,17]; however, concerning the closed-loop system, the added noise $f(k)$ is applied to the input signal through the feedback. Hence invariance between the instrumental matrices and the added noise is not possible. Implementing the IVs method to investigate the closed-loop system is only applicable if the additional input signal is applied [17-19]. Thus, implementing the algorithm proposed in the present work is not recommended for closed systems.

The convergence of the iterative procedure in the AO algorithm based on the IVs method proposed in the present paper is ensured by implementing the non-recurrent method [13,20].

However, this algorithm's main advantage is related to the method used to form the informative matrix. The four sub-matrices Y_{11}, Y_{21}, U_{12}, and U_{22} are used for the formation mentioned, which leads to reducing in the calculation complexity of the matrix G (formed by the sub-matrices G_{11}, G_{12}, G_{21}, G_{22}.) inversion procedure. Regardless of the number N for the coefficients, h_i and a_i estimation inversion of the matrices G_{11} and $(G_{22}-G_{21}M_1G_{12})$, which are always (n x n) dimensional, is only needed. In all other cases, this procedure requires a matrix that is at least (N-n) x (N-n) dimensional to be inverted.

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CERTIFICATE OF PARTICIPATION

Presented to

Nikola Nikolov, Mariela Alexandrova, Shakhnoza Ubaydullayeva, Rano Gaziyeva, Khafizova Zulfiya and Dilorom Ubaydullayeva

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