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ECONOMY. ECONOMIC SCIENCE. OTHER BRANCHES OF THE ECONOMY.

ADAPTIVE OBSERVATION VIA THE INSTRUMENTAL VARIABLES METHOD IN CONTROL SYSTEMS WITH STATE FEEDBACK

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Abstract

When the input signal and the output value of the object of control cannot be measured accurately, the estimation of the state vector is performed. The instrumental variables (IVs) method is one of the commonly used parameter estimation methods [1-10]. The task of adaptive observation is to create state observers containing parameter estimators. In adaptive observers, the matrices A and b or c (depending on the chosen canonical state-space representation form) are assumed to be unknown. In the monitoring process, parameter estimation is performed, the unknown matrices are determined, and then the state vector is calculated. The present paper aims to present a non-recurrent adaptive observation algorithm for SISO linear time-invariant (LTI) discrete systems. The algorithm is based on the instrumental variables (IVs) method and the adaptive state observer (ASO) estimates the parameters, the initial and the current state vectors of the discrete system. The algorithm workability and effectiveness are proved by using simulation data in the MATLAB/Simulink.

Keywords: *adaptive observation, discrete state observer, non-recursive algorithm, initial vector of state, current vector of state, estimation.*

Introduction. For the design of control systems with state feedback, it is often necessary to recover the state vector by using measurements of the output and the input signals of the controlled object.

For reconstruction of the state vector, an implementation of a state observer is necessary. The process of adaptive observation involves creation of an observer with a parameter estimator [11, 12]. The matrices A and b or c (depending on the canonical form for representation of the object in the state space) are considered unknown.

During the observation process, the unknown parameters are estimated, the unknown matrices are determined, and the state vector is calculated.

This paper presents a non-recursive algorithm based on the IVs method [13] for adaptive observation of SISO LTI discrete systems.

The parameters estimator in the adaptive observer is built on the basis of a mathematical procedure with low computational complexity when inverting the information matrix, which is presented in [14].

Materials and methods. Consider a system which is mathematically described in the state space as follows:

$$x(k+1) = Ax(k) + bu(k), \qquad x(0) = x_0$$
$$y(k) = c^T x(k) + f(k), \qquad k = 0,1,2... \qquad (1)$$

where:

$$A = \begin{vmatrix} 0 & \dots & I_{n-1} \\ \dots & \dots & \dots \\ & a^T & \end{vmatrix} \qquad (2)$$

$$a = \begin{vmatrix} a_0 \\ a_1 \\ \dots \\ a_n \end{vmatrix} \qquad b = \begin{vmatrix} b_0 \\ b_1 \\ \dots \\ b_n \end{vmatrix} \qquad c = \begin{vmatrix} 1 \\ 0 \\ \dots \\ 0 \end{vmatrix} \qquad (3)$$

The system order n is a-priori known, $x(0) \in R^n$ is the unknown initial vector of state, $x(k) \in Rn$ is the unknown current vector of state, $u(k) \in R^1$ is a scalar input signal, $y(k) \in R^1$ is a scalar output signal, f(k) is an additionally added noise signal, **a** and **b** are unknown vectors.

The state space description (1) corresponds to the following discrete transfer function:

$$W(z) = \frac{h_1 z^{n-1} + h_2 z^{n-2} + \dots + h_{n-1} z + h_n}{z^n - a_n z^{n-1} - \dots - a_2 z - a_1} \qquad (4)$$

The elements b_i of vector **b** in the chosen phase-coordinate canonical form, are calculated by the coefficients h_i of the polynomial in the numerator and the coefficients a_i of the polynomial in the denominator of (4) as follows [15]:

Tb=h, (5)

where:

$$\mathbf{h}^T = \begin{bmatrix} h_1 & h_2 & \cdots & h_n \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ -a_n & 1 & \cdots & 0 & 0 \\ -a_{n-1} & -a_n & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -a_2 & -a_3 & \cdots & -a_n & 1 \end{bmatrix}. \qquad (5)$$

The vector **a** elements a_i are the coefficients of the polynomial of the denominator of (4), and are presented in reverse order and have the opposite sign.

The purpose is to evaluate the elements of the unknown vectors **a** and **b**, the initial vector **x(0)** and the current vector **x(k)**, k=1, 2, ...

A stage algorithm of the adaptive observer based on the method of instrumental variables (IVs)

A computational procedure of the algorithm, which consists of 12 stages, is developed and is shown below:

Stage 1. Formation of matrices and vectors from input-output data [16,17]:

$$\mathbf{u}_1 = \begin{bmatrix} u(0) & u(1) & \cdots & u(N-2) \end{bmatrix},$$
$$\mathbf{y}_1 = \begin{bmatrix} y(0) & y(1) & \cdots & y(N-1) \end{bmatrix},$$
$$\mathbf{y}_2 = \begin{bmatrix} y(n) & y(n+1) & \cdots & y\left(\frac{N-n}{2}+n-1\right) \end{bmatrix}^T,$$
$$\mathbf{y}_3 = \begin{bmatrix} y\left(\frac{N-n}{2}+n\right) & y\left(\frac{N-n}{2}+n+1\right) & \cdots & y(N-1) \end{bmatrix}^T,$$
$$\mathbf{Y}_{11} = \begin{bmatrix} -y(n-1) & -y(n-2) & \cdots & -y(0) \\ -y(n) & -y(n-1) & \cdots & -y(1) \\ -y(n+1) & -y(n) & \cdots & -y(2) \\ \vdots & \vdots & \ddots & \vdots \\ -y\left(\frac{N-n}{2}+n-2\right) & -y\left(\frac{N-n}{2}+n-3\right) & \cdots & -y\left(\frac{N-n}{2}-1\right) \end{bmatrix},$$

$$\mathbf{Y}_{21}=\begin{bmatrix} -y\left(\frac{N-n}{2}+n-1\right) & -y\left(\frac{N-n}{2}+n-2\right) & \cdots & -y\left(\frac{N-n}{2}\right. \\ -y\left(\frac{N-n}{2}+n\right) & -y\left(\frac{N-n}{2}+n-1\right) & \cdots & -y\left(\frac{N-n}{2}\right. \\ -y\left(\frac{N-n}{2}+n+1\right) & -y\left(\frac{N-n}{2}+n\right) & \cdots & -y\left(\frac{N-n}{2}\right. \\ \vdots & \vdots & \ddots & \vdots \\ -y(N-2) & -y(N-3) & \cdots & -y(N-n \end{bmatrix}$$

$$\mathbf{U}_{12}=\begin{bmatrix} u(n-1) & u(n-2) & \cdots & u(0) \\ u(n) & u(n-1) & \cdots & u(1) \\ u(n+1) & u(n) & \cdots & u(2) \\ \vdots & \vdots & \ddots & \vdots \\ u\left(\frac{N-n}{2}+n-2\right) & u\left(\frac{N-n}{2}+n-3\right) & \cdots & u\left(\frac{N-n}{2}-1\right) \end{bmatrix},$$

$$\mathbf{U}_{22}=\begin{bmatrix} u\left(\frac{N-n}{2}+n-1\right) & u\left(\frac{N-n}{2}+n-2\right) & \cdots & u\left(\frac{N-n}{2}\right) \\ u\left(\frac{N-n}{2}+n\right) & u\left(\frac{N-n}{2}+n-1\right) & \cdots & u\left(\frac{N-n}{2}+1\right) \\ u\left(\frac{N-n}{2}+n+1\right) & u\left(\frac{N-n}{2}+n\right) & \cdots & u\left(\frac{N-n}{2}+2\right) \end{bmatrix} \quad (6)$$

where **Y11, Y21, U12** and **U22** are Toeplitz matrices and N=3n+2l, l=1, 2, 3, ...

Stage 2. Calculate the sub-matrices **G11, G12, G21, G22:**

$$\mathbf{G}_{11}=\mathbf{Y}_{11}^{\mathrm{T}}\mathbf{Y}_{11}+\mathbf{Y}_{21}^{\mathrm{T}}\mathbf{Y}_{21},\quad \mathbf{G}_{12}=\mathbf{Y}_{11}^{\mathrm{T}}\mathbf{U}_{12}+\mathbf{Y}_{21}^{\mathrm{T}}\mathbf{U}_{22},$$
$$\mathbf{G}_{21}=\mathbf{U}_{12}^{\mathrm{T}}\mathbf{Y}_{11}+\mathbf{U}_{22}^{\mathrm{T}}\mathbf{Y}_{21},\quad \mathbf{G}_{22}=\mathbf{U}_{12}^{\mathrm{T}}\mathbf{U}_{12}+\mathbf{U}_{22}^{\mathrm{T}}\mathbf{U}_{22}.$$

Stage 3. Calculate the covariance matrix **C**:

$$\mathbf{C}=\begin{bmatrix} \mathbf{M}_1+\mathbf{M}_1\mathbf{G}_{12}\mathbf{M}_2\mathbf{G}_{21}\mathbf{M}_1 & \vdots & -\mathbf{M}_1\mathbf{G}_{12}\mathbf{M}_2 \\ \cdots\cdots\cdots & \cdots & \cdots\cdots\cdots \\ -\mathbf{M}_2\mathbf{G}_{21}\mathbf{M}_1 & \vdots & \mathbf{M}_2 \end{bmatrix}$$

where:

$$\mathbf{M}_1=\mathbf{G}_{11}^{-1},\quad \mathbf{M}_2=\left(\mathbf{G}_{22}-\mathbf{G}_{21}\mathbf{M}_1\mathbf{G}_{12}\right)^{-1}.$$

Stage 4. Calculate the vector $\hat{\mathbf{h}}$ and $\hat{\mathbf{a}}$ vector and form the estimated system matrix $\hat{\mathbf{A}}$:

$$\hat{\mathbf{p}}=\mathbf{C}\begin{bmatrix} \mathbf{Y}_{11}^{\mathrm{T}}\mathbf{y}_2+\mathbf{Y}_{21}^{\mathrm{T}}\mathbf{y}_3 \\ \mathbf{U}_{12}^{\mathrm{T}}\mathbf{y}_2+\mathbf{U}_{22}^{\mathrm{T}}\mathbf{y}_3 \end{bmatrix},$$

$$\hat{\mathbf{h}}=\begin{bmatrix}\hat{h}_1 & \hat{h}_2 & \cdots & \hat{h}_n\end{bmatrix}^{\mathrm{T}}=\begin{bmatrix}\hat{p}_{n+1} & \hat{p}_{n+2} & \cdots & \hat{p}_{2n}\end{bmatrix}^{\mathrm{T}},$$

$$\hat{\mathbf{a}}=\begin{bmatrix}\hat{a}_1 & \hat{a}_2 & \cdots & \hat{a}_n\end{bmatrix}^{\mathrm{T}}=\begin{bmatrix}-\hat{p}_n & -\hat{p}_{n-1} & \cdots & -\hat{p}_1\end{bmatrix}^{\mathrm{T}},$$

$$\hat{\mathbf{A}}=\begin{bmatrix} \mathbf{0} & \vdots & \mathbf{I}_{n-1} \\ \cdots & \cdots & \cdots \\ & \hat{\mathbf{a}}^{\mathrm{T}} & \end{bmatrix}.$$

Stage 5. Calculate vector **b** estimation by using the linear algebraic system of equations given below:

$$\mathbf{T}\hat{\mathbf{b}}=\hat{\mathbf{h}},$$

where $\mathbf{T}=\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -\hat{a}_n & 1 & 0 & \cdots & 0 & 0 \\ -\hat{a}_{n-1} & -\hat{a}_n & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\hat{a}_2 & -\hat{a}_3 & -\hat{a}_4 & \cdots & -\hat{a}_n & 1 \end{bmatrix}$ is a Toeplitz matrix of lower-triangular type.

Stage 6. Estimate the initial vector of state **x0:**

$$\hat{\mathbf{x}}_0=\left(\mathbf{D}^{\mathrm{T}}\mathbf{D}\right)^{-1}\mathbf{D}^{\mathrm{T}}(\mathbf{y}_1-\mathbf{Q}\mathbf{u}_1)=\begin{bmatrix}\hat{x}_{01} & \hat{x}_{02} & \cdots & \hat{x}_{0n}\end{bmatrix}^{\mathrm{T}}$$

(it is required *det* ($\mathbf{D}^{\mathrm{T}}\mathbf{D}\neq 0$), where

$$\mathbf{D}=\begin{bmatrix} \mathbf{c}^{\mathrm{T}} \\ \mathbf{c}^{\mathrm{T}}\hat{\mathbf{A}} \\ \mathbf{c}^{\mathrm{T}}\hat{\mathbf{A}}^2 \\ \vdots \\ \mathbf{c}^{\mathrm{T}}\hat{\mathbf{A}}^{(N-1)} \end{bmatrix}_{(N\times n)},\quad \mathbf{Q}=\begin{bmatrix} 0 & 0 & \cdots & 0 \\ \mathbf{c}^{\mathrm{T}}\hat{\mathbf{b}} & 0 & \cdots & 0 \\ \mathbf{c}^{\mathrm{T}}\hat{\mathbf{A}}\hat{\mathbf{b}} & \mathbf{c}^{\mathrm{T}}\hat{\mathbf{b}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{c}^{\mathrm{T}}\hat{\mathbf{A}}^{(N-2)}\hat{\mathbf{b}} & \mathbf{c}^{\mathrm{T}}\hat{\mathbf{A}}^{(N-3)}\hat{\mathbf{b}} & \cdots & \mathbf{c}^{\mathrm{T}}\hat{\mathbf{b}} \end{bmatrix}_{(N\times(N-1))}.$$

Stage 7.Compute the output variable **y(k)** estimation:

$$\begin{cases} \hat{\mathbf{x}}(k+1)=\hat{\mathbf{A}}\hat{\mathbf{x}}(k)+\hat{\mathbf{b}}u(k), & \hat{\mathbf{x}}(0)=\hat{\mathbf{x}}_0, \\ \hat{y}(k)=\mathbf{c}^{\mathrm{T}}\hat{\mathbf{x}}(k), & k=0,1,2,\dots,N-1 \end{cases}$$

$$\hat{\mathbf{F}}=\hat{\mathbf{A}}-\mathbf{g}\mathbf{c}^{\mathrm{T}}$$

Stage 8.For the instrumental matrices **V11, V21:**

$$\mathbf{V}_{11}=\begin{bmatrix} -\hat{y}(n-1) & -\hat{y}(n-2) & \cdots & -\hat{y}(0) \\ -\hat{y}(n) & -\hat{y}(n-1) & \cdots & -\hat{y}(1) \\ -\hat{y}(n+1) & -\hat{y}(n) & \cdots & -\hat{y}(2) \\ \vdots & \vdots & \ddots & \vdots \\ -\hat{y}\left(\frac{N-n}{2}+n-2\right) & -\hat{y}\left(\frac{N-n}{2}+n-3\right) & \cdots & -\hat{y}\left(\frac{N-n}{2}-1\right) \end{bmatrix},$$

$$\mathbf{V}_{21}=\begin{bmatrix} -\hat{y}\left(\frac{N-n}{2}+n-1\right) & -\hat{y}\left(\frac{N-n}{2}+n-2\right) & \cdots & -\hat{y}\left(\frac{N-n}{2}\right) \\ -\hat{y}\left(\frac{N-n}{2}+n\right) & -\hat{y}\left(\frac{N-n}{2}+n-1\right) & \cdots & -\hat{y}\left(\frac{N-n}{2}+1\right) \\ -\hat{y}\left(\frac{N-n}{2}+n+1\right) & -\hat{y}\left(\frac{N-n}{2}+n\right) & \cdots & -\hat{y}\left(\frac{N-n}{2}+2\right) \\ \vdots & \vdots & \ddots & \vdots \\ -\hat{y}(N-2) & -\hat{y}(N-3) & \cdots & -\hat{y}(N-n-1) \end{bmatrix}$$

Stage 9. Recalculate the submatrices G11, and G12:

$$\mathbf{G}_{11}=\mathbf{V}_{11}^{\mathrm{T}}\mathbf{Y}_{11}+\mathbf{V}_{21}^{\mathrm{T}}\mathbf{Y}_{21},\quad \mathbf{G}_{12}=\mathbf{V}_{11}^{\mathrm{T}}\mathbf{U}_{12}+\mathbf{V}_{21}^{\mathrm{T}}\mathbf{U}_{22}.$$

Stage 10. Recalculate the vector of the parameters - p:

$$\mathbf{M}_1=\mathbf{G}_{11}^{-1},\quad \mathbf{M}_2=\left(\mathbf{G}_{22}-\mathbf{G}_{21}\mathbf{M}_1\mathbf{G}_{12}\right)^{-1},$$

$$\mathbf{C}=\begin{bmatrix} -\mathbf{M}_2\mathbf{G}_{22}\mathbf{M}_1 & \vdots & \mathbf{M}_2 \\ \cdots\cdots\cdots & \cdots & \cdots\cdots\cdots \\ \mathbf{M}_1+\mathbf{M}_1\mathbf{G}_{11}\mathbf{M}_2\mathbf{G}_{22}\mathbf{M}_1 & \vdots & -\mathbf{M}_1\mathbf{G}_{11}\mathbf{M}_2 \end{bmatrix},$$

$$\hat{\mathbf{h}}=\begin{bmatrix}\hat{h}_1 & \hat{h}_2 & \cdots & \hat{h}_n\end{bmatrix}^{\mathrm{T}}=\begin{bmatrix}\hat{p}_{n+1} & \hat{p}_{n+2} & \cdots & \hat{p}_{2n}\end{bmatrix}^{\mathrm{T}},$$

$$\hat{\mathbf{a}}=\begin{bmatrix}\hat{a}_1 & \hat{a}_2 & \cdots & \hat{a}_n\end{bmatrix}^{\mathrm{T}}=\begin{bmatrix}-\hat{p}_n & -\hat{p}_{n-1} & \cdots & -\hat{p}_1\end{bmatrix}^{\mathrm{T}},$$

$$\hat{\mathbf{A}}=\begin{bmatrix} \mathbf{0} & \vdots & \mathbf{I}_{n-1} \\ \cdots & \cdots & \cdots \\ & \hat{\mathbf{a}}^{\mathrm{T}} & \end{bmatrix}.$$

Stage 11. Repeat stages 7 to 10 four times

Stage 12. Estimate the current vector of state x(k):

$$\hat{\mathbf{x}}(k+1)=\hat{\mathbf{F}}\hat{\mathbf{x}}(k)+\hat{\mathbf{b}}u(k)+\mathbf{g}y(k),\quad \hat{\mathbf{x}}(0)=\hat{\mathbf{x}}_0,\quad \hat{\mathbf{F}}=\hat{\mathbf{A}}-\mathbf{g}\mathbf{c}^{\mathrm{T}}.$$

Vector g can be easily calculated by solving the so called pole assignment problem (PAP), also known as a pole placement problem (PPP). The following options must be taken into consideration during the synthesis of vector g: the eigenvalues of the matrix must lay within the unit circle more inward than the eigenvalues of matrix or must be zero. The implementation of the options mentioned above guarantees good dynamic characteristics of the state observer.

Results and discussion. A computer experiment is performed in MATLAB by performing the following steps:

- The system under investigation is given by a transfer function, with input signal u(k) and the respective output signal y(k);
- To the system output is applied (added) colored noise signal f(k);
- As an input data for the observation algorithm are used: the input signal u(k) and the noise-corrupted output signal y(k);
- The developed algorithm calculates the estimates of the object parameters and the state vector based on the input-output data sequences.

For the simulations is used the discrete transfer function of the system investigated, presented as follows:

$$W_{o6}(z)=\frac{0.6z^{-1}+0.56z^{-2}+0.2125z^{-3}+0.3080z^{-4}+0.5488z^{-5}+0.7221z^{-6}}{1-1.4z^{-1}+0.7875z^{-2}-0.2275z^{-3}+0.035525z^{-4}-0.002835z^{-5}+0.00009z^{-6}}$$

and its corresponding state space representation:

$$\mathbf{a}=\begin{bmatrix}-0.00009\\0.002835\\-0.035525\\0.2275\\-0.7875\\1.4\end{bmatrix};\quad \mathbf{b}=\begin{bmatrix}0.6\\0.2\\0.1\\0.3\\0.4\\0.5\end{bmatrix};\quad \mathbf{c}=\begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix};\quad \mathbf{x}(0)=\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}.$$

The MATLAB function eig (.) is implemented to obtain the matrix A eigenvalues:

$$eig(\mathbf{A})=\begin{bmatrix}0.4 & 0.3 & 0.25 & 0.2 & 0.15 & 0.1\end{bmatrix}^{T}.$$

As an input signal u(k) is used pseudo-random binary sequence (PRBS) which is generated by using the following MATLAB functions: u=(sign(randn(127,1)))*10.

By adding a color noise f(k) the output signal y(k) is noise-corrupted. The following filter

$$W_{\varphi}(z)=\frac{1}{1-1.4z^{-1}+0.7875z^{-2}-0.2275z^{-3}+0.035525z^{-4}-0.002835z^{-5}+0.00009z^{-6}}$$

is used for filtering of white noise in order to obtain the colored noise.

The noise level η is calculated by dividing the noise standard deviation σ_f to the output signal standard deviation σ_y in accordance with the following equation:

$$\eta=\frac{\sigma_f}{\sigma_y}100=0\div 10\% \qquad .(6)$$

Vector a estimation error e_a, vector b estimation error e_b and the state vector x(k) estimation error e_x are relative mean squared errors (RMSE) and could be determined by the equations given below:

$$e_a(k)=-\sqrt{\frac{\sum_{i=1}^{n}\left(a_i(k)-\hat{a}_i(k)\right)^2}{\sum_{i=1}^{n}a_i(k)}},\quad e_b(k)=-\sqrt{\frac{\sum_{i=1}^{n}\left(b_i(k)-\hat{b}_i(k)\right)^2}{\sum_{i=1}^{n}b_i(k)}},\quad e_x(k)=-\sqrt{\frac{\sum_{i=1}^{n}\left(x_i(k)-\hat{x}_i(k)\right)^2}{\sum_{i=1}^{n}x_i(k)}} \qquad .(7)$$

In Fig.1 are presented the results for the case of noise-free output signal (f(k)=0), ⊠ = 0, N=3n=18). With these settings, the algorithm will start working at the 18th clock cycle and in this case in particular the observation errors $e_a(k)$, $e_b(k)$ and $e_x(k)$ are equal to zero.

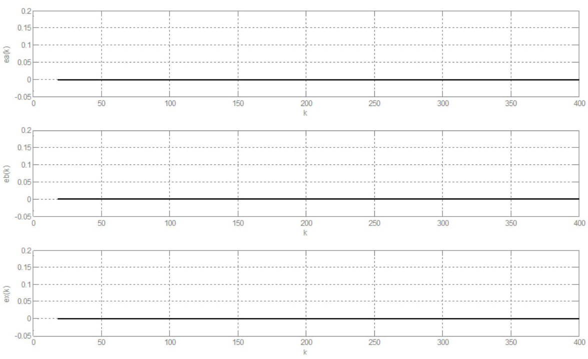

Fig. 1. RMSE for the noise-free output signal case

In the noise-corrupted output signal case an experiment is held for noise level η = 10.018% and N = 3n+2*40 = 98 (*l* = 40). The results are presented in Fig.2. Under the above described initial settings the algorithm will start working at the 98th calculations step and the RMSE are as follows: $e_a(k)<0.033$, $e_b(k)<0.01$, $e_x(k)<0.065$.

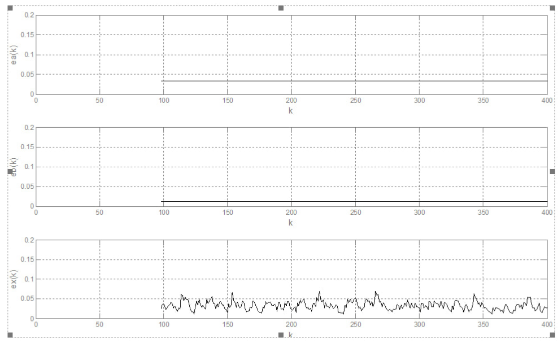

Fig. 2. RMSE for the noise-corrupted output signal case: an experiment is held for noise level ⊠ = 10.018%, N=98

When the output signal is noise-corrupted with noise level η = 10.014% and N = 3n + 2*100 = 218 (*l* = 100). The results are: the algorithm starts at the 218th step of the calculations and the RMSE are: $e_a(k)<0.017$, $e_b(k)<0.0057$, $e_x(k)<0.024$, при 218<k>400 $e_x(k)=0.01$ (shown in Fig.3)

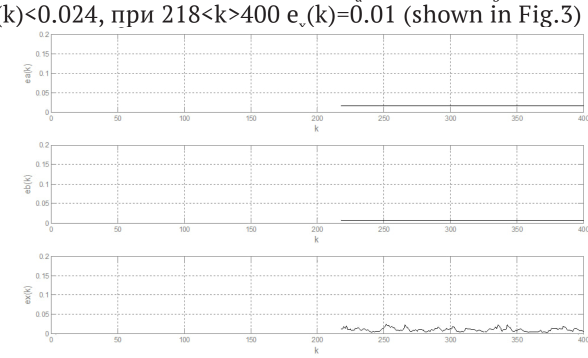

Fig. 3. RMSE for the noise-corrupted output signal case: an experiment is carried out for noise level η = 10.014%, N=218

The results obtained by the computer experiment and the analysis of the graphs lead to the conclusion that as the number of input-output measurements (*N*) increases, the invariance of the algorithm against added noise increases proportionally, but the time required to collect the initial data set increases.

Conclusion. The proposed algorithm implements the IVs method to estimate system parameters, which are the basis for further reconstruction of the current state vector. Only at the zero iteration the Least-Squares Method (Stage 1 to Stage 4 of the suggested calculation procedure) is used.

The developed algorithm also estimates the initial state vector x0 which allows the matrix of instrumental variables to be formed even under non-zero initial conditions.

The obtained results show that the number of input output data measurements (*N*) is of great importance in terms of the accuracy of the estimations in the case of a noise-corrupted output. The highest accuracy should be expected for the highest number of *N* (Fig.2, Fig.3).

The method of the IVs guarantees best results in the case of a-priori collected data estimation [16,17], however in relation to the closed loop system the added noise f(*k*) is applied to the input signal through the feedback. Hence invariance between the instrumental matrices and the added noise is not possible. The implementation of IVs method for investigation of the closed loop system is only applicable if additional input signal is applied [17-19].

Thus the implementation of the algorithm proposed in the present work is not recommended for closed systems.

The convergence of the iterative procedure in the AO algorithm based on the IVs method and proposed in the present paper, is ensured by implementing of non-recurrent method [13, 20].

The main advantage of this algorithm however is related to the method used for formation of the informative matrix. The four sub-matrices **Y11, Y21, U12, and U22** are used for the formation mentioned which leads to reducing of the calculation complexity of the matrix G (formed by the sub-matrices **G11, G12, G21, G22.**) inversion procedure. Regardless the number *N* for the coefficients h_i and a_i estimation inversion of the matrices **G11 and (G22 G21M1G12)**, which are always (*n x n*) dimensional, is only needed. In all other cases this procedure requires a matrix which is at least (*N* - *n*) x (*N* - *n*) dimensional to be inverted.

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