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Theoretical bases of hydraulic mixture in round cylindrical pipelines

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Theoretical bases of hydraulic mixture in round cylindrical pipelines

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Abstract. Hydro transport is characterized by the combined movement of liquid and solid particles, which in a mixture form two-phase or multiphase flows with different physical and mechanical properties. One of the main tasks of hydraulic transport is to study the throughput of pipelines, where taking into account the distribution of sediment concentration over the cross section of the pipeline formed under the influence of gravitational force is of great importance in describing the nature of the two-phase flow. Pressure-bearing suspended flows in hydro transport systems are usually characterized by high volumetric concentrations and a wide range of sizes and densities of solid particles that are part of hydraulic mixtures. The flows under consideration are more complex in their structure than turbulent flows of homogeneous liquids in pipes. Therefore, the methods for calculating these flows are much more complicated than the usual methods of hydraulics of pressure flows of homogeneous liquids. To develop scientifically based methods for calculating the parameters of the movement of hydraulic mixtures in pipes, widely used the averaged equations of the weighted flow and experimental data. When moving slurries through pipes, gravity plays a significant role. The article discusses the influence of gravity on the throughput of pressure systems with different slopes and obtains new dependences of the flow rate of turbid flow on the slope of the pressure pipe.

1.   Introduction

To identify the functional dependence of the kinematic and dynamic parameters of the weighted flow on the values that determine them, to date, much attention has been paid primarily to the experimental study of the hydrotransport of various solid materials. A significant number of works have been devoted to experimental studies and the development of engineering methods for calculating pressure-bearing suspended flows.

V.M. Makkaveev for the first time, derived the general differential equation of turbulent diffusion for the case of motion of suspended particles in a water stream, assuming that the presence of particles does not violate the kinematics of the stream and obtained the following differential dependence:

$$\frac{ds}{dt} = \frac{1}{\rho}\left[\frac{\partial}{\partial x}\left(A\frac{\partial s}{\partial x} \right) + \frac{\partial}{\partial y}\left(A\frac{\partial s}{\partial y} \right) + \frac{\partial}{\partial z}\left(A\frac{\partial s}{\partial z} \right) - w\frac{\partial s}{\partial y} \right] \tag{1}$$

there: s - is the concentration of suspended particles in water;

A - is the coefficient of turbulent exchange proportional to the coefficient of turbulent diffusion $\varepsilon = A/\rho$;

ρ - flow density;

w - hydraulic size of particle.

For the case of a plane uniform stationary flow with the boundary condition $y = 0$, $s = s_0$, integrating twice (1), obtained the formula for the vertical distribution of the concentration of sediment particles:

$$s(y) = s_0 \exp\left\{ - w \int_0^y \frac{dy}{\varepsilon} \right\} \qquad (2)$$

The application of equation (1) or dependence (2) in practice requires the determination of the coefficient of turbulent diffusion ε.

Further developments in the development of the diffusion theory of turbulence were aimed mainly at determining the value of the coefficient of turbulent diffusion. Moreover, the obtained formulas differed only due to the use of different assumptions about the velocity profile.

A. Karaushev and others, developing research on the distribution of sediment concentration, apply the elliptic law of velocity distribution over the flow cross section. M. Velikanov, L. Prandtl, H. Rauz and other researchers apply the logarithmic law of velocity distribution over the vertical flow section.

However, the use of diffusion theory in many cases is limited, due to the use of some assumptions. In these works, the physical nature of the turbulent exchange coefficient is still not completely clear.

Studies by a number of authors [1, 2] and others indicate a change in the parameters (ρ, χ, m, d, etc.) included in the formula for the law of velocity distribution over depth flow over a wide range. And this limits the scope of the obtained formulas. by distribution of sediment concentration. In addition, the analysis of the concentration distribution formula for the depth of flow obtained by the above scientists gives conflicting conclusions.

The main reason for the limited practical application of the above theories is the openness of the system of differential equations and the neglect of the influence of the transported phase on the velocity distribution, and it is of no small importance in studying the laws of the nature of the two-phase flow.

The model of motion of interpenetrating mixtures by Kh. Rakhmatullin made it possible to obtain a closed system of equations of motion, with the help of which one can theoretically study many phenomena that occur in a mixture during its movement. The theory of interpenetrating media was developed in the works and dependences were obtained, confirmed by experimental data.

A. Arifjanov and other [3], based on the convergence of the random motion of particles in a suspended layer under the influence of hydrodynamic flow forces and gravitational field forces with the thermal motion of gas molecules, as a result of processing the experimental data, they proposed a calculation formula for the distribution concentration, reminiscent of the barometric formula of the molecular-kinetic theory of gases:

$$n = n_0 e^{-A_1 h} \qquad (3)$$

there: n – is the number of particles at a certain height h;

n_0 – is the number of particles at the zero level;

A_1 – is a coefficient proportional to the kinetic energy of moving particles.

Starting from a fraction with a diameter of d_0 and a mass of mr_0, the role of gravity becomes much less than the resistance of the flow. As a result of this, in this parameter range, the size of the fraction practically does not affect the concentration distribution. Therefore, to establish the influence of the size of the fractions, the concept of the optimal diameter is introduced.

A particle with a diameter d_0, which velocity is equal to the average flow velocity, will be called an "optimal particle". The depth distribution of the "optimal particles" will be considered "normal" if it obeys an exponential law. The diameter of this particle will be called the "optimal diameter". From the equilibrium conditions of gravity and resistance forces, we determine the optimal particle diameter, for free-flow motion and in the form:

$$d_0 = \sqrt{\frac{18\mu u \sin\alpha}{g(\rho_r - \rho)}} \qquad (4)$$

for pressure movement:

$$d_0 = \sqrt{\frac{18\mu u \frac{d}{dz}\left(\frac{P}{\gamma}\right)}{g(\rho_r - \rho)}} \qquad (5)$$

The above review shows that at present, despite certain successes in the creation of mathematical models for the motion of multiphase (two-phase) flows, the development of a general theory of suspended flows is not completed [4–9].

For the successful application of existing systems of differential equations in practice, it is required to close them with simple additional dependencies [10–14].

The question of the distribution of the averaged longitudinal velocity component over the cross section of the multiphase media flow through horizontal cylindrical pipelines is one of the complex and unsolved problems [15–19].

However, in these works are considered:
- either a uniform distribution of concentration or an implicit effect of the flow rate on the change in the concentration of the transported medium
- the phase interaction coefficient is taken as a constant value;
- for a round horizontal pipe, tasks are reduced to a view symmetrical with respect to the axis or separate flows are considered.

The uneven distribution of sediment concentration significantly complicates the solution of the differential equations of motion compared with a similar problem for symmetric flows relative to the axis of the pipeline. In this regard, many researchers limited themselves, as a rule, to the construction of a profile of averaged velocities.

It is characteristic of these works that they use a single-speed model of the mixture motion, i.e. the slurry in its movement is identified with a fictitious single-speed continuum of a density-variable with respect to height.

2. Methods and Materials

The task is to determine the throughput of the pipeline during the movement of a two-phase mixture through a round cylindrical pipe with a diameter D for given hydrotransport parameters, such as volume concentration of solid particles in the flow, particle size distribution, average linear size and hydraulic size of these particles, density and viscosity of phases.
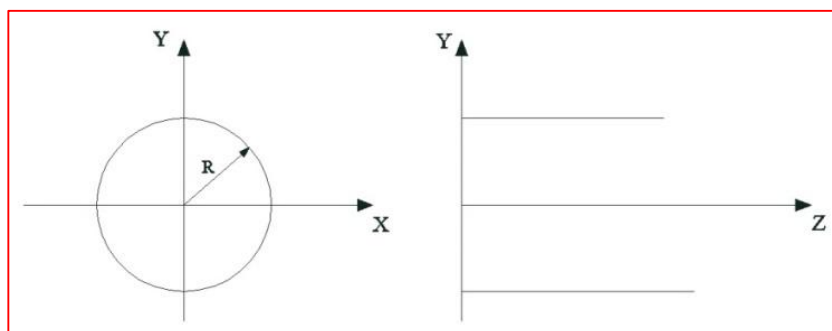
Figure 1. Schematic representation of the used coordinate systems

We consider the steady-state movement of the slurry along a round cylindrical pipe (Fig.1) and the distribution of sediment concentration is uneven in height, which is determined by formula (2) in the cylin-

drical coordinate system and is a function of r and φ, a the solid particles included in the mixture are identical in density.

With one-dimensional motion, the following conditions are satisfied:

$$u_{nr} = u_{n\varphi} = 0$$

$$u_{nz} = u_n \ \ (n = 1,2) \tag{6}$$

Then for the continuity equation is:

$$\frac{\partial(\rho_n u_n)}{\partial z} = 0 \tag{7}$$

there: ρ_n - is the reduced phase density, which is also a function of r and φ, and let ρ_n be independent from the z coordinate.

Then instead of (7) we have:

$$\frac{\partial u_n}{\partial z} = 0 \tag{8}$$

For given initial parameters of hydrotransport, it is necessary to find the distribution of concentrations and longitudinal velocities over the depth of the stream.

The system of differential equations (5) of the interpenetrating motion of a two-phase mixture in a cylindrical coordinate system, taking into account (7) and (8), takes the form:

$$\left. \begin{array}{c} f_n \frac{dp}{dz} = \frac{\mu_n}{r} \frac{\partial}{r} \left(r f_n \frac{\partial u_n}{\partial r} \right) + \frac{\mu_n}{r^2} \frac{\partial}{\partial \varphi} \left(f_n \frac{\partial u_n}{\partial \varphi} \right) + K(u_{2n} - u_n) + \rho_n F_n \\ f_n \frac{\partial p}{\partial r} \\ f_n \frac{\partial p}{\partial \varphi} \end{array} \right\} \tag{9}$$

there: $\frac{\partial p}{\partial r}, \frac{\partial p}{\partial \varphi}$ and $\frac{\partial p}{\partial z}$ – is the differential pressure of the flow along the axes;

$u_{nr}, u_{n\varphi}$ and u_{nz} - are the components of the velocity vector of each phase;

f_n - is the concentration distribution of each phase;

μ_n – is the viscosity coefficient of the phases;

F_{nr}, F and F - projections of mass forces;

K - is the coefficient of the force of interaction between the phases.

In addition, here and hereinafter, with $n = 1$ the parameters of the first carrier (liquid) phase are implied, and with $n = 2$ the parameters of the transported (solid) phase are implied.

From the last two equations (1.4.) We obtain:

$$\frac{dp}{dr} = 0, \ \ \frac{dp}{d\varphi} = 0$$

Hence, the pressure drop is a function of only the z coordinate and does not depend on r and φ, etc:

$$\frac{\partial p}{\partial z} = \frac{dp}{dz}$$

To the differential equations of motion (9) we add the relations between the phase concentrations:

$$f_1 + f_2 = 1 \tag{10}$$

boundary conditions of adhesion at $r = R$:

$$u_1 = 0 \ , \ u_2 = 0 \tag{11}$$

and the condition of symmetry along the vertical axis (y), i.e. with $\varphi = 90^0$ and $\varphi = 270^0$

$$\frac{\partial u_1}{\partial \varphi} = 0, \frac{\partial u_2}{\partial \varphi} = 0 \tag{12}$$

The distribution of sediment concentration is expressed as:

$$f_2 = f_{20} exp\left[\left[-\frac{3(\rho_r-\rho)g}{\rho_r u_i^2} - \frac{3}{2}\rho C_0 \frac{S_r W_r^2}{m_r u_i^2}\right](R + r\sin\varphi)\right] \tag{13}$$

It can be seen from the differential equation of flow motion (9) that the phase velocities of the mixture depend on the concentration distribution f_2 and the interaction coefficient K. At the same time, these parameters depend on the distribution of the velocity of the mixture.

Having integrated all the terms of the equation over the cross-sectional area of the flow, for a steady, all-weighting flow in a one-dimensional formulation from equation (9) we have the following [20, 21]:

$$\frac{dP}{dz} = \rho g i - \frac{\lambda_{CM}\rho Q^2}{2d\omega^2} - \frac{s\pi d}{\omega}\tau_0 \tag{14}$$

When deriving the equations of motion for the density and velocity of the slurry (dispersoid), the following notation was adopted:

$$\rho = (1-s)\rho_1 + s\rho_2 \tag{15}$$

$$v = \frac{(1-s)\rho_1 v_1 + s\rho_2 v_2}{(1-s)\rho_1 + s\rho_2} \tag{16}$$

there: s – is the volume concentration of the solid component;

$\rho_1 \ and \ \rho_2$ - density of liquid and solid particles;

Q - discharge of hydro mixture;

ω - cross-sectional area of the pipeline;

$v_1 \ and \ v_2$ - the fluid and solid particle velocities averaged over the cross section of the

pipeline;

i – is the slope of the flow;

P - hydrodynamic stress, pressure;

χ – is the perimeter of the pipeline;

τ_0 - the initial resistance of the mixture;

λ_{CM} - coefficient of hydraulic friction.

3. Results and Discussion

Solving equation (14) taking into account the boundary conditions (for $z = 0$ $P = P_1$ and for $z = L$, $P = P_2$) $i > 0$ (Fig.2) we obtain:

$$\frac{\lambda_{CM}\rho}{2d\omega^2}Q^2 = \frac{P_2 - P_1}{L} + \rho g i - \frac{s\pi d}{\omega}\tau_0 \tag{17}$$
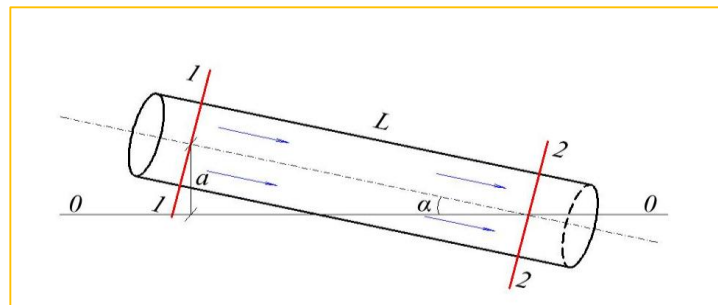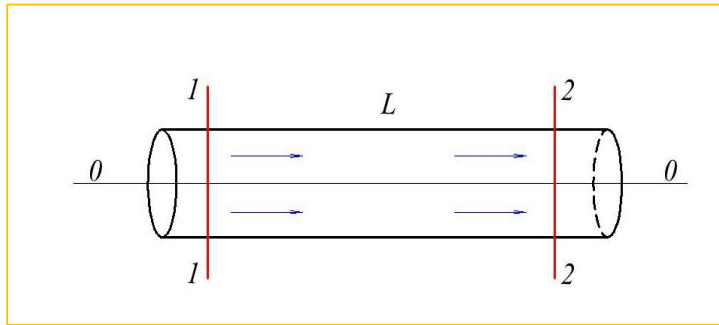

Figure 2. Pipe with is the correct slope

The flow rate is determined by the expression:

$$Q = \sqrt{\frac{2d\omega^2}{\lambda_{CM}\rho}\left(\frac{P_1 - P_2}{L} + \rho g i - \frac{s\pi d}{\omega}\tau_0\right)} \qquad (18)$$

there: $P_1 - P_2 = \Delta P$ - is the pressure drop created by the pump system.
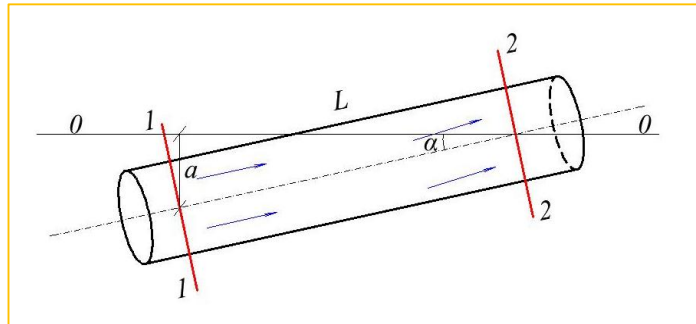For $i = 0$ (Fig.3):

Figure 3. Horizontal pipeline

$$Q = \sqrt{\frac{2d\omega^2}{\lambda_{CM}\rho}\left(\frac{\Delta P}{L} - \frac{s\pi d}{\omega}\tau_0\right)} \qquad (19)$$

The condition under which the mixture begins to move is written as:

$$\frac{P_1 - P_2}{L} > \frac{s}{R}\tau_0 \qquad (20)$$

Therefore, it is necessary to create such a difference in pressure drop ΔP that would exceed the value $\frac{s}{R}\tau_0$.

For a weighted flow in pipelines with a negative slope < 0 (Fig. 4) we have:

Figure 4. Pipeline with negative slope

$$Q = \sqrt{\frac{2d\omega^2}{\lambda_{CM}\rho}\left(\frac{P_1 - P_2}{L} - \rho g i - \frac{s\pi d}{\omega}\tau_0\right)} \qquad (21)$$

Then the condition under which the mixture begins to move is written in the form:

$$\frac{P_1 - P_2}{L} > \rho g i + \frac{s}{R}\tau_0 \qquad (22)$$

A feature of the approach is that here, in addition to the main factors characterizing the movement of the weighted flow, the influence of the slope of the pipeline is taken into account:

$$\Delta P > \rho g i + \frac{s}{R}\tau_0 \qquad (23)$$

Thus, as a mathematical model, a single-speed mixture motion model is used, i.e. the slurry in its movement is identified with a fictitious single-speed continuum of variable density.

4.   Conclusions

1. The limited, and in some cases unacceptably low degree of accuracy of the proposed formulas, the extremely large variety of hydrotransport conditions in practice, do not always allow us to choose a design dependence that meets the conditions of each design object. Unreasonable application of this or that formula in this case can lead to gross errors, making the designed hydrotransport installation uneconomical or even inoperative.

2. The calculated dependences proposed by most researchers to determine the main parameters of hydrotransport often express the results of experiments on the basis of which they are established, and therefore, the field of application of these dependencies is very limited.

3. Based on the model of H. A. Rakhmatullin and further developed in the works of K. Sh. Latipov, A. Arifjanov and other scientists, a model of the motion of a two-phase mixture in a round cylindrical pipe with allowance for the flow slope is proposed. That is, a one-speed model of mixture motion, where the hydraulic mixture in its movement is identified with a fictitious single-speed continuum of variable density.

4. Given the insufficient knowledge of the movement of a two-phase mixture in round cylindrical pipelines with a negative slope, it is necessary to develop a design for the removal of river sediment.

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