6 Waves

6.1 Introduction

Waves are very familiar to everyone, particularly those that toss about at sea during windy, stormy weather and those which roll in rhythmically on the beach. The tides too are waves but they are not so obvious as they slowly build up and subside twice a day and move many millions of tonnes of water across the oceans. But waves are not just confined to the sea. Earlier, in Chapter 5, waves were described on a still pond and on a flowing stream when a stone was thrown into the water to help determine critical flow conditions. Surge waves can sometimes be seen in rivers when control gates are suddenly closed and during high tides when tidal bores, some of which are quite famous, travel upstream from the mouth of rivers. Floods flowing down rivers too are another kind of wave which can be several hundreds of kilometres long.

Waves in rivers depend on gravity for their shape and size in much the same way that channels themselves depend on the force of gravity down the land slope for their energy. Sea waves are different as they depend on the circulation of the atmosphere and the resulting winds for their energy. Although it is possible for winds to create waves on lakes, they are not usually as large as those which can be generated by strong winds blowing over vast stretches of the oceans. These are the waves which play a large part in shaping our shorelines.

Sea waves tend to affect only the water surface. Indeed wave movement is different from the movement of the water in which it is travelling. Watching a single wave, it seems to travel a great distance at a steady speed. But observing the water closely, it hardly moves at all. Just look at any object floating – a seagull – on the water surface to appreciate this. When a wave passes the bird is not swept along with the wave it just bobs up and down. Leonardo da Vinci (1452–1519) compared water waves with the waves you see when the wind blows across a field of corn. The wave pattern seems to travel across the entire field and yet individual stalks only sway back and forth. Another example is that of a rope which is held at one end and shaken. A wave pattern travels along the rope but the rope only moves up and down. So if the water (or the corn or rope) is not being transported then what is? The answer is energy. It is energy which is being transported across the water and through the corn and along the rope.

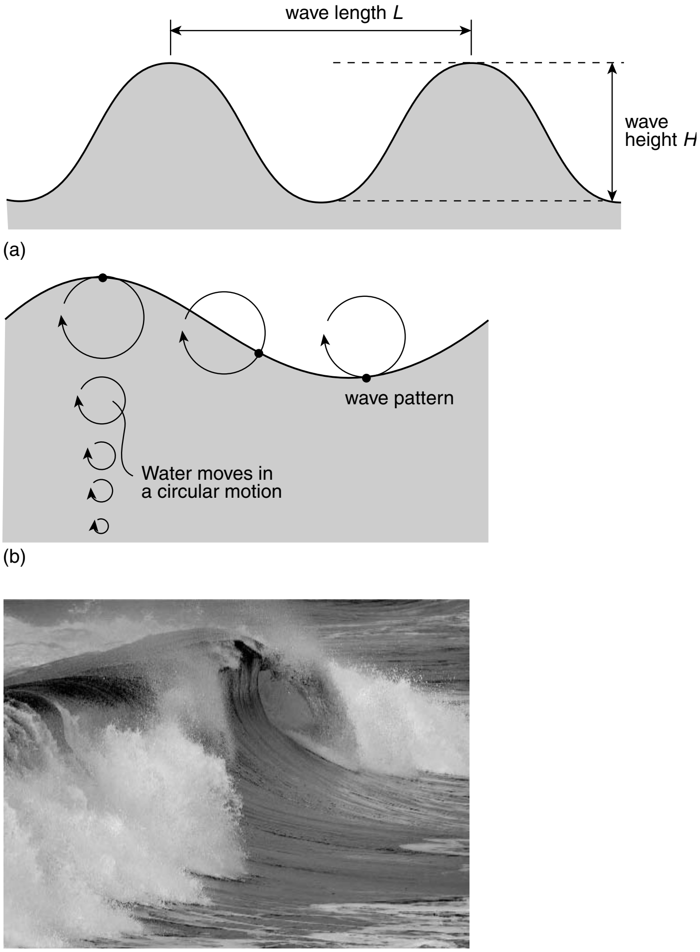
Our understanding of waves and how they are generated is far from complete. This is because is it difficult to observe waves at sea and also because all the mathematical formulae are based on ideal fluids and the sea does not always fit in with the ideal.

6.2 Describing waves

Wave *height H* is the dimension from the crest of a wave (peak) and the wave trough (Figure 6.1a). The height of a wave is twice its *amplitude* and the wave *length L* is the distance between two successive wave peaks.

As well as having dimensions in space, waves also have dimensions in time. So when waves move past an observer the time interval between successive peaks is called the wave *period* (*T)* and the number of peaks which pass an observer each second is the *frequency* (*f*).

The velocity of waves is called the *celerity* (*c*) and this word is used to clearly distinguish the wave velocity from that of the water. If the water is also moving then the wave velocity increases or *f* decreases depending on whether the wave is moving with against the flow. Although water waves have been likened to wave motion along a rope, water is a little different. While a rope is free to rise and fall leaving gaps around it, this is not possible with water and so as the water rises when a wave passes, water nearby flows into the space. This gives rise to a circular motion which extends some distance below the water surface but with diminishing effect (Figure 6.1b). At a depth of half the wave length the effects of surface waves are negligible. A submarine submerged at 150 m would be able to avoid all the severe waves that are produced by a storm at sea.



6.1 Wave dimensions and movement.

6.3 Waves at sea

Sea waves come in many different sizes (Figure 6.2). The babies in the family are the *capillary waves*, which are generated by wind and are only a few millimetres in length. Their shape and size is influenced by surface tension. This is a very small force and so for waves longer than this, the earth's gravity takes over control and these are often referred to as *gravity waves*. These also are generated by the wind. The longer waves are the tsunamis (see Section 6.6) and storm surges. But the longest of all are the tides which are controlled by the movement of the sun and moon. There are no simple dividing lines between the different waves – just a gradual change from one to the other.

The general formula for wave celerity is as follows:

$$C = \sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)}$$

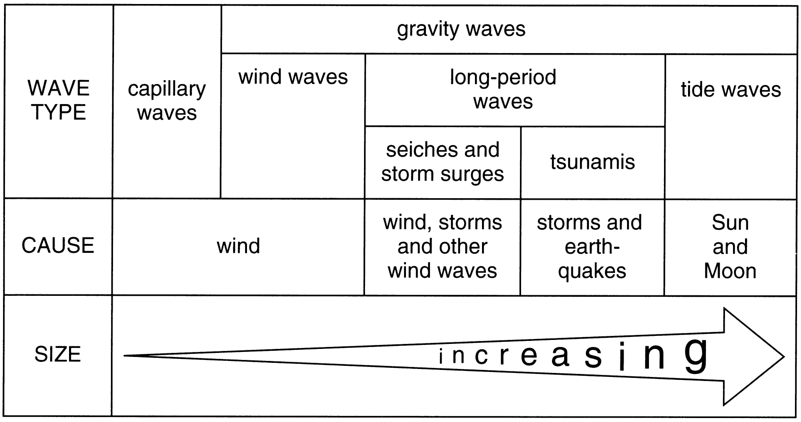
where g is gravity constant (9.81 m/s^2); L is wave length (m); d is water depth (m).

The term tanh is a mathematical function known as the hyperbolic tangent and values can be obtained from mathematical tables in the same way as other functions such as sin and tan.

In deep water (defined as water deeper than half the wave length), as is the case of sea waves, the depth of the water has no effect on wave speed and so the formula simplifies to:

$$C = \sqrt{\frac{gL}{2\pi}}$$

This formula shows that the celerity of sea waves depends only on wave length. So waves of different length travel at different speeds. In an Atlantic storm, for example, waves of many different lengths are generated. As the waves start to move out of the storm area the longer waves move faster and so they begin to overtake the shorter waves. Waves can travel many hundreds of kilometres because there is very little friction to slow them down. When they reach the shores of Europe they are well sorted out with the longer waves arriving first followed eventually by the shorter waves. The rhythmic nature of waves arriving on a beach in this way is often referred to as *swell*. By timing the waves and noting the change in their frequency, it is possible to work out how far the waves have travelled since they were formed. In other words,



<table>
<tr><td rowspan="3">WAVE TYPE</td><td rowspan="3">capillary waves</td><td colspan="4">gravity waves</td></tr>
<tr><td rowspan="2">wind waves</td><td colspan="2">long-period waves</td><td rowspan="2">tide waves</td></tr>
<tr><td>seiches and storm surges</td><td>tsunamis</td></tr>
<tr><td>CAUSE</td><td colspan="2">wind</td><td>wind, storms and other wind waves</td><td>storms and earth-quakes</td><td>Sun and Moon</td></tr>
<tr><td>SIZE</td><td colspan="5">increasing</td></tr>
</table>

6.2 Types of sea waves.

it is possible to link waves to the particular weather events at sea that created them, perhaps several days before.

It was this kind of detailed study of waves and the weather patterns that form them that helped scientists to predict wave heights on the beaches of France during the very successful D-Day landings during the Second World War. Two scientists, Svedrup and Monk, developed a technique for predicting wave heights from weather patterns at sea. They also studied the celerity of waves travelling out from the storm area. From this they were able to predict the wave heights on the Normandy beaches resulting from bad weather conditions out in the Atlantic occurring several days earlier. This information was crucial for those landing on the beaches because the landing craft being used could only land safely at certain limited wave heights. Knowing the way in which the sea can change suddenly in the English channel it would have been a great risk to have gone ahead with such a major operation without some prior knowledge of what the sea was going to be like.

Similar analyses are regularly used today to predict sea conditions when tankers are refuelling ships at sea or ships are taking supplies to oil platforms. Predicting the wave heights that can be expected ensures successful and safe operations.

6.4 Waves in rivers and open channels

Waves which affect only the surface water can also occur in channels and are usually the result of some disturbance – like the waves which characterise super- and sub-critical flow. But in this case the water is shallow and so celerity of waves of this kind are related to the depth of the water. And the general formula is modified accordingly.
When depth d is significant:

$$\tanh\left(\frac{2\pi d}{L}\right) = \frac{2\pi}{L} \quad \text{and so } c = \sqrt{gd}$$

where c is wave celerity (m/s); d is depth of the water (m); g is the gravity constant (9.81 m/s^2).

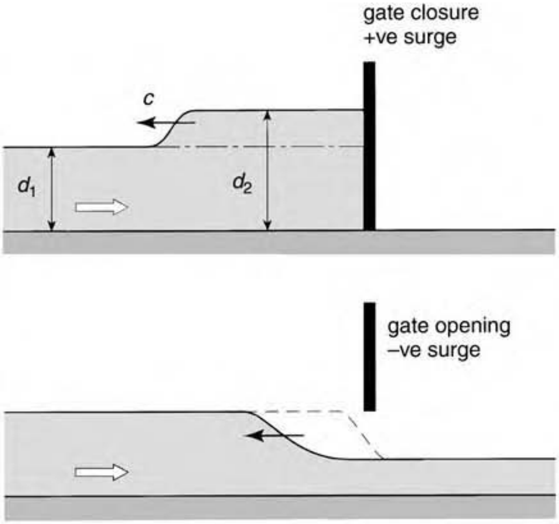
This formula was developed by Joseph Lagrange (1736–1813) and was further verified by John Scott Russel (1808–1882) who spent a great deal of time observing and measuring the speed of bow waves created on canals by horse-drawn barges.

But channels tend to be more associated with waves which have a significant effect on the whole flow and not just on the surface. Such waves include surges (or bores) which are sometimes seen moving upstream on tidal rivers or along channels after the closure of hydraulic gates. The hydraulic jump is sometimes described as a *standing wave*. It is similar to a surge but it stays in one place.

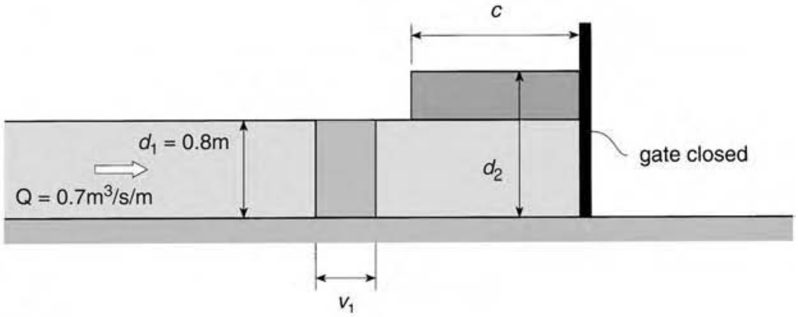
6.4.1 Surges

Surges occur when there is a sudden change in flow such as when a gate is closed or opened (Figure 6.3a). The sudden closure of a gate produces a positive surge whereas a sudden opening produces a negative surge. They are sometimes called *solitary waves*, that is, there is only one wave.

The strength of a surge depends on the change in the flow. When the change is very small the surge is undular (i.e. like a surface wave). When the change is large a much stronger surge develops. If it is a positive surge then it will have a vigorous rolling action and may look like a



(a)



(b)

(c) Tidal bore on R. Severn, UK

6.3 Surges in channels.

hydraulic jump but it is moving along the channel. A negative surge is much weaker and usually produces weak surface waves.

The celerity of a surge in a rectangular channel can be calculated using the formula:

$$C = \sqrt{\frac{gd_2(d_1+d_2)}{2d_1}} - v_1$$

where d_1 is upstream depth (m); d_2 is downstream depth (m); v_1 is upstream velocity (m/s).

This formula can be derived from the momentum equation, but this is not shown here. When the wave celerity is zero, that is, $c = 0$, the formula, if rearranged, is the same as that for the hydraulic jump (see Section 5.7.6). So one way of looking at a hydraulic jump is as a stationary (standing) wave or surge.

EXAMPLE: CALCULATING THE CELERITY AND HEIGHT OF A SURGE WAVE

Calculate the height and celerity of a surge wave resulting from the closure of a sluice gate on a canal when the discharge is 0.7 m^3/s/m width of channel and the normal depth of flow is 0.8 m (Figure 6.3b).

The formula for calculating the celerity is derived from the momentum equation:

$$c = \sqrt{\frac{gd_2(d_1+d_2)}{2d_1}} - v_1$$

but there are two unknown values, wave celerity c and the depth near the gate just after the sudden closure d_2. So another equation is needed before a solution can be found. This is the continuity equation but applied to surge (Figure 6.3b). This is done by calculating the volume of water coming down the channel in one second and equating this to the volume of water in the surge (the two volumes are shown shaded).

So:

$$v_1 d_1 = c(d_2 - d_1)$$

$$c = \frac{v_1 d_1}{d_2 - d_1}$$

Now:

$$d_1 = 0.8 \text{ m}$$

And:

$$v_1 = \frac{q}{d_1} = \frac{0.7}{0.8} = 0.88 \text{ m/s}$$

Put these values into the continuity equation:

$$c = \frac{0.88 \times 0.8}{d_2 - 0.88}$$

Put this equal to the wave celerity in the momentum equation:

$$\frac{0.88 \times 0.8}{d_2 - 0.88} = \sqrt{\frac{gd_2(0.8 + d_2)}{2 \times 0.8}} - 0.88$$

Solve this equation by trial and error, that is, by putting in different values of d_2 until one fits the equation:

$$d_2 = 1.14 \text{ m}$$

And so:

$$c = \frac{0.88 \times 0.8}{1.14 - 0.88 \times 0.8}$$

$$c = 2.07 \text{ m/s}$$

When a surge wave is not very high, that is, the depth d_2 is not significantly greater than the original depth in the channel d_1 then the wave celerity equation can be simplified by assuming that $d_1 = d_{2.}$ So:

$$c = \sqrt{gd_1} - v_1$$

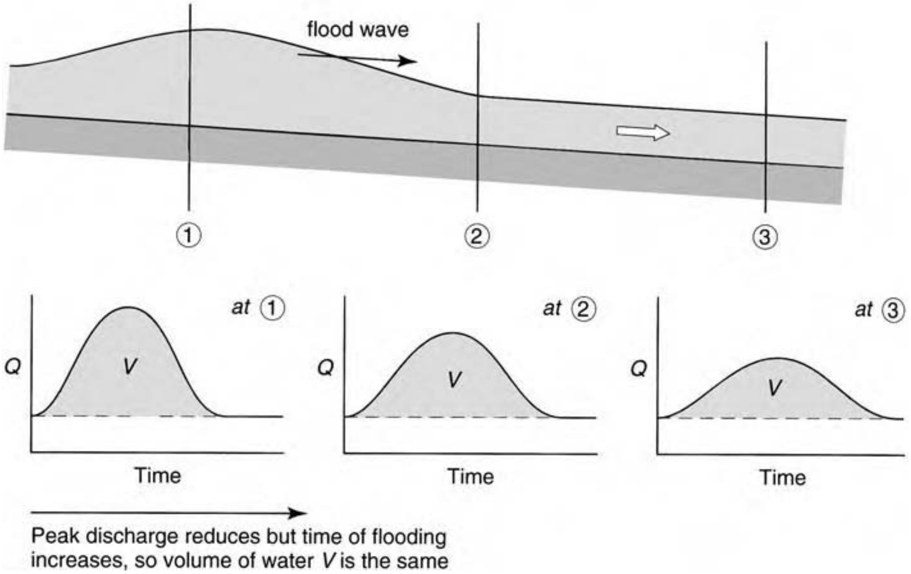
When the water velocity $v_1 = 0$ the celerity becomes $c = \sqrt{gd}$. This is the same formula used to calculate the celerity of gravity waves on a still pond (see Section 5.7.1).

6.4.2 Bores in tidal rivers

Some rivers regularly experience surges – sometimes called *bores* or *eagres*. They occur in tidal reaches close to estuaries. The most prominent surges occur where there is a high tidal range and where wide estuaries converge into a narrow river channel (Figure 6.3c). The rivers Severn and Trent in UK are famous for their bores. They are best observed during the spring tides when the tidal range is greatest and when river flows are at their lowest. The Severn bore starts in the estuary as high tide approaches and moves upstream reaching celerities of 5 m/s with a height of 1.5 m. Another famous surge, which attracts many surfers who like the challenge of riding big waves, is on the Amazon River in Brazil where it meets the Atlantic Ocean. It is up to 4 m high, running at up to 25 km/h and is known locally as the *pororoca* which is the local language for 'the great destructive noise'.

6.5 Flood waves

Flood waves occur on rivers as a result of rainstorms on river catchments. They can be many hundreds of kilometres long; dealing with them is quite different from the much smaller waves already described. Instead of taking account of a whole wave along the entire length of a river, engineers deal with what happens to the discharge and water levels at particular sites over a period of time. A site may have been chosen because it is prone to flooding and there is a need to know what the effects of this will be, or it may be a convenient site for accurately measuring



6.4 Flood waves.

the discharge and water levels. Such observations lead to a graph of the discharge and the water level at that point on the river; it is called a *hydrograph* (Figure 6.4). Hydrographs can be very useful for dealing with river flow and flooding problems. For example, the area under the curve represents the volume of water that has passed the site during the flood. If the normal flow was also known it is possible to determine how much water was brought down by the flood.

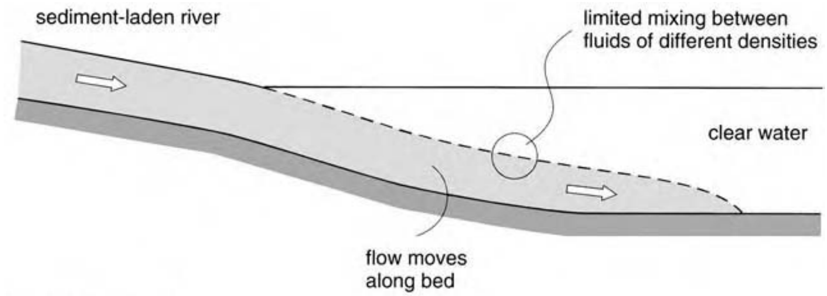
If another site is chosen further downstream and another hydrograph drawn, the two will have different shapes. They will both have the same area under the curve as they are both passing the same amount but the downstream curve will be longer and flatter – remember continuity. This means the maximum discharge is less but the flood duration is longer.

Gathering information on floods in this way provides valuable knowledge for river engineers who will be able to predict the effects of different rainstorms on river catchments and so design and construct engineering works which will help to alleviate and control flooding.

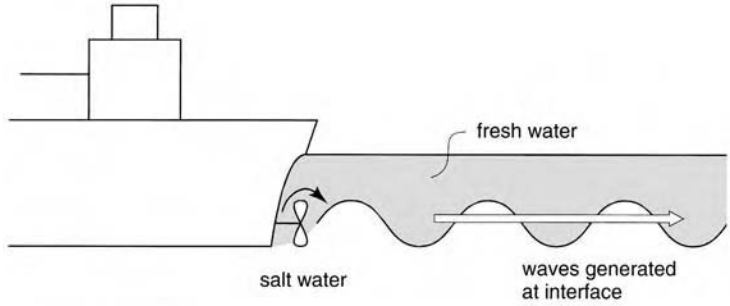
6.6 Some special waves

6.6.1 Density currents

When two fluids of different densities flow together they do not mix easily. The slow movement of sea water up estuaries during a rising tide under the outflowing fresh water from a river is an example of this. Fresh water has a slightly lower density than sea water and so the fresh water sits on the top and does not mix. Similarly, the flow of sediment-laden river water along the bed of the sea or a reservoir does not disperse into the main body of water (Figure 6.5a). Such flows, which occur when there are small differences in density, are called *density currents*. Another example is hot and cold water which do not mix easily because they have slightly different densities. Power stations sometimes have this same problem. They use a lot of cooling water; if the intake is close to the outlet then there is the risk of sucking hot water back into the station before it has had a chance to cool down through turbulent mixing with cooler water.



(a) Density current



(b) Internal waves

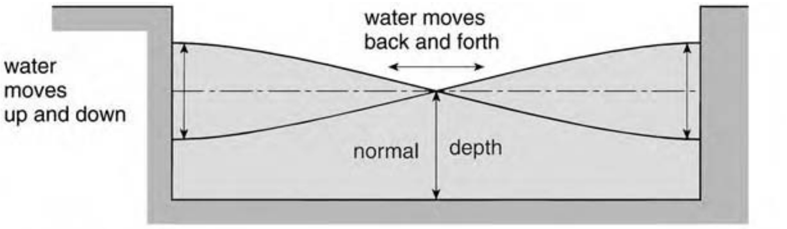
6.5 Density currents.

Waves can also occur at the interface between two fluids of different densities and can be quite intriguing. Oceanographers call these *internal waves* because the waves do not affect the water surface at all. Such waves seem to move in slow motion as if the force of gravity, which is controlling them, has been significantly reduced. This can be explained by the apparent loss in weight of the fluid as it lies on top of the denser lower fluid. The wave celerity equation still works but the value of g is much reduced by this buoyancy effect. When g is reduced the celerity is also reduced which means the waves move very slowly.

One internal wave system which caused some consternation occurred in the north Atlantic when a scientist was observing icebergs. The weather was mild and the ice was melting and there was about one metre of fresh water lying on top of the salt sea water (Figure 6.5b). Because of the slight density difference the two did not mix easily. When the observation ship's engines were started, to the surprise of the crew, the ship did not move, although the propellers were turning. The reason was the ship's propellers just happened to be located at the interface between the fresh and salt water and all the energy was being dissipated as wave energy along the interface. The sea surface was quite calm but just below, the waves were moving up and down at the interface and absorbing all the ship's energy.

6.6.2 Waves in harbours

Harbours are normally designed to keep out waves so that an area of calm water is created for ships to shelter from the sea for servicing and for loading cargo. This is done by having a narrow entrance. Most harbours do this successfully but there are cases when narrowing the harbour entrance has had the opposite effect – known as the *harbour paradox*. The narrow entrance has the effect of tuning certain incoming wave frequencies so rather than stopping the waves, the harbour amplifies them and they become worse than those outside. This is known as *harbour*



(a) Harbour seiche

(b) In the bath

6.6 Waves in harbours.

seiche. When this happens the waves reflect back and forth as there is very little in harbours to absorb the wave energy. It is very much like sliding up and down in the bath tub at home. The sliding action makes the water in the bath slosh back and forth (Figure 6.6). The bath tub has solid vertical sides which are very good at reflecting the waves but not so good at absorbing the wave energy. The waves can continue for some time until they finally settle down as the small amount of friction from the bath and your body take their toll. Harbours are very similar. They have lots of solid concrete surfaces and so waves tend to bounce back and forth rather than be absorbed.

The main concern in harbours is not so much with wave celerity but with the movement of water. As waves are reflected back and forth there is a great deal of water movement from one end of the harbour to the other (remember the bath again). The water velocity at the middle of the harbour may be as much as 0.5 m/s as water flows from one side to the other following the wave. This can create lots of problems. A ship moored at the mid-point could move by as much as 15 m back and forth following the movement of the wave. It is clearly undesirable to have a large ship moving about so much on its moorings when it is being loaded or unloaded. In harbours prone to this problem it is better to moor ships near the edge of the harbour where the water only moves up and down. From experience of harbour operation the most dangerous waves are those with a period of about two minutes. Such waves have been known to resonate in harbours which means that the frequency of the waves and the characteristics of the harbour, ships and their mooring systems combine to amplify the waves which makes the situation very much worse. All these factors are now taken into account when designing new harbour works or in repairing existing ones. Harbour designers wisely opt for model testing before construction to avoid these problems.

6.7 A tsunami wave coming ashore in Thailand 2004.

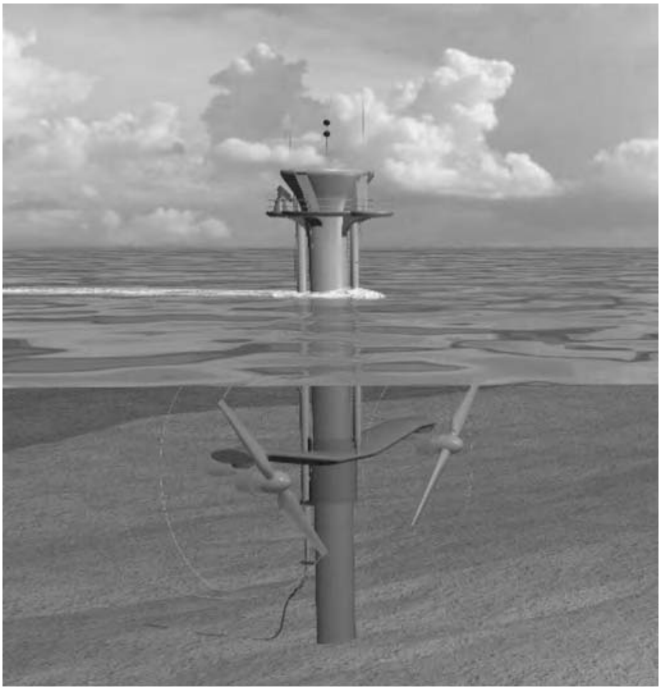
6.6.3 Tsunami wave

The *tsunami* is perhaps the most famous wave of all following the disastrous events in the Indian Ocean on 26 December 2004 (Figure 6.7). The word *tsunami* is Japanese for a long wave caused by earthquakes or landslides under the ocean. There is a common misconception that tsunamis behave like waves or swells which are generated by wind. They may have a similar appearance but they have a very different origin. The sudden movement of the sea bed during an earthquake causes a sudden movement of many millions of tons of water which can literally lift the sea level by a metre or more. The surge that comes from this weight of water has far greater force than any wind generated wave. This was evident from the devastating consequences shown in the many photographs and videos taken of the 2004 event.

Tsunamis are most common in the Pacific ocean. They travel at very high celerity with devastating consequences when they reach land. In the open ocean the wave may be only 0.5 m or so high but the wave celerity can be around 200 m/s when the ocean is say 5000 m deep. This might appear to be a surface wave but because of its great length its celerity is controlled by the depth of the ocean (remember the wave celerity equation $c = \sqrt{gd}$). Even so there is not much water displacement as it passes over the open ocean. A ship meeting this wave would hardly notice its passing. This is one of the reasons why many Indonesian fishermen survived the 2005 tsunami. They were fishing in boats off the coast and hardly noticed the tsunami as it passed underneath them. But nearer the shore was not a good place to be. As the tsunami runs into shallow water, say 100 m deep, the wave slows down to 30 m/s but as the energy in the wave is still the same the wave height increases substantially. At 30 m/s this is still quite fast and with the height increasing further as it runs up the beach it can do a great deal of damage to shore installations. The more destructive tsunamis are those which are funnelled into a narrow space when the width of a beach or harbour is restricted in some way. This has the effect of funnelling the energy of the wave into the harbour with destructive consequences.

6.7 Tidal power

Waves can be very useful for generating power. The most promising method is to hold incoming and outgoing tides behind a dam (or barrage) and use the head to drive turbines to generate



6.8 A new form of tidal turbine for generating electricity.

electricity. A power station using this idea was built on the estuary of the Raunce in Brittany, France in 1966 and there are plans to build a large barrage in the Severn estuary on the west coast of Britain. Although it has been estimated that this station could provide up to 6% of Britain's power needs the scheme is hotly debated because it would affect the pattern of the tides and sediment movements in the estuary and many ecologically sensitive wetland sites could be adversely affected.

Tidal barrages are an economic possibility when the tidal range exceeds 5 m but taking energy out of the tides may cause problems. In estuaries that naturally absorb tidal energy through friction and very little energy is reflected back out to sea there is the chance that a power station can take out the energy from the tides without serious effect. But in cases where there is very little friction and all the tidal energy is reflected back out to sea there may be dangers in extracting tidal energy. The effects of the energy not being reflected back again cannot be predicted.

More recent developments are looking at harnessing tidal energy without having to build large, expensive and environmentally risky structures. One such development is a turbine that sits on a tower in the sea which can be lifted out for routine maintenance (Figure 6.8).

In the 1920s one enterprising engineer undertook a study to establish the feasibility of constructing tidal power stations across the Irish Sea between Ireland and Wales and Ireland and Scotland. Because of the difficulties and high costs of measuring velocities across this large stretch of water he looked around for a hydraulically similar model on which to make measurements. He worked out that that Salisbury plain is hydraulically similar to the Irish sea and that the winds blowing across it produced similar Reynolds Numbers to the flow of water in the Irish sea. The study concluded that the Irish sea was more reflective than energy absorbing and so there were great doubts about the prospects for power generation.