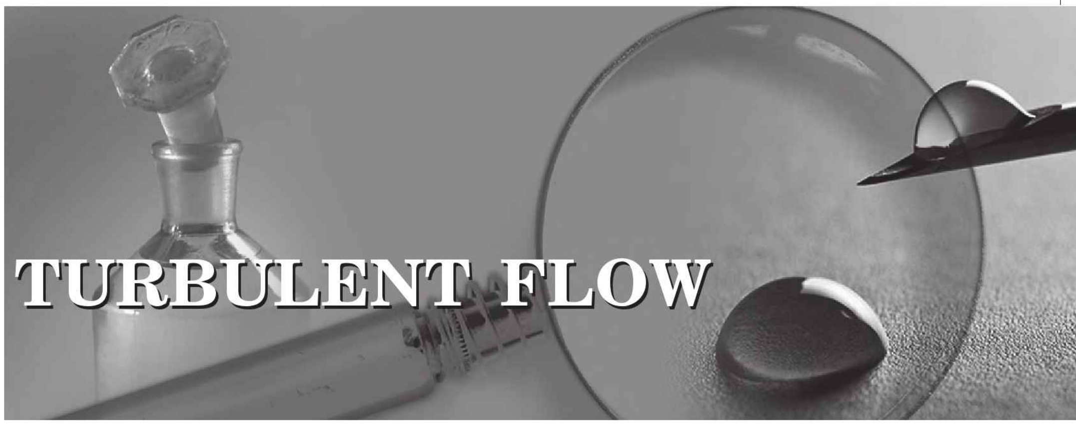
10
CHAPTER



TURBULENT FLOW

▶ 10.1 INTRODUCTION

The laminar flow has been discussed in chapter 9. In laminar flow the fluid particles move along straight parallel path in layers or laminae, such that the paths of individual fluid particles do not cross those of neighbouring particles. Laminar flow is possible only at low velocities and when the fluid is highly viscous. But when the velocity is increased or fluid is less viscous, the fluid particles do not move in straight paths. The fluid particles move in random manner resulting in general mixing of the particles. This type of flow is called turbulent flow.

A laminar flow changes to turbulent flow when (*i*) velocity is increased or (*ii*) diameter of a pipe is increased or (*iii*) the viscosity of fluid is decreased. O. Reynold was first to demonstrate that the transition from laminar to turbulent depends not only on the mean velocity but on the quantity $\frac{\rho VD}{\mu}$. This quantity $\frac{\rho VD}{\mu}$ is a dimensionless quantity and is called Reynolds number (R_e). In case of circular pipe if $R_e < 2000$ the flow is said to be laminar and if $R_e > 4000$, the flow is said to be turbulent. If R_e lies between 2000 to 4000, the flow changes from laminar to turbulent.

▶ 10.2 REYNOLDS EXPERIMENT

The type of flow is determined from the Reynolds number *i.e.*, $\frac{\rho V \times d}{\mu}$. This was demonstrated by O. Reynold in 1883. His apparatus is shown in Fig. 10.1.

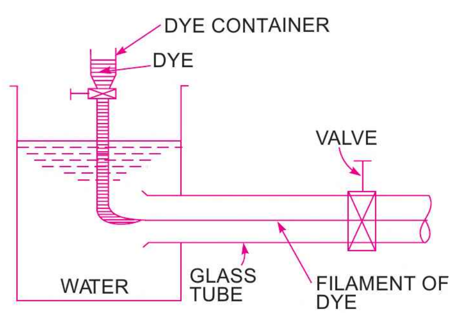


Fig. 10.1 *Reynold apparatus.*

The apparatus consists of :

(*i*) A tank containing water at constant head,

(*ii*) A small tank containing some dye,

(*iii*) A glass tube having a bell-mouthed entrance at one end and a regulating value at other ends.

The water from the tank was allowed to flow through the glass tube. The velocity of flow was varied by the regulating valve. A liquid dye having same specific weight as water was introduced into the glass tube as shown in Fig. 10.1.

The following observations were made by Reynold :

(*i*) When the velocity of flow was low, the dye filament in the glass tube was in the form of a straight line. This straight line of dye filament was parallel to the glass tube, which was the case of laminar flow as shown in Fig. 10.2 (*a*).

(*ii*) With the increase of velocity of flow, the dye-filament was no longer a straight-line but it became a wavy one as shown in Fig. 10.2 (*b*). This shows that flow is no longer laminar.

(*iii*) With further increase of velocity of flow, the wavy dye-filament broke-up and finally diffused in water as shown in Fig. 10.2 (*c*). This means that the fluid particles of the dye at this higher velocity are moving in random fashion, which shows the case of turbulent flow. Thus in case of turbulent flow the mixing of dye-filament and water is intense and flow is irregular, random and disorderly.

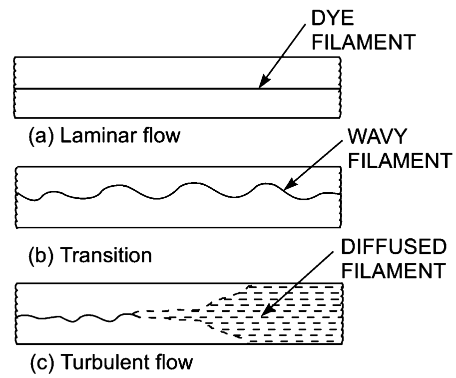


Fig. 10.2 *Different stages of filament.*

In case of laminar flow, the loss of pressure head was found to be proportional to the velocity but in case of turbulent flow, Reynold observed that loss of head is approximately proportional to the square of velocity. More exactly the loss of head, $h_f \propto V^n$, where n varies from 1.75 to 2.0

▶ 10.3 FRICTIONAL LOSS IN PIPE FLOW

When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to the pipe wall is zero. The velocity of liquid goes on increasing from the wall and thus velocity gradient and hence shear stresses are produced in the whole liquid due to viscosity. This viscous action causes loss of energy which is usually known as frictional loss.

On the basis of his experiments, William Froude gave the following laws of fluid fraction for turbulent flow.

The frictional resistance for turbulent flow is :

(*i*) proportional to V^n, where n varies from 1.5 to 2.0,

(*ii*) proportional to the density of fluid,

(*iii*) proportional to the area of surface in contact,

(*iv*) independent of pressure,

(*v*) dependent on the nature of the surface in contact.

10.3.1 Expression for Loss of Head Due to Friction in Pipes. Consider a uniform horizontal pipe, having steady flow as shown in Fig. 10.3. Let 1-1 and 2-2 are two sections of pipe.

Let p_1 = pressure intensity at section 1-1,

V_1 = velocity of flow at section 1-1,

L = length of the pipe between sections 1-1 and 2-2,

d = diameter of pipe,

f' = frictional resistance per unit wetted area per unit velocity,

h_f = loss of head due to friction,

and p_2, V_2 = are values of pressure intensity and velocity at section 2-2.

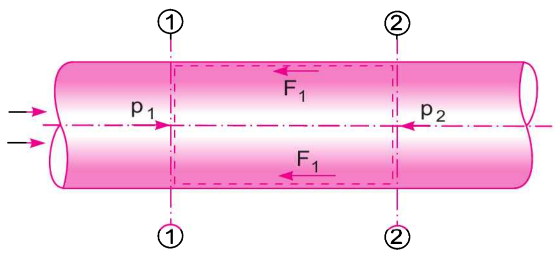


Fig. 10.3 *Uniform horizontal pipe.*

Applying Bernoulli's equations between sections 1-1 and 2-2,

Total head at 1-1 = Total head at 2-2 + loss of head due to friction between 1-1 and 2-2

or $$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

But $z_1 = z_2$ as pipe is horizontal

$V_1 = V_2$ as dia. of pipe is same at 1-1 and 2-2

$$\therefore \quad \frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f \text{ or } h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} \quad ...(i)$$

But h_f is the head lost due to friction and hence intensity of pressure will be reduced in the direction of flow by frictional resistance.

Now frictional resistance = frictional resistance per unit wetted area per unit velocity × wetted area × velocity^2

or $F_1 = f' \times \pi dL \times V^2$ [$\because$ wetted area = $\pi d \times L$, velocity = $V = V_1 = V_2$]

$= f' \times P \times L \times V^2$ [$\because$ πd = Perimeter = P] ...(*ii*)

The forces acting on the fluid between sections 1-1 and 2-2 are :

1. pressure force at section 1-1 = $p_1 \times A$

where A = Area of pipe

2. pressure force at section 2-2 = $p_2 \times A$
3. frictional force F_1 as shown in Fig. 10.3.

Resolving all forces in the horizontal direction, we have

$$p_1 A - p_2 A - F_1 = 0 \quad ...(10.1)$$

or $(p_1 - p_2)A = F_1 = f' \times P \times L \times V^2$ [$\because$ From (*ii*), $F_1 = f'PLV^2$]

or $$p_1 - p_2 = \frac{f' \times P \times L \times V^2}{A}$$

But from equation (*i*), $p_1 - p_2 = \rho g h_f$

Equating the value of $(p_1 - p_2)$, we get

$$\rho g h_f = \frac{f' \times P \times L \times V^2}{A}$$

or

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \qquad ...(iii)$$

In equation (*iii*), $\frac{P}{A} = \frac{\text{Wetted perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$

$\therefore$

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2 = \frac{f'}{\rho g} \times \frac{4LV^2}{d} \qquad ...(iv)$$

Putting $\frac{f'}{\rho} = \frac{f}{2}$, where f is known as co-efficient of friction.

Equation (*iv*), becomes as $h_f = \frac{4 \,.\, f}{2g} \cdot \frac{LV^2}{d} = \frac{4f \,.\, L \,.\, V^2}{d \times 2g}$ ...(10.2)

Equation (10.2) is known as Darcy-Weisbach equation. This equation is commonly used for finding loss of head due to friction in pipes.

Sometimes equation (10.2) is written as

$$h_f = \frac{f \,.\, L \,.\, V^2}{d \times 2g} \qquad ...(10.2A)$$

Then f is known as friction factor.

10.3.2 Expression for Co-efficient of Friction in Terms of Shear Stress. The equation (10.1) gives the forces acting on a fluid between sections 1-1 and 2-2 of Fig. 10.3 in horizontal direction as

$$p_1 A - p_2 A - F_1 = 0$$

or

$$(p_1 - p_2)A = F_1 = \text{force due to shear stress } \tau_0$$
$$= \text{shear stress} \times \text{surface area}$$
$$= \tau_0 \times \pi d \times L$$

or

$$(p_1 - p_2) \frac{\pi}{4} d^2 = \tau_0 \times \pi d \times L \qquad \left\{\because A = \frac{\pi}{4} d^2\right\}$$

Cancelling πd from both sides, we have

$$(p_1 - p_2) \frac{d}{4} = \tau_0 \times L$$

or

$$(p_1 - p_2) = \frac{4\tau_0 \times L}{d} \qquad ...(10.3)$$

Equation (10.2) can be written as $h_f = \frac{p_1 - p_2}{\rho g} = \frac{4f \,.\, L \,.\, V^2}{d \times 2g}$

or $$(p_1 - p_2) = \frac{4f.L.V^2}{d \times 2g} \times \rho g \qquad \text{...(10.4)}$$

Equating the value of $(p_1 - p_2)$ in equations (10.3) and (10.4),

$$\frac{4\tau_0 \times L}{d} = \frac{4f.L.V^2}{d \times 2g} \times \rho g$$

or $$\tau_0 = \frac{fV^2 \times \rho g}{2g} = \frac{fV^2}{2g} \times \rho g$$

or $$\tau_0 = f\frac{\rho V^2}{2} \qquad \text{...(10.5)}$$

$\therefore$ $$f = \frac{2\tau_0}{\rho V^2}. \qquad \text{...(10.6)}$$

▶ 10.4 SHEAR STRESS IN TURBULENT FLOW

The shear stress in viscous flow is given by Newton's law of viscosity as

$$\tau_v = \mu \frac{du}{dy},$$ where τ_v = shear stress due to viscosity.

Similar to the expression for viscous shear, J. Boussinesq expressed the turbulent shear in mathematical form as

$$\tau_t = \eta \frac{d\overline{u}}{dy} \qquad \text{...(10.7)}$$

where τ_t = shear stress due to turbulence

η = eddy viscosity

$\overline{u}$ = average velocity at a distance y from boundary.

The ratio of η (eddy viscosity) and ρ (mass density) is known as kinematic eddy viscosity and is denoted by ε (epsilon). Mathematically it is written as

$$\varepsilon = \frac{\eta}{\rho} \qquad \text{...(10.8)}$$

If the shear stress due to viscous flow is also considered, then the total shear stress becomes as

$$\tau = \tau_v + \tau_t = \mu \frac{du}{dy} + \eta \frac{d\overline{u}}{dy} \qquad \text{...(10.9)}$$

The value of $\eta = 0$ for laminar flow. For other cases the value of η may be several thousand times the value of μ. To find shear stress in turbulent flow, equation (10.7) given by Boussinesq is used. But as the value of η (eddy viscosity) cannot be predicted, this equation is having limited use.

10.4.1 Reynolds Expression for Turbulent Shear Stress.

Reynolds in 1886 developed an expression for turbulent shear stress between two layers of a fluid at a small distance apart, which is given as

$$\tau = \rho u' v' \qquad \text{...(10.10)}$$

where u', v' = fluctuating component of velocity in the direction of x and y due to turbulence.

As u' and v' are varying and hence τ will also vary. Hence to find the shear stress, the time average on both the sides of the equation (10.10) is taken. Then equation (10.10) becomes as

$$\bar{\tau} = \rho \overline{u' v'} \qquad ...(10.11)$$

The turbulent shear stress given by equation (10.11) is known as Reynold stress.

10.4.2 Prandtl Mixing Length Theory for Turbulent Shear Stress. In equation (10.11), the turbulent shear stress can only be calculated if the value of $\overline{u' v'}$ is known. But it is very difficult to measure $\overline{u' v'}$. To overcome this difficulty, *L*. Prandtl in 1925, presented a mixing length hypothesis which can be used to express turbulent shear stress in terms of measurable quantities.

According to Prandtl, the mixing length *l*, is that distance between two layers in the transverse direction such that the lumps of fluid particles from one layer could reach the other layer and the particles are mixed in the other layer in such a way that the momentum of the particles in the direction of *x* is same. He also assumed that the velocity fluctuation in the *x*-direction u' is related to the mixing length *l* as

$$u' = l \frac{du}{dy}$$

and v', the fluctuation component of velocity in *y*-direction is of the same order of magnitude as u' and hence

$$v' = l \frac{du}{dy}$$

Now $\overline{u' \times v'}$ becomes as $\overline{u' v'} = \left(l \frac{du}{dy}\right) \times \left(l \frac{du}{dy}\right) = l^2 \left(\frac{du}{dy}\right)^2$

Substituting the value of $\overline{u' v'}$ in equation (10.11), we get the expression for shear stress in turbulent flow due to Prandtl as

$$\bar{\tau} = \rho l^2 \left(\frac{du}{dy}\right)^2 \qquad ...(10.12)$$

Thus the total shear stress at any point in turbulent flow is the sum of shear stress due to viscous shear and turbulent shear and can be written as

$$\bar{\tau} = \mu \frac{du}{dy} + \rho l^2 \left(\frac{du}{dy}\right)^2 \qquad ...(10.13)$$

But the viscous shear stress is negligible except near the boundary. Equation (10.13) is used for most of turbulent fluid flow problems for determining shear stress in turbulent flow.

▶ 10.5 VELOCITY DISTRIBUTION IN TURBULENT FLOW IN PIPES

In case of turbulent flow, the total shear stress at any point is the sum of viscous shear stress and turbulent shear stress. Also the viscous shear stress is negligible except near the boundary. Hence it can be assumed that the shear stress in turbulent flow is given by equation (10.12). From this equation, the velocity distribution can be obtained if the relation between *l*, the mixing length and *y* is known. Prandtl assumed that the mixing length, *l* is a linear function of the distance *y* from the pipe wall *i.e.*, $l = ky$, where *k* is a constant, known as Karman constant and = 0.4.

Substituting the value of *l* in equation (10.12), we get

$$\bar{\tau} \text{ or } \tau = \rho \times (ky)^2 \times \left(\frac{du}{dy}\right)^2$$

or
$$\tau = \rho k^2 y^2 \left(\frac{du}{dy}\right)^2 \text{ or } \left(\frac{du}{dy}\right)^2 = \tau/\rho k^2 y^2$$

or
$$\frac{du}{dy} = \sqrt{\frac{\tau}{\rho k^2 y^2}} = \frac{1}{ky}\sqrt{\frac{\tau}{\rho}} \qquad \text{...(10.14)}$$

For small values of y that is very close to the boundary of the pipe, Prandtl assumed shear stress τ to be constant and approximately equal to τ_0 which presents the turbulent shear stress at the pipe boundary. Substituting $\tau = \tau_0$ in equation (10.14), we get

$$\frac{du}{dy} = \frac{1}{ky}\sqrt{\frac{\tau_0}{\rho}} \qquad \text{...(10.15)}$$

In equation (10.15), $\sqrt{\frac{\tau_0}{\rho}}$ has the dimensions $\sqrt{\frac{ML^{-1}T^{-2}}{ML^{-3}}} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T}$. But $\frac{L}{T}$ is velocity and hence $\sqrt{\frac{\tau_0}{\rho}}$ has the dimension of velocity, which is known as shear velocity and is denoted by u_*.

Thus $\sqrt{\frac{\tau_0}{\rho}} = u_*$, then equation (10.15) becomes $\frac{du}{dy} = \frac{1}{ky} u_*$.

For a given case of turbulent flow, u_* is constant. Hence integrating above equation, we get

$$u = \frac{u_*}{k} \log_e y + C \qquad \text{...(10.16)}$$

where C = constant of integration.

Equation (10.16) shows that in turbulent flow, the velocity varies directly with the logarithm of the distance from the boundary or in other words the velocity distribution in turbulent flow is logarithmic in nature. To determine the constant of integration, C the boundary condition that at $y = R$ (radius of pipe), $u = u_{\max}$ is substituted in equation (10.16).

Hence
$$u_{\max} = \frac{u_*}{k} \log_e R + C \quad \therefore \quad C = u_{max} - \frac{u_*}{k} \log_e R$$

Substituting the value of C in equation (10.16), we get

$$u = \frac{u_*}{k} \log_e y + u_{\max} - \frac{u_*}{k} \log_e R = u_{\max} + \frac{u_*}{k} (\log_e y - \log_e R)$$

$$= u_{\max} + \frac{u_*}{0.4} \log_e (y/R) \qquad [\because k = 0.4 = \text{Karman constant}]$$

$$= u_{\max} + 2.5\, u_* \log_e (y/R) \qquad \text{...(10.17)}$$

Equation (10.17) is called 'Prandtl's universal velocity distribution equation for turbulent flow in pipes. This equation is applicable to smooth as well as rough pipe boundaries. Equation (10.17) is also written as

$$u_{max} - u = -2.5\, u_* \log_e (y/R) = 2.5\, u_* \log_e (R/y)$$

Dividing by u_*, we get

$$\frac{u_{max} - u}{u_*} = 2.5 \log_e (R/y) = 2.5 \times 2.3 \log_{10} (R/y) \quad [\because \log_e (R/y) = 2.3 \log_{10} (R/y)]$$

or

$$\frac{u_{max} - u}{u_*} = 5.75 \log_{10} (R/y) \qquad ...(10.18)$$

In equation (10.18), the difference between the maximum velocity u_{max}, and local velocity u at any point *i.e.*, $(u_{max} - u)$ is known as '**velocity defect**'.

10.5.1 Hydrodynamically Smooth and Rough Boundaries. Let k is the average height of the irregularities projecting from the surface of a boundary as shown in Fig. 10.4. If the value of k is large for a boundary then the boundary is called rough boundary and if the value of k is less, then boundary is known as smooth boundary, in general. This is the classification of rough and smooth boundary based on boundary characteristics. But for proper classification, the flow and fluid characteristics are also to be considered.

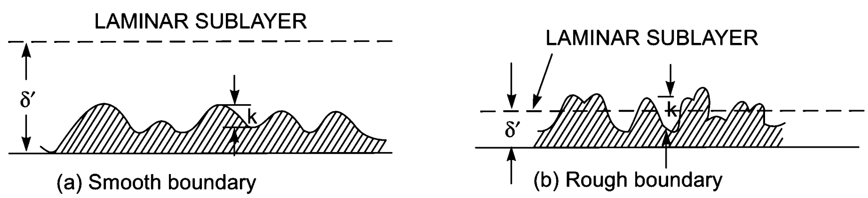


Fig. 10.4 *Smooth and rough boundaries.*

For turbulent flow analysis along a boundary, the flow is divided in two portions. The first portion consists of a thin layer of fluid in the immediate neighbourhood of the boundary, where viscous shear stress predominates while the shear stress due to turbulence is negligible. This portion is known as laminar sub-layer. The height upto which the effect of viscosity predominates in this zone is denoted by δ'. The second portion of flow, where shear stress due to turbulence are large as compared to viscous stress is known as turbulent zone.

If the average height k of the irregularities, projecting from the surface of a boundary is much less than δ', the thickness of laminar sub-layer as shown in Fig. 10.4 (*a*), the boundary is called smooth boundary. This is because, outside the laminar sub-layer the flow is turbulent and eddies of various size present in turbulent flow try to penetrate the laminar sub-layer and reach the surface of the boundary. But due to great thickness of laminar sub-layer the eddies are unable to reach the surface irregularities and hence the boundary behaves as a smooth boundary. This type of boundary is called hydrodynamically smooth boundary.

Now, if the Reynolds number of the flow is increased then the thickness of laminar sub-layer will decrease. If the thickness of laminar sub-layer becomes much smaller than the average height k of irregularities of the surface as shown in Fig. 10.4 (*b*), the boundary will act as rough boundary. This is because the irregularities of the surface are above the laminar sub-layer and the eddies present in turbulent zone will come in contact with the irregularities of the surface and lot of energy will be lost. Such a boundary is called hydrodynamically rough boundary.

From Nikuradse's experiment :

1. If $\frac{k}{\delta'}$ is less than 0.25 or $\frac{k}{\delta'} < 0.25$, the boundary is called smooth boundary.

2. If $\frac{k}{\delta'}$ is greater than 6.0, the boundary is rough,

3. If $0.25 < \left(\frac{k}{\delta'}\right) < 6.0$, the boundary is in transition.

In terms of roughness Reynolds number $\frac{u_* k}{\nu}$:

1. If $\frac{u_* k}{\nu} < 4$, boundary is considered smooth,

2. If $\frac{u_* k}{\nu}$ lies between 4 and 100, boundary is in transition stage, and

3. If $\frac{u_* k}{\nu} > 100$, the boundary is rough.

10.5.2 Velocity Distribution for Turbulent Flow in Smooth Pipes. The velocity distribution for turbulent flow in smooth or rough pipe is given by equation (10.16) as

$$u = \frac{u_*}{k} \log_e y + C$$

It may be seen that at $y = 0$, the velocity u at wall is $-\infty$. This means that velocity u is positive at some distance far away from the wall and $-\infty$ (minus infinity) at the wall. Hence at some finite distance from wall, the velocity will be equal to zero. Let this distance from pipe wall is y'. Now the constant C is determined from the boundary condition *i.e.,* at $y = y'$, $u = 0$. Hence above equation becomes as

$$0 = \frac{u_*}{k} \log_e y' + C \text{ or } C = -\frac{u_*}{k} \log_e y'$$

Substituting the value of C in the above equation, we get

$$u = \frac{u_*}{k} \log_e y - \frac{u_*}{k} \log_e y' = \frac{u_*}{k} \log_e (y/y')$$

Substituting the value of $k = 0.4$, we get

$$u = \frac{u_*}{0.4} \log_e (y/y') = 2.5\, u_* \log_e (y/y')$$

$$\frac{u}{u_*} = 2.5 \times 2.3 \log_{10} (y/y') \qquad [\because \log_e (y/y') = 2.3 \log_{10} (y/y')]$$

or

$$\frac{u}{u_*} = 5.75 \log_{10} (y/y') \qquad \text{...(10.19)}$$

For the smooth boundary, there exists a laminar sub-layer as shown in Fig. 10.4 (*a*). The velocity distribution in the laminar sub-layer is parabolic in nature. Thus in the laminar sub-layer, logarithmic velocity distribution does not hold good. Thus it can be assumed that y' is proportional to δ', where δ' is the thickness of laminar sub-layer. From Nikuradse's experiment the value of y' is given as

$$y' = \frac{\delta'}{107}$$

where $\delta' = \dfrac{11.6\nu}{u_*}$, where ν = kinematic viscosity of fluid.

$$\therefore \qquad y' = \frac{11.6\nu}{u_*} \times \frac{1}{107} = \frac{0.108\nu}{u_*}$$

Substituting this value of y' in equation (10.19), we obtain

$$\frac{u}{u_*} = 5.75 \log_{10} \left(\frac{y}{\frac{.108\nu}{u_*}} \right)$$

$$= 5.75 \log_{10} \left(\frac{y u^*}{.108\, \nu} \right) = 5.75 \log_{10} \left(\frac{u_* y}{\nu} \times 9.259 \right)$$

$$= 5.75 \log_{10} \frac{u_* y}{\nu} + 5.75 \log_{10} 9.259 \qquad \left[\because \frac{1}{0.108} = 9.259 \right]$$

$$= 5.75 \log_{10} \frac{u_* y}{\nu} + 5.55 \qquad \text{...(10.20)}$$

10.5.3 Velocity Distribution for Turbulent Flow in Rough Pipes. In case of rough boundaries, the thickness of laminar sub-layer is very small as shown in Fig. 10.4 (*b*). The surface irregularities are above the laminar sub-layer and hence the laminar sub-layer is completely destroyed. Thus y' can be considered proportional to the height of protrusions k. Nikuradse's experiment shows the value of y' for pipes coated with uniform sand (rough pipes) as $y' = \dfrac{k}{30}$.

Substituting this value of y' in equation (10.19), we get

$$\frac{u}{u_*} = 5.75 \log_{10} \left(\frac{y}{k/30} \right) = 5.75 \, [\log_{10} (y/k) \times 30]$$

$$= 5.75 \log_{10} (y/k) + 5.75 \log_{10} (30.0) = 5.75 \log_{10} (y/k) + 8.5 \qquad \text{...(10.21)}$$

Problem 10.1 *A pipe-line carrying water has average height of irregularities projecting from the surface of the boundary of the pipe as 0.15 mm. What type of boundary is it ? The shear stress developed is 4.9 N/m². The kinematic viscosity of water is .01 stokes.*

Solution. Given :

Average height of irregularities, $k = 0.15 \text{ mm} = 0.15 \times 10^{-3} \text{ m}$

Shear stress developed, $\tau_0 = 4.9 \text{ N/m}^2$

Kinematic viscosity, $\nu = 0.01 \text{ stokes} = .01 \text{ cm}^2/\text{s} = .01 \times 10^{-4} \text{ m}^2/\text{s}$

Density of water, $\rho = 1000 \text{ kg/m}^3$

Shear velocity, $u_* = \sqrt{\tau_0 / \rho} = \sqrt{\dfrac{4.9}{1000}} = \sqrt{0.0049} = 0.07 \text{ m/s}$

Roughness Reynold number $= \dfrac{u_* k}{\nu} = \dfrac{0.07 \times 0.15 \times 10^{-3}}{.01 \times 10^{-4}} = 10.5.$

Since $\dfrac{u_* k}{\nu}$ lies between 4 and 100 and hence pipe surface behaves as in transition.

Problem 10.2 *A rough pipe is of diameter 8.0 cm. The velocity at a point 3.0 cm from wall is 30% more than the velocity at a point 1 cm from pipe wall. Determine the average height of the roughness.*

Solution. Given :

Dia. of rough pipe, $D = 8 \text{ cm} = .08 \text{ m}$

Let velocity of flow at 1 cm from pipe wall $= u$

Then velocity of flow at 3 cm from pipe wall $= 1.3\,u$

The velocity distribution for rough pipe is given by equation (10.21) as

$$\frac{u}{u_*} = 5.75 \log_{10} (y/k) + 8.5, \text{ where } k = \text{height of roughness.}$$

For a point, 1 cm from pipe wall, we have

$$\frac{u}{u_*} = 5.75 \log_{10} (1.0/k) + 8.5 \qquad \text{...}(i)$$

For a point, 3 cm from pipe wall, velocity is 1.3 u and hence

$$\frac{1.3u}{u_*} = 5.75 \log_{10} (3.0/k) + 8.5 \qquad \text{...}(ii)$$

Dividing (*ii*) by (*i*), we get $1.3 = \dfrac{5.75 \log_{10}(3.0/k) + 8.5}{5.75 \log_{10}(1/k) + 8.5}$

or $1.3[5.75 \log_{10} (1/k) + 8.5] = 5.75 \log_{10} (3.0/k) + 8.5$

or $7.475 \log_{10} (1/k) + 11.05 = 5.75 \log_{10} (3.0/k) + 8.5$

or $7.475 \log_{10} (1/k) - 5.75 \log_{10} (3/k) = 8.5 - 11.05 = -2.55$

or $7.475 [\log_{10} 1.0 - \log_{10} k] - 5.75 [\log_{10} 3.0 - \log_{10} k] = -2.55$

or $7.475 [0 - \log_{10} k] - 5.75 [.4771 - \log_{10} k] = -2.55$

or $-7.475 \log_{10} k - 2.7433 + 5.75 \log_{10} k = -2.55$

or $-1.725 \log_{10} k = 2.7433 - 2.55 = 0.1933$

or $\log_{10} k = \dfrac{0.1933}{-1.725} = -0.1120 = \bar{1}.888$

$k =$ **.7726 cm. Ans.**

Problem 10.3 *A smooth pipe of diameter 80 mm and 800 m long carries water at the rate of 0.480 m^3/minute. Calculate the loss of head, wall shearing stress, centre line velocity, velocity and shear stress at 30 mm from pipe wall. Also calculate the thickness of laminar sub-layer. Take kinematic viscosity of water as 0.015 stokes. Take the value of co-efficient of friction 'f' from the relation given as*

$$f = \frac{.0791}{(R_e)^{1/4}}, \text{ where } R_e = \text{Reynolds number.}$$

Solution. Given :

Dia. of smooth pipe, $d = 80 \text{ mm} = .08 \text{ m}$

Length of pipe, $L = 800 \text{ m}$

Discharge, $Q = 0.048 \text{ m}^3/\text{minute} = \dfrac{0.48}{60} = .008 \text{ m}^3/\text{s}$

Kinematic viscosity, $\nu = .015 \text{ stokes} = .015 \times 10^{-4} \text{ m}^2/\text{s}$ [Stokes = cm^2/s]

Density of water, $\rho = 1000 \text{ kg/m}^3$

Mean velocity, $V = \dfrac{Q}{\text{Area}} = \dfrac{0.008}{\frac{\pi}{4}(.08)^2} = 1.591 \text{ m/s}$

∴ Reynolds number, $R_e = \dfrac{V \times d}{\nu} = \dfrac{1.591 \times 0.08}{.015 \times 10^{-4}} = 8.485 \times 10^4$

As the Reynolds number is more than 4000, the flow is turbulent.

Now the value of 'f' is given by $f = \dfrac{.0791}{R_e^{1/4}} = \dfrac{.0791}{\left(8.485 \times 10^4\right)^{1/4}} = .004636$

(i) Head lost is given by equation (10.2) as

$$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g} = \frac{4 \times .004636 \times 800 \times 1.591^2}{.08 \times 2 \times 9.81} = \textbf{23.42 m. Ans.}$$

(ii) Wall shearing stress, τ_0 is given by equation (10.5) as

$$\tau_0 = \frac{f \rho V^2}{2} = .004636 \times \frac{1000}{2} \times 1.591^2 = \textbf{5.866 N/m}^{\mathbf{2}}\textbf{. Ans.}$$

(iii) Centre-line velocity, u_{max} for smooth pipe is given by equation (10.20) as

$$\frac{u}{u_*} = 5.75 \log_{10} \frac{u_* y}{\nu} + 5.55 \qquad ...(i)$$

where u_* is shear velocity and $= \sqrt{\dfrac{\tau_0}{\rho}} = \sqrt{\dfrac{5.866}{1000}} = 0.0765 \text{ m/s}$

The velocity will be maximum when $y = \dfrac{d}{2} = \dfrac{.08}{2} = .04 \text{ m}$.

Hence at $y = .04$ m, $u = u_{max}$. Substituting these values in (i), we get

$$\frac{u_{max}}{.0765} = 5.75 \log_{10} \frac{0.0765 \times .04}{.015 \times 10^{-4}} + 5.55$$

$$= 5.75 \log_{10} 2040 + 5.55$$

$$= 5.75 \times 3.309 + 5.55 = 19.03 + 5.55 = 24.58$$

∴ $u_{max} = .0765 \times 24.58 =$ **1.88 m/s. Ans.**

(iv) The shear stress, τ at any point is given by

$$\tau = -\frac{\partial p}{\partial x} \frac{r}{2} \qquad ...(A)$$

where r = distance from centre of pipe
and hence shear stress at pipe wall where $r = R$ is

$$\tau_0 = -\frac{\partial p}{\partial x}\frac{R}{2} \qquad ...(B)$$

Dividing equation (A) by equation (B), we get

$$\frac{\tau}{\tau_0} = \frac{r}{R}$$

$\therefore$ Shear stress $\qquad \tau = \dfrac{\tau_0 \, r}{R}$

A point 30 mm from pipe wall is having $r = 4 - 3 = 1$ cm $= .01$ m

$$\therefore \qquad \tau \text{ at } (r = .01 \text{ m}) = \frac{\tau_0 \times .01}{.04} = \frac{5.866}{4} = \mathbf{1.4665\ N/m^2.\ Ans.}$$

Velocity at a point 3 cm from pipe wall means $y = 3$ cm $= .03$ m

and is given by equation (10.20) as $\dfrac{u}{u_*} = 5.75 \log_{10} \dfrac{u_* y}{\nu} + 5.55$, where $u_* = .0765$, $y = .03$

$$\therefore \qquad \frac{u}{.0765} = 5.75 \log_{10} \frac{.0765 \times .03}{.015 \times 10^{-4}} + 5.55$$

$$= 5.75 \log_{10} 1530 + 5.55 = 23.86$$

$$\therefore \qquad u = 0.0765 \times 23.86 = \mathbf{1.825\ m/s.\ Ans.}$$

(v) Thickness of laminar sub-layer is given by

$$\delta' = \frac{11.6 \times \nu}{u_*} = \frac{11.6 \times .015 \times 10^{-4}}{.0765} = 2.274 \times 10^{-4} \text{ m}$$

$$= 2.274 \times 10^{-2} \text{ cm} = \mathbf{.02274\ cm.\ Ans.}$$

Problem 10.4 *Determine the wall shearing stress in a pipe of diameter 100 mm which carries water. The velocities at the pipe centre and 30 mm from the pipe centre are 2 m/s and 1.5 m/s respectively. The flow in pipe is given as turbulent.*

Solution. Given :

Dia. of pipe, $\qquad D = 100$ mm $= 0.10$ m

$\therefore$ Radius, $\qquad R = \dfrac{0.10}{2} = 0.05$ m

Velocity at centre, $\qquad u_{max} = 2$ m/s

Velocity at 30 mm or 0.03 m from centre = 1.5 m/s

$\therefore$ Velocity (at $r = 0.03$ m), $u = 1.5$ m/s

Let the wall shearing stress $= \tau_0$

For turbulent flow, the velocity distribution in terms of centre line velocity (u_{max}) is given by equation (10.18) as

$$\frac{u_{max} - u}{u_*} = 5.75 \log_{10}\left(\frac{R}{y}\right)$$

where $u = 1.5$ m/s at $y = (R - r) = 0.05 - 0.03 = .02$ m

$\therefore \quad \dfrac{2.0 - 1.5}{u_*} = 5.75 \log_{10} \dfrac{.05}{.02} = 2.288$ or $\dfrac{0.5}{u_*} = 2.288$

$\therefore \quad u_* = \dfrac{0.5}{2.288} = 0.2185$ m/s

Using the relation $u_* = \sqrt{\tau_0 / \rho}$, where ρ *for* water = 1000 kg/m^3

$\therefore \quad 0.2185 = \sqrt{\dfrac{\tau_0}{1000}}$ or $\dfrac{\tau_0}{1000} = 0.2185^2 = 0.0477$

or $\tau_0 = 0.0477 \times 1000 =$ **47.676 N/m². Ans.**

10.5.4 Velocity Distribution for Turbulent Flow in Terms of Average Velocity. The average velocity $\overline{U}$, through the pipe is obtained by first finding the total discharge Q and then dividing the total discharge by the area of the pipe.

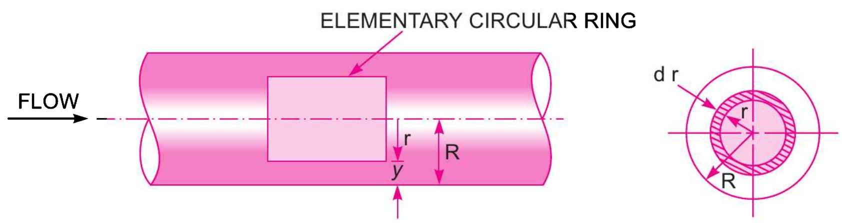


Fig. 10.5 *Average velocity for turbulent flow.*

Consider an elementary circular ring of radius '*r*' and thickness *dr* as shown in Fig. 10.5. The distance of the ring from pipe wall is $y = (R - r)$, where R = radius of pipe.

Then the discharge, dQ, through the ring is given by

$$dQ = \text{area of ring} \times \text{velocity}$$
$$= 2\pi r dr \times u = u \times 2\pi r dr$$

Total discharge,
$$Q = \int dQ = \int_0^R u \times 2\pi r dr \quad \text{...(10.22)}$$

(*a*) **For smooth pipes.** For smooth pipes, the velocity distribution is given by equation (10.20) as

$$\frac{u}{u_*} = 5.75 \log_{10} \frac{u_* y}{\nu} + 5.5$$

or
$$u = \left[5.75 \log_{10} \frac{u_* y}{\nu} + 5.5\right] \times u_*$$

But $y = (R - r)$

$$\therefore \quad u = \left[5.75 \log_{10} \frac{u_*(R - r)}{\nu} + 5.5\right] \times u_*$$

Substituting the value of u in equation (10.22), we get

$$Q = \int_0^R \left[5.75 \log_{10} \frac{u_*(R - r)}{\nu} + 5.5\right] u_* \times 2\pi r dr$$

$\therefore$ Average velocity, $$\overline{U} = \frac{Q}{\text{Area}} = \frac{Q}{\pi R^2}$$

$$= \frac{1}{\pi R^2} \int_0^R \left[5.75 \log_{10} \frac{u_*(R-r)}{\nu} + 5.5\right] u_* \, 2\pi r dr$$

Integration of the above equation and subsequent simplification gives the average velocity for turbulent flow in smooth pipes as

$$\frac{\overline{U}}{u_*} = 5.75 \log_{10} \frac{u_* R}{\nu} + 1.75 \qquad \text{...(10.23)}$$

(*b*) **For rough pipes.** For rough pipes, the velocity at any point in turbulent flow is given by equation (10.21) as

$$\frac{u}{u_*} = 5.75 \log_{10} (y/k) + 8.5$$

But $$y = (R - r)$$

$\therefore$ $$\frac{u}{u_*} = 5.75 \log_{10} \left(\frac{R-r}{k}\right) + 8.5$$

or $$u = u_* \left[5.75 \log_{10} \left(\frac{R-r}{k}\right) + 8.5\right]$$

Substituting the value of u in equation (10.22), we get

$$Q = \int_0^R u_* \left[5.75 \log_{10} \left(\frac{R-r}{k}\right) + 8.5\right] 2\pi r dr$$

$\therefore$ Average velocity, $$\overline{U} = \frac{Q}{\pi R^2} = \frac{\int_0^R u_* \left[5.75 \log_{10} \left(\frac{R-r}{k}\right) + 8.5\right] 2\pi r dr}{\pi R^2}$$

Integration of the above equation and subsequent simplification will give the following relation for average velocity, $\overline{U}$ for turbulent flow in rough pipe as

$$\frac{\overline{U}}{u_*} = 5.75 \log_{10} \frac{R}{k} + 4.75 \qquad \text{...(10.24)}$$

(*c*) **Difference of the velocity at any point and average velocity for smooth and rough pipes.** The velocity at any point for turbulent flow for smooth pipes is given by equation (10.20) as

$$\frac{u}{u_*} = 5.75 \log_{10} \frac{u_*(R-r)}{\nu} + 5.5 \qquad [\because y = R - r]$$

and the average velocity is given by equation (10.23) as

$$\frac{\overline{U}}{u_*} = 5.75 \log_{10} \frac{u_* R}{\nu} + 1.75$$

$\therefore$ Difference of velocity u and $\overline{U}$ for smooth pipe is obtained as

$$\frac{u}{u_*} - \frac{\overline{U}}{u_*} = \left[5.75 \log_{10} \frac{u_*(R-r)}{\nu} + 5.5\right] - \left[5.75 \log_{10} \frac{u_* R}{\nu} + 1.75\right]$$

or

$$\frac{u - \overline{U}}{u_*} = 5.75 \left[\log_{10} \frac{u_*(R-r)}{\nu} - \log_{10} \frac{u_* R}{\nu}\right] + 5.5 - 1.75$$

$$= 5.75 \log_{10} \left[\frac{u_*(R-r)}{\nu} \div \frac{u_* R}{\nu}\right] + 3.75$$

$$= 5.75 \log_{10} \left(\frac{R-r}{\nu}\right) + 3.75$$

$$= 5.75 \log_{10} (y/R) + 3.75 \quad \text{...(10.25)} \quad [\because R - r = y]$$

Similarly the velocity, u at any point for rough pipe is given by equation (10.21) as

$$\frac{u}{u_*} = 5.75 \log_{10} (y/k) + 8.5$$

and average velocity is given by equation (10.24) as

$$\frac{\overline{U}}{u_*} = 5.75 \log_{10} (R/k) + 4.75$$

$\therefore$ Difference of velocity u and $\overline{U}$ for rough pipe is given by

$$\frac{u}{u_*} - \frac{\overline{U}}{u_*} = [5.75 \log_{10} (y/k) + 8.5] - [5.75 \log_{10} (R/k) + 4.75]$$

$$= 5.75 \log_{10} [(y/k) \div (R/k)] + 8.5 - 4.75$$

or

$$\frac{u - \overline{U}}{u_*} = 5.75 \log_{10} (y/R) + 3.75 \quad \text{...(10.26)}$$

Equations (10.25) and (10.26) are the same. This shows that the difference of velocity at any point and the average velocity will be the same in case of smooth as well as rough pipes.

Problem 10.5 *Determine the distance from the pipe wall at which the local velocity is equal to the average velocity for turbulent flow in pipes.*

Solution. Given :

Local velocity at a point = average velocity

or $u = \overline{U}$

For a smooth or rough pipe, the difference of velocity at any point and average velocity is given by equation (10.25) or equation (10.26) as

$$\frac{u - \overline{U}}{u_*} = 5.75 \log_{10} (y/R) + 3.75$$

Substituting the given condition *i.e.,* $u = \overline{U}$, we get

$$\frac{\overline{U} - \overline{U}}{u_*} = 0 = 5.75 \log_{10} (y/R) + 3.75 \quad \text{or} \quad 5.75 \log_{10} (y/R) = 3.75$$

or $$\log_{10}(y/R) = -\frac{3.75}{5.75} = -0.6521 = -\bar{1}.3479$$

$\therefore$ $$y/R = 0.22279 \simeq 0.2228 \text{ or } y = \mathbf{.2228\ R.\ Ans.}$$

Problem 10.6 *For turbulent flow in a pipe of diameter 300 mm, find the discharge when the centre-line velocity is 2.0 m/s and the velocity at a point 100 mm from the centre as measured by pitot-tube is 1.6 m/s.*

Solution. Given :

Dia. of pipe, $D = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Radius, $R = \frac{0.3}{2} = 0.15 \text{ m}$

Velocity at centre, $u_{max} = 2.0 \text{ m/s}$

Velocity (at $r = 100$ mm $= 0.1$ m), $u = 1.6$ m/s

Now $y = R - r = 0.15 - 0.10 = 0.05$ m

$\therefore$ Velocity (at $r = 0.1$ m or at $y = 0.05$ m), $u = 1.6$ m/s

The velocity in terms of centre-line velocity is given by equation (10.18) as

$$\frac{u_{max} - u}{u_*} = 5.75 \log_{10}(R/y)$$

Substituting the values, we get $\frac{2.0 - 1.6}{u_*} = 5.75 \log_{10} \frac{.15}{.05}$ $\left[\because \begin{array}{l} y = .05 \text{ m} \\ R = 0.15 \text{ m} \end{array}\right]$

$$= 5.75 \log_{10} 3.0 = 2.7434$$

or $$\frac{0.4}{u_*} = 2.7434$$

$\therefore$ $$u_* = \frac{0.4}{2.7434} = 0.1458 \text{ m/s} \qquad ...(i)$$

Using equation (10.26) which gives relation between velocity at any point and average velocity, we have

$$\frac{u - \bar{U}}{u_*} = 5.75 \log_{10}(y/R) + 3.75$$

at $y = R$, velocity u becomes $= u_{max}$

$\therefore$ $$\frac{u_{max} - \bar{U}}{u_*} = 5.75 \log_{10}(R/R) + 3.75 = 5.75 \times 0 + 3.75 = 3.75$$

But $u_{max} = 2.0$ and u_* from $(i) = 0.1458$

$\therefore$ $$\frac{2.0 - \bar{U}}{0.1458} = 3.75$$

or $$\bar{U} = 2.0 - .1458 \times 3.75 = 2.0 - 0.5467 = 1.4533 \text{ m/s}$$

$\therefore$ Discharge, $Q = \text{Area} \times \text{average velocity}$

$$= \frac{\pi}{4} D^2 \times \overline{U} = \frac{\pi}{4} (0.3)^2 \times 1.4533 = \mathbf{0.1027\ m^3/s.\ Ans.}$$

10.5.5 Velocity Distribution for Turbulent Flow in Smooth Pipes by Power Law. The velocity distribution for turbulent flow as given by equations (10.18), (10.20) and (10.21) are logarithmic in nature. These equations are not convenient to use. Nikuradse carried out experiments for different Reynolds number to determine the velocity distribution law in smooth pipes. He expressed the velocity distribution in exponential form as

$$\frac{u}{u_{max}} = (y/R)^{1/n} \qquad \text{...(10.27)}$$

where exponent $\frac{1}{n}$ depends on Reynolds number

The value of $\left(\frac{1}{n}\right)$ decreases, with increasing Reynolds number.

For $R_e = 4 \times 10^3, \quad \frac{1}{n} = \frac{1}{6}$

$R_e = 1.1 \times 10^5, \quad \frac{1}{n} = \frac{1}{7}$

$R_e \geq 2 \times 10^6, \quad \frac{1}{n} = \frac{1}{10}$

Thus if $\frac{1}{n} = \frac{1}{7}$, the velocity distribution law becomes as

$$\frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{1/7} \qquad \text{...(10.28)}$$

Equation (10.28) is known as 1/7th power law of velocity distribution for smooth pipes.

▶ 10.6 RESISTANCE OF SMOOTH AND ROUGH PIPES

The loss of head, due to friction in pipes is given by equation (10.2) as

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g}$$

In this equation, the value of co-efficient of friction, f should be known accurately for predicting the loss of head due to friction in pipes. On the basis of dimensional analysis, it can be shown that the pressure loss in a straight pipe of diameter D, length L, roughness k, average velocity of flow $\overline{U}$, viscosity and density of fluid μ and ρ is

$$\Delta p = \frac{\rho \overline{U}^2}{2} \phi\left[R_e, \frac{k}{D}, \frac{L}{D}\right] \quad \text{or} \quad \frac{\Delta p}{\frac{\rho \overline{U}^2}{2}} = \phi\left[R_e, \frac{k}{D}, \frac{L}{D}\right]$$

Experimentally it was found that pressure drop is a function of $\frac{L}{D}$ to the first power and hence

$$\frac{\Delta p}{\frac{\rho \overline{U}^2}{2}} = \frac{L}{D}\phi\left[R_e, \frac{k}{D}\right] \quad \text{or} \quad \frac{\Delta p \times D}{L\frac{\rho \overline{U}^2}{2}} = \phi\left[R_e, \frac{k}{D}\right]$$

The term of the right hand side is called co-efficient of friction f. Thus $f = \phi\left[R_e, \frac{k}{D}\right]$

This equation shows that friction co-efficient is a function of Reynolds number and k/D ratio, where k is the average height of pipe wall roughness protrusions.

(*a*) **Variation of '*f*' for Laminar Flow.** In viscous flow chapter, it is shown that co-efficient of friction 'f' for laminar flow in pipes is given by

$$f = \frac{16}{R_e} \qquad \text{...(10.29)}$$

Thus friction co-efficient is only a function of Reynolds number in case of laminar flow. It is independent of (k/D) ratio.

(*b*) **Variation of '*f*' for Turbulent Flow.** For turbulent flow, the co-efficient of friction is a function of R_e and k/D ratio. For relative roughness (k/D), in the turbulent flow the boundary may be smooth or rough and hence the value of 'f' will be different for these boundaries.

(*i*) **'*f*' for smooth pipes.** For turbulent flow in smooth pipes, co-efficient of friction is a function of Reynolds number only. The value of laminar sub-layer in case of smooth pipe is large as compared to the average height of surface roughness k. The value of 'f' for smooth pipe for Reynolds number varying from 4000 to 100000 is given by the relation

$$f = \frac{.0791}{(R_e)^{1/4}} \qquad \text{...(10.30)}$$

The equation (10.30) is given by Blasius.

The value of 'f' for $R_e > 10^5$ is obtained from equation (10.23) which gives the velocity distribution for smooth pipe in terms of average velocity $(\overline{U})$ as

$$\frac{\overline{U}}{u_*} = 5.75 \log_{10} \frac{(u_* R)}{\nu} + 1.75 \qquad \text{...(10.31)}$$

From equation (10.6), we have $f = \frac{2\tau_0}{\rho V^2}$, where V = average velocity

$$\therefore \quad f = \frac{2\tau_0}{\rho \overline{U}^2} = \frac{2}{\overline{U}^2}\left(\sqrt{\frac{\tau_0}{\rho}}\right)^2 = \frac{2}{\overline{U}^2} \times u_*^2 \qquad \left[\because \sqrt{\frac{\tau_0}{\rho}} = u_*\right]$$

$$\therefore \quad u_*^2 = \frac{f\,\overline{U}^2}{2}$$

or

$$u_* = \overline{U}\sqrt{\frac{f}{2}} \qquad \text{...(10.31}A\text{)}$$

Substituting the value of u_* in equation (10.31), we get

$$\frac{\overline{U}}{\overline{U}\sqrt{f/2}} = 5.75 \log_{10}\left(\frac{\sqrt{f/2}}{\nu}\right) R + 1.75$$

or $$\frac{1}{\sqrt{f/2}} = 5.75 \log_{10}\left(\frac{\bar{U}R}{\nu}\sqrt{f/2}\right) + 1.75$$

Taking $R = D/2$ and simplifying, the above equation is written as

$$\frac{1}{\sqrt{4f}} = 2.03 \log_{10}\left(\frac{\bar{U}D}{\nu}\sqrt{4f}\right) - 0.91$$

But $\frac{\bar{U}D}{\nu} = R_e$ and hence above equation is written as

$$\frac{1}{\sqrt{4f}} = 2.03 \log_{10} (R_e\sqrt{4f}) - 0.91 \qquad ...(10.32)$$

Equation (10.32) is valid upto $R_e = 4 \times 10^6$

Nikuradse's experimental result for turbulent flow in smooth pipe for 'f' is

$$\frac{1}{\sqrt{4f}} = 2.0 \log_{10} (R_e\sqrt{4f}) - 0.8 \qquad ...(10.33)$$

This is applicable upto $R_e = 4 \times 10^7$. But the equation (10.33) is solved by hit and trial method. The value of 'f' (*i.e.*, co-efficient of friction) can alternately be obtained as

$$f = .0008 + \frac{.05525}{(R_e)^{0.237}} \qquad ...(10.34)$$

The value of 'f'[*i.e.*, friction factor which is used in equation (10.2*A*)] is given by

$$f = 0.0032 + \frac{0.221}{(R_e)^{0.237}} \qquad ...(10.34A)$$

(*ii*) **Value of 'f' for rough pipes.** For turbulent flow in rough pipes, the co-efficient of friction is a function of relative roughness (k/D) and it is independent of Reynolds number. This is because the value of laminar sub-layer for rough pipes is very small as compared to the height of surface roughness. The average velocity for rough pipes is given by (10.24) as

$$\frac{\bar{U}}{u_*} = 5.75 \log_{10} (R/k) + 4.75$$

But $$u_* = \bar{U}\sqrt{f/2}$$

Substituting the value of u_* in the above equation, we get

$$\frac{\bar{U}}{\bar{U}\sqrt{f/2}} = 5.75 \log_{10} (R/k) + 4.75$$

which is simplified to the form as $$\frac{1}{\sqrt{4f}} = 2.03 \log_{10} (R/k) + 1.68 \qquad ...(10.35)$$

But Nikuradse's experimental result gave for rough pipe the following relation for 'f' as

$$\frac{1}{\sqrt{4f}} = 2 \log_{10} (R/k) + 1.74 \qquad ...(10.36)$$

(*c*) **Value of 'f' for commercial pipes.** The value of 'f' for commercial pipes such as pipes made of metal, concrete and wood is obtained from Nikuradse's experimental data for smooth and rough pipes. According to Colebrook, by subtracting 2 $\log_{10}$ (R/k) from both sides of equations (10.33) and (10.36), the value of 'f' is obtained for commercial smooth and rough pipes as :

1. Smooth pipes

$$\frac{1}{\sqrt{4f}} - 2\log_{10}(R/k) = 2\log_{10}(R_e\sqrt{4f}) - 0.8 - 2\log_{10}(R/k)$$

$$= 2\log_{10}\left(\frac{R_e\sqrt{4f}}{R/k}\right) - 0.8 \qquad ...(10.37)$$

2. Rough pipes

$$\frac{1}{\sqrt{4f}} - 2\log_{10}(R/k) = 2\log_{10}(R/k) + 1.74 - 2\log_{10}(R/k)$$

$$= 1.74. \qquad ...(10.38)$$

Problem 10.7 *For the problem 10.6, find the co-efficient of friction and the average height of roughness projections.*

Solution. From the solution of problem 10.6, we have

$$R = 0.15 \text{ m}$$

$$u_* = 0.1458 \text{ m/s}$$

$$\overline{U} = 1.4533 \text{ m/s}$$

For co-efficient of friction, we know that

$$u_* = \overline{U}\sqrt{f/2}$$

or $$0.1458 = 1.4533\sqrt{f/2}$$

or $$\sqrt{f/2} = \frac{0.1458}{1.4533} = 0.1$$

∴ $$f = 2.0 \times (.1)^2 = \mathbf{.02.\ Ans.}$$

Height of roughness projection is obtained from equation (10.36) as

$$\frac{1}{\sqrt{4f}} = 2\log_{10}(R/k) + 1.74$$

Substituting the values of R and f, we get

$$\frac{1}{\sqrt{4 \times 0.2}} = 2\log_{10}\left(\frac{0.15}{k}\right) + 1.74 \text{ or } 3.5355 = 2\log_{10}\left(\frac{.15}{k}\right) + 1.74$$

or $$\log_{10}\left(\frac{.15}{k}\right) = \frac{3.5355 - 1.74}{2} = 0.8977 = \log_{10} 7.90$$

∴ $$\frac{0.15}{k} = 7.90$$

∴ $$k = \frac{0.15}{7.90} = 0.01898 \text{ m} = \mathbf{18.98\ mm.\ Ans.}$$

Problem 10.8 *Water is flowing through a rough pipe of diameter 500 mm and length 4000 m at the rate of 0.5 m*^{3}*/s. Find the power required to maintain this flow. Take the average height of roughness as k = 0.40 mm.*

Solution. Given :

Dia. of rough pipe, $D = 500$ mm $= 0.50$ m

$\therefore$ Radius, $R = \frac{D}{2} = 0.25$ m

Length of pipe, $L = 4000$ m

Discharge, $Q = 0.5\ m^3/s$

Average height of roughness, $k = 0.40$ mm $= 0.4 \times 10^{-3}$ m

First find the value of co-efficient of friction. Then calculate the head lost due to friction and then power required.

For a rough pipe, the value of 'f' is given by the equation (10.36) as

$$\frac{1}{\sqrt{4f}} = 2\log_{10}(R/k) + 1.74 = 2\log_{10}\left(\frac{.25}{.4 \times 10^{-3}}\right) + 1.74$$

$$= 2\log_{10}(625.0) + 1.74 = 5.591 + 1.74 = 7.331$$

or $$\sqrt{4f} = \frac{1}{7.331} = 0.1364 \text{ or } f = (0.1364)^2/4 = .00465$$

Also the average velocity, $$\overline{U} = \frac{\text{Discharge}}{\text{Area}} = \frac{0.5}{\frac{\pi}{4}D^2} = \frac{0.5}{\frac{\pi}{4}(.5)^2} = 2.546$$

$\therefore$ Head lost due to friction, $$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g} = \frac{4 \times .00465 \times 4000 \times 2.546^2}{0.5 \times 2 \times 9.81}$$

$$= 49.16 \text{ m} \qquad [\because V = \overline{U} = 2.546,\ d = D = 0.5]$$

$\therefore$ Power required, $$P = \frac{W \times h_f}{1000} = \frac{w.Q.h_f}{1000} = \frac{\rho \times g \times Q \times h_f}{1000} \text{ kW}$$

$$= \frac{1000 \times 9.81 \times 0.5 \times 49.16}{1000} = \mathbf{241.13\ kW.\ Ans.}$$

Problem 10.9 *A smooth pipe of diameter 400 mm and length 800 m carries water at the rate of 0.04 m^3/s. Determine the head lost due to friction, wall shear stress, centre-line velocity and thickness of laminar sub-layer. Take the kinematic viscosity of water as 0.018 stokes.*

Solution. Given :

Dia. of pipe, $D = 400$ mm $= 0.40$ m

$\therefore$ Radius, $R = \frac{D}{2} = 0.20$ m

Length of pipe, $L = 800$ m

Discharge, $Q = 0.04\ m^3/s$

Kinematic viscosity, $\nu = 0.018$ stokes $= 0.018\ cm^2/s = 0.018 \times 10^{-4}\ m^2/s$

Average velocity, $$\overline{U} = \frac{Q}{\text{Area}} = \frac{0.04}{\frac{\pi}{4}(0.4)^2} = 0.3183 \text{ m/s}$$

$\therefore$ Reynolds number, $$R_e = \frac{V \times D}{\nu} = \frac{\overline{U} \times D}{\nu} = \frac{0.3183 \times 0.4}{.018 \times 10^{-4}} = 7.073 \times 10^4$$

The flow is turbulent.

The co-efficient of friction 'f' is obtained from equation (10.30) as

$$f = \frac{.0791}{(R_e)^{1/4}} = \frac{0.0791}{(7.073 \times 10^4)^{1/4}} = \frac{.0791}{16.30} = .00485$$

(*i*) Head lost due to friction, $h_f = \frac{4 \cdot f \cdot L \cdot V^2}{D \times 2g} = \frac{4 \cdot f \cdot L \cdot \overline{U}^2}{D \times 2g}$

$$= \frac{4 \times .00485 \times 800 \times (.3183)^2}{0.40 \times 2 \times 9.81} = \mathbf{0.20\ m.\ Ans.}$$

(*ii*) Wall shear stress (τ_0) is given by equation (10.5) as

$$\tau_0 = \frac{f \cdot \rho \cdot V^2}{2} = \frac{f \cdot \rho \cdot \overline{U}^2}{2} \qquad [\because V = \overline{U}]$$

$$= 0.00485 \times 1000 \times \frac{(.3184)^2}{2.0} \text{ N/m}^2 = \mathbf{0.245\ N/m^2.\ Ans.}$$

(*iii*) The centre-line velocity (u_{max}) for smooth pipe is given by equation (10.20) as in which $u = u_{max}$ at $y = R$

$$\therefore \qquad \frac{u_{max}}{u_*} = 5.75 \log_{10} \frac{u_* R}{\nu} + 5.55 \text{ [Put in equation (10.20), } u = u_{max} \text{ at } y = R]$$

where the shear velocity $\qquad u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{0.245}{1000}} = \sqrt{0.000245} = 0.0156$ m/s

Substituting the values of u_*, R and ν in the above equation, we get

$$\frac{u_{max}}{0.0156} = 5.75 \log_{10} \frac{0.0156 \times 0.20}{.018 \times 10^{-4}} + 5.55 = 24.173$$

or $\qquad u_{max} = 24.173 \times .0156 = \mathbf{0.377\ m/s.\ Ans.}$

(*iv*) The thickness of laminar sub-layer (δ') is given by

$$\delta' = \frac{11.6 \times \nu}{u_*} = \frac{11.6 \times .018 \times 10^{-4}}{.0156} = .001338 \text{ m} = \mathbf{1.338\ mm.\ Ans.}$$

Problem 10.10 *A rough pipe of diameter 400 mm and length 1000 m carries water at the rate of 0.4 m³/s. The wall roughness is 0.012 mm. Determine the co-efficient of friction, wall shear stress, centre-line velocity and velocity at a distance of 150 mm from the pipe wall.*

Solution. Given :

Dia. of rough pipe, $\qquad D = 400 \text{ mm} = 0.4$ m

$\therefore$ Radius, $\qquad R = \frac{D}{2} = \frac{0.4}{2} = 0.20$ m

Length of pipe, $\qquad L = 1000$ m

Discharge, $\qquad Q = 0.4 \text{ m}^3/\text{s}$

Wall roughness, $\qquad k = 0.012 \text{ mm} = 0.012 \times 10^{-3}$ m (*i*) The value of co-efficient of friction 'f' for rough pipe is given by the equation (10.36) as

$$\frac{1.0}{\sqrt{4f}} = 2 \log_{10} (R/k) + 1.74$$

or
$$\frac{1.0}{\sqrt{4f}} = 2 \log_{10} \left(\frac{0.20}{.012 \times 10^{-3}}\right) + 1.74$$

$$= 2 \log_{10} (16666.67) + 1.74 = 10.183$$

$$\therefore \quad 4f = \left(\frac{1}{10.183}\right)^2 = .00964$$

$$\therefore \quad f = \frac{.00964}{4.0} = \mathbf{.00241.\ Ans.}$$

(*ii*) Centre-line velocity (u_{max}) for rough pipe is given by equation (10.21) in which u is made $= u_{max}$ at $y = R$ and hence

$$\frac{u_{max}}{u_*} = 5.75 \log_{10} (R/k) + 8.5 \quad \text{...(i)}$$

where shear velocity, $u_* = \sqrt{\frac{\tau_0}{\rho}}$

and τ_0 = wall shear stress = $\frac{f \cdot \rho \cdot V^2}{2}$

where $V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{\frac{\pi}{4} D^2} = \frac{Q}{\frac{\pi}{4} (.4)^2}$ = **3.183 m/s. Ans.**

(*iii*) $\therefore$
$$\tau_0 = \frac{f \cdot \rho \cdot V^2}{2} = .00241 \times 1000 \times \frac{3.183^2}{2.0} = \mathbf{12.2\ N/m^2.\ Ans.}$$

$$\therefore \quad u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{12.2}{1000}} = 0.11 \text{ m/s}$$

Substituting the value of u_*, R, k in equation (*i*), we get

$$\frac{u_{max}}{0.11} = 5.75 \log_{10} \left(\frac{0.2}{.012 \times 10^{-3}}\right) + 8.5 = 32.77$$

$$\therefore \quad u_{max} = 32.77 \times 0.11 = \mathbf{3.60\ m/s.\ Ans.}$$

(*iv*) Velocity (u) at a distance y = 150 mm = 0.15 m

The velocity (u) at any point for rough pipe is given by equation (10.21) as

$$\frac{u}{u_*} = 5.75 \log_{10} (y/k) + 8.5$$

where u_* = 0.11 m/s and y = 0.15 m, $k = 0.012 \times 10^{-3}$ m

$\because \qquad \frac{u}{0.11} = 5.75 \log_{10}\left(\frac{0.15}{.012 \times 10^{-3}}\right) + 8.5 = 32.05$

$\therefore \qquad u = 32.05 \times 0.11 = \mathbf{3.52\ m/s.\ Ans.}$

Problem 10.11 *A smooth pipe line of 100 mm diameter carries 2.27 m³ per minute of water at 20°C with kinematic viscosity of 0.0098 stokes. Calculate the friction factor, maximum velocity as well as shear stress at the boundary.*

Solution. Given :

Dia. of pipe, $D = 100$ mm $= 0.1$ m

$\therefore$ Radius of pipe, $R = 0.05$ m

Discharge, $Q = 2.27\ \text{m}^3/\text{min} = \frac{2.27}{60}\ \text{m}^3/\text{s} = 0.0378\ \text{m}^3/\text{s}$

Kinematic viscosity, $\nu = 0.0098$ stokes $= 0.0098\ \text{cm}^2/\text{s} = 0.0098 \times 10^{-4}\ \text{m}^2/\text{s}$

Now average velocity is given by $\overline{U} = \frac{Q}{\text{Area}} = \frac{0.0378}{\frac{\pi}{4}(0.1)^2} = \frac{0.0378 \times 4}{\pi \times 0.01} = 4.817$ m/s

$\therefore$ Reynolds number is given by, $R_e = \frac{\overline{U} \times D}{\nu} = \frac{4.817 \times 0.1}{0.0098 \times 10^{-4}} = 4.9154 \times 10^5$.

The flow is turbulent and R_e is more than 10^5. Hence for smooth pipe, the co-efficient of friction 'f' is obtained from equation (10.33) as

$$\frac{1}{\sqrt{4f}} = 2.0 \log_{10} (R_e\sqrt{4f}) - 0.8$$

or

$$\frac{1}{\sqrt{4f}} = 2.0 \log_{10} (4.9154 \times 10^5 \times \sqrt{4f}) - 0.8$$

$$= 2.0 [\log_{10} 4.9154 \times 10^5 + \log_{10} \sqrt{4f}] - 0.8$$

$$= 2.0 [5.6915 + \log_{10} \sqrt{4f}] - 0.8 = 2 \times 5.6915 + 2 \log_{10} \sqrt{4f} - 0.8$$

$$= 11.3830 + \log_{10} \left(\sqrt{4f}\right)^2 - 0.8 = 11.383 + \log_{10} (4f) - 0.8$$

or

$$\frac{1}{\sqrt{4f}} - \log_{10} (4f) = 11.383 - 0.8 = 10.583 \qquad ...(i)$$

(*i*) *Friction factor*

Now, friction factor (f^*) = 4 × co-efficient of friction = $4f$

Substituting the value of '$4f$' in equation (*i*), we get

$$\frac{1}{\sqrt{f^*}} - \log_{10} f^* = 10.583 \qquad ...(ii)$$

The above equation is solved by hit and trial method.

Let $f^* = 0.1$, then L.H.S. of equation (*ii*), becomes as

$$\text{L.H.S.} = \frac{1}{\sqrt{0.1}} - \log_{10} 0.1 = 3.16 - (-1.0) = 4.16$$

Let $f^* = 0.01$, then L.H.S. of equation (*ii*), becomes as

$$\text{L.H.S.} = \frac{1}{\sqrt{0.01}} - \log_{10} 0.01 = 10 - (-2) = 12$$

But for exact solution, L.H.S. should be 10.583. Hence value of f^* lies between 0.1 and 0.01.

Let $f^* = 0.013$ then L.H.S. of equation (*ii*), becomes as

$$\text{L.H.S.} = \frac{1}{\sqrt{0.013}} - \log_{10} 0.013 = 8.77 - (-1.886) = 8.77 + 1.886 = 10.656$$

which is approximately equal to 10.583.

Hence the value of f^* is equal to 0.013.

$\therefore$ Friction factor, $f^* =$ **0.013. Ans.**

(*ii*) *Maximum velocity* (u_{max})

Now we know that $f^* = 4f$

$$\therefore \quad \text{Co-efficient of friction, } f = \frac{f^*}{4} = \frac{0.013}{4} = 0.00325$$

Now the shear velocity (u_*) in terms of co-efficient of friction and average velocity is given by equation (10.31*A*) as

$$u_* = \overline{U}\sqrt{\frac{f}{2}} = 4.817 \times \sqrt{\frac{0.00325}{2}} = 4.817 \times 0.0403 = 0.194$$

For smooth pipe, the velocity at any point is given by equation (10.20)

$$u = u_*\left[5.75 \log_{10} \frac{u_* \times y}{\nu} + 5.55\right]$$

The velocity will be maximum at the centre of the pipe,

where $y = R = 0.05$

i.e., radius of pipe. Hence the above equation becomes as

$$U_{\max} = u_*\left[5.75 \log_{10} \frac{u_* \times R}{\nu} + 5.55\right]$$

$$= 0.194\left[5.75 \log_{10} \frac{0.194 \times 0.05}{0.0098 \times 10^{-4}} + 5.55\right]$$

$$= 0.194\,[22.974 + 5.55] = \mathbf{5.528\ m/s.\ Ans.}$$

(*iii*) *Shear stress at the boundary* (τ_0)

We know that $u_* = \sqrt{\dfrac{\tau_0}{\rho}}$ or $u_*^2 = \dfrac{\tau_0}{\rho}$

$\therefore$ $\tau_0 = \rho\, u_*^2 = 1000 \times 0.194^2 =$ **37.63 N/m². Ans.**

Problem 10.12 *Hydrodynamically smooth pipe carries water at the rate of 300 l/s at 20°C (ρ = 1000 kg/m³, $\nu = 10^{-6}$ m²/s) with a head loss of 3 m in 100 m length of pipe. Determine the pipe diameter. Use* $f = 0.0032 + \dfrac{0.221}{(R_e)^{0.237}}$ *equation for f, where* $h_f = \dfrac{f \times L \times V^2}{D \times 2g}$ *and* $R_e = \dfrac{\rho VD}{\mu}$.

Solution. Given :

Discharge, $Q = 300$ l/s $= 0.3$ m^3/s

Density, $\rho = 1000$ kg/m^3

Kinematic viscosity, $\nu = 10^{-6}$ m^2/s

Head loss, $h_f = 3$ m

Length of pipe, $L = 100$ m

Value of friction factor, $f = 0.0032 + \dfrac{0.221}{(R_e)^{0.237}}$

Reynolds number, $R_e = \dfrac{\rho VD}{\mu} = \dfrac{V \times D}{\nu}$ $\left(\because \dfrac{\mu}{\rho} = \nu\right)$

$$= \frac{V \times D}{10^{-6}} = V \times D \times 10^6$$

Find : Diameter of pipe.

Let D = Diameter of pipe

Head loss in terms of friction factor is given as

$$h_f = \frac{f \times L \times V^2}{D \times 2g}$$

or $$3 = \frac{f \times 100 \times V^2}{D \times 2 \times 9.81} \qquad (\because h_f = 3, L = 100 \text{ m})$$

or $$f = \frac{3 \times D \times 2 \times 9.81}{100\, V^2} \text{ or } f = \frac{0.5886\, D}{V^2} \qquad ...(i)$$

Now $$Q = A \times V$$

or $$0.3 = \frac{\pi}{4} D^2 \times V \text{ or } D^2 \times V = \frac{4 \times 0.3}{\pi} = 0.382$$

$\therefore$ $$V = \frac{0.382}{D^2} \qquad ...(ii)$$

Also $$f = 0.0032 + \frac{0.221}{(R_e)^{0.237}}$$

or $$\frac{0.5886\, D}{V^2} = 0.0032 + \frac{0.221}{\left(V \times D \times 10^6\right)^{0.237}}$$

$$\left(\because \text{From equation } (i),\, f = \frac{0.5886\, D}{V^2} \text{ and } R_e = V \times D \times 10^6\right)$$

or $$\frac{0.5886\, D}{\left(\dfrac{0.382}{D^2}\right)^2} = 0.0032 + \frac{0.221}{\left(\dfrac{0.382}{D^2} \times D \times 10^6\right)^{0.237}}$$

$$\left(\because \text{From equation}(ii),\, V = \frac{0.382}{D^2}\right)$$

or $$\frac{0.5886 \times D^5}{0.382^2} = 0.0032 + \frac{0.221}{\frac{\left(0.382 \times 10^6\right)^{0.237}}{D^{0.237}}}$$

or $$4.033\ D^5 = 0.0032 + 0.0105 \times D^{0.237}$$

or $$4.033\ D^5 - 0.0105\ D^{0.237} - 0.0032 = 0 \qquad ...(iii)$$

The above equation (*iii*) will be solved by hit and trial method.

(*i*) Assume $D = 1$ m, then L.H.S. of equation (*iii*), becomes as

$$\text{L.H.S.} = 4.033 \times 1^5 - 0.0105 \times 1^{0.237} - 0.0032$$
$$= 4.033 - 0.0105 - 0.0032 = 4.0193$$

By increasing the value of D more than 1 m, the L.H.S. will go on increasing. Hence decrease the value of D.

(*ii*) Assume $D = 0.3$ m, then L.H.S. of equation (*iii*),

becomes as $\text{L.H.S.} = 4.033 \times 0.3^5 - 0.0105 \times 0.3^{0.237} - 0.0032$

$$= 0.0098 - 0.00789 - 0.0032 = -0.00129$$

As this value is negative, the value of D will be slightly more than 0.3.

(*iii*) Assume $D = 0.306$ m, then L.H.S. of equation (*iii*), becomes as

$$\text{L.H.S.} = 4.033 \times 0.306^5 - 0.0105 \times 0.306^{0.237} - 0.0032$$
$$= 0.0108 - 0.00793 - 0.0032 = -0.00033$$

This value of L.H.S. is approximately equal to zero. Actually the value of D will be slightly more than 0.306 m say **0.308 m. Ans.**

Problem 10.13 *Water is flowing through a rough pipe of diameter 600 mm at the rate of 600 litres/second. The wall roughness is 3 mm. Find the power lost for 1 km length of pipe.*

Solution. Given :

Dia. of pipe, $D = 600 \text{ mm} = 0.6 \text{ m}$

∴ Radius of pipe, $R = \frac{0.6}{2} = 0.3 \text{ m}$

Discharge, $Q = 600 \text{ litre/s} = 0.6 \text{ m}^3/\text{s}$

Wall roughness, $k = 3 \text{ mm} = 3 \times 10^{-3} \text{ m} = 0.003 \text{ m}$

Length of pipe, $L = 1 \text{ km} = 1000 \text{ m}$

For rough pipes, the co-efficient of friction in terms of wall roughness, k is given by equation (10.36) as

$$\frac{1}{\sqrt{4f}} = 2 \log_{10} (R/k) + 1.74 = 2 \log_{10} \left(\frac{0.3}{0.003}\right) + 1.74 = 5.74$$

or $$\sqrt{4f} = \frac{1}{5.76} = 0.1742 \text{ or } 4f = (0.1742)^2 = 0.03035$$

The head loss due to friction is given by, $h_f = \dfrac{4f \times L \times V^2}{D \times 2g}$

where $V = \dfrac{Q}{A} = \dfrac{0.6}{\frac{\pi}{4}(0.6^2)} = 2.122$ m/s

$$h_f = \frac{0.03035 \times 1000 \times 2.122^2}{0.6 \times 2 \times 9.81} = 11.6 \text{ m}$$

The power* lost is given by, $P = \dfrac{\rho g \times Q \times h_f}{1000} = \dfrac{1000 \times 9.81 \times 0.6 \times 11.6}{1000}$ kW = **68.27 kW. Ans.**

HIGHLIGHTS

1. If the Reynold number is less than 2000 in a pipe, the flow is laminar while if the Reynold number is more than 4000, the flow is turbulent in pipes.
2. Loss of pressure head in a laminar flow is proportional to the mean velocity of flow, while in case of turbulent flow it is approximately proportional to the square of velocity.
3. Expression for head loss due to friction in pipes is given by Darcy-Weisbach equation,

$$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}, \text{ where } f = \text{co-efficient of friction}$$

$$= \frac{f \times L \times V^2}{d \times 2g}, \text{ where } f = \text{friction factor}$$

4. Co-efficient of friction is expressed in terms of shear stress as $= \dfrac{2\tau_0}{\rho V^2}$

   where V = mean velocity of flow, ρ = mass density of fluid.
5. Shear stress in turbulent flow is sum of shear stress due to viscosity and shear stress due to turbulence, *i.e.*,

$$\tau = \tau_v + \tau_t, \quad \text{where} \quad \tau_v = \text{shear stress due to viscosity}, \quad \tau_t = \text{shear stress due to turbulence}$$

$$= \mu \frac{d\bar{u}}{dy} + \eta \frac{d\bar{u}}{dy}$$

6. Turbulent shear stress by Reynolds is given as $\tau = \rho\, u'v'$

   where u' and v' = fluctuating component of velocity.
7. The expression for shear stress in turbulent flow due to Prandtl is $\bar{\tau} = \rho l^2 \left(\dfrac{du}{dy}\right)^2$, where l = mixing length.
8. The velocity distribution in the turbulent flow for pipes is given by the expression

$$u = u_{max} + 2.5\, u^* \log_e (y/R)$$

   where u_{max} = is the centre-line velocity,
   y = distance from the pipe wall,
   R = radius of the pipe,

   and u_* = shear velocity which is equal to $\sqrt{\dfrac{\tau_0}{\rho}}$.

---

* Power = $\rho g \times Q \times h_f$ watt $= \dfrac{\rho g \times Q \times h_f}{1000}$ kW.

9. Velocity defect is the difference between the maximum velocity (u_{max}) and local velocity (u) at any point and is given by $(u_{max} - u) = 5.75 \times u_* \log_{10} (R/y)$.

10. The boundary is known as hydrodynamically smooth if k, the average height of the irregularities projecting from the surface of the boundary is small compared to the thickness of the laminar sub-layer (δ') and boundary is rough if k is large in comparison with the thickness of the sub-layer.

or if $\frac{k}{\delta'} < 0.25$, the boundary is smooth ; if $\frac{k}{\delta'} > 6.0$, the boundary is rough

and if $\frac{k}{\delta'}$ lie between 0.25 to 6.0, the boundary is in transition.

11. Velocity distribution for turbulent flow is

$$\frac{u}{u_*} = 5.75 \log_{10} \frac{u_* y}{\nu} + 5.55 \text{ for smooth pipes}$$

$$= 5.75 \log_{10} (y/k) + 8.5 \text{ for rough pipes}$$

where u = velocity at any point in the turbulent flow,

u_* = shear velocity and = $\sqrt{\frac{\tau_0}{\rho}}$, ν = kinematic viscosity of fluid,

y = distance from pipe wall, and k = roughness factor.

12. Velocity distribution in terms of average velocity is

$$\frac{\bar{U}}{u_*} = 5.75 \log_{10} \frac{u_* R}{\nu} + 1.75 \text{ for smooth pipes,}$$

$$= 5.75 \log_{10} R/k + 4.75 \text{ for rough pipes.}$$

13. Difference of local velocity and average velocity for smooth and rough pipes is

$$\frac{u - \bar{U}}{u_*} = 5.75 \log_{10} (y/R) + 3.75.$$

14. The co-efficient of friction is given by

$$f = \frac{16}{R_e} \text{ ...... for laminar flow,}$$

$$= \frac{0.0791}{(R_e)^{1/4}} \text{ for turbulent flow in smooth pipes for } R_e \geq 4000 \text{ by } \leq 10^5$$

$$= .0008 + \frac{.05525}{(R_e)^{.257}} \text{ for } R_e \leq 10^5 \text{ but } \geq 4 \times 10^7$$

$$\frac{1}{\sqrt{4f}} = 2 \log_{10} (R/k) + 1.74 \text{ for rough pipes where } R_e = \text{Reynolds number.}$$

EXERCISE

(A) THEORETICAL PROBLEMS

1. What do you understand by turbulent flow ? What factor decides the type of flow in pipes ?

2. (*a*) Derive an expression for the loss of head due to friction in pipes.
(*b*) Derive Darcy-Weisbach equation. (*J.N.T.U., Hyderabad, S 2002*)

3. Explain the term co-efficient of friction. On what factors does this co-efficient depend ?

4. Obtain an expression for the co-efficient of friction in the terms of shear stress.
5. What do you mean by Prandtl mixing Length Theory ? Find an expression for shear stress due to Prandtl.
6. Derive an expression for Prandtl's universal velocity distribution for turbulent flow in pipes. Why this velocity distribution is called universal ?
7. What is a velocity defect ? Derive an expression for velocity defect in pipes.
8. How would you distinguish between hydrodynamically smooth and rough boundaries ?
9. Obtain an expression for the velocity distribution for turbulent flow in smooth pipes.
10. Show that velocity distribution for turbulent flow through rough pipe is given by

$$\frac{u}{u_*} = 5.75 \log_{10} (y/k) + 8.5$$

where u_* = shear velocity, y = distance from pipe wall, k = roughness factor.
11. Obtain an expression for velocity distribution in terms of average velocity for (*a*) smooth pipes and (*b*) rough pipes.
12. Prove that the difference of local velocity and average velocity for turbulent flow through rough or smooth pipes is given by

$$\frac{u - \overline{U}}{u_*} = 5.75 \log_{10} (y/R) + 3.75.$$

13. Obtain an expression for velocity distribution in turbulent flow for (*i*) smooth pipes and (*ii*) rough pipes. (*Delhi University, December, 2002*)

(B) NUMERICAL PROBLEMS

1. A pipe-line carrying water has average height of irregularities projecting from the surface of the boundary of the pipe as 0.20 mm. What type of the boundary is it ? The shear stress development is 7.848 N/m^2. Take value of kinematic viscosity for water as 0.01 stokes. [**Ans.** Boundary is in transition]
2. Determine the average height of the roughness for a rough pipe of diameter 10.0 cm when the velocity at a point 4 cm from wall is 40% more than the velocity at a point 1 cm from pipe wall. [**Ans.** 0.94 cm]
3. A smooth pipe of diameter 10 cm and 1000 m long carries water at the rate of 0.70 m^2/minute. Calculate the loss of head, wall shearing stress, centre line velocity, velocity and shear stress at 3 cm from pipe wall. Also calculate the thickness of the laminar sub-layer. Take kinematic viscosity of water as 0.015 stokes and value of co-efficient of friction 'f' as

$$f = \frac{.0791}{(R_e)^{1/4}}, \text{ where } R_e = \text{Reynolds number.}$$

[**Ans.** 20.05 m, 4.9 N/m^2 ; 1.774 m/s ; 1.65 m/s ; 19.62 N/m^2 ; 0.248 mm]
4. The velocities of water through a pipe of diameter 10 cm, are 4 m/s and 3.5 m/s at the centre of the pipe and 2 cm from the pipe centre respectively. Determine the wall shearing stress in the pipe for turbulent flow. [**Ans.** 15.66 kgf/m^2]
5. For turbulent flow in a pipe of diameter 200 mm, find the discharge when the centre-line velocity is 30 m/s and velocity at a point 80 mm from the centre as measured by pitot-tube is 2.0 m/s. [**Ans.** 64.9 litres/s]
6. For problem 5, find the co-efficient of friction and the average height of roughness projections. [**Ans.** 0.029, 25.2 mm]
7. Water is flowing through a rough pipe of diameter 40 cm and length 3000 m at the rate of 0.4 m^3/s. Find the power required to maintain this flow. Take the average height of roughness as K = 0.3 mm. [**Ans.** 278.5 kN]

8. A smooth pipe of diameter 300 mm and length 600 m carries water at rate of 0.04 m^2/s. Determine the head lost due to friction, wall shear stress, centre-line velocity and thickness of laminar sub-layer. Take the kinematic viscosity of water as 0.018 stokes. [**Ans.** 0.588 m, 0.72 N/cm^2, 0.665 m/s, 0.779 mm]

9. A rough pipe of diameter 300 mm and length 800 m carries water at the rate of 0.4 m^2/s. The wall roughness is 0.015 mm. Determine the co-efficient of friction, wall shear stress, centre line velocity and velocity at a distance of 100 mm from the pipe wall.

[**Ans.** f = .00263, τ_0 = 42.08 N/cm^2, u_{max} = 6.457 m/s, u = 6.249 m/s]

10. Determine the distance from the centre of the pipe, at which the local velocity is equal to the average velocity for turbulent flow in pipes. [**Ans.** 0.7772 R]