7 CHAPTER

ORIFICES AND MOUTHPIECES

▶ 7.1 INTRODUCTION

Orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side or at the bottom of a tank, through which a fluid is flowing. A mouthpiece is a short length of a pipe which is two to three times its diameter in length, fitted in a tank or vessel containing the fluid. Orifices as well as mouthpieces are used for measuring the rate of flow of fluid.

▶ 7.2 CLASSIFICATIONS OF ORIFICES

The orifices are classified on the basis of their size, shape, nature of discharge and shape of the upstream edge. The following are the important classifications :

1. The orifices are classified as **small orifice** or **large orifice** depending upon the size of orifice and head of liquid from the centre of the orifice. If the head of liquid from the centre of orifice is more than five times the depth of orifice, the orifice is called small orifice. And if the head of liquids is less than five times the depth of orifice, it is known as large orifice.

2. The orifices are classified as (*i*) Circular orifice, (*ii*) Triangular orifice, (*iii*) Rectangular orifice and (*iv*) Square orifice depending upon their cross-sectional areas.

3. The orifices are classified as (*i*) Sharp-edged orifice and (*ii*) Bell-mouthed orifice depending upon the shape of upstream edge of the orifices.

4. The orifices are classified as (*i*) Free discharging orifices and (*ii*) Drowned or sub-merged orifices depending upon the nature of discharge.

The sub-merged orifices are further classified as (*a*) Fully sub-merged orifices and (*b*) Partially sub-merged orifices.

▶ 7.3 FLOW THROUGH AN ORIFICE

Consider a tank fitted with a circular orifice in one of its sides as shown in Fig. 7.1. Let H be the head of the liquid above the centre of the orifice. The liquid flowing through the orifice forms a jet of liquid whose area of cross-section is less than that of orifice. The area of jet of fluid goes on decreasing and at a section C-C, the area is minimum. This section is approximately at a distance of half of diameter of the orifice. At this section, the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice. This section is called **Vena-contracta**. Beyond this section, the jet diverges and is attracted in the downward direction by the gravity.

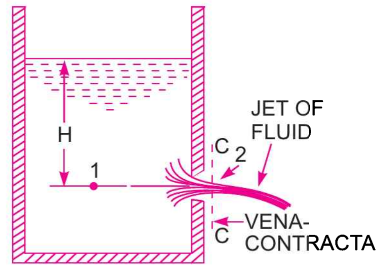

Fig. 7.1 *Tank with an orifice.*

Consider two points 1 and 2 as shown in Fig. 7.1. Point 1 is inside the tank and point 2 at the vena-contracta. Let the flow is steady and at a constant head H. Applying Bernoulli's equation at points 1 and 2.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $$z_1 = z_2$$

$\therefore$ $$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

Now $$\frac{p_1}{\rho g} = H$$

$$\frac{p_2}{\rho g} = 0 \quad \text{(atmospheric pressure)}$$

v_1 is very small in comparison to v_2 as area of tank is very large as compared to the area of the jet of liquid.

$\therefore$ $$H + 0 = 0 + \frac{v_2^2}{2g}$$

$\therefore$ $$v_2 = \sqrt{2gH} \qquad \text{...(7.1)}$$

This is theoretical velocity. Actual velocity will be less than this value.

▶ 7.4 HYDRAULIC CO-EFFICIENTS

The hydraulic co-efficients are

1. Co-efficient of velocity, C_v
2. Co-efficient of contraction, C_c
3. Co-efficient of discharge, C_d.

7.4.1 Co-efficient of Velocity (C_v). It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical velocity of jet. It is denoted by C_v and mathematically, C_v is given as

$$C_v = \frac{\text{Actual velocity of jet at vena-contracta}}{\text{Theoretical velocity}}$$

$$= \frac{V}{\sqrt{2gH}}, \text{ where } V = \text{actual velocity, } \sqrt{2gH} = \text{Theoretical velocity} \qquad \text{...(7.2)}$$

The value of C_v varies from 0.95 to 0.99 for different orifices, depending on the shape, size of the orifice and on the head under which flow takes place. Generally the value of C_v = 0.98 is taken for sharp-edged orifices.

7.4.2 Co-efficient of Contraction (C_c). It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice. It is denoted by C_c.

Let a = area of orifice and

a_c = area of jet at vena-contracta.

Then
$$C_c = \frac{\text{area of jet at vena-contracta}}{\text{area of orifice}}$$

$$= \frac{a_c}{a} \quad \text{...(7.3)}$$

The value of C_c varies from 0.61 to 0.69 depending on shape and size of the orifice and head of liquid under which flow takes place. In general, the value of C_c may be taken as 0.64.

7.4.3 Co-efficient of Discharge (C_d). It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d. If Q is actual discharge and Q_{th} is the theoretical discharge then mathematically, C_d is given as

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$

$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual area}}{\text{Theoretical area}}$$

$$\therefore \quad C_d = C_v \times C_c \quad \text{...(7.4)}$$

The value of C_d varies from 0.61 to 0.65. For general purpose the value of C_d is taken as 0.62.

Problem 7.1 *The head of water over an orifice of diameter 40 mm is 10 m. Find the actual discharge and actual velocity of the jet at vena-contracta. Take C_d = 0.6 and C_v = 0.98.*

Solution. Given :

Head, $H = 10$ cm

Dia. of orifice, $d = 40$ mm $= 0.04$ m

∴ Area, $a = \frac{\pi}{4}(.04)^2 = .001256 \text{ m}^2$

$C_d = 0.6$

$C_v = 0.98$

(*i*) $$\frac{\text{Actual discharge}}{\text{Theoretical discharge}} = 0.6$$

But Theoretical discharge = V_{th} × Area of orifice

V_{th} = Theoretical velocity, where $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10} = 14$ m/s

∴ Theoretical discharge = 14 × .001256 = 0.01758 $\frac{\text{m}^2}{\text{s}}$

∴ Actual discharge = 0.6 × Theoretical discharge

= 0.6 × .01758 = **0.01054 m³/s. Ans.**

(*ii*) $\frac{\text{Actual velocity}}{\text{Theoretical velocity}} = C_v = 0.98$

$\therefore$ Actual velocity = 0.98 × Theoretical velocity

= 0.98 × 14 = **13.72 m/s. Ans.**

Problem 7.2 *The head of water over the centre of an orifice of diameter 20 mm is 1 m. The actual discharge through the orifice is 0.85 litre/s. Find the co-efficient of discharge.*

Solution. Given :

Dia. of orifice, $d = 20 \text{ mm} = 0.02 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4}(0.02)^2 = 0.000314 \text{ m}^2$

Head, $H = 1 \text{ m}$

Actual discharge, $Q = 0.85 \text{ litre/s} = 0.00085 \text{ m}^3/\text{s}$

Theoretical velocity, $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 1} = 4.429 \text{ m/s}$

$\therefore$ Theoretical discharge, $Q_{th} = V_{th} \times \text{Area of orifice}$

$= 4.429 \times 0.000314 = 0.00139 \text{ m}^3/\text{s}$

$\therefore$ Co-efficient of discharge $= \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{0.00085}{0.00139}$ = **0.61. Ans.**

▶ 7.5 EXPERIMENTAL DETERMINATION OF HYDRAULIC CO-EFFICIENTS

7.5.1 Determination of Co-efficient of Discharge (C_d). The water is allowed to flow through an orifice fitted to a tank under a constant head, *H* as shown in Fig. 7.2. The water is collected in a measuring tank for a known time, *t*. The height of water in the measuring tank is noted down. Then actual discharge through orifice,

$$Q = \frac{\text{Area of measuring tank} \times \text{Height of water in measuring tank}}{\text{Time } (t)}$$

and theoretical discharge = area of orifice × $\sqrt{2gH}$

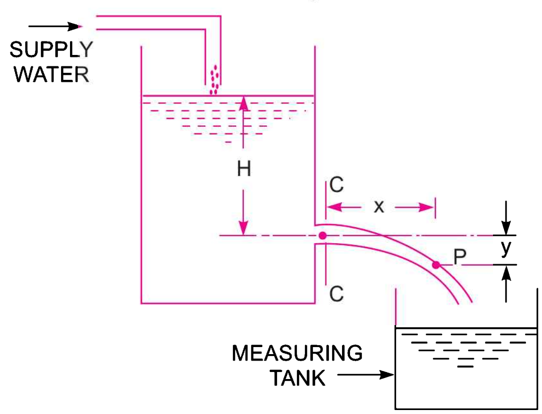

Fig. 7.2 *Value of* C_d.

$$\therefore \quad C_d = \frac{Q}{a \times \sqrt{2gH}} \quad \text{...(7.5)}$$

7.5.2 Determination of Co-efficient of Velocity (C_v). Let C-C represents the vena-contracta of a jet of water coming out from an orifice under constant head H as shown in Fig. 7.2. Consider a liquid particle which is at vena-contracta at any time and takes the position at P along the jet in time 't'.

Let x = horizontal distance travelled by the particle in time 't'

y = vertical distance between P and C-C

V = actual velocity of jet at vena-contracta.

Then horizontal distance, $x = V \times t$ ...(*i*)

and vertical distance, $y = \frac{1}{2} gt^2$ ...(*ii*)

From equation (*i*), $t = \frac{x}{V}$

Substituting this value of 't' in (*ii*), we get

$$y = \frac{1}{2} g \times \frac{x^2}{V^2}$$

$$V^2 = \frac{gx^2}{2y}$$

$$\therefore \quad V = \sqrt{\frac{gx^2}{2y}}$$

But theoretical velocity,

$$V_{th} = \sqrt{2gH}$$

$$\therefore \text{ Co-efficient of velocity, } C_v = \frac{V}{V_{th}} = \sqrt{\frac{gx^2}{2y}} \times \frac{1}{\sqrt{2gH}} = \sqrt{\frac{x^2}{4yH}}$$

$$= \frac{x}{\sqrt{4yH}}. \quad \text{...(7.6)}$$

7.5.3 Determination of Co-efficient of Contraction (C_c). The co-efficient of contraction is determined from the equation (7.4) as

$$C_d = C_v \times C_c$$

$$\therefore \quad C_c = \frac{C_d}{C_v} \quad \text{...(7.7)}$$

Problem 7.3 *A jet of water, issuing from a sharp-edged vertical orifice under a constant head of 10.0 cm, at a certain point, has the horizontal and vertical co-ordinates measured from the vena-contracta as 20.0 cm and 10.5 cm respectively. Find the value of C_v. Also find the value of C_c if C_d = 0.60.*

Solution. Given :

Head, H = 10.0 cm

Horizontal distance, x = 20.0 cm

Vertical distance, y = 10.5 cm

C_d = 0.6

The value of C_v is given by equation (7.6) as

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{20.0}{\sqrt{4 \times 10.5 \times 10.0}} = \frac{20}{20.493} = 0.9759 = \mathbf{0.976.\ Ans.}$$

The value of C_c is given by equation (7.7) as

$$C_c = \frac{C_d}{C_v} = \frac{0.6}{0.976} = 0.6147 = \mathbf{0.615.\ Ans.}$$

Problem 7.4 *The head of water over an orifice of diameter 100 mm is 10 m. The water coming out from orifice is collected in a circular tank of diameter 1.5 m. The rise of water level in this tank is 1.0 m in 25 seconds. Also the co-ordinates of a point on the jet, measured from vena-contracta are 4.3 m horizontal and 0.5 m vertical. Find the co-efficients, C_d, C_v and C_c.*

Solution. Given :

Head, $H = 10$ m

Dia. of orifice, $d = 100$ mm $= 0.1$ m

∴ Area of orifice, $a = \frac{\pi}{4}(.1)^2 = 0.007853\ m^2$

Dia. of measuring tank, $D = 1.5$ m

∴ Area, $A = \frac{\pi}{4}(1.5)^2 = 1.767\ m^2$

Rise of water, $h = 1$ m

Time, $t = 25$ seconds

Horizontal distance, $x = 4.3$ m

Vertical distance, $y = 0.5$ m

Now theoretical velocity, $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10} = 14.0$ m/s

∴ Theoretical discharge, $Q_{th} = V_{th} \times$ Area of orifice $= 14.0 \times 0.007854 = 0.1099\ m^3/s$

Actual discharge, $Q = \frac{A \times h}{t} = \frac{1.767 \times 1.0}{25} = 0.07068$

∴ $$C_d = \frac{Q}{Q_{th}} = \frac{0.07068}{0.1099} = \mathbf{0.643.\ Ans.}$$

The value of C_v is given by equation (7.6) as

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{4.3}{\sqrt{4 \times 0.5 \times 10}} = \frac{4.3}{4.472} = \mathbf{0.96.\ Ans.}$$

C_c is given by equation (7.7) as $C_c = \frac{C_d}{C_v} = \frac{0.643}{0.96} = \mathbf{0.669.\ Ans.}$

Problem 7.5 *Water discharge at the rate of 98.2 litres/s through a 120 mm diameter vertical sharp-edged orifice placed under a constant head of 10 metres. A point, on the jet, measured from the vena-contracta of the jet has co-ordinates 4.5 metres horizontal and 0.54 metres vertical. Find the co-efficient C_v, C_c and C_d of the orifice.*

Solution. Given :

Discharge, $Q = 98.2$ lit/s $= 0.0982$ m^3/s

Dia. of orifice, $d = 120$ mm $= 0.12$ m

∴ Area of orifice, $a = \frac{\pi}{4}(0.12)^2 = 0.01131$ m^2

Head, $H = 10$ m

Horizontal distance of a point on the jet from vena-contracta, $x = 4.5$ m
and vertical distance, $y = 0.54$ m

Now theoretical velocity, $V_{th} = \sqrt{2g \times H} = \sqrt{2 \times 9.81 \times 10} = 14.0$ m/s

Theoretical discharge, $Q_{th} = V_{th} \times$ Area of orifice
$= 14.0 \times 0.01131 = 0.1583$ m^3/s

The value of C_d is given by, $C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{Q}{Q_{th}} = \frac{0.0982}{0.1583} =$ **0.62. Ans.**

The value of C_c is given by equation (7.6),

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{4.5}{\sqrt{4 \times 0.54 \times 10}} = \mathbf{0.968.\ Ans.}$$

The value of C_c is given by equation (7.7) as

$$C_c = \frac{C_d}{C_v} = \frac{0.62}{0.968} = \mathbf{0.64.\ Ans.}$$

Problem 7.6 *A 25 mm diameter nozzle discharges 0.76 m^3 of water per minute when the head is 60 m. The diameter of the jet is 22.5 mm. Determine : (i) the values of co-efficients C_c, C_v and C_d and (ii) the loss of head due to fluid resistance.*

Solution. Given :

Dia. of nozzle, $D = 25$ mm $= 0.025$ m

Actual discharge, $Q_{act} = 0.76$ m^3/minute $= \frac{0.76}{60} = 0.01267$ m^3/s

Head, $H = 60$ m

Dia. of jet, $d = 22.5$ mm $= 0.0225$ m.

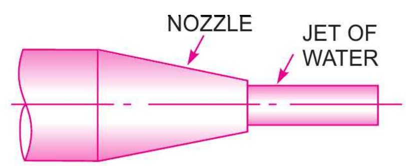

Fig. 7.3

(*i*) *Values of co-efficients :*

Co-efficient of contraction (C_c) is given by,

$$C_c = \frac{\text{Area of jet}}{\text{Area of nozzle}}$$

$$= \frac{\frac{\pi}{4} d^2}{\frac{\pi}{4} D^2} = \frac{d^2}{D^2} = \frac{0.0225^2}{0.025^2} = \mathbf{0.81.\ Ans.}$$

Co-efficient of discharge (C_d) is given by,

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}}$$

$$= \frac{0.01267}{\text{Theoretical velocity} \times \text{Area of nozzle}}$$

$$= \frac{0.01267}{\sqrt{2gH} \times \frac{\pi}{4} D^2} = \frac{0.01267}{\sqrt{2 \times 9.81 \times 60} \times \frac{\pi}{4} (0.025)^2}$$

= **0.752. Ans.**

Co-efficient of velocity (C_v) is given by,

$$C_v = \frac{C_d}{C_c} = \frac{0.752}{0.81} = \mathbf{0.928.\ Ans.}$$

(*ii*) *Loss of head due to fluid resistance* :

Applying Bernoulli's equation at the outlet of nozzle and to the jet of water, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Loss of head}$$

But $\frac{p_1}{\rho g} = \frac{p_2}{\rho g}$ = Atmospheric pressure head

$z_1 = z_2$, $V_1 = \sqrt{2gH}$, V_2 = Actual velocity of jet = $C_v \sqrt{2gH}$

$$\therefore \quad \frac{\left(\sqrt{2gH}\right)^2}{2g} = \frac{\left(C_v\sqrt{2gH}\right)^2}{2g} + \text{Loss of head}$$

or $H = C_v^2 \times H$ + Loss of head

∴ Loss of head $= H - C_v^2 \times H = H(1 - C_v^2)$

$= 60(1 - 0.928^2) = 60 \times 0.1388 =$ **8.328 m. Ans.**

Problem 7.7 *A pipe, 100 mm in diameter, has a nozzle attached to it at the discharge end, the diameter of the nozzle is 50 mm. The rate of discharge of water through the nozzle is 20 litres/s and the pressure at the base of the nozzle is 5.886 N/cm². Calculate the co-efficient of discharge. Assume that the base of the nozzle and outlet of the nozzle are at the same elevation.*

Solution. Given :

Dia. of pipe, D = 100 mm = 0.1 m

$\therefore \quad A_1 = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$

Dia. of nozzle, d = 50 mm = 0.05 m

$\therefore \quad A_2 = \frac{\pi}{4}(.05)^2 = .001963 \text{ m}^2$

Actual discharge, Q = 20 lit/s = 0.02 m³/s

Pressure at the base, $p_1 = 5.886 \text{ N/cm}^2 = 5.886 \times 10^4 \frac{\text{N}}{\text{m}^2}$

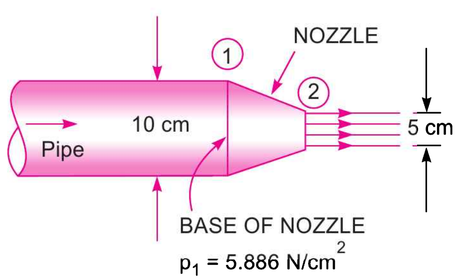

Fig. 7.4

From continuity equation, $A_1V_1 = A_2V_2$

or $.007854\ V_1 = .001963\ V_2$

$$\therefore \qquad V_1 = \frac{.001963 V_2}{.007854} = \frac{V_2}{4}$$

where V_1 and V_2 are theoretical velocity at sections (1) and (2).

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $\qquad z_1 = z_2$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

or $$\frac{5.886 \times 10^4}{1000 \times 9.81} + \frac{\left(\frac{V_2}{4}\right)^2}{2g} = 0 + \frac{V_2^2}{2g} \qquad \left\{\because \frac{p_2}{\rho g} = \text{Atmospheric pressure} = 0\right\}$$

$$6.0 + \frac{V_2^2}{2g \times 16} = \frac{V_2^2}{2g}$$

or $$\frac{V_2^2}{2g}\left[1 - \frac{1}{16}\right] = 6.0 \quad \text{or} \quad \frac{V_2^2}{2g}\left[\frac{15}{16}\right] = 6.0$$

$$\therefore \qquad V_2 = \sqrt{6.0 \times 2 \times 9.81 \times \frac{16}{15}} = 11.205 \text{ m/sec}$$

$\therefore$ Theoretical discharge $= V_2 \times A_2 = 11.205 \times .001963 = 0.022 \text{ m}^3\text{/s}$

$$\therefore \qquad C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{0.02}{0.022} = \mathbf{0.909.\ Ans.}$$

Problem 7.8 *A tank has two identical orifices on one of its vertical sides. The upper orifice is 3 m below the water surface and lower one is 5 m below the water surface. If the value of C_v for each orifice is 0.96, find the point of intersection of the two jets.*

Solution. Given :

Height of water from orifice (1), $H_1 = 3$ m

From orifice (2), $H_2 = 5$ m

C_v for both $= 0.96$

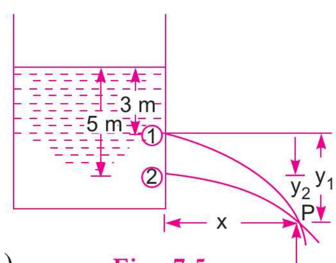

Fig. 7.5

Let P is the point of intersection of the two jets coming from orifices (1) and (2), such that

x = horizontal distance of P

y_1 = vertical distance of P from orifice (1)

y_2 = vertical distance of P from orifice (2)

Then $\qquad y_1 = y_2 + (5 - 3) = y_2 + 2$ m

The value of C_v is given by equation (7.6) as

For orifice (1), $C_{v_1} = \dfrac{x}{\sqrt{4y_1 H_1}} = \dfrac{x}{\sqrt{4y_1 \times 3.0}}$ ...(*i*)

For orifice (2), $C_{v_2} = \dfrac{x}{\sqrt{4y_2 H_2}} = \dfrac{x}{\sqrt{4 \times y_2 \times 5.0}}$ ...(*ii*)

As both the orifices are identical

$\therefore$ $C_{v_1} = C_{v_2}$

or $\dfrac{x}{\sqrt{4y_1 \times 3.0}} = \dfrac{x}{\sqrt{4y_2 \times 5.0}}$ or $3y_1 = 5y_2$

But $y_1 = y_2 + 2.0$

$\therefore$ $3(y_2 + 2.0) = 5y_2$

$\therefore$ $2y_2 = 6.0$

$\therefore$ $y_2 = 3.0$

From (*ii*), $C_{v_2} = \dfrac{x}{\sqrt{4y_2 \times H_2}}$

or $0.96 = \dfrac{x}{\sqrt{4 \times 3.0 \times 5.0}}$

$\therefore$ $x = 0.96 \times \sqrt{4 \times 3.0 \times 5.0}$ = **7.436 m. Ans.**

Problem 7.9 *A closed vessel contains water upto a height of 1.5 m and over the water surface there is air having pressure 7.848 N/cm² (0.8 kgf/cm²) above atmospheric pressure. At the bottom of the vessel there is an orifice of diameter 100 mm. Find the rate of flow of water from orifice. Take C_d = 0.6.*

Solution. Given :

Dia. of orifice, d = 100 mm = 0.1 m

$C_d = 0.6$

Height of water, H = 1.5 m

Air pressure, $p = 7.848 \text{ N/cm}^2 = 7.848 \times 10^4 \text{ N/m}^2$

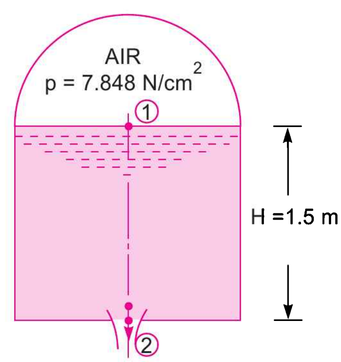

Fig. 7.6

Applying Bernoulli's equation at sections (1) (water surface) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Taking datum line passing through (2) which is very close to the bottom surface of the tank. Then $z_2 = 0$, $z_1 = 1.5$ m

Also $\dfrac{p_2}{\rho g} = 0$ (atmospheric pressure)

and $\dfrac{p_1}{\rho g} = \dfrac{7.848 \times 10^4}{1000 \times 9.81} = 8$ m of water

$\therefore$ $8 + 0 + 1.5 = 0 + \dfrac{V_2^2}{2g} + 0$ {V_1 is negligible}

$\therefore$ $9.5 = \dfrac{V_2^2}{2g}$

$\therefore$ $V_2 = \sqrt{2 \times 9.81 \times 9.5} = 13.652$ m/s

$\therefore$ Rate of flow of water $= C_d \times a_2 \times V_2$

$$= 0.6 \times \frac{\pi}{4} (.1)^2 \times 13.652 \text{ m}^3\text{/s} = \mathbf{0.0643\ m^3/s.}\ \textbf{Ans.}$$

Problem 7.10 *A closed tank partially filled with water upto a height of 0.9 m having an orifice of diameter 15 mm at the bottom of the tank. The air is pumped into the upper part of the tank. Determine the pressure required for a discharge of 1.5 litres/s through the orifice. Take $C_d = 0.62$.*

Solution. Given :

Height of water above orifice, $H = 0.9$ m

Dia. of orifice, $d = 15$ mm $= 0.015$ m

$\therefore$ Area, $a = \frac{\pi}{4}\left[d^2\right] = \frac{\pi}{4} (.015)^2 = 0.0001767 \text{ m}^2$

Discharge, $Q = 1.5$ litres/s $= .0015 \text{ m}^3/\text{s}$

$C_d = 0.62$

Let p is intensity of pressure required above water surface in N/cm^2.

Then pressure head of air$= \frac{p}{\rho g} = \frac{p \times 10^4}{1000 \times 9.81} = \frac{10p}{9.81}$ m of water.

If V_2 is the velocity at outlet of orifice, then

$$V_2 = \sqrt{2g\left(H + \frac{p}{\rho g}\right)} = \sqrt{2 \times 9.81\left(0.9 + \frac{10p}{9.81}\right)}$$

$\therefore$ Discharge $Q = C_d \times a \times \sqrt{2g\,(H + p/\rho g)}$

$$.0015 = 0.6 \times .0001767 \times \sqrt{2 \times 9.81\,(0.9 + p/\rho g)}$$

$$\therefore \quad \sqrt{2 \times 9.81\left(0.9 + \frac{10p}{9.81}\right)} = \frac{.0015}{0.6 \times .0001767} = 14.148$$

or $$2 \times 9.81\left(0.9 + \frac{10p}{9.81}\right) = 14.148 \times 14.148$$

$$\therefore \quad \frac{10p}{9.81} = \frac{14.148 \times 14.148}{2 \times 9.81} - 0.9 = 10.202 - 0.9 = 9.302$$

$$\therefore \quad p = \frac{9.302 \times 9.81}{10} = \mathbf{9.125\ N/cm^2.}\ \textbf{Ans.}$$

▶ 7.6 FLOW THROUGH LARGE ORIFICES

If the head of liquid is less than 5 times the depth of the orifice, the orifice is called large orifice. In case of small orifice, the velocity in the entire cross-section of the jet is considered to be constant and discharge can be calculated by $Q = C_d \times a \times \sqrt{2gh}$. But in case of a large orifice, the velocity is not constant over the entire cross-section of the jet and hence Q cannot be calculated by $Q = C_d \times a \times \sqrt{2gh}$.

7.6.1 Discharge Through Large Rectangular Orifice. Consider a large rectangular orifice in one side of the tank discharging freely into atmosphere under a constant head, H as shown in Fig. 7.7.

Let H_1 = height of liquid above top edge of orifice

H_2 = height of liquid above bottom edge of orifice

b = breadth of orifice

d = depth of orifice = $H_2 - H_1$

C_d = co-efficient of discharge.

Consider an elementary horizontal strip of depth 'dh' at a depth of 'h' below the free surface of the liquid in the tank as shown in Fig. 7.7 (b).

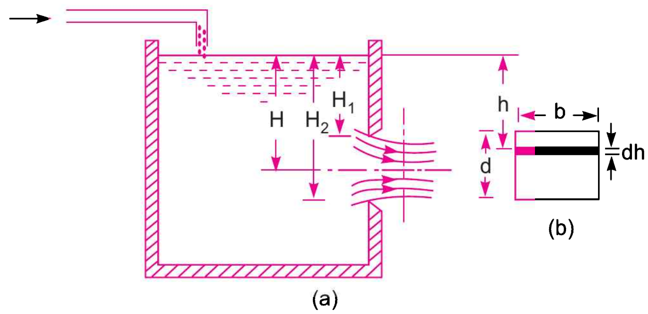

Fig. 7.7 *Large rectangular orifice.*

$\therefore$ Area of strip = $b \times dh$

and theoretical velocity of water through strip = $\sqrt{2gh}$.

$\therefore$ Discharge through elementary strip is given

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity}$$

$$= C_d \times b \times dh \times \sqrt{2gh} = C_d\, b \times \sqrt{2gh}\ dh$$

By integrating the above equation between the limits H_1 and H_2, the total discharge through the whole orifice is obtained

$$\therefore \quad Q = \int_{H_1}^{H_2} C_d \times b \times \sqrt{2gh}\ dh$$

$$= C_d \times b \times \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h}\ dh = C_d \times b \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2}\right]_{H_1}^{H_2}$$

$$= \frac{2}{3} C_d \times b\ \sqrt{2g}\left[H_2^{3/2} - H_1^{3/2}\right]. \quad \text{...(7.8)}$$

Problem 7.11 *Find the discharge through a rectangular orifice 2.0 m wide and 1.5 m deep fitted to a water tank. The water level in the tank is 3.0 m above the top edge of the orifice. Take C_d = 0.62.*

Solution. Given :

Width of orifice, b = 2.0 m

Depth of orifice, d = 1.5 m

Height of water above top edge of the orifice, H_1 = 3 m

Height of water above bottom edge of the orifice,

$$H_2 = H_1 + d = 3 + 1.5 = 4.5 \text{ m}$$

$$C_d = 0.62$$

Discharge Q is given by equation (7.8) as

$$Q = \frac{2}{3} C_d \times b \times \sqrt{2g} \; [H_2^{3/2} - H_1^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 2.0 \times \sqrt{2 + 9.81} [4.5^{1.5} - 3^{1.5}] \text{ m}^3\text{/s}$$

$$= 3.66[9.545 - 5.196] \text{ m}^3\text{/s} = \mathbf{15.917 \text{ m}^3\text{/s. Ans.}}$$

Problem 7.12 *A rectangular orifice, 1.5 m wide and 1.0 m deep is discharging water from a tank. If the water level in the tank is 3.0 m above the top edge of the orifice, find the discharge through the orifice. Take the co-efficient of discharging for the orifice = 0.6.*

Solution. Given :

Width of orifice, $b = 1.5$ m

Depth of orifice, $d = 1.0$ m

$H_1 = 3.0$ m

$H_2 = H_1 + d = 3.0 + 1.0 = 4.0$ m

$C_d = 0.6$

Discharge, Q is given by the equation (7.8) as

$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g} \; [H_2^{3/2} - H_1^{3/2}]$$

$$= \frac{2}{3} \times 0.6 \times 1.5 \times \sqrt{2 + 9.81} [4.0^{1.5} - 3.0^{1.5}] \text{ m}^3\text{/s}$$

$$= 2.657 [8.0 - 5.196] \text{ m}^3\text{/s} = \mathbf{7.45 \text{ m}^3\text{/s. Ans.}}$$

Problem 7.13 *A rectangular orifice 0.9 m wide and 1.2 m deep is discharging water from a vessel. The top edge of the orifice is 0.6 m below the water surface in the vessel. Calculate the discharge through the orifice if C_d = 0.6 and percentage error if the orifice is treated as a small orifice.*

Solution. Given :

Width of orifice, $b = 0.9$ m

Depth of orifice, $d = 1.2$ m

$H_2 = 0.6$ m

$H_2 = H_1 + d = 0.6 + 1.2 = 1.8$ m

$C_d = 0.6$

Discharge Q is given as $Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g} \times [H_2^{3/2} - H_1^{3/2}]$

$$= \frac{2}{3} \times 0.6 \times 2.9 \times \sqrt{2 \times 9.81} \; [1.8^{3/2} - 0.6^{3/2}] \text{ m}^3\text{/s}$$

$$= 1.5946 [2.4149 - .4647] = \mathbf{3.1097 \text{ m}^3\text{/s. Ans.}}$$

Discharging for a small orifice

$$Q_1 = C_d \times a \times \sqrt{2gh}$$

where $h = H_1 + \frac{d}{2} = 0.6 + \frac{1.2}{2} = 1.2$ m and $a = b \times d = 0.9 \times 1.2$

$$Q_1 = 0.6 \times .9 \times 1.2 \times \sqrt{2 \times 9.81 \times 1.2} = 3.1442 \text{ m}^3/\text{s}$$

$$\% \text{ error} = \frac{Q_1 - Q}{Q} = \frac{3.1442 - 3.1097}{3.1097} = \mathbf{0.01109} \text{ or } \mathbf{1.109\%. \ Ans.}$$

▶ 7.7 DISCHARGE THROUGH FULLY SUB-MERGED ORIFICE

Fully sub-merged orifice is one which has its whole of the outlet side sub-merged under liquid so that it discharges a jet of liquid into the liquid of the same kind. It is also called totally drowned orifice. Fig. 7.8 shows the fully sub-merged orifice. Consider two points (1) and (2), point 1 being in the reservoir on the upstream side of the orifice and point 2 being at the vena-contracta as shown in Fig. 7.8.

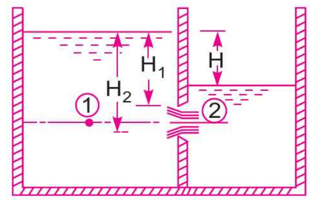

Fig. 7.8 *Fully sub-merged orifice.*

Let H_1 = Height of water above the top of the orifice on the upstream side,
H_2 = Height of water above the bottom of the orifice,
H = Difference in water level,
b = Width of orifice,
C_d = Co-efficient of discharge.

Height of water above the centre of orifice on upstream side

$$= H_1 + \frac{H_2 - H_1}{2} = \frac{H_1 + H_2}{2} \quad \text{...(1)}$$

Height of water above the centre of orifice on downstream side

$$= \frac{H_1 + H_2}{2} - H \quad \text{...(2)}$$

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \quad [\because z_1 = z_2]$$

Now $\frac{p_1}{\rho g} = \frac{H_1 + H_2}{2}$, $\frac{p_2}{\rho g} = \frac{H_1 + H_2}{2} - H$ and V_1 is negligible

$$\therefore \quad \frac{H_1 + H_2}{2} + 0 = \frac{H_1 + H_2}{2} - H + \frac{V_2^2}{2g}$$

$$\therefore \quad \frac{V_2^2}{2g} = H$$

$$\therefore \quad V_2 = \sqrt{2gH}$$

Area of orifice $= b \times (H_2 - H_1)$

$\therefore$ Discharge through orifice $= C_d \times \text{Area} \times \text{Velocity}$

$$= C_d \times b\,(H_2 - H_1) \times \sqrt{2gH}$$

$$\therefore \quad Q = C_d \times b\,(H_2 - H_1) \times \sqrt{2gH}. \quad \text{...(7.9)}$$

Problem 7.14 *Find the discharge through a fully sub-merged orifice of width 2 m if the difference of water levels on both sides of the orifice be 50 cm. The height of water from top and bottom of the orifice are 2.5 m and 2.75 m respectively. Take C_d = 0.6.*

Solution. Given :

Width of orifice, $b = 2$ m

Difference of water level, $H = 50$ cm $= 0.5$ m

Height of water from top of orifice, $H_1 = 2.5$ m

Height of water from bottom of orifice, $H_2 = 2.5$ m

$C_d = 0.6$

Discharge through fully sub-merged orifice is given by equation (7.9)

or
$$Q = C_d \times b \times (H_2 - H_1) \times \sqrt{2gH}$$
$$= 0.6 \times 2.0 \times (2.75 - 2.5) \times \sqrt{2 \times 9.81 \times 0.5} \text{ m}^3/\text{s}$$
$$= \mathbf{0.9396\ m^3/s.\ Ans.}$$

Problem 7.15 *Find the discharge through a totally drowned orifice 2.0 m wide and 1 m deep, if the difference of water levels on both the sides of the orifice be 3 m. Take C_d = 0.62.*

Solution. Given :

Width of orifice, $b = 2.0$ m

Depth of orifice, $d = 1$ m.

Difference of water level on both the sides

$H = 3$ m

$C_d = 0.62$

Discharge through orifice is $Q = C_d \times \text{Area} \times \sqrt{2gH}$
$$= 0.62 \times b \times d \times \sqrt{2gH}$$
$$= 0.62 \times 2.0 \times 1.0 \times \sqrt{2 \times 9.81 \times 3} \text{ m}^3/\text{s} = \mathbf{9.513\ m^3/s.\ Ans.}$$

▶ 7.8 DISCHARGE THROUGH PARTIALLY SUB-MERGED ORIFICE

Partially sub-merged orifice is one which has its outlet side partially sub-merged under liquid as shown in Fig. 7.9. It is also known as partially drowned orifice. Thus the partially sub-merged orifice has two portions. The upper portion behaves as an orifice discharging free while the lower portion behaves as a sub-merged orifice. Only a large orifice can behave as a partially sub-merged orifice. The total discharge Q through partially sub-merged orifice is equal to the discharges through free and the sub-merged portions.

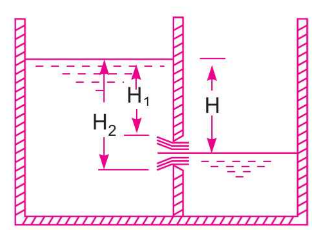

Fig. 7.9 *Partially sub-merged orifice.*

Discharge through the sub-merged portion is given by equation (7.9)
$$Q_1 = C_d \times b \times (H_2 - H) \times \sqrt{2gH}$$

Discharge through the free portion is given by equation (7.8) as

$$Q_2 = \frac{2}{3} C_d \times b \times \sqrt{2g}\ [H_2^{3/2} - H_1^{3/2}]$$

∴ Total discharge $Q = Q_1 + Q_2$

$$= C_d \times b \times (H_2 - H) \times \sqrt{2gH}$$

$$+ \frac{2}{3} C_d \times b \times \sqrt{2g}\ [H_2^{3/2} - H^{3/2}]. \quad ...(7.10)$$

Problem 7.16 *A rectangular orifice of 2 m width and 1.2 m deep is fitted in one side of a large tank. The water level on one side of the orifice is 3 m above the top edge of the orifice, while on the other side of the orifice, the water level is 0.5 m below its top edge. Calculate the discharge through the orifice if $C_d = 0.64$.*

Solution. Given : Width of orifice, $b = 2$ m

Depth of orifice, $d = 1.2$ m

Height of water from top edge of orifice, $H_1 = 3$ m

Difference of water level on both sides, $H = 3 + 0.5 = 3.5$ m

Height of water from the bottom edge of orifice, $H_2 = H_1 + d = 3 + 1.2 = 4.2$ m

The orifice is partially sub-merged. The discharge through sub-merged portion,

$$Q_1 = C_d \times b \times (H_2 - H) \times \sqrt{2gH}$$

$$= 0.64 \times 2.0 \times (4.2 - 3.5) \times \sqrt{2 \times 9.81 \times 3.5} = 7.4249 \text{ m}^3/\text{s}$$

The discharge through free portion is

$$Q_2 = \frac{2}{3} C_d \times b \times \sqrt{2g}\ [H^{3/2} - H_1^{3/2}]$$

$$= \frac{2}{3} \times 0.64 \times 2.0 \times \sqrt{2 \times 9.81}\ [3.5^{3/2} - 3.0^{3/2}]$$

$$= 3.779\ [6.5479 - 5.1961] = 5.108 \text{ m}^3/\text{s}$$

∴ Total discharge through the orifice is

$$Q = Q_1 + Q_2 = 7.4249 + 5.108 = \mathbf{12.5329\ m^3/s.\ Ans.}$$

▶ 7.9 TIME OF EMPTYING A TANK THROUGH AN ORIFICE AT ITS BOTTOM

Consider a tank containing some liquid upto a height of H_1. Let an orifice is fitted at the bottom of the tank. It is required to find the time for the liquid surface to fall from the height H_1 to a height H_2.

Let A = Area of the tank

a = Area of the orifice

H_1 = Initial height of the liquid

H_2 = Final height of the liquid

T = Time in seconds for the liquid to fall from H_1 to H_2.

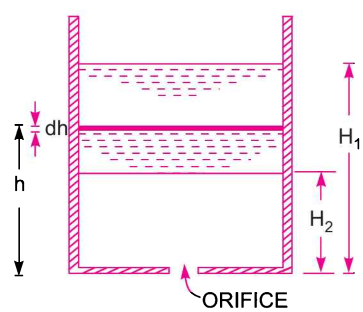

Fig. 7.9. (*a*)

Let at any time, the height of liquid from orifice is h and let the liquid surface fall by a small height dh in time dT. Then

Volume of liquid leaving the tank in time, $dT = A \times dh$

Also the theoretical velocity through orifice, $V = \sqrt{2gh}$

$\therefore$ Discharge through orifice/sec,

$$dQ = C_d \times \text{Area of orifice} \times \text{Theoretical velocity} = C_d \,.\, a \,.\, \sqrt{2gh}$$

$\therefore$ Discharge through orifice in time interval

$$dT = C_d \,.\, a \,.\, \sqrt{2gh} \,.\, dT$$

As the volume of liquid leaving the tank is equal to the volume of liquid flowing through orifice in time dT, we have

$$A(-\,dh) = C_d \,.\, a \,.\, \sqrt{2gh} \,.\, dT$$

– ve sign is inserted because with the increase of time, head on orifice decreases.

$$\therefore \qquad -\,Adh = C_d \,.\, a \,.\, \sqrt{2gh} \,.\, dT \text{ or } dT = \frac{-A\,dh}{C_d \,.\, a \,.\, \sqrt{2gh}} = \frac{-A(h)^{-1/2}}{C_d \,.\, a \,.\, \sqrt{2g}}dh$$

By integrating the above equation between the limits H_1 and H_2, the total time, T is obtained as

$$\int_0^T dT = \int_{H_1}^{H_2} \frac{-Ah^{-1/2}\,dh}{C_d \,.\, a \,.\, \sqrt{2g}} = \frac{-A}{C_d \,.\, a \,.\, \sqrt{2g}} \int_{H_1}^{H_2} h^{-1/2}\,dh$$

or

$$T = \frac{-A}{C_d \,.\, a \,.\, \sqrt{2g}} \left[\frac{h^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_{H_1}^{H_2} = \frac{-A}{C_d \,.\, a \,.\, \sqrt{2g}} \left[\frac{\sqrt{h}}{\frac{1}{2}} \right]_{H_1}^{H_2}$$

$$= \frac{-2A}{C_d \,.\, a \,.\, \sqrt{2g}} \left[\sqrt{H_2} - \sqrt{H_1} \right] = \frac{2A\left[\sqrt{H_1} - \sqrt{H_2}\right]}{C_d \,.\, a \,.\, \sqrt{2g}} \qquad \text{...(7.11)}$$

For emptying the tank completely, $H_2 = 0$ and hence

$$T = \frac{2A\sqrt{H_1}}{C_d \,.\, a \,.\, \sqrt{2g}}. \qquad \text{...(7.12)}$$

Problem 7.17 *A circular tank of diameter 4 m contains water upto a height of 5 m. The tank is provided with an orifice of diameter 0.5 m at the bottom. Find the time taken by water (i) to fall from 5 m to 2 m (ii) for completely emptying the tank. Take C_d = 0.6.*

Solution. Given :

Dia. of tank, $D = 4$ m

$\therefore$ Area, $A = \frac{\pi}{4}(4)^2 = 12.566 \text{ m}^2$

Dia. of orifice, $d = 0.5$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.5)^2 = 0.1963 \text{ m}^2$

Initial height of water, $H_1 = 5$ m

Final height of water, (*i*) $H_2 = 2$ m (*ii*) $H_2 = 0$

First Case. When $H_2 = 2$ m

Using equation (7.11), we have $T = \frac{2A}{C_d \,.\, a \,.\, \sqrt{2g}}\left[\sqrt{H_1} - \sqrt{H_2}\right]$

$$= \frac{2 \times 12.566}{0.6 \times .1963 \times \sqrt{2 \times 9.81}} \left[\sqrt{5} - \sqrt{2.0}\right] \text{ seconds}$$

$$= \frac{20.653}{0.5217} = \mathbf{39.58 \text{ seconds. Ans.}}$$

Second Case. When $H_2 = 0$

$$T = \frac{2A}{C_d \, . \, a \, . \sqrt{2g}} \sqrt{H_1} = \frac{2 \times 12.566 \times \sqrt{5}}{0.6 \times .1963 \times \sqrt{2 \times 9.81}}$$

$$= \mathbf{107.7 \text{ seconds. Ans.}}$$

Problem 7.18 *A circular tank of diameter 1.25 m contains water upto a height of 5 m. An orifice of 50 mm diameter is provided at its bottom. If $C_d = 0.62$, find the height of water above the orifice after 1.5 minutes.*

Solution. Given :

Dia. of tank, $D = 1.25$ m

∴ Area, $A = \frac{\pi}{4}(1.25)^2 = 1.227 \text{ m}^2$

Dia. of orifice, $d = 50 \text{ mm} = .05 \text{ m}$

∴ Area, $a = \frac{\pi}{4} (.05)^2 = .001963 \text{ m}^2$

$C_d = 0.62$

Initial height of water, $H_1 = 5$ m

Time in seconds, $T = 1.5 \times 60 = 90$ seconds

Let the height of water after 90 seconds = H_2

Using equation (7.11), we have $T = \frac{2A\left[\sqrt{H_1} - \sqrt{H_2}\right]}{C_d \, . \, a \, . \sqrt{2g}}$

or
$$90 = \frac{2 \times 1.227\left[\sqrt{5} - \sqrt{H_2}\right]}{0.62 \times 0.001963 \times \sqrt{2 \times 9.81}} = 455.215 \left[2.236 - \sqrt{H_2}\right]$$

∴
$$\sqrt{H_2} = 2.236 - \frac{90}{455.215} = 236 - 0.1977 = 2.0383$$

∴
$$H_2 = 2.0383 \times 2.0383 = \mathbf{4.154 \text{ m. Ans.}}$$

▶ 7.10 TIME OF EMPTYING A HEMISPHERICAL TANK

Consider a hemispherical tank of radius R fitted with an orifice of area 'a' at its bottom as shown in Fig. 7.10. The tank contains some liquid whose initial height is H_1 and in time T, the height of liquid falls to H_2. It is required to find the time T.

Let at any instant of time, the head of liquid over the orifice is h and at this instant let x be the radius of the liquid surface. Then

Area of liquid surface, $A = \pi x^2$

and theoretical velocity of liquid $= \sqrt{2gh}$.

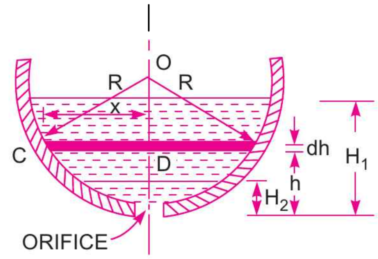

Fig. 7.10 *Hemispherical tank.*

Let the liquid level falls down by an amount of dh in time dT.

∴ Volume of liquid leaving tank in time $dT = A \times dh$

$$= \pi x^2 \times dh \quad \text{...(i)}$$

Also volume of liquid flowing through orifice

$$= C_d \times \text{area of orifice} \times \text{velocity} = C_d . a . \sqrt{2gh} \text{ second}$$

∴ Volume of liquid flowing through orifice in time dT

$$= C_d . a . \sqrt{2gh} \times dT \quad \text{...(ii)}$$

From equations (*i*) and (*ii*), we get

$$\pi x^2 (-dh) = C_d . a . \sqrt{2gh} \; . \; dT$$

–ve sign is introduced, because with the increase of T, h will decrease

$$\therefore \quad -\pi x^2 \, dh = C_d . a . \sqrt{2gh} \; . \; dT \quad \text{...(iii)}$$

But from Fig. 7.10, for ΔOCD, we have $OC = R$

$$DO = R - h$$

$$\therefore \quad CD = x = \sqrt{OC^2 - OD^2} = \sqrt{R^2 - (R-h)^2}$$

$$\therefore \quad x^2 = R^2 - (R-h)^2 = R^2 - (R^2 + h^2 - 2Rh) = 2Rh - h^2$$

Substituting x^2 in equation (*iii*), we get

$$-\pi(2Rh - h^2)dh = C_d . a . \sqrt{2gh} \; . \; dT$$

or
$$dT = \frac{-\pi\left(2Rh - h^2\right)dh}{C_d \, . \, a \, . \sqrt{2gh}} = \frac{-\pi}{C_d \, . \, a \, . \sqrt{2g}}\left(2Rh - h^2\right)h^{-1/2} \; dh$$

$$= \frac{-\pi}{C_d \, . \, a \, . \sqrt{2g}}(2Rh^{1/2} - h^{3/2})dh$$

The total time T required to bring the liquid level from H_1 to H_2 is obtained by integrating the above equation between the limits H_1 and H_2.

$$\therefore \quad T = \int_{H_1}^{H_2} \frac{-\pi}{C_d \, . \, a \, . \sqrt{2g}}(2Rh^{1/2} - h^{3/2})dh$$

$$= \frac{-\pi}{C_d \, . \, a \, . \sqrt{2g}} \int_{H_1}^{H_2} (2Rh^{1/2} - h^{3/2})dh$$

$$= \frac{-\pi}{C_d \times a \times \sqrt{2g}} \left[2R \frac{h^{1/2+1}}{\frac{1}{2}+1} - \frac{h^{3/2}+1}{\frac{3}{2}+1} \right]_{H_1}^{H_2}$$

$$= \frac{-\pi}{C_d \times a \times \sqrt{2g}} \left[2 \times \frac{2}{3} R h^{3/2} - \frac{2}{5} h^{5/2} \right]_{H_1}^{H_2}$$

$$= \frac{-\pi}{C_d \times a \times \sqrt{2g}} \left[\frac{4}{3} R \left(H_2^{3/2} - H_1^{3/2} \right) - \frac{2}{5} \left(H_2^{5/2} - H_1^{5/2} \right) \right]$$

$$= \frac{\pi}{C_d \times a \times \sqrt{2g}} \left[\frac{4}{3} R \left(H_1^{3/2} - H_2^{3/2} \right) - \frac{2}{5} \left(H_1^{5/2} - H_2^{5/2} \right) \right] \quad ...(7.13)$$

For completely emptying the tank, $H_2 = 0$ and hence

$$T = \frac{\pi}{C_d \, . \, a \, . \sqrt{2g}} \left[\frac{4}{3} R H_1^{3/2} - \frac{2}{5} H_1^{5/2} \right]. \quad ...(7.14)$$

Problem 7.19 *A hemispherical tank of diameter 4 m contains water upto a height of 1.5 m. An orifice of diameter 50 mm is provided at the bottom. Find the time required by water (i) to fall from 1.5 m to 1.0 m (ii) for completely emptying the tank. Tank $C_d = 0.6$.*

Solution. Given :

Dia. of hemispherical tank, $D = 4$ m

$\therefore$ Radius, $R = 2.0$ m

Dia. of orifice, $d = 50$ mm $= 0.05$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.05)^2 = 0.001963$ m^2

Initial height of water, $H_1 = 1.5$ m

$C_d = 0.6$

First Case. $H_2 = 1.0$

Time T is given by equation (7.13)

$$\therefore \quad T = \frac{\pi}{C_d \times a \times \sqrt{2g}} \left[\frac{4}{3} R \left(H_1^{3/2} - H_2^{3/2} \right) - \frac{2}{5} \left(H_1^{5/2} - H_2^{5/2} \right) \right]$$

$$= \frac{\pi}{0.6 \times .001963 \times \sqrt{2 \times 9.81}} \times \left[\frac{4}{3} \times 2.0 \left(1.5^{3/2} - 1.0^{3/2} \right) - \frac{2}{5} \left(1.5^{5/2} - 1.0^{5/2} \right) \right]$$

= 602.189 [2.2323 – 0.7022] = 921.4 second

= 15 min 21.4 sec. Ans.

Second Case. $H_2 = 0$ and hence time T is given by equation (7.14)

$$\therefore \quad T = \frac{\pi}{C_d \, . \, a \, . \sqrt{2g}} \left[\frac{4}{3} R H_1^{3/2} - \frac{2}{5} H_1^{5/2} \right]$$

$$= \frac{\pi}{0.6 \times .001963 \sqrt{2 \times 9.81}} \left[\frac{4}{3} \times 2.0 \times 1.5^{3/2} - \frac{2}{5} \times 1.5^{5/2} \right]$$

$= 602.189\ [4.8989 - 1.1022]$ sec $= 2286.33$ sec

$=$ **38 min 6.33 sec. Ans.**

Problem 7.20 *A hemispherical cistern of 6 m radius is full of water. It is fitted with a 75 mm diameter sharp edged orifice at the bottom. Calculate the time required to lower the level in the cistern by 2 metres. Assume co-efficient of discharge for the orifice is 0.6.*

Solution. Given :

Radius of hemispherical cistern, $R = 6$ m

Initial height of water, $H_1 = 6$ m

Dia. of orifice, $d = 75$ mm $= 0.075$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.075)^2 = .004418\ \text{m}^2$

Fall of height of water $= 2$ m

$\therefore$ Final height of water, $H_2 = 6 - 2 = 4$ m

$C_d = 0.6$

The time T is given by equation (7.31)

$$T = \frac{\pi}{C_d \times a \times \sqrt{2g}}\left[\frac{4}{3}R\left(H_1^{3/2} - H_2^{3/2}\right) - \frac{2}{5}\left(H_1^{5/2} - H_2^{5/2}\right)\right]$$

$$= \frac{\pi}{0.6 \times .004418 \times \sqrt{2 \times 9.81}} \times \left[\frac{4}{3} \times 6\left(6.0^{3/2} - 4.0^{3/2}\right) - \frac{2}{5}\left(6.0^{5/2} - 4.0^{5/2}\right)\right]$$

$= 267.56\ [8(14.6969 - 8.0) - 0.4\ (88.18 - 32.0)]$

$= 267.56\ [53.575 - 22.472]$ sec

$= 8321.9$ sec $=$ **2hrs 18 min 42 sec. Ans.**

Problem 7.21 *A cylindrical tank is having a hemispherical base. The height of cylindrical portion is 5 m and diameter is 4 m. At the bottom of this tank an orifice of diameter 200 mm is fitted. Find the time required to completely emptying the tank. Take $C_d = 0.6$.*

Solution. Given :

Height of cylindrical portion (II) $= 5$ m

Dia. of tank $= 4.0$ m

$\therefore$ Area, $A = \frac{\pi}{4}(4)^2 = 12.566\ \text{m}^2$

Dia. of orifice, $d = 200$ mm $= 0.2$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.2)^2 = 0.0314\ \text{m}^2$

$C_d = 0.6$

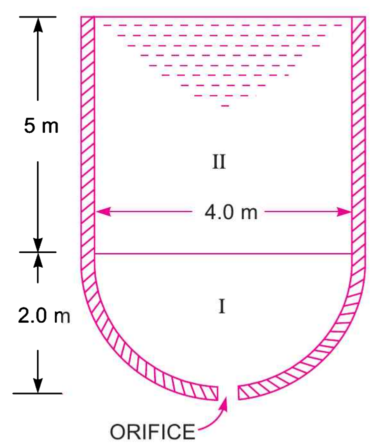

Fig. 7.11

The tank is splitted in two portions. First portion is a hemispherical tank and second portion is cylindrical tank.

Let T_1 = time for emptying hemispherical portion I.

T_2 = time for emptying cylindrical portion II.

Then total time $T = T_1 + T_2$.

For Portion I. $H_1 = 2.0$ m, $H_2 = 0$. Then T_1 is given by equation (7.14) as

$$T_1 = \frac{\pi}{C_d \times a \times \sqrt{2g}}\left[\frac{4}{3}RH_1^{3/2} - \frac{2}{5}H_1^{5/2}\right]$$

$$= \frac{\pi}{0.6 \times .0314 \times \sqrt{2 \times 9.81}}\left[\frac{4}{3} \times 2.0 \times 2.0^{3/2} - \frac{2}{5} \times 2.0^{5/2}\right]$$

$$= 37.646\ [7.5424 - 2.262] \text{ sec} = 198.78 \text{ sec.}$$

For Portion II. $H_1 = 2.0 + 5.0 = 7.0$ m, $H_2 = 2.0$. Then T_2 is given by equation (7.11) as

$$T_2 = \frac{2A\left[\sqrt{H_1} - \sqrt{H_2}\right]}{C_d \times a \times \sqrt{2g}} = \frac{2 \times 12.566\left[\sqrt{7} - \sqrt{2.0}\right]}{0.6 \times .0314 \times \sqrt{2 \times 9.81}} \text{ sec} = 370.92 \text{ sec}$$

∴ Total time, $T = T_1 + T_2 = 198.78 + 370.92 = 569.7$ sec

= **9 min 29 sec. Ans.**

▶ 7.11 TIME OF EMPTYING A CIRCULAR HORIZONTAL TANK

Consider a circular horizontal tank of length L and radius R, containing liquid upto a height of H_1. Let an orifice of area 'a' is fitted at the bottom of the tank. Then the time required to bring the liquid level from H_1 to H_2 is obtained as :

Let at any time, the height of liquid over orifice is 'h' and in time dT, let the height falls by an height of 'dh'. Let at this time, the width of liquid surface = AC as shown in Fig. 7.12.

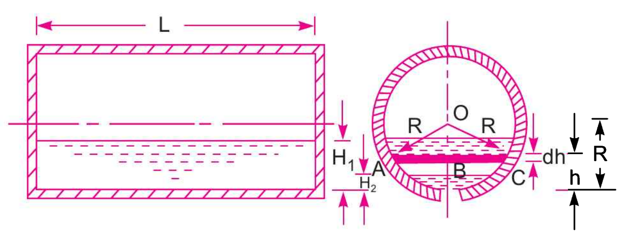

Fig. 7.12

∴ Surface area of liquid = $L \times AC$

But $$AC = 2 \times AB = 2\left[\sqrt{AO^2 - OB^2}\right] = 2\left[\sqrt{R^2 - (R-h)^2}\right]$$

$$= 2\sqrt{R^2 - \left(R^2 + h^2 - 2Rh\right)} = 2\sqrt{2Rh - h^2}$$

$\therefore$ Surface area, $A = L \times 2\sqrt{2Rh - h^2}$

$\therefore$ Volume of liquid leaving tank in time dT

$$= A \times dh = 2L\sqrt{2Rh - h^2} \times dh \qquad ...(i)$$

Also the volume of liquid flowing through orifice in time dT

$$= C_d \times \text{Area of orifice} \times \text{Velocity} \times dT$$

But the velocity of liquid at the time considered = $\sqrt{2gh}$

$\therefore$ Volume of liquid flowing through orifice in time dT

$$= C_d \times a \times \sqrt{2gh} \times dT \qquad ...(ii)$$

Equating (i) and (ii), we get

$$2L\sqrt{2Rh - h^2} \times (-dh) = C_d \times a \times \sqrt{2gh \times dT}$$

– ve sign is introduced as with the increase of T, the height h decreases,

$$\therefore \qquad dT = \frac{-2L\sqrt{2Rh - h^2}\,dh}{C_d \times a \times \sqrt{2gh}} = \frac{-2L\sqrt{(2R - h)}\,dh}{C_d \times a \times \sqrt{2g}}$$

[Taking $\sqrt{h}$ common]

$$\therefore \qquad \text{Total time, } T = \int_{H_1}^{H_2} \frac{-2L(2R - h)^{1/2}\,dh}{C_d \times a \times \sqrt{2g}}$$

$$= \frac{-2L}{C_d \times a \times \sqrt{2g}} \int_{H_1}^{H_2} [2R - h]^{1/2}\,dh$$

$$= \frac{-2L}{C_d \times a \times \sqrt{2g}} \left[\frac{(2R - h)^{1/2+1}}{\frac{1}{2} + 1} \times (-1)\right]_{H_1}^{H_2}$$

$$= \frac{2L}{C_d \times a \times \sqrt{2g}} \times \frac{2}{3} \times \left[(2R - h)^{3/2}\right]_{H_1}^{H_2}$$

$$= \frac{4L}{3C_d \times a \times \sqrt{2g}} \left[(2R - H_2)^{3/2} - (2R - H_1)^{3/2}\right] \qquad ...(7.15)$$

For completely emptying the tank, $H_2 = 0$ and hence

$$T = \frac{4L}{3C_d \times a \times \sqrt{2g}} \left[(2R)^{3/2} - (2R - H_1)^{3/2}\right]. \qquad ...(7.16)$$

Problem 7.22 *An orifice of diameter 100 mm is fitted at the bottom of a boiler drum of length 5 m and of diameter 2 m. The drum is horizontal and half full of water. Find the time required to empty the boiler, given the value of* $C_d = 0.6$.

Solution. Given :

Dia. of orifice, $d = 100$ mm = 0.1 m

∴ Area, $a = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$

Length, $L = 5$ m

Dia. of drum, $D = 2$ m

∴ Radius, $R = 1$ m

Initial height of water, $H_1 = 1$ m

Final height of water, $H_2 = 0$

$C_d = 0.6$

For completely emptying the tank, T is given by equation (7.16)

$$\therefore \quad T = \frac{4L}{3 \times C_d \times a \times \sqrt{2g}} [(2R)^{3/2} - (2R - H_1)^{3/2}]$$

$$= \frac{4 \times 5.0}{3 \times .06 \times .007854 \times \sqrt{2 \times 9.81}} [(2 \times 1)^{3/2} - (2 \times 1 - 1)^{3/2}]$$

= 319.39 [2.8284 – 1.0] = 583.98 sec = **9 min 44 sec. Ans.**

Problem 7.23 *An orifice of diameter 150 mm is fitted at the bottom of a boiler drum of length 8 m and of diameter 3 metres. The drum is horizontal and contains water upto a height of 2.4 m. Find the time required to empty the boiler. Take $C_d = 0.6$.*

Solution. Given :

Dia. of orifice, $d = 150$ mm = 0.15 m

∴ Area, $a = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$

Length, $L = 8.0$ m

Dia. of boiler, $D = 3.0$ m

∴ Radius, $R = 1.5$ m

Initial height of water, $H_1 = 2.4$ m

Find height of water, $H_2 = 0$

$C_d = 0.6.$

For completely emptying the tank, T is given by equation (7.16) as

$$T = \frac{4L}{3C_d \times a \times \sqrt{2g}} [(2\text{R})^{3/2} - (2R - H_1)^{3/2}]$$

$$= \frac{4 \times 8.0}{3 \times .6 \times .01767 \times \sqrt{2 \times 9.81}} [(2 \times 1.5)^{3/2} - (2 \times 1.5 - 2.4)^{3/2}]$$

= 227.14 [5.196 – 0.4647] = 1074.66 sec

= **17 min 54.66 sec. Ans.**

▶ 7.12 CLASSIFICATION OF MOUTHPIECES

1. The mouthpieces are classified as (*i*) External mouthpiece or (*ii*) Internal mouthpiece depending upon their position with respect to the tank or vessel to which they are fitted.

2. The mouthpiece are classified as (*i*) Cylindrical mouthpiece or (*ii*) Convergent mouthpiece or (*iii*) Convergent-divergent mouthpiece depending upon their shapes.

3. The mouthpieces are classified as (*i*) Mouthpieces running full or (*ii*) Mouthpieces running free, depending upon the nature of discharge at the outlet of the mouthpiece. This classification is only for internal mouthpieces which are known Borda's or Re-entrant mouthpieces. A mouthpiece is said to be running free if the jet of liquid after contraction does not touch the sides of the mouthpiece. But if the jet after contraction expands and fills the whole mouthpiece it is known as running full.

▶ 7.13 FLOW THROUGH AN EXTERNAL CYLINDRICAL MOUTHPIECE

A mouthpiece is a short length of a pipe which is two or three times its diameter in length. If this pipe is fitted externally to the orifice, the mouthpiece is called external cylindrical mouthpiece and the discharge through orifice increases.

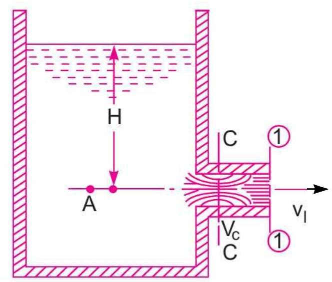

Fig. 7.13 *External cylindrical mouthpieces.*

Consider a tank having an external cylindrical mouthpiece of cross-sectional area a_1, attached to one of its sides as shown in Fig. 7.13. The jet of liquid entering the mouthpiece contracts to form a vena-contracta at a section *C-C*. Beyond this section, the jet again expands and fill the mouthpiece completely.

Let H = Height of liquid above the centre of mouthpiece

v_c = Velocity of liquid at *C-C* section

a_c = Area of flow at vena-contracta

v_1 = Velocity of liquid at outlet

a_1 = Area of mouthpiece at outlet

C_c = Co-efficient of contraction.

Applying continuity equation at *C-C* and (1)-(1), we get

$$a_c \times v_c = a_1 v_1$$

$$\therefore \quad v_c = \frac{a_1 v_1}{a_c} = \frac{v_1}{a_c / a_1}$$

But $$\frac{a_c}{a_1} = C_c = \text{Co-efficient of contraction}$$

Taking $C_c = 0.62$, we get $\frac{a_c}{a_1} = 0.62$

$$\therefore \quad v_c = \frac{v_1}{0.62}$$

The jet of liquid from section *C-C* suddenly enlarges at section (1)-(1). Due to sudden enlargement, there will be a loss of head, h_L* which is given as $h_L = \frac{(v_c - v_1)^2}{2g}$

* Please refer Art. 11.4.1 for loss of head due to sudden enlargement.

But $v_c = \dfrac{v_1}{0.62}$ hence $h_L = \dfrac{\left(\dfrac{v_1}{0.62} - v_1\right)^2}{2g} = \dfrac{v_1^2}{2g}\left[\dfrac{1}{0.62} - 1\right]^2 = \dfrac{0.375\, v_1^2}{2g}$

Applying Bernoulli's equation to point A and (1)-(1)

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

where $z_A = z_1$, v_A is negligible,

$\dfrac{p_1}{\rho g}$ = atmospheric pressure = 0

$$\therefore \quad H + 0 = 0 + \frac{v_1^2}{2g} + .375\,\frac{v_1^2}{2g}$$

$$\therefore \quad H = 1.375\,\frac{v_1^2}{2g}$$

$$\therefore \quad v_1 = \sqrt{\frac{2gH}{1.375}} = 0.855\,\sqrt{2gH}$$

Theoretical velocity of liquid at outlet is $v_{th} = \sqrt{2gH}$

∴ Co-efficient of velocity for mouthpiece

$$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{0.855\sqrt{2gH}}{\sqrt{2gH}} = 0.855.$$

C_c for mouthpiece = 1 as the area of jet of liquid at outlet is equal to the area of mouthpiece at outlet.

Thus $\quad C_d = C_c \times C_v = 1.0 \times .855 = 0.855$

Thus the value of C_d for mouthpiece is more than the value of C_d for orifice, and so discharge through mouthpiece will be more.

Problem 7.24 *Find the discharge from a 100 mm diameter external mouthpiece, fitted to a side of a large vessel if the head over the mouthpiece is 4 metres.*

Solution. Given :

Dia. of mouthpiece = 100 m = 0.1 m

∴ Area, $\quad a = \dfrac{\pi}{4}(0.1)^2 = 0.007854 \text{ m}^2$

Head, $\quad H = 4.0$ m

C_d for mouthpiece $\quad = 0.855$

∴ Discharge $\quad = C_d \times \text{Area} \times \text{Velocity} = 0.855 \times a \times \sqrt{2gH}$

$\quad = .855 \times .007854 \times \sqrt{2 \times 9.81 \times 4.0}$ = **.05948 m³/s. Ans.**

Problem 7.25 *An external cylindrical mouthpiece of diameter 150 mm is discharging water under a constant head of 6 m. Determine the discharge and absolute pressure head of water at vena-contracta. Take C_d = 0.855 and C_c for vena-contracta = 0.62. Atmospheric pressure head = 10.3 m of water.*

Solution. Given :

Dia. of mouthpiece, $d = 150$ mm $= 0.15$ cm

$\therefore$ Area, $a = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$

Head, $H = 6.0$ m

$C_d = 0.855$

C_c at vena-contracta = 0.62

Atmospheric pressure head, $H_a = 10.3$ m

$\therefore$ Discharge $= C_d \times a \times \sqrt{2gH}$

$= 0.855 \times .01767 \times \sqrt{2 \times 9.81 \times 6.0}$ = **0.1639 m³/s. Ans.**

Pressure Head at Vena-contracta

Applying Bernoulli's equation at A and C-C, we get

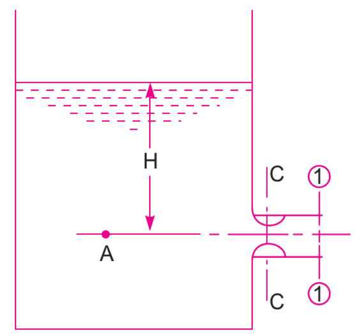

Fig. 7.14

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c$$

But $\frac{p_A}{\rho g} = H_a + H, \; v_A = 0,$

$z_A = z_c$

$$\therefore \quad H_a + H + 0 = \frac{p_c}{\rho g} + \frac{v_c^2}{2g} = H_c + \frac{v_c^2}{2g}$$

$$\therefore \quad H_c = H_a + H - \frac{v_c^2}{2g}$$

But $v_c = \frac{v_1}{0.62}$

$$\therefore \quad H_c = H_a + H - \left(\frac{v_1}{.62}\right)^2 \times \frac{1}{2g} = H_a + H - \frac{v_1^2}{2g} \times \frac{1}{(.62)^2}$$

But $\text{H} = 1.375 \; \frac{v_1^2}{2g}$

$$\therefore \quad \frac{v_1^2}{2g} = \frac{H}{1.375} = 0.7272\, H$$

$$\therefore \quad H_c = H_a + H - .7272\, H \times \frac{1}{(.62)^2}$$

$= H_a + H - 1.89\, H = H_a - .89\, H$

$= 10.3 - .89 \times 6.0$ $\{\because H_a = 10.3 \text{ and } H = 6.0\}$

$= 10.3 - 5.34$ = **4.96 m (Absolute). Ans.**

▶ 7.14 FLOW THROUGH A CONVERGENT-DIVERGENT MOUTHPIECE

If a mouthpiece converges upto vena-contracta and then diverges as shown in Fig. 7.15 then that type of mouthpiece is called Convergent-Divergent Mouthpiece. As in this mouthpiece there is no sudden enlargement of the jet, the loss of energy due to sudden enlargement is eliminated. The co-efficient of discharge for this mouthpiece is unity. Let H is the head of liquid over the mouthpiece.

Applying Bernoulli's equation to the free surface of water in tank and section C-C, we have

$$\frac{p}{\rho g}+\frac{v^2}{2g}+z=\frac{p_c}{\rho g}+\frac{v_c^2}{2g}+z_c$$

Taking datum passing through the centre of orifice, we get

$$\frac{p}{\rho g}=H_a,\ v=0,\ z=H,\ \frac{p_c}{\rho g}=H_c,\ z_c=0$$

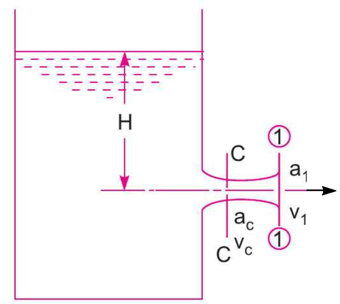

Fig. 7.15 *Convergent-divergent mouthpiece.*

$$\therefore \quad H_a+0+H=H_c+\frac{v_c^2}{2g}+0 \qquad \text{...}(i)$$

$$\therefore \quad \frac{v_c^2}{2g}=H_a+H-H_c \qquad \text{...}(ii)$$

or

$$v_c=\sqrt{2g(H_a+H-H_c)}$$

Now applying Bernoulli's equation at sections C-C and (1)-(1)

$$\frac{p_c}{\rho g}+\frac{v_c^2}{2g}+z_c=\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1$$

But

$$z_c=z_1 \text{ and } \frac{p_1}{\rho g}=H_a$$

$$\therefore \quad H_c+\frac{v_c^2}{2g}=H_a+\frac{v_1^2}{2g}$$

Also from (*i*), $\quad H_c+v_c^2/2g=H+H_a$

$$\therefore \quad H_a+v_1^2/2g=H+H_a$$

$$\therefore \quad v_1=\sqrt{2gH} \qquad \text{...}(iii)$$

Now by continuity equation, $a_c\,v_c=v_1\times a_1$

$$\therefore \quad \frac{a_1}{a_c}=\frac{v_c}{v_1}=\frac{\sqrt{2g(H_a+H-H_c)}}{\sqrt{2gH}}=\sqrt{\frac{H_a}{H}+1-\frac{H_c}{H}}$$

$$=\sqrt{1+\frac{H_a-H_c}{H}} \qquad \text{...(7.17)}$$

The discharge, Q is given as $Q=a_c\times\sqrt{2gH}$ ...(7.18)

where a_c = area at vena-contracta.

Problem 7.26 *A convergent-divergent mouthpiece having throat diameter of 4.0 cm is discharging water under a constant head of 2.0 m, determine the maximum outer diameter for maximum discharge. Find maximum discharge also. Take H_a = 10.3 m of water and H_{sep} = 2.5 m of water (absolute).*

Solution. Given :

Dia. of throat, $d_c = 4.0$ cm

∴ Area, $a_c = \frac{\pi}{4}(4)^2 = 12.566 \text{ cm}^2$

Constant head, $H = 2.0$ m

Find max. dia. at outlet, d_1 and Q_{max}

$H_a = 10.3$ m

$H_{sep} = 2.5$ m (absolute)

The discharge, Q in convergent-divergent mouthpiece depends on the area at throat.

$$\therefore \quad Q_{max} = a_c \times \sqrt{2gH} = 12.566 \times \sqrt{2 \times 9.81 \times 200} = \textbf{7871.5 cm}^3\textbf{/s. Ans.}$$

Now ratio of areas at outlet and throat is given by equation (7.17) as

$$\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}} = \sqrt{1 + \frac{10.3 - 2.5}{2.0}} \qquad \{\because H_c = H_{sep} = 2.5\}$$

$$= 2.2135$$

$$\frac{\pi}{4} d_1^2 \bigg/ \frac{\pi}{4} d_c^2 = 2.2135 \text{ or } \left(\frac{d_1}{d_c}\right)^2 = 2.2135$$

$$\therefore \quad \frac{d_1}{d_c} = \sqrt{2.2135} = 1.4877$$

$$\therefore \quad d_1 = 1.4877 \times d_c = 1.4877 \times 4.0 = \textbf{5.95 cm. Ans.}$$

Problem 7.27 *The throat and exit diameters of convergent-divergent mouthpiece are 5 cm and 10 cm respectively. It is fitted to the vertical side of a tank, containing water. Find the maximum head of a water for steady flow. The maximum vacuum pressure is 8 m of water and take atmospheric pressure = 10.3 m water.*

Solution. Given :

Dia. at throat, $d_c = 5$ cm

Dia. at exit, $d_1 = 10$ cm

Atmospheric pressure head, $H_a = 10.3$ m

The maximum vacuum pressure will be at a throat only

∴ Pressure head at throat = 8 m (vacuum)

or $H_c = H_a - 8.0$ (absolute)

$= 10.3 - 8.0 = 2.3$ m (abs.)

Let maximum head of water over mouthpiece = H m of water.

The ratio of areas at outlet and throat of a convergent-divergent mouthpiece is given by equation (7.17).

$$\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}} \quad \text{or} \quad \frac{\frac{\pi}{4}(d_1)^2}{\frac{\pi}{4}(d_c)^2} = \sqrt{1 + \frac{10.3 - 2.3}{H}}$$

or
$$\frac{10^2}{5^2} = 4 = \sqrt{1 + \frac{8}{H}} \quad \text{or} \quad 16 = 1 + \frac{8}{H} \quad \text{or} \quad 15 = \frac{8}{H}$$

∴
$$H = \frac{8}{15} = 0.5333 \text{ m of water}$$

∴ Maximum head of water = **0.533 m. Ans.**

Problem 7.28 *A convergent-divergent mouthpiece is fitted to the side of a tank. The discharge through mouthpiece under a constant head of 1.5 m is 5 litres/s. The head loss in the divergent portion is 0.10 times the kinetic head at outlet. Find the throat and exit diameters, if separation pressure is 2.5 m and atmospheric pressure head = 10.3 m of water.*

Solution. Given :

Constant head, $H = 1.5$ m

Discharge, $Q = 5$ litres $= .005$ m^3/s

h_L or Head loss in divergent $= 0.1 \times$ kinetic head at outlet

H_c or $H_{sep} = 2.5$ (abs.)

$H_a = 10.3$ m of water

Find (*i*) Dia. at throat, d_c

(*ii*) Dia. at outlet, d_1

(*i*) **Dia. at throat (d_c).** Applying Bernoulli's equation to the free water surface and throat section, we get (See Fig. 7.15).

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c$$

Taking the centre line of mouthpiece as datum, we get

$$H_a + 0 + H = H_c + \frac{v_c^2}{2g}$$

∴
$$\frac{v_c^2}{2g} = H_a + H - H_c = 10.3 + 1.5 - 2.5 = 9.3 \text{ m of water}$$

∴
$$v_c = \sqrt{2 \times 9.81 \times 9.3} = 13.508 \text{ m/s}$$

Now
$$Q = a_c \times v_c \text{ or } .005 = \frac{\pi}{4} d_c^{\,2} \times 13.508$$

∴
$$d_c = \sqrt{\frac{.005 \times 4}{\pi \times 13.508}} = \sqrt{.00047} = .0217 \text{ m} = \textbf{2.17 cm. Ans.}$$

(*ii*) **Dia. at outlet (d_1).** Applying Bernoulli's equation to the free water surface and outlet of mouth-piece (See Fig. 7.15), we get

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

$$H_a + 0 + H = H_a + \frac{v_1^2}{2g} + 0 + 0.1 \times \frac{v_1^2}{2g} \qquad \left\{\because \frac{p_1}{\rho g} = H_a\right\}$$

$\therefore$
$$H = \frac{v_1^2}{2g} + .1 \times \frac{v_1^2}{2g} = 1.1\,\frac{v_1^2}{2g}$$

$\therefore$
$$v_1 = \sqrt{\frac{2gH}{1.1}} = \sqrt{\frac{2 \times 9.81 \times 1.5}{1.1}} = 5.1724$$

Now
$$Q = A_1 v_1 \text{ or } .005 = \frac{\pi}{4} {d_1}^2 \times v_1$$

$\therefore$
$$d_1 = \sqrt{\frac{4 \times .005}{\pi \times v_1}} = \sqrt{\frac{4 \times .005}{\pi \times 5.1724}} = 0.035 \text{ m} = \textbf{3.5 cm. Ans.}$$

▶ 7.15 FLOW THROUGH INTERNAL OR RE-ENTRANT ON BORDA'S MOUTHPIECE

A short cylindrical tube attached to an orifice in such a way that the tube projects inwardly to a tank, is called an internal mouthpiece. It is also called Re-entrant or Borda's mouthpiece. If the length of the tube is equal to its diameter, the jet of liquid comes out from mouthpiece without touching the sides of the tube as shown in Fig. 7.16. The mouthpiece is known as *running free*. But if the length of the tube is about 3 times its diameter, the jet comes out with its diameter equal to the diameter of mouthpiece at outlet as shown in Fig. 7.17. The mouthpiece is said to be *running full*.

(*i*) **Borda's Mouthpiece Running Free.** Fig. 7.16 shows the Borda's mouthpiece running free.

Let H = height of liquid above the mouthpiece,

a = area of mouthpiece,

a_c = area of contracted jet in the mouthpiece,

v_c = velocity through mouthpiece.

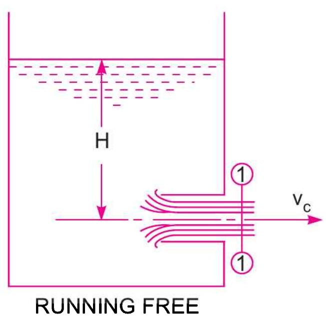

Fig. 7.16

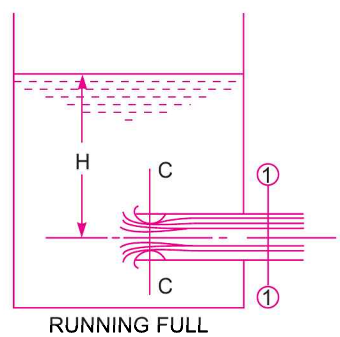

Fig. 7.17

The flow of fluid through mouthpiece is taking place due to the pressure force exerted by the fluid on the entrance section of the mouthpiece. As the area of the mouthpiece is 'a' hence total pressure force on entrance

$$= \rho g \,.\, a.\, h$$

where h = distance of C.G. of area 'a' from free surface = H.

$$= \rho g \,.\, a \,.\, H \qquad ...(i)$$

According to Newton's second law of motion, the net force is equal to the rate of change of momentum.

Now mass of liquid flowing/sec = $\rho \times a_c \times v_c$

The liquid is initially at rest and hence initial velocity is zero but final velocity of fluid is v_c.

∴ Rate of change of momentum = mass of liquid flowing/sec × [final velocity – initial velocity]

$$= \rho a_c \times v_c[v_c - 0] = \rho a_c\, v_c^2 \qquad ...(ii)$$

Equating (*i*) and (*ii*), we get

$$\rho g\ .a.H. = \rho a_c.\ v_c^2 \qquad ...(iii)$$

Applying Bernoulli's equation to free surface of liquid and section (1)-(1) of Fig. 7.16

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1$$

Taking the centre line of mouthpiece as datum, we have

$$z = H,\ z_1 = 0,\ \frac{p}{\rho g} = \frac{p_1}{\rho g} = p_{atmosp.} = 0,$$

$$v_1 = v_c, \qquad v = 0$$

∴ $$0 + 0 + H = 0 + \frac{v_c^2}{2g} + 0 \text{ or } H = \frac{v_c^2}{2g}$$

∴ $$v_c = \sqrt{2gH}$$

Substituting the value of v_c in (*iii*), we get

$$\rho g\ .\ a\ .\ H. = \rho\ .\ a_c\ .\ 2g\ .\ H$$

or $$a = 2a_c \text{ or } \frac{a_c}{a} = \frac{1}{2} = 0.5$$

∴ Co-efficient of contraction, $C_c = \dfrac{a_c}{a} = 0.5$

Since there is no loss of head, co-efficient of velocity, $C_v = 1.0$

∴ Co-efficient of discharge, $C_d = C_c \times C_v = 0.5 \times 1.0 = 0.5$

∴ Discharge $$Q = C_d\, a\sqrt{2gH} \qquad ...(7.19)$$

$$= 0.5 \times a\sqrt{2gH}$$

(*ii*) **Borda's Mouthpiece Running Full.** Fig. 7.17 shows Borda's mouthpiece running full.

Let H = height of liquid above the mouthpiece,

v_1 = velocity at outlet or at (1)-(1) of mouthpiece,

a = area of mouthpiece,

a_c = area of the flow at *C-C*,

v_c = velocity of liquid at vena-contracta or at *C-C*.

The jet of liquid after passing through *C-C*, suddenly enlarges at section (1)-(1). Thus there will be a loss of head due to sudden enlargement.

∴ $$h_L = \frac{(v_c - v_1)^2}{2g} \qquad ...(i)$$

Now from continuity, we have $a_c \times v_c = a_1 \times v_1$

$$\therefore \quad v_c = \frac{a_1}{a_c} \times v_1 = \frac{v_1}{a_c / a_1} = \frac{v_1}{C_c} = \frac{v_1}{0.5} \qquad \{\because C_c = 0.5\}$$

or $$v_c = 2v_1$$

Substituting this value of v_c in (i), we get $h_L = \dfrac{(2v_1 - v_1)^2}{2g} = \dfrac{v_1^2}{2g}$

Applying Bernoulli's equation to free surface of water in tank and section (1)-(1), we get

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

Taking datum line passing through the centre line of mouthpiece

$$0 + 0 + H = 0 + \frac{v_1^2}{2g} + 0 + \frac{v_1^2}{2g}$$

$$\therefore \quad H = \frac{v_1^2}{2g} + \frac{v_1^2}{2g} = \frac{v_1^2}{g}$$

$$\therefore \quad v_1 = \sqrt{gH}$$

Here v_1 is actual velocity as losses have been taken into consideration,

But theoretical velocity, $v_{th} = \sqrt{2gH}$

$\therefore$ Co-efficient of velocity, $C_v = \dfrac{v_1}{v_{th}} = \dfrac{\sqrt{gH}}{\sqrt{2gH}} = \dfrac{1}{\sqrt{2}} = 0.707$

As the area of the jet at outlet is equal to the area of the mouthpiece, hence co-efficient of contraction = 1

$$\therefore \quad C_d = C_c \times C_v = 1.0 \times .707 = 0.707$$

$\therefore$ Discharge, $$Q = C_d \times a \times \sqrt{2gH} = 0.707 \times a \times \sqrt{2gH} \qquad ...(7.20)$$

Problem 7.29 *An internal mouthpiece of 80 mm diameter is discharging water under a constant head of 8 metres. Find the discharge through mouthpiece, when*

(i) The mouthpiece is running free, and (ii) The mouthpiece is running full.

Solution. Given :

Dia. of mouthpiece, d = 80 mm = 0.08 m

$\therefore$ Area, $$a = \frac{\pi}{4}(.08)^2 = .005026 \text{ m}^2$$

Constant head, H = 4 m.

(i) **Mouthpiece running free.** The discharge, Q is given by equation (7.19) as

$$Q = 0.5 \times a \times \sqrt{2gH}$$
$$= 0.5 \times .005026 \times \sqrt{2 \times 9.81 \times 4.0}$$
$$= 0.02226 \text{ m}^3/\text{s} = \textbf{22.26 litres/s. Ans.}$$

(ii) **Mouthpiece running full.** The discharge, Q is given by equation (7.20) as

$$Q = 0.707 \times a \times \sqrt{2gH}$$
$$= 0.707 \times .005026 \times \sqrt{2 \times 9.81 \times 4.0}$$
$$= 0.03147 \text{ m}^3/\text{s} = \textbf{31.47 litre/s. Ans.}$$

HIGHLIGHTS

1. Orifice is a small opening on the side or at the bottom of a tank while mouthpiece is a short length of pipe which is two or three times its diameter in length.
2. Orifices as well as mouthpieces are used for measuring the rate of flow of liquid.
3. Theoretical velocity of jet of water from orifice is given by

$$V = \sqrt{2gH}, \text{ where } H = \text{Height of water from the centre of orifice.}$$

4. There are three hydraulic co-efficients namely :

(*a*) Co-efficient of velocity, $C_v = \dfrac{\text{Actual velocity at vena-contracta}}{\text{Theoretical velocity}} = \dfrac{x}{\sqrt{4yH}}$

(*b*) Co-efficient of contraction, $C_c = \dfrac{\text{Area of jet at vena-contracta}}{\text{Area of orifice}}$

(*c*) Co-efficient of discharge, $C_d = \dfrac{\text{Actual discharge}}{\text{Theoretical discharge}} = C_v \times C_c$

where x and y are the co-ordinates of any point of jet of water from vena-contracta.

5. A large orifice is one, where the head of liquid above the centre of orifice is less than 5 times the depth of orifice. The discharge through a large rectangular orifice is

$$Q = \frac{2}{3} C_d \times b \times \sqrt{2g}\,[H_2^{3/2} - H_1^{3/2}]$$

where b = Width of orifice,
C_d = Co-efficient of discharge for orifice,
H_1 = Height of liquid above top edge of orifice, and
H_2 = Height of liquid above bottom edge of orifice.

6. The discharge through fully sub-merged orifice, $Q = C_d \times b \times (H_2 - H_1) \times \sqrt{2gH}$

where b = Width of orifice,
C_d = Co-efficient of discharge for orifice,
H_2 = Height of liquid above bottom edge of orifice on upstream side,
H_1 = Height of liquid above top edge of orifice on upstream side,
H = Difference of liquid levels on both sides of the orifice.

7. Discharge through partially sub-merged orifice,

$$Q = Q_1 + Q_2$$
$$= C_d b\,(H_2 - H) \times \sqrt{2gH} + 2/3\, C_d b \times \sqrt{2g}\,[H^{3/2} - H_1^{3/2}]$$

where b = Width of orifice
C_d, H_1, H_2 and H are having their usual meaning.

8. Time of emptying a tank through an orifice at its bottom is given by,

$$T = \frac{2A\left[\sqrt{H_1} - \sqrt{H_2}\right]}{C_d \,.\, a \,.\, \sqrt{2g}}$$

where H_1 = Initial height of liquid in tank,
H_2 = Final height of liquid in tank,

A = Area of tank,
a = Area of orifice,
C_d = Co-efficient of discharge.

If the tank is to be completely emptied, then time T,

$$T = \frac{2A\sqrt{H}}{C_d \,.\, a \,.\, \sqrt{2g}}.$$

9. Time of emptying a hemispherical tank by an orifice fitted at its bottom,

$$T = \frac{\pi}{C_d \,.\, a \,.\, \sqrt{2g}}\left[\frac{4}{3}R\left(H_1^{3/2} - H_2^{3/2}\right) - \frac{2}{5}\left(H_1^{5/2} - H_2^{5/2}\right)\right]$$

and for completely emptying the tank, $T = \dfrac{\pi}{C_d \,.\, a \,.\, \sqrt{2g}}\left[\dfrac{4}{3}RH_1^{3/2} - \dfrac{2}{5}H_1^{5/2}\right]$

where R = Radius of the hemispherical tank,
H_1 = Initial height of liquid,
H_2 = Final height of liquid,
a = Area of orifice, and
C_d = Co-efficient of discharge.

10. Time of emptying a circular horizontal tank by an orifice at the bottom of the tank,

$$T = \frac{4L}{3C_d \,.\, a \,.\, \sqrt{2g}}\ [(2R - \text{H}_2)^{3/2} - (2R - H_1)^{3/2}]$$

and for completely emptying the tank, $T = \dfrac{4L}{3C_d \,.\, a \,.\, \sqrt{2g}}\ [(2R)^{3/2} - (2R - H_1)^{3/2}]$

where L = Length of horizontal tank.

11. Co-efficient of discharge for,

(*i*) External mouthpiece, $C_d = 0.855$
(*ii*) Internal mouthpiece, running full, $C_d = 0.707$
(*iii*) Internal mouthpiece, running free, $C_d = 0.50$
(*iv*) Convergent or convergent-divergent, $C_d = 1.0$.

12. For an external mouthpiece, absolute pressure head at vena-contracta

$$H_c = H_a - 0.89\ H$$

where H_a = atmospheric pressure head = 10.3 m of water
H = head of liquid above the mouthpiece.

13. For a convergent-divergent mouthpiece, the ratio of areas at outlet and at vena-contracta is

$$\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}}$$

where a_1 = Area of mouthpiece at outlet
a_c = Area of mouthpiece at vena-contracta
H_a = Atmospheric pressure head
H_c = Absolute pressure head at vena-contracta
H = Height of liquid above mouthpiece.

14. In case of internal mouthpieces, if the jet of liquid comes out from mouthpiece without touching its sides it is known as running free. But if the jet touches the sides of the mouthpiece, it is known as running full.

EXERCISE

(A) THEORETICAL PROBLEMS

1. Define an orifice and a mouthpiece. What is the difference between the two ?
2. Explain the classification of orifices and mouthpieces based on their shape, size and sharpness ?
3. What are hydraulic co-efficients ? Name them.
4. Define the following co-efficients : (*i*) Co-efficient of velocity, (*ii*) Co-efficient of contraction and (*iii*) Co-efficient of discharge.
5. Derive the expression $C_d = C_v \times C_c$.
6. Define vena-contracta.
7. Differentiate between a large and a small orifice. Obtain an expression for discharge through a large rectangular orifice.
8. What do you understand by the terms wholly sub-merged orifice and partially sub-merged orifice ?
9. Prove that the expression for discharge through an external mouthpiece is given by

$$Q = .855 \times a \times v$$

where a = Area of mouthpiece at outlet and
v = Velocity of jet of water at outlet.
10. Distinguish between : (*i*) External mouthpiece and internal mouthpiece, (*ii*) Mouthpiece running free and mouthpiece running full.
11. Obtain an expression for absolute pressure head at vena-contracta for an external mouthpiece.
12. What is a convergent-divergent mouthpiece ? Obtain an expression for the ratio of diameters at outlet and at vena-contracta for a convergent-divergent 'mouthpiece' in terms of absolute pressure head at vena-contracta, head of liquid above mouthpiece and atmospheric pressure head.
13. The length of the divergent outlet part in a venturimeter is usually made longer compared with that of the converging inlet part. Why ?
14. Justify the statement, "In a convergent-divergent mouthpiece the loss of head is practically eliminated".

(B) NUMERICAL PROBLEMS

1. The head of water over an orifice of diameter 50 mm is 12 m. Find the actual discharge and actual velocity of jet at vena-contracta. Take $C_d = 0.6$ and $C_v = 0.98$. [**Ans.** .018 m^3/s ; 15.04 m/s]
2. The head of water over the centre of an orifice of diameter 30 mm is 1.5 m. The actual discharge through the orifice is 2.35 litres/sec. Find the co-efficient of discharge. [**Ans.** 0.613]
3. A jet of water, issuing from a sharp edged vertical orifice under a constant head of 60 cm, has the horizontal and vertical co-ordinates measured from the vena-contracta at a certain point as 10.0 cm and 0.45 cm respectively. Find the value of C_v. Also find the value of C_v if $C_d = 0.60$. [**Ans.** 0.962, 0.623]
4. The head of water over an orifice of diameter 100 mm is 5 m. The water coming out from orifice is collected in a circular tank of diameter 2 m. The rise of water level in circular tank is .45 m in 30 seconds. Also the co-ordinates of a certain point on the jet, measured from vena-contracta are 100 cm horizontal and 5.2 cm vertical. Find the hydraulic co-efficients C_d, C_v and C_c. [**Ans.** 0.605, 0.98, 0.617]
5. A tank has two identical orifices in one of its vertical sides. The upper orifice is 4 m below the water surface and lower one 6 m below the water surface. If the value of C_v for each orifice is 0.98, find the point of intersection of the two jets. [**Ans.** At a horizontal distance of 9.60 cm]
6. A closed vessel contains water upto a height of 2.0 m and over the water surface there is air having pressure 8.829 N/cm^2 above atmospheric pressure. At the bottom of the vessel there is an orifice of diameter 15 cm. Find the rate of flow of water from orifice. Take $C_d = 0.6$. [**Ans.** 0.15575 m^3/s]

7. A closed tank partially filled with water upto a height of 1 m, having an orifice of diameter 20 mm at the bottom of the tank. Determine the pressure required for a discharge of 3.0 litres/s through the orifice. Take $C_d = 0.62$. **[Ans.** 10.88 N/cm^2]
8. Find the discharge through a rectangular orifice 3.0 m wide and 2 m deep fitted to a water tank. The water level in the tank is 4 m above the top edge of the orifice. Take $C_d = 0.62$ **[Ans.** 36.77 m^3/s]
9. A rectangular orifice, 2.0 m wide and 1.5 m deep is discharging water from a tank. If the water level in the tank is 3.0 m above the top edge of the orifice, find the discharge through the orifice. Take $C_d = 0.6$. **[Ans.** 15.40 m^3/s]
10. A rectangular orifice, 1.0 m wide and 1.5 m deep is discharging water from a vessel. The top edge of the orifice is 0.8 m below the water surface in the vessel. Calculate the discharge through the orifice if $C_d = 0.6$. Also calculate the percentage error if the orifice is treated as a small orifice. **[Ans.** 1.058%]
11. Find the discharge through a fully sub-merged orifice of width 2 m if the difference of water levels on both the sides of the orifice be 800 mm. The height of water from top and bottom of the orifice are 2.5 m and 3 m respectively. Take $C_d = 0.6$. **[Ans.** 2.377 m^3/s]
12. Find the discharge through a totally drowned orifice 1.5 m wide and 1 m deep, if the difference of water levels on both the sides of the orifice be 2.5 m. Take $C_d = 0.62$. **[Ans.** 6.513 m^3/s]
13. A rectangular orifice of 1.5 m wide and 1.2 m deep is fitted in one side of a large tank. The water level on one side of the orifice is 2 m above the top edge of the orifice, while on the other side of the orifice, the water level is 0.4 m below its top edge. Calculate the discharge through the orifice if $C_d = 0.62$. **[Ans.** 7.549 m^3/s**]**
14. A circular tank of diameter 3 m contains water upto a height of 4 m. The tank is provided with an orifice of diameter 0.4 m at the bottom. Find the time taken by water : (*i*) to fall from 4 m to 2 m and (*ii*) for completely emptying the tank. Take $C_d = 0.6$. **[Ans.** (*i*) 24.8 s, (*ii*) 84.7 s]
15. A circular tank of diameter 1.5 m contains water upto a height of 4 m. An orifice of 40 mm diameter is provided at its bottom. If $C_d = 0.62$, find the height of water above the orifice after 10 minutes. **[Ans.** 2 m]
16. A hemispherical tank of diameter 4 m contains water upto a height of 2.0 m. An orifice of diameter 50 mm is provided at the bottom. Find the time required by water (*i*) to fall from 2.0 m to 1.0 m (*ii*) for completely emptying the tank. Take $C_d = 0.6$ **[Ans.** (*i*) 30 min 14.34 s, (*ii*) 52 min 59 s]
17. A hemispherical cistern of 4 m radius is full of water. It is fitted with a 60 mm diameter sharp edged orifice at the bottom. Calculate the time required to lower the level in the cistern by 2 metres. Take $C_d = 0.6$. **[Ans.** 1 hr 58 min 45.9 s]
18. A cylindrical tank is having a hemispherical base. The height of cylindrical portion is 4 m and diameter is 3 m. At the bottom of this tank an orifice of diameter 300 mm is fitted. Find the time required to completely emptying the tank. Take $C_d = 0.6$. **[Ans.** 2 min 7.37 s]
19. An orifice of diameter 200 mm is fitted at the bottom of a boiler drum of length 6 m and of diameter 2 m. The drum is horizontal and half full of water. Find the time required to empty the boiler, given the value of $C_d = 0.6$ **[Ans.** 2 min 55.20 s]
20. An orifice of diameter 150 mm is fitted at the bottom of a boiler drum of length 6 m and of diameter 2 m. The drum is horizontal and contains water upto a height of 1.8 m. Find the time required to empty the boiler. Take $C_d = 0.6$. **[Ans.** 7 min 46.64 s]
21. Find the discharge from a 80 mm diameter external mouthpiece, fitted to a side of a large vessel if the head over the mouthpiece is 6 m. **[Ans.** 0.0466 m^3/s]
22. An external cylindrical mouthpiece of diameter 100 mm is discharging water under a constant head of 8 m. Determine the discharge and absolute pressure head of water at vena-contracta. Take $C_d = 0.855$ and C_c for vena-contracta = 0.62. Take atmospheric pressure head = 10.3 m of water. **[Ans.** 0.084 m^3/s ; 3.18 m]
23. A convergent-divergent mouthpiece having throat diameter of 60 mm is discharging water under a constant head of 3.0 m. Determine the maximum outlet diameter for maximum discharge. Find maximum discharge also. Take atmospheric pressure head = 10.3 m of water and separation pressure head = 2.5 m of water absolute. **[Ans.** 6.88 cm, Q_{max} = 0.01506 m^3/s]

24. The throat and exit diameter of a convergent-divergent mouthpiece are 40 mm and 80 mm respectively. It is fitted to the vertical side of a tank, containing water. Find the maximum head of water for steady flow. The maximum vacuum pressure is 8 m of water. Take atmospheric pressure head = 10.3 m of water. [**Ans.** 0.533 m]

25. The discharge through a convergent-divergent mouthpiece fitted to the side of a tank under a constant head of 2 m is 7 litres/s. The head loss in the divergent portion is 0.10 times the kinetic head at outlet. Find the throat and exit diameters, if separation pressure head = 2.5 m and atmospheric pressure head = 10.3 m of water. [**Ans.** 25.3 mm ; 38.6 mm]

26. An internal mouthpiece of 100 mm diameter is discharging water under a constant head of 5 m. Find the discharge through mouthpiece, when
(*i*) the mouthpiece is running free, and (*ii*) the mouthpiece is running full.
[**Ans.** (*i*) 38.8 litres/s, (*ii*) 54.86 litres/s]