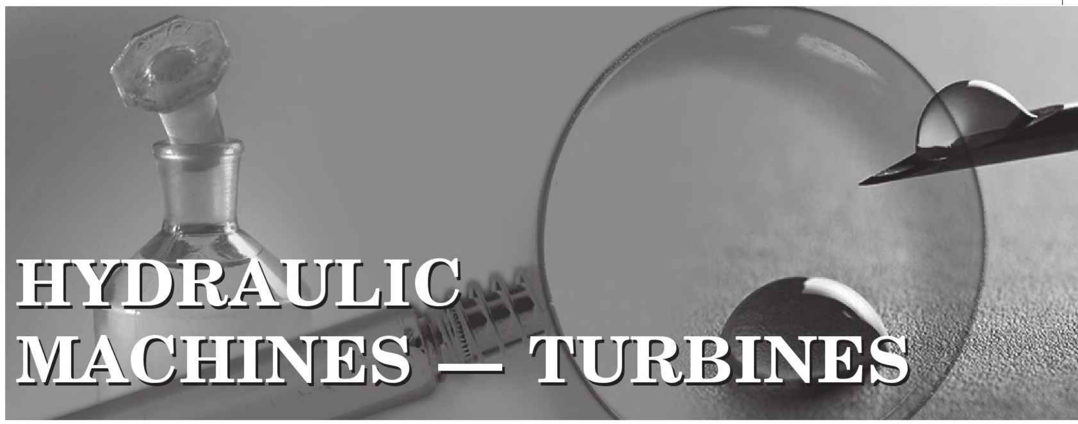

18 CHAPTER HYDRAULIC MACHINES — TURBINES

▶ 18.1 INTRODUCTION

Hydraulic machines are defined as those machines which convert either hydraulic energy (energy possessed by water) into mechanical energy (which is further converted into electrical energy) or mechanical energy into hydraulic energy. The hydraulic machines, which convert the hydraulic energy into mechanical energy, are called *turbines* while the hydraulic machines which convert the mechanical energy into hydraulic energy are called *pumps*. Thus the study of hydraulic machines consists of study of turbines and pumps. Turbines consists of mainly study of Pelton turbine, Francis Turbine and Kaplan Turbine while pumps consist of study of centrifugal pump and reciprocating pumps.

▶ 18.2 TURBINES

Turbines are defined as the hydraulic machines which convert hydraulic energy into mechanical energy. This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine. Thus the mechanical energy is converted into electrical energy. The electric power which is obtained from the hydraulic energy (energy of water) is known as *Hydroelectric power*. At present the generation of hydroelectric power is the cheapest as compared by the power generated by other sources such as oil, coal etc.

▶ 18.3 GENERAL LAYOUT OF A HYDROELECTRIC POWER PLANT

Fig. 18.1 shows a general layout of a hydroelectric power plant which consists of :

(*i*) A dam constructed across a river to store water.

(*ii*) Pipes of large diameters called penstocks, which carry water under pressure from the storage reservoir to the turbines. These pipes are made of steel or reinforced concrete.

(*iii*) Turbines having different types of vanes fitted to the wheels.

(*iv*) Tail race, which is a channel which carries water away from the turbines after the water has worked on the turbines. The surface of water in the tail race channel is also known as tail race.

▶ 18.4 DEFINITIONS OF HEADS AND EFFICIENCIES OF A TURBINE

1. Gross Head. The difference between the head race level and tail race level when no water is flowing is known as Gross Head. It is denoted by 'H_g' in Fig. 18.1.

2. Net Head. It is also called effective head and is defined as the head available at the inlet of the turbine. When water is flowing from head race to the turbine, a loss of head due to friction between the water and penstocks occurs. Though there are other losses also such as loss due to bend, pipe fittings, loss at the entrance of penstock etc., yet they are having small magnitude as compared to head loss due to friction. If 'h_f'is the head loss due to friction between penstocks and water then net heat on turbine is given by

$$H = H_g - h_f \qquad ...(18.1)$$

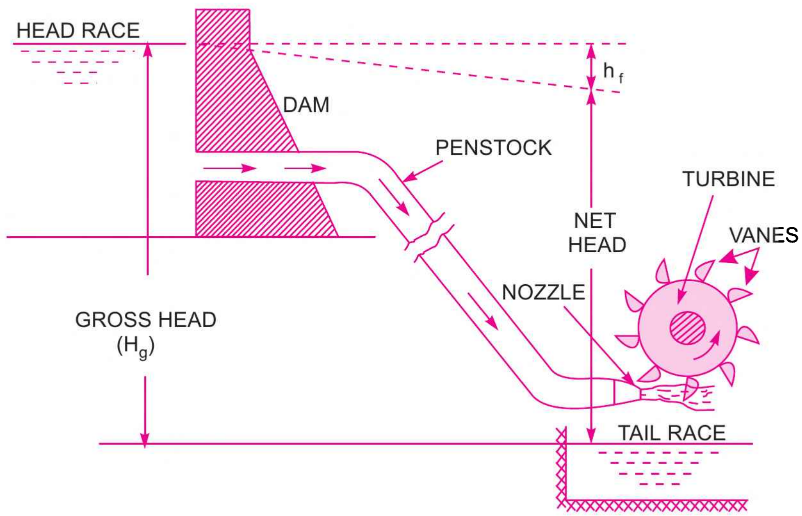

Fig. 18.1 *Layout of a hydroelectric power plant.*

where H_g = Gross head, $h_f = \dfrac{4 \times f \times L \times V^2}{D \times 2g}$,

in which V = Velocity of flow in penstock,

L = Length of penstock,

D = Diameter of penstock.

3. Efficiencies of a Turbine. The following are the important efficiencies of a turbine.

(*a*) Hydraulic Efficiency, η_h (*b*) Mechanical Efficiency, η_m

(*c*) Volumetric Efficiency, η_v and (*d*) Overall Efficiency, η_o

(*a*) **Hydraulic Efficiency (η_h).** It is defined as the ratio of power given by water to the runner of a turbine (runner is a rotating part of a turbine and on the runner vanes are fixed) to the power supplied by the water at the inlet of the turbine. The power at the inlet of the turbine is more and this power goes on decreasing as the water flows over the vanes of the turbine due to hydraulic losses as the vanes are not smooth. Hence, the power delivered to the runner of the turbine will be less than the power available at the inlet of the turbine. Thus, mathematically, the hydraulic efficiency of a turbine is written as

$$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}} = \frac{\text{R.P.}}{\text{W.P.}} \qquad ...(18.2)$$

where R.P. = Power delivered to runner *i.e.*, runner power

$$= \frac{W}{g} \frac{\left[V_{w_1} \pm V_{w_2}\right] \times u}{1000} \text{ kW} \quad \text{...for Pelton Turbine}$$

$$= \frac{W}{g} \frac{\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{1000} \text{ kW} \quad \text{...for a radial flow turbine}$$

W.P. = Power supplied at inlet of turbine and also called water power

$$= \frac{W \times H}{1000} \text{ kW} \quad \text{...(18.3)}$$

where W = Weight of water striking the vanes of the turbine per second

= $\rho g \times Q$ in which Q = Volume of water/s,

V_{w_1} = Velocity of whirl at inlet,

V_{w_2} = Velocity of whirl at outlet,

u = Tangential velocity of vane,

u_1 = Tangential velocity of vane at inlet for radial vane,

u_2 = Tangential velocity of vane at outlet for radial vane,

H = Net head on the turbine.

Power supplied at the inlet of turbine in S.I.units is known as water power. It is given by

$$\text{W.P.} = \frac{\rho \times g \times Q \times H}{1000} \text{ kW} \quad \text{...(18.3}A\text{)}$$

For water $\rho = 1000 \text{ kg/m}^3$

$$\therefore \quad \text{W.P.} = \frac{1000 \times g \times Q \times H}{1000} = g \times Q \times H \text{ kW} \quad \text{...(18.3}B\text{)}$$

The relation (18.3*B*) is only used when the flowing fluid is water. If the flowing fluid is other than the water, then relation (18.3*A*) is used.

(*b*) **Mechanical Efficiency (η_m).** The power delivered by water to the runner of a turbine is transmitted to the shaft of the turbine. Due to mechanical losses, the power available at the shaft of the turbine is less than the power delivered to the runner of a turbine. The ratio of the power available at the shaft of the turbine (known as S.P. or B.P.) to the power delivered to the runner is defined as mechanical efficiency. Hence, mathematically, it is written as

$$\eta_m = \frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by water to the runner}} = \frac{\text{S.P.}}{\text{R.P.}} \quad \text{...(18.4)}$$

(*c*) **Volumetric Efficiency (η_v).** The volume of the water striking the runner of a turbine is slightly less than the vloume of the water supplied to the turbine. Some of the volume of the water is discharged to the tail race without striking the runner of the turbine. Thus the ratio of the volume of the water actually striking the runner to the volume of water supplied to the turbine is defined as volumetric efficiency. It is written as

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine}} \quad \text{...(18.5)}$$

(*d*) **Overall Efficiency (η_o).** It is defined as the ratio of power available at the shaft of the turbine to the power supplied by the water at the inlet of the turbine. It is written as :

$$\eta_o = \frac{\text{Volume available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}} = \frac{\text{Shaft power}}{\text{Water power}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}} \times \frac{\text{R.P.}}{\text{R.P.}} \qquad \text{(where R.P. = Power delivered to runner)}$$

$$= \frac{\text{S.P.}}{\text{R.P.}} \times \frac{\text{R.P.}}{\text{W.P.}}$$

$$= \eta_m \times \eta_h \qquad \left(\begin{array}{l} \because \text{From equation (18.4), } \dfrac{\text{S.P.}}{\text{R.P.}} = \eta_m \\ \text{and from equation (18.2), } \dfrac{\text{R.P.}}{\text{W.P.}} = \eta_h \end{array} \right) \qquad \text{...(18.6)}$$

If shaft power (S.P.) is taken in kW then water power should also be taken in kW. Shaft power is commonly represented by P. But from equation (18.3*A*),

$$\text{Water power in kW} = \frac{\rho \times g \times Q \times H}{1000}, \text{ where } \rho = 1000 \text{ kg/m}^3$$

$$\therefore \qquad \eta_o = \frac{\text{Shaft power in kW}}{\text{Water power in kW}} = \frac{P}{\left(\dfrac{\rho \times g \times Q \times H}{1000}\right)} \qquad \text{...(18.6A)}$$

where P = Shaft power.

▶ 18.5 CLASSIFICATION OF HYDRAULIC TURBINES

The hydraulic turbines are classified according to the type of energy available at the inlet of the turbine, direction of flow through the vanes, head at the inlet of the turbine and specific speed of the turbines. Thus the following are the important classifications of the turbines :

1. According to the type of energy at inlet :
   (*a*) Impulse turbine, and (*b*) Reaction turbine.
2. According to the direction of flow through runner :
   (*a*) Tangential flow turbine, (*b*) Radial flow turbine,
   (*c*) Axial flow turbine, and (*d*) Mixed flow turbine.
3. According to the head at the inlet of turbine :
   (*a*) High head turbine, (*b*) Medium head turbine, and
   (*c*) Low head turbine.
4. According to the specific speed of the turbine :
   (*a*) Low specific speed turbine, (*b*) Medium specific speed turbine, and
   (*c*) High specific speed turbine.

If at the inlet of the turbine, the energy available is only kinetic energy, the turbine is known as **impulse turbine.** As the water flows over the vanes, the pressure is atmospheric from inlet to outlet of the turbine. If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as **reaction turbine**. As the waters flows through the runner, the water is under pressure and the pressure energy goes on changing into kinetic energy. The runner is completely enclosed in an air-tight casing and the runner and casing is completely full of water.

If the water flows along the tangent of the runner, the turbine is known as **tangential flow turbine.** If the water flows in the radial direction through the runner, the turbine is called **radial flow turbine**. If the water flows from outwards to inwards, radially, the turbine is known as **inward** radial flow turbine, on the other hand, if water flows radially from inwards to outwards, the turbine is known as **outward** radial flow turbine. If the water flows through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called **axial flow** turbine. If the water flows through the runner in the radial direction but leaves in the direction parallel to axis of rotation of the runner, the turbine is called **mixed flow** turbine.

▶ 18.6 PELTON WHEEL (OR TURBINE)

The Pelton wheel or Pelton turbine is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and outlet of the turbine is atmospheric. This turbine is used for high heads and is named after L.A. Pelton, an American Engineer.

Fig. 18.1 shows the layout of a hydroelectric power plant in which the turbine is Pelton wheel . The water from the reservoir flows through the penstocks at the outlet of which a nozzle is fitted. The nozzle increases the kinetic energy of the water flowing through the penstock. At the outlet of the nozzle, the water comes out in the form of a jet and strikes the buckets (vanes) of the runner. The main parts of the Pelton turbine are :

1. Nozzle and flow regulating arrangement (spear),
2. Runner and buckets,
3. Casing, and
4. Breaking jet.

1. Nozzle and Flow Regulating Arrangement. The amount of water striking the buckets (vanes) of the runner is controlled by providing a spear in the nozzle as shown in Fig. 18.2. The spear is a conical needle which is operated either by a hand wheel or automatically in an axial direction depending upon the size of the unit. When the spear is pushed forward into the nozzle the amount of water striking the runner is reduced. On the other hand, if the spear is pushed back, the amount of water striking the runner increases.

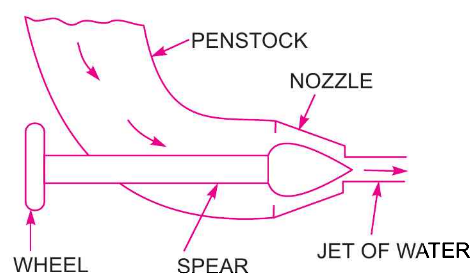

Fig. 18.2 *Nozzle with a spear to regulate flow.*

2. Runner with Buckets. Fig. 18.3 shows the runner of a Pelton wheel. It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the buckets is of a double hemispherical cup or bowl. Each bucket is divided into two symmetrical parts by a dividing wall which is known as splitter.

The jet of water strikes on the splitter. The splitter divides the jet into two equal parts and the jet comes out at the outer edge of the bucket. The buckets are shaped in such a way that the jet gets deflected through 160° or 170°. The buckets are made of cast iron, cast steel bronze or stainless steel depending upon the head at the inlet of the turbine.

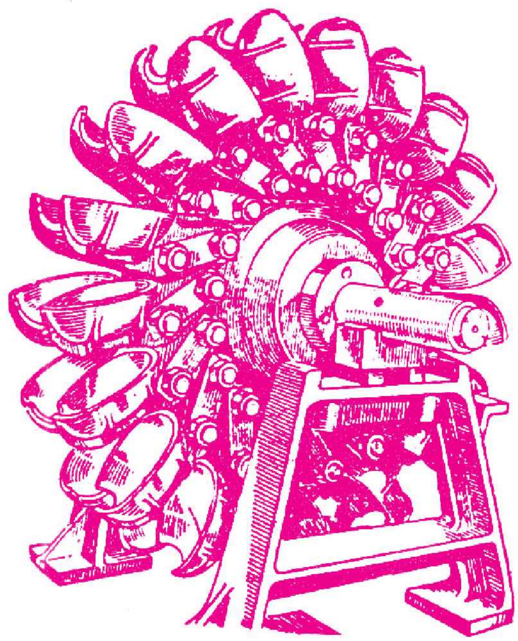

Fig. 18.3 *Runner of a pelton wheel.*

3. Casing. Fig. 18.4 shows a Pelton turbine with a casing. The function of the casing is to prevent the splashing of the water and to discharge water to tail race. It also acts as safeguard against accidents. It is made of cast iron or fabricated steel plates. The casing of the Pelton wheel does not perform any hydraulic function.

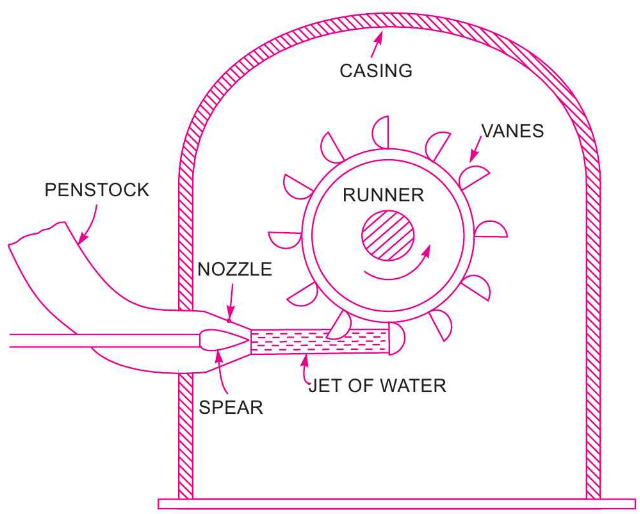

Fig. 18.4 *Pelton turbine.*

4. Breaking Jet. When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on revolving for a long time. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of the vanes. This jet of water is called breaking jet.

18.6.1 Velocity Triangles and Work done for Pelton Wheel.

Fig. 18.5 shows the shape of the vanes or buckets of the Pelton wheel. The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts. These parts of the jet, glides over the inner surfaces and comes out at the outer edge. Fig. 18.5 (*b*) shows the section of the bucket at *Z-Z*. The splitter is the inlet tip and outer edge of the bucket is the outlet tip of the bucket. The inlet velocity triangle is drawn at the splitter and outlet velocity triangle is drawn at the outer edge of the bucket, by the same method as explained in Chapter 17.

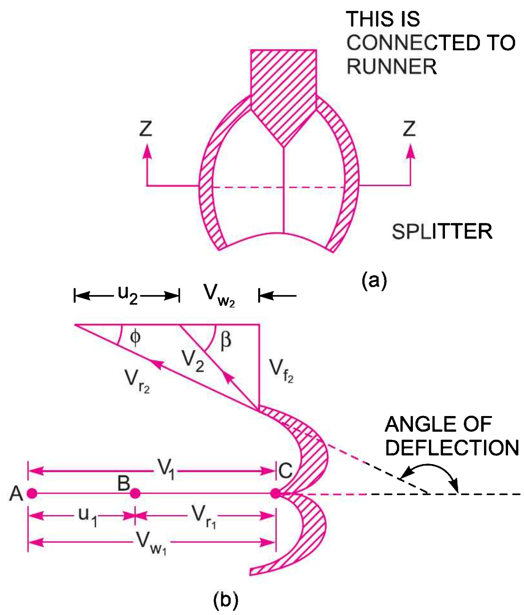

Fig. 18.5 *Shape of bucket.*

Let H = Net head acting on the Pelton wheel

$= H_g - h_f$

where H_g = Gross head and $h_f = \dfrac{4fLV^2}{D^* \times 2g}$

where D^* = Dia. of Penstock, N = Speed of the wheel in r.p.m.,

D = Diameter of the wheel, d = Diameter of the jet.

Then V_1 = Velocity of jet at inlet = $\sqrt{2gH}$ ...(18.7)

$$u = u_1 = u_2 = \frac{\pi DN}{60}.$$

The velocity triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1$$

$$\alpha = 0^\circ \text{ and } \theta = 0^\circ$$

From the velocity triangle at outlet, we have

$$V_{r_2} = V_{r_1} \text{ and } V_{w_2} = V_{r_2} \cos\phi - u_2.$$

The force exerted by the jet of water in the direction of motion is given by equation (17.19) as

$$F_x = \rho a V_1 \, [V_{w_1} + V_{w_2}] \quad \text{...(18.8)}$$

As the angle β is an acute angle, +ve sign should be taken. Also this is the case of series of vanes, the mass of water striking is $\rho a V_1$ and not $\rho a V_{r_1}$. In equation (18.8), 'a' is the area of the jet which is given as

$$a = \text{Area of jet} = \frac{\pi}{4} d^2.$$

Now work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 \, [V_{w_1} + V_{w_2}] \times u \text{ Nm/s} \quad \text{...(18.9)}$$

Power given to the runner by the jet

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{1000} \text{ kW} \quad \text{...(18.10)}$$

Work done/s per unit weight of water striking/s

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\text{Weight of water striking/s}}$$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\rho a V_1 \times g} = \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u \quad \text{...(18.11)}$$

The energy supplied to the jet at inlet is in the form of kinetic energy and is equal to $\frac{1}{2} m V^2$

$\therefore$ K.E. of jet per second $= \frac{1}{2} (\rho a V_1) \times V_1^2$

$\therefore$ **Hydraulic efficiency**, $\eta_h = \frac{\text{Work done per second}}{\text{K.E. of jet per second}}$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\frac{1}{2} (\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} \quad \text{...(18.12)}$$

Now $V_{w_1} = V_1,\ V_{r_1} = V_1 - u_1 = (V_1 - u)$

$\therefore$ $V_{r_2} = (V_1 - u)$

and $V_{w_2} = V_{r_2} \cos\phi - u_2 = V_{r_2} \cos\phi - u = (V_1 - u) \cos\phi - u$

Substituting the values of V_{w_1} and V_{w_2} in equation (18.12),

$$\eta_h = \frac{2\left[V_1 + (V_1 - u)\cos\phi - u\right] \times u}{V_1^2}$$

$$= \frac{2\left[V_1 - u + (V_1 - u)\cos\phi\right] \times u}{V_1^2} = \frac{2(V_1 - u)\,[1 + \cos\phi]\, u}{V_1^2}. \quad \text{...(18.13)}$$

The efficiency will be maximum for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \qquad \text{or} \qquad \frac{d}{du}\left[\frac{2u(V_1 - u)(1 + \cos\phi)}{V_1^2}\right] = 0$$

$$\text{or} \qquad \frac{(1+\cos\phi)}{V_1^2}\frac{d}{du}(2uV_1 - 2u^2) = 0 \qquad \text{or} \qquad \frac{d}{du}[2uV_1 - 2u^2] = 0 \qquad \left(\because \frac{1+\cos\phi}{V_1^2} \neq 0\right)$$

$$\text{or} \qquad 2V_1 - 4u = 0 \qquad \text{or} \qquad u = \frac{V_1}{2} \qquad \text{...(18.14)}$$

Equation (18.14) states that hydraulic efficiency of a Pelton wheel will be maximum when the velocity of the wheel is half the velocity of the jet of water at inlet. The expression for maximum efficiency will be obtained by substituting the value of $u = \frac{V_1}{2}$ in equation (18.13).

$$\therefore \qquad \text{Max. } \eta_h = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1+\cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2 \times \frac{V_1}{2}(1+\cos\phi)\frac{V_1}{2}}{V_1^2} = \frac{(1+\cos\phi)}{2}. \qquad \text{...(18.15)}$$

18.6.2 Points to be Remembered for Pelton Wheel

(*i*) The velocity of the jet at inlet is given by $V_1 = C_v\sqrt{2gH}$

where C_v = Co-efficient of velocity = 0.98 or 0.99

H = Net head on turbine

(*ii*) The velocity of wheel (u) is given by $u = \phi\sqrt{2gH}$

where ϕ = Speed ratio. The value of speed ratio varies from 0.43 to 0.48.

(*iii*) The angle of deflection of the jet through buckets is taken at 165° if no angle of deflection is given.

(*iv*) The mean diameter or the pitch diameter D of the Pelton wheel is given by

$$u = \frac{\pi DN}{60} \text{ or } D = \frac{60u}{\pi N}.$$

(*v*) **Jet Ratio.** It is defined as the ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d). It is denoted by 'm' and is given as

$$m = \frac{D}{d} \text{ (= 12 for most cases)} \qquad \text{...(18.16)}$$

(*vi*) Number of buckets on a runner is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5\text{ m} \qquad \text{...(18.17)}$$

where m = Jet ratio

(*vii*) **Number of Jets.** It is obtained by dividing the total rate of flow through the turbine by the rate of flow of water through a single jet.

Problem 18.1 *A Pelton wheel has a mean bucket speed of 10 metres per second with a jet of water flowing at the rate of 700 litres/s under a head of 30 metres. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.*

Solution. Given :

Speed of bucket, $u = u_1 = u_2 = 10$ m/s

Discharge, $Q = 700$ litres/s $= 0.7$ m^3/s, Head of water, $H = 30$ m

Angle of deflection $= 160°$

∴ Angle, $\phi = 180° - 160° = 20°$

Co-efficient of velocity, $C_v = 0.98$.

The velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77$ m/s

∴ $V_{r_1} = V_1 - u_1 = 23.77 - 10$

$= 13.77$ m/s

$V_{w_1} = V_1 = 23.77$ m/s

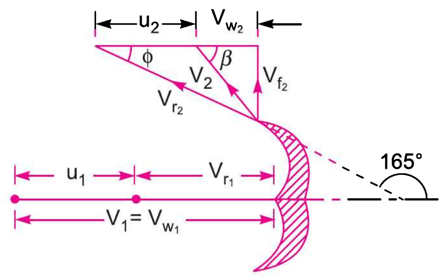

Fig. 18.6

From outlet velocity triangle,

$V_{r_2} = V_{r_1} = 13.77$ m/s

$V_{w_2} = V_{r_2}\cos\phi - u_2$

$= 13.77 \cos 20° - 10.0 = 2.94$ m/s

Work done by the jet per second on the runner is given by equation (18.9) as

$= \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u$

$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10$ (∵ $aV_1 = Q = 0.7$ m^3/s)

$= 186970$ Nm/s

∴ Power given to turbine $= \dfrac{186970}{1000} =$ **186.97 kW. Ans.**

The hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77}$$

$=$ **0.9454** or **94.54%. Ans.**

Problem 18.2 *A Pelton wheel is to be designed for the following specifications : Shaft power = 11,772 kW ; Head = 380 metres ; Speed = 750 r.p.m. ; Overall efficiency = 86% ; Jet diameter is not to exceed one-sixth of the wheel diameter. Determine :*

(i) The wheel diameter, *(ii) The number of jets required, and*

(iii) Diameter of the jet.

Take $K_{v_1} = 0.985$ *and* $K_{u_1} = 0.45$

Solution. Given :

Shaft power, S.P. $= 11,772$ kW

Head, $H = 380$ m

Speed, $N = 750$ r.p.m.

Overall efficiency, $\eta_0 = 86\%$ or 0.86

Ratio of jet dia. to wheel dia. $= \frac{d}{D} = \frac{1}{6}$

Co-efficient of velocity, $K_{v_1} = C_v = 0.985$

Speed ratio, $K_{u_1} = 0.45$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 380} = 85.05$ m/s

The velocity of wheel, $u = u_1 = u_2$

$$= \text{Speed ratio} \times \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 380} = 38.85 \text{ m/s}$$

But $u = \frac{\pi DN}{60} \quad \therefore \quad 38.85 = \frac{\pi DN}{60}$

or $D = \frac{60 \times 38.85}{\pi \times N} = \frac{60 \times 38.85}{\pi \times 750} =$ **0.989 m. Ans.**

But $\frac{d}{D} = \frac{1}{6}$

$\therefore$ Dia. of jet, $d = \frac{1}{6} \times D = \frac{0.989}{6} =$ **0.165 m. Ans.**

Discharge of one jet, q = Area of jet × Velocity of jet

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4}(.165) \times 85.05 \text{ m}^3/\text{s} = 1.818 \text{ m}^3/\text{s} \quad \ldots(i)$$

Now $\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{11772}{\frac{\rho g \times Q \times H}{1000}}$

$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}$, where Q = Total discharge

$\therefore$ Total discharge, $Q = \frac{11772 \times 1000}{1000 \times 9.81 \times 380 \times 0.86} = 3.672 \text{ m}^3/\text{s}$

$\therefore$ Number of jets $= \frac{\text{Total discharge}}{\text{Discharge of one jet}} = \frac{Q}{q} = \frac{3.672}{1.818} =$ **2 jets. Ans.**

Problem 18.3 *The penstock supplies water from a reservoir to the Pelton wheel with a gross head of 500 m. One third of the gross head is lost in friction in the penstock. The rate of flow of water through the nozzle fitted at the end of the penstock is 2.0 m^3/s. The angle of deflection of the jet is 165°. Determine the power given by the water to the runner and also hydraulic efficiency of the Pelton wheel. Take speed ratio = 0.45 and C_v = 1.0.*

Solution. Given :

Gross head, $H_g = 500$ m

Head lost in friction, $h_f = \frac{H_g}{3} = \frac{500}{3} = 166.7$ m

∴ Net head, $H = H_g - h_f = 500 - 166.7 = 333.30$ m

Discharge, $Q = 2.0$ m^3/s

Angle of deflection $= 165°$

∴ Angle, $\phi = 180° - 165° = 15°$

Speed ratio $= 0.45$

Co-efficient of velocity, $C_v = 1.0$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 1.0 \times \sqrt{2 \times 9.81 \times 333.3} = 80.86$ m/s

Velocity of wheel, $u = \text{Speed ratio} \times \sqrt{2gH}$

or $u = u_1 = u_2 = 0.45 \times \sqrt{2 \times 9.81 \times 333.3} = 36.387$ m/s

∴ $V_{r_1} = V_1 - u_1 = 80.86 - 36.387$

$= 44.473$ m/s

Also $V_{w_1} = V_1 = 80.86$ m/s

From outlet velocity triangle, we have

$$V_{r_2} = V_{r_1} = 44.473$$

$$V_{r_2} \cos \phi = u_2 + V_{w_2}$$

or $44.473 \cos 15° = 36.387 + V_{w_2}$

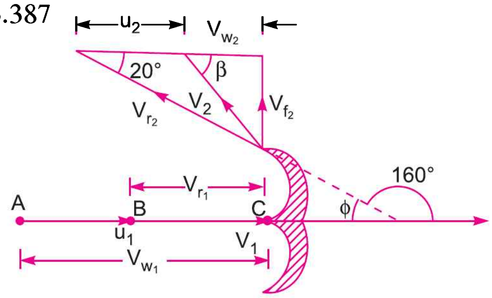

Fig. 18.7

or $V_{w_2} = 44.473 \cos 15° - 36.387 = 6.57$ m/s.

Work done by the jet on the runner per second is given by equation (18.9) as

$$\rho a V_1 [V_{w_1} + V_{w_2}] \times u = \rho Q [V_{w_1} + V_{w_2}] \times u \qquad (\because \quad aV_1 = Q)$$

$$= 1000 \times 2.0 \times [80.86 + 6.57] \times 36.387 = 6362630 \text{ Nm/s}$$

∴ Power given by the water to the runner in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{6362630}{1000} = \textbf{6362.63 kW. Ans.}$$

Hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[80.86 + 6.57] \times 36.387}{80.86 \times 80.86}$$

$$= \textbf{0.9731} \text{ or } \textbf{97.31\%. Ans.}$$

Problem 18.4 *A Pelton wheel is having a mean bucket diameter of 1 m and is running at 1000 r.p.m. The net head on the Pelton wheel is 700 m. If the side clearance angle is 15° and discharge through nozzle is 0.1 m^3/s, find :*

(i) Power available at the nozzle, and (ii) Hydraulic efficiency of the turbine.

Solution. Given :

Diameter of wheel, $D = 1.0$ m

Speed of wheel, $N = 1000$ r.p.m.

∴ Tangential velocity of the wheel, $u = \dfrac{\pi DN}{60} = \dfrac{\pi \times 1.0 \times 1000}{60} = 52.36$ m/s

Net head on turbine $H = 700$ m

Side clearance angle, $\phi = 15°$

Discharge, $Q = 0.1$ m^3/s

Velocity of jet at inlet, $V_1 = C_v\sqrt{2gH} = 1 \times \sqrt{2 \times 9.81 \times 700}$

($\because$ Value of C_v is not given. Take it $= 1.0$)

or $V_1 = 117.19$ m/s

(*i*) Power available at the nozzle is given by equation (18.3) as

$$\text{W.P.} = \frac{W \times H}{1000} = \frac{\rho \times g \times Q \times H}{1000}$$

$$= \frac{1000 \times 9.81 \times 0.1 \times 700}{1000} = \textbf{686.7 kW. Ans.}$$

(*ii*) Hydraulic efficiency is given by equation (18.13) as

$$\eta_h = \frac{2(V_1 - u)(1 + \cos\phi)\, u}{V_1^2}$$

$$= \frac{2(117.19 - 52.36)(1 + \cos 15°) \times 52.36}{117.19 \times 117.19}$$

$$= \frac{2 \times 64.83 \times 1.966 \times 52.36}{117.19 \times 117.19} = 0.9718 = \textbf{97.18 \%. Ans.}$$

Problem 18.5 *A Pelton wheel is working under a gross head of 400 m. The water is supplied through penstock of diameter 1 m and length 4 km from reservoir to the Pelton wheel. The co-efficient of friction for the penstock is given as .008. The jet of water of diameter 150 mm strikes the buckets of the wheel and gets deflected through an angle of 165°. The relative velocity of water at outlet is reduced by 15% due to friction between inside surface of the bucket and water. If the velocity of the buckets is 0.45 times the jet velocity at inlet and mechanical efficiency as 85% determine :*

(i) Power given to the runner, *(ii) Shaft power,*

(iii) Hydraulic efficiency and overall efficiency.

Solution. Given :

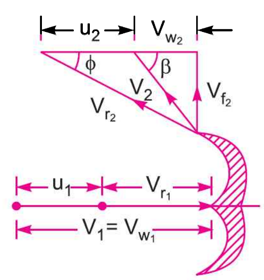

Fig. 18.8

Gross head, $H_g = 400$ m

Diameter of penstock, $D = 1.0$ m

Length of penstock, $L = 4$ km $= 4 \times 1000 = 4000$ m

Co-efficient of friction, $f = .008$

Diameter of jet, $d = 150$ mm $= 0.15$ m

Angle of deflection $= 165°$

$\therefore$ Angle, $\phi = 180° - 165° = 15°$

Relative velocity at outlet, $V_{r_2} = 0.85\, V_{r_1}$

Velocity of bucket, $u = 0.45 \times$ Jet velocity

Mechanical efficiency, $\eta_m = 85\% = 0.85$

Let V^* = Velocity of water in penstock, and

V_1 = Velocity of jet of water.

Using continuity equation, we have

Area of penstock × V^* = Area of jet × V_1

or
$$\frac{\pi}{4} D^2 \times V^* = \frac{\pi}{4} d^2 \times V_1$$

∴
$$V^* = \frac{d^2}{D^2} \times V_1 = \frac{0.15^2}{1.0^2} \times V_1 = .0225\ V_1 \quad ...(i)$$

Applying Bernoulli's equation to the free surface of water in the reservoir and outlet of the nozzle, we get

$$H_g = \text{Head lost due to friction} + \frac{V_1^2}{2g}$$

or
$$400 = \frac{4 fLV^{*2}}{D \times 2g} + \frac{V_1^2}{2g} = \frac{4 \times .008 \times 4000 \times V^{*2}}{1.0 \times 2 \times 9.81} + \frac{V_1^2}{2g}$$

Substituting the value of V^* from equation (*i*), we get

$$400 = \frac{4 \times .008 \times 4000}{2 \times 9.81} \times (0.0225\ V_1)^2 + \frac{V_1^2}{2g}$$

$$= .0033\ V_1^2 + .051\ V_1^2 \text{ or } 400 = .0543\ V_1^2$$

∴
$$V_1 = \sqrt{\frac{400}{.0543}} = 85.83 \text{ m/s.}$$

Now velocity of bucket, $u_1 = 0.45\ V_1 = 0.45 \times 85.83 = 38.62$ m/s

From inlet velocity triangle, $V_{r_1} = V_1 - u_1 = 85.83 - 38.62 = 47.21$ m/s

$$V_{w_1} = V_1 = 85.83 \text{ m/s}$$

From outlet velocity triangle, $V_{r_2} = 0.85 \times V_{r_1} = 0.85 \times 47.21 = 40.13$ m/s

$$V_{w_2} = V_{r_2} \cos \phi - u_2 = 40.13 \cos 15° - 38.62$$

$$= 0.143 \text{ m/s} \qquad (\because\ u = u_1 = u_2 = 38.62)$$

Discharge through nozzle is given as

$$Q = \text{Area of jet} \times \text{Velocity of jet} = a \times V_1$$

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} (.15)^2 \times 85.83 = 1.516 \text{ m}^3\text{/s}$$

Work done on the wheel per second is given by equation (18.9) as

$$= \rho a V_1\ [V_{w_1} + V_{w_2}] \times u = \rho Q \left[V_{w_1} + V_{w_2}\right] \times u$$

$$= 1000 \times 1.516\ [85.83 + .143] \times 38.62 = 5033540 \text{ Nm/s}$$

(*i*) Power given to the runner in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{5033540}{1000} = \mathbf{5033.54\ kW.\ Ans.}$$

(*ii*) Using equation (18.4) for mechanical efficiency,

$$\eta_m = \frac{\text{Power at the shaft}}{\text{Power given to the runner}} = \frac{\text{S.P.}}{5033.54}$$

$\therefore$ S.P. = $\eta_m \times 5033.54 = 0.85 \times 5033.54$ = **4278.5 kW. Ans.**

(*iii*) Hydraulic efficiency is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2}$$

$$= \frac{2\,[85.83 + .143] \times 38.62}{85.83 \times 85.83} = 0.9014 = \mathbf{90.14\%.\ Ans.}$$

Overall efficiency is given by equation (18.6) as

$$\eta_0 = \eta_m \times \eta_h = 0.85 \times .9014 = 0.7662 \text{ or } \mathbf{76.62\%.\ Ans.}$$

Problem 18.6 *A Pelton wheel nozzle, for which C_v = 0.97, is 400 m below the water surface of a lake. The jet diameter is 80 mm, the pipe diameter is 0.6 m, its length is 4 km and f = 0.032 in the formula $h_f = \frac{fLV^2}{2g \times D}$. The buckets, deflect the jet through 165° and they run at 0.48 times the jet speed, bucket friction reducing the relative velocity at outlet by 15% of the relative velocity at inlet. Mechanical efficiency = 90%. Find the flow rate and the shaft power developed by the turbine.*

Solution. Given :

$C_v = 0.97$

Gross head, $H_g = 400$ m

Dia. of jet, $d = 800 \text{ mm} = \frac{80}{1000}$ m $= .08$ m

Dia. of pipe, $D = 0.6$ m

Length of pipe $L = 4$ km $= 4000$ m

$f = .032$

Angle, $\phi = 180° - 165° = 15°$

Bucket speed, $u = 0.48$ times jet speed

Relative velocity at outlet = 0.85 times relative velocity at inlet

or $V_{r_2} = 0.85\ V_{r_1}$

Mechanical efficiency, $\eta_m = 0.90$.

Find. (*i*) Flow rate, and (*ii*) Shaft power, S.P.

Let V = Velocity of water in pipe, and

V_1 = Velocity of jet of water.

From continuity equation, we have

Area of pipe × V = Area of jet × V_1

or $$\frac{\pi}{4} D^2 \times V = \frac{\pi}{4} d^2 \times V_1$$

$$= \frac{d^2}{D^2} \times V_1 = \left(\frac{.08}{0.60}\right)^2 \times V_1 = 0.0177\ V_1 \qquad \text{...}(i)$$

Applying Bernoulli's equation to the free surface of water in the reservoir and the outlet of the nozzle, we get

Head at reservoir = Kinetic head of jet of water + Head lost due to friction in pipe + Head lost in nozzle

$$= \frac{V_1^2}{2g} + \frac{fLV^2}{D \times 2g} + \text{Head lost in nozzle} \qquad ...(ii)$$

Let V* = Theoretical velocity at the outlet of nozzle, V_1 = Actual velocity of jet of water

Then $$\frac{V_1}{V^*} = C_v \text{ or } V^* = \frac{V_1}{C_v}.$$

Now head lost in nozzle = Head corresponding to V^* – Head corresponding to V_1

$$= \frac{V^{*2}}{2g} - \frac{V_1^2}{2g} = \left(\frac{V_1}{C_v}\right)^2 \times \frac{1}{2g} - \frac{V_1^2}{2g} = \frac{V_1^2}{2g}\left(\frac{1}{C_v^2} - 1\right)$$

Substituting this value in equation (*ii*), we get

Head at reservoir $$= \frac{V_1^2}{2g} + \frac{fLV^2}{2g \times D} + \frac{V_1^2}{2g}\left(\frac{1}{C_v^2} - 1\right)$$

or $$400 = \frac{V_1^2}{2g} + \frac{0.032 \times 4000 \times V^2}{0.6 \times 2 \times 9.81} + \frac{V_1^2}{2g} \times \frac{1}{C_v^2} - \frac{V_1^2}{2g}$$

$$= \frac{0.032 \times 4000 \times (.0177V_1)^2}{0.6 \times 2 \times 9.81} + \frac{V_1^2}{2 \times 9.81} \times \frac{1}{.97^2}$$

$$= .0034\ V_1^2 + .054\ V_1^2 \qquad (\because \quad V = .0177\ V_1)$$

$$= 0.0574\ V_1^2$$

$$\therefore \qquad V_1 = \sqrt{\frac{400}{.0574}} = 83.47 \text{ m/s}$$

Now velocity of bucket, $u_1 = 0.48 \times V_1 = 0.48 \times 83.47 = 40.06$ m/s

Refer to Fig. 18.8 (*a*), we have from inlet velocity triangle

$$V_{r_1} = V_1 - u_1 = 83.47 - 40.06 = 43.41 \text{ m/s}$$

$$V_{w_1} = V_1 = 83.47$$

From outlet velocity triangle,

$$V_{r_2} = 0.85\ V_{r_1} = 0.85 \times 43.41 = 36.898 \text{ m/s}$$

$$V_{w_2} = u_2 - V_{r_2} \cos \phi$$

$$= 40.06 - 36.898 \times \cos 15°$$

$$(\because \quad u_1 = u_2 = 40.06)$$

$$= 4.42$$

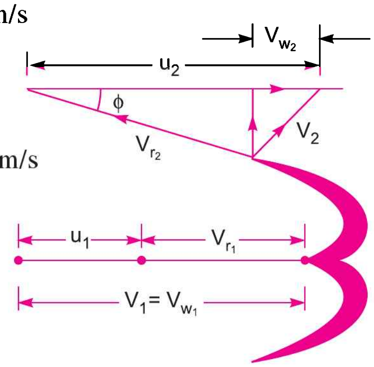

Fig. 18.8 (*a*)

Flow rate, Q = Area of jet × Velocity of jet

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} (.08)^2 \times 83.47 = \mathbf{0.419\ m^3/s.\ Ans.}$$

Now using equation (18.4), $\eta_m = \dfrac{\text{S.P.}}{\text{Power given to the runner}}$

$\therefore$ S.P. $= \eta_m \times$ Power given to the runner

where power given to the runner in kW

$$= \frac{\text{Work done per second}}{1000}$$

Work done per second $= \dfrac{W}{g}\left[V_{w_1} - V_{w_2}\right] \times u$

(Here –ve sign is taken as V_{w_1} and V_{w_2} are in the same direction)

$$= \frac{\rho \times g \times Q}{g} [V_{w_1} - V_{w_2}] \times u_1 \qquad \{\because \quad u = u_1\}$$

$$= \frac{1000 \times 9.81 \times .419}{9.81} [83.47 - 4.42] \times 40.06 = 1326865 \text{ Nm/s}$$

$\therefore$ Power given to the runner $= \dfrac{1326865}{1000} = 1326.865$ kW

$\therefore$ S.P. $= \eta_m \times$ Power given to runner

$= 0.90 \times 1326.865 =$ **1194.18 kW. Ans.**

Problem 18.7 *A 137 mm diameter jet of water issuing from a nozzle impinges on the buckets of a Pelton wheel and the jet is deflected through an angle of 165° by the buckets. The head available at the nozzle is 400 m. Assuming co-efficient of velocity as 0.97, speed ratio as 0.46, and reduction in relative velocity while passing through buckets as 15%, find :*

(i) The force exerted by the jet on buckets in tangential direction,

(ii) The power developed.

Solution. Given :

Dia. of jet, $d = 137$ mm $= 0.137$ m

$\therefore$ Area of jet, $a = \dfrac{\pi}{4} \times 0.137 = 0.01474\ \text{m}^2$

Angle of deflection $= 165°$

$\therefore$ Angle, $\phi = 180° - 165° = 15°$

Head of water, $H = 400$ m

Co-efficient of velocity, $C_v = 0.97$

Speed ratio $= 0.46$

Relative velocity at outlet $= 0.85 \times$ relative velocity at inlet

or $V_{r_2} = 0.85\ V_{r_1}$

Now velocity of jet, $V_1 = C_v \sqrt{2gH} = 0.97 \sqrt{2 \times 9.81 \times 400} = 85.93$ m/s

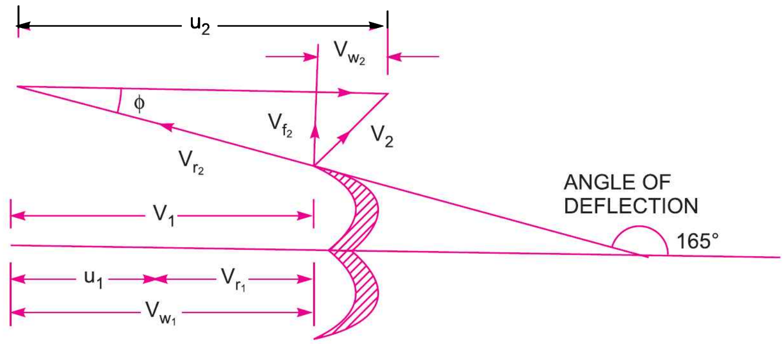

Fig. 18.8 (*b*)

Speed ratio $= \frac{u_1}{\sqrt{2gH}}$ or $0.46 = \frac{u_1}{\sqrt{2 \times 9.81 \times 400}}$

$\therefore$ $u_1 = 0.46 \times \sqrt{2 \times 9.81 \times 400} = 40.75$ m/s

Hence $V_{r_1} = V_1 - u_1 = 85.93 - 40.75 = 45.18$ m/s

and $V_{r_2} = 0.85\, V_{r_1} = 0.85 \times 45.18 = 38.40$ m/s

For Pelton turbine, $u_1 = u_2 = u = 40.75$ m/s

$$V_{r_2} \cos \phi = 38.40 \times \cos 15° = 37.092$$

Here $V_{r_2} \cos \phi$ is less than u_2. Hence velocity triangle at outlet will be as shown in Fig. 18.8 (*b*)

$\therefore$ $V_{w_2} = u_2 - V_{r_2} \cos \phi = 40.75 - 37.092 = 3.658$ m/s.

(*i*) Force exerted by jet on buckets in tangential direction is given by,

$$F_x = \rho a V_1 [V_{w_1} - V_{w_2}]$$

(Here –ve sign is taken as V_{w_1} and V_{w_2} are in the same direction)

$\therefore$ $F_x = 1000 \times 0.01474 \times 85.93\ (85.93 - 3.658)$ N = **104206 N. Ans.**

(*ii*) Power developed is given by,

$$\text{Power} = \frac{F_x \times u}{1000} \text{ kW} = \frac{104206 \times 40.75}{1000} = \textbf{4246.4 kW. Ans.}$$

Problem 18.8 *Two jets strike the buckets of a Pelton wheel, which is having shaft power as 15450 kW. The diameter of each jet is given as 200 mm. If the net head on the turbine is 400 m, find the overall efficiency of the turbine. Take $C_v = 1.0$.*

Solution. Given :

Number of jets = 2

Shaft power, S.P. = 15450 kW

Diameter of each jet, $d = 200$ mm $= 0.20$ m

$\therefore$ Area of each jet, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.2)^2 = 0.031416 \text{ m}^2$

Net head, $H = 400$ m

Co-efficient of velocity, $C_v = 1.0$

Velocity of each jet, $V_1 = C_v\sqrt{2gH} = 1.0 \times \sqrt{2 \times 9.81 \times 400} = 88.58$ m/s

Discharge of each jet $= a \times V_1 = .031416 \times 88.58 = 2.78$ m^3/s

∴ Total discharge, $Q = 2 \times 2.78 = 5.56$ m^3/s

Power at the inlet of turbine,

$$\text{W.P.} = \frac{\rho \times g \times Q \times H}{1000} \text{ kW}$$

$$= \frac{1000 \times 9.81 \times 5.56 \times 400}{1000} = 21817.44 \text{ kW}$$

∴ Overall efficiency is given as

$$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{15450}{21817.44} = 0.708 = \mathbf{70.8\%. \text{ Ans.}}$$

Problem 18.9 *The water available for a Pelton wheel is 4 cumec and the total head from the reservoir to the nozzle is 250 metres. The turbine has two runners with two jets per runner. All the four jets have the same diameters. The pipe line is 3000 metres long. The efficiency of power transmission through the pipe line and the nozzle is 91% and efficiency of each runner is 90%. The velocity co-efficient of each nozzle is 0.975 and co-efficient of friction '4f' for the pipe is 0.0045. Determine :*

(i) The power developed by the turbine, (ii) The diameter of the jet, and

(iii) The diameter of the pipe line.

Solution. Given :

Total discharge, $Q = 4$ cumec $= 4.0$ m^3/s

Total or gross head, $H_g = 250$ m

Total number of jets $= 2 \times 2 = 4$

Length of pipe, $L = 3000$ m

Efficiency of the pipe line and nozzle = 91% or 0.91

Efficiency of runner* or $\eta_h = 90\%$ or 0.90

Co-efficient of velocity, $C_v = 0.975$

Co-efficient of friction, $4f = .0045$

Efficiency of power transmission through pipe lines and nozzle is given by

$$\eta = \frac{H_g - h_f}{H_g} \text{ or } 0.91 = \frac{250 - h_f}{250}$$

where h_f = Head lost due to friction.

∴ $h_f = 250 - 0.91 \times 250 = 22.5$ m

∴ Net head on the turbine, $H = H_g - h_f = 250 - 22.50 = 227.5$ m

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.975\sqrt{2 \times 9.81 \times 227.5} = 65.14$ m/s.

* Efficiency of runner means the ratio of power delivered to the runner to the power at the inlet of turbine *i.e.*, hydraulic efficiency.

(*i*) Power at the inlet of the turbine is given as,

$$\text{W.P.} = \text{Kinetic energy of the jet/s}$$

$$= \frac{\frac{1}{2} m V_1^2}{1000} = \frac{\frac{1}{2}(\rho \times Q) V_1^2}{1000} = \frac{1}{2} \times 1000 \times \frac{4.0 \times 65.14^2}{1000} = 8486.44 \text{ kW}$$

But
$$\eta_h = \frac{\text{Power developed by turbine}}{\text{W.P.}}$$

∴
$$0.90 = \frac{\text{Power developed by turbine}}{8486.44}$$

∴ Power developed by turbine = 0.90 × 8486.44 = **7637.8 kW. Ans.**

(*ii*) Discharge per jet,
$$q = \frac{\text{Total discharge}}{\text{No. of jets}} = \frac{4.0}{4.0} = 1.0 \text{ m}^3/\text{s}$$

But
$$q = \text{Area of one jet} \times \text{Velocity of jet}$$

$$= \frac{\pi}{4} d^2 \times V_1 , \qquad \text{where } d = \text{Diameter of each jet}$$

∴
$$1.0 = \frac{\pi}{4} d^2 \times 65.14$$

∴
$$d = \sqrt{\frac{4 \times 1.0}{\pi \times 65.14}} = \mathbf{0.14 \text{ m. Ans.}}$$

(*iii*) Let
$$D = \text{Diameter of pipe line}$$

Then
$$h_f = \frac{4 \times f \times L \times V^{*2}}{D \times 2g}, \qquad \text{where } V^* = \text{Velocity through pipe}$$

∴
$$V^* = \frac{Q}{\text{Area}} = \frac{Q}{\frac{\pi}{4} D^2} = \frac{4Q}{\pi D^2}$$

And
$$h_f = \frac{.0045 \times 3000 \times \left(\frac{4Q}{\pi D^2}\right)^2}{D \times 2g}$$

or
$$22.50 = \frac{.0045 \times 3000 \times 16 \times Q^2}{D \times 2 \times 9.81 \times \pi^2 \times D^4} = \frac{.0045 \times 3000 \times 16 \times (4)^2}{D^5 \times 2 \times 9.81 \times \pi^2} = \frac{17.84}{D^5}$$

∴
$$D^5 = \frac{17.84}{22.50} = 0.7933$$

∴
$$D = (.7933)^{1/5} = \mathbf{0.955 \text{ m. Ans.}}$$

Problem 18.10 *The following data is related to a Pelton wheel :*

Head at the base of the nozzle = 80 m

Diameter of the jet = 100 mm

Discharge of the nozzle = *0.30* m^3/s
Power at the shaft = *206 kW*
Power absorbed in mechanical resistance= 4.5 kW
Determine (i) Power lost in nozzle and (ii) Power lost due to hydraulic resistance in the runner.

Solution. Given :

Head at the base of the nozzle, $H_1 = 80$ m

Diameter of the jet, $d = 100$ mm $= 0.1$ m

∴ Area of the jet, $a = \frac{\pi}{4} = (0.1)^2 = .007854$

Discharge of the nozzle, $Q = 0.30\ m^3/s$

Shaft power, S.P. = 206 kW

Power absorbed in mechanical resistance = 4.5 kW

Now discharge Q = area of jet × velocity of jet $= a \times V_1$

$$0.30 = .007854 \times V_1$$

∴ $$V_1 = \frac{0.30}{.007854} = 38.197 \text{ m/s}$$

Power at the base of the nozzle in kW

$$= \frac{\rho \times g \times Q \times H_1}{1000} = \frac{1000 \times 9.81 \times 0.30 \times 80}{1000} = 235.44$$

Power corresponding to kinetic energy of the jet in kW

$$= \frac{1}{2}\frac{(m)\,V_1^2}{1000} = \frac{1}{2}\frac{(\rho \times aV_1)\,V_1^2}{1000} = \frac{1}{2}\frac{\rho \times Q \times V_1^2}{1000}$$

$$= \frac{1}{2} \times 1000 \times \frac{0.3 \times 38.197^2}{1000} = 218.85 \text{ kW.}$$

(*i*) Power at the base of the nozzle

= Power of the jet + Power lost in nozzle

or 235.44 = 218.85 + Power lost in nozzle

∴ Power lost in nozzle = 235.44 – 218.85 = **16.59 kW. Ans.**

(*ii*) Also power at the base of nozzle = power at the shaft + power lost in nozzle + power lost in runner + power lost due to mechanical resistance

∴ 235.44 = 206 + 16.59 + Power lost in runner + 4.5

∴ Power lost in runner = 235.44 – (206 + 16.59 + 4.5) = 235.44 – 227.09 = **8.35 kW. Ans.**

18.6.3 Design of Pelton Wheel. Design of Pelton wheel means the following data is to be determined :

1. Diameter of the jet (d),
2. Diameter of wheel (D),
3. Width of the buckets which is $= 5 \times d$,
4. Depth of the buckets which is $= 1.2 \times d$, and
5. Number of buckets on the wheel.

Size of buckets means the width and depth of the buckets.

Problem 18.11 *A Pelton wheel is to be designed for a head of 60 m when running at 200 r.p.m. The Pelton wheel develops 95.6475 kW shaft power. The velocity of the buckets = 0.45 times the velocity of the jet, overall efficiency = 0.85 and co-efficient of the velocity is equal to 0.98.*

Solution. Given :

Head, $H = 60$ m

Speed $N = 200$ r.p.m

Shaft power, S.P. = 95.6475 kW

Velocity of bucket, $u = 0.45 \times$ Velocity of jet

Overall efficiency, $\eta_o = 0.85$

Co-efficient of velocity, $C_v = 0.98$

Design of Pelton wheel means to find diameter of jet (d), diameter of wheel (D), Width and depth of buckets and number of buckets on the wheel.

(*i*) *Velocity of jet,* $V_1 = C_v \times \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 60} = 33.62$ m/s

$\therefore$ Bucket velocity, $u = u_1 = u_2 = 0.45 \times V_1 = 0.45 \times 33.62 = 15.13$ m/s

But $u = \dfrac{\pi DN}{60}$, where D = Diameter of wheel

$$\therefore \quad 15.13 = \frac{\pi \times D \times 200}{60} \quad \text{or} \quad D = \frac{60 \times 15.13}{\pi \times 200} = \textbf{1.44 m. Ans.}$$

(*ii*) *Diameter of the jet* (d)

Overall efficiency $\eta_o = 0.85$

But
$$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{95.6475}{\left(\frac{\text{W.P.}}{1000}\right)} = \frac{95.6475 \times 1000}{\rho \times g \times Q \times H} \qquad (\because \ \text{W.P.} = \rho g Q H)$$

$$= \frac{95.6475 \times 1000}{1000 \times 9.81 \times Q \times 60}$$

$$\therefore \quad Q = \frac{95.6475 \times 1000}{\eta_o \times 1000 \times 9.81 \times 60} = \frac{95.6475 \times 1000}{0.85 \times 1000 \times 9.81 \times 60} = 0.1912 \text{ m}^3/\text{s}.$$

But the discharge, Q = Area of jet $\times$ Velocity of jet

$$\therefore \quad 0.1912 = \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} d^2 \times 33.62$$

$$\therefore \quad d = \sqrt{\frac{4 \times 0.1912}{\pi \times 33.62}} = 0.085 \text{ m} = \textbf{85 mm. Ans.}$$

(*iii*) *Size of buckets*

Width of buckets $= 5 \times d = 5 \times 85 = 425$ mm

Depth of buckets $= 1.2 \times d = 1.2 \times 85 =$ **102 mm. Ans.**

(*iv*) *Number of buckets on the wheel* is given by equation (18.17) as

$$Z = 15 + \frac{D}{2d} = 15 + \frac{1.44}{2 \times .085} = 15 + 8.5 = \textbf{23.5 say 24. Ans.}$$

Problem 18.12 *Determine the power given by the jet of water to the runner of a Pelton wheel which is having tangential velocity as 20 m/s. The net head on the turbine is 50 m and discharge through the jet water is 0.03 m³/s. The side clearance angle is 15° and take C_v = 0.975.*

Solution. Given :

Tangential velocity of wheel, $u = u_1 = u_2 = 20$ m/s

Net head, $H = 50$ m

Discharge , $Q = 0.03\ m^3/s$

Side clearance angle, $\phi = 15°$

Co-efficient of velocity, $C_v = 0.975$

Velocity of the jet, $V_1 = C_v \times \sqrt{2gH}$

$= 0.975 \times \sqrt{2 \times 9.81 \times 50}$

$= 30.54$ m/s

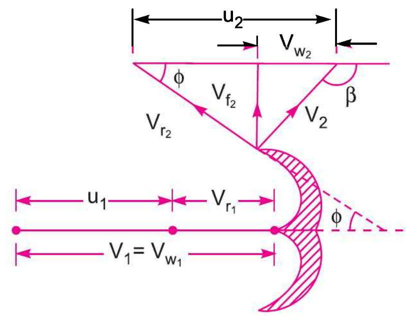

Fig. 18.9

From inlet triangle, $V_{w_1} = V_1 = 30.54$ m/s

$$V_{r_1} = V_{w_1} - u_1 = 30.54 - 20.0 = 10.54 \text{ m/s}$$

From outlet velocity triangle, we have

$$V_{r_2} = V_{r_1} = 10.54 \text{ m/s}$$

$$V_{r_2} \cos \phi = 10.54 \cos 15° = 10.18 \text{ m/s}$$

As $V_{r_2} \cos \phi$ is less than u_2, the velocity triangle at outlet will be as shown in Fig. 18.9.

$\therefore$ $V_{w_2} = u_2 - V_{r_2} \cos \phi = 20 - 10.18 = 9.82$ m/s.

Also as β is an obtuse angle, the work done per second on the runner,

$$= \rho a V_1 \left[V_{w_1} - V_{w_2}\right] \times u = \rho Q \left[V_{w_1} - V_{w_2}\right] \times u$$

$$= 1000 \times .03 \times [30.54 - 9.82] \times 20 = 12432 \text{ Nm/s}$$

$\therefore$ Power given to the runner in kW $= \dfrac{\text{Work done per second}}{1000} = \dfrac{12432}{1000}$ = **12.432 kW. Ans.**

Problem 18.13 *The three-jet Pelton turbine is required to generate 10,000 kW under a net head of 400 m. The blade angle at outlet is 15° and the reduction in the relative velocity while passing over the blade is 5%. If the overall efficiency of the wheel is 80%, C_v = 0.98 and speed ratio = 0.46, then find: (i) the diameter of the jet, (ii) total flow in m³/s and (iii) the force exerted by a jet on the buckets.*

If the jet ratio is not to be less than 10, find the speed of the wheel for a frequency of 50 hertz/sec and the corresponding wheel diameter.

Solution. Given :

No. of jets $= 3$

Total power, $P = 10000$ kW

Net head, $H = 400$ m

Blade angle at outlet, $\phi = 15°$

Relative velocity at outlet $= 0.95$ of relative velocity at inlet

or $V_{r_2} = 0.95\ V_{r_1}$

Overall efficiency, $\eta_o = 0.80$

Value of $C_v = 0.98$

Speed ratio $= 0.46$

Frequency, $f = 50$ hertz/sec

Now using equation (18.6 *A*), $\eta_o = \dfrac{P}{\left(\dfrac{\rho \times g \times Q \times H}{1000}\right)}$

where Q = Total discharge through three nozzles and $\rho = 1000$ kg/m^3

$$\therefore \quad 0.80 = \frac{10000}{\left(\dfrac{1000 \times 9.81 \times Q \times 400}{1000}\right)}$$

$$\therefore \quad Q = \frac{10000}{0.8 \times 9.81 \times 400} = \mathbf{3.18\ m^3/s.\ Ans.}$$

Discharge through one nozzle $= \dfrac{3.18}{3} = 1.06$ m^3/s.

(*i*) **Diameter of the jet (d).**

Discharge through one nozzle = Area of one jet × Velocity

But velocity of jet, $V_1 = C_v \times \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 400} = 87$ m/s

$$\therefore \quad 1.06 = \frac{\pi}{4} d^2 \times 87$$

$$\therefore \quad d = \sqrt{\frac{4 \times 1.06}{\pi \times 87}} = 0.125 \text{ m} = \mathbf{125\ mm.\ Ans.}$$

(*ii*) **Total flow in m^3/s** $= 3.18$ m^3/s.

(*iii*) **Force exerted by a jet on the wheel.**

$$\text{Speed ratio} = \frac{u_1}{\sqrt{2gH}}$$

$$\therefore \quad u_1 = \text{Speed ratio} \times \sqrt{2gH} = 0.46 \times \sqrt{2 \times 9.81 \times 400} = 40.75 \text{ m/s}$$

Now $V_{r_1} = V_1 - u_1 = 87 - 40.75 = 46.25$ m/s

and $V_{r_2} = 0.95\ V_{r_1} = 0.95 \times 46.25 = 44.0$ m/s

$V_{w_1} = V_1 = 87$ m/s

$V_{w_2} = V_{r_2} \cos \phi - u_2 = 44 \times \cos 15° - 40.75$ $(\because \quad u_1 = u_2 = 40.75 \text{ m/s})$

$= 1.75$ m/s

Force exerted by a single jet on the buckets

$= \rho \times$ discharge through one jet $\times (V_{w_1} + V_{w_2})$

$= 1000 \times 1.06\ (87 + 1.75) = 94075$ N $=$ **94.075 kN. Ans.**

(*iv*) **Jet ratio** $= 10$ or $\dfrac{D}{d} = 10$

$\therefore$ Dia. of wheel, $D = 10 \times d = 10 \times 0.125 = 1.25$ m

But, $u_1 = \dfrac{\pi DN}{60}$

$$\therefore \quad N = \frac{60 \times u_1}{\pi \times D} = \frac{60 \times 40.75}{\pi \times 1.25} = 620 \text{ r.p.m.}$$

Now using the relation, $N = \dfrac{60 \times f}{p}$

where f = frequency in hertz per second,
p = pairs of poles, and N = speed.

$$\therefore \quad p = \frac{60 \times f}{N} = \frac{60 \times 50}{620} = 4.85$$

Take the next whole number *i.e.*, 5. Hence, pairs of poles are 5.

Now corresponding to five pairs of poles, the speed of the turbine will become as given below :

$$\text{N} = \frac{60 \times f}{p} = \frac{60 \times 50}{5} = 600 \text{ r.p.m.}$$

But $u = \dfrac{\pi DN}{60}$

As the peripheral velocity is constant. Hence with the change of speed, diameter of wheel will change.

$$\therefore \quad D = \frac{60 \times u}{\pi \times N} = \frac{60 \times 40.75}{\pi \times 600} = 1.3 \text{ m}$$

$\therefore$ Jet ratio becomes $= \dfrac{D}{d} = \dfrac{1.30}{0.125} > 10$

Hence the given condition is satisfied.

▶ 18.7 RADIAL FLOW REACTION TURBINES

Radial flow turbines are those tubines in which the water flows in the radial direction. The water may flow radially from outwards to inwards (*i.e.*, towards the axis of rotation) or from inwards to outwards. If the water flows from outwards to inwards through the runner, the turbine is known as inward radial flow turbine. And if the water flows from inwards to outwards, the turbine is known as outward radial flow turbine.

Reaction turbine means that the water at the inlet of the turbine possesses kinetic energy as well as pressure energy. As the water flows through the runner, a part of pressure energy goes on changing into kinetic energy. Thus the water through the runner is under pressure. The runner is completely enclosed in an air-tight casing and casing and the runner is always full of water.

18.7.1 Main Parts of a Radial Flow Reaction Turbine. The main parts of a radial flow reaction turbine are :

1. Casing,
2. Guide mechanism,
3. Runner, and
4. Draft-tube.

1. Casing. As mentioned above that in case of reaction turbine, casing and runner are always full of water. The water from the penstocks enters the casing which is of spiral shape in which area of cross-section of the casing goes on decreasing gradually. The casing completely surrounds the runner of the turbine. The casing as shown in Fig. 18.10 is made of spiral shape, so that the water may enter the runner at constant velocity throughout the circumference of the runner. The casing is made of concrete, cast steel or plate steel.

2. Guide Mechanism. It consists of a stationary circular wheel all round the runner of the turbine. The stationary guide vanes are fixed on the guide mechanism. The guide vanes allow the water to strike the vanes fixed on the runner without shock at inlet. Also by a suitable arrangement, the width between two adjacent vanes of guide mechanism can be altered so that the amount of water striking the runner can be varied.

3. Runner. It is a circular wheel on which a series of radial curved vanes are fixed. The surface of the vanes are made very smooth. The radial curved vanes are so shaped that the water enters and leaves the runner without shock. The runners are made of cast steel, cast iron or stainless steel. They are keyed to the shaft.

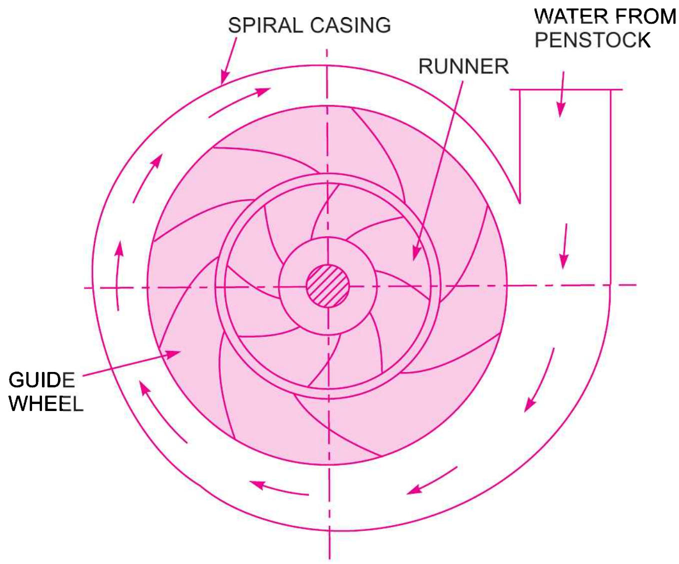

Fig. 18.10 *Main parts of a radial reaction turbines.*

4. Draft-tube. The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure. The water at exit cannot be directly discharged to the tail race. A tube or pipe of gradually increasing area is used for discharging water from the exit of the turbine to the tail race. This tube of increasing area is called draft tube.

18.7.2 Inward Radial Flow Turbine. Fig. 18.11 shows inward radial flow turbine, in which case the water from the casing enters the stationary guiding wheel. The guiding wheel consists of guide vanes which direct the water to enter the runner which consists of moving vanes. The water flows over the moving vanes in the inward radial direction and is discharged at the inner diameter of the runner. The outer diameter of the runner is the inlet and the inner diameter is the outlet.

Velocity Triangles and Work done by Water on Runner. In Chapter 17 (Art. 17.4.6), we have discussed in detail the force exerted by the water on the radial curved vanes fixed on a wheel. From the force exerted on the vanes,the work done by water, the horse power given by the water to the vanes and efficiency of the vanes can be obtained. Also we have drawn velocity triangles at inlet and outlet of the moving radial vanes in Fig. 17.23. From the velocity triangles, the work done by the water on the runners, horse power and efficiency of the turbine can be obtained.

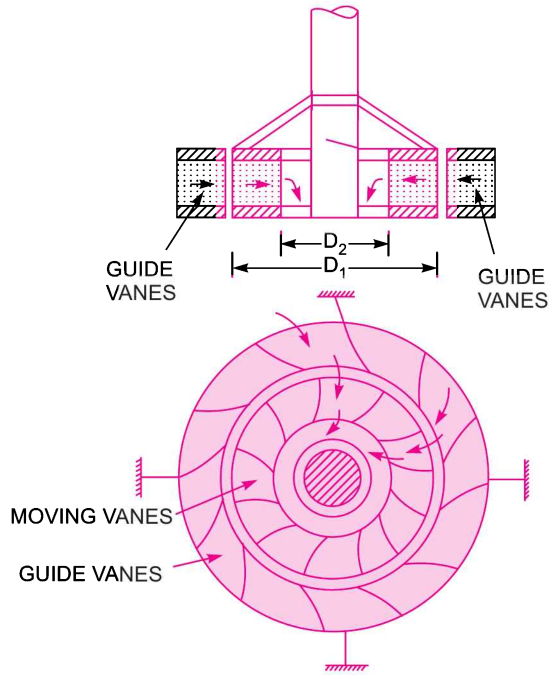

Fig. 18.11 *Inward radial flow turbine.*

The work done per second on the runner by water is given by equation (17.26) as

$$= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]$$

$$= \rho Q [V_{w_1} u_1 \pm V_{w_2} u_2] \quad (\because \; aV_1 = Q) \quad ...(18.18)$$

The equation (18.18) also represents the energy transfer per second to the runner.

where V_{w_1} = Velocity of whirl at inlet,

V_{w_2} = Velocity of whirl at outlet,

u_1 = Tangential velocity of wheel at inlet

$= \dfrac{\pi D_1 \times N}{60}$, where D_1 = Outer dia. of runner,

u_2 = Tangential velocity of wheel at outlet

$= \dfrac{\pi D_2 \times N}{60}$, where D_2 = Inner dia. of runner, N = Speed of the turbine in .r.p.m.

The work done per second per unit weight of water per second.

$$= \frac{\text{Work done per second}}{\text{Weight of water striking per second}}$$

$$= \frac{\rho Q \left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{\rho Q \times g} = \frac{1}{g}\left[V_{w_1} u_1 \pm V_{w_2} u_2\right] \quad ...(18.19)$$

The equation (18.19) represents the energy transfer per unit weight/s to the runner. This equation is known by **Euler's equation** of hydrodynamics machines. This is also known as fundamental equation of hydrodynamic machines. This equation was given by Swiss scientist *L.* Euler.

In equation (18.19), +ve sign is taken if angle β is an acute angle. If β is an obtuse angle then –ve sign is taken. If β = 90°, then $V_{w_2} = 0$ and work done per second per unit weight of water striking/s become as

$$= \frac{1}{g} V_{w_1} u_1 \qquad ...(18.20)$$

Hydraulic efficiency is obtained from equation (18.2) as

$$\eta_h = \frac{\text{R.P.}}{\text{W.P.}} = \frac{\frac{W}{1000g}[V_{w_1} u_1 \pm V_{w_2} u_2]}{\frac{W \times H}{1000}} = \frac{(V_{w_1} u_1 \pm V_{w_2} u_2)}{gH} \qquad ...(18.20A)$$

where R.P. = Runner power *i.e.*, power delivered by water to the runner

W.P. = Water power

If the discharge is radial at outlet, then $V_{w_2} = 0$

$$\eta_h = \frac{V_{w_1} u_1}{gH} \qquad ...(18.20B)$$

18.7.3 Degree of Reaction. Degree of reaction is defined as the ratio of pressure energy change inside a runner to the total energy change inside the runner. It is represented by 'R'. Hence mathematically it can be written as

$$R = \frac{\text{Change of pressure energy inside the runner}}{\text{Change of total energy inside the runner}} \qquad ...(18.20C)$$

The equation (18.19) which is the fundamental equation of hydrodynamic machines, represents the energy transfer per unit weight to the runner. This is also known as the total energy change inside the runner per unit weight.

∴ Change of total energy per unit weight inside the runner

$$= \frac{1}{g}[V_{w_1} u_1 \pm V_{w_2} u_2]$$

Let H_e = Change of total energy per unit weight inside the runner.

Then $$H_e = \frac{1}{g}[V_{w_1} u_1 \pm V_{w_2} u_2] \qquad ...(18.20D)$$

Let us find the values of $V_{w_1} u_1$ and $V_{w_2} u_2$ from inlet and outlet velocity triangles.

Now from inlet velocity triangle, we know that [Refer to Fig. 18.11(*a*)]

$$V_{w_1} = u_1 + V_{r_1 x}, \quad \text{where } V_{r_1 x} = V_{r_1} \cos\theta = \sqrt{V_{r_1}^2 - V_{f_1}^2}$$

$$= u_1 + \sqrt{V_{r_1}^2 - V_{f_1}^2}$$

$$= u_1 + \sqrt{V_{r_1}^2 - (V_1^2 - V_{w_1}^2)} \quad [\because \text{From triangle } ABC,\ V_{f_1}^2 = V_1^2 - V_{w_1}^2]$$

$$\therefore \quad (V_{w_1} - u_1) = \sqrt{V_{r_1}^2 - (V_1^2 - V_{w_1}^2)}$$

Squaring both sides, we get

$$(V_{w_1} - u_1)^2 = V_{r_1}{}^2 - (V_1^2 - V_{w_1}^2)$$

or $$V_{w_1}^2 + u_1{}^2 - 2V_{w_1} u_1 = V_{r_1}{}^2 - V_1^2 + V_{w_1}^2$$

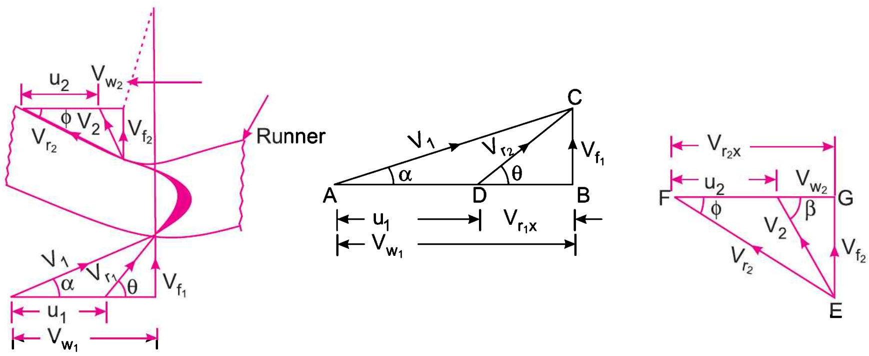

Fig. 18.11 (*a*)

or $$V_{w_1}^2 + u_1{}^2 - V_{r_1}^2 + V_1^2 - V_{w_1}^2 = 2V_{w_1} u_1$$

or $$u_1{}^2 - V_{r_1}{}^2 + V_1^2 = 2V_{w_1} u_1$$

or $$2V_{w_1} u_1 = u_1{}^2 - V_{r_1}^2 + V_1^2$$

or $$V_{w_1} u_1 = \frac{1}{2}[u_1^2 - V_{r_1}^2 + V_1^2] \qquad ...(i)$$

Similarly from outlet triangle, we know that [Refer to Fig. 18.11(*a*)]

$$V_{w_2} = V_{r_2 x} - u_2$$

$$= \sqrt{V_{r_2}{}^2 - V_{f_2}^2} - u_2, \text{ where } V_{r_2 x} = V_{r_2} \cos\theta = \sqrt{V_{r_2}^2 - V_{f_2}^2}$$

$$= \sqrt{V_{r_2}{}^2 - (V_2^2 - V_{w_2}^2)} - u_2 \qquad \therefore \quad V_{f_2}^2 = V_2^2 - V_{w_2}^2$$

$\therefore$ $$V_{w_2} + u_2 = \sqrt{V_{r_2}^2 - V_2^2 + V_{w_2}^2}$$

Squaring both sides, we get

$$(V_{w_2} + u_2)^2 = V_{r_2}^2 - V_2^2 + V_{w_2}^2$$

or $$V_{w_2}^2 + u_2^2 + 2V_{w_2} u_2 = V_{r_2}^2 - V_2^2 + V_{w_2}^2$$

or $$2V_{w_2} u_2 = V_{r_2}^2 - V_2^2 + V_{w_2}^2 - V_{w_2}^2 - u_2^2$$

or $$2V_{w_2} u_2 = V_{r_2}^2 - V_2^2 - u_2^2$$

or $$V_{w_2} u_2 = \frac{1}{2}[V_{r_2}^2 - V_2^2 - u_2^2] \qquad ...(ii)$$

In the above case of velocity triangles under consideration, the change of total energy per unit weight inside the runner is equal to $\frac{1}{g}\,[V_{w_1}u_1 + V_{w_2}u_2]$

Substituting the values of $V_{w_1}u_1$ and $V_{w_2}u_2$ from equations (*i*) and (*ii*) into equation (18.20 *D*), we get Change of total energy per unit weight inside the runner as

$$H_e = \frac{1}{g}\left[\frac{1}{2}\left(u_1^2 - V_{r_1}{}^2 + V_1^2\right) + \frac{1}{2}\left(V_{r_2}{}^2 - V_2^2 - u_2^2\right)\right]$$

$$= \frac{1}{2g}\left[\left(u_1^2 - u_2^2\right) + \left(V_1^2 - V_2^2\right) + \left(V_{r_2}{}^2 - V_{r_1}{}^2\right)\right]$$

$$= \frac{V_1^2 - V_2^2}{2g} + \frac{u_1^2 - u_2^2}{2g} + \frac{V_{r_2}{}^2 - V_{r_1}{}^2}{2g} \qquad \text{...(18.20}E\text{)}$$

The above equation consists of three terms. The first term represents the change in kinetic energy of the fluid per unit weight and the second term represents the change of energy per unit weight due to centrifugal action. The third term represents the change in static pressure energy per unit weight, as per Bernoulli's equation applied to relative flow through runner passage by reducing the rotating system into stationary system. We know that the energy change due to centrifugal action takes place in the form of pressure energy. [When a container containing a liquid is rotated, then due to centrifugal action there is change of pressure energy *i.e.*, $h = \frac{\Delta p}{\rho g} = \frac{u_2^2 - u_1^2}{2g}$]. Hence, the last two terms in equation (18.20*E*) represents the change in pressure energy inside the runner passage per unit weight.

$\therefore$ Change in pressure energy inside the runner per unit weight $= \frac{u_1^2 - u_2^2}{2g} + \frac{V_{r_2}^2 - V_{r_1}^2}{2g}$ ...(*iii*)

Now the equation (18.20*C*) becomes as

$$R = \frac{\text{Change of pressure energy inside the runner per unit weight}}{\text{Change of total energy inside the runner per unit weight}}$$

$$= \left(\frac{\left(u_1^2 - u_2^2\right)}{2g} + \frac{V_{r_2}^2 - V_{r_1}^2}{2g}\right) \Bigg/ \left[\left(\frac{V_1^2 - V_2^2}{2g}\right) + \left(\frac{u_1^2 - u_2^2}{2g}\right) + \left(\frac{V_{r_2}^2 - V_{r_1}^2}{2g}\right)\right]$$

or
$$R = \frac{\left(u_1^2 - u_2^2\right) + \left(V_{r_2}^2 - V_{r_1}^2\right)}{\left(V_1^2 - V_2^2\right) + \left(u_1^2 - u_2^2\right) + \left(V_{r_2}^2 - V_{r_1}^2\right)} \qquad \text{...(18.20}F\text{)}$$

or
$$R = \frac{\left(V_1^2 - V_2^2\right) + \left(u_1^2 - u_2^2\right) + \left(V_{r_2}^2 - V_{r_1}^2\right) - \left(V_1^2 - V_2^2\right)}{\left(V_1^2 - V_2^2\right) + \left(u_1^2 - u_2^2\right) + \left(V_{r_2}^2 - V_{r_1}^2\right)}$$

$$= 1 - \frac{\left(V_1^2 - V_2^2\right)}{\left(V_1^2 - V_2^2\right) + \left(u_1^2 - u_2^2\right) + \left(V_{r_2}^2 - V_{r_1}^2\right)} \qquad \text{...(18.20}G\text{)}$$

From equation (18.20*E*) , we know that

$$H_e = \frac{V_1^2 - V_2^2}{2g} + \frac{u_1^2 - u_2^2}{2g} + \frac{V_{r_2}^2 - V_{r_1}^2}{2g}$$

or $$2gH_e = (V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r_2}^2 - V_{r_1}^2)$$

Now the equation (18.20*G*) can be written as

$$R = 1 - \frac{(V_1^2 - V_2^2)}{2g\ H_e} \quad ...(18.20H)$$

Values of *R* for Pelton turbine and other actual reaction turbines

(*i*) *For a Pelton turbine,*

$$u_1 = u_2 \text{ and } V_{r_2} = V_{r_1}$$

∴ From equation (18.20*G*)

$$R = 1 - \frac{(V_1^2 - V_2^2)}{(V_1^2 - V_2^2)} = 1 - 1 = 0$$

(*ii*) *For an actual reaction turbine,* generally, the angle β is 90° so that the loss of kinetic energy at outlet is minimum (*i.e.*, V_2 is minimum).

Hence in outlet velocity triangle, V_{w_2} becomes zero (*i.e.*, $V_{w_2} = 0$). Also $V_2 = V_{f_2}$ [Refer to Fig. 18.11(*b*)]

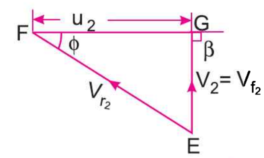

Fig. 18.11 (*b*)

Also there is not much change in velocity of flow. This means $V_{f_1} = V_{f_2}$

From equation (18.20*D*), we know that

$$H_e = \frac{1}{g}[V_{w_1} u_1 + V_{w_2} u_2]$$

$$= \frac{1}{g} V_{w_1} u_1 \qquad (\because\ V_{w_2} = 0)$$

$$= \frac{1}{g}[V_{f_1} \cot \alpha]\,[V_{f_1} \cot \alpha - V_{f_1} \cot \theta] \qquad \text{[Refer to Fig. 18.11(a)]}$$

$$[\because\ V_{w_1} = V_{f_1} \cot \alpha \text{ and } u_1 = V_{w_1} - V_{f_1} \cot \theta = V_{f_1} \cot \alpha - V_{f_1} \cot \theta]$$

$$= \frac{1}{g} V_{f_1}^2 \cot \alpha\,[\cot \alpha - \cot \theta]$$

Now $$V_1^2 - V_2^2 = (V_{f_1} \operatorname{cosec} \alpha)^2 - V_{f_2}^2 \qquad (\because\ V_2 = V_{f_2})$$

$$= V_{f_1}^2 \operatorname{cosec}^2 \alpha - V_{f_1}^2 \qquad (\because\ V_{f_2} = V_{f_1})$$

or $$V_1^2 - V_2^2 = V_{f_1}^2 (\operatorname{cosec}^2 \alpha - 1)$$

$$= V_{f_1}^2 \cot^2 \alpha \qquad (\because\ \operatorname{cosec}^2 \alpha - 1 = \cot^2 \alpha)$$

Substituting the value of H_e and $(V_1^2 - V_2^2)$ in equation (18.20*H*), we get

$$R = 1 - \frac{V_{f_1}^2 \cot^2 \alpha}{2g \times [\frac{1}{g} V_{f_1}^2 \cot \alpha (\cot \alpha - \cot \theta)]}$$

$$= 1 - \frac{\cot \alpha}{2(\cot \alpha - \cot \theta)} \quad ...(18.20I)$$

18.7.4 Definitions. The following terms are generally used in case of reaction radial flow turbines which are defined as :

(*i*) **Speed Ratio.** The speed ratio is defind as = $\frac{u_1}{\sqrt{2gH}}$

where u_1 = Tangential velocity of wheel at inlet.

(*ii*) **Flow Ratio.** The ratio of the velocity of flow at inlet (V_{f_1}) to the velocity given $\sqrt{2gH}$ is known as flow ratio or it is given as

$$= \frac{V_{f_1}}{\sqrt{2gH}}, \text{ where } H = \text{Head on turbine}$$

(*iii*) **Discharge of the Turbine.** The discharge through a reaction radial flow turbine is given by

$$Q = \pi D_1 B_1 \times V_{f_1} = \pi D_2 \times B_2 \times V_{f_2} \quad \text{...(18.21)}$$

where D_1 = Diameter of runner at inlet,
B_1 = Width of runner at inlet,
V_{f_1} = Velocity of flow at inlet, and
D_2, B_2, V_{f_2} = Corresponding values at outlet.

If the thickness of vanes are taken into consideration, then the area through which flow takes place is given by $(\pi D_1 - n \times t)$

where n = Number of vanes on runner and t = Thickness of each vane

The discharge Q, then is given by $Q = (\pi D_1 - n \times t)\ B_1 \times V_{f_1}$ ...(18.22)

(*iv*) The head (H) on the turbine is given by $H = \frac{p_1}{\rho \times g} + \frac{V_1^2}{2g}$ ...(18.23)

where p_1 = Pressure at inlet.

(*v*) **Radial Discharge.** This means the angle made by absolute velocity with the tangent on the wheel is 90° and the component of the whirl velocity is zero. Radial discharge at outlet means β = 90° and V_{w_2} = 0, while radial discharge at outlet means α = 90° and V_{w_1} = 0.

(*vi*) If there is no loss of energy when water flows through the vanes then we have

$$H - \frac{V_2^2}{2g} = \frac{1}{g}[V_{w_1} u_1 \pm V_{w_2} u_2]. \quad \text{...(18.24)}$$

Problem 18.14 *An inward flow reaction turbine has external and internal diameters as 1 m and 0.5 m respectively. The velocity of flow through the runner is constant and is equal to 1.5 m/s. Determine :*

(i) Discharge through the runner, and

(ii) Width of the turbine at outlet if the width of the turbine at inlet = 200 mm.

Solution. Given :

External diameter of turbine, D_1 = 1 m
Internal diameter of turbine, D_2 = 0.5 m
Velocity of flow at inlet and outlet, $V_{f_1} = V_{f_2}$ = 1.5 m/s
Width of turbine at inlet, B_1 = 200 mm = 0.20 m
Let the width at outlet = B_2

Using equation (18.21) for discharge,

$$Q = \pi D_1 B_1 \times V_{f_1} = \pi \times 1 \times 0.20 \times 1.5 = \mathbf{0.9425\ m^3/s.\ Ans.}$$

Also $\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$ or $D_1 B_1 = D_2 B_2$ $\quad (\because \ \pi V_{f_1} = \pi V_{f_2})$

$$\therefore \quad B_2 = \frac{D_1 \times B_1}{D_2} = \frac{1 \times 0.20}{0.5} = 0.40 \text{ m} = \mathbf{400\ mm.\ Ans.}$$

Problem 18.15 *An inward flow reaction turbine has external and internal diameters as 0.9 m and 0.45 m respectively. The turbine is running at 200 r.p.m. and width of turbine at inlet is 200 mm. The velocity of flow through the runner is constant and is equal to 1.8 m/s. The guide blades make an angle of 10° to the tangent of the wheel and the discharge at the outlet of the turbine is radial. Draw the inlet and outlet velocity triangles and determine:*

(*i*) *The absolute velocity of water at inlet of runner,*
(*ii*) *The velocity of whirl at inlet,* (*iii*) *The relative velocity at inlet,*
(*iv*) *The runner blade angles,* (*v*) *Width of the runner at outlet,*
(*vi*) *Mass of water flowing through the runner per second,*
(*vii*) *Head at the inlet of the turbine,*
(*viii*) *Power developed and hydraulic efficiency of the turbine.*

Solution. Given :

External Dia., $D_1 = 0.9$ m
Internal Dia., $D_2 = 0.45$ m
Speed, $N = 200$ r.p.m.
Width at inlet, $B_1 = 200$ mm $= 0.2$ m
Velocity of flow, $V_{f_1} = V_{f_2} = 1.8$ m/s
Guide blade angle, $\alpha = 10°$
Discharge at outlet = Radial
∴ $\beta = 90°$ and $V_{w_2} = 0$

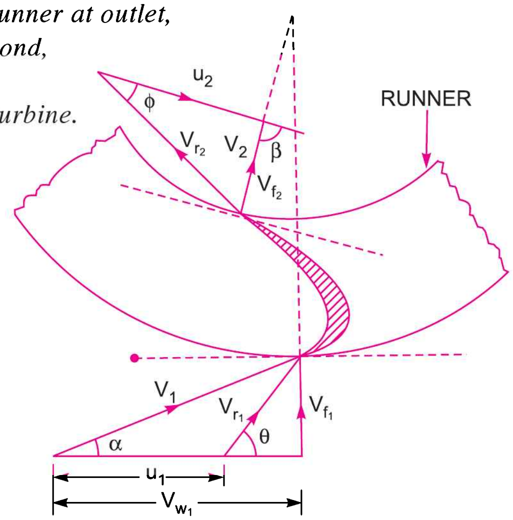

Fig 18.12

Tangential velocity of wheel at inlet and outlet are :

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times .9 \times 200}{60} = 9.424 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times .45 \times 200}{60} = 4.712 \text{ m/s.}$$

(*i*) *Absolute velocity of water at inlet of the runner i.e.,* V_1

From inlet velocity triangle,

$$V_1 \sin \alpha = V_{f_1}$$

$$\therefore \quad V_1 = \frac{V_{f_1}}{\sin \alpha} = \frac{18}{\sin 10°} = \mathbf{10.365 \text{ m/s. Ans.}}$$

(*ii*) *Velocity of whirl at inlet, i.e.,* V_{w_1}

$$V_{w_1} = V_1 \cos \alpha = 10.365 \times \cos 10° = \mathbf{10.207 \text{ m/s. Ans.}}$$

(*iii*) *Relative velocity at inlet, i.e.,* V_{r_1}

$$V_{r_1} = \sqrt{V_{r_1}^{\,3} + (V_{w_1} - u_1)^2} = \sqrt{1.8^2 + (10.207 - 9.424)^2}$$

$$= \sqrt{3.24 + .613} = \mathbf{1.963 \text{ m/s. Ans.}}$$

(*iv*) *The runner blade angles* means the angle θ and φ

Now $$\tan \theta = \frac{V_{f_1}}{(V_{w_1} - u_1)} = \frac{1.8}{(10.207 - 9.424)} = 2.298$$

$$\therefore \quad \theta = \tan^{-1} 2.298 = \mathbf{66.48°} \text{ or } \mathbf{66° \; 29'}. \textbf{ Ans.}$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2} = \frac{1.8}{4.712} = \tan 20.9°$$

$\therefore$ ϕ = **20.9°** or **20° 54.4′. Ans.**

(*v*) *Width of runner at outlet, i.e.,* B_2

From equation (18.21), we have

$$\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2} \text{ or } D_1 B_1 = D_2 B_2 \qquad (\because \ \pi V_{f_1} = \pi V_{f_2} \text{ as } V_{f_1} = V_{f_2})$$

$\therefore$
$$B_2 = \frac{D_1 B_1}{D_2} = \frac{0.90 \times 0.20}{0.45} = 0.40 \text{ m} = \textbf{400 mm. Ans.}$$

(*vi*) *Mass of water flowing through the runner per second.*

The discharge, $Q = \pi D_1 B_1 V_{f_1} = \pi \times 0.9 \times 0.20 \times 1.8 = 1.0178 \text{ m}^3\text{/s}$.

$\therefore$ Mass = $\rho \times Q$ = 1000 × 1.0178 kg/s = **1017.8 kg/s. Ans.**

(*vii*) *Head at the inlet of turbine, i.e., H.*

Using equation (18.24), we have

$$H - \frac{V_2^2}{2g} = \frac{1}{g}(V_{w_1} u_1 \pm V_{w_2} u_2) = \frac{1}{g}(V_{w_1} u_1) \qquad (\because \text{ Here } V_{w_2} = 0)$$

$$H = \frac{1}{g} V_{w_1} u_1 + \frac{V_2^2}{2g} = \frac{1}{9.81} \times 10.207 \times 9.424 + \frac{1.8^2}{2 \times 9.81} \quad (\because V_2 = V_{f_2})$$

$$= 9.805 + 0.165 = \textbf{9.97 m. Ans.}$$

(*viii*) *Power developed, i.e.,*
$$P = \frac{\text{Work done per second on runner}}{1000}$$

$$= \frac{\rho Q \left[V_{w_1} u_1\right]}{1000} \qquad \text{[Using equation (18.18)]}$$

$$= 1000 \times \frac{1.0178 \times 10.207 \times 9.424}{1000} = \textbf{97.9 kW. Ans.}$$

Hydraulic efficiency is given by equation (18.20*B*) as

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{10.207 \times 9.424}{9.81 \times 9.97} = 0.9834 = \textbf{98.34\%. Ans.}$$

Problem 18.16 *A reaction turbine works at 450 r.p.m. under a head of 120 metres. Its diameter at inlet is 120 cm and the flow area is 0.4 m². The angles made by absolute and relative velocities at inlet are 20° and 60° respectively with the tangential velocity. Determine :*

(a) The volume flow rate, (b) The power developed, and

(c) Hydraulic efficiency.

Assume whirl at outlet to be zero.

Solution. Given :

Speed of turbine, N = 450 r.p.m.

Head, H = 120 m

Diameter at inlet, D_1 = 120 cm = 1.2 m

Flow area, $\pi D_1 \times B_1 = 0.4 \text{ m}^2$

Angle made by absolute velocity at inlet, α = 20°

Angle made by the relative velocity at inlet, θ = 60°

Whirl at outlet, $V_{w_2} = 0$

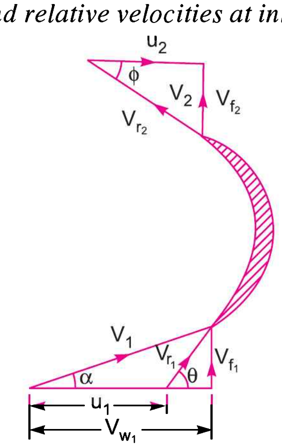

Fig. 18.13

Tangential velocity of the turbine at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 \text{ m/s}$$

From inlet velocity triangle,

$$\tan \alpha = \frac{V_{f_1}}{V_{w_1}} \text{ or } \tan 20° = \frac{V_{f_1}}{V_{w_1}} \text{ or } \frac{V_{f_1}}{V_{w_1}} = \tan 20° = 0.364$$

$\therefore$ $$V_{f_1} = 0.364\ V_{w_1}$$

Also $$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{0.364\ V_{w_1}}{V_{w_1} - 28.27} \qquad (\because\ V_{f_1} = 0.364\ V_{w_1})$$

or $$\frac{0.364\ V_{w_1}}{V_{w_1} - 28.27} = \tan \theta = \tan 60° = 1.732$$

$\therefore$ $$0.364 V_{w_1} = 1.732(V_{w_1} - 28.27) = 1.732 V_{w_1} - 48.96$$

or $$(1.732 - 0.364)\ V_{w_1} = 48.96$$

$\therefore$ $$V_{w_1} = \frac{48.96}{(1.732 - 0.364)} = 35.789 = 35.79 \text{ m/s.}$$

From equation (*i*), $$V_{f_1} = 0.364 \times V_{w_1} = 0.364 \times 35.79 = 13.027 \text{ m/s.}$$

(*a*) Volume flow rate is given by equation (18.21) as $Q = \pi D_1 B_1 \times V_{f_1}$

But $$\pi D_1 \times B_1 = 0.4 \text{ m}^2 \qquad \text{(given)}$$

$$Q = 0.4 \times 13.027 = \mathbf{5.211\ m^3/s.}$$ **Ans.**

(*b*) Work done per sec on the turbine is given by equation (18.18),

$$= \rho Q\ [V_{w_1} u_1] \qquad (\because\ V_{w_2} = 2)$$

$$= 1000 \times 5.211\ [35.79 \times 28.27] = 5272402 \text{ Nm/s}$$

$\therefore$ $$\text{Power developed in kW} = \frac{\text{Work done per second}}{1000} = \frac{5272402}{1000} = \mathbf{5272.402\ kW.}$$ **Ans.**

(*c*) The hydraulic efficiency is given by equation (18.20*B*) as

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{35.79 \times 28.27}{9.81 \times 120} = 0.8595 = \mathbf{85.95\%.}$$ **Ans.**

Problem 18.17 *As inward flow reaction turbine has external and internal diameters as 1.0 m and 0.6 m respectively. The hydraulic efficiency of the turbine is 90% when the head on the turbine is 36 m. The velocity of flow at outlet is 2.5 m/s and discharge at outlet is radial. If the vane angle at outlet is 15° and width of the wheel is 100 mm at inlet and outlet, determine : (i) the guide blade angle, (ii) speed of the turbine, (iii) vane angle of the runner at inlet, (iv) volume flow rate of turbine and (v) power developed.*

Solution. Given :

External diameter, $$D_1 = 1.0 \text{ m}$$

Internal diameter, $D_2 = 0.6$ m
Hydraulic efficiency, $\eta_h = 90\% = 0.90$
Head, $H = 36$ m
Velocity of flow at outlet, $V_{f_2} = 2.5$ m/s
Discharge is radial, $V_{w_2} = 0$
Vane angle at outlet, $\phi = 15°$
Width of wheel, $B_1 = B_2 = 100$ mm $= 0.1$ m

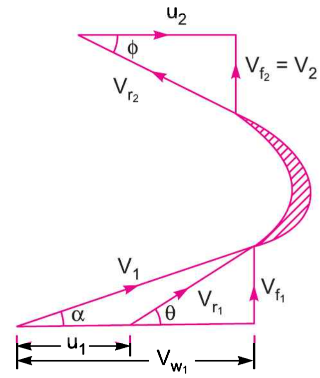

Fig. 18.14

Using equation (18.20 *B*) for hydraulic efficiency as

$$\eta_h = \frac{V_{w_1} u_1}{gH} \text{ or } 0.90 = \frac{V_{w_1} . u_1}{9.81 \times 36}$$

$$\therefore \quad V_{w_1} u_1 = 0.90 \times 9.81 \times 36 = 317.85 \qquad ...(i)$$

From outlet velocity triangle, $\tan \phi = \frac{V_{f_2}}{u_2} = \frac{2.5}{u_2}$

$$\therefore \quad u_2 = \frac{2.5}{\tan \phi} = \frac{2.5}{\tan 15°} = 9.33$$

But

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times N}{60}$$

$$\therefore \quad 9.33 = \frac{\pi \times 0.6 \times N}{60} \text{ or } N = \frac{60 \times 9.33}{\pi \times 0.6} = \textbf{296.98. Ans.}$$

$$\therefore \quad u_1 = \frac{\pi \times D_1 \times N}{60} = \frac{\pi \times 1.0 \times 296.98}{60} = 15.55 \text{ m/s.}$$

Substituting this value of 'u_1' in equation (*i*),

$$V_{w_1} \times 15.55 = 317.85$$

$$\therefore \quad V_{w_1} = \frac{317.85}{15.55} = 20.44 \text{ m/s}$$

Using equation (18.21), $\pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$ or $D_1 V_{f_1} = D_2 V_{f_2}$ $\qquad (\because B_1 = B_2)$

$$\therefore \quad V_{f_1} = \frac{D_2 \times V_{f_2}}{D_1} = \frac{0.6 \times 2.5}{1.0} = 1.5 \text{ m/s.}$$

(*i*) *Guide blade angle* (α).

From inlet velocity triangle, $\tan \alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{1.5}{20.44} = 0.07338$

$$\therefore \quad \alpha = \tan^{-1} 0.07338 = \textbf{4.19° or 4° 11.8'. Ans.}$$

(*ii*) *Speed of the turbine,* N = **296.98 r.p.m. Ans.**

(*iii*) *Same angle of runner at inlet* (θ)

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{1.5}{(20.44 - 15.55)} = 0.3067$$

$\therefore$ $\theta = \tan^{-1} .3067 = 17.05°$ or **17° 3′. Ans.**

(*iv*) *Volume flow rate of turbine* is given by equation (18.21) as

$$= \pi D_1 B_1 V_{f_1} = \pi \times 1.0 \times 0.1 \times 1.5 = \mathbf{0.4712\ m^3/s.\ Ans.}$$

(*v*) *Power developed* (in kW)

$$= \frac{\text{Work done per second}}{1000} = \frac{\rho Q\left[V_{w_1} u_1\right]}{1000}$$

[Using equation (18.18) and $V_{w_2} = 0$]

$$= 1000 \times \frac{0.4712 \times 20.44 \times 15.55}{1000} = \mathbf{149.76\ kW.\ Ans.}$$

Problem 18.18 *An inward flow reaction turbine has an exit diameter of 1 metre and its breadth at inlet is 250 mm. If the velocity of flow at inlet is 2 metres/s, find the mass of water passing through the turbine per second. Assume 10% of the area of flow is blocked by blade thickness. If the speed of the runner is 210 r.p.m. and guide blades make an angle of 10° to the wheel tangent, draw the inlet velocity triangle, and find :*

(i) the runner vane angle at inlet, *(ii) velocity of wheel at inlet,*

(iii) the absolute velocity of water leaving the guide vanes, and

(iv) the relative velocity of water entering the runner blade.

Solution. Given :

Exit or External diameter, $D_1 = 1.0$ m

Breadth at inlet, $B_1 = 250$ mm $= 0.25$ m

Velocity of flow at inlet, $V_{f_1} = 2.0$ m/s

Area blocked by vanes = 10%

Speed, $N = 210$ r.p.m.

Guide blade angle, $\alpha = 10°$

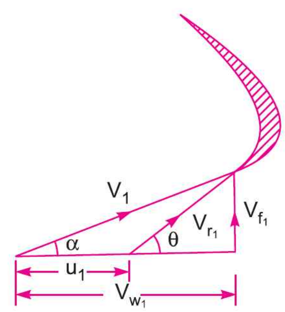

Fig. 18.15

Tangential velocity of wheel at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.0 \times 210}{60} = 10.99 \text{ m/s}$$

$$\text{Area blocked by vane thickness} = \frac{10}{100} \times \pi D_1 B_1 = 0.1\ \pi D_1 B_1$$

$\therefore$ Actual area through which flow takes place,

$$a = \pi D_1 B_1 - 0.1\ \pi D_1 B_1 = 0.9 \pi D_1 B_1$$
$$= 0.9 \times \pi \times 1.0 \times 0.25 = 0.7068 \text{ m}^2$$

$\therefore$ Mass of water passing per second

$$= \rho \times a \times V_{f_1} = 1000 \times .7068 \times 2.0 = \mathbf{1413.6\ kg.\ Ans.}$$

(*i*) *The runner vane angle at inlet* (θ).

From inlet velocity triangle $\tan \alpha = \dfrac{V_{f_1}}{V_{w_1}} = \dfrac{2.0}{V_{w_1}}$

$$\therefore \qquad V_{w_1} = \frac{2.0}{\tan \alpha} = \frac{2.0}{\sin 10} = 11.34 \text{ m/s}$$

$$\therefore \qquad \tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{2.0}{11.34 - 10.99} = 5.714$$

$$\therefore \qquad \theta = \tan^{-1} 5.714 = \mathbf{80.07°} \text{ or } \mathbf{80°4.2'}. \textbf{ Ans.}$$

(*ii*) *Velocity of wheel at inlet*, u_1 = **10.99 m/s. Ans.**

(*iii*) *The absolute velocity of water leaving the guide vanes* (V_1) :

From inlet triangle, $\sin \alpha = \dfrac{V_{f_1}}{V_1}$

$$\therefore \qquad V_1 = \frac{V_{f_1}}{\sin \alpha} = \frac{2.0}{\sin 10°} = \mathbf{11.517 \text{ m/s. Ans.}}$$

(*iv*) *The relative velocity of water entering the runner blade* (V_{r_1})

$$\sin \theta = \frac{V_{f_1}}{V_{r_1}}$$

$$\therefore \qquad V_{r_1} = \frac{V_{f_1}}{\sin \theta} = \frac{2.0}{\sin 80.07°} = \mathbf{2.03 \text{ m/s. Ans.}}$$

Problem 18.19 *The external and internal diameters of an inward flow reaction turbines are 1.20 m and 0.6 m respectively. The head on the turbine is 22 m and velocity of flow through the runner is constant and equal to 2.5 m/s. The guide blade angle is given as 10° and the runner vanes are radial at inlet. If the discharge at outlet is radial, determine :*

(i) The speed of the turbine, (ii) The vane angle at outlet of the runner, and
(iii) Hydraulic efficiency.

Solution. Given :

External diameter, $D_1 = 1.20$ m

Internal diameter, $D_2 = 0.60$ m

Head, $H = 22.0$ m

Velocity of flow, $V_{f_1} = V_{f_2} = 2.5$ m/s

Guide blade angle, $\alpha = 10°$

Runner vanes radial at inlet means, θ = 90°

$$\therefore \qquad V_{w_1} = u_1,\ V_{r_1} = V_{f_1} = 2.5 \text{ m/s}$$

Discharge is radial

$$\therefore \qquad V_{w_2} = 0,\ V_2 = V_{f_2} = 2.5 \text{ m/s}$$

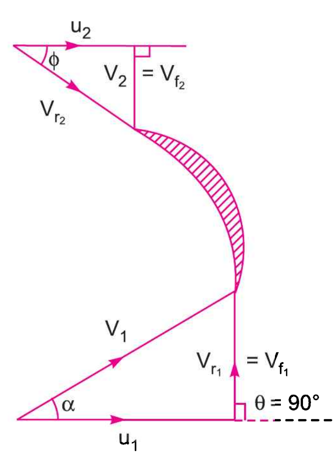

Fig. 18.16

From inlet velocity triangle,

$$\tan \alpha = \frac{V_{f_1}}{u_1}$$

$\therefore \quad u_1 = \dfrac{V_{f_1}}{\tan \alpha} = \dfrac{1.5}{\tan 10°} = 14.178$ m/s

$\therefore \quad V_{w_1} = u_1 = 14.178$ m/s

(*i*) *The speed of the turbine* (*N*)

Using $\quad u_1 = \dfrac{\pi D_1 \times N}{60}$

$\therefore \quad N = \dfrac{60 \times u_1}{\pi D_1} = \dfrac{60 \times 14.178}{\pi \times 1.20} =$ **225.65 r.p.m. Ans.**

(*ii*) *The vane angle at outlet of the runner* (ϕ).

$$u_2 = \frac{\pi D_2 \times N}{60} = \frac{\pi \times 0.6 \times 225.65}{60} = 7.09 \text{ m/s.}$$

From outlet velocity triangle, $\tan \phi = \dfrac{V_{f_2}}{u_2} = \dfrac{2.5}{7.09} = 0.3526$

$\therefore \quad \phi = \tan^{-1} 0.3526 = 19.42°$ or **19° 25.2′. Ans.**

(*iii*) *Hydraulic efficiency* is given by $\eta_h = \dfrac{V_{w_1} u_1}{gH} = \dfrac{14.178 \times 14.178}{9.81 \times 22.0} = 0.9314 =$ **93.14%. Ans.**

Problem 18.20 *233 litres of water per second are supplied to an inward flow reaction turbine. The head available is 11 m. The wheel vanes are radial at inlet and the inlet diameter is twice the outlet diameter. The velocity of flow is constant and equal to 1.83 m/s. The wheel makes 370 r.p.m. Find :*

(a) Guide vane angle, *(b) Inlet and outlet diameter of the wheel,*

(c) The width of the wheel at inlet and exit. Neglect the thickness of the vanes.

Assume that the discharge is radial and there are no losses in the wheel. Take speed ratio = 0.7.

Solution. Given :

Discharge, $\quad Q = 233$ lit/s $= 0.233$ m^3/s

Head, $\quad H = 11$ m

Wheel vanes are radial at inlet. This means angle

$$\theta = 90° \text{ and } V_{r_1} = V_{f_1}$$

Inlet diameter $\quad = 2 \times$ Outlet diameter

$\therefore \quad D_1 = 2D_2$

Velocity of flow at inlet and outlet = 1.83 m/s

$\therefore \quad V_{f_1} = V_{f_2} = 1.83$ m/s

Speed, $\quad N = 370$ r.p.m.

Speed ratio $\quad = 0.7$ or $\dfrac{u_1}{\sqrt{2gH}} = 0.7$

$\therefore \quad u_1 = 0.7 \times \sqrt{2gH}$

$$= 0.7 \times \sqrt{2 \times 9.81 \times 11} = 10.28 \text{ m/s}$$

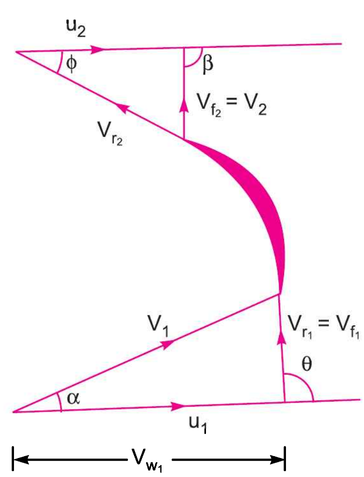

Fig. 18.17

Discharge is radial at outlet. This means angle $\beta = 90°$ and $V_{w_2} = 0$

(*a*) *Guide vane angle* (α)

$$\tan \alpha = \frac{V_{f_1}}{u_1} = \frac{1.83}{10.28} = 0.178$$

$$\therefore \quad \alpha = \tan^{-1} .178 = \mathbf{10° \; 6'. \; Ans.}$$

(*b*) *Inlet and outlet diameter of the wheel*

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times D_1 \times 370}{60}$$

$$\therefore \quad D_1 = \frac{60 \times u_1}{\pi \times 370} = \frac{60 \times 10.28}{\pi \times 370} = .532 \text{ m} = \mathbf{53.2 \text{ cm. Ans.}}$$

$$D_2 = \frac{D_1}{2} = \frac{53.2}{2} = \mathbf{26.6 \text{ cm. Ans.}}$$

(*c*) *Width of wheel at inlet and outlet*

$$Q = \pi D_1 \times B_1 \times V_{f_1} = \pi D_2 \times B_2 \times V_{f_2}$$

But $V_{f_1} = V_{f_2} \quad \therefore \quad D_1 \times B_1 = D_2 \times B_2$

As $D_1 = 2D_2$, $B_2 = 2B_1$

Now $Q = \pi D_1 \times B_1 \times V_{f_1} = \pi \times .532 \times B_1 \times 1.83$

$$\therefore \quad B_1 = \frac{Q}{\pi \times .532 \times 1.83} = \frac{0.233}{\pi \times .532 \times 1.83} = 0.0762 \text{ m} = \mathbf{7.62 \text{ cm. Ans.}}$$

$$B_2 = 2 \times B_1 = 2 \times 7.62 = \mathbf{15.24 \text{ cm. Ans.}}$$

18.7.5 Outward Radial Flow Reaction Turbine. Fig. 18.18 shows outward radial flow reaction turbine in which the water from casing enters the stationary guide wheel. The guide wheel consists of guide

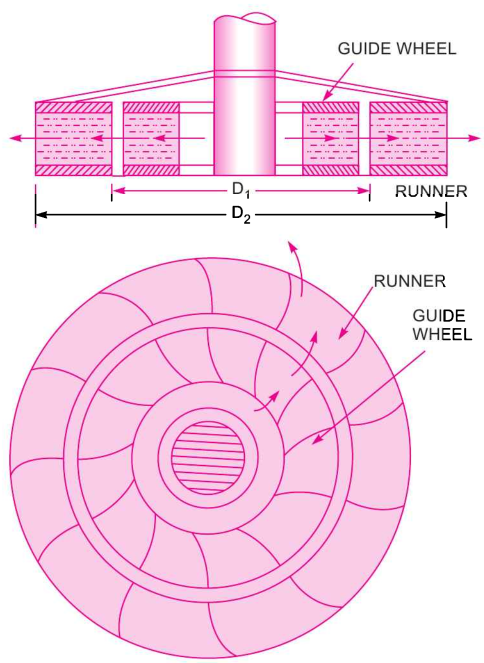

Fig. 18.18 *Outward radial flow turbine.*

vanes which direct water to enter the runner which is around the stationary guide wheel. The water flows through the vanes of the runner in the outward radial direction and is discharged at the outer diameter of the runner. The inner diameter of the runner is inlet and outer diameter is the outlet.

The velocity triangles at inlet and outlet will be drawn by the same procedure as adopted for inward flow turbine. The work done by the water on the runner per second, the horse power developed and hydraulic efficiency will be obtained from the velocity triangles. In this case as inlet of the runner is at the inner diameter of the runner, the tangential velocity at inlet will be less than that of at outlet, *i.e.*,

$$u_1 < u_2 \text{ as } D_1 < D_2.$$

Problem 18.21 *An outward flow reaction turbine has internal and external diameters of the runner as 0.6 m and 1.2 m respectively. The guide blade angle is 15° and velocity of flow through the runner is constant and equal to 4 m/s. If the speed of the turbine is 200 r.p.m., head on the turbine is 10 m and discharge at outlet is radial, determine :*

(i) The runner vane angles at inlet and outlet,
(ii) Work done by the water on the runner per second per unit weight of water striking per second ,
(iii) Hydraulic efficiency, and
(iv) The degree of reaction.

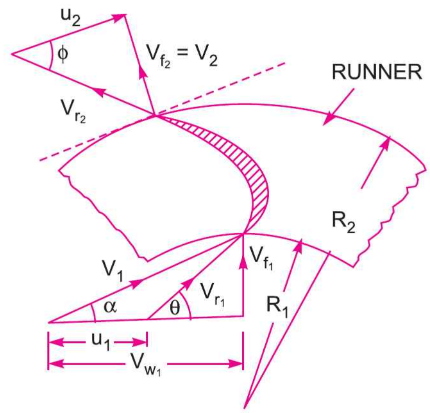

Fig. 18.19

Solution. Given :

Internal diameter, $D_1 = 0.6$ m

External diameter, $D_2 = 1.2$ m

Guide blade angle, $\alpha = 15°$

Velocity of flow, $V_{f_1} = V_{f_2} = 4$ m/s

Speed, $N = 200$ r.p.m.

Head, $H = 10$ m

Discharge at outlet = Radial

$\therefore$ $V_{w_2} = 0,\ V_{f_2} = V_2$

Tangential velocity of runner at inlet and outlet are :

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.6 \times 200}{60} = 6.283 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.566 \text{ m/s.}$$

From the inlet velocity triangle, $\tan \alpha = \dfrac{V_{f_1}}{V_{w_1}}$

$$\therefore \quad V_{w_1} = \frac{V_{f_1}}{\tan \alpha} = \frac{4.0}{\tan 15^\circ} = 14.928 \text{ m/s.}$$

(*i*) *Runner Vane Angles at inlet and outlet* are θ and ϕ

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{4.0}{(14.928 - 6.283)} = 0.4627$$

$$\theta = \tan^{-1} .4627 = 24.83 \text{ or } \mathbf{24° \ 49.8'}. \textbf{ Ans.}$$

From outlet velocity triangle, $\tan\phi = \dfrac{V_{f_2}}{u_2} = \dfrac{4.0}{12.566} = 0.3183$

$\therefore$ $\phi = \tan^{-1} .3183 =$ **17.65° or 17° 39.4′. Ans.**

(*ii*) *Work done by water per second per unit weight of water striking per second*

$$= \frac{1}{g} V_{w_1} u_1 \qquad (\because\ V_{w_2} = 0)$$

$$= \frac{1}{9.81} \times 14.928 \times 6.283 = \textbf{9.561 Nm/N. Ans.}$$

(*iii*) *Hydraulic efficiency* is given by equation (18.20*B*) as

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{14.928 \times 6.283}{9.81 \times 10} = \textbf{0.9561 or 95.61\%. Ans.}$$

(*iv*) Given : In this question, the velocity of flow is constant through the runner (*i.e.*, $V_{f_1} = V_{f_2}$) and the discharge is radial at outlet (*i.e.*, $\beta = 90°$ or $V_{w_2} = 0$), the degree of reaction (R) is given by equation (18.20*I*) as

$$R = 1 - \frac{\cot\alpha}{2(\cot\alpha - \cot\theta)}$$

Here $\alpha = 13.928°$ and $\theta = 41.09°$ (calculated)

Substituting the value of α and θ, we get

$$R = 1 - \frac{\cot 13.928°}{2(\cot 13.928° - \cot 41.09°)} = 1 - \frac{4.032}{2(4.032 - 1.146)}$$

$$= 1 - 0.698 = 0.302 \simeq \textbf{0.3. Ans.}$$

For Francis turbine, the degree of reaction varies from 0 to 1 *i.e.*, $0 \le R \le 1$.

Problem 18.22 *The internal and external diameters of an outward flow reaction turbine are 2 m and 2.75 m respectively. The turbine is running at 250 r.p.m. and rate of flow of water through the turbine is 5 m*^{3}*/s. The width of the runner is constant at inlet and outlet and is equal to 250 mm. The head on the turbine is 150 m. Neglecting thickness of the vanes and taking discharge radial at outlet determine :*

(*i*) *Vane angles at inlet and outlet, and*

(*ii*) *Velocity of flow at inlet and outlet.*

Solution. Given :

Internal diameter,	$D_1 = 2.0$ m
External diameter,	$D_2 = 2.75$ m
Speed of turbine,	$N = 250$ r.p.m.
Discharge,	$Q = 5$ m^3/s
Width at inlet and outlet,	$B_1 = B_2 = 250$ mm $= 0.25$ m
Head,	$H = 150$ m
Discharge at outlet	= radial

$\therefore$ $V_{w_2} = 0$ and $V_{f_2} = V_2$.

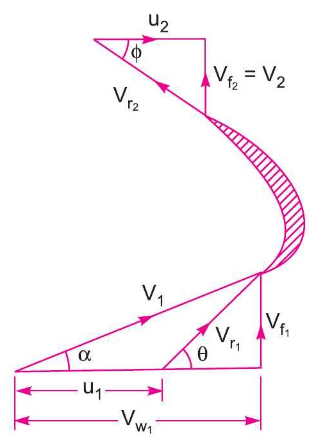

Fig. 18.20

The tangential velocity of the turbine at inlet and outlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 2.0 \times 250}{60} = 26.18 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 2.75 \times 250}{60} = 36.0 \text{ m/s}.$$

The discharge through turbine is given by equation (18.21) as

$$Q = \pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$$

$$\therefore \quad V_{f_1} = \frac{Q}{\pi D_1 B_1} = \frac{5.0}{\pi \times 2.0 \times .25} = 3.183 \text{ m/s}.$$

And $$V_{f_2} = \frac{Q}{\pi D_2 B_2} = \frac{5.0}{\pi \times 2.75 \times .25} = 2.315 \text{ m/s}$$

Using equation (18.24), $H - \dfrac{V_2^2}{2g} = \dfrac{V_{w_1} u_1}{g}$ $\qquad (\because \; V_{w_2} = 0)$

But for radial discharge, $V_2 = V_{f_2} = 2.315$ m/s

$$\therefore \quad 150 - \frac{2.315^2}{2 \times 9.81} = \frac{V_{w_2} \times 26.18}{9.81} \text{ or } 149.73 = \frac{V_{w_1} \times 26.18}{9.81}$$

$$\therefore \quad V_{w_1} = \frac{149.73 \times 9.81}{26.18} = 56.1 \text{ m/s}.$$

(*i*) *Vane angles at inlet and outlet*

From inlet velocity triangle, $\tan \theta = \dfrac{V_{f_1}}{V_{w_1} - u_1} = \dfrac{3.183}{56.1 - 26.18} = 0.1064$

$$\therefore \quad \theta = \tan^{-1} .1064 = \mathbf{6.072^\circ} \text{ or } \mathbf{6^\circ\ 4.32'}. \textbf{ Ans.}$$

From outlet velocity triangle, $\tan \phi = \dfrac{V_{f_2}}{u_2} = \dfrac{2.315}{36.0} = 0.0643$

$$\therefore \quad \phi = \tan^{-1} .0643 = \mathbf{3.68^\circ} \text{ or } \mathbf{3^\circ\ 40.8'}. \textbf{ Ans.}$$

(*ii*) *Velocity of flow at inlet and outlet*

$$V_{f_1} = 3.183 \text{ m/s and } V_{f_2} = \mathbf{2.315 \text{ m/s}}. \textbf{ Ans.}$$

▶ 18.8 FRANCIS TURBINE

The inward flow reaction turbine having radial discharge at outlet is known as Francis Turbine, after the name of J.B. Francis, an American engineer who in the beginning designed inward radial flow reaction type of turbine. In the modern Francis turbine, the water enters the runner of the turbine in the radial direction at outlet and leaves in the axial direction at the inlet of the runner. Thus the modern Francis Turbine is a mixed flow type turbine.

The velocity triangle at inlet and outlet of the Francis turbine are drawn in the same way as in case of inward flow reaction turbine. As in case of Francis turbine, the discharge is radial at outlet, the velocity of whirl at outlet (*i.e.*, V_{w_2}) will be zero. Hence the work done by water on the runner per second will be

$$= \rho Q[V_{w_1} u_1]$$

And work done per second per unit weight of water striking/s = $\frac{1}{g}\left[V_{w_1} u_1\right]$

Hydraulic efficiency will be given by, $\eta_h = \frac{V_{w_1} u_1}{gH}$.

18.8.1 Important Relations for Francis Turbines. The following are the important relations for Francis Turbines :

1. The ratio of width of the wheel to its diameter is given as $n = \frac{B_1}{D_1}$. The value of n varies from 0.10 to .40.

2. The flow ratio is given as,

Flow ratio = $\frac{V_{f_1}}{\sqrt{2gH}}$ and varies from 0.15 to 0.30.

3. The speed ratio = $\frac{u_1}{\sqrt{2gH}}$ varies from 0.6 to 0.9.

Problem 18.23 *A Francis turbine with an overall efficiency of 75% is required to produce 148.25 kW power. It is working under a head of 7.62 m. The peripheral velocity = 0.26 $\sqrt{2gH}$ and the radial velocity of flow at inlet is 0.96$\sqrt{2gH}$. The wheel runs at 150 r.p.m. and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge, determine :*

(i) The guide blade angle,
(ii) The wheel vane angle at inlet,
(iii) Diameter of the wheel at inlet, and
(iv) Width of the wheel at inlet.

Solution. Given :

Overall efficiency $\eta_o = 75\% = 0.75$

Power produced, S.P. = 148.25 kW

Head, $H = 7.62$ m

Peripheral velocity, $u_1 = 0.26\sqrt{2gH} = 0.26 \times \sqrt{2 \times 9.81 \times 7.62} = 3.179$ m/s

Velocity of flow at inlet, $V_{f_1} = 0.96\sqrt{2gH} = 0.96 \times \sqrt{2 \times 9.81 \times 7.62} = 11.738$ m/s.

Speed, $N = 150$ r.p.m.

Hydraulic losses = 22% of available energy

Discharge at outlet = Radial

$$V_{w_2} = 0 \text{ and } V_{f_2} = V_2$$

Hydraulic efficiency is given as

$$\eta_h = \frac{\text{Total head at inlet} - \text{Hydraulic loss}}{\text{Head at inlet}}$$

$$= \frac{H - .22\,H}{H} = \frac{0.78\,H}{H} = 0.78$$

But $$\eta_h = \frac{V_{w_1} u_1}{gH}$$

∴ $$\frac{V_{w_1} u_1}{gH} = 0.78$$

∴ $$V_{w_1} = \frac{0.78 \times g \times H}{u_1}$$

$$= \frac{0.78 \times 9.81 \times 7.62}{3.179} = 18.34 \text{ m/s.}$$

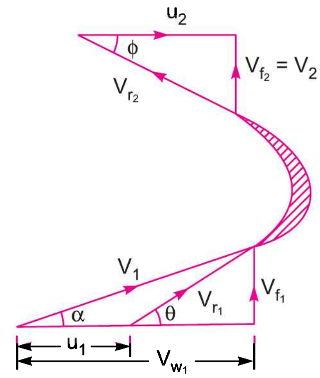

Fig. 18.21

(*i*) *The guide blade angle*, *i.e.*, α. From inlet velocity triangle,

$$\tan \alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{11.738}{18.34} = 0.64$$

∴ $$\alpha = \tan^{-1} 0.64 = \mathbf{32.619°} \text{ or } \mathbf{32° \; 37'}. \textbf{ Ans.}$$

(*ii*) *The wheel vane angle at inlet*, *i.e.*, θ

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{11.738}{18.34 - 3.179} = 0.774$$

∴ $$\theta = \tan^{-1} .774 = \mathbf{37.74} \text{ or } \mathbf{37° \; 44.4'}. \textbf{ Ans.}$$

(*iii*) *Diameter of wheel at inlet* (D_1).

Using the relation, $$u_1 = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{60 \times u_1}{\pi \times N} = \frac{60 \times 3.179}{\pi \times 50} = \mathbf{0.4047 \text{ m.}} \textbf{ Ans.}$$

(*iv*) *Width of the wheel at inlet* (B_1)

$$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{148.25}{\text{W.P.}}$$

But $$\text{W.P.} = \frac{\text{WH}}{1000} = \frac{\rho \times g \times Q \times H}{1000} = \frac{1000 \times 9.81 \times Q \times 7.62}{1000}$$

∴ $$\eta_o = \frac{148.25}{\dfrac{1000 \times 9.81 \times Q \times 7.62}{1000}} = \frac{148.25 \times 1000}{1000 \times 9.81 \times Q \times 7.62}$$

or $$Q = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times \eta_o} = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times 0.75} = 2.644 \text{ m}^3\text{/s}$$

Using equation (18.21), $$Q = \pi D_1 \times B_1 \times V_{f_1}$$

$\therefore$ $$2.644 = \pi \times .4047 \times B_1 \times 11.738$$

$\therefore$ $$B_1 = \frac{2.644}{\pi \times .4047 \times 11.738} = \textbf{0.177 m. Ans.}$$

Problem 18.24 *The following data is given for a Francis Turbine. Net head H = 60 m ; Speed N = 700 r.p.m.; shaft power = 294.3 kW ; η_o = 84% ; η_h = 93%; flow ratio = 0.20 ; breadth ratio n = 0.1; Outer diameter of the runner = 2 × inner diameter of runner. The thickness of vanes occupy 5% of circumferential area of the runner, velocity of flow is constant at inlet and outlet and discharge is radial at outlet. Determine :*

(i) Guide blade angle, *(ii) Runner vane angles at inlet and outlet,*
(iii) Diameters of runner at inlet and outlet, and *(iv) Width of wheel at inlet.*

Solution. Given :

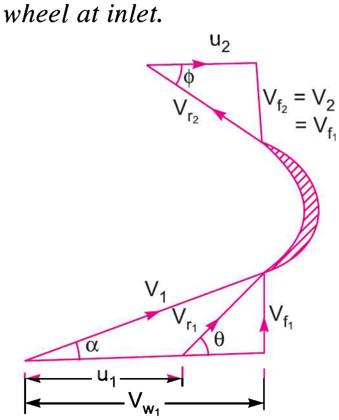

Fig. 18.22

Net head, $H = 60$ m

Speed, $N = 700$ r.p.m.

Shaft power = 294.3 kW

Overall efficiency, $\eta_o = 84\% = 0.84$

Hydraulic efficiency, $\eta_h = 93\% = 0.93$

Flow ratio, $\frac{V_{f_1}}{\sqrt{2gH}} = 0.20$

$\therefore$ $$V_{f_1} = 0.20 \times \sqrt{2gH}$$
$$= 0.20 \times \sqrt{2 \times 9.81 \times 60} = 6.862 \text{ m/s}$$

Breadth ratio, $\frac{B_1}{D_1} = 0.1$

Outer diameter, $D_1 = 2 \times \text{Inner diameter} = 2 \times D_2$

Velocity of flow, $V_{f_1} = V_{f_2} = 6.862$ m/s.

Thickness of vanes = 5% of circumferential area of runner

$\therefore$ Actual area of flow $= 0.95\ \pi D_1 \times B_1$

Discharge at outlet = Radial

$\therefore$ $V_{w_2} = 0$ and $V_{f_2} = V_2$

Using relation, $$\eta_o = \frac{\text{S.P.}}{\text{W.P.}}$$

$$0.84 = \frac{294.3}{\text{W.P.}}$$

$\therefore$ $$\text{W.P.} = \frac{294.3}{0.84} = 350.357 \text{ kW.}$$

But $$\text{W.P.} = \frac{WH}{1000} = \frac{\rho \times g \times Q \times H}{1000} = \frac{1000 \times 9.81 \times Q \times 60}{1000}$$

$\therefore$ $$\frac{1000 \times 9.81 \times Q \times 60}{1000} = 350.357$$

$\therefore$ $$Q = \frac{350.357 \times 1000}{60 \times 1000 \times 9.81} = 0.5952 \text{ m}^3/\text{s}.$$

Using equation (18.21), Q = Actual area of flow × Velocity of flow

$$= 0.95\ \pi D_1 \times B_1 \times V_{f_1}$$

$$= 0.95 \times \pi \times D_1 \times 0.1 D_1 \times V_{f_2} \qquad (\because \ B_1 = 0.1\ D_1)$$

or $$0.5952 = 0.95 \times \pi \times D_1 \times 0.1 \times D_1 \times 6.862 = 2.048\ D_1^2$$

$\therefore$ $$D_1 = \sqrt{\frac{0.5952}{2.048}} = 0.54 \text{ m}$$

But $$\frac{B_1}{D_1} = 0.1$$

$\therefore$ $$B_1 = 0.1 \times D_1 = 0.1 \times .54 = .054 \text{ m} = 54 \text{ mm}$$

Tangential speed of the runner at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.54 \times 700}{60} = 19.79 \text{ m/s}.$$

Using relation for hydraulic efficiency,

$$\eta_h = \frac{V_{w_1} u_1}{gH} \text{ or } 0.93 = \frac{V_{w_1} \times 19.79}{9.81 \times 60}$$

$\therefore$ $$V_{w_1} = \frac{0.93 \times 9.81 \times 60}{19.79} = 27.66 \text{ m/s}.$$

(*i*) *Guide blade angle* (α)

From inlet velocity triangle, $\tan \alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{6.862}{27.66} = 0.248$

$\therefore$ $$\alpha = \tan^{-1} 0.248 = 13.928° \text{ or } \mathbf{13° \ 55.7'}. \textbf{ Ans.}$$

(*ii*) *Runner vane angles at inlet and outlet* (θ *and* ϕ)

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{6.862}{27.66 - 19.79} = 0.872$$

$\therefore$ $$\theta = \tan^{-1} 0.872 = 41.09° \text{ or } \mathbf{41° \ 5.4'}. \textbf{ Ans.}$$

From outlet velocity triangle, $\tan \phi = \frac{V_{f_2}}{u_2} = \frac{V_{f_1}}{u_2} = \frac{6.862}{u_2}$ ...(*i*)

But $$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times D_1}{2} \times \frac{N}{60} \qquad \left(\because \ D_2 = \frac{D_1}{2} \text{ given}\right)$$

$$= \pi \times \frac{.54}{2} \times \frac{700}{60} = 9.896 \text{ m/s}.$$

Substituting the value of u_2 in equation (*i*),

$$\tan \phi = \frac{6.862}{9.896} = 0.6934$$

$\therefore$ $\phi = \tan^{-1} .6934° = 34.74$ or **34° 44.4′. Ans.**

(*iii*) *Diameters of runner at inlet and outlet*

$D_1 = 0.54$ m, $D_2 =$ **0.27 m. Ans.**

(*iv*) *Width of wheel at inlet*

$B_1 =$ **54 mm. Ans.**

Problem 18.24 (A) *For the above problem, find the degree of reaction for the given Francis Turbine.*

Solution. Given :

In this question, the velocity of flow is constant through the runner (*i.e.*, $V_{f_1} = V_{f_2}$) and the discharge is radial at outlet (*i.e.*, $\beta = 90°$ or $V_{w_2} = 0$), the degree of reaction (R) is given by equation (18.20 I) as

$$R = 1 - \frac{\cot \alpha}{2(\cot \alpha - \cot \theta)}$$

Here $\alpha = 13.928°$ and $\theta = 41.09°$ (calculated)

Substituting the value of α and θ, we get

$$R = 1 - \frac{\cot 13.928°}{2(\cot 13.928° - \cot 41.09°)} = 1 - \frac{4.032}{2(4.032 - 1.146)}$$

$= 1 - 0.698 = 0.302 \simeq$ **0.3. Ans.**

For Francis Turbine, the degree of reaction varies from 0 to 1 *i.e.*, $0 \le R \le 1$.

Problem 18.25 *(a) Show that the hydraulic efficiency for a Francis turbine having velocity of flow through runner as constant, is given by the relation.*

$$\eta_h = \frac{1}{1 + \dfrac{\frac{1}{2}\tan^2\alpha}{\left(1 - \dfrac{\tan\alpha}{\tan\theta}\right)}}$$

where α *= Guide blade angle and* θ *= Runner vane angle at inlet.*

The turbine is having radial discharge at outlet.

(b) If vanes are radial at inlet, then show $\eta_h = \dfrac{2}{2 + \tan^2\alpha}$.

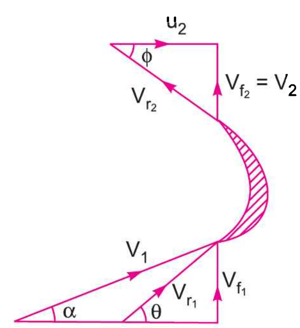

Fig. 18.23

Solution. Given :

Velocity of flow = Constant.

$\therefore$ $V_{f_1} = V_{f_2}$

Discharge is radial at outlet.

$\therefore$ $V_{w_2} = 0$ and $V_{f_2} = V_2$

(*a*) From the inlet velocity triangle,

$$\tan \alpha = \frac{V_{f_1}}{V_{w_1}} \quad \therefore \quad V_{f_1} = V_{w_1} \tan \alpha \qquad ...(i)$$

Also

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1}$$

or

$$\left(V_{w_1} - u_1\right) = \frac{V_{f_1}}{\tan \theta} = \frac{V_{w_1} \tan \alpha}{\tan \theta} \qquad \left(\because \; V_{f_1} = V_{w_1} \tan \alpha\right)$$

$$\therefore \quad u_1 = V_{w_1} - \frac{V_{w_1} \tan \alpha}{\tan \theta} = V_{w_1}\left(1 - \frac{\tan \alpha}{\tan \theta}\right) \qquad ...(ii)$$

Using equation (18.24), we have

$$H - \frac{V_2^2}{2g} = \frac{1}{g}(V_{w_1} u_1) \qquad \left(\because \; V_{w_2} = 0\right)$$

$$\therefore \quad H = \frac{1}{g} V_{w_1} u_1 + \frac{V_2^2}{2g} = \frac{1}{g} V_{w_1} u_1 + \frac{V_{f_1}^2}{2g} \qquad \left(\because \; V_2 = V_{f_2} = V_{f_1}\right)$$

Substituting the values of V_{f_1} and u_1 from equations (*i*) and (*ii*),

$$H = \frac{1}{g} V_{w_1} \times V_{w_1}\left(1 - \frac{\tan \alpha}{\tan \theta}\right) + \frac{\left[V_{w_1} \tan \alpha\right]^2}{2g}$$

$$= \frac{V_{w_1}^2}{g}\left(1 - \frac{\tan \alpha}{\tan \theta}\right) + \frac{V_{w_1}^2}{2g} \tan^2 \alpha = \frac{V_{w_1}^2}{g}\left[1 - \frac{\tan \alpha}{\tan \theta} + \frac{\tan^2 \alpha}{2}\right].$$

Now, hydraulic efficiency is given by

$$\eta_h = \frac{V_{w_1} u_1}{gH} = \frac{V_{w_1} u_1}{g \times \frac{V_{w_1}^2}{g}\left[1 - \frac{\tan \alpha}{\tan \theta} + \frac{\tan^2 \alpha}{2}\right]}$$

$$= \frac{V_{w_1} \times V_{w_1}\left(1 - \frac{\tan \alpha}{\tan \theta}\right)}{V_{w_1}^2\left[1 - \frac{\tan \alpha}{\tan \theta} + \frac{\tan^2 \alpha}{2}\right]} \qquad \left[\because \; u_1 = V_{w_1}\left(1 - \frac{\tan \alpha}{\tan \theta}\right)\right]$$

$$= \frac{\left(1 - \frac{\tan \alpha}{\tan \theta}\right)}{\left[1 - \frac{\tan \alpha}{\tan \theta} + \frac{\tan^2 \alpha}{2}\right]} = \frac{1}{1 + \frac{\frac{1}{2}\tan^2 \alpha}{\left(1 - \frac{\tan \alpha}{\tan \theta}\right)}}. \textbf{ Ans.}$$

(*b*) If vanes are radial at inlet, then $\theta = 90°$

$$\therefore \quad \eta_h = \frac{1}{1+\dfrac{\frac{1}{2}\tan^2\alpha}{\left(1-\dfrac{\tan\alpha}{\tan 90°}\right)}} = \frac{1}{1+\dfrac{\frac{1}{2}\tan^2\alpha}{(1-0)}} = \mathbf{\frac{2}{2+\tan^2\alpha}}. \textbf{ Ans.}$$

Problem 18.26 *A Francis turbine working under a head of 30 m has a wheel diameter of 1.2 m at the entrance and 0.6 m at the exit. The vane angle at the entrance is 90° and guide blade angle is 15°. The water at the exit leaves the vanes without any tangential velocity and the velocity of flow in the runner is constant. Neglecting the effect of draft tube and losses in the guide and runner passages, determine the speed of wheel in r.p.m. and vane angle at the exit. State whether the speed calculated is synchronous or not. If not, what speed would you recommend to couple the turbine with an alternator of 50 cycles ?*

Solution. Given :

Head on turbine, $H = 30$ m

Inlet dia., $D_1 = 1.2$ m

Outlet dia., $D_2 = 0.6$ m

Vane angle at inlet, $\theta = 90°$

Guide blade angle, $\alpha = 15°$

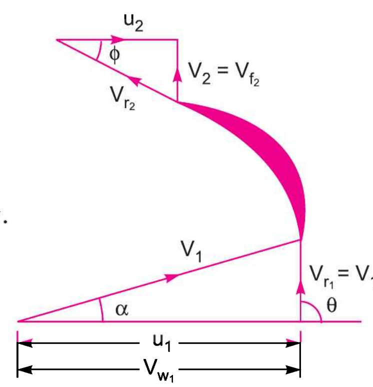

Fig. 18.24

The water at exit leaves the vanes without any tangential velocity.

$$\therefore \quad V_{w_2} = 0 \text{ and } V_2 = V_{f_2}$$

Velocity of flow is constant in runner.

$$\therefore \quad V_{f_1} = V_{f_2}$$

(*i*) *Speed of turbine in r.p.m.*

Using equation (18.24), we have

$$H - \frac{V_2^2}{2g} = \frac{1}{g}\left(V_{w_1}u_1 \pm V_{w_2}u_2\right)$$

$$= \frac{1}{g}\left(V_{w_1} \times u_1\right) \qquad (\because \; V_{w_2} = 0)$$

$$= \frac{1}{g}u_1 \times u_1 \qquad (\because \; V_{w_1} = u_1)$$

or
$$30 - \frac{V_{f_2}^2}{2g} = \frac{1}{g}u_1^2 \qquad (\because \; V_2 = V_{f_2} = V_{f_1}) \quad ...(i)$$

But from inlet velocity triangle, we have

$$\tan\alpha = \frac{V_{f_1}}{u_1} \text{ or } u_1 = \frac{V_{f_1}}{\tan\alpha} = \frac{V_{f_1}}{\tan 15°} = 3.732\,V_{f_1} \quad ...(ii)$$

Substituting the values of u_1 in equation (*i*), we get

$$30 - \frac{V_{f_2}^2}{2g} = \frac{1}{g} \times (3.732\;V_{f_1})^2 \text{ or } 30 - \frac{V_{f_1}^2}{2g} = \frac{13.928\,V_{f_2}^2}{g} \qquad (\because \; V_{f_2} = V_{f_1})$$

or $$30 = \frac{14.928\, V_{f_1}^2}{g}$$

∴ $$V_{f_1} = \sqrt{\frac{30 \times 9.81}{14.928}} = 4.44 \text{ m/s.}$$

Substituting the value of V_{f_1} in equation (*ii*), we get

$$u_1 = 3.732 \times 4.44 = 16.57 \text{ m/s}$$

But $$u_1 = \frac{\pi D_1 N}{60} \text{ or } 16.57 = \frac{\pi \times 1.2 \times N}{60}$$

∴ $$N = \frac{16.57 \times 60}{\pi \times 1.2} = \mathbf{263.72 \text{ r.p.m. Ans.}}$$

(*ii*) *Vane angle at exit* (*i.e.*, ϕ)

$$u_2 = \frac{\pi D_2 \times N}{60} = \frac{\pi \times 0.6 \times 263.72}{60} = 8.285 \text{ m/s}$$

$$V_{f_2} = V_{f_1} = 4.44$$

Now, from velocity triangle at outlet,

$$\tan \phi = \frac{V_{f_2}}{u_2} = \frac{4.44}{8.285} = 0.5359$$

∴ $$\phi = \mathbf{28.187°.\ Ans.}$$

(*iii*) For a turbine, which is directly coupled to the alternator of 50 cycles, the synchronous speed (N^*) is given by

$$f = \frac{p.N^*}{60}$$

where f = Frequency of alternator in cycles/s, p = Number of pair of poles for the alternator.

Assuming the number of pair of poles = 12, we get

$$50 = \frac{12 \times N^*}{60}$$

∴ $$N^* = \frac{60 \times 50}{12} = 250 \text{ r.p.m.}$$

But the speed of turbine is 263.72. And synchronous speed (N^*) is equal to 250. Hence, the speed of turbine is not synchronous. The speed of turbine should be 250 r.p.m.

▶ 18.9 AXIAL FLOW REACTION TURBINE

If the water flows parallel to the axis of the rotation of the shaft, the turbine is known as axial flow turbine. And if the head at the inlet of the turbine is the sum of pressure energy and kinetic energy and during the flow of water through runner a part of pressure energy is converted into kinetic energy, the turbine is known as reaction turbine.

For the axial flow reaction turbine, the shaft of the turbine is vertical. The lower end of the shaft is made larger which is known as 'hub' or 'boss'. The vanes are fixed on the hub and hence hub acts as a runner for axial flow reaction turbine. The following are the important type of axial flow reaction turbines :

1. Propeller Turbine, and
2. Kaplan Turbine.

When the vanes are fixed to the hub and they are not adjustable, the turbine is known as propeller turbine. But if the vanes on the hub are adjustable, the turbine is known as a *Kaplan Turbine*, after the name of V Kaplan, an Austrian Engineer. This turbine is suitable where a large quantity of water at low head is available. Fig. 18.25 shows the runner of a Kaplan turbine, which consists of a hub fixed to the shaft. On the hub, the adjustable vanes are fixed as shown in Fig. 18.25.

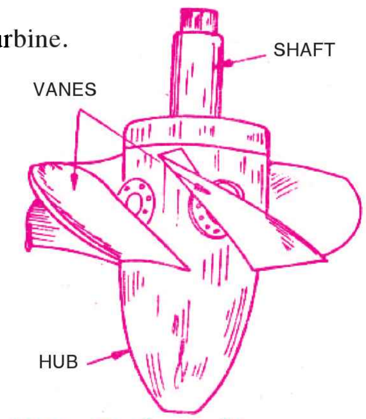

Fig. 18.25 *Kaplan turbine runner.*

The main parts of a Kaplan turbine are :

1. Scroll casing,
2. Guide vanes mechanism,
3. Hub with vanes or runner of the turbine, and
4. Draft tube.

Fig. 18.26 shows all main parts of a Kaplan turbine. The water from penstock enters the scroll casing and then moves to the guide vanes. From the guide vanes, the water turns through 90° and flows axially through the runner as shown in Fig. 18.26. The discharge through the runner is obtained as

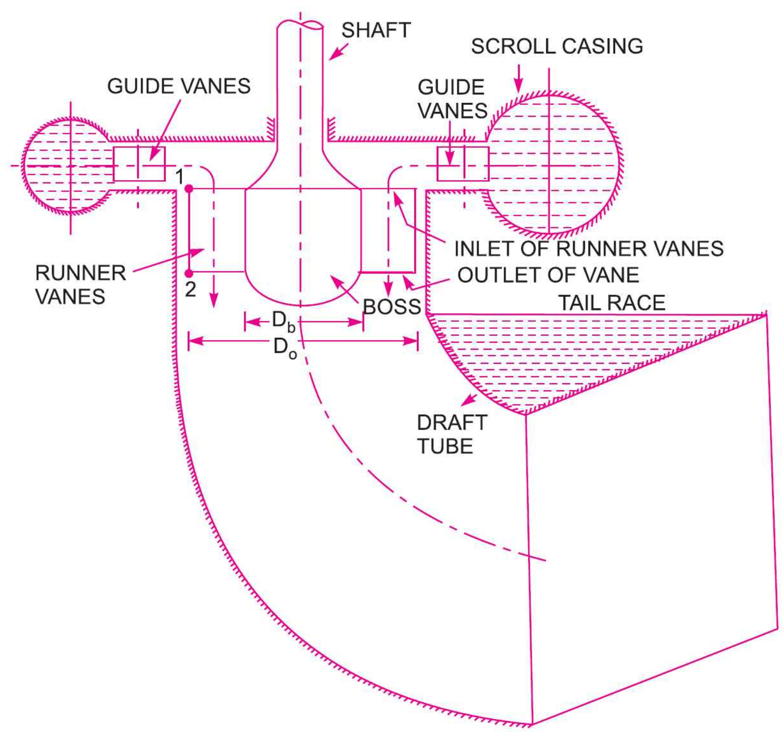

Fig. 18.26 *Main components of Kaplan turbine.*

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^3\right) \times V_{f_1} \qquad \text{...(18.25)}$$

where D_o = Outer diameter of the runner,

D_b = Diameter of hub, and

V_{f_1} = Velocity of flow at inlet.

The inlet and outlet velocity triangles are drawn at the extreme edge of the runner vane corresponding to the points 1 and 2 as shown in Fig. 18.26.

18.9.1 Some Important Point for Propeller (Kaplan Turbine). The following are the important points for propeller or Kaplan turbine :

1. The peripheral velocity at inlet and outlet are equal

$$\therefore \qquad u_1 = u_2 = \frac{\pi D_o N}{60}, \text{ where } D_o = \text{Outer dia. of runner}$$

2. Velocity of flow at inlet and outlet are equal

$$\therefore \qquad V_{f_1} = V_{f_2}.$$

3. Area of flow at inlet = Area of flow at outlet

$$= \frac{\pi}{4}\left(D_o^2 - D_b^2\right).$$

Problem 18.27 *A Kaplan turbine working under a head of 20 m develops 11772 kW shaft power. The outer diameter of the runner is 3.5 m and hub diameter is 1.75 m. The guide blade angle at the extreme edge of the runner is 35°. The hydraulic and overall efficiencies of the turbines are 88% and 84% respectively. If the velocity of whirl is zero at outlet, determine :*

(i) Runner vane angles at inlet and outlet at the extreme edge of the runner, and

(ii) Speed of the turbine.

Solution. Given :

Head, $H = 20$ m

Shaft power, S.P. = 11772 kW

Outer dia. of runner, $D_o = 3.5$ m

Hub diameter, $D_b = 1.75$ m

Guide blade angle, $\alpha = 35°$

Hydraulic efficiency, $\eta_h = 88\%$

Overall efficiency, $\eta_o = 84\%$

Velocity of whirl at outlet = 0.

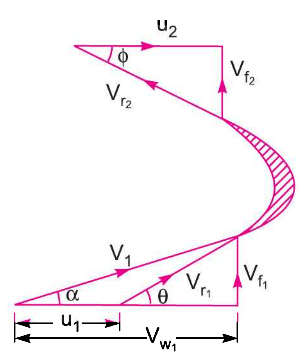

Fig. 18.27

Using the relation, $\eta_o = \dfrac{\text{S.P.}}{\text{W.P.}}$

where $\text{W.P.} = \dfrac{\text{W.P.}}{1000} = \dfrac{\rho \times g \times Q \times H}{1000}$, we get

$$0.84 = \frac{11772}{\dfrac{\rho \times g \times Q \times H}{1000}}$$

$$= \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 20} \qquad (\because \rho = 1000)$$

$$\therefore \qquad Q = \frac{11772 \times 1000}{0.84 \times 1000 \times 9.81 \times 20} = 71.428 \text{ m}^3\text{/s.}$$

Using equation (18.25), $Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$

or

$$71.428 = \frac{\pi}{4}(3.5^2 - 1.75^2) \times V_{f_1} = \frac{\pi}{4}(12.25 - 3.0625)\, V_{f_1}$$

$$= 7.216\, V_{f_1}$$

$$\therefore \qquad V_{f_1} = \frac{71.428}{7.216} = 9.9 \text{ m/s.}$$

From inlet velocity triangle, $\tan \alpha = \frac{V_{f_2}}{V_{w_1}}$

$$\therefore \qquad V_{w_1} = \frac{V_{f_1}}{\tan \alpha} = \frac{9.9}{\tan 35^\circ} = \frac{9.9}{.7} = 14.14 \text{ m/s}$$

Using the relation for hydraulic efficiency,

$$\eta_h = \frac{V_{w_1} u_1}{gH} \qquad (\because V_{w_2} = 0)$$

$$0.88 = \frac{14.14 \times u_1}{9.81 \times 20}$$

$$\therefore \qquad u_1 = \frac{0.88 \times 9.81 \times 20}{14.14} = 12.21 \text{ m/s.}$$

(*i*) Runner vane angles at inlet and outlet at the extreme edge of the runner are given as :

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{9.9}{(14.14 - 12.21)} = 5.13$$

$$\therefore \qquad \theta = \tan^{-1} 5.13 = \mathbf{78.97^\circ} \text{ or } \mathbf{78^\circ\, 58'}. \textbf{ Ans.}$$

For Kaplan turbine, $u_1 = u_2 = 12.21$ m/s and $V_{f_1} = V_{f_2} = 9.9$ m/s

$\therefore$ From outlet velocity triangle, $\tan \phi = \frac{V_{f_2}}{u_2} = \frac{9.9}{12.21} = 0.811$

$$\therefore \qquad \phi = \tan^{-1} .811 = \mathbf{39.035^\circ} \text{ or } \mathbf{39^\circ\, 2'}. \textbf{ Ans.}$$

(*ii*) Speed of turbine is given by $u_1 = u_2 = \frac{\pi D_o N}{60}$

$$12.21 = \frac{\pi \times 3.5 \times N}{60}$$

$$\therefore \qquad N = \frac{60 \times 12.21}{\pi \times 3.50} = \mathbf{66.63 \text{ r.p.m.}} \textbf{ Ans.}$$

Problem 18.28 *A Kaplan turbine develops 24647.6 kW power at an average head of 39 metres. Assuming a speed ratio of 2, flow ratio of 0.6, diameter of the boss equal to 0.35 times the diameter of the runner and an overall efficiency of 90%, calculate the diameter, speed and specific speed of the turbine.*

Solution. Given :

Shaft power, S.P. = 24647.6 kW

Head, $H = 39$ m

Speed ratio, $u_1\sqrt{2gH} = 2.0$

$\therefore \quad u_1 = 2.0 \times \sqrt{2gH} = 2.0 \times \sqrt{2 \times 9.81 \times 39} = 55.32$ m/s

Flow ratio, $\dfrac{V_{f_1}}{\sqrt{2gH}} = 0.6$

$\therefore \quad V_{f_1} = 0.6 \times \sqrt{2gH} = 0.6 \times \sqrt{2 \times 9.81 \times 39} = 16.59$ m/s

Diameter of boss = 0.35 × Diameter of runner

$\therefore \quad D_b = 0.35 \times D_o$

Overall efficiency, $\eta_o = 90\% = 0.90$

Using the relation, $\eta_o = \dfrac{\text{S.P.}}{\text{W.P.}}$, where W.P. $= \dfrac{\rho \times g \times Q \times H}{1000}$

$$\therefore \quad 0.90 = \frac{24647.6}{\dfrac{\rho \times g \times Q \times H}{1000}} = \frac{24647.6 \times 1000}{1000 \times 9.81 \times Q \times 39}$$

$$\therefore \quad Q = \frac{24647.6 \times 1000}{0.9 \times 1000 \times 9.81 \times 39} = 71.58 \text{ m}^3\text{/s.}$$

But from equation (18.25), we have

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

$$\therefore \quad 71.58 = \frac{\pi}{4}\left[D_o^2 - (.35\, D_o)^2\right] \times 16.59 \qquad \left(\because \ D_b = 0.35\, D_o,\ V_{f_1} = 16.59\right)$$

$$= \frac{\pi}{4}\left[D_o^2 - .1225\, D_o^2\right] \times 16.59$$

$$= \frac{\pi}{4} \times .8775\, D_o^2 \times 16.59 = 11.433\, D_o^2$$

(*i*) $\therefore \quad D_o = \sqrt{\dfrac{71.58}{11.433}} = $ **2.5 m. Ans.**

$\therefore \quad D_b = 0.35 \times D_o = 0.35 \times 2.5 = $ **0.875 m. Ans.**

(*ii*) Speed of the turbine is given by $u_1 = \dfrac{\pi D_o N}{60}$

$$\therefore \quad 55.32 = \frac{\pi \times 2.5 \times N}{60}$$

$\therefore$ $$N = \frac{60 \times 55.32}{\pi \times 2.5} = \textbf{422.61 r.p.m. Ans.}$$

(*iii*) Specific speed * is given by $N_s = \frac{N\sqrt{P}}{H^{5/4}}$, where P = Shaft power in kW

$\therefore$ $$N_s = \frac{422.61 \times \sqrt{24647.6}}{(39)^{5/4}} = \frac{422.61 \times 156.99}{97.461} = \textbf{680.76 r.p.m. Ans.}$$

Problem 18.29 *A Kaplan turbine runner is to be designed to develop 9100 kW. The net available head is 5.6 m. If the speed ratio = 2.09, flow ratio = 0.68, overall efficiency = 86% and the diameter of the boss is 1/3 the diameter of the runner. Find the diameter of the runner, its speed and the specific speed of the turbine.*

Solution. Given :

Power, $P = 9100$ kW

Net head, $H = 5.6$ m

Speed ratio $= 2.09$

Flow ratio $= 0.68$

Overall efficiency, $\eta_o = 86\% = 0.86$

Diameter of boss $= \frac{1}{3}$ of diameter of runner

or $$D_b = \frac{1}{3} D_o$$

Now, speed ratio $= \frac{u_1}{\sqrt{2gH}}$

$\therefore$ $$u_1 = 2.09 \times \sqrt{2 \times 9.81 \times 5.6} = 21.95 \text{ m/s}$$

Flow ratio $= \frac{V_{f_1}}{\sqrt{2gH}}$

$\therefore$ $$V_{f_1} = 0.68 \times \sqrt{2 \times 9.81 \times 5.6} = 7.12 \text{ m/s}$$

The overall efficiency is given by, $\eta_o = \frac{P}{\left(\frac{\rho \times g.Q.H}{1000}\right)}$

or $$Q = \frac{P \times 1000}{\rho \times g \times H \times \eta_o} = \frac{9100 \times 1000}{1000 \times 9.81 \times 5.6 \times 0.86}$$

$$(\because \ \rho g = 1000 \times 9.81 \text{ N/m}^3)$$

$$= 192.5 \text{ m}^3/\text{s}.$$

The discharge through a Kaplan turbine is given by

$$Q = \frac{\pi}{4}[D_o^2 - D_b^2] \times V_{f_1}$$

* For the definition and derivation, please refer to page 920 Arts. 18.11 and 18.11.1.

or $$192.5 = \frac{\pi}{4}\left[D_o^2 - \left(\frac{D_o}{3}\right)^2\right] \times 7.12 \qquad \left(\because\ D_b = \frac{D_o}{3}\right)$$

$$= \frac{\pi}{4}\left[1 - \frac{1}{9}\right] D_o^2 \times 7.12$$

∴ $$D_o = \sqrt{\frac{4 \times 192.5 \times 9}{\pi \times 8 \times 7.12}} = \textbf{6.21 m. Ans.}$$

The speed of turbine is given by, $u_1 = \frac{\pi D N}{60}$

∴ $$N = \frac{60 \times u_1}{\pi \times D} = \frac{60 \times 21.95}{\pi \times 6.21} = \textbf{67.5 r.p.m. Ans.}$$

The specific speed is given by, $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{67.5 \times \sqrt{9100}}{5.6^{5/4}} = \textbf{746. Ans.}$

Problem 18.30 *The hub diameter of a Kaplan turbine, working under a head of 12 m, is 0.35 times the diameter of the runner. The turbine is running at 100 r.p.m. If the vane angle of the extreme edge of the runner at outlet is 15° and flow ratio is 0.6, find :*

(i) Diameter of the runner, *(ii) Diameter of the boss, and*

(iii) Discharge through the runner.

The velocity of whirl at outlet is given as zero.

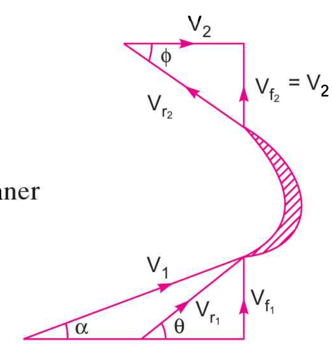

Fig. 18.28

Solution. Given :

Head, $H = 12$ m

Hub diameter, $D_b = 0.35 \times D_o$, where D_o = Dia. of runner

Speed, $N = 100$ r.p.m.

Vane angle at outlet, $\phi = 15°$

Flow ratio $= \frac{V_{f_1}}{\sqrt{2gH}} = 0.6$

∴ $$V_{f_1} = 0.6 \times \sqrt{2gH} = 0.6 \times \sqrt{2 \times 9.81 \times 12}$$

$$= 9.2 \text{ m/s.}$$

From the outlet velocity triangle, $V_{w_2} = 0$

$$\tan \phi = \frac{V_{f_2}}{u_2} = \frac{V_{f_1}}{u_2} \qquad (\because\ V_{f_2} = V_{f_1} = 9.2)$$

∴ $$\tan 15° = \frac{9.2}{u_2}$$

∴ $$u_2 = \frac{9.2}{\tan 15°} = 34.33 \text{ m/s.}$$

But for Kaplan turbine, $u_1 = u_2 = 34.33$

Now, using the relation, $u_1 = \frac{\pi D_o \times N}{60}$ or $34.33 = \frac{\pi \times D_o \times 100}{60}$

$$D_o = \frac{60 \times 34.33}{\pi \times 100} = \textbf{6.55 m. Ans.}$$

$\therefore$ $$D_b = 0.35 \times D_o = 0.35 \times 6.35 = \textbf{2.3 m. Ans.}$$

Discharge through turbine is given by equation (18.25) as

$$Q = \frac{\pi}{4}\left[D_o^2 - D_b^2\right] \times V_{f_1} = \frac{\pi}{4}\left[6.55^2 - 2.3^2\right] \times 9.2$$

$$= \frac{\pi}{4}(42.9026 - 5.29) \times 9.2 = \textbf{271.77 m}^3\textbf{/s. Ans.}$$

Problem 18.31 *A Kaplan turbine runner is to be designed to develop 7357.5 kW shaft power. The net available head is 5.50 m. Assume that the speed ratio is 2.09 and flow ratio is 0.68, and the overall efficiency is 60%. The diameter of the boss is $\frac{1}{3}$rd of the diameter of the runner. Find the diameter of the runner, its speed and its specific speed.*

Solution. Given :

Shaft power, $P = 7357.5$ kW

Head, $H = 5.50$ m

Speed ratio $= \dfrac{u_1}{\sqrt{2gH}} = 2.09$

$\therefore$ $u_1 = 2.09 \times \sqrt{2 \times 9.81 \times 5.50} = 21.71$ m/s

Flow ratio $= \dfrac{V_{f_1}}{\sqrt{2gH}} = 0.68$

$\therefore$ $V_{f_1} = 2.68 \times \sqrt{2 \times 9.81 \times 5.50} = 7.064$ m/s

Overall efficiency, $\eta_o = 60\% = 0.60$

Diameter of boss, $D_b = \dfrac{1}{3} \times D_o$

Using relation, $$\eta_o = \frac{\text{Shaft power}}{\text{Water power}} = \frac{7357.5}{\dfrac{\rho \times g \times Q \times H}{1000}}$$

or $$0.60 = \frac{7357.5 \times 1000}{\rho \times g \times Q \times H} = \frac{7357.5 \times 1000}{1000 \times 9.81 \times Q \times 5.5}$$

$\therefore$ $$Q = \frac{7357.5 \times 1000}{1000 \times 9.81 \times 5.5 \times 0.60} = 227.27 \text{ m}^3\text{/s.}$$

Using equation (18.25) for discharge,

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

or $$227.27 = \frac{\pi}{4}\left[D_o^2 - \left(\frac{D_o}{3}\right)^2\right] \times 7.064 \qquad \left(\because\ D_b = \frac{D_o}{3}\right)$$

$$= \frac{\pi}{4} \times \frac{8}{9} D_o^2 \times 7.064 = 4.9316\ D_o^2$$

$\therefore$ $$D_o = \sqrt{\frac{227.27}{4.9316}} = \mathbf{6.788\ m.\ Ans.}$$

And $$D_b = \frac{1}{3} \times 6.788 = \mathbf{2.262\ m.\ Ans.}$$

Using the relation, $$u_1 = \frac{\pi D_o \times N}{60}$$

$\therefore$ $$N = \frac{60 \times u_1}{\pi D_o} = \frac{60 \times 21.71}{\pi \times 6.788} = \mathbf{61.08\ r.p.m.\ Ans.}$$

The specific speed (N_s) is given by,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{61.08 \times \sqrt{7357.5}}{5.50^{5/4}} = \mathbf{622\ r.p.m.\ Ans.}$$

Problem 18.32 *In a tidal power plant, a bulb turbine (which is basically an axial flow turbine) operates a 5 MW generator at 150 r.p.m. under a head of 5.5 m. The generator efficiency is 93% and the overall efficiency of the turbine is 88%. The tip diameter of the runner is 4.5 m and hub diameter is 2 m. Assuming hydraulic efficiency of 94% and no exit whirl, determine the runner vane angles at inlet and exit at the mean diameter of the vanes.*

Solution. Given :

Output of generator $= 5$ M* W $= 5 \times 10^6$ W

Speed of turbine, $N = 150$ r.p.m.

Head on turbine, $H = 5.5$ m

Generator efficiency, $\eta_g = 93\%$ or 0.93

Overall efficiency of the turbine, $\eta_o = 88\%$ or 0.88

Tip dia. of runner, $D_o = 4.5$ m

Hub dia. of runner, $D_b = 2$ m

Hydraulic efficiency, $\eta_h = 94\%$ or 0.94

No exit whirl means the velocity of whirl at outlet is zero *i.e.*, $V_{w_2} = 0$. And hence angle $\beta = 90°$ as shown in Fig. 18.29.

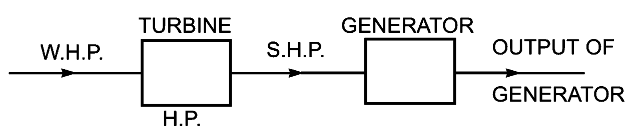

Fig. 18.29

Now, generator efficiency is given by,

$$\eta_g = \frac{\text{Output of generator}}{\text{Input of generator}}$$

$$= \frac{\text{Output of generator}}{\text{Output of turbine}} \quad (\because \text{ Output of turbine = Input of generator})$$

or $$0.93 = \frac{5 \times 10^6}{\text{S.P.}}$$

$\therefore$ $$\text{S.P.} = \frac{5 \times 10^6}{0.93} \text{ W} \qquad ...(i)$$

* MW stands for Mega Watt which is equal to 10^6 Watt or 10^6 W.

Now, overall efficiency of turbine is given by, $\eta_o = \dfrac{\text{S.P.}}{\text{W.P.}}$

But W.P. in Watt $= \rho \times Q \times H \times 9.81 = 1000 \times Q \times 5.5 \times 9.81$ W

$$\therefore \quad \eta_o = \frac{\text{S.P.}}{1000 \times Q \times 5.5 \times 9.81}$$

$$\therefore \quad \text{S.P.} = \eta_o \times 1000 \times Q \times 5.5 \times 9.81 \qquad ...(ii)$$

Equating the two values of S.P. given by equations (*i*) and (*ii*), we get

$$\frac{5 \times 10^6}{0.93} = 0.88 \times 1000 \times Q \times 5.5 \times 9.81 \qquad (\because \eta_o = 0.88)$$

$$\therefore \quad Q = \frac{5 \times 10^6}{0.93 \times 0.88 \times 1000 \times 5.5 \times 9.81} = 113.23 \text{ m}^3/\text{s}$$

The vane angles are to be calculated at the mean diameter of the runner.

$$\therefore \text{ Mean diameter,} \quad D_m = \frac{D_o + D_b}{2} = \frac{4.5 + 2.0}{2} = 3.25 \text{ m}$$

Inlet vane velocity corresponding to mean dia. is given by,

$$u_1 = \frac{\pi D_m \times N}{60} = \frac{\pi \times 3.25 \times 150}{60} = 25.52 \text{ m/s}$$

For axial flow turbine, $u_1 = u_2 = 25.52$ m/s and $V_{f_1} = V_{f_2}$

For no whirl velocity at outlet, the hydraulic efficiency is given by,

$$\eta_h = \frac{V_{w_2} \times u_1}{g \times H} \text{ or } 0.94 = \frac{V_{w_1} \times 25.52}{9.81 \times 5.5}$$

$$\therefore \quad V_{w_1} = \frac{0.94 \times 9.81 \times 5.5}{25.52} = 1.987 \text{ m/s}$$

This value of V_{w_1} is less than u_1. Hence, the velocity triangle at inlet will be as shown in Fig. 18.30.

Now, using equation (18.25), we get

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

or

$$113.23 = \frac{\pi}{4}(4.5^2 - 2^2) \times V_{f_1} = \frac{\pi}{4} \times 16.25 \times V_{f_1}$$

$$\therefore \quad V_{f_1} = \frac{113.23 \times 4}{\pi \times 16.25} = 8.87 \text{ m/s}$$

$$\therefore \quad V_{f_2} = V_{f_1} = 8.87 \text{ m/s.}$$

Let θ = Runner vane angle at inlet and

ϕ = Runner vane angle at outlet.

From inlet velocity triangle,

$$\tan\theta = \frac{V_{f_1}}{\left(u_1 - V_{w_1}\right)} = \frac{8.87}{25.52 - 1.987}$$

$$= 0.3769$$

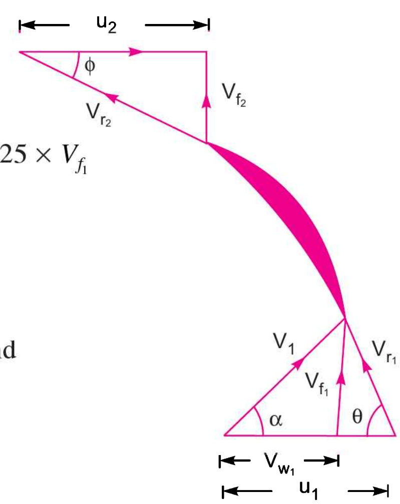

Fig. 18.30

$\therefore$ $\theta = \tan^{-1} 0.3769 = $ **20.65°. Ans.**

Now, from outlet velocity triangle,

$$\tan \phi = \frac{V_{f_2}}{u_2} = \frac{8.87}{25.52} \qquad (\because V_{f_2} = V_{f_1} \text{ and } u_1 = u_2)$$

$$= 0.347$$

$\therefore$ $\phi = \tan^{-1} 0.347 = $ **19.16°. Ans.**

Problem 18.33 *A propeller reaction turbine of runner diameter 4.5 m is running at 40 r.p.m. The guide blade angle at inlet is 145° and runner blade angle at outlet is 25° to the direction of vane. The axial flow area of water through runner is 25 m². If the runner blade angle at inlet is radial determine :*

(i) Hydraulic efficiency of the turbine, *(ii) Discharge through turbine,*

(iii) Power developed by the turbine, and *(iv) Specific speed of the turbine.*

Solution. Given :

Runner diameter, $D_o = 4.5$ m

Speed, $N = 40$ r.p.m.

Guide blade angle, $\alpha = 145°$

Runner blade angle at outlet, $\phi = 25°$

Flow area, $a = 25 \text{ m}^2$

Runner blade angle at inlet is radial

$\therefore$ $\theta = 90°$, $V_{r_1} = V_{f_1}$ and $u_1 = V_{w_1}$

For Kaplan turbine, the discharge is given by the product of area of flow and velocity of flow.

As area of flow is constant and hence $V_{f_1} = V_{f_2}$ ($\because Q$ = Area of flow $\times V_{f_1}$ = Area of flow $\times V_{f_2}$)

The tangential speed of turbine at inlet,

$$u_1 = \frac{\pi D_o N}{60} = \frac{\pi \times 4.5 \times 40}{60}$$

$$= 9.42 \text{ m/s}$$

Also $u_2 = u_1 = 9.42$ m/s.

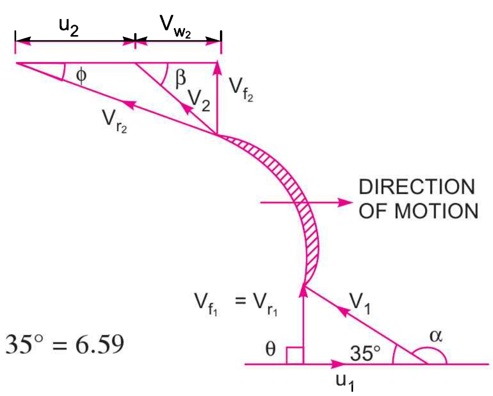

Fig. 18.31

From inlet velocity triangle,

$$\tan(180° - \alpha) = \frac{V_{f_1}}{u_1}$$

or $$\tan(180° - 145°) = \tan 35° = \frac{V_{f_1}}{u_1}$$

$\therefore$ $V_{f_1} = u_1 \tan 35° = 9.42 \tan 35° = 6.59$

Also $V_{w_1} = u_1 = 9.42$ m/s.

From outlet velocity triangle,

$$\tan \phi = \frac{V_{f_2}}{u_2 + V_{w_2}} \qquad (\text{where } V_{f_2} = V_{f_1} = 6.59 \text{ and } u_2 = u_1 = 9.42)$$

$\therefore$ $$\tan 25° = \frac{6.59}{9.42 + V_{w_2}}$$

$\therefore \qquad V_{w_2} + 9.42 = \dfrac{6.59}{\tan 25^\circ} = 14.13$

$\therefore \qquad V_{w_2} = 14.13 - 9.42 = 4.71$ m/s

$\therefore \qquad V_2 = \sqrt{V_{f_2}^2 + V_{w_2}^2} = \sqrt{6.59^2 + 4.71^2} = \sqrt{43.43 + 22.18} = 8.1$ m/s.

Using equation (18.24),

$$H - \frac{V_2^2}{2g} = \frac{1}{g}\left[V_{w_1} u_1 - V_{w_2} u_2\right].$$

Here –ve sign is taken as the absolute velocity at inlet and outlet (*i.e.*, V_1 and V_2) are in the same direction and hence change of velocity will be with a –ve sign

$$\therefore \qquad H - \frac{8.1^2}{2 \times 9.81} = \frac{1}{9.81}\,[9.42 \times 9.42 - 4.71 \times 9.42]$$

$$H - 3.344 = \frac{1}{9.81}\,[88.736 - 44.368] = 4.522 \text{ m}$$

$$\therefore \qquad H = 4.522 + 3.344 = 7.866 \text{ m.}$$

(*i*) Hydraulic efficiency is given by equation (18.20*A*) as

$$\eta_h = \frac{V_{w_1} u_1 - V_{w_2} u_2}{g \times H}$$

$$= \frac{(9.42 \times 9.42 - 4.71 \times 9.42)}{9.81 \times 7.866} = 0.575 = \mathbf{57.5\%.\ Ans.}$$

(*ii*) Discharge through turbine is given by,

$$Q = \text{Area of flow} \times \text{Velocity of flow}$$

$$= 25 \times V_{f_1} = 25 \times 6.59 = \mathbf{164.75\ m^3/s.\ Ans.}$$

(*iii*) Power developed by turbine $= \dfrac{\text{Work done per second}}{1000}$

$$= \frac{1}{g}\frac{\left[V_{w_1} u_1 - V_{w_2} u_2\right]}{1000} \times \text{Weight of water}$$

$$= \frac{1}{9.81}\left[\frac{9.42 \times 9.42 - 4.71 \times 9.42}{1000}\right] \times \rho \times g \times Q$$

$$= \frac{1}{9.81}\left[\frac{9.42 \times 9.42 - 4.71 \times 9.42}{1000}\right] \times 1000 \times 9.81 \times 164.75$$

= **6867 kW. Ans.**

(*iv*) Specific speed is given by the relation,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{N\sqrt{6867}}{7.866^{5/4}} = \frac{40 \times \sqrt{6867}}{7.866^{5/4}}$$

$$= \frac{40 \times 82.867}{13.173} = \mathbf{251.62\ r.p.m.\ Ans.}$$

▶ 18.10 DRAFT-TUBE

The draft-tube is a pipe of gradually increasing area which connects the outlet of the runner to the tail race. It is used for discharging water from the exit of the turbine to the tail race. This pipe of gradually increasing area is called a draft-tube. One end of the draft-tube is connected to the outlet of the runnner while the other end is sub-merged below the level of water in the tail race. The draft-tube, in addition to serve a passage for water discharge, has the following two purposes also :

1. It permits a negative head to be established at the outlet of the runner and thereby increase the net head on the turbine. The turbine may be placed above the tail race without any loss of net head and hence turbine may be inspected properly.

2. It converts a large proportion of the kinetic energy $(V_2^2/2g)$ rejected at the outlet of the turbine into useful pressure energy. Without the draft tube, the kinetic energy rejected at the outlet of the turbine will go waste to the tail race.

Hence by using draft-tube, the net head on the turbine increases. The turbine develops more power and also the efficiency of the turbine increases.

If a reaction turbine is not fitted with a draft-tube, the pressure at the outlet of the runner will be equal to atmospheric pressure. The water from the outlet of the runner will discharge freely into the tail race. The net head on the turbine will be less than that of a reaction turbine fitted with a draft-tube. Also without a draft-tube, the kinetic energy $\left(\frac{V_2^2}{2g}\right)$ rejected at the outlet of the runner will go waste to the tail race.

18.10.1 Types of Draft-Tubes. The following are the important types of draft-tubes which are commonly used :

1. Conical draft-tubes,
2. Simple elbow tubes,
3. Moody spreading tubes, and
4. Elbow draft-tubes with circular inlet and rectangular outlet.

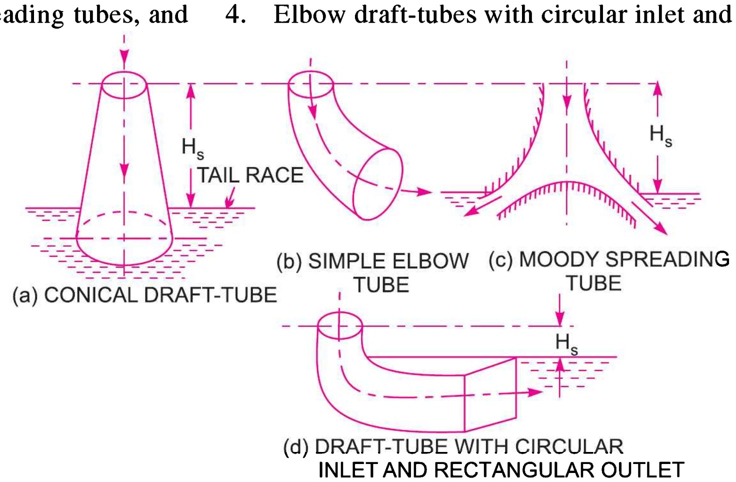

Fig. 18.32 *Types of draft-tubes.*

These different types of draft-tubes are shown in Fig. 18.32. The conical draft-tubes and Moody spreading draft-tubes are most efficient while simple elbow tubes and elbow draft-tubes with circular inlet and rectangular outlet require less space as compared to other draft-tubes.

18.10.2 Draft-Tube Theory. Consider a capital draft-tube as shown in Fig. 18.33.

Let H_s = Vertical height of draft-tube above the tail race,

y = Distance of bottom of draft-tube from tail race.

Applying Bernoulli's equation to inlet (section 1-1) and outlet (section 2-2) of the draft-tube and taking section 2-2 as the datum line,we get

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+(H_s+y)=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+0+h_f \qquad ...(i)$$

where h_f = loss of energy between sections 1-1 and 2-2.

But $\frac{p_2}{\rho g}$ = Atmospheric pressure head + y

$$= \frac{p_a}{\rho g}+y.$$

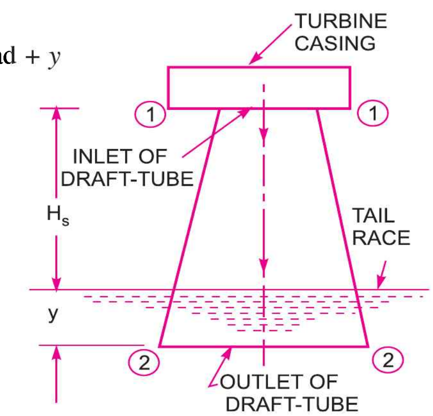

Fig. 18.33 *Draft-tube theory.*

Substituting this value of $\frac{p_2}{\rho g}$ in equation (i), we get

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+(H_s+y)=\frac{p_a}{\rho g}+y+\frac{V_2^2}{2g}+h_f$$

or

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+H_s=\frac{p_a}{\rho g}+\frac{V_2^2}{2g}+h_f$$

$$\therefore \quad \frac{p_1}{\rho g}=\frac{p_a}{\rho g}+\frac{V_2^2}{2g}+h_f-\frac{V_1^2}{2g}-H_s$$

$$=\frac{p_a}{\rho g}-H_s-\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}-h_f\right) \qquad ...(18.26)$$

In equation (18.26), $\frac{p_1}{\rho g}$ is less than atmospheric pressure.

18.10.3 Efficiency of Draft-Tube. The efficiency of a draft-tube is defined as the ratio of actual conversion of kinetic head into pressure head in the draft-tube to the kinetic head at the inlet of the draft-tube. Mathematically, it is written as

$$\eta_d=\frac{\text{Actual conversion of kinetic head into pressure head}}{\text{Kinetic head at the inlet of draft-tube}}$$

Let V_1 = Velocity of water at inlet of draft-tube,

V_2 = Velocity of water at outlet of draft-tube, and

h_f = Loss of head in the draft-tube.

Theoretical conversion of kinetic head into pressure head in draft-tube = $\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right)$.

Actual conversion of kinetic head into pressure head = $\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right)-h_f$

$$\therefore \qquad \eta_d = \frac{\left(\dfrac{V_1^2}{2g} - \dfrac{V_2^2}{2g}\right) - h_f}{\left(\dfrac{V_1^2}{2g}\right)} \qquad ...(18.27)$$

Problem 18.33 (A) *A water turbine has a velocity of 6 m/s at the entrance to the draft-tube and a velocity of 1.2 m/s at the exit. For friction losses of 0.1 m and a tail water 5 m below the entrance to the draft-tube, find the pressure head at the entrance.*

Solution. Given :

Velocity at inlet, $V_1 = 6$ m/s

Velocity at outlet, $V_2 = 1.2$ m/s

Friction loss, $h_f = 0.1$ m

Vertical height between tail race and inlet of draft-tube = 5 m

Let y = Vertical height between tail race and outlet of draft-tube.

Applying Bernoulli's equation at the inlet and outlet of draft-tube and taking reference line passing through section (2-2), we get

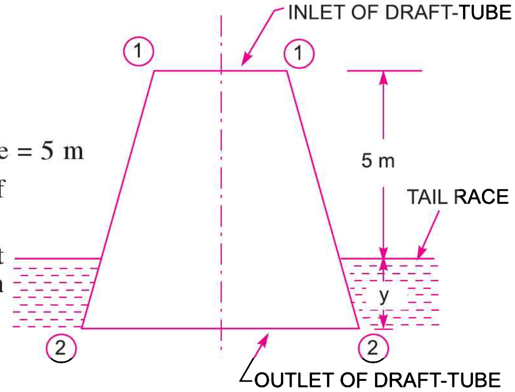

Fig. 18.33 (*a*)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{\rho g} + Z_2 + h_f$$

where $Z_1 = (5 + y)$; $V_1 = 6$ m/s ; $V_2 = 1.2$ m/s, $h_f = 0.1$

$$\frac{p_2}{\rho g} = \text{Atmospheric pressure head} + y = \frac{p_a}{\rho g} + y$$

$$Z_2 = 0$$

Substituting the values, we get

$$\frac{p_1}{\rho g} + \frac{6^2}{2 \times 9.81} + (5 + y) = \left(\frac{p_a}{\rho g} + y\right) + \frac{1.2^2}{2 \times 9.81} + 0 + 0.1$$

or
$$\frac{p_1}{\rho g} + 1.835 + 5 + y = \frac{p_a}{\rho g} + y + 0.0734 + 0.1$$

or
$$\frac{p_1}{\rho g} + 6.835 = \frac{p_a}{\rho g} + 0.1734 \qquad ...(i)$$

If $\frac{p_a}{\rho g}$ (*i.e.,* atmospheric pressure head) is taken zero, then we will get $\frac{p_1}{\rho g}$ as vacuum pressure head at inlet of draft-tube.

But if $\frac{p_a}{\rho g} = 10.3$ m of water, then we will get $\frac{p_1}{\rho g}$ as absolute pressure head at inlet of draft-tube.

Taking $\frac{p_a}{\rho g} = 0$ and substituting this value in equation (*i*), we get

$$\frac{p_1}{\rho g} + 6.835 = 0 + 0.1734$$

$\therefore$ $$\frac{p_1}{\rho g} = -6.835 + 0.1734 = \mathbf{-6.6616\ m.\ Ans.}$$

Negative sign means vacuum pressure head.

Problem 18.34 *A conical draft-tube having diameter at the top as 2.0 m and pressure head at 7 m of water (vacuum), discharges water at the outlet with a velocity of 1.2 m/s at the rate of 25 m³/s. If atmospheric pressure head is 10.3 m of water and losses between the inlet and outlet of the draft-tubes are negligible, find the length of draft-tube immersed in water. Total length of tube is 5 m.*

Solution. Given :

Diameter at top, $D_1 = 2.0$ m

Pressure head, $\frac{p_1}{\rho g} = 7$ m (Vacuum)

$= 10.3 - 7.0 = 3.3$ m (abs.)

Velocity at outlet, $V_2 = 1.2$ m/s

Discharge, $Q = 25\ m^3/s$

Loss of energy, h_f = Negligible

Let the length of the tube immersed in water = y m.

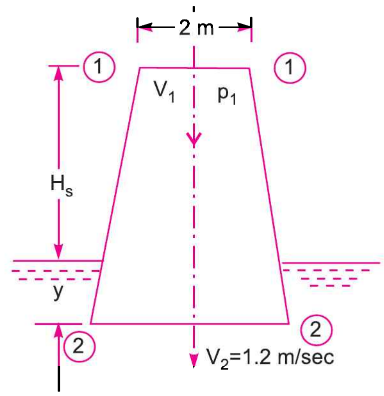

Fig. 18.34

Total length of the tube = 5 m

The velocity at inlet, $$V_1 = \frac{\text{Discharge}}{\text{Area at inlet}}$$

$$= \frac{Q}{\frac{\pi}{4}D_1^2} = \frac{25}{\frac{\pi}{4}(2.0)^2} = 7.957 \text{ m/s.}$$

Using equation (18.26), we have

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - H_s - \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f\right)$$

$$3.30 = 10.3 - H_s - \left(\frac{7.957^2}{2 \times 9.81} - \frac{1.2^2}{2 \times 9.81} - 0\right)$$

$$\left(\because\ h_f = 0 \text{ and } \frac{p_a}{\rho g} = 10.3\right)$$

$$= 10.3 - H_s - (3.227 - .0734)$$

or $$3.3 = 10.3 - H_s - 3.1536$$

$\therefore$ $$H_s = 10.3 - 3.1536 - 3.3 = 3.8464 \text{ m}$$

$\therefore$ $$y = \text{Total length} - H_s = 5 - 3.8464 = \mathbf{1.1536\ m.\ Ans.}$$

Problem 18.35 *A conical draft-tube having inlet and outlet diameters 1 m and 1.5 m discharges water at outlet with a velocity of 2.5 m/s. The total length of the draft-tube is 6 m and 1.20 m of the length of draft-tube is immersed in water . If the atmospheric pressure head is 10.3 m of water and loss of head due to friction in the draft-tube is equal to 0.2 × velocity head at outlet of the tube, find :*

(i) Pressure head at inlet, and (ii) Efficiency of the draft-tube.

Solution. Given :

Diameter at inlet, $D_1 = 1.0$ m

Diameter at outlet $D_2 = 1.5$ m

Velocity at outlet, $V_2 = 2.5$ m/s

Total length of tube, $H_s + y = 6.0$ m

Length of tube in water, $y = 1.20$ m

$\therefore$ $H_s = 6.0 - 1.20 = 4.80$ m

Atmospheric pressure head, $\frac{p_a}{\rho g} = 10.3$ m

Loss of head due to friction, $h_f = 0.2 \times$ Velocity head at outlet

$$= 0.2 \times \frac{V_2^2}{2g}$$

Discharge through tube, $Q = A_2 V_2 = \frac{\pi}{4} D_2^2 \times 2.5 = \frac{\pi}{4} (1.5)^2 \times 2.5 = 4.4178 \text{ m}^3/\text{s}$

Velocity at inlet, $V_1 = \frac{Q}{A_1} = \frac{4.4178}{\frac{\pi}{4} \times 1^2} = 5.625$ m/s

(i) Pressure head at inlet $\left(\frac{p_1}{\rho g}\right)$.

Using equation (18.26),
$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - H_s - \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_f\right)$$

$$= 10.3 - 4.8 - \left(\frac{5.625^2}{2 \times 9.81} - \frac{2.5^2}{2 \times 9.81} - 0.2 \times \frac{V_2^2}{2g}\right)$$

$$= 10.3 - 4.8 - \left(1.6126 - .3185 - \frac{0.2 \times 2.5^2}{2 \times 9.81}\right)$$

$$= 10.3 - 4.8 - (1.6126 - .3185 - .0637) = 5.5 - (1.2304) = 4.269$$

$\simeq$ **4.27 m (abs.) Ans.**

(ii) Efficiency of Draft-tube (η_d)

Using equation (18.27),
$$\eta_d = \frac{\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - h_f}{\frac{V_1^2}{2g}} = \frac{\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{0.2\,V_2^2}{2g}}{\frac{V_1^2}{2g}}$$

$$= \frac{V_1^2 - 1.2\,V_2^2}{V_1^2} = 1 - 1.2\left(\frac{V_2}{V_1}\right)^2 = 1 - 1.2\left(\frac{2.5}{5.625}\right)^2 = 1 - 0.237$$

= **0.763** or **76.3%. Ans.**

▶ 18.11 SPECIFIC SPEED

It is defined as the speed of a turbine which is identical in shape, geometrical dimensions, blade angles, gate opening etc., with the actual turbine but of such a size that it will develop unit power when working under unit head. It is denoted by the symbol N_s. The specific speed is used in comparing the different types of turbines as every type of turbine has different specific speed.

In M.K.S. units, unit power is taken as one horse power and unit head as one metre. But in S.I. units, unit power is taken as one kilowatt and unit head as one metre.

18.11.1 Derivation of the Specific Speed. The overall efficiency (η_o) of any turbine is given by,

$$\eta_o = \frac{\text{Shaft power}}{\text{Water power}} = \frac{\text{Power developed}}{\frac{\rho \times g \times Q \times H}{1000}} = \frac{P}{\frac{\rho \times g \times Q \times H}{1000}} \quad \text{...}(i)$$

where H = Head under which the turbine is working,

Q = Discharge through turbine,

P = Power developed or shaft power.

From equation (*i*),

$$P = \eta_o \times \frac{\rho \times g \times Q \times H}{1000}$$

$$\propto Q \times H \text{ (as } \eta_o \text{ and } \rho \text{ are constant)} \quad \text{...}(ii)$$

Now let D = Diameter of actual turbine,

N = Speed of actual turbine,

u = Tangential velocity of the turbine,

N_s = Specific speed of the turbine,

V = Absolute velocity of water.

The absolute velocity, tangential velocity and head on the turbine are related as,

$$u \propto V, \text{ where } V \propto \sqrt{H}$$

$$\propto \sqrt{H} \quad \text{...}(iii)$$

But the tangential velocity u is given by

$$u = \frac{\pi DN}{60}$$

$$\propto DN \quad \text{...}(iv)$$

∴ From equations (*iii*) and (*iv*), we have

$$\sqrt{H} \propto DN \text{ or } D \propto \frac{\sqrt{H}}{N} \quad \text{...}(v)$$

The discharge through turbine is given by

$$Q = \text{Area} \times \text{Velocity}$$

But $\text{Area} \propto B \times D$ (where B = Width)

$\propto D^2$ $(\because \quad B \propto D)$

And $\text{Velocity} \propto \sqrt{H}$

∴ $Q \propto D^2 \times \sqrt{H}$

$$\propto \left(\frac{\sqrt{H}}{N}\right)^2 \times \sqrt{H} \qquad \left(\because \text{ From equation } (v), D \propto \frac{\sqrt{H}}{N}\right)$$

$$\propto \frac{H}{N^2} \times \sqrt{H} \propto \frac{H^{3/2}}{N^2} \qquad ...(vi)$$

Substituting the value of Q in equation (*ii*), we get

$$P \propto \frac{H^{3/2}}{N^2} \times H \propto \frac{H^{5/2}}{N^2}$$

$\therefore$ $$P = K \frac{H^{5/2}}{N^2}, \text{ where } K = \text{Constant of proportionality.}$$

If $P = 1$, $H = 1$, the speed N = Specific speed N_s. Substituting these values in the above equation, we get

$$1 = \frac{K \times 1^{5/2}}{N_s^2} \quad \text{or} \quad N_s^2 = K$$

$\therefore$ $$P = N_s^2 \frac{H^{5/2}}{N^2} \quad \text{or} \quad N_s^2 = \frac{N^2 P}{H^{5/2}}$$

$\therefore$ $$N_s = \sqrt{\frac{N^2 P}{H^{5/2}}} = \frac{N\sqrt{P}}{H^{5/4}} \qquad ...(18.28)$$

In equation (18.28), if P is taken in metric horse power the specific speed is obtained in M.K.S. units. But if P is taken in kilowatts, the specific speed is obtained in S.I. units.

18.11.2 Significance of Specific Speed. Specific speed plays an important role for selecting the type of the turbine. Also the performance of a turbine can be predicted by knowing the specific speed of the turbine. The type of turbine for different specific speed is given in Table 18.1 as :

Table 18.1

S. No.	*Specific speed*		*Types of turbine*
	(M.K.S.)	*(S.I.)*	
1.	10 to 35	8.5 to 30	Pelton wheel with single jet
2.	35 to 60	30 to 51	Pelton wheel with two or more jets
3.	60 to 300	51 to 225	Francis turbine
4.	300 to 1000	255 to 860	Kaplan or Propeller turbine

Problem 18.36 *A turbine develops 7225 kW power under a head of 25 metres at 135 r.p.m. Calculate the specific speed of the turbine and state the type of the turbine.*

Solution. Given :

Power developed, $P = 7225$ kW

Head, $H = 25$ m

Speed, $N = 135$ r.p.m.

Specific speed of the turbine (N_s)

Using equation (18.28),

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{135 \times \sqrt{7225}}{25^{5/4}} = \textbf{205.28. Ans.}$$

From Table 18.1, for specific speeds (S.I.) between 51 and 255 the type of turbine is Francis. As the specific speed 205.28 lies in this range and hence type of turbine is Francis. **Ans.**

Problem 18.37 *A turbine is to operate under a head of 25 m at 200 r.p.m. The discharge is 9 cumec. If the efficiency is 90%, determine :*

(i) Specific speed of the machine, *(ii) Power generated, and*
(iii) Type of turbine.

Solution. Given :

Head, $H = 25$ m
Speed, $N = 200$ r.p.m.
Discharge, $Q = 9$ cumec $= 9\ m^3/s$
Efficiency, $\eta_o = 90\% = 0.90$ (Take the efficiency as overall η)

Now using relation,
$$\eta_o = \frac{\text{Power developed}}{\text{Water power}} = \frac{P}{\dfrac{\rho \times g \times Q \times H}{1000}}$$

$$\therefore \quad P = \eta_o \times \frac{\rho \times g \times Q \times H}{1000}$$
$$= \frac{0.90 \times 9.81 \times 1000 \times 9 \times 25}{1000} = 1986.5 \text{ kW}$$

(*i*) *Specific speed of the machine* (N_s)

Using equation (18.28),
$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{200 \times \sqrt{1986.5}}{25^{5/4}} = \textbf{159.46 r.p.m. Ans.}$$

(*ii*) *Power generated*
$$P = \textbf{1986.5 kW. Ans.}$$

(*iii*) As the specific speed lies between 51 and 255, the turbine is a Francis turbine. **Ans.**

Problem 18.38 *A turbine develops 9000 kW when running at a speed of 140 r.p.m. and under a head of 30 m. Determine the specific speed of the turbine.*

Solution. Given :

Power developed, $P = 9000$ kW
Head, $H = 30$ m
Speed , $N = 140$ r.p.m.

The specific speed is given by equation (18.28) as
$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{140 \times \sqrt{9000}}{30^{5/4}} = \frac{13281.56}{70.21}$$
$$= \textbf{189.167 (S.I. units). Ans.}$$

Problem 18.39 *A Pelton wheel develops 8000 kW under a net head of 130 m at a speed of 200 r.p.m. Assuming the co-efficient of velocity for the nozzle 0.98, hydraulic efficiency 87%, speed ratio 0.46 and jet diameter to wheel diameter ratio* $\frac{1}{9}$, *determine :*

(i) the discharge required, *(ii) the diameter of the wheel,*
(iii) the diameter and number of jets required, and *(iv) the specific speed.*
Mechanical efficiency is 75%.

Solution. Given :

Power developed, $P = 8000$ kW

Net head, $H = 130$ m

Speed, $N = 200$ r.p.m.

Co-efficient of velocity, $C_v = 0.98$

Hydraulic efficiency, $\eta_h = 87\% = 0.87$

Speed ratio, $\dfrac{u_1}{\sqrt{2gH}} = 0.46$

$$\therefore \quad u_1 = 0.46 \times \sqrt{2gH} = 0.46 \times \sqrt{2 \times 9.81 \times 130} = 23.23 \text{ m/s.}$$

Jet diameter to wheel diameter $= \dfrac{d}{D} = \dfrac{1}{9}$

Mechanical efficiency, $\eta_m = 75\% = 0.75$

Overall efficiency is given by equation (18.6) as

$$\eta_o = \eta_h \times \eta_m = 0.87 \times 0.75 = 0.6525$$

Also $\eta_o = \dfrac{\text{Power developed}}{\text{Water power}} = \dfrac{8000}{\text{W.P. in kW}}$ or $0.6525 = \dfrac{8000}{\text{W.P. in kW}}$

$\therefore$ W.P. in kW $= \dfrac{8000}{0.6525} = 12260.536$ kW

But W.P. in kW $= \dfrac{\rho \times g \times Q \times H}{1000}$

$$= \frac{1000 \times 9.81 \times Q \times H}{1000} \qquad (\because \rho g \text{ in S.I.} = 1000 \times 9.81)$$

$$= Q \times H \times 9.81 = Q \times 130 \times 9.81$$

$$\therefore \quad 12260.536 = Q \times 130 \times 9.81$$

$$\therefore \quad Q = \frac{12260.536}{130 \times 9.81} = \mathbf{9.614\ m^3/s.\ Ans.}$$

(*i*) *Discharge required*

$$Q = \mathbf{9.614\ m^3/s.\ Ans.}$$

(*ii*) *Diameter of wheel* (*D*)

Using the relation, $u_1 = \dfrac{\pi D N}{60}$

$$\therefore \quad D = \frac{60 \times u_1}{\pi \times N} = \frac{60 \times 23.23}{\pi \times 200} = \mathbf{2.218\ m.\ Ans.}$$

(*iii*) *Diameter of jet* (*d*) *and number of jets required*

$$\frac{d}{D} = \frac{1}{9}$$

$$\therefore \quad d = \frac{D}{9} = \frac{2.218}{9} = 0.2464 \text{ m} = \mathbf{246.4\ mm.\ Ans.}$$

$\therefore$ Area of jet, $a = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4}(.2464)^2 = .04768 \text{ m}^2.$

Velocity of jet is given by, $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 130} = 49.49$ m/s

∴ Discharge through one jet = Area of jet × Velocity of jet = $a \times V_1$

$= .04768 \times 49.49 = 2.359 \text{ m}^3/\text{s}$

∴ Number of jets $= \dfrac{\text{Total discharge}}{\text{Discharge through one jet}}$

$= \dfrac{Q}{2.359} = \dfrac{9.614}{2.359} = 4.07$ say **4.0. Ans.**

(*iv*) *Specific speed* is given by equation (18.28) as

N_s (S.I. units) $= \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{200 \times \sqrt{8000}}{130^{5/4}} = \dfrac{17888.54}{438.96} = \mathbf{40.75.}$ **Ans.**

Problem 18.40 *A Pelton turbine develops 3000 kW under a head of 300 m. The overall efficiency of the turbine is 83%. If speed ratio = 0.46, C_v = 0.98 and specific speed is 16.5, then find :*

(i) Diameter of the turbine, and (ii) Diameter of the jet.

Solution. Given :

Power, $P = 3000$ kW

Net head, $H = 300$ m

Overall efficiency, $\eta_o = 83\%$ or 0.83

Speed ratio = 0.46

Value of C_v, = 0.98

Specific speed*, $N_s = 16.5$

Using equation, $N_s = \dfrac{N\sqrt{P}}{H^{5/4}}$ or $N = \dfrac{N_s H^{5/4}}{\sqrt{P}} = \dfrac{16.5 \times 300^{5/4}}{\sqrt{3000}} = 375$ r.p.m.

The velocity (V) at the outlet of nozzle is given by,

$$V = C_v\sqrt{2 \times g \times H} = 0.98\sqrt{2 \times 9.81 \times 300} = 75.1 \text{ m/s}$$

Now speed ratio $= \dfrac{u}{\sqrt{2gH}}$ or u = Speed ratio × $\sqrt{2gH}$

$= 0.46 \times \sqrt{2 \times 9.81 \times 300} = 34.95$ m/s.

(*i*) *Diameter of the turbine (D)*

Using, $u = \dfrac{\pi DN}{60}$ or $D = \dfrac{60 \times u}{\pi \times N} = \dfrac{60 \times 34.95}{\pi \times 375} = \mathbf{1.78 \text{ m.}}$ **Ans.**

(*ii*) *Diameter of the jet (d)*

Let Q = Discharge through turbine in m^3/s

Using the relation, $\eta_o = \dfrac{P}{\left(\dfrac{\rho \times g \times Q \times H}{1000}\right)}$, where $\rho \times g = 1000 \times 9.81 \text{ N/m}^3$ for water

∴ $0.83 = \dfrac{3000}{\left(\dfrac{1000 \times 9.81 \times Q \times 300}{1000}\right)}$

* Specific speed is the speed of the turbine working under a unit head and develops one kilowatt power.

$$\therefore \qquad Q = \frac{3000}{9.81 \times 300 \times 0.83} = 1.23 \text{ m}^3\text{/s}$$

But discharge through a Pelton turbine is given by,

$$Q = \text{Area of jet} \times \text{Velocity}$$

or
$$1.23 = \frac{\pi}{4} d^2 \times 75.1$$

$$\therefore \qquad d = \sqrt{\frac{4 \times 1.23}{\pi \times 75.1}} = 0.142 \text{ m} = \textbf{142 mm. Ans.}$$

Problem 18.41 *Water under a head of 300 m is available for a hydel-plant situated at a distance of 2.35 km from the source. The frictional losses of energy for transporting water is equivalent to 26 (J/N). A number of Pelton wheels are to be installed generating a total output of 18 MW. Determine the number of units to be installed, diameter of Pelton wheel and the jet diameter when the following are available : Wheel speed 650 r.p.m.; ratio of bucket to jet speed 0.46 ; specific speed not to exceed 30 (m, kW, r.p.m.) ; C_v and C_d for the nozzle 0.97 and 0.94 respectively and the overall efficiency of the wheel 87%.*

Solution. Given :

Total head = 300 m

Length = 2.35 km = 2350 m

Frictional losses = 26 (J/N) = 26 (Nm/N) (as J = Nm) = 26 m

∴ Net head, $H = 300 - 26 = 274$ m

Total output = 18 MW = 18×10^3 kW

$N = 650$ r.p.m.

Ratio of bucket to jet speed = 0.46

$C_v = 0.97$, $C_d = 0.94$

$\eta_o = 87\% = 0.87$

and $N_s = 30$, where H is in m, P in kW and N in r.p.m.

Find : (*i*) Number of units to be installed

(*ii*) Dia. of Pelton wheel (D)

(*iii*) Dia. of jet of water (d)

(*i*) *Number of units to be installed*

Let P = Power output of each unit in kW

Using equation (18.28) as $N_s = \dfrac{N\sqrt{P}}{H^{5/4}}$ or $30 = \dfrac{650 \times \sqrt{P}}{274^{5/4}}$ or $\sqrt{P} = \dfrac{30 \times 274^{5/4}}{650}$

Squaring both sides, we get $P = \dfrac{30^2 \times 274^{5/2}}{650^2} = 2647.2$ kW

$$\therefore \text{ No. of units} = \frac{\text{Total output in kW}}{\text{Output of one unit in kW}}$$

$$= \frac{18 \times 10^3}{2647.2} = 6.799 \simeq \textbf{7 units. Ans.}$$

(*ii*) *Dia. of Pelton wheel (D)*

Velocity of jet is given by, $V_1 = C_v \times \sqrt{2gH}$

$$= 0.97 \times \sqrt{2 \times 9.81 \times 274} = 71.12 \text{ (m/s)}$$

But ratio of bucket to jet speed $= 0.46$*

or $$\frac{\text{Speed of bucket}}{\text{Speed of jet}} = 0.46 \text{ or } \frac{u_1}{V_1} = 0.46$$

$\therefore$ $$u_1 = 0.46 \times V_1 = 0.46 \times 71.12 = 32.715 \text{ m/s.}$$

But $$u_1 = \frac{\pi DN}{60}$$

$\therefore$ $$32.715 = \frac{\pi \times D \times 650}{60} \text{ or } \frac{32.715 \times 60}{\pi \times 650} = D \text{ or } 0.945 \text{ m} = D$$

$\therefore$ Dia. of Pelton wheel $=$ **0.945. Ans.**

(*iii*) *Dia. of jet* (*d*)

We know $$\eta_o = \frac{\text{Total power output}}{\text{Total water power in kW}}$$

or $$0.87 = \frac{18 \times 10^3}{\text{Total water power in kW}}$$

$\therefore$ Total water power in kW $$= \frac{18 \times 10^3}{0.87} = 20.689 \times 10^3 \text{ kW}$$

$\therefore$ Water power in kW per unit $$= \frac{\text{Total water power}}{\text{No. of units}} = \frac{20.689 \times 10^3}{7} = 2.955 \times 10^3 \text{ kW}$$

But water power in kW per unit is given by equation (18.3 *A*) as,

Water power $$= \frac{\rho \times g \times Q \times H}{1000} \text{ kW}$$

$\therefore$ $$2.955 \times 10^3 = \frac{\rho \times g \times Q \times H}{1000} = \frac{(1000 \times 9.81) \times Q \times H}{1000}$$

$(\because \rho \times g = 1000 \times 9.81)$

$$= 9.81 \times Q \times 274$$

$\therefore$ $$Q = \frac{2.955 \times 10^3}{9.81 \times 274} = 1.099 \text{ m}^3/\text{s}$$

But discharge (Q) through one unit is also given by

$$Q = C_d \times \frac{\pi}{4} d^2 \times \sqrt{2gH}$$

* It is not speed ratio. It is the ratio of bucket speed to jet speed *i.e.*, ratio of u_1 and V_1. Speed ratio is $u_1\sqrt{2gH}$.

or $$1.099 = 0.94 \times \frac{\pi}{4} d^2 \times \sqrt{2 \times 9.81 \times 274}$$

or $$d^2 = \frac{1.099 \times 4}{0.94 \times \pi \times \sqrt{2 \times 9.81 \times 274}} = 0.0203 \text{ m}$$

$\therefore$ $$d = \sqrt{0.0203} = 0.1424 \text{ m} = \mathbf{142.4 \text{ mm.}} \ \textbf{Ans.}$$

▶ 18.12 UNIT QUANTITIES

In order to predict the behaviour of a turbine working under varying conditions of head, speed, output and gate opening, the results are expressed in terms of quantities which may be obtained when the head on the turbine is reduced to unity. The conditions of the turbine under unit head are such that the efficiency of the turbine remains unaffected. The following are the three important unit quantities which must be studied under unit head :

1. Unit speed, 2. Unit discharge, and 3. Unit power.

18.12.1 Unit Speed. It is defined as the speed of a turbine working under a unit head (*i.e.*, under a head of 1 m). It is denoted by 'N_u'. The expression for unit speed (N_u) is obtained as :

Let N = Speed of a turbine under a head H,

H = Head under which a turbine is working,

u = Tangential velocity.

The tangential velocity, absolute velocity of water and head on the turbine are related as

$$u \propto V, \quad \text{where } V \propto \sqrt{H}$$

$$\propto \sqrt{H} \quad \text{...}(i)$$

Also tangential velocity (u) is given by

$$u = \frac{\pi D N}{60}, \quad \text{where } D = \text{Diameter of turbine.}$$

For a given turbine, the diameter (D) is constant.

$\therefore$ $$u \propto N \text{ or } N \propto u \text{ or } N \propto \sqrt{H} \quad (\because \text{ From } (i),\ u \propto \sqrt{H})$$

$\therefore$ $$N = K_1 \sqrt{H} \quad \text{...}(ii)$$

where K_1 is a constant of proportionality.

If head on the turbine becomes unity, the speed becomes unit speed or

when $H = 1, N = N_u$

Substituting these values in equation (*ii*), we get

$$N_u = K_1 \sqrt{1.0} = K_1$$

Substituting the value of K_1 in equation (*ii*),

$$N = N_u \sqrt{H} \text{ or } N_u = \frac{N}{\sqrt{H}}. \quad \text{...(18.29)}$$

18.12.2 Unit Discharge. It is defined as the discharge passing through a turbine, which is working under a unit head (*i.e.*, 1 m). It is denoted by the symbol 'Q_u'. The expression for unit discharge is given as :

Let H = Head of water on the turbine,

Q = Discharge passing through turbine when head is H on the turbine,

a = Area of flow of water.

The discharge passing through a given turbine under a head 'H' is given by,

$$Q = \text{Area of flow} \times \text{Velocity}$$

But for a turbine, area of flow is constant and velocity is proportional to $\sqrt{H}$.

$$\therefore \quad Q \propto \text{Velocity} \propto \sqrt{H}$$

or

$$Q = K_2 \sqrt{H} \quad \text{...(iii)}$$

where K_2 is constant of proportionality.

If $H = 1, Q = Q_u$. (By definition)

Substituting these values in equation (*iii*), we get

$$Q_u = K_2 \sqrt{1.0} = K_2.$$

Substituting the value of K_2 in equation (*iii*), we get

$$Q = Q_u \sqrt{H}$$

$$\therefore \quad Q_u = \frac{Q}{\sqrt{H}} \quad \text{...(18.30)}$$

18.12.3 Unit Power. It is defined as the power developed by a turbine, working under a unit head (*i.e.*, under a head or 1 m). It is denoted by the symbol 'P_u'. The expression for unit power is obtained as :

Let H = Head of water on the turbine,

P = Power developed by the turbine under a head of H,

Q = Discharge through turbine under a head H.

The overall efficiency (η_o) is given as

$$\eta_o = \frac{\text{Power developed}}{\text{Water power}} = \frac{P}{\dfrac{\rho \times g \times Q \times H}{1000}}$$

$$\therefore \quad P = \eta_o \times \frac{\rho \times g \times Q \times H}{1000}$$

$$\propto Q \times H$$

$$\propto \sqrt{H} \times H \quad (\because \quad Q \propto \sqrt{H})$$

$$\propto H^{3/2}$$

$$\therefore \quad P = K_3 H^{3/2} \quad \text{...(iv)}$$

where K_3 is a constant of proportionality.

When $H = 1$ m $\quad P = P_u$

$$\therefore \quad P_u = K_3 (1)^{3/2} = K_3.$$

Substituting the value of K_3 in equation (*iv*), we get

$$P = P_u H^{3/2}$$

$$\therefore \quad P_u = \frac{P}{H^{3/2}}. \quad \text{...(18.31)}$$

18.12.4 Use of Unit Quantities (N_u, Q_u, P_u). If a turbine is working under different heads, the behaviour of the turbine can be easily known from the values of the unit quantities, *i.e.*, from the values of unit speed, unit discharge and unit power.

Let H_1, H_2, ...are the heads under which a turbine works,
N_1, N_2, ...are the corresponding speeds,
Q_1, Q_2, ...are the discharge, and
P_1, P_2, ...are the power developed by the turbine.

Using equations (18.29), (18.30) and (18.31) respectively,

$$\left.\begin{aligned} N_u &= \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}} \\ Q_u &= \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}} \\ P_u &= \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}} \end{aligned}\right\} \quad \text{...(18.32)}$$

Hence, if the speed, discharge and power developed by a turbine under a head are known, then by using equation (18.32) the speed, discharge and power developed by the same turbine under a different head can be obtained easily.

Problem 18.41 (A) *A turbine develops 9000 kW when running at 10 r.p.m. The head on the turbine is 30 m. If the head on the turbine is reduced to 18 m, determine the speed and power developed by the turbine.*

Solution. Given :

Power developed, $P_1 = 9000$ kW
Speed, $N_1 = 100$ r.p.m.
Head, $H_1 = 30$ m
Let for a head, $H_2 = 18$ m
Speed $= N_2$
Power $= P_2$

Using equation (18.32), $\dfrac{N_1}{\sqrt{H_1}} = \dfrac{N_2}{\sqrt{H_2}}$

$$N_2 = \frac{N_1\sqrt{H_2}}{\sqrt{H_1}} = \frac{100\sqrt{18}}{\sqrt{30}} = \frac{100 \times 4.2426}{5.4772} = \textbf{77.46 r.p.m. Ans.}$$

Also we have $\dfrac{P_1}{H_1^{3/2}} = \dfrac{P_2}{H_2^{3/2}}$

$$\therefore \quad P_2 = \frac{P_1 H_2^{3/2}}{H_1^{3/2}} = \frac{9000 \times 18^{3/2}}{30^{3/2}} = \frac{687307.78}{164.316} = \textbf{4182.84 kW. Ans.}$$

Problem 18.42 *A turbine develops 500 kW power under a head of 100 metres at 200 r.p.m. What would be its normal speed and output under a head of 81 metres ?*

Solution. Given :

Power, $P_1 = 500$ kW
Head, $H_1 = 100$ m

Speed, $N_1 = 200$ r.p.m.

For a head, $H_2 = 81$ m

Let, N_2 = Speed

P_2 = Power

Using equation (18.32) for speed, we have

$$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$\therefore \quad N_2 = \sqrt{H_2} \times \frac{N_1}{\sqrt{H_1}} = \sqrt{\frac{H_2}{H_1}} \times N_1 = \sqrt{\frac{81}{100}} \times 200$$

$$= \frac{9}{10} \times 20 = \textbf{180 r.p.m. Ans.}$$

Using equation (18.32) for power, we have

$$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$\therefore \quad P_2 = H_2^{3/2} \times \frac{P_1}{H_1^{3/2}} = \frac{81^{3/2}}{100^{3/2}} \times 500$$

$$= \frac{729}{1000} \times 500 = \textbf{364.5 kW. Ans.}$$

Problem 18.43 *A turbine is to operate under a head of 25 m at 200 r.p.m. The discharge is 9 cumec. If the efficiency is 90%, determine the performance of the turbine under a head of 20 metres.*

Solution. Given :

Head on turbine, $H_1 = 25$ m

Speed, $N_1 = 200$ r.p.m.

Discharge, $Q_1 = 9\ m^3/s$

Overall efficiency, $\eta_o = 90\%$ or 0.90.

Performance of the turbine under a head, $H_2 = 20$ m, means to find the speed, discharge and power developed by the turbine when working under the head of 20 m.

Let for the head, $H_2 = 20$ m, Speed = N_2, discharge = Q_2 and power = P_2

Using the relation, $$\eta_o = \frac{P}{\text{W.P.}} = \frac{p_1}{\frac{\rho \times g \times Q_1 \times H_1}{1000}}$$

$$\therefore \quad P_1 = \frac{\eta_o \times \rho \times g \times Q_1 \times H_1}{1000} = \frac{0.90 \times 1000 \times 9.81 \times 9 \times 25}{1000} = 1986.5 \text{ kW}$$

Using equation (18.32), $$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$\therefore \quad N_2 = \frac{N_1\sqrt{H_2}}{\sqrt{H_1}} = 200 \times \frac{\sqrt{20}}{\sqrt{25}} = \textbf{178.88 r.p.m. Ans.}$$

Also $$\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

∴ $$Q_2 = Q_1 \times \frac{\sqrt{H_2}}{\sqrt{H_1}} = 9.0 \times \sqrt{\frac{20}{25}} = \mathbf{8.05\ m^3/s.\ Ans.}$$

And $$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

∴ $$P_2 = \frac{P_1 H_2^{3/2}}{H_1^{3/2}} = P_1 \left(\frac{H_2}{H_1}\right)^{3/2} = 1986.5 \left(\frac{20}{25}\right)^{3/2} = \mathbf{1421.42\ kW.\ Ans.}$$

Problem 18.44 *A Pelton wheel is revolving at a speed of 190 r.p.m. and develops 5150.25 kW when working under a head of 220 m with an overall efficiency of 80%. Determine unit speed, unit discharge and unit power. The speed ratio for the turbine is given as 0.47. Find the speed, discharge and power when this turbine is working under a head of 140 m.*

Solution. Given :

Speed, $N_1 = 190$ r.p.m.

Power, $P_1 = 5150.25$ kW

Head, $H_1 = 220$ m

Overall efficiency, $\eta_o = 80\% = 0.80$

Speed ratio $= 0.47$

New head of water, $H_2 = 140$ m

Overall efficiency is given by $$\eta_o = \frac{P_1}{\frac{\rho \times g \times Q_1 \times H_1}{1000}} = \frac{1000 \times P_1}{\rho \times g \times Q_1 \times H_1}$$

∴ $$Q_1 = \frac{1000 \times P_1}{\eta_o \times \rho \times g \times H_1} = \frac{1000 \times 5150.25}{0.80 \times 1000 \times 9.81 \times 220} = 2.983\ m^3/s$$

Unit speed is given by equation (18.29),

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{190}{\sqrt{220}} = \mathbf{12.81\ r.p.m.\ Ans.}$$

Unit discharge is given by equation (18.30),

$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{2.983}{\sqrt{220}} = \mathbf{0.201\ m^3/s.\ Ans.}$$

Unit power is given by equation (18.31),

$$P_u = \frac{P_1}{H_1^{3/2}} = \frac{5150.25}{220^{3/2}} = \mathbf{1.578\ kW.\ Ans.}$$

When the turbine is working under a new head of 140 m, the speed, discharge and power are given by equation (18.32) as

For speed, $$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

∴ $$N_2 = \frac{N_1\sqrt{H_2}}{\sqrt{H_1}} = N_1\sqrt{\frac{H_2}{H_1}} = 190\sqrt{\frac{140}{220}} = \mathbf{151.56\ r.p.m.\ Ans.}$$

For discharge, $\dfrac{Q_1}{\sqrt{H_1}} = \dfrac{Q_2}{\sqrt{H_2}}$

$\therefore$ $Q_2 = \dfrac{Q_1\sqrt{H_2}}{\sqrt{H_1}} = Q_1\sqrt{\dfrac{H_2}{H_1}} = 2.983\sqrt{\dfrac{140}{220}}$ **= 2.379 m³/s. Ans.**

For power, $\dfrac{P_1}{H_1^{3/2}} = \dfrac{P_2}{H_2^{3/2}}$

$\therefore$ $P_2 = P_1\dfrac{H_2^{3/2}}{H_1^{3/2}} = P_1\left(\dfrac{H_2}{H_1}\right)^{3/2} = 5150.25\left(\dfrac{140}{220}\right)^{3/2}$ **= 2614.48 kW. Ans.**

Problem 18.45 *A Pelton wheel is supplied with water under a head of 35 m at the rate of 40.5 kilo litre/min. The bucket deflects the jet through an angle of 160° and the mean bucket speed is 13 m/s. Calculate the power and hydraulic efficiency of the turbine.*

Solution. Given :

Net head, $H = 35$ m

Discharge, $Q = 40.5$ kilo litre/min

$= 40.5 \times 1000$ litre/min

$= \dfrac{40.5 \times 1000}{1000}$ m³/min $\quad \left(1 \text{ litre} = \dfrac{1}{1000} \text{m}^3\right)$

$= 40.5 \text{ m}^3/\text{min} = \dfrac{40.5}{60}$ m³/s = 0.675 m³/s

Angle of deflection = 160°

$\therefore$ Angle, $\phi = 180° - 160° = 20°$

Mean bucket speed, $u = u_1 = u_2 = 13$ m/s

Calculate : (*i*) Power at runner and (*ii*) Hydraulic efficiency.

Taking the value of $C_v = 1.0$

The velocity of jet, $V_1 = C_v\sqrt{2gH} = 1 \times \sqrt{2 \times 9.81 \times 35} = 26.2$ m/s

$\therefore$ $V_{r_1} = V_1 - u_1 = 26.2 - 13 = 13.2$ m/s

Also $V_{w_1} = V_1 = 26.2$ m/s

$V_{r_2} = V_{r_1} = 13.2$ m/s

and $V_{w_2} = V_{r_2}\cos\phi - u_2$

$= 13.2 \times \cos 20° - 13 = 12.554 - 13 = -0.446$ m/s

(*i*) *Power at runner*

Using equation (18.9), we get the work done by the jet on the runner per second.

$\therefore$ Work done/s $= \rho \times a \times V_1\left[V_{w_1} + V_{w_2}\right] \times u$

$= \rho \times Q \times \left[V_{w_1} + V_{w_2}\right] \times u \quad (\because \ a \times V_1 = Q)$

$$= 1000 \times 0.675\ [26.2 + (-\ 0.446)] \times 13\ \frac{\text{Nm}}{\text{s}} = 225991\ \text{W}$$

$(\because\ \ \text{Nm/s} = \text{W})$

$$= 225.991\ \text{kW}$$

$\therefore$ Power at runner = **225.991 kW. Ans.**

(*ii*) *Hydraulic efficiency*

Input power in kW is given by equation (18.3*A*).

$\therefore$ Input power $= \dfrac{\rho \times g \times Q \times H}{1000}$, where $\rho = 1000\ \text{kg/m}^3$

$$= \frac{1000 \times 9.81 \times 0.675 \times 35}{1000} = 231.761\ \text{kW}$$

$\therefore$ Hydraulic efficiency $= \dfrac{\text{Power at runner}}{\text{Input power}}$

$$= \frac{225.991}{231.761} = 0.975 = 0.975 \times 100 = \mathbf{97.5\%.\ Ans.}$$

▶ 18.13 CHARACTERISTIC CURVES OF HYDRAULIC TURBINES

Characteristic curves of a hydraulic turbine are the curves, with the help of which the exact behaviour and performance of the turbine under different working conditions, can be known. These curves are plotted from the results of the tests performed on the turbine under different working conditions.

The important parameters which are varied during a test on a turbine are :

1. Speed (N) 2. Head (H) 3. Discharge (Q)
4. Power (P) 5. Overall efficiency (η_o) and 6. Gate opening.

Out of the above six parameters, three parameters namely speed (N), head (H) and discharge (Q) are independent parameters.

Out of the three independent parameters, (N, H, Q) one of the parameter is kept constant (say H) and the variation of the other four parameters with respect to any one of the remaining two independent variables (say N and Q) are plotted and various curves are obtained. These curves are called characteristic curves. The following are the important characteristic curves of a turbine.

1. Main Characteristic Curves or Constant Head Curves.
2. Operating Characteristic Curves or Constant Speed Curves.
3. Muschel Curves or Constant Efficiency Curves.

18.13.1 Main Characteristic Curves or Constant Head Curves.

Main characteristic curves are obtained by maintaining a constant head and a constant gate opening (G.O.) on the turbine. The speed of the turbine is varied by changing load on the turbine. For each value of the speed, the corresponding values of the power (P) and discharge (Q) are obtained. Then the overall efficiency (η_o) for each value of the speed is calculated. From these readings the values of unit speed (N_u), unit power (P_u) and unit discharge (O_u) are determined. Taking N_u as abscissa, the values of Q_u, P_u, P and η_o are plotted as shown in Figs. 18.35 and 18.36. By changing the gate opening, the values of Q_u, P_u and η_o and N_u are determined and taking N_u as abscissa, the values of Q_u, P_u and η_o are plotted. Fig. 18.35

shows the main characteristic curves for Pelton wheel and Fig. 18.36 shows the main characteristic curves for reaction (Francis and Kaplan) turbines.

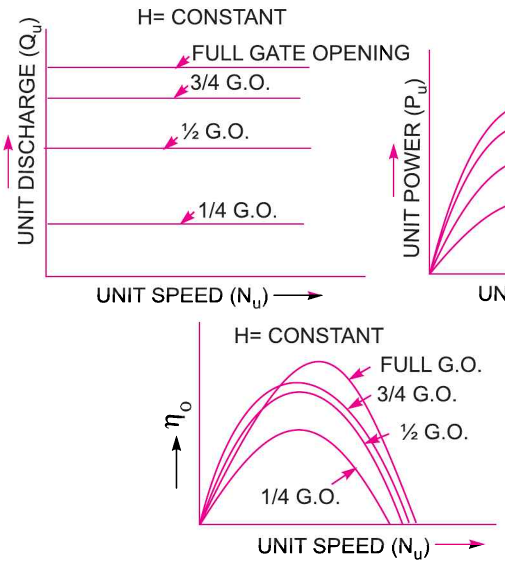

Fig. 18.35 *Main characteristic curves for a Pelton wheel.*

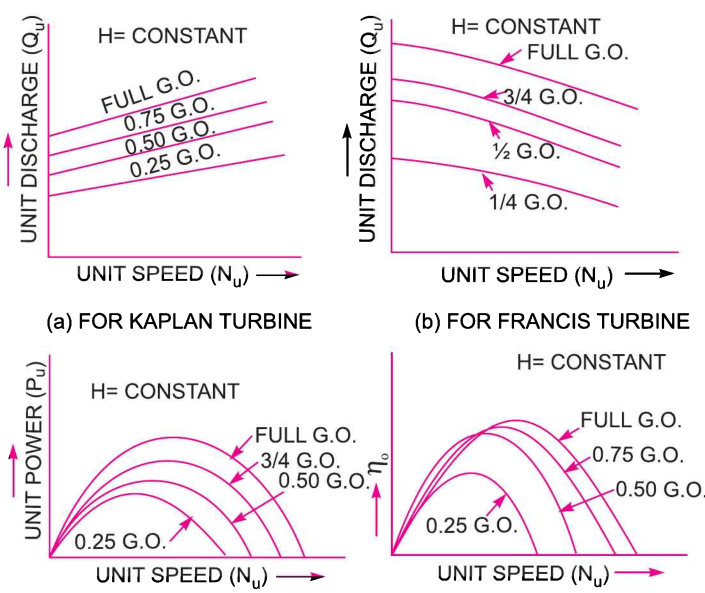

Fig. 18.36 *Main characteristic curves for reaction turbine.*

18.13.2 Operating Characteristic Curves or Constant Speed Curves. Operating characteristic curves are plotted when the speed on the turbine is constant. In case of turbines, the head is generally constant. As mentioned in Art. 18.13, there are three independent parameters namely N, H and Q. For operating characteristics N and H are constant and hence the variation of power and efficiency with respect to discharge Q are plotted. The power curve for turbines shall not pass through the origin because certain amount of discharge is needed to produce power to overcome initial friction. Hence the power and efficiency curves will be slightly away from the origin on the *x*-axis, as to overcome initial friction certain amount of discharge will be required. Fig. 18.37 shows the variation of power and efficiency with respect to discharge.

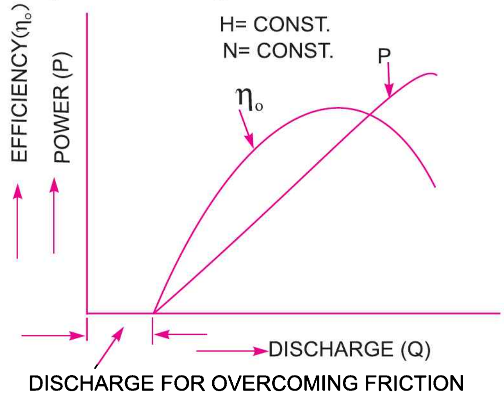

Fig. 18.37 *Operating characteristic curves.*

18.13.3 Constant Efficiency Curves or Muschel Curves or Iso-Efficiency Curves. These curves are obtained from the speed *vs*. efficiency and speed *vs*. discharge curves for different gate openings. For a given efficiency from the N_u *vs*. η_o curves, there are two speeds. From the N_u *vs*. Q_u curves, corresponding to two values of speeds there are two values of discharge. Hence for a given efficiency there are two values of discharge for a particular gate opening. This means for a given efficiency there are two values of speeds and two values of the discharge for a given gate opening. If the efficiency is maximum there is only one value. These two values of speed and two values of discharge corresponding to a particular gate opening are plotted as shown in Fig. 18.38 (*b*). The procedure is repeated for different gate openings and the curves *Q vs*. *N* are plotted. The points having the same efficiencies are joined. The curves having same efficiency are called iso-efficiency curves. These curves are helpful for determining the zone of constant efficiency and for predicating the performance of the turbine at various efficiencies.

For plotting the iso-efficiency curves, horizontal lines representing the same efficiency are drawn on the η_o ~ speed curves. The points at which these lines cut the efficiency curves at various gate openings are transferred to the corresponding *Q* ~ speed curves. The points having the same efficiency are then joined by a smooth curves. These smooth curves represents the iso-efficiency curve.

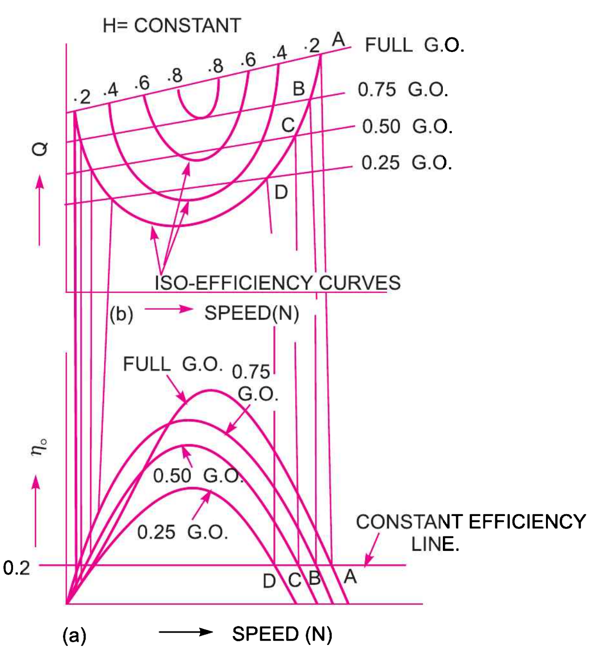

Fig. 18.38 *Constant efficiency curve.*

▶ 18.14 GOVERNING OF TURBINES

The governing of a turbine is defined as the operation by which the speed of the turbine is kept constant under all conditions of working. It is done automatically by means of a governor, which regulates the rate of flow through the turbines according to the changing load conditions on the turbine.

Governing of a turbine is necessary as a turbine is directly coupled to an electric generator, which is required to run at constant speed under all fluctuating load conditions. The frequency of power generation by a generator of constant number of pair of poles under all varying conditions should be constant. This is only possible when the speed of the generator, under all changing load condition, is constant. The speed of the generator will be constant, when the speed of the turbine (which is coupled to the generator) is constant.

When the load on the generator decreases, the speed of the generator increases beyond the normal speed (constant speed). Then the speed of the turbine also increases beyond the normal speed. If the turbine or the generator is to run at constant (normal) speed, the rate of flow of water to the turbine should be decreased till the speed becomes normal. This process by which the speed of the turbine (and hence of generator) is kept constant under varying condition of load is called governing.

Governing of Pelton Turbine (Impulse Turbine)

Governing of Pelton turbine is done by means of oil pressure governor, which consists of the following parts :

1. Oil sump.
2. Gear pump also called oil pump, which is driven by the power obtained from turbine shaft.
3. The Servomotor also called the relay cylinder.
4. The control valve or the distribution valve or relay valve.
5. The centrifugal governor or pendulum which is driven by belt or gear from the turbine shaft.
6. Pipes connecting the oil sump with the control valve and control valve with servomotor and
7. The spear rod or needle.

Fig. 18.39 shows the position of the piston in the relay cylinder, position of control or relay valve and fly-balls of the centrifugal governor, when the turbine is running at the normal speed.

When the load on the generator decreases, the speed of the generator increases. This increases the speed of the turbine beyond the normal speed. The centrifugal governor, which is connected to the turbine main shaft, will be rotating at an increased speed . Due to increase in the speed of the centrifugal governor, the fly-balls move upward due to the increased centrifugal force on them. Due to the upward movement of the fly-balls, the sleeve will also move upward. A horizontal lever, supported over a fulcrum, connects the sleeve and the piston rod of the control valve. As the sleeve moves up, the lever turns about the fulcrum and the piston rod of the control valve moves downward. This closes the valve V_1 and opens the valve V_2 as shown in Fig. 18.39.

The oil, pumped from the oil pump to the control valve or relay valve, under pressure will flow through the valve V_2 to the servomotor (or relay cylinder) and will exert force on the face A of the piston of the relay cylinder. The piston along with piston rod and spear will move towards right. This will decrease the area of flow of water at the outlet of the nozzle. This decrease of area of flow will reduce the rate of flow of water to the turbine which consequently reduces the speed of the turbine. When the speed of the turbine becomes normal, the fly-balls, sleeve, lever and piston rod of control valve come to its normal position as shown in Fig. 18.39.

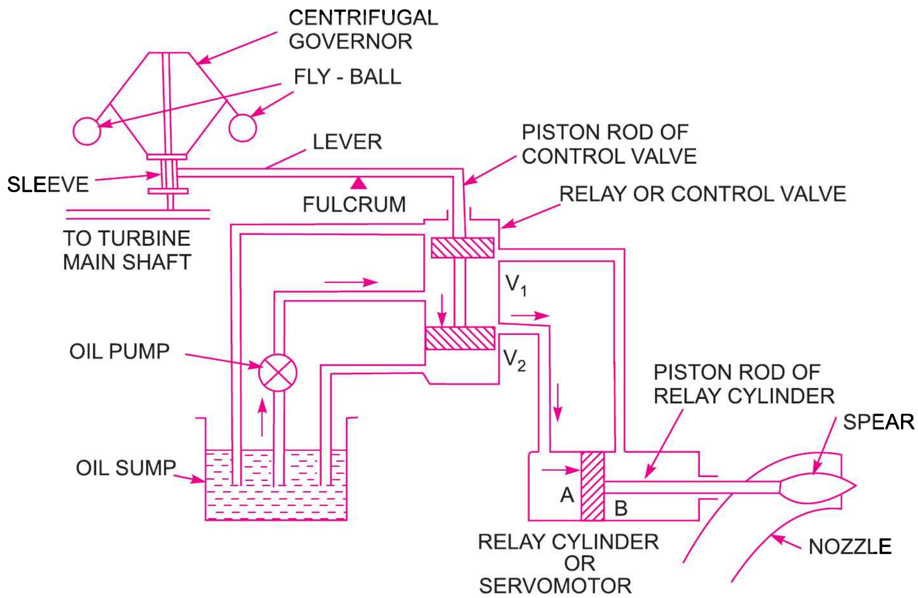

Fig. 18.39. *Governing of Pelton turbine.*

When the load on the generator increases, the speed of the generator and hence of the turbine decreases. The speed of the centrifugal governor also decreases and hence centrifugal force acting on the fly-balls also reduces. This brings the fly-balls in the downward direction. Due to this, the sleeve moves downward and the lever turns about the fulcrum, moving the piston rod of the control valve in the upward direction. This closes the valve V_2 and opens the valve V_1. The oil under pressure from the control valve, will move through valve V_1 to the servomotor and will exert a force on the face B of the piston. This will move the piston along with the piston rod and spear towards left, increasing the area of flow of water at the outlet of the nozzle. This will increase the rate of flow of water to the turbine and consequently, the speed of the turbine will also increase, till the speed of the turbine becomes normal.

HIGHLIGHTS

1. The hydraulic machines, which convert the hydraulic energy into mechanical energy, are called turbines.
2. Gross head is the vertical difference between the head race and tail race levels. Net head or effective head is the head, available at the inlet of the turbine. It is given by

$$H = H_g - h_f$$

where H_g = Gross head, and

h_f = Loss of head due to friction in penstocks

$$= \frac{4f \times L \times V^2}{D \times 2g}$$

where D = Dia. of penstock.

3. The efficiencies of a turbine are : (*i*) Hydraulic efficiency, η_h, (*ii*) Mechanical efficiency, η_m, and (*iii*) Overall efficiency, η_o.
4. Hydraulic efficiency, η_h is given by

$$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}} = \frac{\text{R.P.}}{\text{W.P.}}$$

$$= \frac{W}{g}\frac{(V_{w_1}u_1 \pm V_{w_2}u_2)}{1000} \bigg/ \frac{(W \times H)}{1000} = \frac{(V_{w_1}u_1 \pm V_{w_2}u_2)}{gH}$$

where W.P. = Water power,
R.P. = Runner power *i.e.*, power available at the runner of the turbine,
S.P. = Shaft power *i.e.*, power at the shaft of the turbine.

5. Mechanical efficiency, η_m is given by $\eta_m = \frac{\text{S.P.}}{\text{R.P.}}$

6. Overall efficiency, η_o is given by $\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \eta_m \times \eta_h$.

7. If at the inlet of a turbine, the energy available is only kinetic energy, the turbine is known as impulse turbine. But if at the inlet of the turbine, the energy available is kinetic energy as well as pressure energy, the turbine is called reaction turbine.

8. Pelton wheel (or turbine) is a tangential flow impulse turbine and is used for high head. In this turbine,

$$V_1 = C_v\sqrt{2gH}\ ,\ u_1 = u_2 = u.$$

9. For the maximum efficiency of Pelton wheel the condition is $u = \frac{V_1}{2}$

Max. efficiency is given by $\eta_{max} = \frac{(1+\cos\phi)}{2}$, where ϕ = Vane angle at outlet.

10. The jet ratio (m) is defined as ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d) or $m = \frac{D}{d}$.

11. Francis turbine is an inward radial flow reaction turbine having discharge radial at outlet, which means the angle made by absolute velocity at outlet is 90°, *i.e.*, β = 90°. Then $V_{w_2} = 0$ and work done by water on the runner per second per unit weight of water becomes as $= \frac{1}{g}V_{w_1} \times u_1$.

12. Speed ratio is the ratio of the velocity of wheel at inlet to the velocity given by $\sqrt{2gH}$ whereas the flow ratio is the ratio of velocity of flow at inlet to the velocity given by $\sqrt{2gH}$.

13. Kaplan turbine is an axial flow reaction turbine in which the vanes on the hub are adjustable. The peripheral velocity at inlet and outlet are equal, *i.e.*, $u_1 = u_2$.

14. The discharge through a turbine is given by

$$Q = \frac{\pi}{4}d^2 \times \sqrt{2gH} \quad \text{...For a Pelton wheel}$$

$$= \pi D_1 B_1 \times V_{f_1} \quad \text{...For a Francis turbine}$$

$$= \frac{\pi}{4}(D_o^2 - D_b^2) \times V_{f_1} \quad \text{...For a Kaplan turbine.}$$

15. Draft-tube is a pipe of gradually increasing area used for discharging water from the exit of a reaction turbine. They may be conical or simple elbow type. The efficiency of the draft-tube is given by

$$\eta_d = \frac{\left(\frac{V_1}{2g} - \frac{V_2^2}{2g}\right) - h_f}{\left(\frac{V_1^2}{2g}\right)}$$

where V_1 = Velocity of water at the inlet of the draft-tube,
V_2 = Velocity of water at the outlet of the draft-tube,
h_f = Loss of head in draft-tube.

16. Specific speed of a turbine is defined as the speed at which a turbine runs when it is working under a unit head and develops unit power. The expression for specific speed (N_s) is given as

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

where P = shaft power in kW, H = Net head on the turbine.

17. Unit quantities are the quantities (like speed, discharge, power, etc.) which are obtained when the head on the turbine is unity. They are unit speed (N_u), unit power (P_u) and unit discharge (Q_u). They are given as

$$N_u = \frac{N}{\sqrt{H}},\ Q_u = \frac{Q}{\sqrt{H}},\ P_u = \frac{P}{H^{3/2}}\ .$$

18. The important characteristic curves of a turbine are :
(*a*) Main characteristic curves or Constant head curves.
(*b*) Operating characteristic curves or Constant speed curves, and
(*c*) Muschel curves or Constant efficiency curves.

19. Governing of a turbine is defined as the operation by which the speed of the turbine is kept constant under all conditions of working. It is done by oil pressure governor.

EXERCISE

(A) THEORETICAL PROBLEMS

1. Define the terms : Hydraulic machines, Turbines and Pumps.

2. Differentiate between the turbines and pumps.

3. (*a*) What do you mean by gross head, net head and efficiency of turbine ? Explain the different types of the efficiency of a turbine.
(*b*) Explain clearly the following terms as they are applied to a Pelton wheel :
(*i*) Gross head ; (*ii*) Net head.

4. How will you classify the turbines ?

5. Differentiate between : (*a*) The impulse and reaction turbines, (*b*) Radial and axial flow turbines, (*c*) Inward and outward radial flow turbine, and (*d*) Kaplan and propeller turbines.

6. Obtain an expression for the work done per second by water on the runner of a Pelton wheel. Hence derive an expression for maximum efficiency of the Pelton wheel giving the relationship between the jet speed and bucket speed.
Draw inlet and outlet velocity triangles for a Pelton turbine and indicate the direction of various velocities.

7. Prove that the work done per second per unit weight of water in a reaction turbine is given as

$$= \frac{1}{g}(V_{w_1}u_1 \pm V_{w_2}u_2)$$

where V_{w_1} and V_{w_2} = Velocities of whirl at inlet and outlet,

u_1 and u_2 = Peripheral velocities at inlet and outlet.

8. Define the terms : speed ratio, flow ratio and jet ratio.

9. (*a*) What is a draft-tube ? Why is it used in a reaction turbine ? Describe with sketch two different types of draft-tubes.

(*b*) What are the uses of a draft-tube ? Describe with neat sketches different types of draft-tubes. (*J.N.T.U., Hyderabad S 2002*).

10. What is the basis of selection of a turbine at a particular place ?

11. Define the specific speed of a turbine ? Derive an expression for the specific speed. What is the significance of the specific speed?

12. What are unit quantities ? Define the unit quantities for a turbine. Why are they important ?

13. Obtain an expression for unit speed, unit discharge and unit power for a turbine.

14. What do you understand by the characteristic curves of a turbine ? Name the important types of characteristic curves.

15. Define the term 'Governing of a turbine'. Describe with a neat sketch the working of an oil pressure governor.

16. Give the range of specific speed values of the Kaplan, Francis turbines and Pelton wheels.

What factors decide whether Kaplan, Francis, or a Pelton type turbine would be used in a hydroelectric project ?

17. (*a*) Draw neat sketches of the Pelton turbine and Francis Turbine.

(*b*) Describe briefly the function of various main components of Pelton turbine with neat sketches.

18. What is cavitation ? How can it be avoided in reaction turbine ?

19. Define the terms 'unit power', 'unit speed ' and 'unit discharge' with reference to a hydraulic turbine. Also derive expressions for these terms.

20. (*a*) Define specific speed of a turbine and derive an expression for the same. Show that Pelton turbine is a low specific speed turbine.

(*b*) What is specific speed ? State its significance in the study of hydraulic machines.

21. (*a*) By means of a neat sketch explain the governing mechanism of Francis Turbine.

(*b*) Explain the difference between Kaplan turbine and propeller turbine.

22. Define and explain hydraulic efficiency, mechanical efficiency and overall efficiency of a turbine.

23. Define the terms : specific speed of a turbine, unit speed, unit power and unit rate of flow of a turbine. Derive the expressions for specific speed and unit speed.

24. (*a*) What is meant by the speed ratio of a Pelton wheel ?

(*b*) What is a draft-tube ? What are its functions ?

(*c*) Differentiate between an inward and an outward flow reaction turbine.

(B) NUMERICAL PROBLEMS

1. A Pelton wheel has a mean bucket speed of 35 m/s with a jet of water flowing at the rate of 1 m^3/s under a head of 270 m. The buckets deflect the jet through an angle of 170° . Calculate the power delivered to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.

[**Ans.** 2523.8 kW, 95.3%]

2. A Pelton wheel is to be designed for the following specifications. Power = 735.75 kW, S.P. Head = 200 m, Speed = 800 r.p.m. , η_o = 0.86 and jet diameter is not to exceed one-tenth the wheel diameter.

Determine : (*i*) Wheel diameter, (*ii*) The number of jets required, and (*iii*) Diameter of the jet. Take C_v = 0.98 and speed ratio = 0.45. [**Ans.** (*i*) 0.673 m, (*ii*) 2, (*iii*) 67.3 mm]

3. A Pelton wheel is having a mean bucket diameter of 0.8 m and is running at 1000 r.p.m. The net head on the Pelton wheel is 400 m. If the side clearance angle is 15° and discharge through nozzle is 150 litres/s, find : (*i*) Power available at the nozzle, and (*ii*) Hydraulic efficiency of the turbine.

[**Ans.** (*i*) 588.6 kW, (*ii*) 98%]

4. Two jets strike at buckets of a Pelton wheel, which is having shaft power as 14,715 kW. The diameter of each jet is given as 150 mm. If the net head on the turbine is 500 m, find the overall efficiency of the turbine. Take $C_v = 1.0$. [**Ans.** 85.7%]

5. The following data is related to the Pelton wheel :

Head at the base of the nozzle = 110 m,
Diameter of the jet = 7.5 cm,
Discharge of the nozzle = 200 litres/s,
Shaft power = 191.295 kW
Power absorbed in mechanical resistance = 3.675 kW.

Determine : (*i*) Power lost in nozzle and, (*ii*) Power lost due to hydraulic resistance in the runner.

[**Ans.** (*i*) 10.874 kW, (*ii*) 9.97 kW]

6. Design a Pelton wheel for a head of 80 m and speed 300 r.p.m. The Pelton wheel develops 103 kW S.P. Take $C_v = 0.98$, speed ratio = 0.45 and overall efficiency = 0.80.

[**Ans.** $D = 1.135$ m, $d = 72.6$ mm, size = 36.3 × 8.7, $Z = 23$]

7. An inward flow reaction turbine has external and internal diameters as 1.2 m and 0.6 m respectively. The velocity of flow through the runner is constant and is equal to 1.8 m/s. Determine : (*i*) Discharge through the runner, and (*ii*) Width at outlet if the width at inlet = 200 mm. [**Ans.** (*i*) 1.357 m^3/s, (*ii*) 400 mm]

8. A reaction turbine works at 500 r.p.m. under a head of 100 m. The diameter of turbine at inlet is 100 cm and flow area is 0.35 m^2. The angles made by absolute and relative velocities at inlet are 15° and 60° respectively with the tangential velocity. Determine :

(*i*) The volume flow rate, (*ii*) The power developed, and (*iii*) Efficiency. Assume whirl at outlet to be zero.

[**Ans.** (*i*) 2.905 m^3/s, (*ii*) 2355.35 kW, (*iii*) 82.6%]

9. An inward flow reaction turbine has an external diameter of 1 m and its breadth at inlet is 200 mm. If the velocity of flow at inlet is 1.5 m/s, find the mass of water passing through the turbine per second. Assume 15% of the area of flow is blocked by blade thickness. If the speed of the runner is 200 r.p.m. and guide blades make an angle of 15° to the wheel tangent, draw the inlet velocity triangle and find : (*i*) The runner vane angle at inlet (*ii*) Velocity of wheel at inlet, (*iii*) The absolute velocity of water leaving the guide vanes, and (*iv*) The relative velocity of water entering the runner blade.

[**Ans.** 1602.2 kg/s, (*i*) 76.19, (*ii*) 10.47 m/s, (*iii*) 11.59 m/s, (*iv*) 3.087 m/s]

10. An outward flow reaction turbine has internal and external diameters of the runner as 0.5 m and 1.0 m respectively. The guide blade angle is 15° and velocity of flow through the runner is constant and equal to 3 m/s. If the speed of the turbine is 250 r.p.m., head on turbine is 10 m and discharge at outlet is radial, determine : (*i*) The runner vane angles at inlet and outlet, (*ii*) Work done by the water on the runner per second per unit weight of water striking per second and (*iii*) Hydraulic efficiency.

[**Ans.** (*i*) 32° 48′, 12° 55′, (*ii*) 7.47 m, (*iii*) 7.47%]

11. A Francis turbine with an overall efficiency of 70% is required to produce 147.15 kW. It is working under a head of 8 m. The peripheral velocity = 0.30 $\sqrt{2gH}$ and the radial velocity of flow at inlet is 0.96 $\sqrt{2gH}$. The wheel runs at 200 r.p.m. and the hydraulic losses in the turbine are 20% of the available energy. Assume radial discharge, determine : (*i*) The guide blade angle, (*ii*) The wheel vane angle at inlet, (*iii*) Diameter of the wheel at inlet, and (*iv*) Width of wheel at inlet.

[**Ans.** (*i*) 35° 45′,(*ii*) 42° 54′, (*iii*) 35.9 cm, (*iv*) 19.75 cm]

12. The following data is given for a Francis turbine : Net head = 70 m, speed = 600 r.p.m., shaft power = 367.875 kW, η_o = 85% , η_h = 95%, flow ratio = 0.25, breadth ratio = 0.1, outer diameter of the runner = 2 × inner diameter of runner. The thickness of vanes occupy 10% of the circumferential area of the runner. Velocity of flow is constant at inlet and outlet and discharge is radial at outlet. Determine : (*i*) Guide blade angle, (*ii*) Runner vane angles at inlet and outlet, (*iii*) Diameters of runner at inlet and outlet, and (*iv*) Width of wheel at inlet.

[**Ans.** (*i*) 12° 20′, (*ii*) 18° 57′, 50° 17′, (*iii*) .49,.245 m, (*iv*) 49 mm]

13. A Kaplan turbine working under a head of 15 m develops 7357.5 kW shaft power. The outer diameter of the runner is 4 m and hub diameter is 2 m. The guide blade angle at the extreme edge of the runner is 30°. The hydraulic and overall efficiencies of the turbine are 90% and 85% respectively. If the velocity of whirl is zero at outlet, detemine : (*i*) runner vane angles at inlet and outlet at the extreme edge of the runner and (*ii*) speed of the turbine. [**Ans.** (*i*) 103° 10′, 26° 58.5′, (*ii*) 58.5]

14. A Kaplan turbine runner is to be designed to develop 7357.5 kW S.P. The net available head is 10 m. Assume that the speed ratio is 1.8 and flow ratio is 0.6. If the overall efficiency is 70% and diameter of the boss is 0.4 times the diameter of the runner, find the diameter of the runner, its speed and specific speed.

[**Ans.** 4.39 m, 109.63 r.p.m., 528.82]

15. A conical draft-tube having inlet and outlet diameters 0.8 m and 1.2 m discharges water at outlet with a velocity of 3 m/s. The total length of the draft-tube is 8 m and 2 m of the length of draft-tube is immersed in water. If the atmospheric pressure head is 10.3 m of water and loss of head due to friction in the draft-tube is equal to 0.25 times the velocity head at outlet of the tube, find : (*i*) Pressure head at inlet, and (*ii*) Efficiency of the draft-tube. [**Ans.** (*i*) 2.551 m (abs.), (*ii*) 75.3%]

16. A turbine is to operate under a head of 30 m at 300 r.p.m. The discharge is 10 m^3/s. If the efficiency is 90%, determine : (*i*) specific speed of the machine, (*ii*) power generated, and (*iii*) types of the turbine.

[**Ans.** (*i*) 219.9, (*ii*) 2648.7 kW (*iii*) Francis]

17. A turbine develops 7357.5 kW S.P. when running at 200 r.p.m. The head on the turbine is 40 m. If the head on the turbine is reduced to 25 m, determine the speed and power developed by the turbine.

[**Ans.** 158.11, 3635.34 kW]

18. A Pelton wheel is revolving at a speed of 200 r.p.m. and develops 5886 kW S.P. when working under a head of 200 m with an overall efficiency of 80%. Determine unit speed, unit discharge and unit power . The speed ratio for the turbine is given as 0.48. Find the speed, discharge and power when this turbine is working under a head of 150 m.

[**Ans.** 14.14, 0.265 m^3/s, 2.08 kW and 173.2 r.p.m., 3.247 m^3/s, 3823 kW]

19. A Kaplan turbine working under a head of 29 m develops 1287.5 kW S.P. If the speed ratio is equal to 2.1, flow ratio = 0.62, diameter of boss = 0.34 times the diameter of the runner and overall efficiency of the turbine = 89%, find the diameter of the runner and the speed of turbine. [**Ans.** 0.705 m, 1162.3]

20. A Kaplan turbine working under a head of 25 m develops 16000 kW shaft power. The outer diameter of the runner is 4 m and hub diameter is 2 m . The guide blade angle is 35°. The hydraulic and overall efficiency are 90% and 85% respectively. If the velocity of whirl is zero at outlet, determine runner vane angles at inlet and outlet, and speed of turbine. [**Ans.** 35° 27′, 88° 36′, 9.236]

21. A Kaplan turbine develops 9000 kW under a net head of 7.5 m. Mechanical efficiency of the wheel is 86%. The speed ratio based on the outer diameter is 2.2 and the flow ratio is 0.66. Diameter of the boss is 0.35 times the external diameter of the wheel. Determine the diameter of the runner and the specific speed of the runner. (*J.N.T.U., Hyderabad, S 2002*)

[**Hint.** Given : S.P. = 9000 kW; H = 7.5 m ; η_m = 86% = 0.86; Speed ratio = 2.2 ;
flow ratio = 0.66 ; Dia. of boss = 0.35 × External dia. of wheel *i.e.*, $D_b = 0.35\, D_o$.

$$\frac{u_1}{\sqrt{2gH}} = 2.2 \quad \therefore \quad u_1 = 2.2 \times \sqrt{2gH} = 2.2 \times \sqrt{2 \times 9.81 \times 7.5} = 26.68 \text{ m/s}$$

$$\frac{V_{f_1}}{\sqrt{2gH}} = 0.66 \; \therefore \; V_{f_1} = 0.66 \times \sqrt{2gH} = 0.66 \times \sqrt{2 \times 9.81 \times 7.5} = 8 \text{ m/s}$$

Note. In this question either data is incomplete or instead of mechanical efficiency it should be overall efficiency. The question is solved taking the given efficiency as overall efficiency.

Now $\eta_o = 0.86$ But $\eta_o = \frac{\text{S.P.}}{\text{W.P.}}$ $\therefore$ $\text{W.P.} = \frac{\text{S.P.}}{\eta_o} = \frac{9000}{0.86}$ kW

But $\text{W.P.} = \frac{\rho \times Q \times g \times H}{1000}\ \text{kW} = \frac{1000 \times Q \times 9.81 \times 7.5}{1000} = Q \times 9.81 \times 7.5$ kW

$\therefore Q \times 9.81 \times 7.5 = \frac{9000}{0.86}$ or $Q = \frac{9000}{0.86} \times \frac{1}{9.81 \times 7.5} = 142.237\ \text{m}^3/\text{s}$

But $Q = \frac{\pi}{4}\left[D_o^2 - D_b^2\right] \times V_{f_1}$ $\therefore$ $142.237 = \frac{\pi}{4}\left[D_o^2 - D_b^2\right] \times V_{f_1} = \frac{\pi}{4}[D_o^2 - (0.35D_o)^2] \times 8$

or $142.237 = \frac{\pi}{4} \times 0.8775\, D_o^2 \times 8$ or $D_o = \sqrt{\frac{142.237 \times 4}{\pi \times 0.8775 \times 8}} = 5.079\ \text{m} \simeq$ **5 m.**

Specific speed of turbine, $N_s = \frac{N\sqrt{\text{S.P.}}}{H^{5/4}}$, where N is obtained from u_1

$\therefore$ $u_1 = \frac{\pi D_o N}{60}$ or $26.68 = \frac{\pi \times 5 \times N}{60}$ or $N = \frac{26.68 \times 60}{\pi \times 5} = 101.91$ r.p.m.

$\therefore$ $N_s = \frac{101.91 \times \sqrt{9000}}{7.5^{1.25}}$ = **778.95]**