9 CHAPTER

VISCOUS FLOW

▶ 9.1 INTRODUCTION

This chapter deals with the flow of fluids which are viscous and flowing at very low velocity. At low velocity the fluid moves in layers. Each layer of fluid slides over the adjacent layer. Due to relative velocity between two layers the velocity gradient $\frac{du}{dy}$ exists and hence a shear stress $\tau = \mu \frac{du}{dy}$ acts on the layers.

The following cases will be considered in this chapter :

1. Flow of viscous fluid through circular pipe.
2. Flow of viscous fluid between two parallel plates.
3. Kinetic energy correction and momentum correction factors.
4. Power absorbed in viscous flow through
   (*a*) Journal bearings, (*b*) Foot-step bearings, and (*c*) Collar bearings.

▶ 9.2 FLOW OF VISCOUS FLUID THROUGH CIRCULAR PIPE

For the flow of viscous fluid through circular pipe, the velocity distribution across a section, the ratio of maximum velocity to average velocity, the shear stress distribution and drop of pressure for a given length is to be determined. The flow through the circular pipe will be viscous or laminar, if the Reynolds number (R_e*) is less than 2000. The expression for Reynold number is given by

$$R_e = \frac{\rho V D}{\mu}$$

where ρ = Density of fluid flowing through pipe
V = Average velocity of fluid
D = Diameter of pipe and
μ = Viscosity of fluid.

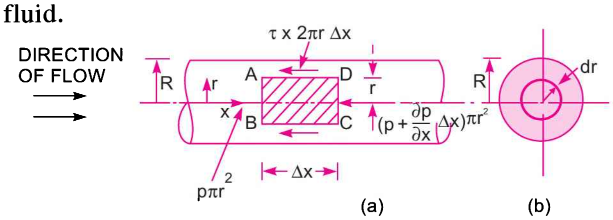

Fig. 9.1 *Viscous flow through a pipe.*

* For derivation, please refer to Art. 12.8.1.

Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe as shown in Fig. 9.1 (a). Consider a fluid element of radius r, sliding in a cylindrical fluid element of radius $(r + dr)$. Let the length of fluid element be Δx. If 'p' is the intensity of pressure on the face AB, then the intensity of pressure on face CD will be $\left(p + \frac{\partial p}{\partial x}\Delta x\right)$. Then the forces acting on the fluid element are :

1. The pressure force, $p \times \pi r^2$ on face AB.
2. The pressure force, $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2$ on face CD.
3. The shear force, $\tau \times 2\pi r \Delta x$ on the surface of fluid element. As there is no acceleration, hence the summation of all forces in the direction of flow must be zero *i.e.*,

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x} . r - 2\tau = 0$$

$\therefore$
$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2} \quad \text{...(9.1)}$$

The shear stress τ across a section varies with 'r' as $\frac{\partial p}{\partial x}$ across a section is constant. Hence shear stress distribution across a section is linear as shown in Fig. 9.2 (a).

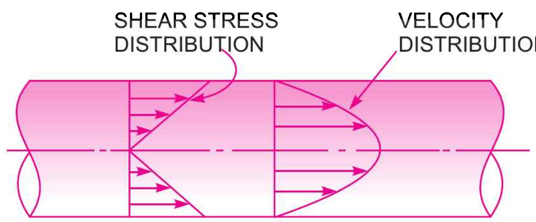

Fig. 9.2 *Shear stress and velocity distribution across a section.*

(i) **Velocity Distribution.** To obtain the velocity distribution across a section, the value of shear stress $\tau = \mu \frac{du}{dy}$ is substituted in equation (9.1).

But in the relation $\tau = \mu \frac{du}{dy}$, y is measured from the pipe wall. Hence

$$y = R - r \quad \text{and} \quad dy = -dr$$

$\therefore$
$$\tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

Substituting this value in (9.1), we get

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2} \quad \text{or} \quad \frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x} r$$

Integrating this above equation w.r.t. 'r', we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + C \qquad \text{...(9.2)}$$

where C is the constant of integration and its value is obtained from the boundary condition that at $r = R$, $u = 0$.

$$\therefore \qquad 0 = \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 + C$$

$$\therefore \qquad C = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

Substituting this value of C in equation (9.2), we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \qquad \text{...(9.3)}$$

In equation (9.3), values of μ, $\frac{\partial p}{\partial x}$ and R are constant, which means the velocity, u varies with the square of r. Thus equation (9.3) is a equation of parabola. This shows that the velocity distribution across the section of a pipe is parabolic. This velocity distribution is shown in Fig. 9.2 (*b*).

(*ii*) **Ratio of Maximum Velocity to Average Velocity.** The velocity is maximum, when $r = 0$ in equation (9.3). Thus maximum velocity, U_{max} is obtained as

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 \qquad \text{...(9.4)}$$

The average velocity, $\bar{u}$, is obtained by dividing the discharge of the fluid across the section by the area of the pipe (πR^2). The discharge (Q) across the section is obtained by considering the flow through a circular ring element of radius r and thickness dr as shown in Fig. 9.1 (*b*). The fluid flowing per second through this elementary ring

$$dQ = \text{velocity at a radius } r \times \text{area of ring element}$$

$$= u \times 2\pi r\, dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \times 2\pi r\, dr$$

$$\therefore \qquad Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2) \times 2\pi r\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 - r^2)\, r dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 r - r^3)\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \times \frac{R^4}{4} = \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4$$

∴ Average velocity, $$\bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4}{\pi R^2}$$

or $$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2 \qquad ...(9.5)$$

Dividing equation (9.4) by equation (9.5),

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2} = 2.0$$

∴ Ratio of maximum velocity to average velocity = 2.0.

(*iii*) **Drop of Pressure for a given Length (*L*) of a pipe**

From equation (9.5), we have

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2 \quad \text{or} \quad \left(\frac{-\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$

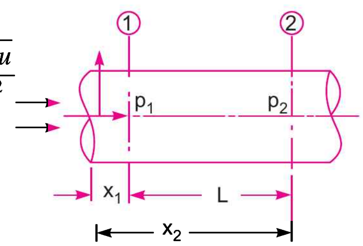

Fig. 9.3

Integrating the above equation w.r.t. x, we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\, dx$$

$$\therefore \quad -[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2] \text{ or } (p_1 - p_2) = \frac{8\mu\bar{u}}{R^2}[x_2 - x_1]$$

$$= \frac{8\mu\bar{u}}{R^2} L \qquad \{\because \; x_2 - x_1 = L \text{ from Fig. 9.3}\}$$

$$= \frac{8\mu\bar{u}L}{(D/2)^2} \qquad \left\{\because \; R = \frac{D}{2}\right\}$$

or $$(p_1 - p_2) = \frac{32\mu\bar{u}L}{D^2},$$ where $p_1 - p_2$ is the drop of pressure.

∴ Loss of pressure head $$= \frac{p_1 - p_2}{\rho g}$$

$$\therefore \quad \frac{p_1 - p_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2} \qquad ...(9.6)$$

Equation (9.6) is called **Hagen Poiseuille Formula.**

Problem 9.1 *A crude oil of viscosity 0.97 poise and relative density 0.9 is flowing through a horizontal circular pipe of diameter 100 mm and of length 10 m. Calculate the difference of pressure at the two ends of the pipe, if 100 kg of the oil is collected in a tank in 30 seconds.*

Solution. Given : $\mu = 0.97 \text{ poise} = \frac{0.97}{10} = 0.097 \text{ Ns/m}^2$

Relative density $= 0.9$

$\therefore$ ρ_0, or density, $= 0.9 \times 1000 = 900 \text{ kg/m}^3$

Dia. of pipe, $D = 100 \text{ mm} = 0.1 \text{ m}$

$L = 10 \text{ m}$

Mass of oil collected, $M = 100 \text{ kg}$

Time, $t = 30$ seconds

Calculate difference of pressure or $(p_1 - p_2)$.

The difference of pressure $(p_1 - p_2)$ for viscous or laminar flow is given by

$$p_1 - p_2 = \frac{32\mu\bar{u}L}{D^2}, \text{ where } \bar{u} = \text{average velocity} = \frac{Q}{\text{Area}}$$

Now, mass of oil/sec $= \frac{100}{30}$ kg/s

$$= \rho_0 \times Q = 900 \times Q \qquad (\because \rho_0 = 900)$$

$$\therefore \quad \frac{100}{30} = 900 \times Q$$

$$\therefore \quad Q = \frac{100}{30} \times \frac{1}{900} = 0.0037 \text{ m}^3/\text{s}$$

$$\therefore \quad \bar{u} = \frac{Q}{\text{Area}} = \frac{.0037}{\frac{\pi}{4}D^2} = \frac{.0037}{\frac{\pi}{4}(.1)^2} = 0.471 \text{ m/s.}$$

For laminar or viscous flow, the Reynolds number (R_e) is less than 2000. Let us calculate the Reynolds number for this problem.

Reynolds number, $R_e{}^* = \frac{\rho VD}{\mu}$

where $\rho = \rho_0 = 900$, $V = \bar{u} = 0.471$, $D = 0.1$ m, $\mu = 0.097$

$$\therefore \quad R_e = 900 \times \frac{.471 \times 0.1}{0.097} = 436.91$$

As Reynolds number is less than 2000, the flow is laminar.

$$\therefore \quad p_1 - p_2 = \frac{32\mu\bar{u}L}{D^2} = \frac{32 \times 0.097 \times .471 \times 10}{(.1)^2} \text{ N/m}^2$$

$$= 1462.28 \text{ N/m}^2 = 1462.28 \times 10^{-4} \text{ N/cm}^2 = \mathbf{0.1462 \text{ N/cm}^2}. \textbf{ Ans.}$$

* For derivation, please refer to Art. 12.8.1

Problem 9.2 *An oil of viscosity 0.1 Ns/m² and relative density 0.9 is flowing through a circular pipe of diameter 50 mm and of length 300 m. The rate of flow of fluid through the pipe is 3.5 litres/s. Find the pressure drop in a length of 300 m and also the shear stress at the pipe wall.*

Solution. Given : Viscosity, $\mu = 0.1$ Ns/m²

Relative density $= 0.9$

$\therefore$ ρ_0 or density of oil $= 0.9 \times 1000 = 900$ kg/m³ ($\because$ Density of water = 1000 kg/m³)

$D = 50$ mm $= .05$ m

$L = 300$ m

$$Q = 3.5 \text{ litres/s} = \frac{3.5}{1000} = .0035 \text{ m}^3\text{/s}$$

Find (*i*) Pressure drop, $p_1 - p_2$

(*ii*) Shear stress at pipe wall, τ_0

(*i*) **Pressure drop ($p_1 - p_2$)** $= \dfrac{32\mu\bar{u}L}{D^2}$, where $\bar{u} = \dfrac{Q}{\text{Area}} = \dfrac{.0035}{\frac{\pi}{4}D^2} = \dfrac{.0035}{\frac{\pi}{4}(.05)^2} = 1.782$ m/s

The Reynolds number (R_e) is given by, $R_e = \dfrac{\rho VD}{\mu}$

where $\rho = 900$ kg/m³, V = average velocity $= \bar{u} = 1.782$ m/s

$$\therefore \qquad R_e = 900 \times \frac{1.782 \times .05}{0.1} = 801.9$$

As Reynolds number is less than 2000, the flow is viscous or laminar

$$\therefore \qquad p_1 - p_2 = \frac{32 \times 0.1 \times 1.782 \times 3000}{(.05)^2}$$

$= 684288$ N/m² $= 68428 \times 10^{-4}$ N/cm² $=$ **68.43 N/cm². Ans.**

(*ii*) **Shear Stress at the pipe wall (τ_0)**

The shear stress at any radius r is given by the equation (9.1)

i.e.,
$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

$\therefore$ Shear stress at pipe wall, where $r = R$ is given by

$$\tau_0 = \frac{-\partial p}{\partial x}\frac{R}{2}$$

Now
$$\frac{-\partial p}{\partial x} = \frac{-(p_2 - p_1)}{x_2 - x_1} = \frac{p_1 - p_2}{x_2 - x_1} = \frac{p_1 - p_2}{L}$$

$$= \frac{684288}{300}\frac{\text{N/m}^2}{\text{m}} = 2280.96 \text{ N/m}^3$$

and
$$R = \frac{D}{2} = \frac{.05}{2} = .025 \text{ m}$$

$$\tau_0 = 2280.96 \times \frac{.025}{2}\frac{\text{N}}{\text{m}^2} = \mathbf{28.512 \text{ N/m}^2. \text{ Ans.}}$$

Problem 9.3 *A laminar flow is taking place in a pipe of diameter 200 mm. The maximum velocity is 1.5 m/s. Find the mean velocity and the radius at which this occurs. Also calculate the velocity at 4 cm from the wall of the pipe.*

Solution. Given : Dia. of pipe, D = 200 mm = 0.20 m

U_{max} = 1.5 m/s

Find (*i*) Mean velocity, $\bar{u}$

(*ii*) Radius at which $\bar{u}$ occurs

(*iii*) Velocity at 4 cm from the wall.

(*i*) **Mean velocity,** $\bar{u}$

Ratio of $$\frac{U_{max}}{\bar{u}} = 2.0 \quad \text{or} \quad \frac{1.5}{\bar{u}} = 2.0 \quad \therefore \quad \bar{u} = \frac{1.5}{2.0} = \mathbf{0.75\ m/s.\ Ans.}$$

(*ii*) **Radius at which** $\bar{u}$ occurs

The velocity, u, at any radius 'r' is given by (9.3)

$$u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2\left[1 - \frac{r^2}{R^2}\right]$$

But from equation (9.4) U_{max} is given by

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 \quad \therefore \quad u = U_{max}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad \text{...(1)}$$

Now, the radius r at which $u = \bar{u} = 0.75$ m/s

$$\therefore \qquad 0.75 = 1.5\left[1 - \left(\frac{r}{D/2}\right)^2\right]$$

$$= 1.5\left[1 - \left(\frac{r}{0.2/2}\right)^2\right] = 1.5\left[1 - \left(\frac{r}{0.1}\right)^2\right]$$

$$\therefore \qquad \frac{0.75}{0.50} = 1 - \left(\frac{r}{0.1}\right)^2$$

$$\therefore \qquad \left(\frac{r}{0.1}\right)^2 = 1 - \frac{.75}{1.50} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore \qquad \frac{r}{0.1} = \sqrt{\frac{1}{2}} = \sqrt{0.5}$$

$$\therefore \qquad r = 0.1 \times \sqrt{.5} = 0.1 \times .707 = .0707 \text{ m}$$

$$= \mathbf{70.7\ mm.\ Ans.}$$

(*iii*) **Velocity at 4 cm from the wall**

$$r = R - 4.0 = 10 - 4.0 = 6.0 \text{ cm} = 0.06 \text{ m}$$

∴ The velocity at a radius = 0.06 m

or 4 cm from pipe wall is given by equation (1)

$$= U_{max}\left[1-\left(\frac{r}{R}\right)^2\right] = 1.5\left[1-\left(\frac{.06}{.1}\right)^2\right]$$

$$= 1.5[1.0 - .36] = 1.5 \times .64 = \mathbf{0.96\ m/s.\ Ans.}$$

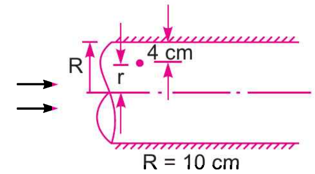

Fig. 9.4

Problem 9.4 *Crude oil of μ = 1.5 poise and relative density 0.9 flows through a 20 mm diameter vertical pipe. The pressure gauges fixed 20 m apart read 58.86 N/cm² and 19.62 N/cm² as shown in Fig. 9.5. Find the direction and rate of flow through the pipe.*

Solution. Given : $\mu = 1.5 \text{ poise} = \frac{1.5}{10} = 0.15 \text{ Ns/m}^2$

Relative density = 0.9

∴ Density of oil = $0.9 \times 1000 = 900 \text{ kg/m}^3$

Dia. of pipe, $D = 20 \text{ mm} = 0.02 \text{ m}$

$L = 20 \text{ m}$

$p_A = 58.86 \text{ N/cm}^2 = 58.86 \times 10^4 \text{ N/m}^2$

$p_B = 19.62 \text{ N/cm}^2 = 19.62 \times 10^4 \text{ N/m}^2$.

Find (*i*) Direction of flow

(*ii*) Rate of flow.

(*i*) **Direction of flow.** To find the direction of flow, the total energy $\left(\frac{p}{\rho g} + \frac{v^2}{2g} + Z\right)$ at the lower end *A* and at the upper end *B* is to be calculated. The direction of flow will be given from the higher energy to the lower energy. As here the diameter of the pipe is same and hence kinetic energy at *A* and *B* will be same. Hence to find the direction of flow, calculate $\left(\frac{p}{\rho g} + Z\right)$ at *A* and *B*.

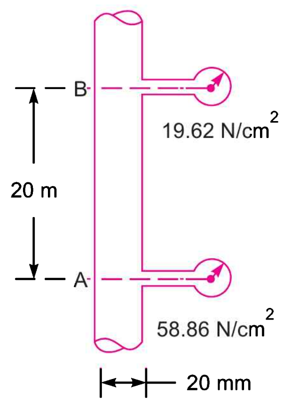

Fig. 9.5

Taking the level at *A* as datum. The value of $\left(\frac{p_A}{\rho g} + Z\right)$ at

$$A = \frac{p_A}{\rho g} + Z_A$$

$$= \frac{6 \times 10^4 \times 9.81}{900 \times 9.81} + 0 \quad \{\because \ r = 900 \text{ kg/cm}^2\}$$

$$= 66.67 \text{ m}$$

The value of $\left(\frac{p}{\rho g} + Z\right)$ at $B = \frac{p_B}{\rho g} + Z_B$

$$= \frac{2 \times 10^4 \times 9.81}{900 \times 9.81} + 20 = 22.22 + 20 = 42.22 \text{ m}$$

As the value of $\left(\frac{p}{\rho g} + Z\right)$ is higher at *A* and hence flow takes place from *A* to *B*. **Ans.**

(*ii*) **Rate of flow.** The loss of pressure head for viscous flow through circular pipe is given by

$$h_f = \frac{32\mu \bar{u} L}{\rho g D^2}$$

For a vertical pipe h_f = Loss of peizometric head

$$= \left(\frac{p_A}{\rho g} + Z_A\right) - \left(\frac{p_B}{\rho g} + Z_B\right) = 66.67 - 42.22 = 24.45 \text{ m}$$

$$\therefore \quad 24.45 = \frac{32 \times 0.15 \times \bar{u} \times 20.0}{900 \times 9.81 \times (.02)^2}$$

or

$$\bar{u} = \frac{24.45 \times 900 \times 9.81 \times .0004}{32 \times 0.15 \times 20.0} = 0.889 \simeq 0.9 \text{ m/s}.$$

The Reynolds number should be calculated. If Reynolds number is less than 2000, the flow will be laminar and the above expression for loss of pressure head for laminar flow can be used.

Now Reynolds number $= \dfrac{\rho V D}{\mu}$

where $\rho = 900$ kg/m^3 and $V = \bar{u}$

$\therefore$ Reynolds number $= 900 \times \dfrac{0.9 \times .02}{0.15} = 108$

As Reynolds number is less than 2000, the flow is laminar.

$\therefore$ Rate of flow = average velocity × area

$$= \bar{u} \times \frac{\pi}{4} D^2 = 0.9 \times \frac{\pi}{4} \times (.02)^2 \text{ m}^3\text{/s} = 2.827 \times 10^{-4} \text{ m}^3\text{/s}$$

= **0.2827 litres/s. Ans.** $(\because \ 10^{-3} \text{ m}^3 = 1 \text{ litre})$

Problem 9.5 *A fluid of viscosity 0.7 Ns/m^2 and specific gravity 1.3 is flowing through a circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is given as 196.2 N/m^2, find (i) the pressure gradient, (ii) the average velocity, and (iii) Reynolds number of the flow.*

Solution. Given : $\mu = 0.7 \dfrac{\text{Ns}}{\text{m}^2}$

Sp. gr. = 1.3

$\therefore$ Density $= 1.3 \times 1000 = 1300$ kg/m^3

Dia. of pipe, $D = 100$ mm $= 0.1$ m

Shear stress, $\tau_0 = 196.2$ N/m^2

Find (*i*) Pressure gradient, $\dfrac{dp}{dx}$

(*ii*) Average velocity, $\bar{u}$

(*iii*) Reynolds number, R_e

(*i*) **Pressure gradient,** $\dfrac{dp}{dx}$

The maximum shear stress (τ_0) is given by

$$\tau_0 = -\frac{\partial p}{\partial x}\frac{R}{2} \text{ or } 196.2 = -\frac{\partial p}{\partial x} \times \frac{D}{4} = -\frac{\partial p}{\partial x} \times \frac{0.1}{4}$$

$\therefore \quad \dfrac{\partial p}{\partial x} = -\dfrac{196.2 \times 4}{0.1} = -7848 \text{ N/m}^2 \text{ per m}$

$\therefore$ Pressure Gradient = **– 7848 N/m² per m. Ans.**

(*ii*) **Average velocity,** $\bar{u}$

$$\bar{u} = \frac{1}{2}U_{max} = \frac{1}{2}\left[-\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2\right] \qquad \left\{\because\ U_{max} = -\frac{1}{8\mu}\frac{\partial p}{\partial x}R^2\right\}$$

$$= \frac{1}{8\mu} \times \left(-\frac{\partial p}{\partial x}\right)R^2$$

$$= \frac{1}{8 \times 0.7} \times (7848) \times (.05)^2 \qquad \left\{\because\ R = \frac{D}{2} = \frac{1}{2} = .05\right\}$$

$$= 3.50 \text{ m/s}$$

(*iii*) **Reynolds number,** R_e

$$R_e = \frac{\bar{u} \times D}{\nu} = \frac{\bar{u} \times D}{\mu / \rho} = \frac{\rho \times \bar{u} \times D}{\mu}$$

$$= 1300 \times \frac{3.50 \times 0.1}{0.7} = \mathbf{650.00.\ Ans.}$$

Problem 9.6 *What power is required per kilometre of a line to overcome the viscous resistance to the flow of glycerine through a horizontal pipe of diameter 100 mm at the rate of 10 litres/s ? Take* $\mu = 8$ *poise and kinematic viscosity* (ν) = 6.0 *stokes.*

Solution. Given :

Length of pipe, $L = 1 \text{ km} = 1000 \text{ m}$

Dia. of pipe, $D = 100 \text{ mm} = 0.1 \text{ m}$

Discharge, $Q = 10 \text{ litres/s} = \dfrac{10}{1000} \text{ m}^3\text{/s} = .01 \text{ m}^3\text{/s}$

Viscosity, $\mu = 8 \text{ poise} = \dfrac{8}{10}\dfrac{\text{Ns}}{\text{m}^2} = 0.8 \text{ N s/m}^2$

Kinematic Viscosity, $\nu = 6.0$ stokes $\qquad \left(\because\ 1 \text{ poise} = \dfrac{1}{10}\text{Ns/m}^2\right)$

$= 6.0 \text{ cm}^2\text{/s} = 6.0 \times 10^{-4} \text{ m}^2\text{/s}$

Loss of pressure head is given by equation (9.6) as $h_f = \dfrac{32\mu \bar{u} L}{\rho g D^2}$

Power required = $W \times h_f$ watts ...(*i*)

where W = weight of oil flowing per sec = $\rho g \times Q$

Substituting the values of W and h_f in equation (*i*),

Power required $= (\rho g \times Q) \times \dfrac{(32\,\mu \bar{u} L)}{\rho g D^2}$ watts $= \dfrac{Q \times 32\,\mu \bar{u} L}{D^2}$ (cancelling ρg)

But $\bar{u} = \dfrac{Q}{\text{Area}} = \dfrac{.01}{\frac{\pi}{4}D^2} = \dfrac{.01 \times 4}{\pi \times (.1)^2} = 1.273 \text{ m/s}$

$\therefore$ Power required $= \dfrac{.01 \times 32 \times 0.8 \times 1.273 \times 1000}{(.1)^2}$

$= 32588.8$ W = **32.588 kW. Ans.**

▶ 9.3 FLOW OF VISCOUS FLUID BETWEEN TWO PARALLEL PLATES

In this case also, the shear stress distribution, the velocity distribution across a section ; the ratio of maximum velocity to average velocity and difference of pressure head for a given length of parallel plates, are to be calculated.

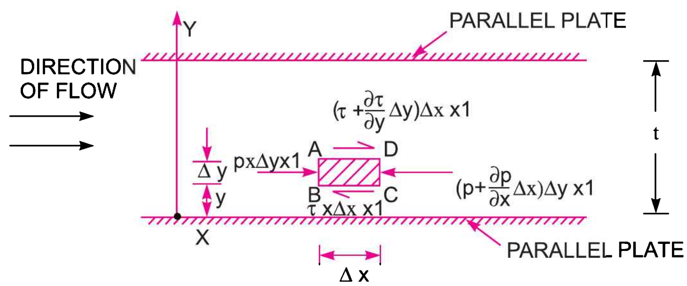

Fig. 9.6 *Viscous flow between two parallel plates.*

Consider two parallel fixed plates kept at a distance 't' apart as shown in Fig. 9.6. A viscous fluid is flowing between these two plates from left to right. Consider a fluid element of length Δx and thickness Δy at a distance y from the lower fixed plate. If p is the intensity of pressure on the face AB of the fluid element then intensity of pressure on the face CD will be $\left(p + \frac{\partial p}{\partial x}\Delta x\right)$. Let τ is the shear stress acting on the face BC then the shear stress on the face AD will be $\left(\tau + \frac{\partial \tau}{\partial y}\Delta y\right)$. If the width of the element in the direction perpendicular to the paper is unity then the forces acting on the fluid element are :

1. The pressure force, $p \times \Delta y \times 1$ on face AB.
2. The pressure force, $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\Delta y \times 1$ on face CD.
3. The shear force, $\tau \times \Delta x \times 1$ on face BC.
4. The shear force, $\left(\tau + \frac{\partial \tau}{\partial y}\Delta y\right)\Delta x \times 1$ on face AD.

For steady and uniform flow, there is no acceleration and hence the resultant force in the direction of flow is zero.

$$\therefore \quad p\Delta y \times 1 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\Delta y \times 1 - \tau\Delta x \times 1 + \left(\tau + \frac{\partial \tau}{\partial y}\Delta y\right)\Delta x \times 1 = 0$$

or
$$-\frac{\partial p}{\partial x}\Delta x\Delta y + \frac{\partial \tau}{\partial x}\Delta y\Delta x = 0$$

Dividing by $\Delta x\Delta y$, we get $-\dfrac{\partial p}{\partial x} + \dfrac{\partial \tau}{\partial y} = 0$ or $\dfrac{\partial p}{\partial x} = \dfrac{\partial \tau}{\partial y}$ ...(9.7) (*i*) **Velocity Distribution.** To obtain the velocity distribution across a section, the value of shear stress $\tau = \mu \dfrac{du}{dy}$ from Newton's law of viscosity for laminar flow is substituted in equation (9.7).

$$\therefore \qquad \frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu \frac{du}{dy}\right) = \mu \frac{\partial^2 u}{dy^2}$$

$$\therefore \qquad \frac{\partial^2 u}{dy^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

Integrating the above equation w.r.t. y, we get

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x} y + C_1 \qquad \left\{\because \frac{\partial p}{\partial x} \text{ is constant}\right\}$$

Integrating again

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + C_1 y + C_2 \qquad \text{...(9.8)}$$

where C_1 and C_2 are constants of integration. Their values are obtained from the two boundary conditions that is (*i*) at $y = 0$, $u = 0$ (*ii*) at $y = t$, $u = 0$.

The substitution of $y = 0$, $u = 0$ in equation (9.8) gives

$$0 = 0 + C_1 \times 0 + C_2 \text{ or } C_2 = 0$$

The substitution of $y = t$, $u = 0$ in equation (9.8) gives

$$0 = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{t^2}{2} + C_1 \times t + 0$$

$$\therefore \qquad C_1 = -\frac{1}{\mu}\frac{\partial p}{\partial x}\frac{t^2}{2 \times t} = -\frac{1}{2\mu}\frac{\partial p}{\partial x} t$$

Substituting the values of C_1 and C_2 in equation (9.8)

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + y\left(-\frac{1}{2\mu}\frac{\partial p}{\partial x} t\right)$$

or

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2] \qquad \text{...(9.9)}$$

In the above equation, μ, $\dfrac{\partial p}{\partial x}$ and t are constant. It means u varies with the square of y. Hence equation (9.9) is a equation of a parabola. Hence velocity distribution across a section of the parallel plate is parabolic. This velocity distribution is shown in Fig. 9.7 (*a*).

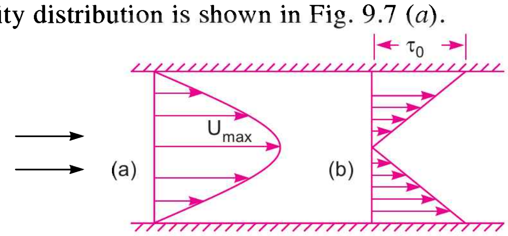

Fig. 9.7 *Velocity distribution and shear stress distribution across a section of parallel plates.*

(*ii*) **Ratio of Maximum Velocity to Average Velocity.** The velocity is maximum, when $y = t/2$. Substituting this value in equation (9.9), we get

$$U_{max} = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[t \times \frac{t}{2} - \left(\frac{t}{2}\right)^2\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^2}{2} - \frac{t^2}{4}\right] = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{t^2}{4} = -\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2 \qquad \text{...(9.10)}$$

The average velocity, $\bar{u}$, is obtained by dividing the discharge (Q) across the section by the area of the section ($t \times 1$). And the discharge Q is obtained by considering the rate of flow of fluid through the strip of thickness dy and integrating it. The rate of flow through strip is

$$dQ = \text{Velocity at a distance } y \times \text{Area of strip}$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2] \times dy \times 1$$

$$\therefore \qquad Q = \int_0^t dQ = \int_0^t -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]\,dy$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{ty^2}{2} - \frac{y^3}{3}\right]_0^t = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^3}{2} - \frac{t^3}{3}\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{t^3}{6} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^3$$

$$\therefore \qquad \bar{u} = \frac{Q}{\text{Area}} = -\frac{\frac{1}{12\mu}\frac{\partial p}{\partial x}.t^3}{t \times 1} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2 \qquad \text{...(9.11)}$$

Dividing equation (9.10) by equation (9.11), we get

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2}{-\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2} = \frac{12}{8} = \frac{3}{2} \qquad \text{...(9.12)}$$

(*iii*) **Drop of Pressure head for a given Length.** From equation (9.11), we have

$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2 \quad \text{or} \quad \frac{\partial p}{\partial x} = -\frac{12\mu\bar{u}}{t^2}$$

Integrating this equation w.r.t. x, we get

$$\int_2^1 dp = \int_2^1 -\frac{12\mu\bar{u}}{t^2}\,dx$$

or

$$p_1 - p_2 = -\frac{12\mu\bar{u}}{t^2}[x_1 - x_2] = \frac{12\mu\bar{u}}{t^2}[x_2 - x_1]$$

or $$p_1 - p_2 = \frac{12\mu\bar{u}L}{t^2} \qquad [\because x_1 - x_2 = L]$$

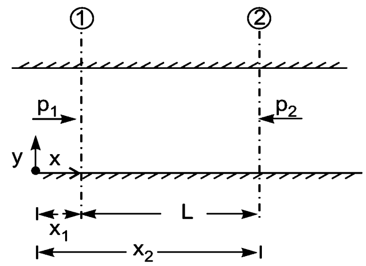

Fig. 9.8

If h_f is the drop of pressure head, then

$$h_f = \frac{p_1 - p_2}{\rho g} = \frac{12\mu\bar{u}L}{\rho g t^2} \qquad ...(9.13)$$

(*iv*) **Shear Stress Distribution.** It is obtained by substituting the value of u from equation (9.9) into

$$\tau = \mu \frac{\partial u}{\partial y}$$

$$\therefore \qquad \tau = \mu \frac{\partial u}{\partial y} = \mu \frac{\partial}{\partial y}\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(ty - y^2\right)\right] = \mu\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(t - 2y)\right]$$

$$\tau = -\frac{1}{2}\frac{\partial p}{\partial x}[t - 2y] \qquad ...(9.14)$$

In equation (9.14), $\frac{\partial p}{\partial x}$ and t are constant. Hence τ varies linearly with y. The shear stress distribution is shown in Fig. 9.7 (*b*). Shear stress is maximum, when $y = 0$ or t at the walls of the plates. Shear stress is zero, when $y = t/2$ that is at the centre line between the two plates. Max. shear stress (τ_0) is given by

$$\tau_0 = -\frac{1}{2}\frac{\partial p}{\partial x} t. \qquad ...(9.15)$$

Problem 9.7 *Calculate : (i) the pressure gradient along flow, (ii) the average velocity, and (iii) the discharge for an oil of viscosity 0.02 Ns/m² flowing between two stationary parallel plates 1 m wide maintained 10 mm apart. The velocity midway between the plates is 2 m/s.*

Solution. Given :

Viscosity, $\mu = .02$ N s/m^2

Width, $b = 1$ m

Distance between plates, $t = 10$ mm $= .01$ m

Velocity midway between the plates, $U_{max} = 2$ m/s.

(*i*) **Pressure gradient** $\left(\frac{dp}{dx}\right)$

Using equation (9.10), $U_{max} = -\frac{1}{8\mu}\frac{dp}{dx}t^2$ or $2.0 = -\frac{1}{8 \times .02}\left(\frac{dp}{dx}\right)(.01)^2$

$$\therefore \qquad \frac{dp}{dx} = -\frac{2.0 \times 8 \times .02}{.01 \times .01} = \mathbf{-3200\ N/m^2\ per\ m.\ Ans.}$$

(*ii*) **Average velocity** ($\bar{u}$)

Using equation (9.12), $\frac{U_{max}}{\bar{u}} = \frac{3}{2} \quad \therefore \quad \bar{u} = \frac{2U_{max}}{3} = \frac{2 \times 2}{3} =$ **1.33 m/s. Ans.**

(*iii*) **Discharge** (Q) $=$ Area of flow $\times \bar{u} = b \times t \times \bar{u} = 1 \times .01 \times 1.33 =$ **.0133 m³/s. Ans.**

Problem 9.8 *Determine (i) the pressure gradient, (ii) the shear stress at the two horizontal parallel plates and (iii) the discharge per metre width for the laminar flow of oil with a maximum velocity of 2 m/s between two horizontal parallel fixed plates which are 100 mm apart. Given* $\mu = 2.4525$ *N s/m²*.

Solution. Given :

$$U_{max} = 2 \text{ m/s}, t = 100 \text{ mm} = 0.1 \text{ m}, \mu = 2.4525 \text{ N/m}^2$$

Find (*i*) Pressure gradient, $\frac{dp}{dx}$

(*ii*) Shear stress at the wall, τ_0

(*iii*) Discharge per metre width, Q.

(*i*) **Pressure gradient,** $\frac{dp}{dx}$

Maximum velocity, U_{max}, is given by equation (9.10)

$$U_{max} = -\frac{1}{8\mu}\frac{dp}{dx}t^2$$

Substituting the values

or $$2.0 = -\frac{1}{8 \times 2.4525} \times \frac{dp}{dx} \times (.1)^2$$

$$\therefore \quad \frac{dp}{dx} = -\frac{2.0 \times 8 \times 2.4525}{.1 \times .1} = -\ \mathbf{3924\ N/m^2\ per\ m.\ Ans.}$$

(*ii*) **Shear stress at the wall,** τ_0

τ_0 is given by equation (9.15) as $\tau_0 = -\frac{1}{2}\frac{dp}{dx} \times t = -\frac{1}{2}(-3924) \times 0.1 = \mathbf{196.2\ N/m^2.\ Ans.}$

(*iii*) **Discharge per metre width,** Q

$$= \text{Mean velocity} \times \text{Area}$$

$$= \frac{2}{3}U_{max} \times (t \times 1) = \frac{2}{3} \times 2.0 \times 0.1 \times 1 = \mathbf{0.133\ m^3/s.\ Ans.}$$

Problem 9.9 *An oil of viscosity 10 poise flows between two parallel fixed plates which are kept at a distance of 50 mm apart. Find the rate of flow of oil between the plates if the drop of pressure in a length of 1.2 m be 0.3 N/cm³. The width of the plates is 200 mm.*

Solution. Given :

$$\mu = 10 \text{ poise}$$

$$= \frac{10}{10} \text{ N s/m}^2 = 1 \text{ N s/m}^2 \qquad \left(\because\ 1 \text{ poise} = \frac{1}{10}\frac{\text{Ns}}{\text{m}^2}\right)$$

$$t = 50 \text{ mm} = 0.05 \text{ m}$$

$$p_1 - p_2 = 0.3 \text{ N/m}^2 = 0.3 \times 10^4 \text{ N/m}^2$$

$$L = 1.20 \text{ m}$$

Width, $B = 200$ mm $= 0.20$ m.

Find Q, rate of flow

The difference of pressure is given by equation (9.13)

$$p_1 - p_2 = \frac{12\mu \bar{u} L}{t^2}$$

Substituting the values, we get

$$0.3 \times 10^4 = 12 \times 1.0 \times \frac{\bar{u} \times 1.20}{.05 \times .05}$$

$$\therefore \quad \bar{u} = \frac{0.3 \times 10^4 \times 1.0 \times .05 \times .05}{12 \times 1.20} = 0.52 \text{ m/s}$$

$\therefore$ Rate of flow $= \bar{u} \times \text{Area} = 0.52 \times (B \times t)$

$= 0.52 \times 0.20 \times .05 \text{ m}^3\text{/s} = .0052 \text{ m}^3\text{/s}$

$= 0.0052 \times 10^3$ litre/s = **5.2 litre/s. Ans.**

Problem 9.10 *Water at 15°C flows between two large parallel plates at a distance of 1.6 mm apart. Determine (i) the maximum velocity (ii) the pressure drop per unit length and (iii) the shear stress at the walls of the plates if the average velocity is 0.2 m/s. The viscosity of water at 15°C is given as 0.01 poise.*

Solution. Given : $t = 1.6 \text{ mm} = 1.6 \times 10^{-3} \text{ m}$

$= 0.0016$ m

$$\bar{u} = 0.2 \text{ m/sec}, \ \mu = .01 \text{ poise} = \frac{.01}{10} = 0.001 \text{ N s/m}^2$$

(*i*) **Maximum velocity**, U_{max} is given by equation (9.12)

i.e., $U_{max} = \frac{3}{2}\bar{u} = 1.5 \times 0.2 =$ **0.3 m/s. Ans.**

(*ii*) **The pressure drop**, $(p_1 - p_2)$ is given by equation (9.13)

$$p_1 - p_2 = \frac{12\mu \bar{u} L}{t^2}$$

or pressure drop per unit length $= \frac{12\mu\bar{u}}{t^2}$

or $$\frac{\partial p}{\partial x} = 12 \times \frac{.01}{10} \times \frac{0.2}{(.0016)^2} = 937.44 \text{ N/m}^2 \text{ per m.}$$

(*iii*) **Shear stress at the walls** is given by equation (9.15)

$$\tau_0 = -\frac{1}{2}\frac{\partial p}{\partial x} \times t = \frac{1}{2} \times 937.44 \times .0016 = \mathbf{0.749 \ N/m^2.\ Ans.}$$

Problem 9.11 *There is a horizontal crack 40 mm wide and 2.5 mm deep in a wall of thickness 100 mm. Water leaks through the crack. Find the rate of leakage of water through the crack if the difference of pressure between the two ends of the crack is 0.02943 N/cm². Take the viscosity of water equal to 0.01 poise.*

Solution. Given :

Width of crack, $b = 40 \text{ mm} = 0.04$ m

Depth of crack, $t = 2.5 \text{ mm} = .0025$ m

Length of crack, $L = 100 \text{ mm} = 0.1$ m

$$p_1 - p_2 = 0.02943 \text{ N/cm}^2 = 0.02943 \times 10^4 \text{ N/m}^2 = 294.3 \text{ N/m}^2$$

$$\mu = .01 \text{ poise} = \frac{.01}{10}\frac{\text{Ns}}{\text{m}^2}$$

Find rate of leakage (Q)

($p_1 - p_2$) is given by equation (9.13) as

$$p_1 - p_2 = \frac{12\mu \bar{u} L}{t^2} \quad \text{or} \quad 294.3 = 12 \times \frac{.01}{10} \times \frac{\bar{u} \times 0.1}{(.0025 \times .0025)}$$

$$\therefore \qquad \bar{u} = \frac{294.3 \times 10 \times .0025 \times .0025}{12 \times .01 \times 0.1} = 1.5328 \text{ m/s}$$

$\therefore$ Rate of leakage = $\bar{u}$ × area of cross-section of crack

= 1.538 × (b × t)

= $1.538 \times .04 \times .0025 \text{ m}^3/\text{s} = 1.538 \times 10^{-4} \text{ m}^3/\text{s}$

= $1.538 \times 10^{-4} \times 10^3$ litre/s = **0.1538 litre/s. Ans.**

Problem 9.12 *The radial clearance between a hydraulic plunger and the cylinder walls is 0.1 mm; the length of the plunger is 300 mm and diameter 100 mm. Find the velocity of leakage and rate of leakage past the plunger at an instant when the difference of the pressure between the two ends of the plunger is 9 m of water. Take μ = 0.0127 poise.*

Solution. Given :

The flow through the clearance area will be the same as the flow between two parallel surfaces,

t = 0.1 mm = 0.0001 m

L = 300 mm = 0.3 m

Diameter, D = 100 mm = 0.1 m

Difference of pressure $= \dfrac{p_1 - p_2}{\rho g}$ = 9 m of water

$$\therefore \qquad p_1 - p_2 = 9 \times 1000 \times 9.81 \text{ N/m}^2 = 88290 \text{ N/m}^2$$

Viscosity, $\mu = .0127 \text{ poise} = \dfrac{.0127}{10}\dfrac{\text{Ns}}{\text{m}^2}$

Find (*i*) Velocity of leakage, *i.e.*, mean velocity $\bar{u}$

(*ii*) Rate of leakage, Q

(*i*) **Velocity of leakage** ($\bar{u}$). The average velocity ($\bar{u}$) is given by equation (9.11)

$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x} t^2$$

$$= \frac{1}{12 \times \frac{.0127}{10}} \times \frac{p_1 - p_2}{L} \times (.0001) \times (.0001)$$

$$= \frac{1}{12 \times .0127} \times \frac{88290}{0.3} \times (.0001) \times (.0001)$$

= .193 m/s = **19.3 cm/s. Ans.**

(*ii*) **Rate of leakage,** Q

$$Q = \bar{u} \times \text{area of flow}$$
$$= 0.193 \times \pi D \times t \text{ m}^3/\text{s} = 0.193 \times \pi \times .1 \times .0001 \text{ m}^3/\text{s}$$
$$= 6.06 \times 10^{-6} \text{ m}^3/\text{s} = 6.06 \times 10^{-6} \times 10^3 \text{ litre/s}$$
$$= \mathbf{6.06 \times 10^{-3} \text{ litre/s. Ans.}}$$

▶ 9.4 KINETIC ENERGY CORRECTION AND MOMENTUM CORRECTION FACTORS

Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy of the flow per second based on average velocity across the same section. It is denoted by α. Hence mathematically,

$$\alpha = \frac{\text{K.E./sec based on actual velocity}}{\text{K.E./sec based on average velocity}} \qquad \text{...(9.16)}$$

Momentum Correction Factor. It is defined as the ratio of momentum of the flow per second based on actual velocity to the momentum of the flow per second based on average velocity across a section. It is denoted by β. Hence mathematically,

$$\beta = \frac{\text{Momentum per second based on actual velocity}}{\text{Momentum per second based on average velocity}}. \qquad \text{...(9.17)}$$

Problem 9.13 *Show that the momentum correction factor and energy correction factor for laminar flow through a circular pipe are 4/3 and 2.0 respectively.*

Solution. (*i*) **Momentum Correction Factor or** β

The velocity distribution through a circular pipe for laminar flow at any radius r is given by equation (9.3)

or
$$u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2) \qquad \text{...(1)}$$

Consider an elementary area dA in the form of a ring at a radius r and of width dr, then

$$dA = 2\pi r\, dr$$

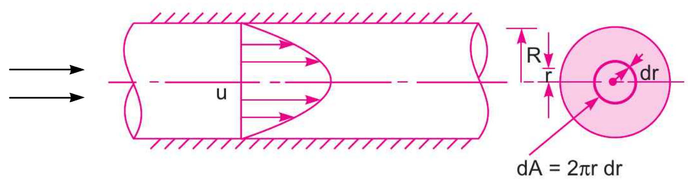

Fig. 9.9

Rate of fluid flowing through the ring

$$= dQ = \text{velocity} \times \text{area of ring element}$$
$$= u \times 2\pi r\, dr$$

Momentum of the fluid through ring per second

$$= \text{mass} \times \text{velocity}$$
$$= \rho \times dQ \times u = \rho \times 2\pi r\, dr \times u \times u = 2\pi\rho\, u^2 r\, dr$$

$\therefore$ Total actual momentum of the fluid per second across the section

$$= \int_0^R 2\pi\rho\, u^2 r\, dr$$

Substituting the value of u from (1)

$$= 2\pi\rho \int_0^R \left[\frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)\left(R^2 - r^2\right)\right]^2 rdr$$

$$= 2\pi\rho\left[\frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right)\right]^2 \int_0^R [R^2 - r^2]^2\, rdr$$

$$= 2\pi\rho \frac{1}{\left(16\mu^2\right)}\left(\frac{\partial p}{\partial x}\right)^2 \int_0^R (R^4 + r^4 - 2R^2r^2)\, rdr$$

$$= \frac{\pi\rho}{8\mu^2}\left(\frac{\partial p}{\partial x}\right)^2 \int_0^R (R^4r + r^5 - 2R^2r^3)\, dr$$

$$= \frac{\pi\rho}{8\mu^2}\left(\frac{\partial p}{\partial x}\right)^2\left[\frac{R^4r^2}{2} + \frac{r^6}{6} - \frac{2R^2r^4}{4}\right]_0^R = \frac{\pi\rho}{8\mu^2}\left(\frac{\partial p}{\partial x}\right)^2\left[\frac{R^6}{2} + \frac{R^6}{6} - \frac{2R^6}{4}\right]$$

$$= \frac{\pi\rho}{8\mu^2}\left(\frac{\partial p}{\partial x}\right)^2 \frac{6R^6 + 2R^6 - 6R^6}{12}$$

$$= \frac{\pi\rho}{8\mu^2}\left(\frac{\partial p}{\partial x}\right)^2 \times \frac{R^6}{6} = \frac{\pi\rho}{48\mu^2}\left(\frac{\partial p}{\partial x}\right)^2 R^6 \qquad \text{...(2)}$$

Momentum of the fluid per second based on average velocity

$$= \frac{\text{mass of fluid}}{\text{sec}} \times \text{average velocity}$$

$$= \rho A\bar{u} \times \bar{u} = \rho A\bar{u}^2$$

where A= Area of cross-section = πR^2, $\bar{u}$ = average velocity = $\frac{U_{max}}{2}$

$$= \frac{1}{2} \times \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) R^2 \qquad \left\{\because \quad U_{max} = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)R^2\right\}$$

$$= \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$

∴ Momentum/sec based on average velocity

$$= \rho \times \pi R^2 \times \left[\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2\right]^2 = \rho \times \pi R^2 \times \frac{1}{64\mu^2}\left(-\frac{\partial p}{\partial x}\right)^2 R^4$$

$$= \frac{\rho\pi\left(-\frac{\partial p}{\partial x}\right)^2 R^6}{64\mu^2} \qquad \text{...(3)}$$

∴
$$\beta = \frac{\text{Momentum / sec based on actual velocity}}{\text{Momentum / sec based on average velocity}}$$

$$= \frac{\frac{\pi\rho}{48\mu^2}\left(\frac{\partial p}{\partial x}\right)^2 R^6}{\frac{\pi\rho}{64\mu^2}\left(-\frac{\partial p}{\partial x}\right)^2 R^6} = \frac{64}{48} = \mathbf{\frac{4}{3}}. \textbf{ Ans.}$$

(*ii*) **Energy Correction Factor**, α. Kinetic energy of the fluid flowing through the elementary ring of radius 'r' and of width 'dr' per sec

$$= \frac{1}{2} \times \text{mass} \times u^2 = \frac{1}{2} \times \rho dQ \times u^2$$

$$= \frac{1}{2} \times \rho \times (u \times 2\pi r\, dr) \times u^2 = \frac{1}{2}\rho \times 2\pi r u^3\, dr = \pi\rho r u^3\, dr$$

$\therefore$ Total actual kinetic energy of flow per second

$$= \int_0^R \pi\rho r u^3\, dr = \int_0^R \pi\rho r \left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\left(R^2 - r^2\right)\right]^3 dr$$

$$= \pi\rho \times \left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\right]^3 \int_0^R [R^2 - r^2]^3\, r dr$$

$$= \pi\rho \times \frac{1}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \int_0^R (R^6 - r^6 - 3R^4r^2 + 3R^2r^4) r\, dr$$

$$= \frac{\pi\rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \int_0^R (R^6 r - r^7 - 3R^4r^3 + 3R^2r^5)\, dr$$

$$= \frac{\pi\rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \left[\frac{R^6r^2}{2} - \frac{r^8}{8} - \frac{3R^4r^4}{4} + \frac{3R^2r^6}{6}\right]_0^R$$

$$= \frac{\pi\rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \left[\frac{R^8}{2} - \frac{R^8}{8} - \frac{3R^8}{4} + \frac{3R^8}{6}\right]$$

$$= \frac{\pi\rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 R^8 \left[\frac{12 - 3 - 18 + 12}{24}\right]$$

$$= \frac{\pi\rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \frac{R^8}{8} \qquad \text{...(4)}$$

Kinetic energy of the flow based on average velocity

$$= \frac{1}{2} \times \text{mass} \times \bar{u}^2 = \frac{1}{2} \times \rho A\bar{u} \times \bar{u}^2 = \frac{1}{2} \times \rho A\bar{u}^3$$

Substituting the value of $A = \pi R^2$

and $\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2$

$\therefore$ Kinetic energy of the flow/sec

$$= \frac{1}{2} \times \rho \times \pi R^2 \times \left[\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2\right]^3$$

$$= \frac{1}{2} \times \rho \times \pi R^2 \times \frac{1}{64 \times 8\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \times R^6$$

$$= \frac{\rho\pi}{128 \times 8\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \times R^8 \qquad \text{...(5)}$$

$$\therefore \qquad \alpha = \frac{\text{K.E./sec based on actual velocity}}{\text{K.E./sec based on average velocity}} = \frac{\text{Equation (4)}}{\text{Equation (5)}}$$

$$= \frac{\dfrac{\pi\rho}{64\mu^3}\left(-\dfrac{\partial p}{\partial x}\right)^3 \times \dfrac{R^8}{8}}{\dfrac{\rho}{128 \times 8\mu^3}\left(-\dfrac{\partial p}{\partial x}\right)^3 \times R^8} = \frac{128 \times 8}{64 \times 8} = \textbf{2.0. Ans.}$$

▶ 9.5 POWER ABSORBED IN VISCOUS FLOW

For the lubrication of the machine parts, an oil is used. Flow of oil in bearings is an example of viscous flow. If a highly viscous oil is used for lubrication of bearings, it will offer great resistance and thus a greater power loss will take place. But if a light oil is used, a required film between the rotating part and stationary metal surface will not be possible. Hence, the wear of the two surface will take place. Hence an oil of correct viscosity should be used for lubrication. The power required to overcome the viscous resistance in the following cases will be determined :

1. Viscous resistance of Journal Bearings,
2. Viscous resistance of Foot-step Bearings,
3. Viscous resistance of Collar Bearings.

9.5.1 Viscous Resistance of Journal Bearings. Consider a shaft of diameter D rotating in a journal bearing. The clearance between the shaft and journal bearing is filled with a viscous oil. The oil film in contact with the shaft rotates at the same speed as that of shaft while the oil film in contact with journal bearing is stationary. Thus the viscous resistance will be offered by the oil to the rotating shaft.

Let N = speed of shaft in r.p.m.
t = thickness of oil film
L = length of oil film

$\therefore$ Angular speed of the shaft, $\omega = \dfrac{2\pi N}{60}$

$\therefore$ Tangential speed of the shaft $= \omega \times R$ or $V = \dfrac{2\pi N}{60} \times \dfrac{D}{2} = \dfrac{\pi DN}{60}$

The shear stress in the oil is given by, $\tau = \mu \dfrac{du}{dy}$

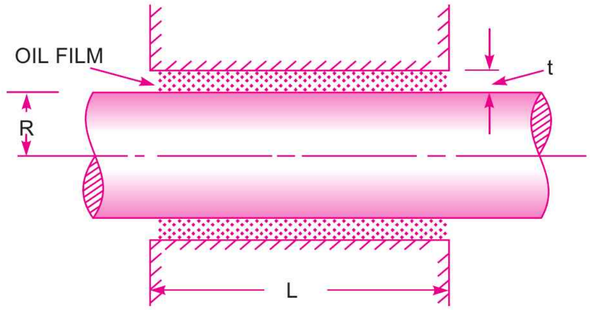

Fig. 9.10 *Journal bearing.*

As the thickness of oil film is very small, the velocity distribution in the oil film can be assumed as linear.

Hence $$\frac{du}{dy} = \frac{V - 0}{t} = \frac{V}{t} = \frac{\pi DN}{60 \times t}$$

∴ $$\tau = \mu \frac{\pi DN}{60 \times t}$$

∴ Shear force or viscous resistance = τ × Area of surface of shaft

$$= \frac{\mu \pi DN}{60t} \times \pi DL = \frac{\mu \pi^2 D^2 NL}{60t}$$

∴ Torque required to overcome the viscous resistance,

$$T = \text{Viscous resistance} \times \frac{D}{2}$$

$$= \frac{\mu \pi^2 D^2 NL}{60t} \times \frac{D}{2} = \frac{\mu \pi^2 D^3 NL}{120t}$$

∴ Power absorbed in overcoming the viscous resistance

$$^*P = \frac{2\pi NT}{60} = \frac{2\pi N}{60} \times \frac{\mu \pi^2 D^3 NL}{120t}$$

$$= \frac{\mu \pi^3 \mathbf{D^3 N^2 L}}{\mathbf{60 \times 60 \times t}} \textbf{ watts. Ans.} \quad ...(9.18)$$

Problem 9.14 *A shaft having a diameter of 50 mm rotates centrally in a journal bearing having a diameter of 50.15 mm and length 100 mm. The angular space between the shaft and the bearing is filled with oil having viscosity of 0.9 poise. Determine the power absorbed in the bearing when the speed of rotation is 60 r.p.m.*

Solution. Given :

Dia. of shaft, D = 50 mm or .05 m

Dia. of bearing, D_1 = 50.15 mm or 0.05015 m

Length, L = 100 mm or 0.1 m

---

*Power, $P = T \times \omega = T \times \frac{2\pi N}{60} = \frac{2\pi NT}{60}$ watts $= \frac{2\pi NT}{60{,}000}$ kW.

μ of oil = 0.9 poise = $\frac{0.9}{10}\frac{\text{Ns}}{\text{m}^2}$

N = 600 r.p.m.

Power = ?

$\therefore$ Thickness of oil film, $t = \frac{D_1 - D}{2} = \frac{50.15 - 50}{2}$

$= \frac{0.15}{2} = 0.075 \text{ mm} = 0.075 \times 10^{-3} \text{ m}$

Tangential speed of shaft, $V = \frac{\pi DN}{60} = \frac{\pi \times 0.05 \times 600}{60} = 0.5 \times \pi \text{ m/s}$

Shear stress $\tau = \mu \frac{du}{dy} = \mu \frac{V}{t} = \frac{0.9}{10} \times \frac{0.5 \times \pi}{0.075 \times 10^{-3}} = 1883.52 \text{ N/m}^2$

$\therefore$ Shear force (F) $= \tau \times \text{Area} = 1883.52 \times \pi D \times L$

$= 1883.52 \times \pi \times .05 \times 0.1 = 29.586 \text{ N}$

Resistance torque $T = F \times \frac{D}{2} = 29.586 \times \frac{.05}{2} = 0.7387 \text{ Nm}$

Power $= \frac{2\pi NT}{60} = \frac{2\pi \times 600 \times 0.7387}{60} =$ **46.41 W. Ans.**

Problem 9.15 *A shaft of 100 mm, diameter rotates at 60 r.p.m. in a 200 mm long bearing. Taking that the two surfaces are uniformly separated by a distance of 0.5 mm and taking linear velocity distribution in the lubricating oil having dynamic viscosity of 4 centipoises, find the power absorbed in the bearing.*

Solution. Given :

Dia. of shaft, D = 100 mm = 0.1 m

Length of bearing, L = 200 mm = 0.2 m

$t = 0.5 \text{ mm} = .5 \times 10^{-3} \text{ m}$

μ = 4 centipoise = .04 poise = $\frac{0.04}{10}\frac{\text{Ns}}{\text{m}^2}$

N = 60 r.p.m.

Find power absorbed

Using equation (9.18), $P = \frac{\mu \pi^3 D^3 N^2 L}{60 \times 60 \times t}$

$= \frac{.04}{10} \times \frac{\pi^3 \times (.1)^3 \times (60)^2 \times 0.2}{60 \times 60 \times 0.5 \times 10^{-3}} =$ **4.961×10^{-2} W. Ans.**

Problem 9.16 *A shaft of diameter 0.35 m rotates at 200 r.p.m. inside a sleeve 100 mm long. The dynamic viscosity of lubricating oil in the 2 mm gap between sleeve and shaft is 8 poises. Calculate the power lost in the bearing.*

Solution. Given :

Dia. of shaft, D = 0.35 m

Speed of shaft, $N = 200$ r.p.m.
Length of sleeve, $L = 100$ mm $= 0.1$ m
Distance between sleeve and shaft, $t = 2$ mm $= 2 \times 10^{-3}$ m

Viscosity, $\mu = 8 \text{ poise} = \frac{8}{10} \frac{\text{Ns}}{\text{m}^2}$

The power lost in the bearing is given by equation (9.18) as

$$P = \frac{\mu\pi^3 D^3 N^2 L}{60 \times 60 \times t} \text{ watts}$$

$$= \frac{8}{10} \times \frac{\pi^3 \times (.35)^3 \times (200)^2 \times 0.1}{60 \times 60 \times 2 \times 10^{-3}} = \mathbf{590.8\ W = 0.59\ kW.\ Ans.}$$

Problem 9.17 *A sleeve, in which a shaft of diameter 75 mm, is running at 1200 r.p.m., is having a radial clearance of 0.1 mm. Calculate the torque resistance if the length of sleeve is 100 mm and the space is filled with oil of dynamic viscosity 0.96 poise.*

Solution. Given :

Dia. of shaft, $D = 75$ mm $= 0.075$ m
$N = 1200$ r.p.m.
$t = 0.1$ mm $= 0.1 \times 10^{-3}$ m
Length of sleeve, $L = 100$ mm $= 0.1$ m

$$\mu = 0.96 \text{ poise} = \frac{0.96}{10} \frac{\text{Ns}}{\text{m}^2}$$

Tangential velocity of shaft, $V = \frac{\pi DN}{60} = \frac{\pi \times .075 \times 1200}{60} = 4.712$ m/s

Shear stress, $\tau = \mu \frac{V}{t} = \frac{.96}{10} \times \frac{4.712}{.1 \times 10^{-3}} = 4523.5 \text{ N/m}^2$

Shear force, $F = \tau \times \pi DL$

$$= 4523.5 \times \pi \times .075 \times .1 = 106.575 \text{ N}$$

$\therefore$ Torque resistance $= F \times \frac{D}{2}$

$$= 106.575 \times \frac{.075}{2} = \mathbf{3.996\ Nm.\ Ans.}$$

Problem 9.18 *A shaft of 100 mm diameter runs in a bearing of length 200 mm with a radial clearance of 0.025 mm at 30 r.p.m. Find the velocity of the oil, if the power required to overcome the viscous resistance is 183.94 watts.*

Solution. Given :

$D = 100$ mm $= 0.1$ m
$L = 200$ mm $= 0.2$ m
$t = .025$ mm $= 0.025 \times 10^{-3}$ m
$N = 30$ r.p.m. ; H.P. $= 0.25$

Find viscosity of oil, μ.

The h.p. is given by equation (9.18) as

$$P = \frac{\mu\pi^3 D^3 N^2 L}{60 \times 60 \times t} \quad \text{or} \quad 183.94 = \frac{\mu\pi^3 \times (.1)^3 \times (30)^2 \times 0.2}{60 \times 60 \times 0.025 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{183.94 \times 60 \times 60 \times .025 \times 10^{-3}}{\pi^3 \times .001 \times 900 \times 0.2} \frac{\text{Ns}}{\text{m}^2}$$

$$= 2.96 \frac{\text{Ns}}{\text{m}^2} = 2.96 \times 10 = \textbf{29.6 poise. Ans.}$$

9.5.2 Viscous Resistance of Foot-Step Bearing. Fig. 9.11 shows the foot-step bearing, in which a vertical shaft is rotating. An oil film between the bottom surface of the shaft and bearing is provided, to reduce the wear and tear. The viscous resistance is offered by the oil to the shaft. In this case the radius of the surface of the shaft in contact with oil is not constant as in the case of the journal bearing. Hence, viscous resistance in foot-step bearing is calculated by considering an elementary circular ring of radius r and thickness dr as shown in Fig. 9.11.

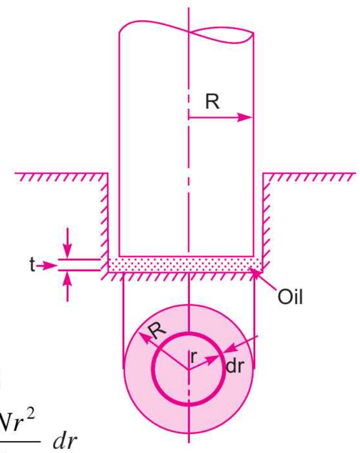

Fig. 9.11 *Foot-step bearing.*

Let N = speed of the shaft

t = thickness of oil film

R = radius of the shaft

Area of the elementary ring $= 2\pi r dr$

Now shear stress is given by $t = \mu \dfrac{du}{dy} = \mu \dfrac{V}{t}$

where V is the tangential velocity of shaft at radius r and is equal to

$$\omega \times r = \frac{2\pi N}{60} \times r$$

$\therefore$ Shear force on the ring $= dF = \tau \times$ area of elementary ring

$$= \mu \times \frac{2\pi N}{60} \times \frac{r}{t} \times 2\pi r\, dr = \frac{\mu}{15} \frac{\pi^2 N r^2}{t} dr$$

$\therefore$ Torque required to overcome the viscous resistance,

$$dT = dF \times r$$

$$= \frac{\mu}{15t} \pi^2 N r^2 dr \times r = \frac{\mu}{15t} \pi^2 N r^3 dr \qquad \text{...(9.19)}$$

$\therefore$ Total torque required to overcome the viscous resistance,

$$T = \int_0^R dT = \int_0^R \frac{\mu}{15t} \pi^2 N r^3 dr$$

$$= \frac{\mu}{15t} \pi^2 N \int_0^R r^3 dr = \frac{\mu}{15t} \pi^2 N \left[\frac{r^4}{4}\right]_0^R = \frac{\mu}{15t} \pi^2 N \frac{R^4}{4}$$

$$= \frac{\mu}{60t} \pi^2 N R^4 \qquad \text{...(9.19A)}$$

$\therefore$ Power absorbed, $\quad P = \dfrac{2\pi N T}{60}$ watts

$$= \frac{2\pi N}{60} \times \frac{\mu}{60t} \pi^2 N R^4 = \frac{\mu\pi^3 N^2 R^4}{60 \times 30t} \qquad \text{...(9.20)}$$

Problem 9.19 *Find the torque required to rotate a vertical shaft of diameter 100 mm at 750 r.p.m. The lower end of the shaft rests in a foot-step bearing. The end of the shaft and surface of the bearing are both flat and are separated by an oil film of thickness 0.5 mm. The viscosity of the oil is given 1.5 poise.*

Solution. Given :

Dia. of shaft, $D = 100 \text{ mm} = 0.1 \text{ m}$

$\therefore$ $R = \frac{D}{2} = \frac{0.1}{2} = 0.05 \text{ m}$

$N = 750 \text{ r.p.m.}$

Thickness of oil film, $t = 0.5 \text{ mm} = 0.0005 \text{ m}$

$\mu = 1.5 \text{ poise} = \frac{1.5}{10} \frac{\text{Ns}}{\text{m}^2}$

The torque required is given by equation (9.19) as

$$T = \frac{\mu}{60t} \pi^2 N R^4 \text{ Nm}$$

$$= \frac{1.5}{10} \times \frac{\pi^2 \times 750 \times (.05)^4}{60 \times .0005} = \mathbf{0.2305 \text{ Nm. Ans.}}$$

Problem 9.20 *Find the power required to rotate a circular disc of diameter 200 mm at 1000 r.p.m. The circular disc has a clearance of 0.4 mm from the bottom flat plate and the clearance contains oil of viscosity 1.05 poise.*

Solution. Given :

Dia. of disc, $D = 200 \text{ mm} = 0.2 \text{ m}$

$\therefore$ $R = \frac{D}{2} = \frac{0.2}{2} = 0.1 \text{ m}$

$N = 1000 \text{ r.p.m.}$

Thickness of oil film, $t = 0.4 \text{ mm} = 0.0004 \text{ m}$

$\mu = 1.05 \text{ poise} = \frac{1.05}{10} \text{ N s/m}^2$

The power required to rotate the disc is given by equation (9.20) as

$$P = \frac{\mu \pi^3 N^2 R^4}{60 \times 30 \times t} \text{ watts}$$

$$= \frac{1.05}{10} \times \frac{\pi^3 \times 1000^2 \times (0.1)^4}{60 \times 30 \times .0004} = \mathbf{452.1 \text{ W. Ans.}}$$

9.5.3 Viscous Resistance of Collar Bearing. Fig. 9.12 shows the collar bearing, where the face of the collar is separated from bearing surface by an oil film of uniform thickness.

Let N = Speed of the shaft in r.p.m.

R_1 = Internal radius of the collar

R_2 = External radius of the collar
t = Thickness of oil film.

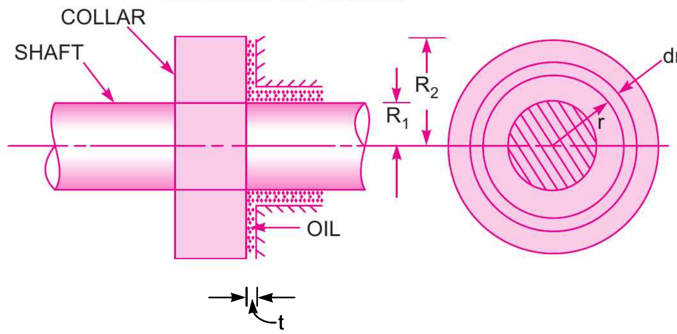

Fig. 9.12 *Collar bearing.*

Consider an elementary circular ring of radius 'r' and width dr of the bearing surface. Then the torque (dT) required to overcome the viscous resistance on the elementary circular ring is the same as given by equation (9.19A) or

$$dT = \frac{\mu}{15t} \pi^2 N r^3 \, dr$$

∴ Total torque, required to overcome the viscous resistance, on the whole collar is

$$T = \int_{R_1}^{R_2} dT = \int_{R_1}^{R_2} \frac{\mu}{15t} \pi^2 N r^3 \, dr = \frac{\mu}{15t} \pi^2 N \left[\frac{r^4}{4}\right]_{R_1}^{R_2}$$

$$= \frac{\mu}{15t \times 4} \pi^2 N [R_2^4 - R_1^4] = \frac{\mu}{60t} \pi^2 N [R_2^4 - R_1^4] \quad ...(9.21)$$

∴ Power absorbed in overcoming viscous resistance

$$P = \frac{2\pi NT}{60} = \frac{2\pi N}{60} \times \frac{\mu}{60t} \pi^2 N [R_2^4 - R_1^4]$$

$$= \frac{\mu \pi^3 N^2}{60 \times 30t} [R_2^4 - R_1^4] \text{ watts.} \quad ...(9.22)$$

Problem 9.21 *A collar bearing having external and internal diameters 150 mm and 100 mm respectively is used to take the thrust of a shaft. An oil film of thickness 0.25 mm is maintained between the collar surface and the bearing. Find the power lost in overcoming the viscous resistance when the shaft rotates at 300 r.p.m. Take μ = 0.91 poise.*

Solution. Given :

External Dia. of collar, $D_2 = 150 \text{ mm} = 0.15 \text{ m}$

∴ $R_2 = \frac{D_2}{2} = \frac{.15}{2} = 0.075 \text{ m}$

Internal Dia. of collar, $D_1 = 100 \text{ mm} = 0.1 \text{ m}$

∴ $R_1 = \frac{D_1}{2} = \frac{0.1}{2} = 0.05 \text{ m}$

Thickness of oil film, $t = 0.25 \text{ mm} = 0.00025 \text{ m}$

$N = 300$ r.p.m.

$$\mu = 0.91 \text{ poise} = \frac{0.91}{10} \frac{\text{Ns}}{\text{m}^2}$$

The power required is given by equation (9.22) or

$$P = \frac{\mu\pi^3 N^2}{60 \times 30 \times t} [R_2^4 - R_1^4]$$

$$= \frac{0.91}{10} \times \frac{\pi^3 \times 300^2 \times [.075^4 - .05^4]}{60 \times 30 \times .00025}$$

$$= 564314 \, [.00003164 - .00000625]$$

$$= 564314 \times .00002539 = \mathbf{14.327 \ W.\ Ans.}$$

Problem 9.22 *The external and internal diameters of a collar bearing are 200 mm and 150 mm respectively. Between the collar surface and the bearing, an oil film of thickness 0.25 mm and of viscosity 0.9 poise, is maintained. Find the torque and the power lost in overcoming the viscous resistance of the oil when the shaft is running at 250 r.p.m.*

Solution. Given :

$$D_2 = 200 \text{ mm} = 0.2 \text{ m}$$

$$\therefore \quad R_2 = \frac{D_2}{2} = \frac{0.2}{2} = 0.1 \text{ m}$$

$$D_1 = 150 \text{ mm} = 0.15 \text{ m}$$

$$\therefore \quad R_1 = \frac{D_1}{2} = \frac{0.15}{2} = .075 \text{ m}$$

$$t = 0.25 \text{ mm} = .00025 \text{ m}$$

$$\mu = 0.9 \text{ poise} = \frac{0.9}{10} \frac{\text{Ns}}{\text{m}^2}$$

Torque required is given by equation (9.21)

$$\therefore \quad T = \frac{\mu}{60t} \pi^2 N [R_2^4 - R_1^4] = \frac{0.9}{10} \times \frac{\pi^2 \times 250 [0.1^4 - .075^4]}{60 \times 0.00025} \text{ Nm}$$

$$= 14804.4 \, [\,.0001 - .00003164] = \mathbf{1.0114 \ Nm.\ Ans.}$$

$\therefore$ Power lost in viscous resistance

$$= \frac{2\pi NT}{60} = \frac{2\pi \times 250 \times 1.0114}{60} = \mathbf{26.48 \ W.\ Ans.}$$

▶ 9.6 LOSS OF HEAD DUE TO FRICTION IN VISCOUS FLOW

The loss of pressure head, h_f in a pipe of diameter D, in which a viscous fluid of viscosity μ is flowing with a velocity $\bar{u}$ is given by Hagen Poiseuille formula *i.e.*, by equation (9.6) as

$$h_f = \frac{32\mu\bar{u}L}{\rho g D^2} \qquad ...(i)$$

where L = length of pipe

The loss of head due to friction* is given by

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{D \times 2g} = \frac{4 \cdot f \cdot L \cdot \bar{u}^2}{D \times 2g} \qquad ...(ii)$$

$\{\because$ velocity in pipe is always average velocity $\quad \therefore \quad V = \bar{u}\}$

*For derivation, please refer to Art. 10.3.1.

where f = co-efficient of friction between the pipe and fluid.

Equating (*i*) and (*ii*), we get $\frac{32\mu \bar{u} L}{\rho g D^2} = \frac{4 \cdot f \cdot L \cdot \bar{u}^2}{D \times 2g}$

$$f = \frac{32\mu \bar{u} L \times D \times 2g}{4 \cdot L \cdot \bar{u}^2 \cdot \rho g \cdot D^2} = \frac{16\mu}{\bar{u} \cdot \rho \cdot D} \qquad \{\because \bar{u} = V\}$$

$$= 16 \times \frac{\mu}{\rho V D} = 16 \times \frac{1}{R_e}$$

where $\frac{\mu}{\rho V D} = \frac{1}{R_e}$ and R_e = Reynolds number = $\frac{\rho V D}{\mu}$

$$\therefore \quad f = \frac{16}{R_e}. \qquad \text{...(9.23)}$$

Problem 9.23 *Water is flowing through a 200 mm diameter pipe with coefficient of friction f = 0.04. The shear stress at a point 40 mm from the pipe axis is 0.00981 N/cm². Calculate the shear stress at the pipe wall.*

Solution. Given :

Dia. of pipe, D = 200 mm = 0.20 m

Co-efficient of friction, f = 0.04

Shear stress at r = 40 mm, τ = 0.00981 N/cm^2

Let the shear stress at pipe wall = τ_0.

First find whether the flow is viscous or not. The flow will be viscous if Reynolds number R_e is less than 2000.

Using equation (9.23), we get $f = \frac{16}{R_e}$ or $.04 = \frac{16}{R_e}$

$$\therefore \quad R_e = \frac{16}{.04} = 400$$

This means flow is viscous. The shear stress in case of viscous flow through a pipe is given by the equation (9.1) as

$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

But $\frac{\partial p}{\partial x}$ is constant across a section. Across a section, there is no variation of x and there is no variation of p.

$$\therefore \quad \tau \propto r$$

At the pipe wall, radius = 100 mm and shear stress is τ_0

$$\therefore \quad \frac{\tau}{r} = \frac{\tau_0}{100} \quad \text{or} \quad \frac{0.00981}{40} = \frac{\tau_0}{100}$$

$$\therefore \quad \tau_0 = \frac{100 \times 0.00981}{40} = \mathbf{0.0245\ N/cm^2.\ Ans.}$$

Problem 9.24 *A pipe of diameter 20 cm and length 10^4 m is laid at a slope of 1 in 200. An oil of sp. gr. 0.9 and viscosity 1.5 poise is pumped up at the rate of 20 litres per second. Find the head lost due to friction. Also calculate the power required to pump the oil.*

Solution. Given :

Dia. of pipe, $D = 20 \text{ cm} = 2.50 \text{ m}$

Length of pipe, $L = 10000 \text{ m}$

Slope of pipe, $i = 1 \text{ in } 200 = \frac{1}{200}$

Sp. gr. of oil, $S = 0.9$

∴ Density of oil, $\rho = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Viscosity of oil, $\mu = 1.5 \text{ poise} = \frac{1.5}{10} \frac{\text{Ns}}{\text{m}^2}$

Discharge, $Q = 20 \text{ litre/s} = 0.02 \text{ m}^3\text{/s}$ $\{\because \; 1000 \text{ litres} = 1 \text{ m}^3\}$

∴ Velocity of flow, $\bar{u} = \frac{Q}{\text{Area}} = \frac{0.020}{\frac{\pi}{4} D^2} = \frac{0.020}{\frac{\pi}{4}(.2)^2} = 0.6366 \text{ m/s}$

∴ $R_e = \text{Reynolds number}$

$$= \frac{\rho V D}{\mu} = \frac{900 \times 0.6366 \times .2}{\frac{1.5}{10}}$$

$$= \frac{900 \times .6366 \times .2 \times 10}{1.5} \qquad \{\because \; V = \bar{u} = 0.6366\}$$

$= 763.89$

As the Reynolds number is less than 2000, the flow is viscous. The co-efficient of friction for viscous flow is given by equation (9.23) as

$$f = \frac{16}{R_e} = \frac{16}{763.89} = 0.02094$$

∴ Head lost due to friction, $h_f = \frac{4 \,.\, f \,.\, L \,.\, \bar{u}^2}{D \times 2g}$

$$= \frac{4 \times .02094 \times 10000 \times (.6366)^2}{0.2 \times 2 \times 9.81} \text{ m} = \mathbf{86.50 \text{ m. Ans.}}$$

Due to slope of pipe 1 in 200, the height through which oil is to be raised by pump

$= \text{Slope} \times \text{Length of pipe}$

$$= i \times L = \frac{1}{200} \times 10000 = 50 \text{ m}$$

∴ Total head against which pump is to work,

$$H = h_f + i \times L = 86.50 + 50 = 136.50 \text{ m}$$

∴ Power required to pump the oil

$$= \frac{\rho g \,.\, Q \,.\, H}{1000} = \frac{900 \times 9.81 \times 0.20 \times 136.50}{1000} = \textbf{24.1 kW. Ans.}$$

▶ 9.7 MOVEMENT OF PISTON IN DASH-POT

Consider a piston moving in a vertical dash-pot containing oil as shown in Fig. 9.13.

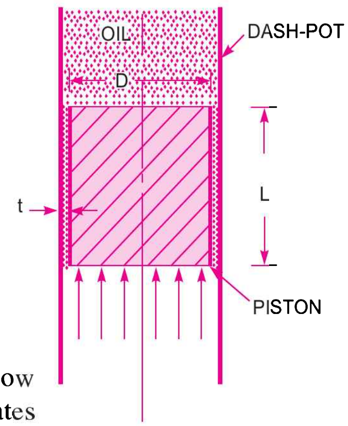

Fig. 9.13

Let D = Diameter of piston,
L = Length of piston,
W = Weight of piston,
μ = Viscosity of oil,
V = Velocity of piston,
$\bar{u}$ = Average velocity of oil in the clearance,
l = Clearance between the dash-pot and piston,
Δp = Difference of pressure intensities between the two ends of the piston.

The flow of oil through clearance is similar to the viscous flow between two parallel plates. The difference of pressure for parallel plates for length 'L' is given by

$$\Delta p = \frac{12\mu \bar{u} L}{t^2} \quad \text{...}(i)$$

Also the difference of pressure at the two ends of piston is given by,

$$\Delta p = \frac{\text{Weight of piston}}{\text{Area of piston}} = \frac{W}{\frac{\pi}{4} D^2} = \frac{4W}{\pi D^2} \quad \text{...}(ii)$$

Equating (i) and (ii), we get $\dfrac{12\mu \bar{u} L}{t^2} = \dfrac{4W}{\pi D^2}$

$$\therefore \quad \bar{u} = \frac{4W}{\pi D^2} \times \frac{t^2}{12\mu L} = \frac{Wt^2}{3\pi\mu L D^2} \quad \text{...}(iii)$$

V is the velocity of piston or the velocity of oil in dash-pot in contact with piston. The rate of flow of oil in dash-pot

$$= \text{velocity} \times \text{area of dash-pot} = V \times \frac{\pi}{4} D^2$$

Rate of flow through clearance = velocity through clearance × area of clearance = $\bar{u} \times \pi D \times t$

Due to continuity equation, rate of flow through clearance must be equal to rate of flow through dash-pot.

$$\therefore \quad \bar{u} \times \pi D \times t = V \times \frac{\pi}{4} D^2$$

$$\therefore \quad \bar{u} = V \times \frac{\pi}{4} D^2 \times \frac{1}{\pi D \times t} = \frac{VD}{4t} \quad \text{...}(iv)$$

Equating the value of $\bar{u}$ from (iii) and (iv), we get

$$\frac{Wt^2}{3\pi\mu L D^2} = \frac{VD}{4t}$$

$$\mu = \frac{4t^3W}{3\pi LD^3V} = \frac{4Wt^3}{3\pi LD^3V}. \quad ...(9.24)$$

Problem 9.25 *An oil dash-pot consists of a piston moving in a cylinder having oil. This arrangement is used to damp out the vibrations. The piston falls with uniform speed and covers 5 cm in 100 seconds. If an additional weight of 1.36 N is placed on the top of the piston, it falls through 5 cm in 86 seconds with uniform speed. The diameter of the piston is 7.5 cm and its length is 10 cm. The clearance between the piston and the cylinder is 0.12 cm which is uniform throughout. Find the viscosity of oil.*

Solution. Given :

Distance covered by piston due to self weight, = 5 cm

Time taken, = 100 sec

Additional weight, = 1.36 N

Time taken to cover 5 cm due to additional weight, = 86 sec

Dia. of piston, D = 7.5 cm = 0.075 m

Length of piston, L = 10 cm = 0.1 m

Clearance, t = 0.12 cm = 0.0012 m

Let the viscosity of oil = μ

W = Weight of piston,

V = Velocity of piston without additional weight,

V^* = Velocity of piston with additional weight.

Using equation (9.24), we have

$$\mu = \frac{4Wt^3}{3\pi D^3LV} = \frac{4[W+1.36]t^3}{3\pi D^3LV^*}$$

or
$$\frac{W}{V} = \frac{W+1.36}{V^*} \quad \left(\text{Cancelling } \frac{4Wt^3}{3\pi D^3L}\right)$$

or
$$\frac{V}{V^*} = \frac{W}{W+1.36} \quad ...(i)$$

But V = Velocity of piston due to self weight of piston

$$= \frac{\text{Distance covered}}{\text{Time taken}} = \frac{5}{100} \text{ cm/s}$$

Similarly,
$$V^* = \frac{\text{Distance covered due to self weight + additional weight}}{\text{Time taken}}$$

$$= \frac{5}{86} \text{ cm/s}$$

$\therefore$
$$\frac{V}{V^*} = \frac{5}{100} \times \frac{86}{5} = 0.86 \quad ...(ii)$$

Equating (*i*) and (*ii*), we get $\frac{W}{W+1.36} = 0.86$

or $W = 0.86\text{ W} + .86 \times 1.36$

or $W - 0.86\,W = 0.14\text{ W} = .86 \times 1.36$

$\therefore \quad W = \dfrac{0.86 \times 1.36}{0.14} = 8.354$ N

Using equation (9.24), we get $\mu = \dfrac{4Wt^3}{3\pi D^3 LV}$

$$= \frac{4 \times 8.354 \times (.0012)^3}{3\pi \times (0.075)^3 \times .10 \times \left(\frac{5}{100} \times \frac{1}{100}\right)} \quad \left\{\because V = \frac{5}{100} \text{ cm/s} = \frac{5}{100} \times \frac{1}{100} \text{ m/s}\right\}$$

$= 0.29$ N s/m^2 = 0.29×10 poise = **2.9 poise. Ans.**

▶ 9.8 METHODS OF DETERMINATION OF CO-EFFICIENT OF VISCOSITY

The following are the experimental methods of determining the co-efficient of viscosity of a liquid:

1. Capillary tube method,
2. Falling sphere resistance method,
3. By rotating cylinder method, and
4. Orifice type viscometer.

The apparatus used for determining the viscosity of a liquid is called viscometer.

9.8.1 Capillary Tube Method. In capillary tube method, the viscosity of a liquid is calculated by measuring the pressure difference for a given length of the capillary tube. The Hagen Poiseuille law is used for calculating viscosity.

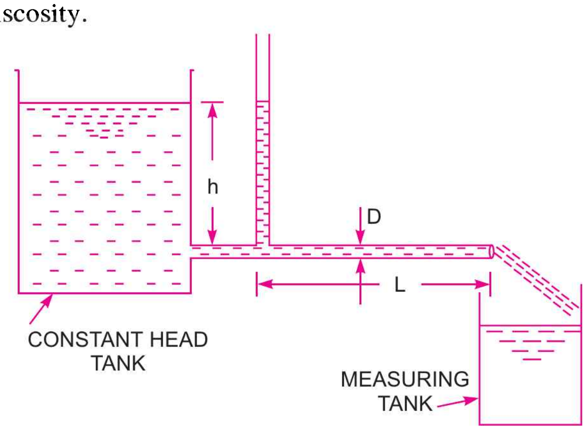

Fig. 9.14 *Capillary tube viscometer.*

Fig. 9.14 shows the capillary tube viscometer. The liquid whose viscosity is to be determined is filled in a constant head tank. The liquid is maintained at constant temperature and is allowed to pass through the capillary tube from the constant head tank. Then, the liquid is collected in a measuring tank for a given time. Then the rate of liquid collected in the tank per second is determined. The pressure head 'h' is measured at a point far away from the tank as shown in Fig. 9.14.

Then $\quad h$ = Difference of pressure head for length L.

The pressure at outlet is atmospheric.

Let $\quad D$ = Diameter of capillary tube,

L = Length of tube for which difference of pressure head is known,

ρ = Density of fluid,

and μ = Co-efficient of viscosity.

Using Hagen Poiseuille's Formula, $h = \dfrac{32\mu\bar{u}L}{\rho g D^2}$

But $\bar{u} = \dfrac{Q}{\text{Area}} = \dfrac{Q}{\frac{\pi}{4}D^2}$

where Q is rate of liquid flowing through tube.

$$h = \frac{32\mu \times \dfrac{Q}{\frac{\pi}{4}D^2} \times L}{\rho g D^2} = \frac{128\,\mu\, Q\,.\,L}{\pi \rho g D^4}$$

or $$\mu = \frac{\pi \rho g h \overline{D}^4}{128\, Q\,.\,L} \qquad \text{...(9.25)}$$

Measurement of D should be done very accurately.

9.8.2 Falling Sphere Resistance Method.

Theory. This method is based on Stoke's law, according to which the drag force, F on a small sphere moving with a constant velocity, U through a viscous fluid of viscosity, μ for viscous conditions is given by

$$F = 3\pi\mu U d \qquad \text{...}(i)$$

where d = diameter of sphere

U = velocity of sphere.

When the sphere attains a constant velocity U, the drag force is the difference between the weight of sphere and buoyant force acting on it.

Let L = distance travelled by sphere in viscous fluid,

t = time taken by sphere to cover distance l,

ρ_s = density of sphere,

ρ_f = density of fluid,

W = weight of sphere,

and F_B = buoyant force acting on sphere.

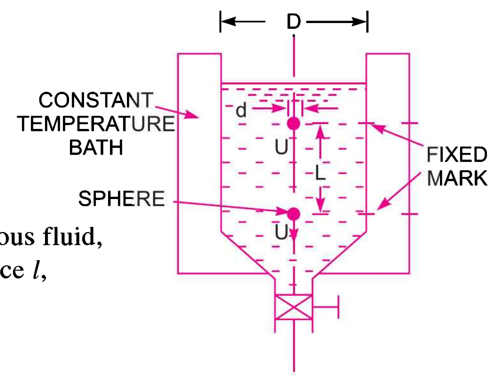

Fig. 9.15 *Falling sphere resistance method.*

Then constant velocity of sphere, $U = \dfrac{L}{t}$

Weight of sphere, W = volume × density of sphere × g

$$= \frac{\pi}{6} d^3 \times \rho_s \times g \qquad \left\{\because \text{ volume of sphere } = \frac{\pi}{6} d^3\right\}$$

and buoyant force, F_B = weight of fluid displaced

= volume of liquid displaced × density of fluid × g

$$= \frac{\pi}{6} d^3 \times \rho_f \times g \qquad \{\text{volume of liquid displaced = volume of sphere}\}$$

For equilibrium,

$$\text{Drag force} = \text{Weight of sphere} - \text{buoyant force}$$

or $$F = W - F_B$$

Substituting the values of F, W and F_B, we get

$$3\pi\mu Ud = \frac{\pi}{6} d^3 \times \rho_s \times g - \frac{\pi}{6} d^3 \times \rho_f \times g = \frac{\pi}{6} d^3 \times g\, [\rho_s - \rho_f]$$

or $$\mu = \frac{\pi}{6} \frac{d^3 \times g[\rho_s - \rho_f]}{3\pi Ud} = \frac{gd^2}{18U} [\rho_s - \rho_f] \qquad ...(9.26)$$

where ρ_f = Density of liquid

Hence in equation (9.26), the values of d, U, ρ_s and ρ_f are known and hence the viscosity of liquid can be determined.

Method. Thus this method consists of a tall vertical transparent cylindrical tank, which is filled with the liquid whose viscosity is to be determined. This tank is surrounded by another transparent tank to keep the temperature of the liquid in the cylindrical tank to be constant.

A spherical ball of small diameter 'd' is placed on the surface of liquid. Provision is made to release this ball. After a short distance of travel, the ball attains a constant velocity. The time to travel a known vertical distance between two fixed marks on the cylindrical tank is noted to calculate the constant velocity U of the ball. Then with the known values of d, ρ_s, ρ_f the viscosity μ of the fluid is calculated by using equation (9.26).

9.8.3 Rotating Cylinder Method. This method consists of two concentric cylinders of radii R_1 and R_2 as shown in Fig. 9.16. The narrow space between the two cylinders is filled with the liquid whose viscosity is to be determined. The inner cylinder is held stationary by means of a torsional spring while outer cylinder is rotated at constant angular speed ω. The torque T acting on the inner cylinder is measured by the torsional spring. The torque on the inner cylinder must be equal and opposite to the torque applied on the outer cylinder.

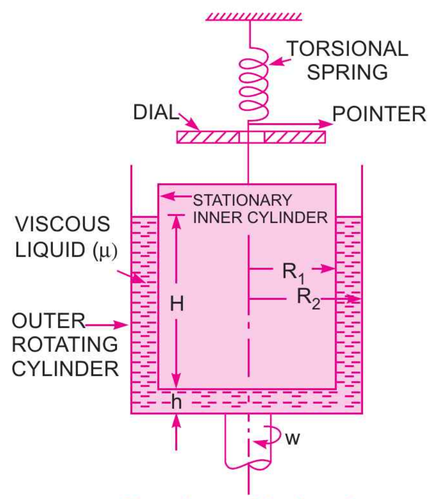

Fig. 9.16 *Rotating cylinder viscometer.*

The torque applied on the outer cylinder is due to viscous resistance provided by liquid in the annular space and at the bottom of the inner cylinder.

Let ω = angular speed of outer cylinder.

Tangential (peripheral) speed of outer cylinder

$$= \omega \times R_2$$

Tangential velocity of liquid layer in contact with outer cylinder will be equal to the tangential velocity of outer cylinder.

$\therefore$ Velocity of liquid layer with outer cylinder = $\omega \times R_2$

Velocity of liquid layer with inner cylinder = 0 {$\because$ Inner cylinder is stationary}

$\therefore$ Velocity gradient over the radial distance $(R_2 - R_1)$

$$= \frac{du}{dy} = \frac{\omega R_2 - 0}{R_2 - R_1} = \frac{\omega R_2}{R_2 - R_1}$$

$\therefore$ Shear stress (τ) $$= \mu \frac{du}{dy} = \mu \frac{\omega R_2}{(R_2 - R_1)}$$

$\therefore$ Shear force (F) $= \text{shear stress} \times \text{area of surface}$

$= \tau \times 2\pi R_1 H$

{$\because$ shear stress is acting on surface area $= 2\pi R_1 \times H$}

$$= \mu \frac{\omega R_2}{(R_2 - R_1)} \times 2\pi R_1 H$$

The torque T_1 on the inner cylinder due to shearing action of the liquid in the annular space is

$$T_1 = \text{shear force} \times \text{radius}$$

$$= \mu \frac{\omega R_2}{(R_2 - R_1)} \times 2\pi R_1 H \times R_1$$

$$= \frac{2\pi\mu\omega H R_1^2 R_2}{(R_2 - R_1)} \quad \text{...}(i)$$

If the gap between the bottom of the two cylinders is 'h', then the torque applied on inner cylinder (T_2) is given by equation (9.19A) as

$$T_2 = \frac{\mu}{60t} \pi^2 NR^4$$

But here $R = R_1$, $t = h$ then $T_2 = \dfrac{\mu}{60h} \pi^2 N{R_1}^4$

$$\omega = \frac{2\pi N}{60} \text{ or } N = \frac{60\omega}{2\pi}$$

$$\therefore \quad T_2 = \frac{\mu}{60h} \times \pi^2 \times \frac{60\omega}{2\pi} \times {R_1}^4 = \frac{\pi\mu\omega}{2h} {R_1}^4 \quad \text{...}(ii)$$

$\therefore$ Total torque T acting on the inner cylinder is

$$T = T_1 + T_2$$

$$= \frac{2\pi\mu\omega H R_1^2 R_2}{(R_2 - R_1)} + \frac{\pi\mu\omega}{2h} {R_1}^4 = 2\pi\mu {R_1}^2 \left[\frac{R_2 H}{R_2 - R_1} + \frac{R_1^2}{4h}\right] \times \omega$$

$$\therefore \quad \mu = \frac{2(R_2 - R_1)hT}{\pi R_1^2 \omega \left[4HhR_2 + R_1^2 (R_2 - R_1)\right]} \quad \text{...(9.27)}$$

where T = torque measured by the strain of the torsional spring,

R_1, R_2 = radii of inner and outer cylinder,

h = clearance at the bottom of cylinders,

H = height of liquid in annular space,

μ = co-efficient of viscosity to be determined.

Hence, the value of μ can be calculated from equation (9.27).

9.8.4 Orifice Type Viscometer. In this method, the time taken by a certain quantity of the liquid whose viscosity is to be determined, to flow through a short capillary tube is noted down. The co-efficient of viscosity is then obtained by comparing with the co-efficient of viscosity of a liquid whose viscosity is known or by the use conversion factors.

Viscometers such as Saybolt, Redwood or Engler are usually used. The principle for all the three viscometer is same. In the United Kingdom, Redwood viscometer is used while in U.S.A., Saybolt viscometer is commonly used.

Fig. 9.17 shows that Saybolt viscometer, which consists of a tank at the bottom of which a short capillary tube is fitted. In this tank the liquid whose viscosity is to be determined is filled. This tank is surrounded by another tank, called constant temperature bath. The liquid is allowed to flow through capillary tube at a standard temperature. The time taken by 60 c.c. of the liquid to flow through the capillary tube is noted down. The initial height of liquid in the tank is previously adjusted to a standard height. From the time measurement, the kinematic viscosity of liquid is known from the relation,

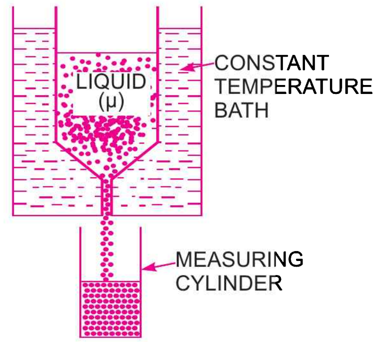

Fig. 9.17 *Saybolt viscometer.*

$$\nu = At - \frac{B}{t}$$

where $A = 0.24$, $B = 190$, t = time noted in seconds, ν = kinematic viscosity in stokes.

Problem 9.26 *The viscosity of an oil of sp. gr. 0.9 is measured by a capillary tube of diameter 50 mm. The difference of pressure head between two points 2 m apart is 0.5 m of water. The mass of oil collected in a measuring tank is 60 kg in 100 seconds. Find the viscosity of oil.*

Solution. Given :

Sp. gr. of oil $= 0.9$

Dia. of capillary tube, $D = 50$ mm $= 5$ cm $= 0.05$ m

Length of tube, $L = 2$ m

Difference of pressure head, $h = 0.5$ m

Mass of oil, $M = 60$ kg

Time, $t = 100$ s

Mass of oil per second $= \frac{60}{100} = 0.6$ kg/s

Density of oil, ρ = sp. gr. of oil $\times 1000 = 0.9 \times 1000 = 900$ kg/m^3

$\therefore$ Discharge, $Q = \frac{\text{Mass of oil / s}}{\text{Density}} = \frac{0.6}{900}$ m^3/s $= 0.000667$ m^3/s

Using equation (9.25), we get viscosity,

$$\mu = \frac{\pi \rho g h D^4}{128\, Q \,.\, L} \qquad [\text{here } h = h_f = 0.5]$$

$$= \frac{\pi \times 900 \times 9.81 \times 0.5 \times (.05)^4}{128 \times 0.000667 \times 2.0} = 0.5075 \text{ (SI Units) N s/m}^2$$

$= 0.5075 \times 10$ poise $=$ **5.075 poise. Ans.**

Problem 9.27 *A capillary tube of diameter 2 mm and length 100 mm is used for measuring viscosity of a liquid. The difference of pressure between the two ends of the tube is 0.6867 N/cm^2 and the viscosity of liquid is 0.25 poise. Find the rate of flow of liquid through the tube.*

Solution. Given :

Dia. of capillary tube, $D = 2$ m $= 2 \times 10^{-3}$ m

Length of tube, $L = 100$ mm $= 10$ cm $= 0.1$ m

Difference of pressure, $\Delta p = 0.6867$ N/cm^2 $= 0.6867 \times 10^4$ N/m^2

$\therefore$ Difference of pressure head, $h = \frac{\Delta p}{\rho g} = \frac{0.6867 \times 10^4}{\rho g}$

Viscosity, $\mu = 0.25$ poise

$$= \frac{0.25}{10} \text{ Ns/m}^2$$

Let the rate of flow of liquid = Q

Using equation (9.25), we get $\mu = \dfrac{\pi\rho g h D^4}{128 . Q . L} = \pi\rho g \times \dfrac{\dfrac{0.6867 \times 10^4}{\rho g} \times \left(2 \times 10^{-3}\right)^4}{128 \times Q \times 0.1}$

or $$\frac{0.25}{10} = \frac{\pi \times 0.6867 \times 10^4 \times \left(2 \times 10^{-3}\right)^4}{128 \times Q \times 0.1}$$

or $$Q = \frac{\pi \times 0.6867 \times 10^4 \times 2^4 \times 10^{-12} \times 10}{128 \times 0.1 \times 0.25} \text{ m}^3\text{/s}$$

$= 107.86 \times 10^{-8}$ m^3/s $= 107.86 \times 10^{-8} \times 10^6$ cm^3/s

$= 107.86 \times 10^{-2}$ cm^3/s = **1.078 cm^3/s. Ans.**

Problem 9.28 *A sphere of diameter 2 mm falls 150 mm in 20 seconds in a viscous liquid. The density of the sphere is 7500 kg/m^3 and of liquid is 900 kg/m^3. Find the co-efficient of viscosity of the liquid.*

Solution. Given :

Dia. of sphere, $d = 2$ mm $= 2 \times 10^{-3}$ m

Distance travelled by sphere = 150 mm = 0.15 m

Time taken, $t = 20$ seconds

Velocity of sphere, $U = \dfrac{\text{Distance}}{\text{Time}} = \dfrac{0.15}{20} = .0075$ m/s

Density of sphere, $\rho_s = 7500$ kg/m^3

Density of liquid, $\rho_f = 900$ kg/m^3

Using relation (9.26), we get $\mu = \dfrac{gd^2}{18U}[\rho_s - \rho_f] = \dfrac{9.81 \times \left[2 \times 10^{-3}\right]^2}{18 \times 0.0075}[7500 - 900]$

$$= \frac{9.81 \times 4 \times 10^{-6} \times 6600}{18 \times 0.0075} = 1.917 \frac{\text{Ns}}{\text{m}^2}$$

$= 1.917 \times 10 =$ **19.17 poise. Ans.**

Problem 9.29 *Find the viscosity of a liquid of sp. gr. 0.8, when a gas bubble of diameter 10 mm rises steadily through the liquid at a velocity of 1.2 cm/s. Neglect the weight of the bubble.*

Solution. Given :

Sp. gr. of liquid = 0.8

∴ Density of liquid, $\rho_f = 0.8 \times 1000 = 800$ kg/m^3

Dia. of gas bubble, $D = 10$ mm = 1 cm = 0.01 m

Velocity of bubble, $U = 1.2$ cm/s = .012 m/s

As weight of bubble is neglected and density of bubble

$$\rho_s = 0$$

Now using the relation, $\mu = \frac{gd^2}{18U}[\rho_s - \rho_f]$ which is for a falling sphere.

For a rising bubble, the relation will become as

$$\mu = \frac{gd^2}{18U}[\rho_f - \rho_s]$$

Substituting the values, we get $\mu = \frac{9.81 \times .01 \times .01}{18 \times .012}[800 - 0]\frac{\text{Ns}}{\text{m}^2} = 3.63\frac{\text{Ns}}{\text{m}^2}$

$= 3.63 \times 10 =$ **36.3 poise. Ans.**

Problem 9.30 *The viscosity of a liquid is determined by rotating cylinder method, in which case the inner cylinder of diameter 20 cm is stationary. The outer cylinder of diameter 20.5 cm, contains the liquid upto a height of 30 cm. The clearance at the bottom of the two cylinders is 0.5 cm. The outer cylinder is rotated at 400 r.p.m. The torque registered on the torsion meter attached to the inner cylinder is 5.886 Nm. Find the viscosity of fluid.*

Solution. Given :

Dia. of inner cylinder, $D_1 = 20$ cm

∴ Radius of inner cylinder, $R_1 = 10$ cm $= 0.1$ m

Dia. of outer cylinder, $D_2 = 20.5$ cm

∴ Radius of outer cylinder, $R_2 = \frac{20.5}{2} = 10.25$ cm $= .1025$ m

Height of liquid from bottom of outer cylinder = 30 cm

Clearance at the bottom of two cylinders, $h = 0.5$ cm $= .005$ m

∴ Height of inner cylinder immersed in liquid

$= 30 - h = 30 - 0.5 = 29.5$ m

or $H = 29.5$ cm $= .295$ m

Speed of outer cylinder, $N = 400$ r.p.m.

∴ $\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 400}{60} = 41.88$

Torque measured, $T = 5.886$ Nm

Using equation (9.27), we get $\mu = \frac{2(R_2 - R_1) \times h \times T}{\pi R_1^2 \omega \left[4HhR_2 + R_1^2(R_2 - R_1)\right]}$

$$= \frac{2(.1025 - 0.1) \times .005 \times 5.886}{\pi \times (.1)^2 \times 41.88\left[4 \times .295 \times .005 \times .1025 + .1^2\,(.1025 - .1)\right]}$$

$$= \frac{2 \times .0025 \times .005 \times 5.886}{\pi \times .01 \times 41.88\,[.0006047 - .000025]}$$

$= 0.19286$ Ns/m^2 $= 0.19286 \times 10 =$ **1.9286 poise. Ans.**

Problem 9.31 *A sphere of diameter 1 mm falls through 335 m in 100 seconds in a viscous fluid. If the relative densities of the sphere and the liquid are 7.0 and 0.96 respectively, determine the dynamic viscosity of the liquid.*

Solution. Given :

Dia. of sphere, $d = 1$ mm $= 0.001$ m

Distance travelled by sphere $= 335$ mm $= 0.335$ m

Time taken, $t = 100$ seconds

$\therefore$ Velocity of sphere, $U = \dfrac{\text{Distance}}{\text{Time}} = \dfrac{0.335}{100} = 0.00335$ m/sec

Relative density of sphere $= 7$

$\therefore$ Density of sphere, $\rho_s = 7 \times 1000 = 7000$ kg/m^3

Relative density of liquid $= 0.96$

$\therefore$ Density of liquid, $\rho_f = 0.96 \times 1000 = 960$ kg/m^3

Using the relation (9.26), we get $\mu = \dfrac{gd^2}{18U}\,[\rho_s - \rho_f] = \dfrac{9.81 \times 0.001^2}{18 \times 0.00335}\,[7000 - 960]$

$$= \frac{0.00000981 \times 6040}{18 \times 0.00335} = 0.981 \text{ Ns/m}^2$$

$= 0.981 \times 10 =$ **9.81 poise. Ans.**

Problem 9.32 *Determine the fall velocity of 0.06 mm sand particle (specific gravity = 2.65) in water at 20°C, take $\mu = 10^{-3}$ kg/ms.*

Solution. Given :

Dia. of sand particle, $d = 0.06$ mm $= 0.06 \times 10^{-3}$ m

Specific gravity of sand $= 2.65$

$\therefore$ Density of sand, $\rho_s = 2.65 \times 1000$ kg/m^3 $= 2650$ kg/m^3 ($\because$ ρ for water in S.I. unit $= 1000$ kg/m^3)

Viscosity of water, $\mu^* = 10^{-3}$ kg/ms $= 10^{-3}$ Ns/m^2 $\left[\because \dfrac{\text{Ns}}{\text{m}^2} = \left(\text{kg} \times \dfrac{\text{m}}{\text{s}^2}\right) \times \dfrac{\text{s}}{\text{m}^2} = \dfrac{\text{kg}}{\text{ms}}\right]$

Density of water, $\rho_f = 1000$ kg/m^3

Sand particle is just like a sphere.

For equilibrium of sand particle,

Drag force = Weight of sand particle – buoyant force

or $F_D = W - F_B$ ...(*i*)

But $F_D = 3\pi\mu \times U \times d$, where U = Velocity of particle

$= 3\pi \times 10^{-3} \times U \times 0.06 \times 10^{-3}$ N

W = Weight of sand particle

$$= \frac{\pi}{6} \times d^3 \times \rho_s \times g = \frac{\pi}{6} \times (0.06 \times 10^{-3})^3 \times 2650 \times 9.81 \text{ N}$$

F_B = Buoyant force = Weight of water displaced

*Viscosity in S.I. unit = N s/m^2. But 1 N = 1 kg × 1 m/s^2

Hence viscosity $= \left(\dfrac{1\text{ kg} \times 1\text{ m}}{\text{s}^2}\right) \times \dfrac{\text{s}}{\text{m}^2}$ = kg/ms. Hence kg/ms $= \dfrac{\text{Ns}}{\text{m}^2}$.

$$= \frac{\pi}{6} \times d^3 \times \rho_f \times g = \frac{\pi}{6} \times (0.06 \times 10^{-3})^3 \times 1000 \times 9.81 \text{ N}$$

Substituting the above values in equation (*i*), we get

$$3\pi \times 10^{-3} \times U \times 0.06 \times 10^{-3} = \frac{\pi}{6} \times (0.06 \times 10^{-3})^3 \times 2650 \times 9.81 - \frac{\pi}{6} \times (0.06 \times 10^{-3})^3 \times 1000 \times 9.81$$

Cancelling $(\pi \times 0.06 \times 10^{-3})^2$ throughout, we get

$$3 \times U = \frac{1}{6} \times 0.06^2 \times 10^{-3} \times 2650 \times 9.81 - \frac{1}{6} \times 0.06^2 \times 10^{-3} \times 1000 \times 9.81$$

$$= \frac{1}{6} \times 0.06^2 \times 10^{-3} \times 9.81\ (2650 - 1000)$$

$$= \frac{1}{6} \times 0.0036 \times 10^{-3} \times 9.81 \times 1650 = 0.009712$$

$\therefore$ $U = 0.009712/3 =$ **0.00323 m/sec. Ans.**

HIGHLIGHTS

1. A flow is said to be viscous if the Reynolds number is less than 2000, or the fluid flows in layers.
2. For the viscous flow through circular pipes,

   (*i*) Shear stress ...... $\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$

   (*ii*) Velocity ......$u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$

   (*iii*) Ratio of velocities $\frac{U_{max}}{\bar{u}} = 2.0$

   (*iv*) Loss of pressure head, $h_f = \frac{32\mu\bar{u}L}{\rho g D^2}$

   where $\frac{\partial p}{\partial x}$ = pressure gradient, r = radius at any point,
   R = radius of the pipe, U_{max} = maximum velocity or velocity at $r = 0$,
   $\bar{u}$ = average velocity = $\frac{Q}{\pi R^2}$, μ = co-efficient of viscosity,
   D = diameter of the pipe.
3. For the viscous flow between two parallel plates,

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2) \quad \text{... Velocity distribution}$$

$$\frac{U_{max}}{\bar{u}} = 1.5 \quad \text{... Ratio of maximum and average velocity}$$

$$h_f = \frac{12\mu\bar{u}L}{\rho g t^2} \quad \text{... Loss of pressure head}$$

$$\tau = -\frac{1}{2}\frac{\partial p}{\partial x}[t - 2y] \quad \text{... Shear stress distribution}$$

where t = thickness or distance between two plates,
y = distance in the vertical direction from the lower plate,
τ = shear stress at any point in flow.

4. The kinetic energy correction factor α is given as

$$\alpha = \frac{\text{K.E. per second based on actual velocity}}{\text{K.E. per second based on average velocity}}$$

$= 2.0$ ... for a circular pipe.

5. Momentum correction factor, β is given by

$$\beta = \frac{\text{Momentum per second based on actual velocity}}{\text{Momentum per second based on average velocity}}$$

$= \frac{4}{3}$ ... for a circular pipe.

6. For the viscous resistance of Journal Bearing.

$$V = \frac{\pi DN}{60}, \frac{du}{dy} = \frac{V}{t} = \frac{\pi DN}{60t}$$

$$\tau = \frac{\mu \pi dN}{60t}, \text{ Shear force} = \frac{\mu \pi^2 D^2 NL}{60t}$$

Torude, $T = \frac{\mu \pi^2 D^3 NL}{120t}$ and power $= \frac{\mu \pi^3 D^3 N^2 L}{60 \times 60 \times t}$

where L = length of bearing, N = speed of shaft

t = clearance between the shaft and bearing.

7. For the Foot-Step Bearing, the shear force, torque and h.p. absorbed are given as :

Shear force, $F = \frac{\mu}{15} \frac{\pi^2 N}{t} \frac{R^3}{3}$

Torque, $T = \frac{\mu}{60t} \pi^2 NR^4$

Power $= \frac{\mu \pi^3 N^2 R^4}{60 \times 30 \times t}$

where R = radius of the shaft, N = speed of the shaft.

8. For the collar bearing the torque and power absorbed are given as

$$T = \frac{\mu}{60t} \pi^2 N [R_2^4 - R_1^4], \qquad P = \frac{\mu \pi^3 N^2}{60 \times 30t} [R_2^4 - R_1^4]$$

where R_1 = internal radius of the collar, R_2 = external radius of the collar,

t = thickness of oil film, P = power in watts.

9. For the viscous flow the co-efficient of friction is given by, $f = \frac{16}{R_e}$

where R_e = the Reynolds number $= \frac{\rho VD}{\mu} = \frac{VD}{\nu}$.

10. The co-efficient of viscosity is determined by dash-pot arrangement as $\mu = \frac{4Wt^3}{3\pi LD^3 V}$

where W = weight of the piston, t = clearance between dash-pot and piston,

L = length of the piston, D = diameter of the piston,

V = velocity of the piston.

11. The co-efficient of viscosity of a liquid is also determined experimentally by the following method :

(*i*) Capillary tube method, $\mu = \dfrac{\pi\rho g h D^4}{128\,QL}$

(*ii*) Falling sphere method, $\mu = \dfrac{gd^2\left[\rho_s - \rho_f\right]}{18U}$

(*iii*) Rotating cylinder method, $\mu = \dfrac{2\left(R_2 - R_1\right)hT}{\pi R_1^2 \omega\left[4HhR_2 + R_1^2\left(R_2 - R_1\right)\right]}$

where w = specific weight of fluid, L = length of the tube,
D = diameter of the capillary tube, Q = rate of flow of fluid through capillary tube,
d = diameter of the sphere, ρ_s = density of sphere,
ρ_f = density of fluid, U = velocity of sphere,
R_2 = radius of outer rotating cylinder, R_1 = radius of inner stationary cylinder,
T = torque.

EXERCISE

(A) THEORETICAL PROBLEMS

1. Define the terms : Viscosity, kinematic viscosity, velocity gradient and pressure gradient.
2. What do you mean by 'Viscous Flow'?
3. Derive an expression for the velocity distribution for viscous flow through a circular pipe. Also sketch the velocity distribution and shear stress distribution across a section of the pipe.
4. Prove that the maximum velocity in a circular pipe for viscous flow is equal to two times the average velocity of the flow. (*Delhi University, December 2002*)
5. Find an expression for the loss of head of a viscous fluid flowing through a circular pipe.
6. What is Hagen Poiseuille's Formula ? Derive an expression for Hagen Poiseuille's Formula.
7. Prove that the velocity distribution for viscous flow between two parallel plates when both plates are fixed across a section is parabolic in nature. Also prove that maximum velocity is equal to one and a half times the average velocity.
8. Show that the difference of pressure head for a given length of the two parallel plates which are fixed and through which viscous fluid is flowing is given by

$$h_f = \frac{12\mu \bar{u} L}{\rho g t^2}$$

where μ = Viscosity of fluid, $\bar{u}$ = Average velocity,
t = Distance between the two parallel plates, L = Length of the plates.
9. Define the terms : Kinetic energy correction factor and momentum correction factor.
10. Prove that for viscous flow through a circular pipe the kinetic energy correction factor is equal to 2 while momentum correction factor = $\dfrac{4}{3}$.
11. A shaft is rotating in a journal bearing. The clearance between the shaft and the bearing is filled with a viscous oil. Find an expression for the power absorbed in overcoming viscous resistance.
12. Prove that power absorbed in overcoming viscous resistance in foot-step bearing is given by

$$P = \frac{\mu \pi^3 N^2 R^4}{60 \times 30t}$$

where R = Radius of the shaft, N = Speed of the shaft,
t = Clearance between shaft and foot-step bearing, μ = Viscosity of fluid.

13. Show that the value of the co-efficient of friction for viscous flow through a circular pipe is given by,

$$f = \frac{16}{R_e}, \quad \text{where} \quad R_e = \text{Reynolds number.}$$

14. Prove that the co-efficient of viscosity by the dash-pot arrangement is given by,

$$\mu = \frac{4Wt^3}{3\pi LD^3 V}$$

where W = Weight of the piston, t = Clearance between dash-pot and piston,
L = Length of piston, D = Diameter of piston,
V = Velocity of piston.

15. What are the different methods of determining the co-efficient of viscosity of a liquid ? Describe any two method in details.

16. Prove that the loss of pressure head for the viscous flow through a circular pipe is given by

$$h_f = \frac{32\mu \bar{u} L}{\rho g d^2}$$

where $\bar{u}$ = Average velocity, w = Specific weight.

17. For a laminar steady flow, prove that the pressure gradient in a direction of motion is equal to the shear gradient normal to the direction of motion.

18. Describe Reynolds experiments to demonstrate the two types of flow.

19. For the laminar flow through a circular pipe, prove that :

(*i*) the shear stress variation across the section of the pipe is linear and

(*ii*) the velocity variation is parabolic.

(B) NUMERICAL PROBLEMS

1. A crude oil of viscosity 0.9 poise and sp. gr. 0.8 is flowing through a horizontal circular pipe of diameter 80 mm and of length 15 m. Calculate the difference of pressure at the two ends of the pipe, if 50 kg of the oil is collected in a tank in 15 seconds. [**Ans.** 0.559 N/cm^2]

2. A viscous flow is taking place in a pipe of diameter 100 mm. The maximum velocity is 2 m/s. Find the mean velocity and the radius at which this occurs. Also calculate the velocity at 30 mm from the wall of the pipe. [**Ans.** 1 m/s, r = 35.35 mm, u = 1.68 m/s]

3. A fluid of viscosity 0.5 poise and specific gravity 1.20 is flowing through a circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is given as 147.15 N/m^2, find : (*a*) the pressure gradient, (*b*) the average velocity, and (*c*) the Reynolds number of the flow.

[**Ans.** (*a*) – 64746 N/m^2 per m, (*b*) 3.678 m/s, (*c*) 882.72]

4. Determine (*a*) the pressure gradient, (*b*) the shear stress at the two horizontal parallel plates and (*c*) the discharge per metre width for the laminar flow of oil with a maximum velocity of 1.5 m/s between two horizontal parallel fixed plates which are 80 mm apart. Take viscosity of oil as $\frac{1.962 \text{ Ns}}{\text{m}^2}$.

[**Ans.** (*a*) – 3678.7 N/m^2 per m, (*b*) 147.15 N/m^2, (*c*) .08 m^3/s]

5. Water is flowing between two large parallel plates which are 2.0 mm apart. Determine : (*a*) maximum velocity, (*b*) the pressure drop per unit length and (*c*) the shear stress at walls of the plate if the average velocity is 0.4 m/s. Take viscosity of water as 0.01 poise.

[**Ans.** (*a*) 0.6 m/s, (*b*) 1199.7 N/m^2 per m, (*c*) 1.199 N/m^3]

6. There is a horizontal crack 50 mm wide and 3 mm deep in a wall of thickness 150 mm. Water leaks through the crack. Find the rate of leakage of water through the crack if the difference of pressure between the two ends of the crack is 245.25 N/m^2. Take the viscosity of water as 0.01 poise. [**Ans.** 183.9 cm^3/s]

7. A shaft having a diameter of 10 cm rotates centrally in a journal bearing having a diameter of 10.02 cm and length 20 cm. The annular space between the shaft and the bearing is filled with oil having viscosity of 0.8 poise. Determine the power absorbed in the bearing when the speed of rotation is 500 r.p.m.

[**Ans.** 343.6 W]

8. A shaft 150 mm diameter runs in a bearing of length 300 mm, with a radial clearance of 0.04 mm at 40 r.p.m. Find the viscosity of the oil, if the power required to overcome the viscous resistance is 220.725 W.

[**Ans.** 6.32 poise]

9. Find the torque required to rotate a vertical shaft of diameter 8 cm at 800 r.p.m. The lower end of the shaft rests in a foot-step bearing. The end of the shaft and surface of the bearing are both flat and are separated by an oil film of thickness 0.075 cm. The viscosity of the oil is given as 1.2 poise. [**Ans.** 0.0538 Nm]

10. A collar bearing having external and internal diameters 20 cm and 10 cm respectively is used to take the thrust of a shaft. An oil film of thickness 0.03 cm is maintained between the collar surface and the bearing. Find the power lost in overcoming the viscous resistance when the shaft rotates at 250 r.p.m. Take $\mu = 0.9$ poise. [**Ans.** 30.165 W]

11. Water is flowing through a 150 mm diameter pipe with a co-efficient of friction $f = .05$. The shear stress at a point 40 mm from the pipe wall is 0.01962 N/cm^2. Calculate the shear stress at the pipe wall.

[**Ans.** 0.04198 N/cm^2]

12. An oil dash-pot consists of a piston moving in a cylinder having oil. The piston falls with uniform speed and covers 4.5 cm in 80 seconds. If an additional weight of 1.5 N is placed on the top of the piston, it falls through 4.5 cm in 70 seconds with uniform speed. The diameter of the piston is 10 cm and its length is 15 cm. The clearance between the piston and the cylinder is 0.15 cm, which is uniform throughout. Find the viscosity of oil. [**Ans.** 0.177 poise]

13. The viscosity of oil of sp. gr. 0.8 is measured by a capillary tube of diameter 40 mm. The difference of pressure head between two points 1.5 m apart is 0.3 m of water. The mass of oil collected in a measuring tank is 40 kg in 120 seconds. Find the viscosity of the oil. [**Ans.** 2.36 poise]

14. A capillary tube of diameter 4 mm and length 150 mm is used for measuring viscosity of a liquid. The difference of pressure between the two ends of the tube is 0.7848 N/cm^2 and the viscosity of the liquid is 0.2 poise. Find the rate of flow of liquid through the tube. [**Ans.** 16.43 cm^3/s]

15. A sphere of diameter 3 mm falls 100 mm in 1.5 seconds in a viscous liquid. The density of the sphere is 7000 kg/m^3 and of liquid is 800 kg/m^3. Find the co-efficient of viscosity of the liquid. [**Ans.** 45.61 poise]

16. The viscosity of a liquid is determined by rotating cylinder method, in which case the inner cylinder of diameter 25 cm is stationary. The outer cylinder of diameter 25.5 cm contains the liquid upto a height of 40 cm. The clearance at the bottom of the two cylinders is 0.6 cm. The outer cylinder is rotated at 300 r.p.m. The torque registered on the torsion metre attached to the inner cylinder is 4.905 Nm. Find the viscosity of liquid. [**Ans.** .77 poise]

17. Calculate : (*a*) the pressure gradient along the flow, (*b*) the average velocity, and (*c*) the discharge for an oil of viscosity 0.02 Ns/m^2 flowing between two stationary parallel plates 1 m wide maintained 10 mm apart. The velocity midway between the plates is 2.5 m/s.

[**Ans.** (a) – 4000 N/m^2 per m, (b) 1.667 m/s, (c) .01667 m^3/s]

18. Calculate :

(*i*) the pressure gradient along the flow,

(*ii*) the average velocity, and

(*iii*) the discharge for an oil of viscosity 0.03 N s/m^2 flowing between two stationary plates which are parallel and are at 10 mm apart. Width of plates is 2 m. The velocity midway between the plates is 2.0 m/s.

19. A cylinder of 100 mm diameter, 0.15 m length and weighing 10 N slides axially in a vertical pipe of 104 mm dia. If the space between cylinder surface and pipe wall is filled with liquid of viscosity μ and the cylinder slides downwards at a velocity of 0.45 m/s, determine μ.

[**Hint.** D = 100 mm = 0.1, L = 0.15 m, W = 10 N, D_p = 1.4 mm = 0.104 m,
V = 0.45 m/s. Hence t = (0.104 – 0.1)/2 = 0.002 m.

$$\mu = \frac{4Wt^3}{3\pi D^3 LV} = \frac{4 \times 10 \times 0.002^3}{3\pi \times 0.1^3 \times 0.15 \times .45} = \mathbf{503 \times 10^{-6}\ N\ s/m^2}]$$

20. A liquid is pumped through a 15 cm diameter and 300 m long pipe at the rate of 20 tonnes per hour. The density of liquid is 910 kg/m^3 and kinematic viscosity = 0.002 m^2/s. Determine the power required and show that the flow is viscous.

[**Hint.** D = 15 cm = 0.15 m, L = 300 m, W = 20 tonnes/hr

= 20 × 1000 kgf/60 × 60 sec = 5.555 kgf/sec = 5.555 × 9.81 N/s.

$$Q = \frac{W}{\rho g} = \frac{5.555 \times 9.81}{910 \times 9.81} = 0.0061\ \text{m}^3/\text{s}.\ V = \frac{Q}{A} = \frac{0.0061}{\frac{\pi}{4}(.15^2)}$$

= 0.345 m/s, v = 0.002 m^2/s.

Now $$R_e = \frac{\rho VD}{\mu} = \frac{V \times D}{\nu} = \frac{0.345 \times 0.15}{0.002} = 25.87$$

which is less than 2000. Hence flow is viscous.

$$h_f = 32\ \mu LV/\rho g D^2,\ \text{where } \nu = \frac{\mu}{\rho}\ \therefore\ \mu = \nu \times \rho = 0.002 \times 910 = 1.82$$

Hence, $$h_f = \frac{32 \times 1.82 \times 300 \times 0.345}{(910 \times 9.81 \times 0.15^2)} = 30$$

∴ $P = \rho g.Q.h_f/1000 = 910 \times 9.81 \times 0.0061 \times 30/1000 =$ **1.633 kW.**]

21. An oil of specific gravity 0.9 and viscosity 10 poise is flowing through a pipe of diameter 110 mm. The velocity at the centre is 2 m/s, find : (*i*) pressure gradient in the direction of flow, (*ii*) shear stress at the pipe wall ; (*iii*) Reynolds number, and (*iv*) velocity at a distance of 30 mm from the wall.

[**Hint.** ρ = 900 kg/m^3 ; μ = 10 poise = 1 N s/m^2 ; D = 110 mm = 0.11 m,

$$U_{max} = 2\ \text{m/s}\ ;\ \bar{u} = 1\ \text{m/s}\ ;\ U_{max} = \frac{1}{4\mu}\left(\frac{-dp}{dx}\right)R^2$$

(*i*) $$\left(\frac{-dp}{dx}\right) = \frac{4\mu \times U_{max}}{R^2} = \frac{4 \times 1 \times 2}{0.055^2} = 2644.6\ \text{N/m}^3\ ;$$

(*ii*) $$\tau_0 = \left(\frac{-dp}{dx}\right) \times \frac{R}{2} = 2644.6 \times \frac{0.055}{2} = 72.72\ \text{N/m}^2\ ;$$

(*iii*) $$R_e = \frac{\rho \times \bar{u} \times D}{\mu} = \frac{900 \times 1 \times 0.11}{1} = 99\ ;\ \text{and}$$

(*iv*) $$u = \frac{1}{4\mu}\left(\frac{-dp}{dx}\right)(R^2 - r^2) = \frac{1}{4 \times 1}(2644.6)(0.055^2 - 0.025^2) = 1.586\ \text{m/s.}]$$

22. Determine (*i*) the pressure gradient, (*ii*) the shear stress at the two horizontal plates, (*iii*) the discharge per metre width for laminar flow of oil with a maximum velocity of 2 m/s between two plates which are 150 mm apart. Given : μ = 2.5 N s/m^2. (*Delhi University, December 2002*)

[**Hint.** U_{max} = 2 m/s, t = 150 mm = 0.15 m, μ = 2.5 N s/m^2

(*i*) $$U_{max} = -\frac{1}{8\mu}\frac{dp}{dx}t^2\quad \therefore\quad \frac{dp}{dx} = \frac{-8\mu\, U_{max}}{t^2} = \frac{-8 \times 2.5 \times 2}{0.15^2} = \mathbf{-1777.77\ N/m^2.}$$

(*ii*) $$\tau_0 = -\frac{1}{2}\frac{dp}{dx} \times t = -\frac{1}{2}(-1777.77) \times 0.15 = \mathbf{133.33\ N/m^2.}$$

(*iii*) $$Q = \text{Mean velocity} \times \text{Area} = \left(\frac{2}{3}U_{max}\right) \times (t \times 1) = \left(\frac{2}{3} \times 2\right) \times (0.15 \times 1) = \mathbf{0.2\ m^3/s.}]$$