8

CHAPTER

NOTCHES AND WEIRS

▶ 8.1 INTRODUCTION

A **notch** is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A **weir** is a concrete or masonary structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel. The notch is of small size while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of concrete or masonary structure.

1. Nappe or Vein. The sheet of water flowing through a notch or over a weir is called Nappe or Vein.

2. Crest or Sill. The bottom edge of a notch or a top of a weir over which the water flows, is known as the sill or crest.

▶ 8.2 CLASSIFICATION OF NOTCHES AND WEIRS

The notches are classified as :

1. According to the shape of the opening :
   (*a*) Rectangular notch,
   (*b*) Triangular notch,
   (*c*) Trapezoidal notch, and
   (*d*) Stepped notch.
2. According to the effect of the sides on the nappe :
   (*a*) Notch with end contraction.
   (*b*) Notch without end contraction or suppressed notch.

Weirs are classified according to the shape of the opening, the shape of the crest, the effect of the sides on the nappe and nature of discharge. The following are important classifications.

(*a*) According to the shape of the opening :
   (*i*) Rectangular weir, (*ii*) Triangular weir, and
   (*iii*) Trapezoidal weir (Cipolletti weir)

(*b*) According to the shape of the crest :
   (*i*) Sharp-crested weir, (*ii*) Broad-crested weir,
   (*iii*) Narrow-crested weir, and (*iv*) Ogee-shaped weir.

(*c*) According to the effect of sides on the emerging nappe :
 (*i*) Weir with end contraction, and (*ii*) Weir without end contraction.

▶ 8.3 DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

The expression for discharge over a rectangular notch or weir is the same.

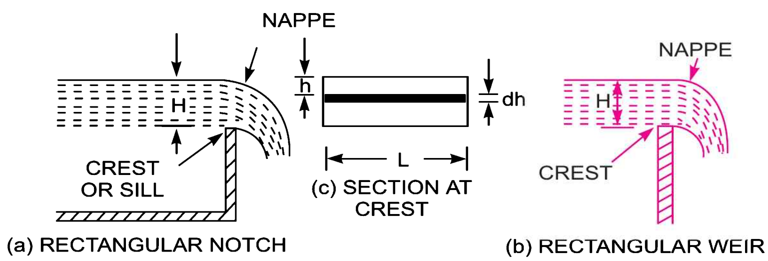

Fig. 8.1 *Rectangular notch and weir.*

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig. 8.1.

Let H = Head of water over the crest

L = Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water as shown in Fig. 8.1(*c*).

The area of strip $= L \times dh$

and theoretical velocity of water flowing through strip $= \sqrt{2gh}$

The discharge dQ, through strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \quad ...(i)$$

where C_d = Co-efficient of discharge.

The total discharge, Q, for the whole notch or weir is determined by integrating equation (*i*) between the limits 0 and H.

$$\therefore \quad Q = \int_0^H C_d \,.\, L \,.\, \sqrt{2gh}\,.\, dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1}\right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2}\right]_0^H$$

$$= \frac{2}{3} C_d \times L \times \sqrt{2g}\; [H]^{3/2}. \quad ...(8.1)$$

Problem 8.1 *Find the discharge of water flowing over a rectangular notch of 2 m length when the constant head over the notch is 300 mm. Take C_d = 0.60.*

Solution. Given :

Length of the notch, L = 2.0 m

Head over notch, $H = 300 \text{ m} = 0.30 \text{ m}$

$C_d = 0.60$

Discharge,
$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\left[H^{3/2}\right]$$
$$= \frac{2}{3} \times 0.6 \times 2.0 \times \sqrt{2 \times 9.81} \times [0.30]^{1.5} \text{ m}^3\text{/s}$$
$$= 3.5435 \times 0.1643 = \mathbf{0.582\ m^3/s.\ Ans.}$$

Problem 8.2 *Determine the height of a rectangular weir of length 6 m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8 m and discharge is 2000 litres/s. Take C_d = 0.6 and neglect end contractions.*

Solution. Given :

Length of weir, $L = 6$ m

Depth of water, $H_1 = 1.8$ m

Discharge, $Q = 2000$ lit/s $= 2 \text{ m}^3$/s

$C_d = 0.6$

Let H is height of water above the crest of weir, and H_2 = height of weir (Fig. 8.2)

The discharge over the weir is given by the equation (8.1) as

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\ H^{3/2}$$

or
$$2.0 = \frac{2}{3} \times 0.6 \times 6.0 \times \sqrt{2 \times 9.81} \times H^{3/2}$$
$$= 10.623\ H^{3/2}$$

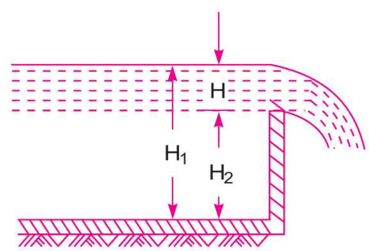

Fig. 8.2

$\therefore$
$$H^{3/2} = \frac{2.0}{10.623}$$

$\therefore$
$$H = \left(\frac{2.0}{10.623}\right)^{2/3} = 0.328 \text{ m}$$

$\therefore$ Height of weir,
$$H_2 = H_1 - H$$
= Depth of water on upstream side – H
= 1.8 – .328 = **1.472 m. Ans.**

Problem 8.3 *The head of water over a rectangular notch is 900 mm. The discharge is 300 litres/s. Find the length of the notch, when C_d = 0.62.*

Solution. Given :

Head over notch, $H = 90 \text{ cm} = 0.9$ m

Discharge, $Q = 300$ lit/s $= 0.3 \text{ m}^3$/s

$C_d = 0.62$

Let length of notch $= L$

Using equation (8.1), we have

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

or $$0.3 = \frac{2}{3} \times 0.62 \times L \times \sqrt{2 \times 9.81} \times (0.9)^{3/2}$$

$$= 1.83 \times L \times 0.8538$$

$\therefore$ $$L = \frac{0.3}{1.83 \times .8538} = .192 \text{ m} = \mathbf{192 \text{ mm. Ans.}}$$

▶ 8.4 DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR

The expression for the discharge over a triangular notch or weir is the same. It is derived as :

Let H = head of water above the V- notch

θ = angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water as shown in Fig. 8.3.

From Fig. 8.3 (b), we have

$$\tan \frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

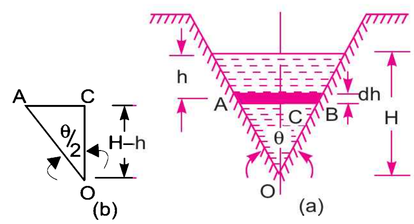

Fig. 8.3 *The triangular notch.*

$\therefore$ $$AC = (H-h) \tan \frac{\theta}{2}$$

Width of strip $$= AB = 2AC = 2\,(H-h) \tan \frac{\theta}{2}$$

$\therefore$ Area of strip $$= 2\,(H-h) \tan \frac{\theta}{2} \times dh$$

The theoretical velocity of water through strip = $\sqrt{2gh}$

$\therefore$ Discharge, through the strip,

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times 2\,(H-h) \tan \frac{\theta}{2} \times dh \times \sqrt{2gh}$$

$$= 2C_d\,(H-h) \tan \frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$\therefore$ Total discharge, $$Q = \int_0^H 2C_d\,(H-h) \tan \frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$$= 2C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \int_0^H (H-h)h^{1/2}\,dh$$

$$= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \int_0^H (Hh^{1/2} - h^{3/2})\,dh$$

$$= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2} \right]_0^H$$

$$= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{2}{3} H.H^{3/2} - \frac{2}{5} H^{5/2} \right]$$

$$= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2} \right]$$

$$= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{4}{15} H^{5/2} \right]$$

$$= \frac{8}{15} C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2} \quad \text{...(8.2)}$$

For a right-angled V-notch, if $C_d = 0.6$

$$\theta = 90°, \quad \therefore \quad \tan \frac{\theta}{2} = 1$$

Discharge,

$$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2} \quad \text{...(8.3)}$$

$$= 1.417\ H^{5/2}.$$

Problem 8.4 *Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m. Assume $C_d = 0.6$.*

Solution. Given :

Angle of V-notch, $\theta = 60°$

Head over notch, $H = 0.3$ m

$C_d = 0.6$

Discharge, Q over a V-notch is given by equation (8.2)

$$Q = \frac{8}{15} \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \tan \frac{60°}{2} \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$= 0.8182 \times 0.0493 =$ **0.040 m³/s. Ans.**

Problem 8.5 *Water flows over a rectangular weir 1 m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Taking C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively, find the depth over the triangular weir.*

Solution. Given :

For rectangular weir, length, $L = 1$ m

Depth of water, $H = 150$ mm $= 0.15$ m

$C_d = 0.62$

For triangular weir, $\theta = 90°$

$C_d = 0.59$

Let depth over triangular weir $= H_1$

The discharge over the rectangular weir is given by equation (8.1) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 1.0 \times \sqrt{2 \times 9.81} \times (.15)^{3/2} \text{ m}^3/\text{s} = 0.10635 \text{ m}^3/\text{s}$$

The same discharge passes through the triangular right-angled weir. But discharge, Q, is given by equation (8.2) for a triangular weir as

$$Q = \frac{8}{15} \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$\therefore \quad 0.10635 = \frac{8}{15} \times .59 \times \tan \frac{90^\circ}{2} \times \sqrt{2g} \times H_1^{5/2} \qquad \{\because \theta = 90^\circ \text{ and } H = H_1\}$$

$$= \frac{8}{15} \times .59 \times 1 \times 4.429 \times H_1^{5/2} = 1.3936\, H_1^{5/2}$$

$$\therefore \quad H_1^{5/2} = \frac{0.10635}{1.3936} = 0.07631$$

$$\therefore \quad H_1 = (.07631)^{0.4} = \mathbf{0.3572 \text{ m. Ans.}}$$

Problem 8.5A *Water flows through a triangular right-angled weir first and then over a rectangular weir of 1 m width. The discharge co-efficients of the triangular and rectangular weirs are 0.6 and 0.7 respectively. If the depth of water over the triangular weir is 360 mm, find the depth of water over the rectangular weir.*

Solution. Given :

For triangular weir : $\theta = 90^\circ$, $C_d = 0.6$, $H = 360$ mm $= 0.36$ m

For rectangular weir : $L = 1$ m, $C_d = 0.7$, $H = ?$

The discharge for a triangular weir is given by equation (8.2) as

$$Q = \frac{8}{15} \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \times \tan \left(\frac{90^\circ}{2}\right) \times \sqrt{2 \times 9.81} \times (0.36)^{5/2} = 0.1102 \text{ m}^3/\text{s}$$

The same discharge is passing through the rectangular weir. But discharge for a rectangular weir is given by equation (8.1) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$\text{or} \quad 0.1102 = \frac{2}{3} \times 0.7 \times 1 \times \sqrt{2 \times 9.81} \times H^{3/2} = 2.067\, H^{3/2}$$

$$\text{or} \quad H^{3/2} = \frac{0.1102}{2.067} = 0.0533$$

$$\therefore \quad H = (0.0533)^{2/3} = 0.1415 \text{ m} = \mathbf{141.5 \text{ mm. Ans.}}$$

Problem 8.6 *A rectangular channel 2.0 m wide has a discharge of 250 litres per second, which is measured by a right-angled V-notch weir. Find the position of the apex of the notch from the bed of the channel if maximum depth of water is not to exceed 1.3 m. Take C_d = 0.62.*

Solution. Given :

Width of rectangular channel, L = 2.0 m

Discharge, Q = 250 lit/s = 0.25 m^3/s

Depth of water in channel = 1.3 m

Let the height of water over V-notch = H

The rate of flow through V-notch is given by equation (8.2) as

$$Q = \frac{8}{15} \times C_d \times \sqrt{2g} \times \tan\frac{\theta}{2} \times H^{5/2}$$

where $C_d = 0.62$, $\theta = 90°$

$$\therefore \quad Q = \frac{8}{15} \times .62 \times \sqrt{2 \times 9.81} \times \tan\frac{90°}{2} \times H^{5/2}$$

or
$$0.25 = \frac{8}{15} \times .62 \times 4.429 \times 1 \times H^{5/2}$$

or
$$H^{5/2} = \frac{.25 \times 15}{8 \times .62 \times 4.429} = 0.1707$$

$$\therefore \quad H = (.1707)^{2/5} = (.1707)^{0.4} = 0.493 \text{ m}$$

Position of apex of the notch from the bed of channel

= depth of water in channel–height of water over V-notch

= 1.3 – .493 = **0.807 m. Ans.**

▶ 8.5 ADVANTAGES OF TRIANGULAR NOTCH OR WEIR OVER RECTANGULAR NOTCH OR WEIR

A triangular notch or weir is preferred to a rectangular weir or notch due to following reasons :

1. The expression for discharge for a right-angled V-notch or weir is very simple.
2. For measuring low discharge, a triangular notch gives more accurate results than a rectangular notch.
3. In case of triangular notch, only one reading, *i.e.*, H is required for the computation of discharge.
4. Ventilation of a triangular notch is not necessary.

▶ 8.6 DISCHARGE OVER A TRAPEZOIDAL NOTCH OR WEIR

As shown in Fig. 8.4, a trapezoidal notch or weir is a combination of a rectangular and triangular notch or weir. Thus the total discharge will be equal to the sum of discharge through a rectangular weir or notch and discharge through a triangular notch or weir.

Let H = Height of water over the notch

L = Length of the crest of the notch

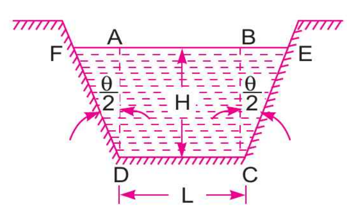

Fig. 8.4 *The trapezoidal notch.*

C_{d_1} = Co-efficient of discharge for rectangular portion *ABCD* of Fig. 8.4.

C_{d_2} = Co-efficient of discharge for triangular portion [*FAD* and *BCE*]

The discharge through rectangular portion *ABCD* is given by (8.1)

or
$$Q_1 = \frac{2}{3} \times C_{d_1} \times L \times \sqrt{2g} \times H^{3/2}$$

The discharge through two triangular notches *FDA* and *BCE* is equal to the discharge through a single triangular notch of angle θ and it is given by equation (8.2) as

$$Q_2 = \frac{8}{15} \times C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

∴ Discharge through trapezoidal notch or weir *FDCEF* = $Q_1 + Q_2$

$$= \frac{2}{3} C_{d_1} L \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\theta/2 \times \sqrt{2g} \times H^{5/2}. \quad ...(8.4)$$

Problem 8.7 *Find the discharge through a trapezoidal notch which is 1 m wide at the top and 0.40 m at the bottom and is 30 cm in height. The head of water on the notch is 20 cm. Assume C_d for rectangular portion = 0.62 while for triangular portion = 0.60.*

Solution. Given :

Top width, $AE = 1$ m

Base width, $CD = L = 0.4$ m

Head of water, $H = 0.20$ m

For rectangular portion, $C_{d_1} = 0.62$

For triangular portion, $C_{d_2} = 0.60$

From ΔABC, we have

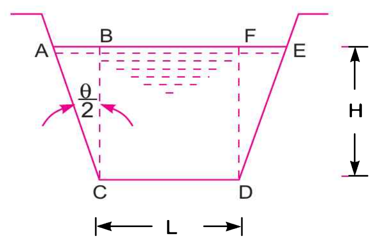

Fig. 8.5

$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{(AE - CD)/2}{H}$$

$$= \frac{(1.0 - 0.4)/2}{0.3} = \frac{0.6/2}{0.3} = \frac{0.3}{0.3} = 1$$

Discharge through trapezoidal notch is given by equation (8.4)

$$Q = \frac{2}{3} C_{d_1} \times L \times \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.4 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2} + \frac{8}{15} \times .60 \times 1 \times \sqrt{2 \times 9.81} \times (0.2)^{5/2}$$

$= 0.06549 + 0.02535 = 0.09084\ m^3/s$ = **90.84 litres/s. Ans.**

▶ 8.7 DISCHARGE OVER A STEPPED NOTCH

A stepped notch is a combination of rectangular notches. The discharge through stepped notch is equal to the sum of the discharges through the different rectangular notches.

Consider a stepped notch as shown in Fig. 8.6.

Let H_1 = Height of water above the crest of notch 1,

L_1 = Length of notch 1,

H_2, L_2 and H_3, L_3 are corresponding values for notches 2 and 3 respectively.

C_d = Co-efficient of discharge for all notches

∴ Total discharge $Q = Q_1 + Q_2 + Q_3$

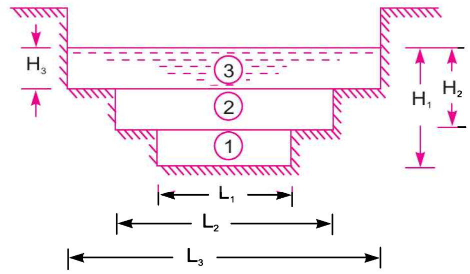

Fig. 8.6 *The stepped notch.*

or
$$Q = \frac{2}{3} \times C_d \times L_1 \times \sqrt{2g}\ [H_1^{3/2} - H_2^{3/2}]$$

$$+ \frac{2}{3}\ C_d \times L_2 \times \sqrt{2g}\,[H_2^{3/2} - H_3^{3/2}] + \frac{2}{3}\ C_d \times L_3 \times \sqrt{2g}\ \times H_3^{3/2}. \quad \text{...(8.5)}$$

Problem 8.8 *Fig. 8.7 shows a stepped notch. Find the discharge through the notch if C_d for all section = 0.62.*

Solution. Given :

L_1 = 40 cm, L_2 = 80 cm,

L_3 = 120 cm

H_1 = 50 + 30 + 15 = 95 cm,

H_2 = 80 cm, H_3 = 50 cm,

C_d = 0.62

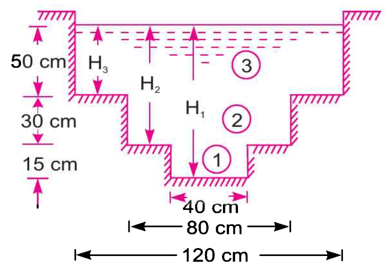

Fig. 8.7

Total discharge, $Q = Q_1 + Q_2 + Q_3$

where
$$Q_1 = \frac{2}{3} \times C_d \times L_1 \times \sqrt{2g}\ [H_1^{3/2} - H_2^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 40 \times \sqrt{2 \times 981} \times [95^{3/2} - 80^{3/2}]$$

$$= 732.26[925.94 - 715.54] = 154067 \text{ cm}^3\text{/s} = 154.067 \text{ lit/s}$$

$$Q_2 = \frac{2}{3} \times C_d \times L_2 \times \sqrt{2g} \times [H_2^{3/2} - H_3^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 80 \times \sqrt{2 \times 981} \times [80^{3/2} - 50^{3/2}]$$

$$= 1464.52[715.54 - 353.55] \text{ cm}^3\text{/s} = 530141 \text{ cm}^3\text{/s} = 530.144 \text{ lit/s}$$

and
$$Q_3 = \frac{2}{3} \times C_d \times L_3 \times \sqrt{2g} \times H_3^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 120 \times \sqrt{2 \times 981} \times 50^{3/2} = 776771 \text{ cm}^3\text{/s} = 776.771 \text{ lit/s}$$

∴
$$Q = Q_1 + Q_2 + Q_3 = 154.067 + 530.144 + 776.771$$

= **1460.98 lit/s. Ans.**

▶ 8.8 EFFECT ON DISCHARGE OVER A NOTCH OR WEIR DUE TO ERROR IN THE MEASUREMENT OF HEAD

For an accurate value of the discharge over a weir or notch, an accurate measurement of head over the weir or notch is very essential as the discharge over a triangular notch is proportional to $H^{5/2}$ and in case of rectangular notch it is proportional to $H^{3/2}$. A small error in the measurement of head, will affect the discharge considerably. The following cases of error in the measurement of head will be considered :

(*i*) For Rectangular Weir or Notch.
(*ii*) For Triangular Weir or Notch.

8.8.1 For Rectangular Weir or Notch. The discharge for a rectangular weir or notch is given by equation (8.1) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$
$$= KH^{3/2} \qquad \text{...}(i)$$

where $K = \frac{2}{3} C_d \times L \times \sqrt{2g}$

Differentiating the above equation, we get

$$dQ = K \times \frac{3}{2} H^{1/2}\, dH \qquad \text{...}(ii)$$

Dividing (*ii*) by (*i*),
$$\frac{dQ}{Q} = \frac{K \times \frac{3}{2} \times H^{1/2} dH}{KH^{3/2}} = \frac{3}{2}\frac{dH}{H} \qquad \text{...(8.6)}$$

Equation (8.6) shows that an error of 1% in measuring H will produce 1.5% error in discharge over a rectangular weir or notch.

8.8.2 For Triangular Weir or Notch. The discharge over a triangular weir or notch is given by equation (8.2) as

$$Q = \frac{8}{15} C_d . \tan\frac{\theta}{2}\sqrt{2g} \times H^{5/2}$$
$$= KH^{5/2} \qquad \text{...}(iii)$$

where $K = \frac{8}{15} C_d . \tan\frac{\theta}{2}\sqrt{2g}$

Differentiating equation (*iii*), we get

$$dQ = K\frac{5}{2} H^{3/2} \times dH \qquad \text{...}(iv)$$

Dividing (*iv*) by (*iii*), we get
$$\frac{dQ}{Q} = \frac{K\frac{5}{2} H^{3/2}\, dH}{KH^{5/2}} = \frac{5}{2}\frac{dH}{H} \qquad \text{...(8.7)}$$

Equation (8.7) shows that an error of 1% in measuring H will produce 2.5% error in discharge over a triangular weir or notch.

Problem 8.9 *A rectangular notch 40 cm long is used for measuring a discharge of 30 litres per second. An error of 1.5 mm was made, while measuring the head over the notch. Calculate the percentage error in the discharge. Take C_d = 0.60.*

Solution. Given :

Length of notch, $L = 40$ cm

Discharge, $Q = 30$ lit/s $= 30000$ cm^3/s

Error in head, $dH = 1.5$ mm $= 0.15$ cm

$C_d = 0.60$

Let the height of water over rectangular notch = H

The discharge through a rectangular notch is given by (8.1)

or $$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

or $$30000 = \frac{2}{3} \times 0.60 \times 40 \times \sqrt{2 \times 981} \times H^{3/2}$$

or $$H^{3/2} = \frac{3 \times 30000}{2 \times .60 \times 40 \times \sqrt{2 \times 981}} = 42.33$$

$\therefore$ $$H = (42.33)^{2/3} = 12.16 \text{ cm}$$

Using equation (8.6), we get

$$\frac{dQ}{Q} = \frac{3}{2}\frac{dH}{H} = \frac{3}{2} \times \frac{0.15}{12.16} = 0.0185 = \mathbf{1.85\%.\ Ans.}$$

Problem 8.10 *A right-angled V-notch is used for measuring a discharge of 30 litres/s. An error of 1.5 mm was made while measuring the head over the notch. Calculate the percentage error in the discharge. Take C_d = 0.62.*

Solution. Given :

Angle of *V*-notch, $\theta = 90°$

Discharge, $Q = 30$ lit/s $= 30000$ cm^3/s

Error in head, $dH = 1.5$ mm $= 0.15$ cm

$C_d = 0.62$

Let the head over the *V*-notch = H

The discharge Q through a triangular notch is given by equation (8.2)

$$Q = \frac{8}{15} C_d \,.\, \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

or $$30000 = \frac{8}{15} \times 0.62 \times \tan\left(\frac{90°}{2}\right) \times \sqrt{2 \times 981} \times H^{5/2}$$

$$= \frac{8}{15} \times .62 \times 1 \times 44.29 \times H^{5/2}$$

$\therefore$ $$H^{5/2} = \frac{30000 \times 15}{8 \times .62 \times 44.29} = 2048.44$$

$\therefore$ $$H = (2048.44)^{2/5} = 21.11 \text{ cm}$$

Using equation (8.7), we get

$$\frac{dQ}{Q} = \frac{5}{2}\frac{dH}{H} = 2.5 \times \frac{0.15}{21.11} = 0.01776 = \mathbf{1.77\%.} \quad \textbf{Ans.}$$

Problem 8.11 *The head of water over a triangular notch of angle 60° is 50 cm and co-efficient of discharge is 0.62. The flow measured by it is to be within an accuracy of 1.5% up or down. Find the limiting values of the head.*

Solution. Given :

Angle of V-notch, $\theta = 60°$

Head of water, $H = 50$ cm

$C_d = 0.62$

$$\frac{dQ}{Q} = \pm\ 1.5\% = \pm\ 0.015$$

The discharge Q over a triangular notch is

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{5/2}$$

$$= \frac{8}{15} \times 0.62 \times \sqrt{2 \times 981} \times \tan\frac{60°}{2} \times (50)^{5/2}$$

$$= 14.64 \times 0.5773 \times 17677.67 = 149405.86 \text{ cm}^3/\text{s}$$

Now applying equation (8.7), we get

$$\frac{dQ}{Q} = \frac{5}{2}\frac{dH}{H} \quad \text{or} \quad \pm .015 = 2.5\frac{dH}{H} \quad \text{or} \quad \frac{dH}{H} = \pm\frac{.015}{2.5}$$

$$\therefore \quad dH = \pm\frac{.015}{2.5} \times H = \pm\frac{.015}{2.5} \times 50 = \pm\ 0.3$$

∴ The limiting values of the head

$= H \pm dH = 50 \pm 0.3 = 50.3$ cm, 49.7 cm

= 50.3 cm and 49.7 cm. Ans.

▶ 8.9. (a) TIME REQUIRED TO EMPTY A RESERVOIR OR A TANK WITH A RECTANGULAR WEIR OR NOTCH

Consider a reservoir or tank of uniform cross-sectional area A. A rectangular weir or notch is provided in one of its sides.

Let L = Length of crest of the weir or notch

C_d = Co-efficient of discharge

H_1 = Initial height of liquid above the crest of notch

H_2 = Final height of liquid above the crest of notch

T = Time required in seconds to lower the height of liquid from H_1 to H_2.

Let at any instant, the height of liquid surface above the crest of weir or notch be h and in a small time dT, let the liquid surface falls by 'dh'. Then,

$$-Adh = Q \times dT$$

–ve sign is taken, as with the increase of T, h decreases.

But $$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times h^{3/2}$$

$\therefore$ $$-Adh = \frac{2}{3} C_d \times L \times \sqrt{2g} \,.\, h^{3/2} \times dT \text{ or } dT = \frac{-Adh}{\frac{2}{3} C_d \times L \times \sqrt{2g} \times h^{3/2}}$$

The total time T is obtained by integrating the above equation between the limits H_1 and H_2.

$$\int_0^T dT = \int_{H_1}^{H_2} \frac{-Adh}{\frac{2}{3} C_d \times L \times \sqrt{2g} \times h^{3/2}}$$

or $$T = \frac{-A}{\frac{2}{3} C_d \times L \times \sqrt{2g}} \int_{H_1}^{H_2} h^{-3/2}\, dh = \frac{-3A}{2\, C_d \times L \times \sqrt{2g}} \left[\frac{h^{-3/2+1}}{-\frac{3}{2}+1} \right]_{H_1}^{H_2}$$

$$= \frac{-3A}{2\, C_d \times L \times \sqrt{2g}} \left[\frac{h^{-1/2}}{-\frac{1}{2}} \right]_{H_1}^{H_2} = \frac{-3A}{2\, C_d \times L \times \sqrt{2g}} \left(-\frac{2}{1} \right) \left[\frac{1}{\sqrt{h}} \right]_{H_1}^{H_2}$$

$$= \frac{3A}{C_d \times L \times \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right]. \qquad \text{...(8.8)}$$

(b) TIME REQUIRED TO EMPTY A RESERVOIR OR A TANK WITH A TRIANGULAR WEIR OR NOTCH

Consider a reservoir or tank of uniform cross-sectional area A, having a triangular weir or notch in one of its sides.

Let θ = Angle of the notch

C_d = Co-efficient of discharge

H_1 = Initial height of liquid above the apex of notch

H_2 = Final height of liquid above the apex of notch

T = Time required in seconds, to lower the height from H_1 to H_2 above the apex of the notch.

Let at any instant, the height of liquid surface above the apex of weir or notch be h and in a small time dT, let the liquid surface falls by 'dh'. Then

$$-Adh = Q \times dT$$

–ve sign is taken, as with the increase of T, h decreases.

And Q for a triangular notch is

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \sqrt{2g} \times h^{5/2}$$

$\therefore$ $$-Adh = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times h^{5/2} \times dT$$

$$\therefore \quad dT = \frac{Adh}{\frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times h^{5/2}}$$

The total time T is obtained by integrating the above equation between the limits H_1 and H_2.

$$\therefore \quad \int_0^T dT = \int_{H_1}^{H_2} \frac{-Adh}{\frac{8}{15} C_d \tan\frac{\theta}{2} \sqrt{2g}\, h^{5/2}}$$

or

$$T = \frac{-A}{\frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} \int_{H_1}^{H_2} h^{-5/2}\, dh$$

$$= \frac{-15A}{8 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} \left[\frac{h^{-3/2}}{-\frac{3}{2}}\right]_{H_1}^{H_2}$$

$$= \frac{-15A}{8 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} \times \left(-\frac{2}{3}\right) \left[\frac{1}{h^{3/2}}\right]_{H_1}^{H_2}$$

$$= \frac{5A}{4 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}} \left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right]. \quad \text{...(8.9)}$$

Problem 8.12 *Find the time required to lower the water level from 3 m to 2 m in a reservoir of dimension 80 m × 80 m, by a rectangular notch of length 1.5 m. Take $C_d = 0.62$.*

Solution. Given :

Initial height of water, $H_1 = 3$ m

Final height of water, $H_2 = 2$ m

Dimension of reservoir = 80 m × 80 m

or Area, $A = 80 \times 80 = 6400 \text{ m}^2$

Length of notch, $L = 1.5$ m, $C_d = 0.62$

Using the relation given by the equation (8.8)

$$T = \frac{3A}{C_d \times L \times \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right]$$

$$= \frac{3 \times 6400}{0.62 \times 1.5 \times \sqrt{2 \times 9.81}} \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right]$$

= 4661.35 [0.7071 – 0.5773] seconds

= 605.04 seconds = **10 min 5 sec. Ans.**

Problem 8.13 *If in problem 8.12, instead of a rectangular notch, a right-angled V-notch is used, find the time required. Take all other data same.*

Solution. Given :

Angle of notch, $\theta = 90°$

Initial height of water, $H_1 = 3$ m

Final height of water, $H_2 = 2$ m

Area of reservoir, $A = 80 \times 80 = 6400 \text{ m}^2$

$C_d = 0.62$

Using the relation given by equation (8.9)

$$T = \frac{5A}{4 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}}\left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right]$$

$$= \frac{5 \times 6400}{4 \times .62 \times \tan\frac{90°}{2} \times \sqrt{2 \times 9.81}}\left[\frac{1}{2^{1.5}} - \frac{1}{3^{1.5}}\right] \qquad \left\{\because \tan\frac{90°}{2} = 1\right\}$$

$$= 2913.34 \times \left[\frac{1}{2.8284} - \frac{1}{5.1961}\right]$$

$= 2913.34$ [0.3535 – 0.1924] seconds

$= 469.33$ seconds = **7 min 49.33 sec. Ans.**

Problem 8.14 *A right-angled V-notch is inserted in the side of a tank of length 4 m and width 2.5 m. Initial height of water above the apex of the notch is 30 cm. Find the height of water above the apex if the time required to lower the head in tank from 30 cm to final height is 3 minutes. Take $C_d = 0.60$.*

Solution. Given :

Angle of notch, $\theta = 90°$

Area of tank, $A = \text{Length} \times \text{width} = 4 \times 2.5 = 10.0 \text{ m}^2$

Initial height of water, $H_1 = 30 \text{ cm} = 0.3$ m

Time, $T = 3 \text{ min} = 3 \times 60 = 180$ seconds

$C_d = 0.60$

Let the final height of water above the apex of notch $= H_2$

Using the relation given by equation (8.9)

$$T = \frac{5A}{4 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}}\left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right]$$

$$180 = \frac{5 \times 10}{4 \times .60 \times \tan\left(\frac{90°}{2}\right) \times \sqrt{2 \times 9.81}}\left[\frac{1}{H_2^{3/2}} - \frac{1}{(0.3)^{3/2}}\right]$$

$$= \frac{50}{4 \times .60 \times 1 \times 4.429}\left[\frac{1}{H_2^{3/2}} - \frac{1}{(0.3)^{3/2}}\right]$$

or
$$\frac{1}{H_2^{1.5}} - \frac{1}{0.3^{1.5}} = \frac{180 \times 4 \times 0.60 \times 4.429}{50} = 38.266.$$

or $$\frac{1}{H_2^{1.5}} - 6.0858 = 38.266$$

$\therefore$ $$\frac{1}{H_2^{1.5}} = 38.266 + 6.0858 = 44.35 \text{ or } {H_2}^{1.5} = \frac{1}{44.35} = 0.0225$$

$\therefore$ $$H_2 = (0.0225)^{1/1.5} = (0.0225)^{.6667} = 0.0822 \text{ m} = \mathbf{8.22\ cm.\ Ans.}$$

▶ 8.10 VELOCITY OF APPROACH

Velocity of approach is defined as the velocity with which the water approaches or reaches the weir or notch before it flows over it. Thus if V_a is the velocity of approach, then an additional head h_a equal to $\frac{V_a^2}{2g}$ due to velocity of approach, is acting on the water flowing over the notch. Then initial height of water over the notch becomes $(H + h_a)$ and final height becomes equal to h_a. Then all the formulae are changed taking into consideration of velocity of approach.

The velocity of approach, V_a is determined by finding the discharge over the notch or weir neglecting velocity of approach. Then dividing the discharge by the cross-sectional area of the channel on the upstream side of the weir or notch, the velocity of approach is obtained. Mathematically,

$$V_a = \frac{Q}{\text{Area of channel}}$$

This velocity of approach is used to find an additional head $\left(h_a = \frac{{V_a}^2}{2g}\right)$. Again the discharge is calculated and above process is repeated for more accurate discharge.

Discharge over a rectangular weir, with velocity of approach

$$= \frac{2}{3} \times C_d \times L \times \sqrt{2g}\ [(H_1 + h_a)^{3/2} - h_a^{3/2}] \qquad \text{...(8.10)}$$

Problem 8.15 *Water is flowing in a rectangular channel of 1 m wide and 0.75 m deep. Find the discharge over a rectangular weir of crest length 60 cm, if the head of water over the crest of weir is 20 cm and water from channel flows over the weir. Take $C_d = 0.62$. Neglect end contractions. Take velocity of approach into consideration.*

Solution. Given :

Area of channel, $A = \text{Width} \times \text{depth} = 1.0 \times 0.75 = 0.75 \text{ m}^2$

Length of weir, $L = 60 \text{ cm} = 0.6 \text{ m}$

Head of water, $H_1 = 20 \text{ cm} = 0.2 \text{ m}$

$C_d = 0.62$

Discharge over a rectangular weir without velocity of approach is given by

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times {H_1}^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.6 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2} \text{ m}^3\text{/s}$$

$$= 1.098 \times 0.0894 = 0.0982 \text{ m}^3/\text{s}$$

Velocity of approach, $V_a = \frac{Q}{A} = \frac{.0982}{0.75} = 0.1309$ m/s

$\therefore$ Additional head, $h_a = \frac{V_a^2}{2g} = (.1309)^2/2 \times 9.81 = .0008733$ m

Then discharge with velocity of approach is given by equation (8.10)

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\ [(H_1 + h_a)^{3/2} - h_a^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 0.6 \times \sqrt{2 \times 9.81}\ [(0.2 + .00087)^{3/2} - (.00087)^{3/2}]$$

$$= 1.098\ [0.09002 - .00002566]$$

$$= 1.098 \times 0.09017 = \mathbf{.09881\ m^3/s.\ Ans.}$$

Problem 8.16 *Find the discharge over a rectangular weir of length 100 m. The head of water over the weir is 1.5 m. The velocity of approach is given as 0.5 m/s. Take $C_d = 0.60$.*

Solution. Given :

Length of weir, $L = 100$ m

Head of water, $H_1 = 1.5$ m

Velocity of approach, $V_a = 0.5$ m/s

$C_d = 0.60$

$\therefore$ Additional head, $h_a = \frac{V_a^2}{2g} = \frac{0.5 \times 0.5}{2 \times 9.81} = 0.0127$ m

The discharge, Q over a rectangular weir due to velocity of approach is given by equation (8.10)

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\ [(H_1 + h_a)^{3/2} - h_a^{3/2}]$$

$$= \frac{2}{3} \times 0.6 \times 100 \times \sqrt{2 \times 9.81}\ [(1.5 + .0127)^{3/2} - .0127^{3/2}]$$

$$= 177.16\ [\ 1.5127^{3/2} - .0127^{3/2}]$$

$$= 177.16\ [1.8605 - .00143] = \mathbf{329.35\ m^3/s.\ Ans.}$$

Problem 8.17 *A rectangular weir of crest length 50 cm is used to measure the rate of flow of water in a rectangular channel of 80 cm wide and 70 cm deep. Determine the discharge in the channel if the water level is 80 mm above the crest of weir. Take velocity of approach into consideration and value of $C_d = 0.62$.*

Solution. Given :

Length of weir, $L = 50$ cm $= 0.5$ m

Area of channel, $A =$ Width $\times$ depth $= 80$ cm $\times$ 70 cm $= 0.80 \times 0.70 = 0.56$ m^2

Head over weir, $H = 80$ mm $= 0.08$ m

$C_d = 0.62$

The discharge over a rectangular weir without velocity of approach is given by equation (8.1)

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.5 \times \sqrt{2 \times 9.81} \times (0.08)^{3/2} \text{ m}^3/\text{s}$$

$$= 0.9153 \times .0226 = .0207 \text{ m}^3/\text{s}$$

Velocity of approach, $V_a = \frac{Q}{A} = \frac{.0207}{0.56} = .0369$ m/s

$\therefore$ Head due to V_a, $h_a = V_a^2/2g = \frac{(.0369)^2}{2 \times 9.81} = .0000697$ m

Discharge with velocity of approach is

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \; [(H_1 + h_a)^{3/2} - h_a^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 0.5 \times \sqrt{2 \times 9.81} \; [(.08 + .0000697)^{3/2} - .0000697^{3/2}]$$

$$= 0.9153 \times [.0800697^{1.5} - .0000697^{1.5}]$$

$$= .9153 \; [.02265 - .000000582] = \mathbf{0.2073 \text{ m}^3/\text{s}.} \textbf{ Ans.}$$

Problem 8.18 *A suppressed rectangular weir is constructed across a channel of 0.77 m width with a head of 0.39 m and the crest 0.6 m above the bed of the channel. Estimate the discharge over it. Consider velocity of approach and assume C_d = 0.623.*

Solution. Given :

Width of channel, $b = 0.77$ m

Head over weir, $H = 0.39$ m

Height of crest from bed of channel = 0.6 m

$\therefore$ Depth of channel $= 0.6 + 0.39 = 0.99$

Value of $C_d = 0.623$

Suppressed weir means that the width of channel is equal to width of weir *i.e.*, there is no end contraction.

$\therefore$ Width of channel = Width of weir = 0.77 m

Now area of channel, A = Width of channel $\times$ Depth of channel

$= 0.77 \times 0.99$

The discharge over a rectangular weir without velocity of approach is given by equation (8.1).

$$\therefore \quad Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g} \times H^{3/2} \qquad (\because \text{ Here } b = L)$$

$$= \frac{2}{3} \times 0.623 \times 0.77 \times \sqrt{2 \times 9.81} \times 0.39^{3/2} = 0.345 \text{ m}^3/\text{s}$$

Now velocity of approach, $V_a = \frac{Q}{\text{Area of channel}} = \frac{0.345}{0.77 \times 0.99} = 0.4526$ m/s

Head due to velocity of approach,

$$h_a = \frac{V_a^2}{2g} = \frac{0.4526^2}{2 \times 9.81} = 0.0104 \text{ m}$$

Now the discharge with velocity of approach is given by,

$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g}\ [(H + h_a)^{3/2} - h_a^{3/2}]$$

$$= \frac{2}{3} \times 0.623 \times 0.77 \times \sqrt{2 \times 9.81}\ [(0.39 + 0.0104)^{3/2} - (0.0104)^{3/2}]$$

$$= \frac{2}{3} \times 0.623 \times 0.77 \times 4.43\ [0.2533 - 0.00106]$$

$$= \mathbf{0.3573\ m^3/s.\ Ans.}$$

Problem 8.19 *A sharp crested rectangular weir of 1 m height extends across a rectangular channel of 3 m width. If the head of water over the weir is 0.45 m, calculate the discharge. Consider velocity of approach and assume $C_d = 0.623$.*

Solution. Given :

Width of channel, $b = 3$ m

Height of weir $= 1$ m

Head of water over weir, $H = 0.45$ m

∴ Depth of channel = Height of weir + Head of water over weir
= 1 + 0.45 = 1.45 m

Value of $C_d = 0.623$

The discharge over a rectangular weir without velocity of approach is given by equation (8.1) as

$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.623 \times 3 \times \sqrt{2 \times 9.81} \times 0.45^{3/2} = 1.665 \text{ m}^3/\text{s}$$

Now velocity of approach is given by

$$V_a = \frac{Q}{\text{Area of channel}}$$

$$= \frac{1.665}{\text{Width of channel} \times \text{Depth of channel}} = \frac{1.665}{3 \times 1.45} = 0.382 \text{ m/s}$$

Head due to velocity of approach is given by,

$$h_a = \frac{V_a^2}{2g} = \frac{0.382^2}{2 \times 9.81} = 0.0074 \text{ m}$$

Now the discharge with velocity of approach is given by,

$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g}\ [(H + h_a)^{3/2} - (h_a)^{3/2}]$$

$$= \frac{2}{3} \times 0.623 \times 3 \times \sqrt{2 \times 9.81}\ [(0.45 + 0.0074)^{3/2} - (0.0074)^{3/2}]$$

$$= \mathbf{1.703\ m^3/s.\ Ans.}$$

▶ 8.11 EMPIRICAL FORMULAE FOR DISCHARGE OVER RECTANGULAR WEIR

The discharge over a rectangular weir is given by

$$Q = \frac{2}{3} C_d \sqrt{2g} \times L \times [H^{3/2}] \text{ without velocity of approach} \quad ...(i)$$

$$= \frac{2}{3} C_d \sqrt{2g} \times L \times [(H + h_a)^{3/2} - h_a^{3/2}] \text{ with velocity of approach} \quad ...(ii)$$

Equations (*i*) and (*ii*) are applicable to the weir or notch for which the crest length is equal to the width of the channel. This type of weir is called *Suppressed weir*. But if the weir is not suppressed, the effect of end contraction will be taken into account.

(*a*) **Francis's Formula.** Francis on the basis of his experiments established that end contraction decreases the effective length of the crest of weir and hence decreases the discharge. Each end contraction reduces the crest length by $0.1 \times H$, where H is the head over the weir. For a rectangular weir there are two end contractions only and hence effective length

$$L = (L - 0.2\,H)$$

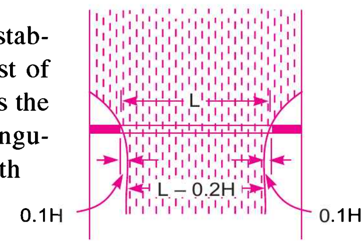

Fig. 8.8

and

$$Q = \frac{2}{3} \times C_d \times [L - 0.2 \times H] \times \sqrt{2g}\; H^{3/2}$$

If $C_d = 0.623$, $g = 9.81$ m/s^2, then

$$Q = \frac{2}{3} \times .623 \times \sqrt{2 \times 9.81} \times [L - 0.2 \times H] \times H^{3/2}$$

$$= 1.84\,[L - 0.2 \times H] H^{3/2} \quad ...(8.11)$$

If end contractions are suppressed, then

$$H = 1.84\,LH^{3/2} \quad ...(8.12)$$

If velocity of approach is considered, then

$$Q = 1.84\,L\,[(H + h_a)^{3/2} - h_a^{3/2}] \quad ...(8.13)$$

(*b*) **Bazin's Formula.** On the basis of results of a series of experiments, Bazin's proposed the following formula for the discharge over a rectangular weir as

$$Q = m \times L \times \sqrt{2g} \times H^{3/2} \quad ...(8.14)$$

where $m = \frac{2}{3} \times C_d = 0.405 + \frac{.003}{H}$

H = height of water over the weir

If velocity of approach is considered, then

$$Q = m_1 \times L \times \sqrt{2g}\; [(H + h_a)^{3/2}] \quad ...(8.15)$$

where $m_1 = 0.405 + \frac{.003}{(H + h_a)}$.

Problem 8.20 *The head of water over a rectangular weir is 40 cm. The length of the crest of the weir with end contraction suppressed is 1.5 m. Find the discharge using the following formulae : (i) Francis's Formula and (ii) Bazin's Formula.*

Solution. Given :

Head of water, $H = 40 \text{ cm} = 0.40 \text{ m}$

Length of weir, $L = 1.5 \text{ m}$

(*i*) Francis's Formula for end contraction suppressed is given by equation (8.12).

$$Q = 1.84\, L \times H^{3/2} = 1.84 \times 1.5 \times (.40)^{3/2}$$
$$= 0.6982 \text{ m}^3/\text{s}$$

(*ii*) Bazin's Formula is given by equation (8.14)

$$Q = m \times L \times \sqrt{2g} \times H^{3/2}$$

where $m = 0.405 + \dfrac{.003}{H} = 0.405 + \dfrac{.003}{.40} = 0.4125$

$$\therefore \quad Q = .4125 \times 1.5 \times \sqrt{2 \times 9.81} \times (.4)^{3/2}$$
$$= \mathbf{0.6932 \text{ m}^3/\text{s}.\ Ans.}$$

Problem 8.21 *A weir 36 metres long is divided into 12 equal bays by vertical posts, each 60 cm wide. Determine the discharge over the weir if the head over the crest is 1.20 m and velocity of approach is 2 metres per second.*

Solution. Given :

Length of weir, $L_1 = 36 \text{ m}$

Number of bays, $= 12$

For 12 bays, no. of vertical post $= 11$

Width of each post $= 60 \text{ cm} = 0.6 \text{ m}$

∴ Effective length, $L = L_1 - 11 \times 0.6 = 36 - 6.6 = 29.4 \text{ m}$

Head on weir, $H = 1.20 \text{ m}$

Velocity of approach, $V_a = 2 \text{ m/s}$

∴ Head due to V_a, $h_a = \dfrac{V_a^2}{2g} = \dfrac{2^2}{2 \times 9.81} = 0.2038 \text{ m}$

Number of end contraction, $n = 2 \times 12 = 24$ {Each bay has two end contractions}

∴ Discharge by Francis Formula with end contraction and velocity of approach is

$$Q = 1.84\,[L - 0.1 \times n(H + h_a)][(H + h_a)^{3/2} - h_a^{3/2}]$$
$$= 1.84[29.4 - 0.1 \times 24(1.20 + .2038)] \times [(1.2 + .2038)^{1.5} - .2038^{1.5}]$$
$$= 1.84[29.4 - 3.369][1.663 - .092]$$
$$= \mathbf{75.246 \text{ m}^3/\text{s}.\ Ans.}$$

Problem 8.22 *A discharge of 2000 m^3/s is to pass over a rectangular weir. The weir is divided into a number of openings each of span 10 m. If the velocity of approach is 4 m/s, find the number of openings needed in order the head of water over the crest is not to exceed 2 m.*

Solution. Given :

Total discharge, $Q = 2000 \text{ m}^3/\text{s}$

Length of each opening, $L = 10$

Velocity of approach, $V_a = 4$ m/s

Head over weir, $H = 2$ m

Let number of openings $= N$

Head due to velocity of approach,

$$h_a = \frac{V_a^2}{2g} = \frac{4 \times 4}{2 \times 9.81} = 0.8155 \text{ m}$$

For each opening, number of end contractions are two. Hence discharge for each opening considering velocity of approach is given by Francis Formula

i.e., $$Q = 1.84[L - 0.1 \times 2 \times (H + h_a)][(H + h_a)^{3/2} - h_a^{3/2}]$$
$$= 1.84[10.0 - 0.2 \times (2 + .8155)][2.8155^{1.5} - .8155^{1.5}]$$
$$= 17.363[4.7242 - 0.7364] = 69.24 \text{ m}^3\text{/s}$$

$\therefore$ Number of opening $$= \frac{\text{Total discharge}}{\text{Discharge for one opening}} = \frac{2000}{69.24}$$
$$= 28.88 \text{ (say 29)} = \mathbf{29.\ Ans.}$$

▶ 8.12 CIPOLLETTI WEIR OR NOTCH

Cipolletti weir is a trapezoidal weir, which has side slopes of 1 horizontal to 4 vertical as shown in Fig. 8.9. Thus in ΔABC,

$$\tan \frac{\theta}{2} = \frac{AB}{BC} = \frac{H/4}{H} = \frac{1}{4}$$

$$\therefore \qquad \frac{\theta}{2} = \tan \frac{1}{4} = 14° \, 2'.$$

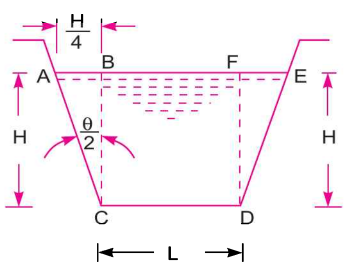

Fig. 8.9 *The cipolletti weir.*

By giving this slope to the sides, an increase in discharge through the triangular portions *ABC* and *DEF* of the weir is obtained. If this slope is not provided the weir would be a rectangular one, and due to end contraction, the discharge would decrease. Thus in case of cipolletti weir, the factor of end contraction is not required which is shown below.

The discharge through a rectangular weir with two end contractions is

$$Q = \frac{2}{3} \times C_d \times (L - 0.2\,H)\,\sqrt{2g} \times H^{3/2}$$
$$= \frac{2}{3} \times C_d \times L \times \sqrt{2g}\; H^{3/2} - \frac{2}{15} \times C_d \times \sqrt{2g} \times H^{5/2}$$

Thus due to end contraction, the discharge decreases by $\frac{2}{15} \times C_d \times \sqrt{2g} \times H^{5/2}$. This decrease in discharge can be compensated by giving such a slope to the sides that the discharge through two triangular portions is equal to $\frac{2}{15} \times C_d \times \sqrt{2g} \times H^{5/2}$. Let the slope is given by $\theta/2$. The discharge through a *V*-notch of angle θ is given by

$$= \frac{8}{15} \times C_d \times \sqrt{2g} \times \tan \frac{\theta}{2}\, H^{5/2}$$

Thus $$\frac{8}{15} \times C_d \times \sqrt{2g} \times \tan \frac{\theta}{2} H^{5/2} = \frac{2}{15} \times C_d \times \sqrt{2g} \times H^{5/2}$$

$\therefore$ $$\tan \frac{\theta}{2} = \frac{2}{15} \times \frac{15}{8} = \frac{1}{4} \quad \text{or} \quad \theta/2 = \tan^{-1} \frac{1}{4} = 14° \, 2'.$$

Thus discharge through cipolletti weir is

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \; H^{3/2} \qquad \text{...(8.16)}$$

If velocity of approach, V_a is to be taken into consideration,

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \; [(H + h_a)^{3/2} - h_a^{3/2}] \qquad \text{...(8.17)}$$

Problem 8.23 *Find the discharge over a cipolletti weir of length 2.0 m when the head over the weir is 1 m. Take $C_d = 0.62$.*

Solution. Given :

Length of weir, $L = 20$ m

Head over weir, $H = 1.0$ m

$C_d = 0.62$

Using equation (8.16), the discharge is given as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 2.0 \times \sqrt{2 \times 9.81} \times (1)^{3/2} = \mathbf{3.661 \; m^3/s. \; Ans.}$$

Problem 8.24 *A cipolletti weir of crest length 60 cm discharges water. The head of water over the weir is 360 mm. Find the discharge over the weir if the channel is 80 cm wide and 50 cm deep. Take $C_d = 0.60$.*

Solution. Given :

$C_d = 0.60$

Length of weir, $L = 60$ cm $= 0.60$ m

Head of water, $H = 360$ mm $= 0.36$ m

Channel width $= 80$ cm $= 0.80$ m

Channel depth $= 50$ cm $= 0.50$ m

A = cross-sectional area of channel $= 0.8 \times 0.5 = 0.4$ m^2

To find velocity of approach, first determine discharge over the weir as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

The velocity of approach, $V_a = \dfrac{Q}{A}$

$\therefore$ $$Q = \frac{2}{3} \times 0.60 \times 0.60 \times \sqrt{2 \times 9.81} \times (0.36)^{3/2} \text{ m}^3/\text{s} = 0.2296 \text{ m}^3/\text{s}$$

$\therefore$ $$V_a = \frac{.2296}{0.40} = 0.574 \text{ m/s}$$

Head due to velocity of approach,

$$h_a = V_a^2/2g = \frac{(0.574)^2}{2 \times 9.81} = 0.0168 \text{ m}$$

Thus the discharge is given by equation (8.17) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\ [(H + h_a)^{1.5} - h_a^{1.5}]$$

$$= \frac{2}{3} \times 0.60 \times .6 \times \sqrt{2 \times 9.81}\ [(.36 + .0168)^{1.5} - (.0168)^{1.5}]$$

$$= 1.06296 \times [.2313 - .002177] = \mathbf{0.2435\ m^3/s.\ Ans.}$$

▶ 8.13 DISCHARGE OVER A BROAD-CRESTED WEIR

A weir having a wide crest is known as broad-crested weir.

Let H = height of water above the crest

L = length of the crest

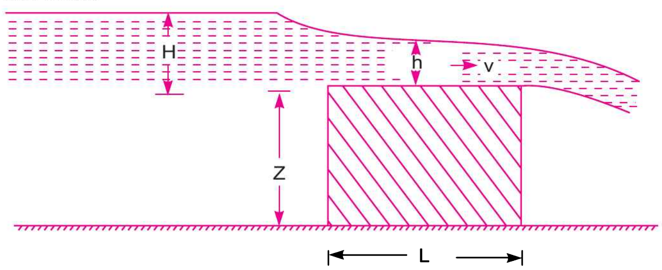

Fig. 8.10 *Broad-crested weir.*

If $2L > H$, the weir is called broad-crested weir

If $2L < H$, the weir is called a narrow-crested weir

Fig. 8.10 shows a broad-crested weir.

Let h = head of water at the middle of weir which is constant

v = velocity of flow over the weir

Applying Bernoulli's equation to the still water surface on the upstream side and running water at the end of weir,

$$0 + 0 + H = 0 + \frac{v^2}{2g} + h$$

$$\therefore \quad \frac{v^2}{2g} = H - h$$

$$\therefore \quad v = \sqrt{2g(H - h)}$$

∴ The discharge over weir $Q = C_d \times$ Area of flow $\times$ Velocity

$$= C_d \times L \times h \times \sqrt{2g(H - h)}$$

$$= C_d \times L \times \sqrt{2g\left(Hh^2 - h^3\right)} \qquad \text{...(8.18)}$$

The discharge will be maximum, if $(Hh^2 - h^3)$ is maximum

or $$\frac{d}{dh}(Hh^2 - h^3) = 0 \text{ or } 2h \times H - 3h^2 = 0 \text{ or } 2H = 3h$$

$$\therefore \qquad h = \frac{2}{3}H$$

Q_{max} will be obtained by substituting this value of h in equation (8.18) as

$$Q_{max} = C_d \times L \times \sqrt{2g\left[H \times \left(\frac{2}{3}H\right)^2 - \left(\frac{2}{3}H\right)^3\right]}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{H \times \frac{4}{9} \times H^2 - \frac{8}{27}H^3}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{\frac{4}{9}H^3 - \frac{8}{27}H^3} = C_d \times L \times \sqrt{2g}\sqrt{\frac{(12-8)H^3}{27}}$$

$$= C_d \times L \times \sqrt{2g}\sqrt{\frac{4}{27}H^3} = C_d \times L \times \sqrt{2g} \times 0.3849 \times H^{3/2}$$

$$= .3849 \times \sqrt{2 \times 9.81} \times C_d \times L \times H^{3/2} = 1.7047 \times C_d \times L \times H^{3/2}$$

$$= 1.705 \times C_d \times L \times H^{3/2}. \qquad \text{...(8.19)}$$

▶ 8.14 DISCHARGE OVER A NARROW-CRESTED WEIR

For a narrow-crested weir, $2L < H$. It is similar to a rectangular weir or notch hence, Q is given by

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2} \qquad \text{...(8.20)}$$

▶ 8.15 DISCHARGE OVER AN OGEE WEIR

Fig. 8.11 shows an Ogee weir, in which the crest of the weir rises upto maximum height of $0.115 \times H$ (where H is the height of water above inlet of the weir) and then falls as shown in Fig. 8.11. The discharge for an Ogee weir is the same as that of a rectangular weir, and it is given by

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2} \qquad \text{...(8.21)}$$

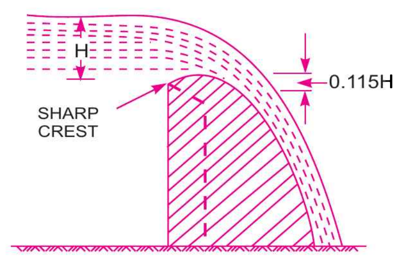

Fig. 8.11 *An Ogee weir.*

▶ 8.16 DISCHARGE OVER SUB-MERGED OR DROWNED WEIR

When the water level on the downstream side of a weir is above the crest of the weir, then the weir is called to be a sub-merged or drowned weir. Fig. 8.12 shows a sub-merged weir. The total discharge, over the weir is obtained by dividing the weir into two parts. The portion between upstream and downstream water surface may be treated as free weir and portion between downstream water surface and crest of weir as a drowned weir.

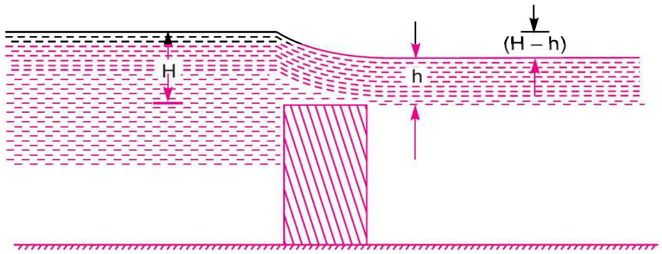

Fig. 8.12 *Sub-merged weir.*

Let H = height of water on the upstream side of the weir

h = height of water on the downstream side of the weir

Then Q_1 = discharge over upper portion

$$= \frac{2}{3} \times C_{d_1} \times L \times \sqrt{2g}\ [H - h]^{3/2}$$

Q_2 = discharge through drowned portion

$$= C_{d_2} \times \text{Area of flow} \times \text{Velocity of flow}$$

$$= C_{d_2} \times L \times h \times \sqrt{2g(H - h)}$$

∴ Total discharge, $Q = Q_1 + Q_2$

$$= \frac{2}{3} C_{d_1} \times L \times \sqrt{2g}\ [H - h]^{3/2} + C_{d_2} \times L \times h \times \sqrt{2g(H - h)}\ . \quad ...(8.22)$$

Problem 8.25 *(a) A broad-crested weir of 50 m length, has 50 cm height of water above its crest. Find the maximum discharge. Take C_d = 0.60. Neglect velocity of approach. (b) If the velocity of approach is to be taken into consideration, find the maximum discharge when the channel has a cross-sectional area of 50 m² on the upstream side.*

Solution. Given :

Length of weir, $L = 50$ m

Head of water, $H = 50$ cm $= 0.5$ m

$C_d = 0.60$

(*i*) **Neglecting velocity of approach.** Maximum discharge is given by equation (8.19) as

$$Q_{max} = 1.705 \times C_d \times L \times H^{3/2}$$

$$= 1.705 \times 0.60 \times 50 \times (.5)^{3/2} = \mathbf{18.084\ m^3/s.\ Ans.}$$

(*ii*) **Taking velocity of approach into consideration**

Area of channel, $A = 50\ m^2$

Velocity of approach, $V_a = \frac{Q}{A} = \frac{18.084}{50} = 0.36$ m/s

∴ Head due to V_a, $h_a = \frac{V_a^2}{2g} = \frac{0.36 \times .36}{2 \times 9.81} = .0066$ m

Maximum discharge, Q_{max} is given by

$$Q_{max} = 1.705 \times C_d \times L \times [(H + h_a)^{3/2} - h_a^{3/2}]$$

$$= 1.705 \times 0.6 \times 50 \times [(.50 + .0066)^{1.5} - (.0066)^{1.5}]$$

$$= 51.15[0.3605 - .000536] = \mathbf{18.412\ m^3/s.\ Ans.}$$

Problem 8.26 *An Ogee weir 5 metres long has a head of 40 cm of water. If C_d = 0.6, find the discharge over the weir.*

Solution. Given :

Length of weir, $L = 5$ m

Head of water, $H = 40$ cm $= 0.40$ m

$C_d = 0.6$

Discharge over Ogee weir is given by equation (8.21) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.60 \times 5.0 \times \sqrt{2 \times 9.81} \times (0.4)^{3/2} = \mathbf{2.2409\ m^3/s.\ Ans.}$$

Problem 8.27 *The heights of water on the upstream and downstream side of a sub-merged weir of 3 m length are 20 cm and 10 cm respectively. If C_d for free and drowned portions are 0.6 and 0.8 respectively, find the discharge over the weir.*

Solution. Given :

Height of water on upstream side, $H = 20$ cm $= 0.20$ m

Height of water on downstream side, $h = 10$ cm $= 0.10$ m

Length of weir, $L = 3$ m

$C_{d_1} = 0.6$

$C_{d_2} = 0.8$

Total discharge Q is the sum of discharge through free portion and discharge through the drowned portion. This is given by equation (8.22) as

$$Q = \frac{2}{3} \times C_{d_1} \times L \times \sqrt{2g}\ [H - h]^{3/2} + C_{d_2} \times L \times h \times \sqrt{2g(H - h)}$$

$$= \frac{2}{3} \times 0.6 \times 3 \times \sqrt{2 \times 9.81}\ [.20 - .10]^{1.5} + 0.8 \times 3 \times .10 \times \sqrt{2 \times 9.81(.2 - .1)}$$

$$= 0.168 + 0.336 = \mathbf{0.504\ m^3/s.\ Ans.}$$

HIGHLIGHTS

1. A notch is a device used for measuring the rate of flow of a liquid through a small channel. A weir is a concrete or masonary structure placed in the open channel over which the flow occurs.

2. The discharge through a rectangular notch or weir is given by

$$Q = \frac{2}{3}\ C_d \times L \times H^{3/2}$$

where C_d = Co-efficient of discharge,
L = Length of notch or weir,
H = Head of water over the notch or weir.

3. The discharge over a triangular notch or weir is given by

$$Q = \frac{8}{15}\ C_d \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

where θ = total angle of triangular notch.

4. The discharge through a trapezoidal notch or weir is equal to the sum of discharge through a rectangular notch and the discharge through a triangular notch. It is given as

$$Q = \frac{2}{3} C_{d_1} \times L \times \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

where C_{d_1} = co-efficient of discharge for rectangular notch,

C_{d_2} = co-efficient of discharge for triangular notch,

$\theta/2$ = slope of the side of trapezoidal notch.

5. The error in discharge due to the error in the measurement of head over a rectangular and triangular notch or weir is given by

$$\frac{dQ}{Q} = \frac{3}{2}\frac{dH}{H} \quad \text{... For a rectangular weir or notch}$$

$$= \frac{5}{2}\frac{dH}{H} \quad \text{... For a triangular weir or notch}$$

where Q = discharge through rectangular or triangular notch or weir

H = head over the notch or weir.

6. The time required to empty a reservoir or a tank by a rectangular or a triangular notch is given by

$$H = \frac{3A}{C_d L \sqrt{2g}}\left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right] \quad \text{... By a rectangular notch}$$

$$= \frac{5A}{4C_d \tan\frac{\theta}{2} \times \sqrt{2g}}\left[\frac{1}{H_2^{3/2}} - \frac{1}{H_1^{3/2}}\right] \quad \text{... By a triangular notch}$$

where A = cross-sectional area of a tank or a reservoir

H_1 = initial height of liquid above the crest or apex of notch

H_2 = final height of liquid above the crest or apex of notch.

7. The velocity with which the water approaches the weir or notch is called the velocity of approach. It is denoted by V_a and is given by

$$V_a = \frac{\text{Discharge over the notch or weir}}{\text{Cross-sectional area of channel}}.$$

8. The head due to velocity of approach is given by $h_a = \frac{V_a^2}{2g}$.

9. Discharge over a rectangular weir, with velocity of approach,

$$Q = \frac{2}{3} C_d L \sqrt{2g}\,[(H_1 + h_a)^{3/2} - h_a^{3/2}].$$

10. Francis's Formula for a rectangular weir is given by

$$Q = 1.84[L - 0.2\,H]\,H^{3/2} \quad \text{... For two end contractions}$$

$$= 1.84\,L\,H^{3/2} \quad \text{... If end contractions are suppressed}$$

$$= 1.84\,L\,[(H + h_a)^{3/2} - h_a^{3/2}] \quad \text{... If velocity of approach is considered}$$

where L = length of weir,

H = height of water above the crest of the weir,

h_a = head due to velocity of approach.

11. Bazin's Formula for discharge over a rectangular weir,

$$Q = m\,L\,\sqrt{2g}H^{3/2} \quad \text{... without velocity of approach}$$

$$= m\,L\,\sqrt{2g}\,[(H + h_a)^{3/2}] \quad \text{... with velocity of approach}$$

where $m = \frac{2}{3}\,C_d = 0.405 + \frac{.003}{H}$ ... without velocity of approach

$$= 0.405 + \frac{.003}{(H + h_a)} \quad \text{... with velocity of approach.}$$

12. *A* trapezoidal weir, with side slope or 1 horizontal to 4 vertical, is called Cipolletti weir. The discharge through Cipolletti weir is given by

$$Q = \frac{2}{3}\,C_d \times L \times \sqrt{2g}\,H^{3/2} \quad \text{... without velocity of approach}$$

$$= \frac{2}{3}\,C_d \times L \times \sqrt{2g}\,[(H + h_a)^{3/2} - h_a^{3/2}] \quad \text{... with velocity of approach.}$$

13. The discharge over a broad-crested weir is given by,

$$Q = C_d L \sqrt{2g\left(Hh^2 - h^3\right)}$$

where H = height of water above the crest
h = head of water at the middle of the weir which is constant
L = length of the weir.

14. The condition for maximum discharge over a broad-crested weir is $h = \frac{2}{3}\,H$

and maximum discharge is given by $Q_{max} = 1.705\,C_d\,L\,H^{3/2}$.

15. The discharge over an Ogee weir is given by $Q = \frac{2}{3}\,C_d L \times \sqrt{2g} \times H^{3/2}$.

16. The discharge over sub-merged or drowned weir is given by

Q = discharge over upper portion + discharge through downed portion

$$= \frac{2}{3} C_{d_1}\,L \times \sqrt{2g}\,(H - h)^{3/2} + C_{d_2}\,Lh \times \sqrt{2g\left(H - h\right)}$$

where H = height of water on the upstream side of the weir,
h = height of water on the downstream side of the weir.

EXERCISE

(A) THEORETICAL PROBLEMS

1. Define the terms : notch, weir, nappe and crest.
2. How are the weirs and notches classified ?
3. Find an expression for the discharge over a rectangular weir in terms of head of water over the crest of the weir.
4. Prove that the discharge through a triangular notch or weir is given by

$$Q = \frac{8}{15}\,C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\,H^{3/2}$$

where H = head of water over the notch or weir
θ = angle of notch or weir.

5. What are the advantages of triangular notch or weir over rectangular notch ?
6. Prove that the error in discharge due to the error in the measurement of head over a rectangular notch is given by

$$\frac{dQ}{Q} = \frac{3}{2}\frac{dH}{H}$$

where Q = discharge through rectangular notch
and H = head over the rectangular notch.
7. Find an expression for the time required to empty a tank of area of cross-section A, with a rectangular notch.
8. What do you understand by 'Velocity of Approach' ? Find an expression for the discharge over a rectangular weir with velocity of approach.
9. Define 'end contraction' of a weir. What is the effect of end contraction on the discharge through a weir ?
10. What is a Cipolletti Weir ? Prove that the discharge through Cipolletti weir is given by

$$Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{3/2}$$

where L = length of weir, and H = head of water over weir.
11. Differentiate between Broad-crested weir and Narrow-crested weir. Find the condition for maximum discharge over a Broad-crested weir and hence derive an expression for maximum discharge over a broad-crested weir.
12. What do you mean by a drowned weir ? How will you determine the discharge for the downed weir ?
13. Discuss 'end contraction' of a weir.
14. State the different devices that can be used to measure the discharge through a pipe also through an open channel. Describe one of such devices with a neat sketch and explain how one can obtain the actual discharge with its help.
15. What is the difference between a notch and a weir ?
16. Define velocity of approach. How does the velocity of approach affect the discharge over a weir ?

(B) NUMERICAL PROBLEMS

1. Find the discharge of water flowing over rectangular notch of 3 m length when the constant head of water over the notch is 40 cm. Take $C_d = 0.6$. [**Ans.** 1.344 m^3/s]
2. Determine the height of a rectangular weir of length 5 m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.5 m and discharge is 1.5 m^3 per second. Take $C_d = 0.6$ and neglect end contractions. [**Ans.** 1.194 m]
3. Find the discharge over a triangular notch of angle 60° when the head over the triangular notch is 0.20 m. Take $C_d = 0.6$. [**Ans.** 0.0164 m^3/s]
4. A rectangular channel 1.5 m wide has a discharge of 200 litres per second, which is measured by a right-angled V-notch weir. Find the position of the apex of the notch from the bed of the channel if maximum depth of water is not be exceed 1 m. Take $C_d = 0.62$. [**Ans.** .549 m]
5. Find the discharge through a trapezoidal notch which is 1.2 m wide at the top and 0.50 m at the bottom and is 40 cm in height. The head of water on the notch is 30 cm. Assume C_d for rectangular portion as 0.62 while for triangular portion = 0.60. [**Ans.** 0.22 m^3/s]
6. A rectangular notch 50 cm long is used for measuring a discharge of 40 litres per second. An error of 2 mm was made in measuring the head over the notch. Calculate the percentage error in the discharge. Take $C_d = 0.6$. [**Ans.** 2.37%]
7. A right-angled V-notch is used for measuring a discharge of 30 litres/s. An error of 2 mm was made in measuring the head over the notch. Calculate the percentage error in the discharge. Take $C_d = 0.62$. [**Ans.** 2.37%]

8. Find the time required to lower the water level from 3 m to 1.5 m in a reservoir of dimension 70 m × 70 m, by a rectangular notch of length 2.0 m. Take $C_d = 0.60$. [**Ans.** 11 min 1 s]

9. If in the problem 8, instead of a rectangular notch, a right angled *V*-notch is used, find the time required. Take all other data same. [**Ans.** 13 min 31 s]

10. Water is flowing in a rectangular channel of 1.2 m wide and 0.8 m deep. Find the discharge over a rectangular weir of crest length 70 cm if the head of water over the crest of weir is 25 cm and water from channel flows over the weir. Take $C_d = 0.60$. Neglect end contractions but consider velocity of approach. [**Ans.** 0.1557 m^3/s]

11. Find the discharge over a rectangular weir of length 80 m. The head of water over the weir is 1.2 m. The velocity of approach is given as 1.5 m/s. Take $C_d = 3.6$. [**Ans.** 208.11 m^3/s]

12. The head of water over a rectangular weir is 50 cm. The length of the crest of the weir with end contraction suppressed is 1.4 m. Find the discharge using following formulae : (*i*) Francis's Formula and (*ii*) Bazin's Formula. [**Ans.** (*i*) 0.91 m^3/s, (*ii*) .901 m^3/s]

13. A discharge of 1500 m^3/s is to pass over a rectangular weir. The weir is divided into a number of openings each of span 7.5 m. If the velocity of approach is 3 m/s, find the number of openings needed in order the head of water over the crest is not to exceed 1.8. [**Ans.** 37.5 say 38]

14. Find the discharge over a cipolletti weir of length 1.8 m when the head over the weir is 1.2 m. Take $C_d = 0.62$ [**Ans.** 4.331 m^3/s]

15. (*a*) A broad-crested weir of length 40 m, has 400 mm height of water above its crest. Find the maximum discharge. Take $C_d = 0.6$. Neglect velocity of approach. [**Ans.** 10.352 m^3/s]

(*b*) If the velocity of approach is to be taken into consideration, find the maximum discharge when the channel has a cross-sectional area of 40 m^2 on the upstream side. [**Ans.** 10.475 m^3/s]

16. An Ogee weir 4 m long has a head of 500 mm of water. If $C_d = 0.6$, find the discharge over the weir. [**Ans.** 2.505 m^3/s]

17. The heights of water on the upstream and downstream side of a sub-merged weir of length 3.5 m are 300 mm and 150 mm respectively. If C_d for free and drowned portion are 0.6 and 0.8 respectively, find the discharge over the weir. [**Ans.** 1.0807 m^3/s]