# Hydraulics and Pneumatics <br> A technician's and engineer's guide 

## Second edition

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## Preface

> Machines should work, people should think
> The IBM Pollyanna Principle

Practically every industrial process requires objects to be moved, manipulated or be subjected to some form of force. This is generally accomplished by means of electrical equipment (such as motors or solenoids), or via devices driven by air (pneumatics) or liquids (hydraulics).

Traditionally, pneumatics and hydraulics are thought to be a mechanical engineer's subject (and are generally taught as such in colleges). In practice, techniques (and, more important, the faultfinding methodology) tend to be more akin to the ideas used in electronics and process control.

This book has been written by a process control engineer as a guide to the operation of hydraulic and pneumatics systems. It is intended for engineers and technicians who wish to have an insight into the components and operation of a pneumatic or hydraulic system. The mathematical content has been deliberately kept simple with the aim of making the book readable rather than rigorous. It is not, therefore, a design manual and topics such as sizing of pipes and valves have been deliberately omitted.

This second edition has been updated to include recent developments such as the increasing use of proportional valves, and includes an expanded section on industrial safety.

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## 1

## Fundamental principles

## Industrial prime movers

Most industrial processes require objects or substances to be moved from one location to another, or a force to be applied to hold, shape or compress a product. Such activities are performed by Prime Movers; the workhorses of manufacturing industries.

In many locations all prime movers are electrical. Rotary motions can be provided by simple motors, and linear motion can be obtained from rotary motion by devices such as screw jacks or rack and pinions. Where a pure force or a short linear stroke is required a solenoid may be used (although there are limits to the force that can be obtained by this means).

Electrical devices are not, however, the only means of providing prime movers. Enclosed fluids (both liquids and gases) can also be used to convey energy from one location to another and, consequently, to produce rotary or linear motion or apply a force. Fluidbased systems using liquids as transmission media are called hydraulic systems (from the Greek words hydra for water and aulos for a pipe; descriptions which imply fluids are water although oils are more commonly used). Gas-based systems are called Pneumatic systems (from the Greek pneumn for wind or breath). The most common gas is simply compressed air. although nitrogen is occasionally used.

The main advantages and disadvantages of pneumatic or hydraulic systems both arise out of the different characteristics of low density compressible gases and (relatively) high density
incompressible liquids. A pneumatic system, for example, tends to have a 'softer' action than a hydraulic system which can be prone to producing noisy and wear inducing shocks in the piping. A liquid-based hydraulic system, however, can operate at far higher pressures than a pneumatic system and, consequently, can be used to provide very large forces.

To compare the various advantages and disadvantages of electrical pneumatic and hydraulic systems, the following three sections consider how a simple lifting task could be handled by each.

## A brief system comparison

The task considered is how to lift a load by a distance of about 500 mm . Such tasks are common in manufacturing industries.

## An electrical system

With an electrical system we have three basic choices; a solenoid, a DC motor or the ubiquitous workhorse of industry, the AC induction motor. Of these, the solenoid produces a linear stroke directly but its stroke is normally limited to a maximum distance of around 100 mm .

Both DC and AC motors are rotary devices and their outputs need to be converted to linear motion by mechanical devices such as wormscrews or rack and pinions. This presents no real problems; commercial devices are available comprising motor and screw.

The choice of motor depends largely on the speed control requirements. A DC motor fitted with a tacho and driven by a thyristor drive can give excellent speed control, but has high maintenance requirements for brushes and commutator.

An AC motor is virtually maintenance free, but is essentially a fixed speed device (with speed being determined by number of poles and the supply frequency). Speed can be adjusted with a variable frequency drive, but care needs to be taken to avoid overheating as most motors are cooled by an internal fan connected directly to the motor shaft. We will assume a fixed speed raise/lower is required, so an AC motor driving a screwjack would seem to be the logical choice.

Neither type of motor can be allowed to stall against an end of travel stop, (this is not quite true; specially-designed DC motors, featuring good current control on a thyristor drive together with an external cooling fan, can be allowed to stall), so end of travel limits are needed to stop the drive.

We have thus ended up with the system shown in Figure 1.1 comprising a mechanical jack driven by an AC motor controlled by a reversing starter. Auxiliary equipment comprises two limit switches, and a motor overload protection device. There is no practical load limitation provided screw/gearbox ratio, motor size and contactor rating are correctly calculated.


Figure 1.1 Electrical solution, based on three phase motor

## A hydraulic system

A solution along hydraulic lines is shown in Figure 1.2. A hydraulic linear actuator suitable for this application is the ram, shown schematically in Figure 1.2a. This consists of a movable piston connected directly to the output shaft. If fluid is pumped into pipe A the piston will move up and the shaft will extend; if fluid is pumped into pipe B, the shaft will retract. Obviously some method of retrieving fluid from the non-pressurised side of the piston must be incorporated.

The maximum force available from the cylinder depends on fluid pressure and cross sectional area of the piston. This is discussed further in a later section but, as an example, a typical hydraulic pressure of 150 bar will lift $150 \mathrm{~kg} \mathrm{~cm}^{-2}$ of piston area. A load of 2000 kg could thus be lifted by a 4.2 cm diameter piston.

A suitable hydraulic system is shown in Figure 1.2b. The system requires a liquid fluid to operate; expensive and messy and, consequently, the piping must act as a closed loop, with fluid transferred from a storage tank to one side of the piston, and returned from the other side of the piston to the tank. Fluid is drawn from the tank by a pump which produces fluid flow at the required 150 bar. Such high pressure pumps, however, cannot operate into a dead-end load as they deliver constant volumes of fluid from input to output ports for each revolution of the pump shaft. With a dead-end load, fluid pressure rises indefinitely, until a pipe or the pump itself fails. Some form of pressure regulation, as shown, is therefore required to spill excess fluid back to the tank.

Cylinder movement is controlled by a three position changeover valve. To extend the cylinder, port A is connected to the pressure line and port B to the tank. To reverse the motion, port B is connected to the pressure line and port A to the tank. In its centre position the valve locks the fluid into the cylinder (thereby holding it in position) and dead-ends the fluid lines (causing all the pump output fluid to return to the tank via the pressure regulator).

There are a few auxiliary points worthy of comment. First, speed control is easily achieved by regulating the volume flow rate to the cylinder (discussed in a later section). Precise control at low speeds is one of the main advantages of hydraulic systems.

Second, travel limits are determined by the cylinder stroke and cylinders, generally, can be allowed to stall at the ends of travel so no overtravel protection is required.

(a) Hydraulic cylinder

(b) Physical components

Figure 1.2 Hydraulic solution
Third, the pump needs to be turned by an external power source; almost certainly an AC induction motor which, in turn, requires a motor starter and overload protection.

Fourth, hydraulic fluid needs to be very clean, hence a filter is needed (shown in Figure 1.2b) to remove dirt particles before the fluid passes from the tank to the pump.

One final point worth mentioning is that leaks of fluid from the system are unsightly, slippery (hence hazardous) and environmentally very undesirable A major failure can be catastrophic.

At first sight Figure 1.2b appears inordinately complicated compared with the electrical system of Figure 1.1, but it should be remembered all parts enclosed in the broken-lined box in Figure 1.2 are common to an area of plant and not usually devoted to just one motion as we have drawn.

## A pneumatic system

Figure 1.3 shows the components of a pneumatic system. The basic actuator is again a cylinder, with maximum force on the shaft being determined by air pressure and piston cross sectional area. Operating pressures in pneumatic systems are generally much lower than those in a hydraulic systems; 10 bar being typical which will lift $10 \mathrm{~kg} \mathrm{~cm}^{-2}$ of piston area, so a 16 cm diameter piston is required to lift the 2000 kg load specified in the previous section. Pneumatic systems therefore require larger actuators than hydraulic systems for the same load.

The valve delivering air to the cylinder operates in a similar way to its hydraulic equivalent. One notable difference arises out of the simple fact that air is free; return air is simply vented to atmosphere.


Figure 1.3 Pneumatic solution

Air is drawn from the atmosphere via an air filter and raised to required pressure by an air compressor (usually driven by an AC motor). The air temperature is raised considerably by this compressor. Air also contains a significant amount of water vapour. Before the air can be used it must be cooled, and this results in the formation of condensation So, the air compressor must be followed by a cooler and air treatment unit.

Compressibility of a gas makes it necessary to store a volume of pressurised gas in a reservoir, to be drawn on by the load. Without this reservoir, a slow exponential rise of pressure results in a similar slow cylinder movement when the valve is first opened. The air treatment unit is thus followed by an air reservoir.

Hydraulic systems require a pressure regulator to spill excess fluid back to the tank, but pressure control in a hydraulic system is much simpler. A pressure switch, fitted to the air reservoir, starts the compressor motor when pressure falls and stops it again when pressure reaches the required level.

The general impression is again one of complexity, but units in the broken-lined box are again common to one plant or even a whole site. Many factories produce compressed air at one central station and distribute an air ring main to all places on the site in a similar way to other services such as electricity, water or gas.

## A comparison

Table 1.1 gives superficial comparisons of the various systems discussed in the previous sections.

## Definition of terms

There is an almost universal lack of standardisation of units used for measurement in industry, and every engineer will tell tales of gauges indicating, say, velocity in furlongs per fortnight. Hydraulics and pneumatic systems suffer particularly from this characteristic, and it is by no means unusual to find pressure indicated at different locations in the same system in bar, kpascal and psi.

There is, however, a welcome (and overdue) movement to standardisation on the International System (SI) of units, but it will be some time before this is complete. The engineer will therefore encounter many odd-ball systems in the years to come.

Table 1.1 Comparisons of electrical, hydraulic and pneumatic systems

|  | Electrical | Hydraulic | Pneumatic |
| :--- | :--- | :--- | :--- |
| Energy source | Usually from <br> outside supplier | Electric motor or <br> diesel driven | Electric motor or <br> diesel driven |
| Energy storage | Limited (batteries) | Limited <br> (accumulator) | Good (reservoir) |
| sistribution | Excellent, with <br> minimal loss | Limited basically <br> a local facility | Good. can be <br> treated as a plant <br> wide service |
| Energy cost | Lowest | Medium | Highest |
| Rotary actuators | AC \& DC motors. <br> Good control on | Low speed. Good <br> control. Can be <br> stalled | Wide speed range. <br> Accurate speed <br> control difficult |
|  | DC motors. AC <br> motors cheap | Cylinders. Very | Cylinders. <br> Medium force |
| Cinear actuator motion via | Sholenoid. <br> high force | Otherwise via <br> mechanical <br> conversion | Controllable high | | Controllable |
| :--- |
| medium force |

Any measurement system requires definition of the six units used to measure:

- length:
- mass;
- time;
- temperature;
- electrical current;
- light intensity.

Of these, hydraulic/pneumatic engineers are primarily concerned with the first three. Other units (such as velocity, force, pressure)
can be defined in terms of these basic units. Velocity, for example, is defined in terms of length/time.

The old British Imperial system used units of foot, pound and second (and was consequently known as the fps system). Early metric systems used centimetre, gramme and second (known as the cgs system), and metre, kilogramme and second (the mks system). The mks system evolved into the SI system which introduces a more logical method of defining force and pressure (discussed in later sections). Table 1.2 gives conversions between basic simple units.

Table 1.2 Fundamental mechanical units

Mass
$1 \mathrm{~kg}=2.2046$ pound $(\mathrm{lb})=1000 \mathrm{gm}$
$1 \mathrm{lb}=0.4536 \mathrm{~kg}$
1 ton (imperial) $=2240 \mathrm{lb}=1016 \mathrm{~kg}=1.12 \mathrm{ton}$ (US)
1 tonne $=1000 \mathrm{~kg}=2204.6 \mathrm{lb}=0.9842$ ton (imperial)
1 ton (US) $=0.8929$ ton (imperial)
Length
1 metre $=3.281$ foot $(\mathrm{ft})=1000 \mathrm{~mm}=100 \mathrm{~cm}$
1 inch $=25.4 \mathrm{~mm}=2.54 \mathrm{~cm}$
1 yard $=0.9144 \mathrm{~m}$

## Volume

1 litre $=0.2200$ gallon $($ imperial $)=0.2642$ gallon (US)
1 gallon $($ imperial $)=4.546$ litre $=1.2011$ gallon $($ US $)$ $=0.161$ cubic ft
1 gallon $(\mathrm{US})=3.785$ litre $=0.8326$ gallon (imperial)
1 cubic meter $=220$ gallon (imperial $)=35.315$ cubic feet
1 cubic inch $=16.387$ cubic centimetres

## Mass and force

Pneumatic and hydraulic systems generally rely on pressure in a fluid. Before we can discuss definitions of pressure, though, we must first be clear what is meant by everyday terms such as weight, mass and force.

We all are used to the idea of weight, which is a force arising from gravitational attraction between the mass of an object and the earth. The author weighs 75 kg on the bathroom scales; this is equivalent to saying there is 75 kg force between his feet and the ground.

Weight therefore depends on the force of gravity. On the moon, where gravity is about one sixth that on earth, the author's weight would be about 12.5 kg ; in free fall the weight would be zero. In all cases, though, the author's mass is constant.

The British Imperial fps system and the early metric systems link mass and weight (force) by defining the unit of force to be the gravitational attraction of unit mass at the surface of the earth. We thus have a mass defined in pounds and force defined in pounds force (lbs f) in the fps system, and mass in kilogrammes and force in kg f in the mks system.

Strictly speaking, therefore, bathroom scales which read 75 kg are measuring 75 kg f , not the author's mass. On the moon they would read 12.5 kg f, and in free fall they would read zero.

If a force is applied to a mass, acceleration (or deceleration) will result as given by the well known formula:

$$
\begin{equation*}
\mathrm{F}=\mathrm{ma} . \tag{1.1}
\end{equation*}
$$

Care must be taken with units when a force $F$ is defined in lbs $f$ or kg f and mass is defined in lbs or kg , because resulting accelerations are in units of $g$; acceleration due to gravity. A force of 25 kg f applied to the author's mass of 75 kg produces an acceleration of 0.333 g .

The SI unit of force, the newton ( N ), is defined not from earth's gravity, but directly from expression 1.1. A newton is defined as the force which produces an acceleration of $1 \mathrm{~m} \mathrm{~s}^{-2}$ when applied to a mass of 1 kg .

One kgf produces an acceleration of $1 \mathrm{~g}\left(9.81 \mathrm{~ms}^{-2}\right)$ when applied to a mass of 1 kg . One newton produces an acceleration of $1 \mathrm{~ms}^{-2}$ when applied to mass of 1 kg . It therefore follows that:

$$
1 \mathrm{~kg} \mathrm{f}=9.81 \mathrm{~N}
$$

but as most instruments on industrial systems are at best $2 \%$ accurate it is reasonable (and much simpler) to use:

$$
1 \mathrm{~kg} \mathrm{f}=10 \mathrm{~N}
$$

for practical applications.
Table 1.3 gives conversions between various units of force.

Table 1.3 Units of force
1 newton $(\mathrm{N})=0.2248$ pound force ( lb f )
$=0.1019$ kilogram force $(\mathrm{kg} \mathrm{f})$
$1 \mathrm{lb} \mathrm{f}=4.448 \mathrm{~N}=0.4534 \mathrm{~kg} \mathrm{f}$
$1 \mathrm{~kg} \mathrm{f}=9.81 \mathrm{~N}=2.205 \mathrm{lb}$
Other units are
dynes (cgs unit); $1 \mathrm{~N}=10^{5}$ dynes
ponds (gram force); $1 \mathrm{~N}=102$ ponds
SI unit is the newton:
$\mathrm{N}=\mathrm{kg} \mathrm{ms}^{-2}$

## Pressure

Pressure occurs in a fluid when it is subjected to a force. In Figure 1.4 a force F is applied to an enclosed fluid via a piston of area A . This results in a pressure $P$ in the fluid. Obviously increasing the force increases the pressure in direct proportion. Less obviously, though, decreasing piston area also increases pressure. Pressure in the fluid can therefore be defined as the force acting per unit area, or:

$$
\begin{equation*}
\mathrm{P}=\frac{\mathrm{F}}{\mathrm{~A}} . \tag{1.2}
\end{equation*}
$$

Although expression 1.2 is very simple, there are many different units of pressure in common use. In the Imperial fps system. for example, F is given in lbs f and A is given in square inches to give pressure measured in pound force per square inch (psi).


Figure 1.4 Pressure in a fluid subjected to a force

In metric systems, F is usually given in kgf and A in square centimetres to give pressure in kilogram/force per square centimetre ( kg $\mathrm{f} \mathrm{cm}^{-2}$ ).

The SI system defines pressure as the force in newtons per square metre $\left(\mathrm{N} \mathrm{m}^{-2}\right)$. The SI unit of pressure is the pascal (with $1 \mathrm{~Pa}=$ $1 \mathrm{~N} \mathrm{~m}^{-2}$ ). One pascal is a very low pressure for practical use, however, so the kilopascal ( $1 \mathrm{kPa}=10^{3} \mathrm{~Pa}$ ) or the megapascal ( $1 \mathrm{MPa}=10^{6} \mathrm{~Pa}$ ) are more commonly used.

Pressure can also arise in a fluid from the weight of a fluid. This is usually known as the head pressure and depends on the height of fluid. In Figure 1.5 the pressure at the bottom of the fluid is directly proportional to height $h$.


## Figure 1.5 Head pressure in a fluid

In the Imperial and metric systems head pressure is given by:

$$
\begin{equation*}
\mathrm{P}=\rho \mathrm{h} . \tag{1.3}
\end{equation*}
$$

where $\rho$ is the density and h the height (both in the correct units) to give $P$ in psi or $\mathrm{kg} \mathrm{cm}^{-2}$.

In the SI system expression 1.3. is re-arranged as:

$$
\begin{equation*}
\mathrm{P}=\rho \mathrm{gh} . \tag{1.4}
\end{equation*}
$$

where $g$ is the acceleration due to gravity $\left(9.81 \mathrm{~ms}^{-2}\right)$ to give the pressure in pascal.

Pressure in a fluid can, however, be defined in terms of the equivalent head pressure. Common units are millimetres of mercury and centimetres, inches, feet or metres of water. The suffix wg (for water gauge) is often used when pressure is defined in terms of an equivalent head of water.

We live at the bottom of an ocean of air, and are consequently subject to a substantial pressure head from the weight of air above
us. This pressure, some $15 \mathrm{psi}, 1.05 \mathrm{~kg} \mathrm{f} \mathrm{cm}^{-2}$, or 101 kPa , is called an atmosphere, and is sometimes used as a unit of pressure.

It will be noted that 100 kPa is, for practical purposes, one atmosphere As this is a convenient unit for many applications 100 kPa $\left(10^{5} \mathrm{~Pa}\right.$ or 0.1 MPa ) has been given the name bar. Within the accuracy of instrumentation generally found in industry 1 bar = 1 atmosphere.

There are three distinct ways in which pressure is measured, shown in Figure 1.6. Almost all pressure transducers or transmitters measure the pressure difference between two input ports. This is known as differential pressure, and the pressure transmitter in Figure 1.6a indicates a pressure of $\mathrm{P}_{1}-\mathrm{P}_{2}$.

In Figure 1.6b the low pressure input port is open to atmosphere, so the pressure transmitter indicates pressure above atmospheric pressure. This is known as gauge pressure, and is usually denoted by a $g$ suffix (e.g. psig). Gauge pressure measurement is almost universally used in hydraulic and pneumatic systems (and has been implicitly assumed in all previous discussions in this chapter).

(c) Absolute pressure

Figure 1.6 Different forms of pressure measurement


Figure 1.7 Relationship between absolute and gauge pressures

Figure 1.6 c shows the pressure transmitter measuring pressure with respect to a vacuum. This is known as absolute pressure and is of importance when the compression of gases is considered. The relationship between absolute and gauge pressure is illustrated in Figure 1.7. Pressure measurement and gas compression are discussed in later sections. Table 1.4 compares units of pressure. A typical hydraulic system operates at 150 bar, while typical pneumatic systems operate at 10 bar.

## Work, energy and power

Work is done (or energy is transferred) when an object is moved against a force, and is defined as:

$$
\begin{equation*}
\text { work }=\text { force } \times \text { distance moved. } \tag{1.5}
\end{equation*}
$$

In the Imperial fps system expression 1.5 gives a unit of ft lb f . For metric systems the unit is cm kg . The SI unit of work is the joule, where $1 \mathrm{~J}=1 \mathrm{Nm}\left(=1 \mathrm{~m}^{2} \mathrm{~kg} \mathrm{~s}^{-2}\right)$. Table 1.5 compares these, and other, units of work.

Power is the rate at which work is performed:

$$
\begin{equation*}
\text { power }=\frac{\text { work }}{\text { time }} . \tag{1.6}
\end{equation*}
$$

The SI unit of power is the watt, defined as $1 \mathrm{~J} \mathrm{~s}^{-1}$. This is by far the most common unit of power, as it is almost universally used for the measurement of electrical power.

The Imperial system uses horse power ( Hp ) which was used historically to define motor powers. One horse power is defined as $550 \mathrm{ft} \mathrm{lb} \mathrm{f} \mathrm{s}^{-1}$. Table 1.6 compares units of power.

Table 1.4 Units of pressure

$$
\begin{aligned}
& 1 \mathrm{bar}=100 \mathrm{kPa} \\
& =14.5 \mathrm{psi} \\
& =750 \mathrm{mmHg} \\
& =401.8 \text { inches } \mathrm{W} \text { G } \\
& =1.0197 \mathrm{kgf} \mathrm{~cm}^{-2} \\
& =0.9872 \text { atmosphere } \\
& 1 \text { kilopascal }=1000 \mathrm{~Pa} \\
& =0.01 \mathrm{bar} \\
& =0.145 \mathrm{psi} \\
& =1.0197 \times 10^{-3} \mathrm{kgf} \mathrm{~cm}^{-2} \\
& =4.018 \text { inches } \mathrm{W} \text { G } \\
& =9.872 \times 10^{-3} \text { atmosphere } \\
& 1 \text { pound per square inch (psi) }=6.895 \mathrm{kPa} \\
& =0.0703 \mathrm{kgf} \mathrm{~cm}^{-2} \\
& =27.7 \text { inches } \mathrm{W} \text { G } \\
& 1 \text { kilogram force per square } \mathrm{cm}(\mathrm{kgf} \mathrm{~cm}-2)=98.07 \mathrm{kPa} \\
& =14.223 \mathrm{psi} \\
& 1 \text { Atmosphere }=1.013 \text { bar } \\
& =14.7 \mathrm{psi} \\
& =1.033 \mathrm{kgf} \mathrm{~cm}^{-2} \\
& \text { SI unit of pressure is the pascal }(\mathrm{Pa}) \mathrm{Pa}=1 \mathrm{~N} \mathrm{~m}^{-2} \\
& \text { Practical units are the bar and the psi. }
\end{aligned}
$$

Table 1.5 Units of work (energy)

$$
\begin{aligned}
& 1 \text { joule }(\mathrm{J})=2.788 \times 10^{-4} \mathrm{~Wh}\left(2.788 \times 10^{-7} \mathrm{kWh}\right) \\
&=0.7376 \mathrm{ft} \mathrm{lbf} \\
&=0.2388 \text { calories } \\
&=9.487 \times 10^{-4} \text { British thermal units }(\mathrm{BTu}) \\
&=0.102 \mathrm{kgf} \mathrm{~m} \\
&=10^{7} \mathrm{ergs}(\mathrm{cgs} \text { unit) } \\
& \text { SI unit of work is the joule }(\mathrm{J}) \\
& \begin{aligned}
\mathrm{J} & =1 \mathrm{~N} \mathrm{~m} \\
& =1 \mathrm{~m}^{2} \mathrm{~kg} \mathrm{~s}^{-2}
\end{aligned}
\end{aligned}
$$

Table 1.6 Units of power

$$
\begin{aligned}
1 \text { kwatt }(\mathrm{kw}) & =1.34 \mathrm{Hp} \\
& =1.36 \text { metric Hp } \\
& =102 \mathrm{kgf} \mathrm{~m} \mathrm{~s}^{-1} \\
& =1000 \mathrm{~W} \\
1 \text { horse power }(\mathrm{Hp}) & =0.7457 \mathrm{kw} \\
& =550 \mathrm{Ft} \mathrm{lb} \mathrm{~s}^{-1} \\
& =2545 \mathrm{BTU} \mathrm{~h}
\end{aligned}
$$

Work can be considered as the time integral of power (often described loosely as total power used). As electrical power is measured in watts or kilowatts ( $1 \mathrm{~kW}=10^{3} \mathrm{~W}$ ), the kilowatt hour ( kW h ) is another representation of work or energy.

## Torque

The term torque is used to define a rotary force. and is simply the product of the force and the effective radius as shown in Figure 1.8. We thus have:

$$
\begin{equation*}
\mathrm{T}=\mathrm{F} \times \mathrm{d} . \tag{1.7}
\end{equation*}
$$

In the Imperial system the unit is lbf ft , in metric systems the unit is kgf m or kgf cm , and in SI the unit is Nm .


Figure 1.8 Definition of torque

## Pascal's law

Pressure in an enclosed fluid can be considered uniform throughout a practical system. There may be small differences arising from head pressures at different heights, but these will generally be negligible compared with the system operating pressure. This equality of pressure is known as Pascal's law, and is illustrated in Figure 1.9 where a force of 5 kgf is applied to a piston of area $2 \mathrm{~cm}^{2}$. This produces a pressure of $2.5 \mathrm{kgf} \mathrm{cm}^{-2}$ at every point within the fluid, which acts with equal force per unit area on the walls of the system.

(a) Forces and pressure in closed tanks

(b) Pressure in a bottle

Figure 1.9 Pressure in an enclosed fluid

Suppose the base of the left hand tank is $0.1 \times 0.1 \mathrm{~m}$ to give a total area of $100 \mathrm{~cm}^{2}$. The total force acting on the base will be 250 kgf . If the top of the right hand $\operatorname{tank}$ is $1 \mathrm{~m} \times 1.5 \mathrm{~m}$, a surprisingly large upwards force of $37,500 \mathrm{kgf}$ is developed. Note, the size of the connecting pipe has no effect. This principle explains why it is possible to shear the bottom off a bottle by applying a small force to the cork, as illustrated in Figure 1.9b.

The applied force develops a pressure, given by the expression:

$$
\begin{equation*}
P=\frac{f}{a} . \tag{1.8}
\end{equation*}
$$

The force on the base is:

$$
\begin{equation*}
\mathrm{F}=\mathrm{P} \times \mathrm{A} . \tag{1.9}
\end{equation*}
$$

from which can be derived:

$$
\begin{equation*}
\mathrm{F}=\mathrm{f} \times \frac{\mathrm{A}}{\mathrm{a}} . \tag{1.10}
\end{equation*}
$$

Expression 1.10 shows an enclosed fluid may be used to magnify a force. In Figure 1.10 a load of 2000 kg is sitting on a piston of area $500 \mathrm{~cm}^{2}$ (about 12 cm radius). The smaller piston has an area of $2 \mathrm{~cm}^{2}$. An applied force $f$ given by:

$$
\begin{equation*}
\mathrm{f}=2000 \times \frac{2}{500}=8 \mathrm{kgf} . \tag{1.11}
\end{equation*}
$$

will cause the 2000 kg load to rise. There is said to be a mechanical advantage of 250 .

Energy must, however, be conserved. To illustrate this, suppose the left hand piston moves down by 100 cm (one metre). Because


Figure 1.10 Mechanical advantage
we have assumed the fluid is incompressible, a volume of liquid $200 \mathrm{~cm}^{2}$ is transferred from the left hand cylinder to the right hand cylinder, causing the load to rise by just 0.4 cm . So, although we have a force magnification of 250 , we have a movement reduction of the same factor. Because work is given by the product of force and the distance moved, the force is magnified and the distance moved reduced by the same factor, giving conservation of energy. The action of Figure 1.10 is thus similar to the mechanical systems of Figure 1.11 which also exhibit mechanical advantage.


Figure 1.11 Examples of mechanical advantage where a small input force $f$ produces a larger output force $F$

The principle of Figure 1.10 is widely used where a large force is required with small movement. Typical examples are clamps, presses, hydraulic jacks and motor car brake and clutch operating mechanisms.

It should be noted that pressure in, say, a cylinder is determined solely by load and piston area in the steady state, and is not dependent on velocity of the piston once a constant speed has been achieved. Relationships between force, pressure, flow and speed are illustrated in Figure 1.12.

In Figure 1.12a, fluid is delivered to a cylinder at a rate of $\mathrm{Q} \mathrm{cm}{ }^{3} \mathrm{~s}^{-1}$. When the inlet valve is first opened, a pressure spike is observed as the load accelerates, but the pressure then settles back


Figure 1.12 The relationships between force, pressure, flow and speed
to a steady value of $P=F / A \mathrm{kgf} \mathrm{cm}^{-2}$ where $A$ is the area of the piston in $\mathrm{cm}^{2}$ and F is measured in kgf. The load rises with a velocity $V=Q / A \mathrm{~cm} \mathrm{~s}^{-1}$ and velocity can obviously be controlled by adjusting flow rate Q .

In Figure 1.12 b , the inlet valve has been closed, and the outlet valve opened allowing $\mathrm{R} \mathrm{cm}^{-3} \mathrm{~s}^{-1}$ to flow out of the cylinder. There is again a pressure spike (negative this time) as the load accelerates downwards, but the pressure reverts to $\mathrm{P}=\mathrm{F} / \mathrm{A}$ once the steady speed $V=R / A \mathrm{~cm} \mathrm{~s}^{-1}$ is achieved.

Finally, in Figure 1.12c both valves are open. The net flow is ( $\mathrm{Q}-\mathrm{R}$ ) giving a cylinder velocity ( $\mathrm{Q}-\mathrm{R}$ )/A which can be positive (rising) or negative (falling) dependent on which flow is the largest. The steady state pressure, however, is unchanged at $\mathrm{P}=\mathrm{F} / \mathrm{A}$.

## Pressure measurement

Behaviour of a fluid can generally be deduced from measurements of flow or pressure. A flow transducer or transmitter has to be plumbed, in line, into a pipe, whereas pressure transmitters can be added non-intrusively as tappings to the side of a pipe. The basic fault-finding tool in both pneumatic or hydraulic systems is therefore a pressure gauge. Often this is a simple gauge which can be plugged into various parts of the system via a flexible connection.

These test pressure gauges invariably measure gauge pressure with the simple Bourdon pressure gauge shown in Figure 1.13. This consists of a flattened C shaped tube which is fixed at one end, shown in Figure 1.13a. When pressure is applied to the tube it tends to straighten, with the free end moving up and to the right. For low pressure ranges a spiral tube is used to increase the sensitivity.

This movement is converted to a circular pointer movement by a mechanical quadrant and pinion. If an electrical output signal is required for remote indication, the pointer can be replaced by a potentiometer, as shown in Figure 1.13b.

Hydraulic and pneumatic systems tend to exhibit large pressure spikes as loads accelerate or decelerate (a typical example being shown on Figure 1.12c.) These spikes can be irritating to the observer, can mislead, and in extreme cases could damage a pressure indicator. The response of a pressure sensor can be dampened by inclusion of a snubber restriction, as shown in Figure 1.13c.

Bourdon gauge-based transducers are generally robust but are low accuracy (typically $\pm 2 \%$ ) devices. As the limit of visual resolution of a pointer position is no better than $\pm 2 \%$ anyway, ruggedness of these transducers makes them ideal for plant mounted monitoring.

Where more accurate pressure measurement is required, transducers based on the force balance principle of Figure 1.14 are generally used. This is essentially a differential pressure transducer, in which the low pressure inlet (LP) is left open to atmosphere and the high pressure (HP) inlet connects to the system. The signal given (HP-LP) is thus gauge pressure.

A pressure increase in the system deflects the pressure sensitive diaphragm to the left. This movement is detected by the displace-


Figure 1.13 The Bourdon pressure gauge


Figure 1.14 Force balance pressure transducer
ment transducer which, via a servo amplifier, leads to an increase in current in the balance coil.

Because the force from the balance coil always exactly balances the force arising from the pressure difference between LP and HP, current through the transducer is directly proportional to the differential pressure.

Remote indicating transducers are generally arranged with a remote power supply and the indicator and/or recorder connected into one line as Figure 1.15 to give a two-wire system. A signal range of 4 to 20 mA is commonly used, with the 4 mA zero level providing a current supply for the transducer's servo amplifier and also indicating circuit continuity ( 0 mA indicating a open circuit fault condition).

## Fluid flow

Hydraulic and pneumatic systems are both concerned with the flow of a fluid (liquid or gas) down a pipe. Flow is a loose term that generally has three distinct meanings:

- volumetric flow is used to measure volume of fluid passing a point per unit of time. Where the fluid is a compressible gas, temperature and pressure must be specified or flow normalised to


Figure 1.15 Advantages of two-wire transducers
some standard temperature and pressure (a topic discussed later). Volumetric flow is the most common measurement in process control

- mass flow measures the mass of fluid passing the point in unit time
- velocity of flow measures linear speed (in $\mathrm{m} \mathrm{s}^{-1}$, say) past the point of measurement. Flow velocity is of prime importance in the design of hydraulic and pneumatic systems.

Types of fluid flow are illustrated in Figure 1.16. At low flow velocities, the flow pattern is smooth and linear with low velocities at the pipe walls and the highest flow at the centre of the pipe. This is known as laminar or streamline flow.

As flow velocity increases, eddies start to form until at high flow velocities complete turbulence results as shown in Figure 1.16b. Flow velocity is now virtually uniform across the pipe.

(a) Laminar or streamline flow

(b) Turbulent flow

Figure 1.16 Types of fluid flow

The nature of the flow is determined by the Reynolds number, $\mathrm{R}_{\mathrm{c}}$, given by the expression:

$$
\begin{equation*}
\mathrm{R}_{\mathrm{c}}=\frac{v \mathrm{~d} \rho}{\eta} . \tag{1.12}
\end{equation*}
$$

where $v$ is flow velocity, d is pipe diameter, $\rho$ the fluid density and $\eta$ the viscosity. The Reynolds number is a ratio and hence dimensionless. If $R_{c}<2000$, flow is laminar. If $R_{c}>10^{5}$, flow is turbulent.

A turbulent flow is generally preferred for products in process control as it simplifies volumetric flow measurement (with differential pressure flowmeters - see later). Turbulent flow, however, increases energy loss through friction and may lead to premature wear. Cavitation (formation and collapse of vapour bubbles) occurs with turbulent liquid flow and may result in pitting on valve surfaces. Laminar flow is therefore specified for hydraulic and pneumatic systems. This results in a desired flow velocity of about $5 \mathrm{~m} \mathrm{~s}^{-2}$.

Energy in a unit mass of fluid has three components:

- kinetic energy from its motion, given by $v^{2} / 2$ where $v$ is flow velocity
- potential energy from the height of the fluid
- energy arising from the pressure of the fluid, given by $\mathrm{P} / \rho$ where P is the pressure and $\rho$ the density.

Fluid is passing along a pipe in Figure 1.17. Neglecting energy losses from friction, energies at points $\mathrm{X}, \mathrm{Y}$ and Z will be equal. The flow velocity at point Y , however, is higher than at points X and Z


Figure 1.17 Relationship between flow and pressure
because of the smaller pipe diameter. Potential energy at each point is constant because the pipe is horizontal. so we can write:

$$
\begin{equation*}
\frac{v_{\mathrm{x}}^{2}}{2}+\frac{\mathrm{P}_{\mathrm{x}}}{\rho}=\frac{v_{\mathrm{y}}^{2}}{2}+\frac{\mathrm{P}_{\mathrm{y}}}{\rho}=\frac{v_{\mathrm{z}}^{2}}{2}+\frac{\mathrm{P}_{\mathrm{z}}}{\rho} \tag{1.13}
\end{equation*}
$$

We have implied an incompressible fluid by assuming the density, $\rho$, is constant throughout. Expression 1.13 becomes more complicated for a gas as different densities have to be used at each point.

The net result of the expression is fluid pressure falls as flow velocity rises. Note, though, that the pressure recovers as flow velocity falls again at point $Z$.

The simplest method of measuring flow (known as a variable area flowmeter) uses a float in a vertical tube arranged as Figure 1.18. The obstruction of the float causes a local increase in the fluid velocity which causes a differential pressure drop across the float, resulting in an upward force. The weight of the float obviously causes a downward force. The float therefore rises or falls depending on which force is the largest. The area around the float, however, increases the higher the float rises because of the tube taper. This increase in area decreases the pressure drop across the float and the upwards force. The float therefore settles at a vertical


Figure 1.18 Variable area flowmeter
position where the weight of the float and the upwards force from the differential pressure exactly match. Flow rate can therefore be determined from the float position.

A remote indicating flowmeter can be constructed from a pipe mounted turbine, as shown in Figure 1.19. Fluid flow causes the propeller to rotate, its rotational speed being proportional to flow rate. Blade rotation is counted electronically by an external inductive proximity detector to give an electrical signal for remote indication of the flow rate.


Figure 1.19 Turbine flowmeter
Finally, the classical method of measuring flow returns directly to expression 1.13 by locally increasing flow velocity with a deliberately introduced restriction as shown in Figure 1.20. Typical obstructions are an orifice plate or a venturi. These increase flow velocity, causing a pressure drop which can be measured to give a differential pressure related to the flow. Unfortunately, the differential pressure is proportional to the square of the flow rate, so a linearising square root extractor circuit is required to give a linear signal. Although differential pressure flow measurement is widely used to measure the flow rates of process material, the technique is not widely used in hydraulic and pneumatic systems.

It will be apparent that all flow measurement systems are intrusive to various degrees, and cannot be tapped in as easily as pressure measurement can. Fault finding in hydraulic and pneumatic systems is therefore generally based on pressure readings at strategic points.


Figure 1.20 Orifice plate flowmeter

## Temperature

Fluid behaviour is determined to some extent by its temperature. A later section discusses the relationship between pressure and temperature in a gas.

## Temperature scales

A temperature scale is established by choosing two observable physical effects which are dependent upon temperature and assigning numerical values to them. The Fahrenheit and Celsius (previously known as Centigrade) scales use the freezing and boiling points of water as the two reference points:

|  | Fahrenheit | Celsius |
| :--- | :---: | :---: |
| Freezing point | 32 | 0 |
| Boiling point | 212 | 100 |

From which:

$$
\begin{equation*}
F=\left(9 \times \frac{C}{5}\right)+32 \tag{1.14}
\end{equation*}
$$

and:

$$
\begin{equation*}
\mathrm{C}=(\mathrm{F}-32) \times \frac{5}{9} . \tag{1.15}
\end{equation*}
$$

The SI unit of temperature is the Kelvin. This defines the lowest theoretical temperature (called absolute zero) as 0 K , and the triple point of water $\left(0.01^{\circ} \mathrm{C}\right)$ as 273.16 K . It should be noted that temperatures in Kelvin do not use the degree $\left({ }^{\circ}\right)$ symbol. These apparently odd numerical values make a temperature change of 1 K the same as $1^{\circ} \mathrm{C}$, and:

$$
\begin{equation*}
\mathrm{K}={ }^{\circ} \mathrm{C}+273.1 \tag{1.16}
\end{equation*}
$$

The Celsius scale is most widely used in industry, but the Kelvin scale is important in determining the changes in gas pressure or volume with temperature.

## Temperature measurement

There are four basic ways of measuring temperature based on tem-perature-dependent physical properties.

Expansion of a substance with temperature can be used to produce a change in volume, length or pressure. This is probably the most common type of temperature measurement in the form of mercury or alcohol-in-glass thermometers. A variation is the bimetallic strip shown in Figure 1.21. where two dissimilar metals have different coefficients of expansion which cause the strip to


Cold


Hot, metal A expands more than metal B

Figure 1.21 Bimetallic strip
bend according to the temperature. This technique is the basis of most on/off thermostats used for temperature control or alarm annunciation. A bimetallic spiral can be used to construct an indicating thermometer.

Electrical resistance changes with temperature. A platinum wire with resistance 100 ohms at $0^{\circ} \mathrm{C}$ will have a resistance of 138.5 ohms at $100^{\circ} \mathrm{C}$. Temperature sensors based on this principle are known as RTDs (for resistance temperature detector) or PT100 sensors (from PT, for platinum, and 100 for 100 ohms at $0^{\circ} \mathrm{C}$ ). Semiconductor devices called thermistors have more dramatic changes, the characteristics of a typical device being shown in Figure 1.22. The response, however, is non-linear which makes thermistors more suitable for alarm/control application than temperature indication.


Figure 1.22 Typical resistance temperature curve for NTC thermistor

Thermocouples, the principle of which is shown in Figure 1.23, use the small difference in contact potentials between different metals to give a voltage which depends on the temperature difference between the measurement and reference points. Although widely used in process control, the technique is rarely encountered in pneumatic and hydraulic systems.

The final method, called pyrometry, uses the change in radiated energy with temperature. As this has a minimum temperature measurement of about $400^{\circ} \mathrm{C}$, it is totally unsuitable for the systems we shall be discussing.


Figure 1.23 The thermocouple

## Gas laws

For all practical purposes, liquids used in hydraulic systems can be considered incompressible and insensitive to changes in temperature (provided the temperature remains within some quite broad limits). The gas in a pneumatic system is very sensitive to changes in pressure and temperature, and its behaviour is determined by the gas laws described below.

In the following expressions it is important to note that pressures are given in absolute, not gauge, terms and temperatures are given in absolute degrees Kelvin, not in degrees Celsius. If we discuss, say, a litre of air at atmospheric pressure and $20^{\circ} \mathrm{C}$ being compressed to three atmospheres gauge pressure, its original pressure was one atmosphere, its original temperature was 293 K and its final pressure is four atmospheres absolute.

Pressure and volume are related by Boyle's law. In Figure 1.24 we have a volume of gas $V_{1}$ at pressure $P_{1}$ (in absolute units,


Figure 1.24 Boyle's law
remember). This gas is compressed to volume $V_{2}$, which will result in a rise of pressure to $P_{2}$, where:

$$
\begin{equation*}
\mathrm{P}_{1} \mathrm{~V}_{1}=\mathrm{P}_{2} \mathrm{~V}_{2} . \tag{1.17}
\end{equation*}
$$

provided the temperature of the gas does not change during the compression. A reduction of pressure similarly leads to an increase in volume.

In practice, compression of a gas is always accompanied by a rise in temperature (as is commonly noticed when pumping up a bicycle tyre) and a reduction in pressure produces a temperature fall (the principle of refrigeration). For expression 1.17 to apply, the gas must be allowed to return to its original temperature.


Figure 1.25 Relationship between temperature and pressure

In Figure 1.25 , on the other hand, the temperature of a fixed volume of gas is controlled by a heater. A rise in temperature from $T_{1}$ to $T_{2}$ results in an increase in pressure from $P_{1}$ to $P_{2}$, where:

$$
\begin{equation*}
\frac{\mathrm{P}_{1}}{\mathrm{~T}_{1}}=\frac{\mathrm{P}_{2}}{\mathrm{~T}_{2}} . \tag{1.18}
\end{equation*}
$$

Again it should be remembered pressure and temperature are in absolute terms. Although expression 1.18 gives the change in pressure resulting from a change in temperature, it also applies to changes of temperature resulting from a change in pressure provided no heat is lost from the system. In a pneumatic air compressor, the temperature of the outgoing compressed air is considerably elevated by the increase in pressure, resulting in the need for the compressor to be followed by an air cooler.

Expressions 1.17 and 1.18 are combined to give the general gas law:

$$
\begin{equation*}
\frac{P_{1} V_{1}}{T_{1}}=\frac{P_{2} V_{2}}{T_{2}} \tag{1.1}
\end{equation*}
$$

where $\mathrm{P}_{1}, \mathrm{~V}_{1}, \mathrm{~T}_{1}$ are initial conditions and $\mathrm{P}_{2}, \mathrm{~V}_{2}, \mathrm{~T}_{2}$ are final conditions. As before, expression 1.19 assumes no heat is lost to, or gained from, the environment.

