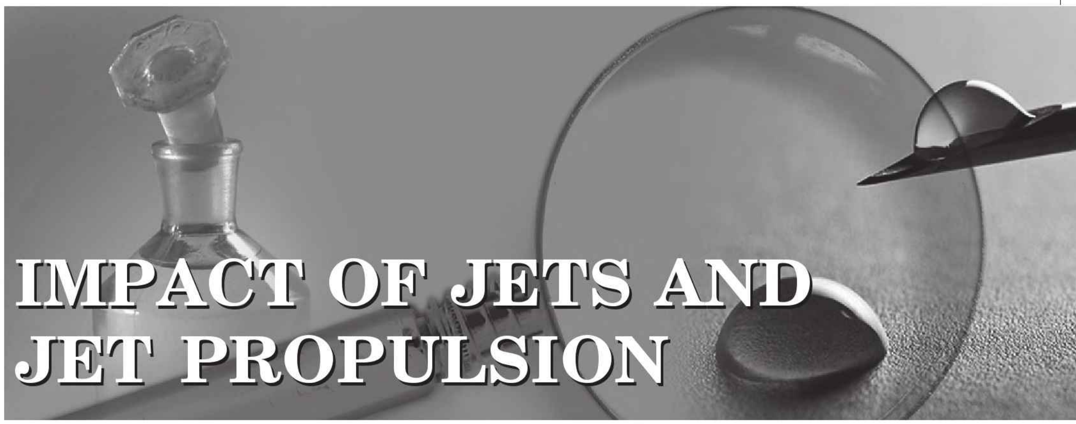
17
CHAPTER



IMPACT OF JETS AND JET PROPULSION

▶ 17.1 INTRODUCTION

The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressure. If some plate, which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate. This force is obtained from Newton's second law of motion or from impulse-momentum equation. Thus impact of jet means the force exerted by the jet on a plate which may be stationary or moving. In this chapter, the following cases of the impact of jet *i.e.,* the force exerted by the jet on a plate, will be considered :

1. Force exerted by the jet on a stationary plate when
   (*a*) Plate is vertical to the jet, (*b*) Plate is inclined to the jet, and (*c*) Plate is curved.
2. Force exerted by the jet on a moving plate, when
   (*a*) Plate is vertical to the jet, (*b*) Plate is inclined to the jet, and (*c*) Plate is curved.

▶ 17.2 FORCE EXERTED BY THE JET ON A STATIONARY VERTICAL PLATE

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate as shown in Fig. 17.1

Let V = velocity of the jet, d = diameter of the jet,

a = area of cross-section of the jet = $\frac{\pi}{4} d^2$.

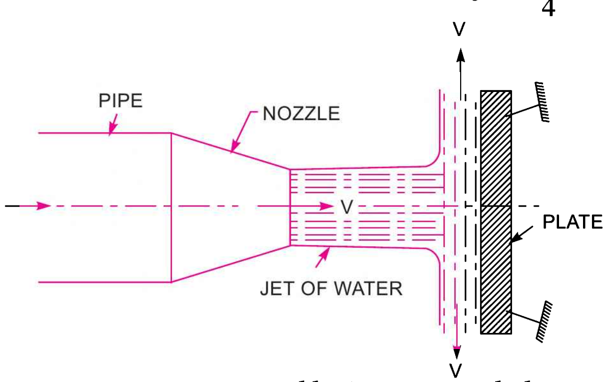


Fig. 17.1 *Force exerted by jet on vertical plate.*

The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking, will get deflected through 90°. Hence the component of the velocity of jet, in the direction of jet, after striking will be zero.

The force exerted by the jet on the plate in the direction of jet,

$$F_x = \text{Rate of change of momentum in the direction of force}$$

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} [\text{Initial velocity} - \text{Final velocity}]$$

$$= (\text{Mass/sec}) \times (\text{velocity of jet before striking} - \text{velocity of jet after striking})$$

$$= \rho a V [V - 0] \qquad (\because \text{ mass/sec} = \rho \times a\, V)$$

$$= \rho a V^2 \qquad \ldots(17.1)$$

For deriving above equation, we have taken initial velocity minus final velocity and not final velocity minus initial velocity. If the force exerted on the jet is to be calculated then final minus initial velocity is taken. But if the force exerted by the jet on the plate is to be calculated, then initial velocity minus final velocity is taken.

Note. In equation (17.1), if the value of density (ρ) is taken in S.I. units (*i.e.,* kg/m^3), the force (F_x) will be in Newton (N). The value of ρ for water in S.I. units is equal to 1000 kg/m^3.

17.2.1 Force Exerted by a Jet on Stationary Inclined Flat Plate. Let a jet of water, coming out from the nozzle, strikes an inclined flat plate as shown in Fig. 17.2.

Let V = Velocity of jet in the direction of x,

θ = Angle between the jet and plate,

a = Area of cross-section of the jet.

Then mass of water per sec striking the plate = $\rho \times aV$.

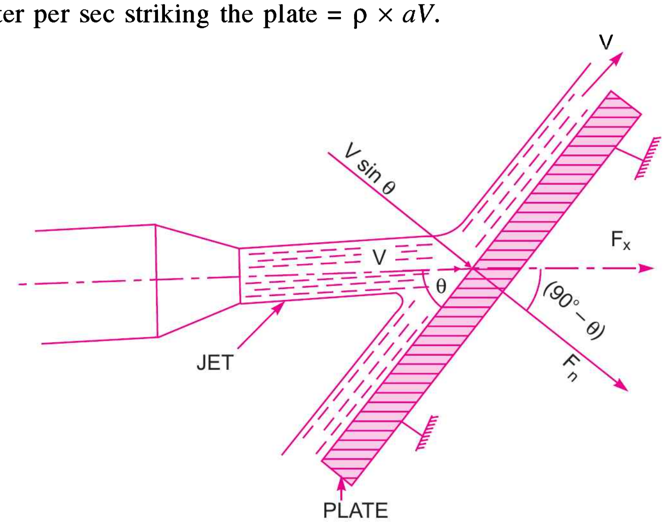


Fig. 17.2 *Jet striking stationary inclined plate.*

If the plate is smooth and if it is assumed that there is no loss of energy due to impact of the jet, then jet will move over the plate after striking with a velocity equal to initial velocity *i.e.*, with a velocity V. Let us find the force exerted by the jet on the plate in the direction normal to the plate. Let this force is represented by F_n

Then F_n = mass of jet striking per second
× [Initial velocity of jet before striking in the direction of n
– Final velocity of jet after striking in the direction of n]

$$= \rho aV [V \sin\theta - 0] = \rho aV^2 \sin\theta \quad \text{...(17.2)}$$

This force can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of flow. Then we have,

F_x = component of F_n in the direction of flow

$$= F_n \cos(90° - \theta) = F_n \sin\theta = \rho A V^2 \sin\theta \times \sin\theta \quad (\because F_n = \rho aV^2 \sin\theta)$$

$$= \rho AV^2 \sin^2\theta \quad \text{...(17.3)}$$

And, F_y = component of F_n, perpendicular to flow

$$= F_n \sin(90° - \theta) = F_n \cos\theta = \rho AV^2 \sin\theta \cos\theta. \quad \text{...(17.4)}$$

17.2.2 Force Exerted by a Jet on Stationary Curved Plate

(A) Jet strikes the curved plate at the centre. Let a jet of water strikes a fixed curved plate at the centre as shown in Fig. 17.3. The jet after striking the plate, comes out with the same velocity if the plate is smooth and there is no loss of energy due to impact of the jet, in the tangential direction of the curved plate. The velocity at outlet of the plate can be resolved into two components, one in the direction of jet and other perpendicular to the direction of the jet.

Component of velocity in the direction of jet = $- V \cos\theta$.

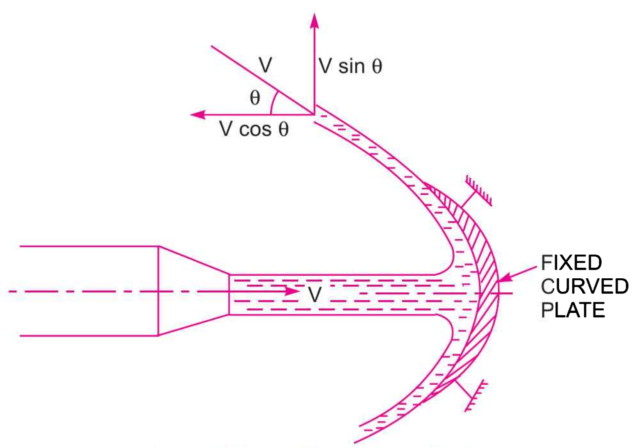


Fig. 17.3 *Jet striking a fixed curved plate at centre.*

(–ve sign is taken as the velocity at outlet is in the opposite direction of the jet of water coming out from nozzle).

Component of velocity perpendicular to the jet = $V \sin\theta$

Force exerted by the jet in the direction of jet,

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

where V_{1x} = Initial velocity in the direction of jet = V

V_{2x} = Final velocity in the direction of jet = $- V \cos\theta$

$\therefore \quad F_x = \rho aV[V - (- V \cos \theta)] = \rho aV[V + V \cos \theta]$

$= \rho aV^2[1 + \cos \theta]$ ...(17.5)

Similarly, F_y = Mass per sec × $[V_{1y} - V_{2y}]$

where V_{1y} = Initial velocity in the direction of $y = 0$

V_{2y} = Final velocity in the direction of $y = V \sin \theta$

$\therefore \quad F_y = \rho aV[0 - V \sin \theta] = - \rho aV^2 \sin \theta$ ...(17.6)

–ve sign means that force is acting in the downward direction. In this case the angle of deflection of the jet $= (180° - \theta)$ ...[17.6 (*A*)]

(B) Jet strikes the curved plate at one end tangentially when the plate is symmetrical. Let the jet strikes the curved fixed plate at one end tangentially as shown in Fig. 17.4. Let the curved plate is symmetrical about *x*-axis. Then the angle made by the tangents at the two ends of the plate will be same.

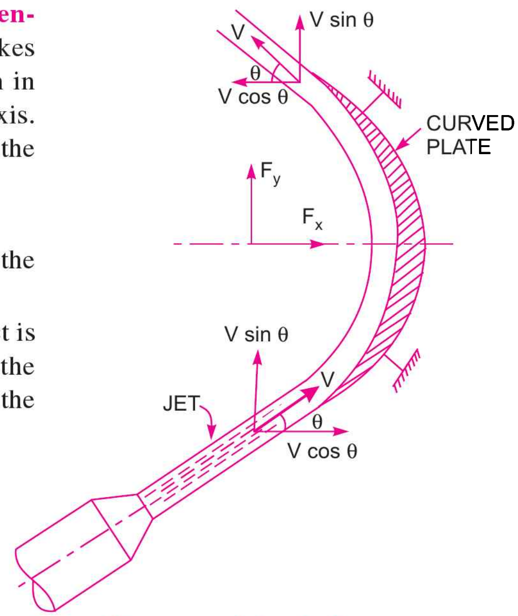


Fig. 17.4 *Jet striking curved fixed plate at one end.*

Let V = Velocity of jet of water,

θ = Angle made by jet with *x*-axis at inlet tip of the curved plate.

If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the outlet tip of the curved plate will be equal to V. The forces exerted by the jet of water in the directions of x and y are

F_x = (mass/sec) × $[V_{1x} - V_{2x}]$

$= \rho aV[V \cos \theta - (- V \cos \theta)]$

$= \rho aV[V \cos \theta + V \cos \theta]$

$= 2\rho aV^2 \cos \theta$ ...(17.7)

$F_y = \rho aV[V_{1y} - V_{2y}]$

$= \rho aV[V \sin \theta - V \sin \theta] = 0$

(C) Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical. When the curved plate is unsymmetrical about *x*-axis, then angle made by the tangents drawn at the inlet and outlet tips of the plate with *x*-axis will be different.

Let θ = angle made by tangent at inlet tip with *x*-axis,

ϕ = angle made by tangent at outlet tip with *x*-axis.

The two components of the velocity at inlet are

$V_{1x} = V \cos \theta$ and $V_{1y} = V \sin \theta$

The two components of the velocity at outlet are

$V_{2x} = - V \cos \phi$ and $V_{2y} = V \sin \phi$

$\therefore$ The forces exerted by the jet of water in the directions of x and y are

$F_x = \rho aV[V_{1x} - V_{2x}] = \rho aV[V \cos \theta - (-V \cos \phi)]$

$= \rho aV[V \cos \theta + V \cos \phi] = \rho aV^2 [\cos \theta + \cos \phi]$ ...(17.8)

$F_y = \rho aV[V_{1y} - V_{2y}] = \rho aV[V \sin \theta - V \sin \phi]$

$= \rho aV^2 [\sin \theta - \sin \phi]$. ...(17.9)

Problem 17.1 *Find the force exerted by a jet of water of diameter 75 mm on a stationary flat plate, when the jet strikes the plate normally with velocity of 20 m/s.*

Solution. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.075)^2 = .004417 \text{ m}^2$

Velocity of jet, $V = 20 \text{ m/s.}$

The force exerted by the jet of water on a stationary vertical plate is given by equation (17.1) as

$$F = \rho a V^2 \quad \text{where } \rho = 1000 \text{ kg/m}^3$$

$$\therefore \quad F = 1000 \times .004417 \times 20^2 \text{ N} = \mathbf{1766.8 \text{ N. Ans.}}$$

Problem 17.2 *Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and the head of water at the centre nozzle is 100 m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95.*

Solution. Given :

Diameter of nozzle, $d = 100 \text{ mm} = 0.1 \text{ m}$

Head of water, $H = 100 \text{ m}$

Co-efficient of velocity, $C_v = 0.95$

Area of nozzle, $a = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$

Theoretical velocity of jet of water is given as

$$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 100} = 44.294 \text{ m/s}$$

But $$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}}$$

$\therefore$ Actual velocity of jet of water, $V = C_v \times V_{th} = 0.95 \times 44.294 = 42.08 \text{ m/s.}$

Force on a fixed vertical plate is given by equation (17.1) as

$$F = \rho a V^2 = 1000 \times .007854 \times 42.08^2 \quad (\because \text{ In S.I. units } \rho \text{ for water} = 1000 \text{ kg/m}^3)$$

$$= 13907.2 \text{ N} = \mathbf{13.9 \text{ kN. Ans.}}$$

Problem 17.3 *A jet of water of diameter 75 mm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by the jet on the plate (i) in the direction normal to the plate and (ii) in the direction of the jet.*

Solution. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.075)^2 = 0.004417 \text{ m}^2$

Velocity of jet, $V = 25 \text{ m/s.}$

Angle between jet and plate $\theta = 60°$

(*i*) The force exerted by the jet of water in the direction normal to the plate is given by equation (17.2) as

$$F_n = \rho a V^2 \sin \theta$$

$$= 1000 \times .004417 \times 25^2 \times \sin 60° = \mathbf{2390.7 \text{ N. Ans.}}$$

(ii) The force in the direction of the jet is given by equation (17.3),

$$F_x = \rho a V^2 \sin^2 \theta$$
$$= 1000 \times .004417 \times 25^2 \times \sin^2 60° = \mathbf{2070.4\ N.\ Ans.}$$

Problem 17.4 *A jet of water of diameter 50 mm strikes a fixed plate in such a way that the angle between the plate and the jet is 30°. The force exerted in the direction of the jet is 1471.5 N. Determine the rate of flow of water.*

Solution. Given :

Diameter of jet, $d = 50$ mm $= 0.05$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.05)^2 = .001963\ m^2$

Angle, $\theta = 30°$

Force in the direction of jet, $F_x = 1471.5$ N

Force in the direction of jet is given by equation (17.3) as $F_x = \rho a V^2 \sin^2 \theta$

As the force is given in Newton, the value of ρ should be taken equal to 1000 kg/m^3

$$\therefore \quad 1471.5 = 1000 \times .001963 \times V^2 \times \sin^2 30° = .05\ V^2$$

$$\therefore \quad V^2 = \frac{150}{.05} = 3000.0$$

$$V = 54.77\ m/s$$

$\therefore$ Discharge, Q = Area $\times$ Velocity

$$= .001963 \times 54.77 = 0.1075\ m^3/s = \mathbf{107.5\ liters/s.\ Ans.}$$

Problem 17.5 *A jet of water of diameter 50 mm moving with a velocity of 40 m/s, strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate.*

Solution. Given :

Diameter of the jet, $d = 50$ mm $= 0.05$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.05)^2 = 0.001963\ m^2$

Velocity of jet, $V = 40$ m/s

Angle of deflection $= 120°$

From equation [17.6 (A)], the angle of deflection $= 180° - \theta$

$$\therefore \quad 180° - \theta = 120° \text{ or } \theta = 180° - 120° = 60°$$

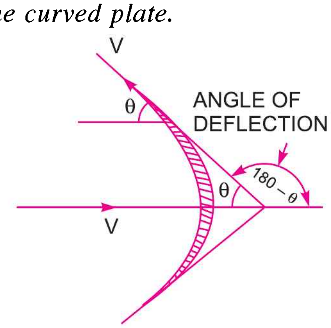


Fig. 17.5

Force exerted by the jet on the curved plate in the direction of the jet is given by equation (17.5) as

$$F_x = \rho a V^2 [1 + \cos \theta]$$
$$= 1000 \times .001963 \times 40^2 \times [1 + \cos 60°] = \mathbf{4711.15\ N.\ Ans.}$$

Problem 17.6 *A jet of water of diameter 75 mm moving with a velocity of 30 m/s, strikes a curved fixed plate tangentially at one end at an angle of 30° to the horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal and vertical direction.*

Solution. Given :

Diameter of the jet, $d = 75$ mm $= 0.075$ m

∴ Area, $a = \frac{\pi}{4} (.075)^2 = .004417 \text{ m}^2$

Velocity of jet, $V = 30$ m/s

Angle made by the jet at inlet tip with horizontal, $\theta = 30°$

Angle made by the jet at outlet tip with horizontal, $\phi = 20°$

The force exerted by the jet of water in the direction of x is given by equation (17.8) and in the direction of y by equation (17.9),

$$\therefore \quad F_x = \rho a V^2 [\cos\theta + \cos\phi]$$

$$= 1000 \times .004417 [\cos 30° + \cos 20°] \times 30^2 = \mathbf{7178.2 \text{ N. Ans.}}$$

and

$$F_y = \rho a V^2 [\sin\theta - \sin\phi]$$

$$= 1000 \times .004417 [\sin 30° - \sin 20°] \times 30^2 = \mathbf{628.13 \text{ N. Ans.}}$$

▶ 17.3 FORCE EXERTED BY A JET ON A HINGED PLATE

Consider a jet of water striking a vertical plate at the centre which is hinged at O. Due to the force exerted by the jet on the plate, the plate will swing through some angle about the hinge as shown in Fig. 17.6

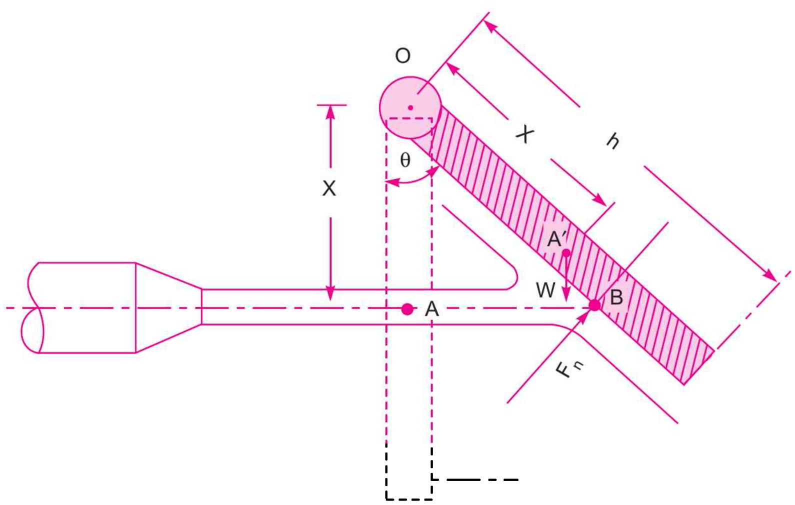


Fig. 17.6 *Force on a hinged plate.*

Let x = distance of the centre of jet from hinge O,

θ = angle of swing about hinge,

W = weight of plate acting at C.G. of the plate.

The dotted lines show the position of the plate, before the jet strikes the plate. The point A on the plate will be at A' after the jet strikes the plate. The distance $OA = OA' = x$. Let the weight of the plate is acting at A'. When the plate is in equilibrium after the jet strikes the plate, the moment of all the forces about the hinge must be zero. Two forces are acting on the plate. They are :

1. Force due to jet of water, normal to the plate,

$$F_n = \rho a V^2 \sin \theta'$$

where θ' = Angle between jet and plate = $(90° - \theta)$

2. Weight of the plate, W

Moment of force F_n about hinge $= F_n \times OB = \rho a V^2 \sin (90° - \theta) \times OB = \rho a V^2 \cos \theta \times OB$

$$= \rho a V^2 \cos \theta \times \frac{OA}{\cos \theta} = \rho a V^2 \times OA = \rho a V^2 \times x$$

Moment of weight W about hinge $= W \times OA' \sin \theta = W \times x \times \sin \theta$

For equilibrium of the plate, $\rho a V^2 \times x = W \times x \times \sin \theta$

$$\therefore \quad \sin \theta = \frac{\rho a V^2}{W} \quad \text{...(17.10)}$$

From equation (17.10), the angle of swing of the plate about hinge can be calculated.

Problem 17.7 *A jet of water of 2.5 cm diameter, moving with a velocity of 10 m/s, strikes a hinged square plate of weight 98.1 N at the centre of the plate. The plate is of uniform thickness. Find the angle through which the plate will swing.*

Solution. Given :

Diameter of jet, $d = 2.5 \text{ cm} = 0.025 \text{ m}$

Velocity of jet, $V = 10 \text{ m/s}$

Weight of plate, $W = 98.1 \text{ N}$

Area of jet, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (.025)^2 = .00049 \text{ m}^2$

The angle through which the plate will swing is given by equation (17.10) as

$$\sin \theta = \frac{\rho a V^2}{W} = 1000 \times \frac{.00049 \times 10^2}{98.1} \quad (\because \ \rho = 1000)$$

$$= .499$$

$\therefore$ $\theta =$ **29.96°. Ans.**

Problem 17.8 *A jet of water of 30 mm diameter strikes a hinged square plate at its centre with a velocity of 20 m/s. The plate is deflected through an angle of 20°. Find the weight of the plate.*

If the plate is not allowed, to swing, what will be the force required at the lower edge of the plate to keep the plate in vertical position.

Solution. Given :

Diameter of the jet, $d = 30 \text{ mm} = 3 \text{ cm} = 0.03 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.03)^2 = .0007068 \text{ m}^2$

Velocity of jet, $V = 20 \text{ m/s}$

Angle of swing, $\theta = 20°$

Using equation (17.10) for angle of swing,

$$\sin\theta = \frac{\rho a V^2}{W}$$

or $$\sin 20^\circ = 1000 \times \frac{.0007068 \times 20^2}{W} = \frac{282.72}{W}$$

$\therefore$ $$W = \frac{282.72}{\sin 20^\circ} = 826.6 \text{ N}$$

If the plate is not allowed to swing, a force P will be applied at the lower edge of the plate as shown in Fig. 17.7. The weight of the plate is acting vertically downward through the C.G. of the plate.

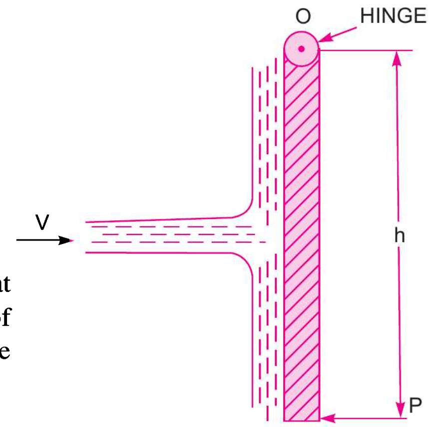


Fig. 17.7

Let F = Force exerted by jet of water

h = Height of plate

= Distance of P from the hinge.

The jet strikes at the centre of the plate and hence distance of the centre of the jet from hinge = $\frac{h}{2}$.

Taking moments* about the hinge, O, $P \times h = F \times \frac{h}{2}$.

$\therefore$ $$P = \frac{F \times h}{2 \times h} = \frac{F}{2} = \frac{\rho a V^2}{2} \qquad (\because \; F = \rho a V^2)$$

$$= 1000 \times \frac{.0007068 \times 20^2}{2} = \textbf{141.36 N. Ans.}$$

Problem 17.9 *A rectangular plate, weighing 58.86 N is suspended vertically by a hinge on the top of horizontal edge. The centre of gravity of the plate is 10 cm from the hinge. A horizontal jet of water 2 cm diameter, whose axis is 15 cm below the hinge impinges normally on the plate with a velocity of 5 m/s. Find the horizontal force applied at the centre of the gravity to maintain the plate in its vertical position. Find the corresponding velocity of the jet, if the plate is deflected through 30° and the same force continues to act at the centre of gravity of the plate.*

Solution. Given :

Weight of plate, W = 58.86 N

Distance of W from hinge, x = 10 cm = 0.1 m

Diameter of jet, d = 2 cm = 0.02 m

$\therefore$ Area, $$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times .02^2 = .000314 \text{ m}^2$$

Distance of the axis of the jet of water from hinge = 15 cm = 0.15 m

Velocity of jet, V = 5 m/s

(*i*) Let the force applied at the centre of gravity of the plate to keep the plate in vertical position = P as shown in Fig. 17.8 (*a*).

* The weight of the plate is passing through the hinge O. Hence moment of W about hinge is zero.

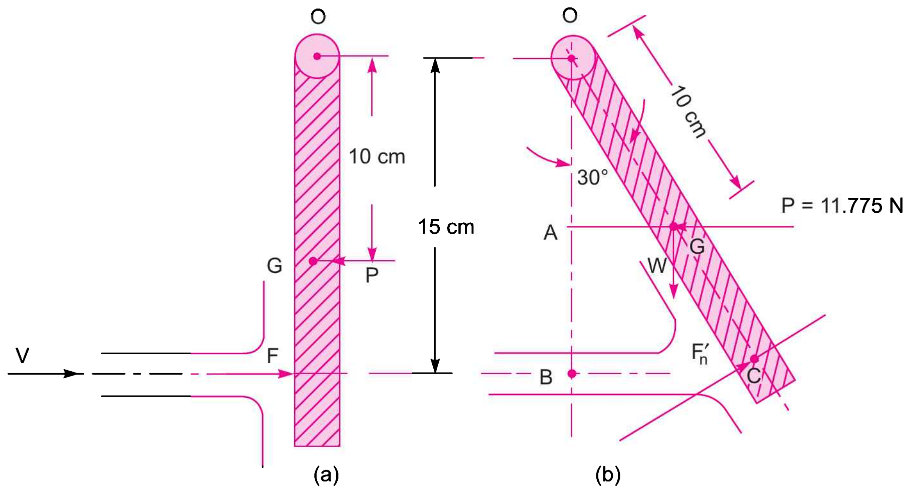


Fig. 17.8

The force exerted by a jet of water on the vertical plate,

$$F = \rho a V^2 = 1000 \times .000314 \times 5^2 = 7.85 \text{ N}$$

This force F is acting at a distance of 15 cm or 0.15 m from the hinge. Taking moments about hinge, we get

$$F \times 0.15 = P \times 0.10$$

$$\therefore \quad P = \frac{F \times 0.15}{0.10} = \frac{7.85 \times .15}{.10} = \textbf{11.775 N. Ans.}$$

(*ii*) The plate is deflected through an angle of 30° as shown in Fig. 17.8(*b*).

$\therefore$ Angle of swing = 30°

The force at the C.G. = P = 11.775 N

Let the velocity of the jet in this position = V m/s

For the deflected position of the plate as shown in Fig. 17.8 (*b*), the plate is in equilibrium under the action of three forces, which are :

(*i*) Weight of the plate, W acting at G at a distance 10 cm from O.

(*ii*) Horizontal force, P acting at G.

(*iii*) Normal force F_n' on the plate due to jet of water.

The angle between the jet and the plate, $\theta = 90° - 30° = 60°$

Hence, F_n' is given by equation (17.2) as

$$F_n' = \rho a V^2 \sin \theta = \rho a V^2 \sin 60°$$

$$= 1000 \times .000314 \times V^2 \times \sin 60° = 0.2717\ V^2$$

Taking moments of all forces about hinge O, we get

$$F_n' \times OC = P \times OA + W \times AG \qquad \ldots(i)$$

where $OB = OC \cos 30°$

$$\therefore \quad OC = \frac{OB}{\cos 30^\circ} = \frac{15}{\cos 30^\circ} = 17.32 \text{ cm} = 0.1732 \text{ m}$$

$$OA = OG \cos 30° = 10 \times .866 = 8.66 \text{ cm} = 0.0866 \text{ m}$$

$$AG = OG \sin 30° = 10 \times \frac{1}{2} = 5 \text{ cm} = .05 \text{ m}$$

Substituting these values in equation (*i*), we get

$$0.2717\, V^2 \times .1732 = 11.775 \times .0866 + 58.86 \times 0.05 = 3.962$$

$$\therefore \qquad V = \sqrt{\frac{3.962}{0.2717 \times .1732}} = 9.175 \text{ m/s.}$$

Problem 17.10 *A jet of water of diameter 25 mm strikes a 20 cm × 20 cm square plate of uniform thickness with a velocity of 10 m/s at the centre of the plate which is suspended vertically by a hinge on its top horizontal edge. The weight of the plate is 98.1 N. The jet strikes normal to the plate. What force must be applied at the lower edge of the plate so that plate is kept vertical ? If the plate is allowed to deflect freely, what will be the inclination of the plate with vertical due to the force exerted by jet of water?*

Solution. Given :

Diameter of the jet, $\quad d = 25 \text{ mm} = 25 \times 10^{-3} \text{ m} = .025 \text{ m}$

$\therefore$ Area, $\quad a = \frac{\pi}{4} (.025)^2 = .00049 \text{ m}^2$

Size of the plate, $\quad = 20 \text{ cm} \times 20 \text{ cm}$

Weight of the plate, $\quad W = 98.1 \text{ N}$

Velocity of jet, $\quad V = 10 \text{ m/s}$

(*i*) Let the force applied at the lower edge to keep the plate in vertical position is = P . See Fig.17.9 (*a*).

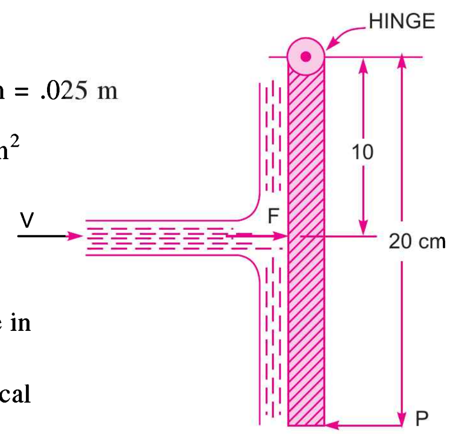


Fig. 17.9 (*a*)

Force exerted by the jet of water at the centre of the vertical plate,

$$F = \rho a V^2$$
$$= 1000 \times .00049 \times 10^2 = 49 \text{ N.}$$

This force is acting at a distance of $\frac{20}{2}$ = 10 cm from the hinge. The force P is acting at a distance of 20 cm from the hinge.

Taking moments about hinge,

$$F \times 10 = P \times 20$$

$$\therefore \qquad 49 \times 10 = P \times 20$$

$$\therefore \qquad P = \frac{49 \times 10}{20} = \mathbf{24.5 \text{ N. Ans.}}$$

(*ii*) When the plate is allowed to deflect freely about hinge.

Let the inclination of the plate with vertical = θ

In this position, the angle between the plate and jet will be

$$= (90° - \theta) .$$

$\therefore$ Force exerted by water normal to the plate is given by equation (17.2) as

$$F_n = \rho a V^2 \sin (90° - \theta) = \rho a V^2 \cos \theta$$

The distance $OB = \frac{OA}{\cos\theta} = \frac{10}{\cos\theta}$

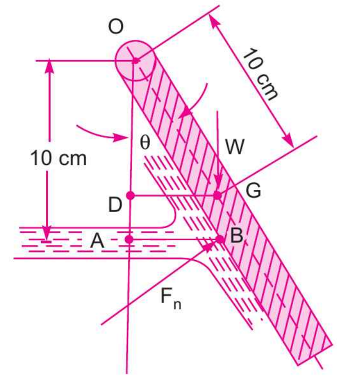


Fig. 17.9. (*b*)

The weight W of the plate is acting at a distance 10 cm from hinge. Distance

$$DG = OG \sin\theta = 10 \times \sin\theta$$

Taking moments about hinge, we get

$$F_n \times OB = W \times GD$$

or
$$\rho a V^2 \cos\theta \times \frac{10}{\cos\theta} = W \times 10 \times \sin\theta$$

$\therefore$
$$\rho a V^2 = W \times \sin\theta$$

$\therefore$
$$\sin\theta = \frac{\rho a V^2}{W} = 1000 \times \frac{.00049 \times 10^2}{98.1} = 0.5$$

$\therefore$
$$\theta = \mathbf{30°.\ Ans.}$$

Problem 17.10 (A) *A square plate of uniform thickness and length of side 300 mm hangs vertically from hinge at its top edge. When a horizontal water jet strikes the plate at its centre, the plate is deflected and comes to rest at angle of 30° to the vertical. The jet is 25 mm in diameter and has a velocity of 6 m/s. Determine the weight of the plate.*

Solution. Given :

Length of plate, $L = 300$ mm $= 0.3$ m

Angle of swing, or angle made by deflected plate with the vertical, $\theta = 30°$

Dia. of the jet, $d = 25$ mm $= 0.025$ m

$\therefore$ Area of jet, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.025^2)\ m^2$

Velocity of jet, $V = 6$ m/s

Let W = Weight of plate

Using equation (17.10), we get $\sin\theta = \frac{\rho \times a \times V^2}{W}$

$\therefore$
$$W = \frac{\rho^* \times a \times V^2}{\sin\theta} = \frac{1000 \times \left(\frac{\pi}{4} \times 0.025^2\right) \times 6^2}{\sin 30°} = \mathbf{35.33\ N.\ Ans.}$$

▶ 17.4 FORCE EXERTED BY A JET ON MOVING PLATES

The following cases of the moving plates will be considered :

1. Flat vertical plate moving in the direction of the jet and away from the jet,
2. Inclined plate moving in the direction of the jet, and
3. Curved plate moving in the direction of the jet or in the horizontal direction.

* If $\rho = 1000\ kg/m^3$, then weight W will be in Newton.

17.4.1 Force on Flat Vertical Plate Moving in the Direction of Jet. Fig. 17.10 shows a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

Let V = Velocity of the jet (absolute),
a = Area of cross-section of the jet,
u = Velocity of the flat plate.

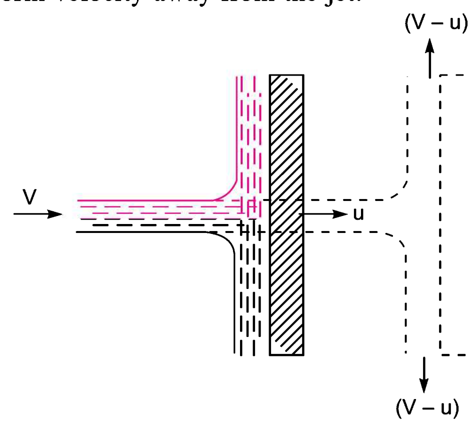


Fig. 17.10 *Jet striking a flat vertical moving plate.*

In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus the velocity of the plate.

Hence relative velocity of the jet with respect to plate

$$= (V - u)$$

Mass of water striking the plate per sec

= ρ × Area of jet × Velocity with which jet strikes the plate

$$= \rho a \times [V - u]$$

∴ Force exerted by the jet on the moving plate in the direction of the jet,

F_x = Mass of water striking per sec × [Initial velocity with which water strikes – Final velocity]

$$= \rho a(V - u)\,[(V - u) - 0] \qquad (\because \text{ Final velocity in the direction of jet is zero})$$

$$= \rho a(V - u)^2 \qquad ...(17.11)$$

In this case, the work will be done by the jet on the plate, as plate is moving. For the stationary plates, the work done is zero.

∴ Work done per second by the jet on the plate

$$= \text{Force} \times \frac{\text{Distance in the direction of force.}}{\text{Time}}$$

$$= F_x \times u = \rho a(V - u)^2 \times u \qquad ...(17.12)$$

In equation (17.12), if the value of ρ for water is taken in S.I. units (*i.e.,* 1000 kg/m^3), the work done will be in N m/s. The term $\frac{\text{'Nm'}}{\text{s}}$ is equal to W (watt).

17.4.2 Force on the Inclined Plate Moving in the Direction of the Jet. Let a jet of water strikes an inclined plate, which is moving with a uniform velocity in the direction of the jet as shown in Fig. 17.11.

Let V = Absolute velocity of jet of water,
u = Velocity of the plate in the direction of jet,
a = Cross-sectional area of jet, and
θ = Angle between jet and plate.

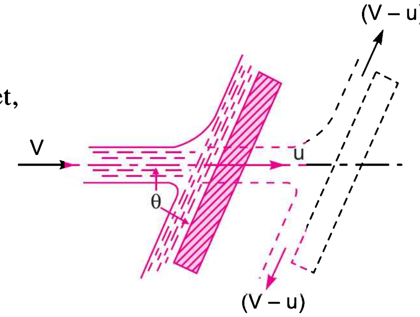


Fig. 17.11 *Jet striking an inclined moving plate.*

Relative velocity of jet of water = $(V - u)$

∴ The velocity with which jet strikes = $(V - u)$

Mass of water striking per second

$$= \rho \times a \times (V - u)$$

If the plate is smooth and loss of energy due to impact of the jet is assumed zero, the jet of water will leave the inclined plate with a velocity equal to $(V - u)$.

The force exerted by the jet of water on the plate in the direction normal to the plate is given as

$$F_n = \text{Mass striking per second} \times [\text{Initial velocity in the normal direction with which jet strikes} - \text{Final velocity}]$$

$$= \rho a (V - u) [(V - u) \sin \theta - 0] = \rho a (V - u)^2 \sin \theta \qquad ...(17.13)$$

This normal force F_n is resolved into two components namely F_x and F_y in the direction of the jet and perpendicular to the direction of the jet respectively.

$$\therefore \quad F_x = F_n \sin \theta = \rho a (V - u)^2 \sin^2 \theta \qquad ...(17.14)$$

$$F_y = F_n \cos \theta = \rho a (V - u)^2 \sin \theta \cos \theta \qquad ...(17.15)$$

$\therefore$ Work done per second by the jet on the plate

$$= F_x \times \text{Distance per second in the direction of } x$$

$$= F_x \times u = \rho a(V - u)^2 \sin^2 \theta \times u = \rho a (V - u)^2 u \sin^2 \theta \text{ N m/s.} \qquad ...(17.16)$$

Problem 17.11 *A jet of water of diameter 10 cm strikes a flat plate normally with a velocity of 15 m/s. The plate is moving with a velocity of 6 m/s in the direction of the jet and away from the jet. Find:*

(i) the force exerted by the jet on the plate

(ii) work done by the jet on the plate per second.

Solution. Given :

Diameter of the jet, $d = 10 \text{ cm} = 0.1 \text{ m}$

$\therefore$ Area, $a = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$

Velocity of jet, $V = 15$ m/s

Velocity of the plate, $u = 6$ m/s.

(*i*) The force exerted by the jet on a moving flat vertical plate is given by equation (17.11),

$$F_x = \rho a (V - u)^2$$

$$= 1000 \times .007854 \times (15 - 6)^2 \text{ N} = \mathbf{636.17 \text{ N. Ans.}}$$

(*ii*) Work done per second by the jet

$$= F_x \times u = 636.17 \times 6 = \mathbf{3817.02 \text{ Nm/s. Ans.}}$$

Problem 17.12 *For Problem 17.11, find the power and efficiency of the jet.*

Solution. The given data from Problem 17.11 is

$$a = .007854 \text{ m}^2, V = 15 \text{ m/s}, u = 6 \text{ m/s}$$

Also work done per second by the jet = 3817.02 Nm/s

(*i*) Power of the jet in kW $= \dfrac{\text{Work done per second}}{1000} = \dfrac{3817.02}{1000} = \mathbf{3.817 \text{ kW. Ans.}}$

(*ii*) Efficiency of the jet (η) $= \dfrac{\text{Output of the jet per second}}{\text{Input of the jet per second}}$ ...(*i*)

where output of jet/sec = Work done by jet per second = 3817.02 Nm/s

And input per second = Kinetic energy of the jet/sec

$$= \frac{1}{2}\left(\frac{\text{mass}}{\text{sec}}\right) V^2 = \frac{1}{2}(\rho a V) \times V^2 = \frac{1}{2}\rho a\, V^3$$

$$= \frac{1}{2} \times 1000 \times .007854 \times 15^3 \text{ Nm/s} = 13253.6 \text{ Nm/s}$$

$\therefore$ $$\eta \text{ of the jet} = \frac{3817.02}{13253.6} = 0.288 = \mathbf{28.8\%.\ Ans.}$$

Problem 17.12 (A) *A nozzle of 50 mm diameter delivers a stream of water at 20 m/s perpendicular to a plate that moves away from the jet at 5 m/s. Find :*

(i) the force on the plate,

(ii) the work done, and

(iii) the efficiency of jet. (J.N.T.U., Hyderabad S 2002)

Solution. Given :

Dia. of jet = 50 mm = 0.05 m

$\therefore$ Area, $$a = \frac{\pi}{4}(0.05^2) = 0.0019635 \text{ m}^2$$

Velocity of jet, $V = 20$ m/s, Velocity of plate, $u = 5$ m/s

(*i*) The force on the plate is given by equation (17.11) as,

$$F_x = \rho a\,(V - u)^2$$

$$= 1000 \times 0.0019635 \times (20 - 5)^2 = \mathbf{441.78\ N.\ Ans.}$$

(*ii*) The work done by the jet

$$= F_x \times u = 441.78 \times 5 = \mathbf{2208.9\ Nm/s.\ Ans.}$$

(*iii*) The efficiency of the jet, $$\eta = \frac{\text{Output of jet}}{\text{Input of jet}}$$

$$= \frac{\text{Work done/s}}{\text{K.E. of jet/s}} = \frac{F_x \times u}{\frac{1}{2} m V^2}$$

$$= \frac{F_x \times u}{\frac{1}{2}(\rho A V) \times V^2}$$

$$= \frac{2208.9}{\frac{1}{2}(1000 \times 0.0019635 \times 20) \times 20^2} = \frac{2208.9}{6540}$$

$$= 0.3377 = \mathbf{33.77\%.\ Ans.}$$

Problem 17.13 *A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate : (i) when the plate is stationary , and (ii) when the plate is moving with a velocity of 15 m/s and away from the jet. Also determine the power and efficiency of the jet when the plate is moving.*

Solution. Given :

Diameter of the jet, $d = 7.5$ cm $= 0.075$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.075)^2 = .004417 \text{ m}^2$

Angle between the jet and plate $\theta = 90° - 45° = 45°$

Velocity of jet, $V = 30$ m/s.

(*i*) When the plate is stationary, the normal force on the plate is given by equation (17.2) as

$$F_n = \rho a V^2 \sin\theta = 1000 \times .004417 \times 30^2 \times \sin 45^\circ = \mathbf{2810.96 \text{ N. Ans.}}$$

(*ii*) When the plate is moving with a velocity 15 m/s and away from the jet, the normal force on the plate is given by equation (17.13) as

$$F_n = \rho a\,(V - u)^2 \sin\theta \qquad \text{where } u = 15 \text{ m/s.}$$

$$= 1000 \times .004417 \times (30 - 15)^2 \times \sin 45^\circ = \mathbf{702.74 \text{ N. Ans.}}$$

(*iii*) The power and efficiency of the jet when plate is moving is obtained as

Work done per second by the jet

= Force in the direction of jet × Distance moved by the plate in the direction of jet/sec

$= F_x \times u$, where $F_x = F_n \sin\theta = 702.74 \times \sin 45^\circ = 496.9$ N

Work done/sec $= 496.9 \times 15 = 7453.5$ Nm/s

$$\therefore \quad \text{Power in kW} = \frac{\text{Work done per second}}{1000} = \frac{7453.5}{1000} = \mathbf{7.453 \text{ kW. Ans.}}$$

$$\text{Efficiency of the jet} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done per second}}{\text{Kinetic energy of the jet}}$$

$$= \frac{7453.5}{\frac{1}{2}(\rho a V) \times V^2} = \frac{7453.5}{\frac{1}{2}\rho a V^3} = \frac{7453.5}{\frac{1}{2} \times 1000 \times .004417 \times 30^3}$$

$$= 0.1249 \simeq 0.125 = \mathbf{12.5\%. \text{ Ans.}}$$

17.4.3 Force on the Curved Plate when the Plate is Moving in the Direction of Jet. Let a jet of water strikes a curved plate at the centre of the plate which is moving with a uniform velocity in the direction of the jet as shown in Fig. 17.12.

Let V = Absolute velocity of jet,

a = Area of jet,

u = Velocity of the plate in the direction of the jet.

Relative velocity of the jet of water or the velocity with which jet strikes the curved plate $= (V - u)$.

If plate is smooth and the loss of energy due to impact of jet is zero, then the velocity with which the jet will be leaving the curved vane $= (V - u)$.

This velocity can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of the jet.

Component of the velocity in the direction of jet

$= -(V - u)\cos\theta$

(–ve sign is taken as at the outlet, the component is in the opposite direction of the jet).

Component of the velocity in the direction perpendicular to the direction of the jet $= (V - u)\sin\theta$.

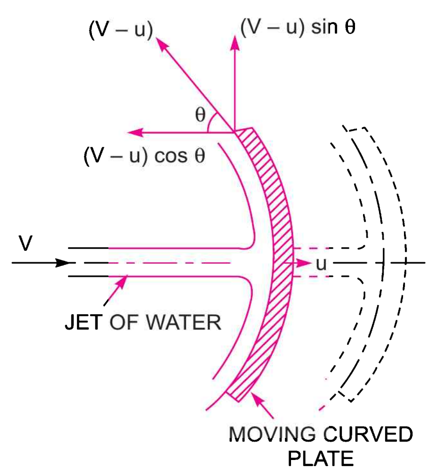


Fig. 17.12 *Jet striking a curved moving plate.*

Mass of the water striking the plate = $\rho \times a \times$ Velocity with which jet strikes the plate

$$= \rho a(V - u)$$

$\therefore$ Force exerted by the jet of water on the curved plate in the direction of the jet,

F_x = Mass striking per sec × [Initial velocity with which jet strikes the plate in the direction of jet – Final velocity]

$$= \rho a(V - u)\,[(V - u) - (-(V - u)\cos\theta)]$$

$$= \rho a(V - u)\,[(V - u) + (V - u)\cos\theta]$$

$$= \rho a(V - u)^2\,[1 + \cos\theta] \quad ...(17.17)$$

Work done by the jet on the plate per second

= F_x × Distance travelled per second in the direction of x

$$= F_x \times u = \rho a(V - u)^2\,[1 + \cos\theta] \times u$$

$$= \rho a(V - u)^2 \times u\,[1 + \cos\theta] \quad ...(17.18)$$

Problem 17.14 *A jet of water of diameter 7.5 cm strikes a curved plate at its centre with a velocity of 20 m/s. The curved plate is moving with a velocity of 8 m/s in the direction of the jet. The jet is deflected through an angle of 165°. Assuming the plate smooth find :*

(i) Force exerted on the plate in the direction of jet, (ii) Power of the jet, and (iii) Efficiency of the jet.

Solution. Given :

Diameter of the jet, $d = 7.5$ cm $= 0.075$ m

$\therefore$ Area, $a = \dfrac{\pi}{4}(.075)^2 = 0.004417$

Velocity of the jet, $V = 20$ m/s

Velocity of the plate, $u = 8$ m/s

Angle of deflection of the jet, = 165°

$\therefore$ Angle made by the relative velocity at the outlet of the plate,

$$\theta = 180° - 165° = 15°.$$

(*i*) Force exerted by the jet on the plate in the direction of the jet is given by equation (17.17) as

$$= F_x = \rho a(V - u)^2 (1 + \cos\theta)$$

$$= 1000 \times .004417 \times (20 - 8)^2\,[1 + \cos 15°] = \mathbf{1250.38\ N.\ Ans.}$$

(*ii*) Work done by the jet on the plate per second

$$= F_x \times u = 1250.38 \times 8 = 10003.04 \text{ N m/s}$$

$\therefore$ Power of the jet $= \dfrac{10003.04}{1000} = \mathbf{10\ kW.\ Ans.}$

(*iii*) Efficiency of the jet $= \dfrac{\text{Output}}{\text{Input}} = \dfrac{\text{Work done by jet/sec}}{\text{Kinetic energy of jet/sec}}$

$$= \frac{1250.38 \times 8}{\frac{1}{2}(\rho a V) \times V^2} = \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times V^3}$$

$$= \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times 20^3} = 0.564 = \mathbf{56.4\%.\ Ans}$$

Problem 17.15 *A jet of water from a nozzle is deflected through 60° from its original direction by a curved plate which it enters tangentially without shock with a velocity of 30 m/s and leaves with a mean velocity of 25 m/s. If the discharge from the nozzle is 0.8 kg/s, calculate the magnitude and direction of the resultant force on the vane, if the vane is stationary.*

Solution. Given :

Velocity at inlet, $V_1 = 30$ m/s

Velocity at outlet, $V_2 = 25$ m/s

Mass per second = 0.8 kg/s

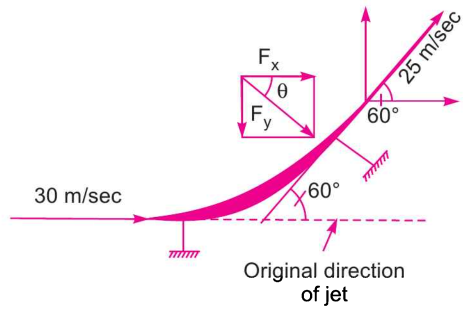


Fig. 17.13 (*a*)

Force in the direction of jet,

$$F_x = \text{Mass per second} \times (V_{1x} - V_{2x})$$

where V_{1x} = Initial velocity in the direction of x

= 30 m/s

V_{2x} = Final velocity in the direction of x

$$= 25 \cos 60^\circ = 25 \times \frac{1}{2} = 12.5 \text{ m/s}$$

$$\therefore \quad F_x = 0.8[30 - 12.5] = 0.8 \times 17.5 = 14.0 \text{ N}$$

Similarly, force normal to the jet,

$$F_y = \text{Mass per second} \times (V_{1y} - V_{2y})$$
$$= 0.8\,[0 - 25 \sin 60^\circ] = -17.32 \text{ N}$$

–ve sign means the force, F_y, is acting in the vertically downward direction.

$$\therefore \quad \text{Resultant force on the vane} = \sqrt{F_x^2 + F_y^2} = \sqrt{14^2 + (-17.32)^2} = \textbf{22.27 N. Ans.}$$

The angle made by the resultant with x-axis

$$\tan\theta = \frac{F_y}{F_x} = \frac{-17.32}{14.0} = -1.237$$

–ve sign means the angle θ is in the clockwise direction with x- axis as shown in Fig. 17.13 (*a*)

$$\therefore \quad \theta = \tan^{-1} 1.237 = \textbf{51° 2.86′. Ans.}$$

Problem 17.16 (*a*) *A stationary vane having an inlet angle of zero degree and an outlet angle of 25° as shown in Fig. 17.13(b), receives water at a velocity of 50 m/s. Determine the components of force acting on it in the direction of the jet velocity and normal to it. Also find the resultant force in magnitude and direction per unit weight of the flow.*

(b) If the vane stated above is moving with a velocity of 20 m/s in the direction of the jet, calculate the force components in the direction of the vane velocity and across it, also the resultant force in magnitude and direction. Calculate the work done and power developed per unit weight of the flow.

Solution. Given :

(*a*) Velocity of jet, $V = 50$ m/s

Angle at outlet, = 25°

For the stationary vane, the force in the direction of jet is given as

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

where $V_{1x} = 50$ m/s

$V_{2x} = -50 \cos 25^\circ = -45.315$

∴ Force in the direction of jet per unit weight of water

$$= \frac{\text{Mass/sec}\,[50 - (-45.315)]}{\text{Weight of water/sec}}$$

or $$F_x = \frac{(\text{Mass/sec})\,[50 + 45.315]}{(\text{Mass/sec}) \times g}$$

$$= \frac{1}{g}\,[50 + 45.315] \text{ N/N} = \frac{95.315}{9.81} = 9.716 \text{ N/N}$$

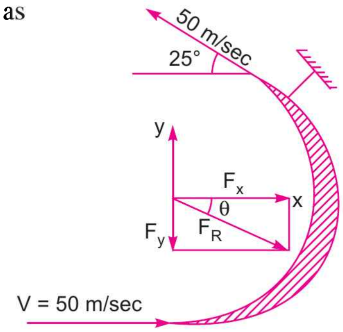


Fig. 17.13 (*b*)

Force exerted by jet in the direction perpendicular to the direction of the jet per unit weight of the flow,

$$F_y = \frac{(\text{Mass per sec})\,[V_{1y} - V_{2y}]}{g \times \text{Mass per sec}}$$

$$= \frac{1}{g}\,[V_{1y} - V_{2y}] = \frac{1}{g}\,[0 - 50 \sin 25^\circ] \quad (\because V_{1y} = 0,\ V_{2y} = 50 \sin 25^\circ)$$

$$= \frac{-50 \sin 25^\circ}{9.81} = \mathbf{-\ 2.154.\ \ Ans.}$$

–ve sign means the force F_y is acting in the downward direction.

∴ Resultant force per unit weight of water = $\sqrt{F_x^2 + F_y^2}$

or $$\mathbf{F_R} = \sqrt{(9.716)^2 + (2.154)^2} = \mathbf{9.952\ N.\ \ Ans.}$$

The angle made by the resultant with the x-axis,

$$\tan\theta = \frac{F_y}{F_x} = \frac{2.154}{9.716} = 0.2217$$

∴ $$\theta = \tan^{-1} .2217 = \mathbf{12.50^\circ.\ \ Ans.}$$

(*b*) Velocity of the vane = 20 m/s.

When the vane is moving in the direction of the jet, the force exerted by the jet on the plate in the direction of jet,

$$F_x' = [\text{Mass of water striking/sec}] \times [V_{1x} - V_{2x}]$$

where V_{1x} = Initial velocity of the striking water

$= (V - u) = 50 - 20 = 30$ m/s

V_{2x} = Final velocity in the direction of x

$= -(V - u) \cos 25^\circ = 30 \times \cos 25^\circ = -27.189$ m/s.

∴ F_x = Mass per sec [30 + 27.189]

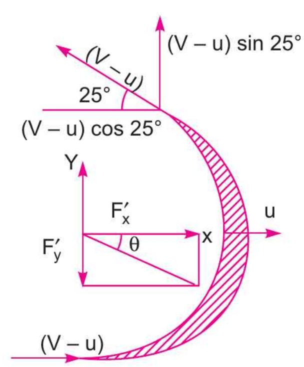


Fig. 17.14

Force in the direction of jet per unit weight,

$$F_x' = \frac{\text{Mass per sec }[30 + 27.189]}{\text{Mass per sec} \times g}$$

$$= \frac{(30 + 27.189)}{9.81} = 5.829 \text{ N.}$$

Force exerted by the jet in the direction perpendicular to direction of jet, per unit weight,

$$F_y' = \frac{1}{g}\,[V_{1y} - V_{2y}]$$

where $V_{1y} = 0$; $V_{2y} = (V - u) \sin 25^\circ = (50 - 20) \sin 25^\circ = 30 \sin 25^\circ$

$$F_y' = \frac{1}{9.81}\,[0 - 30 \sin 25^\circ] = -\,1.292 \text{ N}$$

$\therefore$ Resultant force $= \sqrt{(5.829)^2 + (1.292)^2} = 5.917$ N

The angle made by the resultant with x-axis, $\tan\theta = \dfrac{1.292}{5.829} = 0.2217$

$\therefore$ $\theta = \tan^{-1} .2217 = 12.30°$

$\therefore$ Work done per second per unit weight of flow

$$= F_x' \times u = 5.829 \times 20 = 116.58 \text{ N m/s}$$

$\therefore$ Power developed $= \dfrac{\text{Work done per second}}{1000} = \dfrac{116.58}{1000} =$ **0.116 kW. Ans.**

Problem 17.17 *A jet of water of diameter 50 mm moving with a velocity of 25 m/s impinges on a fixed curved plate tangentially at one end at an angle of 30° to the horizontal. Calculate the resultant force of the jet on the plate if the jet is deflected through an angle of 50°. Take g = 10 m/s²*

Solution. Given :

Dia. of jet, $d = 50 \text{ mm} = 0.05 \text{ m}^2$

$\therefore$ Area of jet, $a = \dfrac{\pi}{4}(0.05)^2 \text{m}^2$

Velocity of jet, $V = 25$ m/s

The angle made by the jet at inlet with horizontal, $\theta = 30°$

Angle of deflection = 50°

$\therefore$ Angle made by the jet at outlet with horizontal is given by,

$$\phi = \theta + \text{Angle of deflection}$$
$$= 30^\circ + 50^\circ = 80^\circ$$

Value of g $= 10 \text{ m/s}^2$

The force exerted by the jet of water in the direction of x is given by,

$$F_x = \rho a V\,[V_{1x} - V_{2x}] \qquad \ldots(i)$$

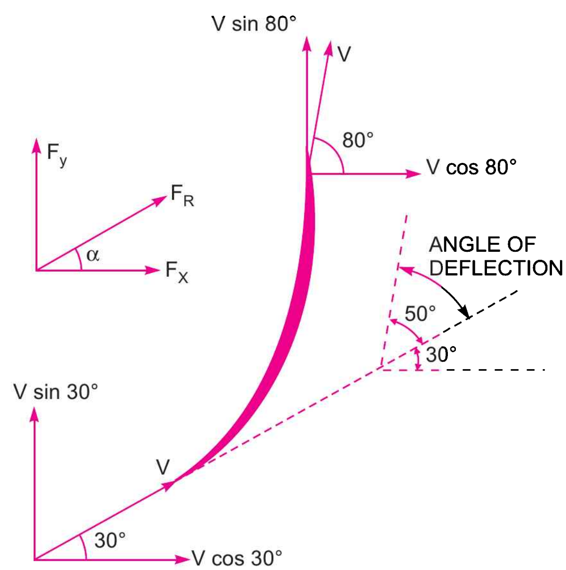


Fig. 17.14 (*a*)

where $\rho = 1000$ ($\because$ g is given as 10 m/s^2)

$$a = \frac{\pi}{4}(0.05)^2; \; V = 25 \text{ m/s};$$
$$V_{1x} = V\cos 30° = 25\cos 30°,$$
$$V_{2x} = V\cos 80° = 25\cos 80°.$$

Substituting these values in equation (*i*), we get

$$F_x = 1000 \times \frac{\pi}{4}(0.05)^2 \times 25\,[25\cos 30° - 25\cos 80°] = 849.7 \text{ N}$$

The force in the direction of y is given by,

$$F_y = \rho a V[V_{1y} - V_{2y}]$$
$$= 1000 \times \frac{\pi}{4}(0.05)^2 \times 25[25\sin 30° - 25\sin 80°] = -594.9 \text{ N}$$

The –ve sign shows that force F_y is acting in the downward direction.

The resultant force is given by,

$$F_R = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{849.7^2 + 594.9^2} = \mathbf{1037\ N.}\ \ \textbf{Ans.}$$

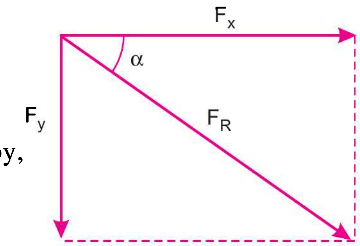


Fig. 17.14 (*b*)

And the angle made by the resultant with the horizontal is given by,

$$\tan\alpha = \frac{F_y}{F_x} = \frac{594.9}{849.7} = 0.7$$

$\therefore$ $\alpha = \tan^{-1} 0.7 = \mathbf{35°}.$ **Ans.**

17.4.4 Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the Tips. Fig. 17.15 shows a jet of water striking a moving curved plate (also called vane) tangentially, at one of its tips. As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. In this case as plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate. Also as the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of the velocity of jet and velocity of the plate at inlet.

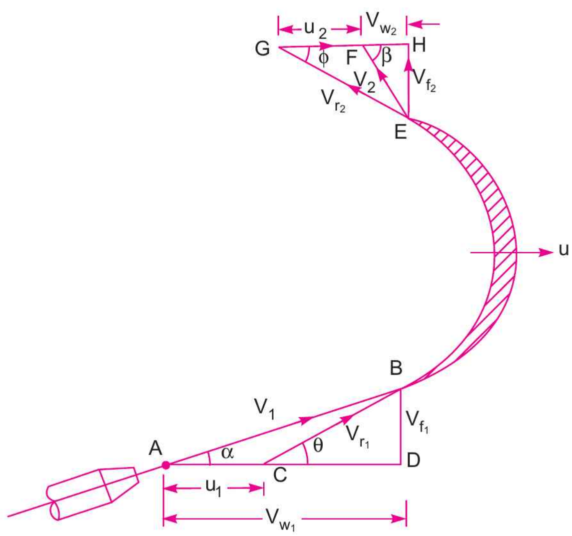


Fig. 17.15 *Jet striking a moving curved vane at one of the tips.*

Let V_1 = Velocity of the jet at inlet.

u_1 = Velocity of the plate (vane) at inlet.

V_{r_1} = Relative velocity of jet and plate at inlet.

α = Angle between the direction of the jet and direction of motion of the plate, also called guide blade angle.

θ = Angle made by the relative velocity (V_{r_2}) with the direction of motion at inlet also called vane angle at inlet.

V_{w_1} and V_{f_1} = The components of the velocity of the jet V_1, in the direction of motion and perpendicular to the direction of motion of the vane respectively.

V_{w_1} = It is also known as velocity of whirl at inlet.

V_{f_1} = It is also known as velocity of flow at inlet.

V_2 = Velocity of the jet, leaving the vane or velocity of jet at outlet of the vane.

u_2 = Velocity of the vane at outlet.

V_{r_2} = Relative velocity of the jet with respect to the vane at outlet.

β = Angle made by the velocity V_2 with the direction of motion of the vane at outlet.

ϕ = Angle made by the relative velocity V_{r_2} with the direction of motion of the vane at outlet and also called vane angle at outlet.

V_{w_1} and V_{f_1} = Components of the velocity V_2, in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet.

V_{w_2} = It is also called the velocity of whirl at outlet.

V_{f_2} = Velocity of flow at outlet.

The triangles *ABD* and *EGH* are called the velocity triangles at inlet and outlet. These velocity triangles are drawn as given below :

1. Velocity Triangle at Inlet. Take any point *A* and draw a line $AB = V_1$ in magnitude and direction which means line *AB* makes an angle α with the horizontal line *AD*. Next draw a line $AC = u_1$ in magnitude. Join *C* to *B*. Then *CB* represents the relative velocity of the jet at inlet. If the loss of energy at inlet due to impact is zero, then *CB* must be in the tangential direction to the vane at inlet. From *B* draw a vertical line *BD* in the downward direction to meet the horizontal line *AC* produced at *D*.

Then BD = Represents the velocity of flow at inlet = V_{f_1}

AD = Represents the velocity of whirl at inlet = V_{w_1}

$\angle BCD$ = Vane angle at inlet = θ.

2. Velocity Triangle at Outlet. If the vane surface is assumed to be very smooth, the loss of energy due to friction will be zero. The water will be gliding over the surface of the vane with a relative velocity equal to V_{r_1} and will come out of the vane with a relative relocity V_{r_2}. This means that the relative velocity at outlet $V_{r_2} = V_{r_1}$. And also the relative velocity at outlet should be in tangential direction to the vane at outlet.

Draw *EG* in the tangential direction of the vane at outlet and cut $EG = V_{r_2}$. From *G*, draw a line *GF* in the direction of vane at outlet and equal to u_2, the velocity of the vane at outlet. Join *EF*. Then *EF* represents the absolute velocity of the jet at outlet in magnitude and direction. From *E* draw a vertical line *EH* to meet the line *GF* produced at *H*. Then

EH = Velocity of flow at outlet = V_{f_2}

FH = Velocity of whirl at outlet = V_{w_2}

ϕ = Angle of vane at outlet.

β = Angle made by V_2 with the direction of motion of vane at outlet.

If the vane is smooth and is having velocity in the direction of motion at inlet and outlet equal then we have

$u_1 = u_2 = u$ = Velocity of vane in the direction of motion and

$V_{r_1} = V_{r_2}$.

Now mass of water striking vane per sec = $\rho a V_{r_1}$ ...(*i*)

where a = Area of jet of water, V_{r_1} = Relative velocity at inlet.

∴ Force exerted by the jet in the direction of motion

F_x = Mass of water striking per sec × [Initial velocity with which jet strikes in the direction of motion – Final velocity of jet in the direction of motion] ...(*ii*)

But initial velocity with which jet strikes the vane = V_{r_1}

The component of this velocity in the direction of motion

$= V_{r_1} \cos\theta = (V_{w_1} - u_1)$ (See Fig. 17.15)

Similarly, the component of the relative velocity at outlet in the direction of motion = $-V_{r_2}\cos\phi$

$= -[u_2 + V_{w_2}]$

–ve sign is taken as the component of V_{r_2} in the direction of motion is in the opposite direction.

Substituting the equation (*i*) and all above values of the velocities in equation (*ii*), we get

$$F_x = \rho a V_{r_1}\,[(V_{w_1} - u_1) - \{-(u_2 + V_{w_2})\}] = \rho a V_{r_1}\,[V_{w_1} - u_1 + u_2 + V_{w_2}]$$

$$= \rho a V_{r_1}\,[V_{w_1} + V_{w_2}] \qquad (\because\ u_1 = u_2) \quad ...(iii)$$

Equation (*iii*) is true only when angle β shown in Fig. 17.15 is an acute angle. If $\beta = 90°$, the $V_{w_2} = 0$, then equation (*iii*) becomes as,

$$F_x = \rho a V_{r_1}[V_{w_1}]$$

If β is an obtuse angle, the expression for F_x will become

$$F_x = \rho a V_{r_1}\,[V_{w_1} - V_{w_2}]$$

Thus in general, F_x is written as $F_x = \rho a V_{r_1}\,[V_{w_1} \pm V_{w_2}]$ ...(17.19)

Work done per second on the vane by the jet

= Force × Distance per second in the direction of force

$$= F_x \times u = \rho a V_{r_1}\,[V_{w_1} \pm V_{w_2}] \times u \quad ...(17.20)$$

∴ Work done per second per unit weight of fluid striking per second

$$= \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\text{Weight of fluid striking/s}}\ \frac{\text{Nm/s}}{\text{N/s}} = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{g \times \rho a V_{r_1}} = \text{Nm/N}$$

$$= \frac{1}{g}\,[V_{w_1} \pm V_{w_2}] \times u \text{ Nm/N} \quad ...(17.21)$$

Work done/sec per unit mass of fluid striking per second

$$= \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right]\times u}{\text{Mass of fluid striking / s}} \frac{\text{Nm / s}}{\text{kg / s}} = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right]\times u}{\rho a V_{r_1}} \text{ Nm/kg}$$

$$= (V_{w_1} \pm V_{w_2}) \times u \text{ Nm/kg} \qquad \text{...[17.21(A)]}$$

Note. Equation (17.21) gives the work done per unit weight whereas equation [17.21(*A*)] gives the work done per unit mass.

3. Efficiency of Jet. The work done by the jet on the vane given by equation (17.20), is the output of the jet whereas the initial kinetic energy of the jet is the input. Hence, the efficiency (η) of the jet is expressed as

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done per second on the vane}}{\text{Initial K. E. per second of the jet}} = \frac{\rho a V_{r_1}(V_{w_1} \pm V_{w_2})\times u}{\frac{1}{2} m V_1^2}$$

where m = mass of the fluid per second in the jet = $\rho a V_1$

V_1 = initial velocity of jet

$$\therefore \qquad \eta = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right]\times u}{\frac{1}{2}(\rho a V_1)\times V_1^2}. \qquad \text{...[17.21(B)]}$$

Problem 17.18 *A jet of water having a velocity of 20 m/s strikes a curved vane, which is moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of motion of vane an outlet. Calculate :*

(i) Vane angles, so that the water enters and leaves the vane without shock.

(ii) Work done per second per unit weight of water striking (or work done per unit weight of water striking) the vane per second.

Solution. Given :

Velocity of jet, $V_1 = 20$ m/s

Velocity of vane, $u_1 = 10$ m/s

Angle made by jet at inlet, with direction of motion of vane,

$\alpha = 20°$

Angle made by the leaving jet, with the direction of motion

= 130°

$\therefore \quad \beta = 180° - 130° = 50°$

In this problem, $u_1 = u_2 = 10$ m/s

$V_{r_1} = V_{r_2}$

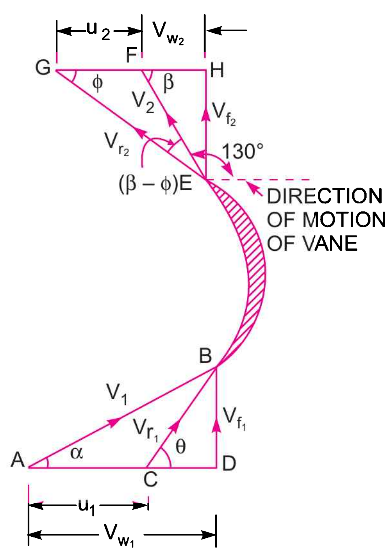


Fig. 17.16

(*i*) **Vane Angle** means angle made by the relative velocities at inlet and outlet, *i.e.*, θ and ϕ.

From Fig. 17.16, in ΔABD, we have $\tan\theta = \frac{BD}{CD}$

$$= \frac{V_{f_1}}{AD - AC} = \frac{V_{f_1}}{V_{w_1} - u_1} \qquad \text{...(i)}$$

where $V_{f_1} = V_1 \sin\alpha = 20 \times \sin 20° = 6.84$ m/s

$V_{w_1} = V_1 \cos \alpha = 20 \times \cos 20° = 18.794$ m/s.

$u_1 = 10$ m/s

$$\therefore \quad \tan \theta = \frac{6.84}{18.794 - 10} = .7778 \text{ or } \theta = 37.875°$$

$$\therefore \quad \theta = \mathbf{37° \ 52.5'. \ Ans.}$$

From, ΔABC, $\quad \sin \theta = \frac{V_{f_1}}{V_{r_1}}$ or $V_{r_1} = \frac{V_{f_1}}{\sin \theta} = \frac{6.84}{\sin 37.875°} = 11.14$

$$\therefore \quad V_{r_2} = V_{r_1} = 11.14 \text{ m/s.}$$

From, ΔEFG, applying sine rule, we have

$$\frac{V_{r_2}}{\sin (180° - \beta)} = \frac{u_2}{\sin (\beta - \phi)}$$

or
$$\frac{11.14}{\sin \beta} = \frac{10}{\sin [\beta - \phi]} \quad \text{or} \quad \frac{11.14}{\sin 50°} = \frac{10}{\sin [50° - \phi]} \qquad (\because \ \beta = 50°)$$

$$\therefore \quad \sin (50° - \phi) = \frac{10 \times \sin 50°}{11.14} = 0.6876 = \sin 43.44°$$

$$\therefore \quad 50° - \phi = 43.44° \quad \text{or} \quad \phi = 50° - 43.44° = 6.56°$$

$$\therefore \quad \phi = \mathbf{6° \ 33.6'. \ Ans.}$$

(*ii*) Work done per second per unit weight of the water striking the vane per second is given by equation (17.21) as

$$= \frac{1}{g} [V_{w_1} + V_{w_2}] \times u \text{ Nm/N (+ ve sign is taken as } \beta \text{ is an acute angle)}$$

where $V_{w_1} = 18.794$ m/s, $V_{w_2} = GH - GF = V_{r_2} \cos \phi - u_2 = 11.14 \times \cos 6.56° - 10 = 1.067$ m/s

$u = u_1 = u_2 = 10$ m/s

$\therefore$ Work done per unit weight of water

$$= \frac{1}{9.81} [18.794 + 1.067] \times 10 \text{ Nm/N} = \mathbf{20.24 \ Nm/N. \ Ans.}$$

Problem 17.19 *A jet of water having a velocity of 40 m/s strikes a curved vane, which is moving with a velocity of 20 m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves at an angle of 90° to the direction of motion of vane at outlet. Draw the velocity triangles at inlet and outlet and determine the vane angles at inlet and outlet so that the water enters and leaves the vane without shock.*

Solution. Given :

Velocity of jet, $\quad V_1 = 40$ m/s

Velocity of vane, $\quad u_1 = 20$ m/s

Angle made by jet at inlet, $\quad \alpha = 30°$

Angle made by leaving jet $\quad = 90°$

$\therefore \quad \beta = 180° - 90° = 90°$

For this problem, we have

$$u_1 = u_2 = u = 20 \text{ m/s}$$

Vane angles at inlet and outlet are θ and ϕ respectively.

From ΔBCD, we have

$$\tan\theta = \frac{BD}{CD} = \frac{BD}{AD - AC} = \frac{V_{f_1}}{V_{w_1} - u_1}$$

where $V_{f_1} = V_1 \sin\alpha = 40 \times \sin 30^\circ = 20$ m/s

$V_{w_1} = V_1 \cos\alpha = 40 \times \cos 30^\circ = 34.64$ m/s

$u_1 = 20$ m/s

$$\therefore \quad \tan\theta = \frac{20}{34.64 - 20} = \frac{20}{14.64} = 1.366 = \tan 53.79^\circ$$

$\therefore$ $\theta = 53.79^\circ$ or **53° 47.4′. Ans.**

Also from ΔBCD,

$$\sin\theta = \frac{V_{f_1}}{V_{r_1}} \quad \text{or} \quad V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{20}{\sin 53.79^\circ}$$

$\therefore$ $V_{r_1} = 24.78$

But $V_{r_2} = V_{r_1} = 24.78$

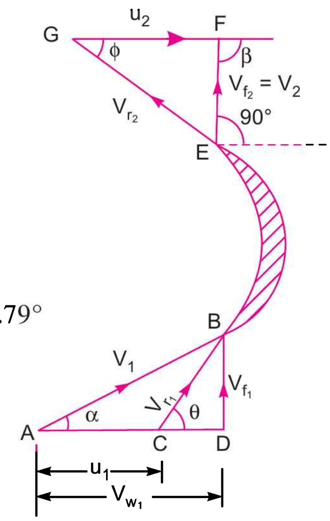


Fig. 17.17

Hence, from ΔEFG, $\cos\phi = \dfrac{u_2}{V_{r_2}} = \dfrac{20}{24.78} = 0.8071 = \cos 36.18^\circ$

$\therefore$ $\phi = 36.18^\circ$ or **36° 10.8′. Ans.**

Problem 17.20 *A jet of water of diameter 50 mm, having a velocity of 20 m/s strikes a curved vane which is moving with a velocity of 10 m/s in the direction of the jet. The jet leaves the vane at an angle of 60° to the direction of motion of vane at outlet. Determine :*

(i) The force exerted by the jet on the vane in the direction of motion.

(ii) Work done per second by the jet.

Solution. Given :

Diameter of the jet, $d = 50$ mm $= 0.05$ m

$\therefore$ Area, $a = \dfrac{\pi}{4} (.05)^2 = .001963 \text{ m}^2$

Velocity of jet, $V_1 = 20$ m/s

Velocity of vane, $u_1 = 10$ m/s

As jet and vane are moving in the same direction,

$\therefore$ $\alpha = 0$

Angle made by the leaving jet, with the direction of motion $= 60^\circ$

$\therefore$ $\beta = 180^\circ - 60^\circ = 120^\circ$

For this problem, we have

$u_1 = u_2 = u = 10$ m/s

$V_{r_1} = V_{r_2}$

From Fig. 17.18, we have

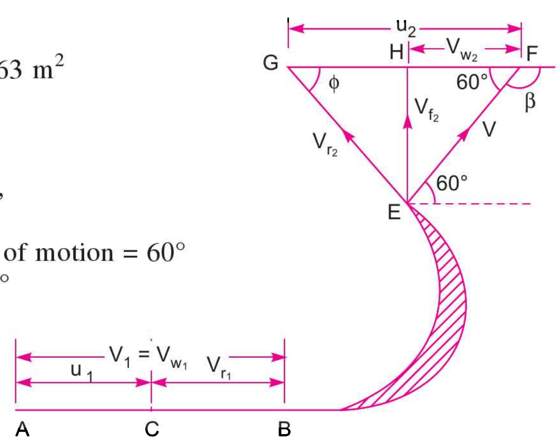


Fig. 17.18

$$V_{r_1} = AB - AC = V_1 - u_1$$
$$= 20 - 10 = 10 \text{ m/s}$$
$$V_{w_1} = V_1 = 20 \text{ m/s}$$

$\therefore$ $V_{r_2} = V_{r_1} = 10$ m/s

Now in ΔEFG, $EG = V_{r_2} = 10$ m/s,

$GF = u_2 = 10$ m/s

$\angle GEF = 180° - (60° + \phi) = (120° - \phi)$

From sine rule, we have

$$\frac{EG}{\sin 60°} = \frac{GF}{\sin(120° - \phi)} \quad \text{or} \quad \frac{10}{\sin 60°} = \frac{10}{\sin(120° - \phi)}$$

or $\sin 60° = \sin(120° - \phi)$

$\therefore$ $60° = 120° - \phi$ or $\phi = 120° - 60° = 60°$

Now $V_{w_2} = HF = GF - GH$

$= u_2 - V_{r_2} \cos\phi = 10 - 10 \times \cos 60° = 10 - 5 = 5$ m/s.

(*i*) The force exerted by the jet on the vane in the direction of motion is given by equation (17.19) as

$F_x = \rho a V_{r_1} [V_{w_1} - V_{w_2}]$ (–ve sign is taken as β is an obtuse angle)

$= 1000 \times .001963 \times 10 [20 - 5]$ N = **294.45 N. Ans.**

(*ii*) Work done per second by the jet

$= F_x \times u = 294.45 \times 10 = 2944.5$ N m/s

= **2944.5 W. Ans.** $[\because \text{Nm/s} = \text{W (watt)}]$

Problem 17.21 *A jet of water having a velocity of 15 m/s strikes a curved vane which is moving with a velocity of 5 m/s. The vane is symmetrical and is so shaped that the jet is deflected through 120°. Find the angle of the jet at inlet of the vane so that there is no shock. What is the absolute velocity of the jet at outlet in magnitude and direction and the work done per unit weight of water. Assume the vane to be smooth.*

Solution. Given :

Velocity of jet, $V_1 = 15$ m/s

Velocity of vane, $u_1 = 5$ m/s

As vane is symmetrical. Hence angle $\theta = \phi$

Angle of deflection of the jet $= 120° = 180° - (\theta + \phi)$

$\therefore$ $\theta + \phi = 60°$ or each angle, *i.e.*, $\theta = \phi = 30°$

Let the angle of jet at inlet $= \alpha$

Absolute velocity of jet at outlet $= V_2$

Angle made by V_2 at outlet with direction of motion of vane $= \beta$*.

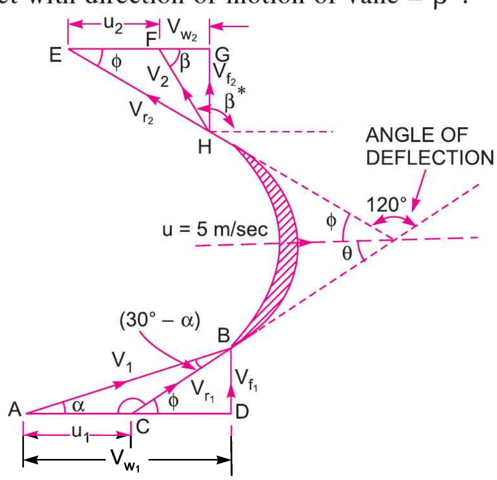


Fig. 17.19

For this problem, $u_1 = u_2 = u = 5$ m/s

$V_{r_1} = V_{r_2}$ (as vane is smooth)

Applying the sine rule to ΔACB,

$$\frac{AB}{\sin(180° - \theta)} = \frac{AC}{\sin(30° - \alpha)} \quad \text{or} \quad \frac{V_1}{\sin(180° - 30°)} = \frac{u_1}{\sin(30° - \alpha)}$$

or $$\frac{15}{\sin 30°} = \frac{5}{\sin(30° - \alpha)} \quad \text{or} \quad \sin(30° - \alpha) = \frac{5 \sin 30°}{15}$$

$$= \frac{1}{3} \times 0.5 = .1667 = \sin 9.596°$$

$\therefore$ $30° - \alpha = 9.596°$ or $\alpha = 30° - 9.596° = 20.404°$ or **20° 24′. Ans.**

Also from sine rule to ΔACB, we have

$$\frac{AB}{\sin(180° - 30°)} = \frac{CB}{\sin \alpha} \quad \text{or} \quad \frac{V_1}{\sin 30°} = \frac{V_{r_1}}{\sin 20.404°}$$

$\therefore$ $$V_{r_1} = \frac{V_1 \sin 20.404°}{\sin 30°} = 10.46 \text{ m/s}$$

$\therefore$ $V_{r_2} = V_{r_1} = 10.46$ m/s

From velocity ΔHEG at outlet,

$$V_{r_2} \cos \phi = u_2 + V_{w_2} \quad \text{or} \quad 10.46 \cos 30° = 5.0 + V_{w_2}$$

$\therefore$ $V_{w_2} = 10.46 \cos 30° - 5.0 = 4.06$ m/s

Also, we have $V_{r_2} \sin \phi = V_{f_2}$ or $V_{f_2} = 10.46 \sin 30° = 5.23$ m/s

In ΔHFG, $$V_2 = \sqrt{V_{f_2}^2 + V_{w_2}^2} = \sqrt{5.23^2 + 4.06^2}$$

$$= \sqrt{27.353 + 16.483} = \textbf{6.62 m/s. Ans.}$$

$$\tan \beta = \frac{V_{f_2}}{V_{w_2}} = \frac{5.23}{4.06} = 1.288 = \tan 52.17°$$

$\therefore$ $\beta = 52.17°$ or $52° \; 10.2′$

$\therefore$ Angle made by absolute velocity at outlet with the direction of motion β^*

$= 180° - \beta = 180° - (52° \; 10.2′) = 127° \; 49.8′$

$\therefore$ $\beta^* =$ **127° 49.8. Ans.**

Work done* per unit weight of the water striking

$$= \frac{1}{g} [V_{w_1} + V_{w_2}] \times u \text{ Nm} \quad (\because \text{ +ve sign taken as } \beta \text{ is an acute angle})$$

$$= \frac{1}{9.81} [V_1 \cos \alpha + 4.06] \times 5 \qquad (\because \; V_{w_1} = V_1 \cos \alpha)$$

$$= \frac{5}{9.81} [15 \cos 20.404° + 4.06] = \textbf{9.225 Nm/N. Ans.}$$

---

* Work done per unit weight of water striking is the same as work done per second per unit weight of water striking per second refer to equation (17.21).

Problem 17.22 *A jet of water moving at 12 m/s impinges on vane shaped to deflect the jet through 120° when stationary. If the vane is moving at 5 m/s, find the angle of the jet so that there is no shock at inlet. What is the absolute velocity of the jet at exit in magnitude and direction and the work done per second per unit weight of water striking per second? Assume that the vane is smooth.*

Solution. Given :

Velocity of jet, $V_1 = 12$ m/s

Velocity of vane, $u = u_1 = u_2 = 5$ m/s

Angle of deflection of jet = 120°

$\therefore \quad \theta + \phi = 180° - 120° = 60°.$

It is not given that the vane is symmetrical and without this condition problem cannot be solved. Assuming vane to be symmetrical, we have $\theta = \phi$

Then $\theta = \phi = 30°$

(*i*) Angle of jet at inlet with the direction of motion of vane = α

In ΔABC, applying sine rule, we have

$$\frac{AB}{\sin(180° - \theta)} = \frac{AC}{\sin(30° - \alpha)} \text{ or } \frac{V_1}{\sin\theta} = \frac{u_1}{\sin(30° - \alpha)} \text{ or } \frac{12}{\sin 30°} = \frac{5}{\sin(30° - \alpha)}$$

$$\therefore \quad \sin(30° - \alpha) = \frac{5 \sin 30°}{12} = 0.2083 = \sin 12.02°$$

$$\therefore \quad 30° - \alpha = 12.02° \text{ or } \alpha = 30° - 12.02° = 17.98° \text{ or } \mathbf{17° \, 59'.} \textbf{ Ans.}$$

Again applying sine rule to ΔABC, we have

$$\frac{V_1}{\sin(180° - \theta)} = \frac{V_{r_1}}{\sin\alpha} \text{ or } \frac{12}{\sin\theta} = \frac{V_{r_1}}{\sin 17.98°}$$

$$\therefore \quad V_{r_1} = \frac{12 \sin 17.98°}{\sin\theta} = \frac{12 \times \sin 17.98°}{\sin 30°} = 7.41 \text{ m/s}$$

In ΔABD, $\quad V_{w_1} = V_1 \times \cos\alpha = 12 \cos 17.98° = 11.41$ m/s

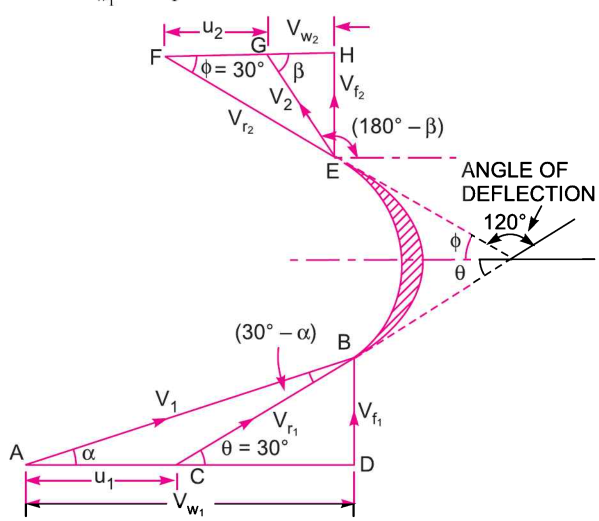


Fig. 17.20

(*ii*) The absolute velocity of jet at outlet = V_2

The angle made by V_2 at outlet with the direction of the motion of vane = 180° – β

Now as vane is given smooth, $V_{r_2} = V_{r_1} = 7.41$ m/s

At outlet, from ΔEFH, we have $V_{r_2}\cos\phi = u_2 + V_{w_2}$ or $7.41\cos 30° = 5 + V_{w_2}$

$$\therefore \quad V_{w_2} = 7.41\cos 30° - 5.0 = 1.417 \text{ m/s}$$

Also $$V_{f_2} = V_{r_2}\sin 30° = 7.41\sin 30° = 3.705 \text{ m/s}$$

And $$\tan\beta = \frac{V_{f_2}}{V_{w_2}} = \frac{3.705}{1.417} = 2.614 = \tan 69.07°$$

$$\therefore \quad \beta = 69.07° \text{ or } 69° \ 4.2'$$

∴ Angle made by V_2 at outlet with the direction of motion of vane

$$= 180° - \beta = 180° - (69° \ 4.2') = \mathbf{110° \ 55.8'}. \textbf{ Ans.}$$

Also $$V_2 = \sqrt{V_{f_2}^2 + V_{w_2}^2} = \sqrt{(3.705)^2 + (1.417)^2} = \sqrt{13.727 + 2.007}$$

$$= \mathbf{3.96 \text{ m/s. Ans.}}$$

(*iii*) Work done per second per unit weight of water striking per second

$$= \frac{1}{g}[V_{w_1} + V_{w_2}]u = \frac{1}{9.81}[11.41 + 1.417] \times 5 = \mathbf{6.537 \text{ Nm/N. Ans.}}$$

Problem 17.23 *A jet of water having a velocity of 15 m/s, strikes a curved vane which is moving with a velocity of 5 m/s in the same direction as that of the jet at inlet. The vane is so shaped that the jet is deflected through 135°. The diameter of jet is 100 mm. Assuming the vane to be smooth , find :*

(i) Force exerted by the jet on the vane in the direction of motion,

(ii) Power exerted on the vane, and

(iii) Efficiency of the vane.

Solution. Given :

Velocity of jet, $V_1 = 15$ m/s

Velocity of vane, $u = u_1 = u_2 = 5$ m/s

At inlet jet and vane are in the same direction, hence $\alpha = 0$

Diameter of jet, $d = 100$ mm $= 0.1$ m

∴ Area, $$a = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$$

Angle of deflection of the jet $= 135° = 180° - \phi$ $\quad$ ($\because \theta = 0°$)

$$\therefore \quad \phi = 180° - 135° = 45°$$

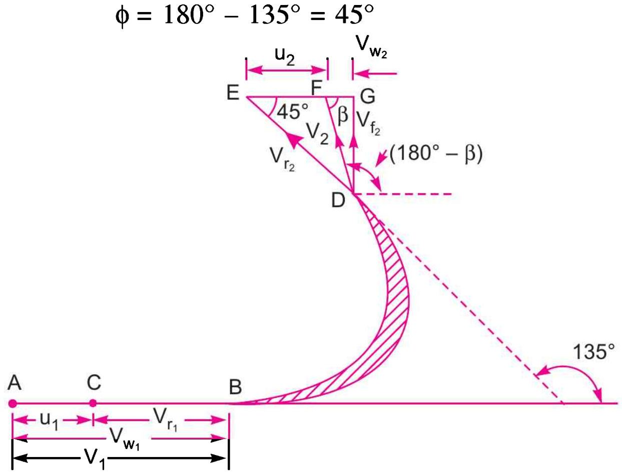


Fig. 17.21

As vane is given smooth hence $V_{r_1} = V_{r_2}$

From the inlet velocity triangle, which is a straight line in this case, we have

$$V_{r_1} = V_1 - u_1 = 15 - 5 = 10 \text{ m/s}$$

$$V_{w_1} = V_1 = 15 \text{ m/s}$$

From the outlet velocity triangle *DEG*, we have

$$V_{r_2} = V_{r_1} = 10 \text{ m/s}$$

$$u_2 = u_1 = u = 5 \text{ m/s}$$

$$V_{r_2} \cos \phi = u_2 + V_{w_2} \quad \text{or} \quad 10 \cos 45° = 5 + V_{w_2}$$

$$\therefore \quad V_{w_2} = 10 \cos 45° - 5 = 7.07 - 5 = 2.07 \text{ m/s.}$$

(*i*) Force exerted by the jet on the vane in the direction of motion is given by equation (17.19) as

$$F_x = \rho a V_{r_1}[V_{w_1} + V_{w_2}] \text{ (+ve sign is taken as } \beta \text{ is an acute angle)}$$

$$= 1000 \times .007854 \times 10[15 + 2.07] = \mathbf{1340.6 \text{ N. Ans.}}$$

(*ii*) Power of the vane is given as

$$= F_x \times u \text{ N m/s} = 1340.6 \times 5 = 6703 \text{ W} = \mathbf{6.703 \text{ kW. Ans.}}$$

(*iii*) Efficiency of the vane

$$= \frac{\text{Work done per second on vane}}{\text{Kinetic energy supplied by jet per second}}$$

$$= \frac{F_x \times u}{\frac{1}{2} \times (\text{mass per second}) \times V^2} = \frac{F_x \times u}{\frac{1}{2}(\rho a V_1) \times V_1^2}$$

$$= \frac{1340.6 \times 5.0}{\frac{1}{2} \times (1000 \times .007854 \times 15) \times 15^2} = \mathbf{0.505 = 50.5\%. \text{ Ans.}}$$

17.4.5 Force Exerted by a Jet of Water on a Series of Vanes. The force exerted by a jet of water on a *single* moving plate (which may be flat or curved) is not practically feasible. This case is only a theoretical one. In actual practice, a large number of plates are mounted on the circumference of a wheel at a fixed distance apart as shown in Fig. 17.22. The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving and the 2nd plate mounted on the wheel appears before the jet, which again exerts the force on the 2nd plate. Thus each plate appears successively before the jet and the jet exerts force on each plate. The wheel starts moving at a constant speed.

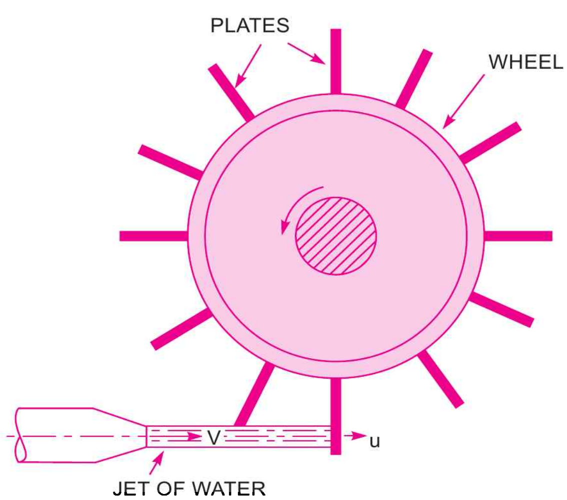


Fig. 17.22 *Jet striking a series of vanes.*

Let V = Velocity of jet,
d = Diameter of jet,
a = Cross-sectional area of jet,

$$= \frac{\pi}{4} d^2$$

u = Velocity of vane.

In this case the mass of water coming out from the nozzle per second is always in contact with the plates, when all the plates are considered. Hence mass of water per second striking the series of plates = ρaV.

Also the jet strikes the plate with a velocity = $(V - u)$.

After striking, the jet moves tangential to the plate and hence the velocity component in the direction of motion of plate is equal to zero.

∴ The force exerted by the jet in the direction of motion of plate,

$$F_x = \text{Mass per second [Initial velocity – Final velocity]}$$
$$= \rho aV[(V - u) - 0] = \rho aV[V - u] \qquad ...(17.22)$$

Work done by the jet on the series of plates per second

$$= \text{Force} \times \text{Distance per second in the direction of force}$$
$$= F_x \times u = \rho aV[V - u] \times u$$

Kinetic energy of the jet per second

$$= \frac{1}{2} mV^2 = \frac{1}{2} (\rho aV) \times V^2 = \frac{1}{2} \rho aV^3$$

∴ Efficiency,

$$\eta = \frac{\text{Work done per second}}{\text{Kinetic energy per second}} = \frac{\rho aV[V-u]\times u}{\frac{1}{2}\rho aV^3} = \frac{2u[V-u]}{V^2} \qquad ...(17.23)$$

Condition for Maximum Efficiency. Equation (17.23) gives the value of the efficiency of the wheel. For a given jet velocity V, the efficiency will be maximum when

$$\frac{d\eta}{du} = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2u(V-u)}{V^2}\right] = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2uV - 2u^2}{V^2}\right] = 0$$

or

$$\frac{2V - 2 \times 2u}{V^2} = 0 \quad \text{or} \quad 2V - 4u = 0 \quad \text{or} \quad V = \frac{4u}{2} = 2u \quad \text{or} \quad u = \frac{V}{2}. \qquad ...(17.24)$$

Maximum Efficiency. Substituting the value of $V = 2u$ in equation (17.23), we get the maximum efficiency as

$$\eta_{max} = \frac{2u\,[2u - u]}{(2u)^2} = \frac{2u \times u}{2u \times 2u} = \frac{1}{2} = 0.5 \text{ or } 50\%. \qquad ...(17.25)$$

17.4.6 Force Exerted on a Series of Radial Curved Vanes.

For a radial curved vane, the radius of the vane at inlet and outlet is different and hence the tangential velocities of the radial vane at inlet and outlet will not be equal. Consider a series of radial curved vanes mounted on a wheel as shown in Fig. 17.23. The jet of water strikes the vanes and the wheel starts rotating at a constant angular speed.

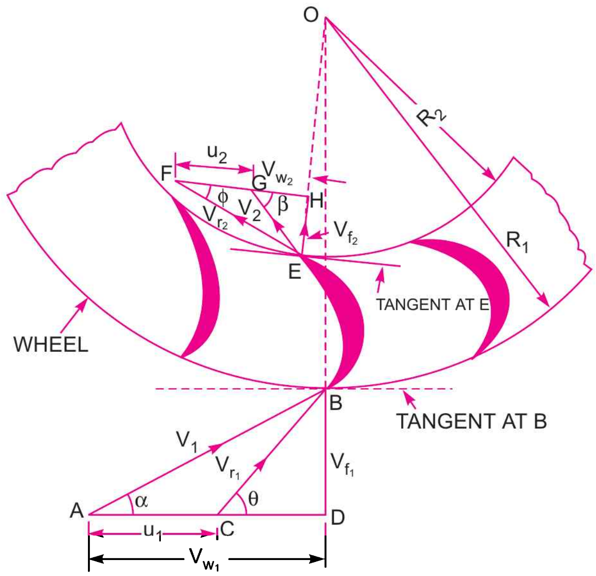


Fig. 17.23 *Series of radial curved vanes mounted on a wheel.*

Let R_1 = Radius of wheel at inlet of the vane,

R_2 = Radius of the wheel at the outlet of the vane,

ω = Angular speed of the wheel.

Then $u_1 = \omega R_1$ and $u_2 = \omega R_2$

The velocity triangles at inlet and outlet are drawn as shown in Fig. 17.23.

The mass of water striking per second for a series of vanes

= Mass of water coming out from nozzle per second

= $\rho a V_1$, where a = Area of jet and V_1 = Velocity of jet.

Momentum of water striking the vanes in the tangential direction per sec at inlet

= Mass of water per second × Component of V_1 in the tangential direction

= $\rho a V_1 \times V_{w_1}$ ($\because$ Component of V_1 in tangential direction = $V_1 \cos \alpha = V_{w_1}$)

Similarly, momentum of water at outlet per sec

= $\rho a V_1$ × Component of V_2 in the tangential direction

= $\rho a V_1 \times (-V_2 \cos \beta) = -\rho a V_1 \times V_{w_2}$ ($\because$ $V_2 \cos \beta = V_{w_2}$)

–ve sign is taken as the velocity V_2 at outlet is in opposite direction.

Now, angular momentum per second at inlet

= Momentum at inlet × Radius at inlet

= $\rho a V_1 \times V_{w_1} \times R_1$

Angular momentum per second at outlet

= Momentum of outlet × Radius at outlet

= $-\rho a V_1 \times V_{w_2} \times R_2$

Torque exerted by the water on the wheel,

T = Rate of change of angular momentum

= [Initial angular momentum per second – Final angular momentum per second]

$= \rho a V_1 \times V_{w_1} \times R_1 - (-\rho a V_1 \times V_{w_2} \times R_2) = \rho a V_1 [V_{w_1} \times R_1 + V_{w_2} R_2]$

Work done per second on the wheel

= Torque × Angular velocity = $T \times \omega$

$= \rho a V_1 [V_{w_1} \times R_1 + V_{w_2} R_2] \times \omega = \rho a V_1 [V_{w_1} \times R_1 \times \omega + V_{w_2} R_2 \times \omega]$

$= \rho a V_1 [V_{w_1} u_1 + V_{w_2} \times u_2]$ $(\because u_1 = \omega R_1 \text{ and } u_2 = \omega R_2)$

If the angle β in Fig. 17.23 is an obtuse angle then work done per second will be given as

$= \rho a V_1 [V_{w_1} u_1 - V_{w_2} u_2]$

∴ The general expression for the work done per second on the wheel

$$= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2] \quad ...(17.26)$$

If the discharge is radial at outlet, then $\beta = 90°$ and work done becomes as

$$= \rho a V_1 [V_{w_1} u_1] \quad (\because V_{w_2} = 0) \quad ...(17.27)$$

Efficiency of the Radial Curved Vane

The work done per second on the wheel is the output of the system whereas the initial kinetic energy per second of the jet is the input. Hence, efficiency of the system is expressed as

$$\text{Efficiency,} \quad \eta = \frac{\text{Work done per second}}{\text{Kinetic energy per second}} = \frac{\rho a V_1 \left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{\frac{1}{2}(\text{mass/sec}) \times V_1^2}$$

$$= \frac{\rho a V_1 \left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{\frac{1}{2}(\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{V_1^2}. \quad ...(17.28)$$

If there is no loss of energy when water is flowing over the vanes, the work done on the wheel per second is also equal to the change in kinetic energy of the jet per second. Hence, the work done per second on the wheel is also given as

Work done per second on the wheel

= Change of K.E. per second of the jet

= (Initial K.E. per second – Final K.E. per second) of the jet

$$= \left(\frac{1}{2} m V_1^2 - \frac{1}{2} m V_2^2\right)$$

$$= \frac{1}{2} m \left(V_1^2 - V_2^2\right) = \frac{1}{2}\left(\rho a V_1^2\right)\left(V_1^2 - V_2^2\right) \quad (\because \text{mass/second} = \rho a V_1)$$

$$\text{Hence efficiency, } \eta = \frac{\text{Work done per second on the wheel}}{\text{Initial K.E. per second of the jet}}$$

$$= \frac{\frac{1}{2} \rho a V_1^2 \left(V_1^2 - V_2^2\right)}{\frac{1}{2}\left(\rho a V_1^2\right) \cdot V_1^2}$$

$$= \frac{V_1^2 - V_2^2}{V_1^2} = \left(1 - \frac{V_2^2}{V_1^2}\right) \qquad ...(17.28A)$$

From the above equation, it is clear that for a given initial velocity of the jet (*i.e.*, V_1), the efficiency will be maximum, when V_2 is minimum. But V_2 cannot be zero as in that case the incoming jet will not move out of the vane. Equation (17.28) also gives the efficiency of the system. From this equation, it is clear that efficiency will be maximum when V_{w_2} is added to V_{w_1}. This is only posible if β is an acute* angle. Also for maximum efficiency V_{w_2} should also be maximum. This is only possible if $\beta = 0$. In that case $V_{w_2} = V_2$ and angle ϕ will be zero. But in actual practice ϕ cannot be zero. Hence for maximum efficiency, the angle ϕ should be minimum.

Problem 17.24 *If in Problem 17.23, the jet of water instead of striking a single plate, strikes a series of curved vanes, find for the data given Problem 17.23,*

(i) Force exerted by the jet on the vane in the direction of motion,
(ii) Power exerted on the vane, and
(iii) Efficiency of the vane.

Solution. Given :

From Problem 17.23, $V_1 = 15$ m/s, $u = u_1 = u_2 = 5$m/s,
$\alpha = 0$, $a = .007854\ \text{m}^2$
$\phi = 45°$, $V_{w_1} = 15$ m/s and $V_{w_2} = 2.07$ m/s.

For the series of vanes, mass of water striking per second
= Mass of water coming out from nozzle
$= \rho a V_1 = 1000 \times .007854 \times 15 = 117.72$

(*i*) Force exerted by the jet on the vane in the direction of motion

$F_x = \rho a V_1\ [V_{w_1} + V_{w_2}] = 117.72\ [15 + 2.07] =$ **2009.5 N. Ans.**

(*ii*) Power of the vane in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{F_x \times u}{1000}\text{kW} = \frac{2009.5 \times 5}{1000}$$

= **10.05 kW. Ans.**

(*iii*) Efficiency,
$$\eta = \frac{\text{Work done per second}}{\frac{1}{2}(\text{mass of water per sec}) \times V_1^2}$$

$$= \frac{2009.5 \times 5.0}{\frac{1}{2} \times 117.72 \times 15^2} = 0.7586 \text{ or } \mathbf{75.86\%}. \quad \textbf{Ans.}$$

Problem 17.25 *A jet of water having a velocity of 35 m/s impinges on a series of vanes moving with a velocity of 20 m/s. The jet makes an angle of 30° to the direction of motion of vanes when entering and leaves at an angle of 120°. Draw the triangles of velocities at inlet and outlet and find :*

(a) the angles of vanes tips so that water enters and leaves without shock,
(b) the work done per unit weight of water entering the vanes, and
(c) the efficiency.

Solution. Given :

Velocity of jet, $V_1 = 35$ m/s
Velocity of vane, $u_1 = u_2 = 20$ m/s
Angle of jet at inlet, $\alpha = 30°$

* The work done is equal to torque multiplied by ω(angular velocity). Torque is the rate of change of angular momentum. Due to change of angular momentum (*i.e.*, initial angular momentum – final angular momentum), V_{w_2} should be in opposite direction so that it can be added to V_{w_1}. This is possible if $\beta \angle 90°$.

Angle made by the jet at outlet with the direction of motion of vanes = 120°

∴ Angle $\beta = 180° - 120° = 60°$

(*a*) *Angle of vanes tips.*

From inlet velocity triangle

$$V_{w_1} = V_1 \cos \alpha = 35 \cos 30° = 30.31 \text{ m/s}$$

$$V_{f_1} = V_1 \sin \alpha = 35 \sin 30° = 17.50 \text{ m/s}$$

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{17.50}{30.31 - 20} = 1.697$$

∴ $\theta = \tan^{-1} 1.697 = \mathbf{60°. \ Ans.}$

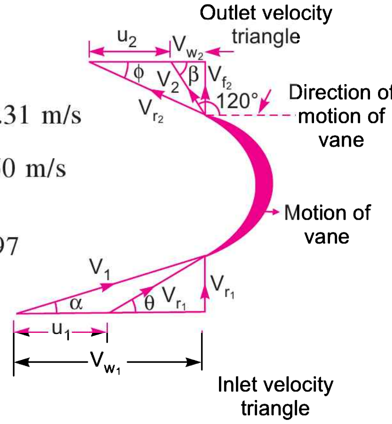


Fig. 17.23(*a*)

By sine rule, $\dfrac{V_{r_1}}{\sin 90°} = \dfrac{V_{f_1}}{\sin \theta}$ or $\dfrac{V_{r_1}}{1} = \dfrac{17.50}{\sin 60°}$

∴ $V_{r_1} = \dfrac{17.50}{.866} = 20.25$ m/s.

Now, $V_{r_2} = V_{r_1} = 20.25$ m/s

From outlet velocity triangle, by sine rule

$$\frac{V_{r_2}}{\sin 120°} = \frac{u_2}{\sin (60° - \phi)} \quad \text{or} \quad \frac{20.25}{0.886} = \frac{20}{\sin (60° - \phi)}$$

∴ $\sin (60° - \phi) = \dfrac{20 \times 0.866}{20.25} = 0.855 = \sin (58.75°)$

$60° - \phi = 58.75°$

∴ $\phi = 60° - 58.75° = \mathbf{1.25°. \ Ans.}$

(*b*) *Work done per unit weight of water entering* $= \dfrac{1}{g}(V_{w_1} + V_{w_2}) \times u_1$ ...(*i*)

$$V_{w_1} = 30.31 \text{ m/s and } u_1 = 30 \text{ m/s}$$

The value of V_{w_2} is obtained from outlet velocity triangle

$$V_{w_2} = V_{r_2} \cos \phi - u_2 = 20.25 \cos 1.25° - 20.0 = 0.24 \text{ m/s}$$

∴ Work done/unit weight $= \dfrac{1}{9.81}[30.31 + 0.24] \times 20 = \mathbf{62.28 \ Nm/N. \ Ans.}$

(*c*) *Efficiency* $= \dfrac{\text{Work done per kg}}{\text{Energy supplied per kg}}$

$$= \frac{62.28}{\dfrac{V_1^2}{2g}} = \frac{62.28 \times 2 \times 9.81}{35 \times 35} = \mathbf{99.74\% \ Ans.}$$

Problem 17.26 *A jet of water having a velocity of 30 m/s strikes a series of radial curved vanes mounted on a wheel which is rotating at 200 r.p.m. The jet makes an angle of 20° with the tangent to the wheel at inlet and leaves the wheel with a velocity of 5 m/s at an angle of 130° to the tangent to the wheel at outlet. Water is flowing from outward in a radial direction. The outer and inner radii of the wheel are 0.5 m and 0.25 m respectively. Determine :*

(i) Vane angles at inlet and outlet, *(ii) Work done per unit weight of water, and*
(iii) Efficiency of the wheel.

Solution. Given :

Velocity of jet, $V_1 = 30$ m/s

Speed of wheel, $N = 200$ r.p.m.

$\therefore$ Angular speed, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 200}{60} = 20.94$ rad/s

Angle of jet at inlet, $\alpha = 20°$

Velocity of jet at outlet, $V_2 = 5$ m/s

Angle made by the jet at outlet with the tangent to wheel = 130°

$\therefore$ Angle, $\beta = 180° - 130° = 50°$

Outer radius, $R_1 = 0.5$ m

Inner radius, $R_2 = 0.25$ m

$\therefore$ Velocity $u_1 = \omega \times R_1 = 20.94 \times 0.5 = 10.47$ m/s

And $u_2 = \omega \times R_2 = 20.94 \times 0.25 = 5.235$ m/s.

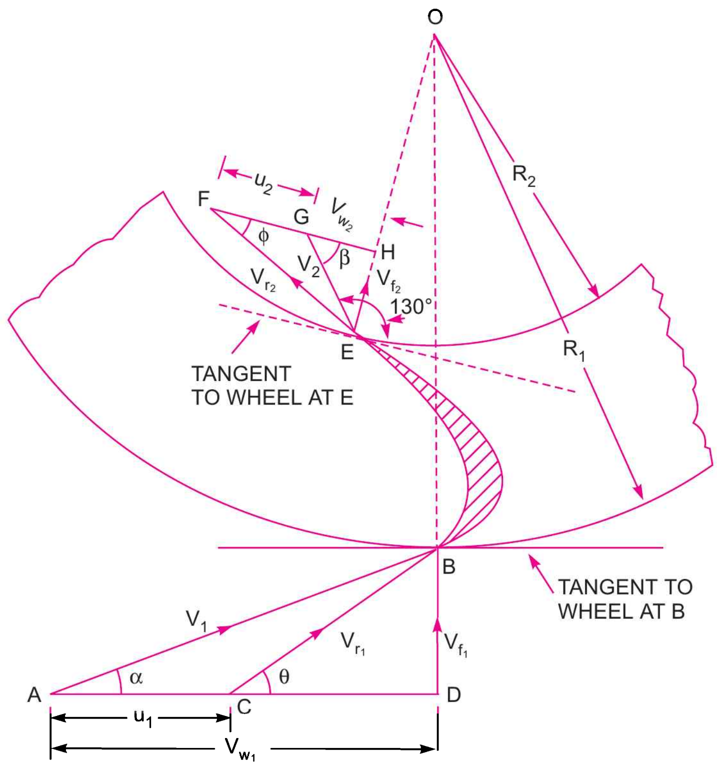


Fig. 17.24

(*i*) **Vane angles** at inlet and outlet means the angle made by the relative velocities V_{r_1} and V_{r_2}, *i.e.*, angle θ and ϕ.

From ΔABD,
$$V_{w_1} = V_1 \cos \alpha = 30 \times \cos 20° = 28.19 \text{ m/s}$$
$$V_{f_1} = V_1 \sin \alpha = 30 \times \sin 20° = 10.26 \text{ m/s}$$

In ΔCBD,
$$\tan \theta = \frac{BD}{CD} = \frac{V_{f_1}}{AD - AC} = \frac{10.26}{V_{w_1} - u_1} = \frac{10.26}{28.19 - 10.47} = 0.579 = \tan 30.07$$

$\therefore$ $\theta = 30.07°$ or **30° 4.2′. Ans.**

From outlet velocity Δ,
$$V_{w_2} = V_2 \cos \beta = 5 \times \cos 50° = 3.214 \text{ m/s}$$
$$V_{f_2} = V_2 \times \sin \beta = 5 \sin 50° = 3.83 \text{ m/s}$$

In ΔEFH,
$$\tan \phi = \frac{V_{f_2}}{u_2 + V_{w_2}} = \frac{3.83}{5.235 + 3.214} = 0.453 = \tan 24.385°$$

$\therefore$ $\phi = 24.385°$ or **24° 23.1′. Ans.**

(*ii*) Work done per second by water is given by equation (17.26)
$$= \rho a V_1 \left[V_{w_1} u_1 + V_{w_2} u_2\right]$$
(+ ve sign is taken as β is acute angle in Fig.17.24)

$\therefore$ Work done* per second per unit weight of water striking per second
$$= \frac{\rho a V_1\left[V_{w_1} u_1 + V_{w_2} u_2\right]}{\text{Weight of water/s}} = \frac{\rho a V_1\left[V_{w_1} u_1 + V_{w_2} u_2\right]}{\rho a V_1 \times g}$$
$$= \frac{1}{g}\left[V_{w_1} u_1 + V_{w_2} u_2\right] \text{Nm/N} = \frac{1}{9.81}\,[28.19 \times 10.47 + 3.214 \times 5.235]$$
$$= \frac{1}{9.81}\,[295.15 + 16.82] = \mathbf{31.8\ Nm/N.\ Ans.}$$

(*iii*) Efficiency, η is given by equation (17.28) as
$$\eta = \frac{2\left[V_{w_1} u_1 + V_{w_2} u_2\right]}{V_1^2} = \frac{2[28.19 \times 10.47 + 3.214 \times 5.235]}{30^2}$$
$$= \frac{2[295.15 + 16.82]}{30 \times 30} = 0.6932 \text{ or } \mathbf{69.32\%.\ Ans.}$$

▶ 17.5 JET PROPULSION

Jet propulsion means the propulsion or movement of the bodies such as ships, aircrafts, rocket etc., with the help of jet. The reaction of the jet coming out from the orifice provided in the bodies is used to move the bodies. This is explained as given below.

* Work done per second per unit weight striking per second is same as work done per unit weight of water.

A jet of fluid coming out from an orifice or nozzle, when strikes a plate, exerts a force on the plate. The magnitude of the force exerted on the plate can be determined depending upon whether plate is flat, inclined, curved, stationary or moving. This force exerted by the jet on the plate is called as '*action of the jet*'. But according to Newtons third law of motion, every action is accompanied by an equal and opposite reaction. Hence the jet while coming out of the orifice or nozzle, exerts a force on the orifice or nozzle in the opposite direction in which jet is coming out. The magnitude of the force exerted is equal to the 'action of the jet'. This force which is acting on the orifice or nozzle in the opposite direction is called the 'reaction of the jet'. If the body in which orifice or nozzle is fitted, is free to move, the body will start moving in the direction opposite to the jet. The following cases are important where this principle is used :

(*a*) Jet propulsion of a tank to which orifice is fitted, and

(*b*) Jet propulsion of ships.

17.5.1 Jet Propulsion of a Tank with an Orifice. Consider a large tank fitted with an orifice in one of its sides as shown in Fig. 17.25.

Let H = Constant head of water in tank from the centre of orifice,

a = Area of orifice,

V = Velocity of the jet of water,

C_v = Co-efficient of the velocity of orifice.

Then $V = C_v \sqrt{2gH}$

And mass of water coming out from the orifice per second

$= \rho \times$ Volume per second $= \rho \times$ (Area $\times$ Velocity)

$= \rho \times a \times V$

Force acting on the water is equal to the rate of change of momentum.

or F = Mass per second $\times$ [Change of velocity]

= Mass per second $\times$ [Final velocity – Initial velocity].

Note. Here change of velocity is to be taken as final minus initial as we are finding force on water and not force exerted by water.

Initial velocity of water in the tank is zero and final velocity of water when it comes out in the form of jet is equal to V.

$$\therefore \quad F = \rho a V\,[V - 0] = \rho a V^2 \qquad \ldots(17.29)$$

Thus, F is the force exerted on the jet of water. This jet of water will exert a force on the tank which is equal to F but opposite in direction as shown in Fig. 17.25. The force will be acting at A, the point on the tank in the horizontal line of the centre of the orifice. If the tank is free to move or the tank is fitted with frictionless wheel, it will start moving with some velocity say, 'u' in the direction opposite to the direction of the jet. When the tank starts moving, the velocity of the jet with which it comes out of the orifice will not be equal to V but it will be equal to the relative velocity of the jet with respect to tank.

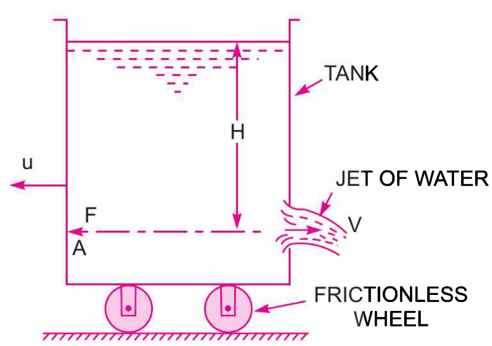


Fig. 17.25 *Jet propulsion of a tank with an orifice.*

Hence if V = Absolute velocity of jet,

u = Velocity of tank,

V_r = Velocity of jet with respect to tank

Then V_r = Vectorial difference of absolute velocity (V) and velocity of tank (u)

$= V - (-u)$(as u is in opposite direction to V hence velocity of tank is taken as $-u$)

$= V + u$

Hence when the tank is moving, the velocity with which jet comes out from the orifice is $(V + u)$.

Mass of water coming out from the orifice per sec

$= \rho \times a \times$ Velocity with which water comes out

$= \rho \times a \times (V_r) = \rho a\,(V + u)$

∴ Force exerted on the tank is given as

F_x = Mass of water coming out from orifice per second × [Change of velocity]*

$= \rho a(V + u) \times [(V + u) - u] = \rho a\,[V + u]\,[V]$

$$= \rho a[V + u] \times V \qquad \text{...(17.30)}$$

Thus, the force given by equation (17.30) is used for propelling the tank.

∴ Work done on the moving tank by jet per second

$= F_x \times u = \rho a(V + u) \times V \times u$

∴ Efficiency of propulsion is given as,

$$\eta = \frac{\text{Work done per second}}{\text{Kinetic energy of the issuing jet per second}}$$

$$= \frac{\rho a(V+u)\times V\times u}{\frac{1}{2}(\text{Mass of water issuing per second})\times(\text{Velocity of issuing jet})}$$

$$= \frac{\rho a(V+u)\times V\times u}{\frac{1}{2}\left[\rho a(V+u)\right]\times(V+u)^2} = \frac{2Vu}{(V+u)^2} \qquad \text{...(17.31)}$$

Condition for Maximum Efficiency and Expression for Maximum η. For a given value of V, the efficiency will be maximum when $\dfrac{d\eta}{du} = 0$

or $$\frac{d}{du}\left[\frac{2Vu}{(V+u)^2}\right] = 0 \quad \text{or} \quad \frac{d}{du}\left[2Vu\times(V+u)^{-2}\right] = 0$$

or $$2Vu \times (-2)\,(V + u)^{-3} + (V + u)^{-2} \times 2V = 0$$

or $$\frac{-4Vu}{(V+u)^3} + \frac{2V}{(V+u)^2} = 0 \quad \text{or} \quad -4Vu + 2V\,(V + u) = 0$$

Dividing by $2V$, $-2u + (V + u) = 0$ or $-u + V = 0$ or $u = V$ ...(17.32)

Equation (17.32) is the condition for maximum efficiency. Substituting equation (17.32) in equation (17.31), the value of maximum efficiency is obtained as

$$\eta_{max} = \frac{2\times u\times u}{(u+u)^2} = \frac{2u^2}{4u^2} = \frac{1}{2} = 0.5 \text{ or } 50\%. \qquad \text{...(17.32A)}$$

---

* Change of velocity is the final velocity minus initial velocity of jet of water coming out from the orifice. The final velocity of the jet with respect to tank is $(V + u)$. This velocity is obtained by applying a velocity u to the whole system (*i.e.*, tank, water in the tank and jet of water) in a direction opposite to the motion of tank. Then final velocity of jet becomes as $(V + u)$ and initial velocity of water as u. Hence change of velocity is $(V + u) - u$.

Problem 17.27 *The head of water from the centre of the orifice which is fitted to one side of the tank is maintained at 2 m of water. The tank is not allowed to move and the diameter of orifice is 100 mm. Find the force exerted by the jet of water on the tank. Take C_v = 0.97 cm.*

Solution. Given :

Head of water, $H = 2$ m

Diameter of orifice, $d = 100$ mm $= 0.1$ m

∴ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$

Value of $C_v = 0.97$

∴ Velocity of jet, $V = C_v \times \sqrt{2gH} = 0.97 \times \sqrt{2 \times 9.81 \times 2.0} = 6.07$ m/s

Force exerted on the tank is given by equation (17.29) as

$$F = \rho a V^2 = 1000 \times .007854 \times 6.072 = \textbf{289.3 N. Ans.}$$

Problem 17.28 *If in the above problem, the tank is fitted with frictionless wheels and allowed to move, determine*

(i) Propelling force on tank, *(ii) Work done by the propelling force per second, and*

(iii) Efficiency of propulsion.

The tank is moving with a velocity of 2 m/s.

Solution. Given :

$H = 2$ m, $d = 100$ mm, $a = .007854 \text{ m}^2$, $C_v = 0.97$

and velocity of jet, $V = 6.07$ m/s.

Velocity of tank, $u = 2$ m/s.

(*i*) Propelling force is given by equation (17.30) as

$$F_x = \rho a(V + u) \times V$$

$$= 1000 \times .007854 \times (6.07 + 2.0) \times 6.07 = \textbf{384.65 N. Ans.}$$

(*ii*) Work done by the propelling force per second

$$= F_x \times u = 384.65 \times 2.0 = \textbf{769.3 N m/s. Ans.}$$

(*iii*) Efficiency of propulsion is given by equation (17.31) as

$$\eta = \frac{2Vu}{(V+u)^2} = \frac{2 \times 6.07 \times 2.0}{(6.07 + 2.0)^2} = \textbf{0.3728 or 37.28\%. Ans.}$$

17.5.2 Jet Propulsion of Ships. By the application of the jet propulsion principle, a ship is driven through water. A jet of water which is discharged at the back (also called stern) of the ship, exerts a propulsive force on the ship. The ship carries centrifugal pumps which draw water from the surrounding sea. This water is discharged through the orifice provided at the back of the ship in the form of a jet. The reaction of the jet coming out at the back of the ship propels the ship in the opposite direction of the jet. The water from the surrounding sea by the centrifugal pump is taken by the following two ways :

1. Through inlet orifices which are at right angles to the direction of the motion of the ship, and
2. Through the inlet orifices, which are facing the direction of motion of the ship.

1st Case. Jet propulsion of the ship when the inlet orifices are at right angles to the direction of the motion of the ship.

Fig. 17.26 shows a ship which is having the inlet orifices at right angles to its direction.

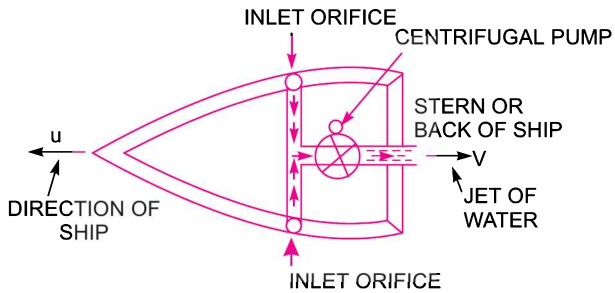


Fig. 17.26 *Inlet orifices are at right angles.*

Let V = Absolute velocity of jet of water coming at the back of the ship,

u = Velocity of the ship,

V_r = Relative velocity of jet with respect to ship $= (V + u)$.

As the velocity V and u are in opposite direction and hence relative velocity will be equal to the sum of these two velocities.

Mass of water issuing from the orifice at the back of the ship $= \rho a V_r = \rho a(V + u)$,

where a = Area of the jet of water

∴ Propulsive force exerted on the ship

$$F = \text{Mass of water issuing per sec} \times \text{Change of velocity*}$$

$$= \rho a(V + u)\,[V_r - u] = \rho a(V + u)\,[(V + u) - u] = \rho a(V + u) \times V \quad ...(17.33)$$

Work done per second $= F \times u = \rho a\,(V + u) \times V \times u$ ...(17.34)

The efficiency of propulsion, the condition of maximum efficiency and expression for maximum efficiency are given by equations (17.31), (17.32) and (17.32 *A*) respectively.

Note. (*i*) When the inlet orifices are at right angles to the direction of motion of the ship, then this case is also known as water is drawn **AMID SHIP** which means the water is drawn at the middle of the ship.

(*ii*) The centrifugal pump draws the water from the surrounding sea and discharges through orifice. The kinetic energy of the issuing jet is $\frac{1}{2} \times \text{mass} \times \text{velocity}^2$ *i.e.*, $\frac{1}{2}\,[\rho a(V + u)] \times [V + u]^2 = \frac{1}{2}\,\rho a(V + u)^3$. This energy is provided by centrifugal pump *i.e.*, work is done by pump to provide this energy.

Problem 17.29 *Find the propelling force acting on a ship which takes water through inlet orifices which are at right angles to the direction of motion of ship, and discharges at the back through orifices having effective areas of 0.04 m². The water is flowing at the rate of 1000 litres/s and ship is moving with a velocity of 8 m/s.*

Solution. Given :

Effective areas of orifices, $a = 0.04\ \text{m}^2$

Discharge of water, $Q = 1000\ \text{litres/s} = 1\ \text{m}^3/\text{s}$

∴ Velocity of jet relative to water $= \dfrac{Q}{a} = \dfrac{1}{.04} = \dfrac{100}{4} = 25$ m/s or $V_r = 25$ m/s

---

* To find the change of velocity, apply a velocity u to the whole system (*i.e.*, ship, jet of water and surrounding water in the sea) in a direction opposite to the motion of ship. Then final velocity of jet of water becomes as $(V + u)$. And velocity of water in sea becomes as u. Hence change of velocity becomes $(V + u) - u$.

Velocity of ship, $u = 8$ m/s

Now, $V_r = u + V$, where V = Absolute velocity of jet

$\therefore$ $25 = 8 + V$ or $V = 25 - 8 = 17$ m/s

Propelling force is given by equation (17.33) as,

$$F = \rho a\,(V + u) \times V$$
$$= 1000 \times .04 \times (17 + 8) \times 17 = \mathbf{16999.94\ N.\ Ans.}$$

Problem 17.30 *The water in a jet propelled boat is drawn amid-ship and discharged at the back with an absolute velocity of 20 m/s. The cross-sectional area of the jet at the back is 0.02 m*^{2} *and the boat is moving in sea water with a speed of 30 km/hour. Determine :*

(i) Propelling force on the boat, (ii) Power required to drive the pump, and

(iii) Efficiency of the jet propulsion.

Solution. Given :

'Water is drawn amid-ship' means water is drawn at the middle of the ship and inlet orifices are at right angles to the motion of ship.

Absolute velocity of jet, $V = 20$ m/s

Area of the jet, $a = 0.02\ \text{m}^2$

Speed of boat, $u = 30 \text{ km/hr} = \dfrac{30 \times 1000}{60 \times 60} = 8.33$ m/s.

(*i*) Propelling force is given by equation (17.33) as

$$F = \rho a(V + u) \times V$$
$$= 1000 \times .02(20 + 8.33) \times 20 = \mathbf{11332\ N.\ Ans.}$$

(*ii*) Power required to drive the pump in kW

$$= \frac{\text{Work done per sec}}{1000} = \frac{F \times u}{1000} = \frac{11332 \times 8.33}{1000} = \mathbf{94.395\ kW.\ Ans.}$$

(*iii*) Efficiency of the jet propulsion is given by (17.31) as

$$\eta = \frac{2Vu}{(V + u)^2} = \frac{2 \times 20 \times 8.33}{(20 + 8.33)^2} = \mathbf{0.415\ or\ 41.5\%.\ Ans.}$$

Problem 17.31 *A small ship is fitted with jets of total area 0.65 m*^{2}*. The velocity through the jet is 9 m/s and speed of the ship is 18 km p.h. in sea-water. The efficiencies of the engine and pump are 85% and 65% respectively. If the water is taken amid-ships, determine the propelling force and the overall efficiency, assuming the pipe losses to be 10% of the kinetic energy of the jets.*

Solution. Given :

Total area of jets, $a = 0.65\ \text{m}^2$

Velocity through the jet relative to ship, $V_r = 9$ m/s

Speed of ship, $u = 18 \text{ km/hour} = \dfrac{18 \times 1000}{60 \times 60} \text{ m/s} = 5$ m/s

Efficiency of the engine, $\eta_E = 85\% = 0.85$

Efficiency of the pump, $\eta_P = 65\% = 0.65$

Pipe losses, h_f = 10% of kinetic energy of the jet

$$= \frac{10}{100} \times \frac{V_r^2}{2g} = \frac{V_r^2}{20g}$$

Now, $V_r = u + V$, where V = Absolute velocity of jet

$\therefore$ $9 = 5 + V$ or $V = 9 - 5 = 4$ m/s.

(*i*) Propelling force is given by equation (17.33) as

$$F = \rho a\,(V + u) \times V$$

$$= 1000 \times 0.65 \times (4 + 5) \times 4 = \mathbf{23400\ N.\ Ans.}$$

(*ii*) Work done by the jets per second

$$= F \times u = 23400 \times 5 = 117000 \text{ Nm/s}$$

Weight of water issuing from the jets per second

$$= g \times \text{Mass of water per second}$$

$$= g \times \rho a V_r = 9.81 \times 1000 \times 0.65 \times 9 = 57388.5 \text{ N/s.}$$

The pump should have the output which will give the jet a relative velocity (V_r) and also overcome the pipe losses.

$\therefore$ Output of the pump per unit weight of water

= Kinetic energy of jet + Pipe losses

$$= \frac{V_r^2}{2g} + \frac{V_r^2}{20g} = \frac{V_r^2}{2g}(1 + 0.1) = \frac{V_r^2}{2g} \times 1.1$$

Input to the pump per unit weight of water

$$= \frac{\text{Output of pump}}{\text{Efficiency of pump}} = \frac{1.1\, V_r^2}{2g \times 0.65}$$

The input to the pump is equal to the output of the engine. Hence input to the engine per unit weight of water

$$= \frac{1.1\, V_r^2}{2g \times 0.65 \times \text{Efficiency of engine}}$$

$$= \frac{1.1\, V_r^2}{2g \times 0.65 \times 0.85} = \frac{1.1 \times 9^2}{2 \times 9.81 \times 0.65 \times 0.85} = 8.22 \text{ Nm/N}$$

$\therefore$ Total input to the engine = Weight of water × Input per unit weight of water

$$= 57388.5 \times 8.22 = 471733.5 \text{ Nm}$$

$\therefore$ Overall efficiency, $$\eta_o = \frac{\text{Work done by jets}}{\text{Total input to engine}} = \frac{117000}{471733.5} = 0.248 = \mathbf{24.80\%.\ Ans.}$$

Problem 17.32 *A jet propelled boat, moving with a velocity of 5 m/s, draws water amid-ship. The water is discharged through two jets provided at the back of the ship. The diameter of each jet is 150 mm. The total resistance offered to the motion of the boat is 4905 N (500 × 9.81 N). Determine :*

(i) Volume of water drawn by the pump per second, and

(ii) Efficiency of the jet propulsion.

Solution. Given :

Velocity of boat, $u = 5$ m/s

Diameter of each jet, $d = 150$ mm $= 0.15$ m

Area of each jet $= \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$

$\therefore$ Total area of the jets, $a = 2 \times .01767 = .03534 \text{ m}^2$

Total resistance to motion $= 4905$ N $(500 \times 9.81$ N)

The propelling force must be equal to the resistance to the motion.

$\therefore$ Propelling force, $F = 4905$ N or $(500 \times 9.81$ N)

Propelling force is given by equation (17.33) as

$$F = \rho a\,(V + u)\,V$$

$$500 \times 9.81 = 1000 \times 0.03534 \times (V + 5) \times V$$

or
$$500 = \frac{1000}{9.81} \times .03534 \times (V + 5) \times V$$

$$= 3.6\,(V + 5)\,V = 3.6V^2 + 3.6 \times 5V = 3.6\,V^2 + 18V$$

or $3.6\,V^2 + 18V - 500 = 0$

The above equation is quadratic and its solution is

$$V = \frac{-18 \pm \sqrt{18^2 + 4 \times 3.6 \times 500}}{2 \times 3.6} = \frac{-18 \pm 86.74}{7.2}$$

$$= \frac{86.74 - 18}{2} = 34.37 \text{ m/s} \qquad \text{[–ve value is not possible]}$$

(*i*) Volume of water drawn by the pump per second is equal to the volume of water discharged through the orifices at the back in the form of jets and this volume

$$= aV_r = a(V + u)$$

$$= .03534 \times (34.37 + 5.0) = \mathbf{1.39\ m^3/s.\ Ans.}$$

(*ii*) Efficiency of the jet propulsion is given by equation (17.31) as

$$\eta = \frac{2Vu}{(V + u)^2} = \frac{2 \times 34.37 \times 5.0}{(34.37 + 5.0)^2} = .2217 \text{ or } \mathbf{22.17\%.\ Ans.}$$

2nd Case. Jet propulsion of ship when the inlet orifices face the direction of motion of the ship. Fig. 17.27 shows a ship which is having the inlet orifices facing the direction of the motion of the ship. In this case the expression for propelling force and work done per second will be same as in the 1st case in which inlet orifices are at right angles to the ship. But the energy supplied by the jet will be different,

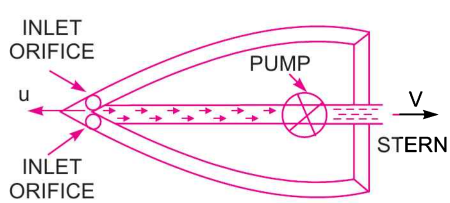


Fig. 17.27 *Inlet orifices facing the direction of ship.*

as in this case the water enters with a velocity equal to the velocity of the ship, *i.e.,* with a velocity u. Hence the expression for the energy supplied by the jet.

$$= \frac{1}{2} \text{(Mass of water supplied per sec)} \times [V_r^2 - u^2]$$

$$= \frac{1}{2} (\rho a V_r) \times [V_r^2 - u^2]$$

where $V_r = (V + u)$ as in the previous case

$\therefore$ K.E. supplied by jet $= \frac{1}{2}\rho a(V + u)\ [(V + u)^2 - u^2]$ ...(17.35)

$\therefore$ Efficiency of propulsion, $\eta = \dfrac{\text{Work done per sec by jet}}{\text{Energy supplied by jet}}$

$$= \frac{\rho a(V+u) \times V \times u}{\frac{1}{2}\rho a(V+u)\left[(V+u)^2 - u^2\right]}$$

$\left\{\because \text{ From equation (17.34) work done} = \rho a\,(V+u)\,V \times u\right\}$

$$= \frac{2V \times u}{(V+u)^2 - u^2} = \frac{2Vu}{V^2 + u^2 + 2Vu - u^2} = \frac{2Vu}{V^2 + 2Vu} = \frac{2u}{V + 2u} \quad ...(17.36)$$

Problem 17.33 *The water in a jet propelled boat is drawn through inlet openings facing the direction of motion of the ship. The boat is moving in sea-water with a speed of 30 km/hour. The absolute velocity of the jet of the water discharged at the back is 20 m/s and the area of the jet of water is 0.03 m*^{2}. *Find the propelling force and efficiency of propulsion.*

Solution. Given :

Speed of boat, $u = 30 \text{ km/hr} = \dfrac{30 \times 1000}{60 \times 60} = 8.33 \text{ m/s}$

Absolute velocity of jet, $V = 20$ m/s

Area of the jet, $a = .03\ \text{m}^2$.

(*i*) Propelling force is given by equation (17.33) as

$$F = \rho a\ (V + u) \times V = 1000 \times .03 \times (20 + 8.33) \times 20 = 16997.98 \text{ N.}$$

(*ii*) Efficiency of propulsion is given by equation (17.36) as

$$\eta = \frac{2u}{V + 2u} = \frac{2 \times 8.33}{20 + 2 \times 8.33} = \mathbf{0.4544} \text{ or } \mathbf{45.44\%.}\ \textbf{Ans.}$$

HIGHLIGHTS

1. The force exerted by a jet of water on a stationary plate in the direction of the jet is given by

$F_x = \rho a V^2$ ...for a vertical plate

$= \rho a V^2 \sin^2 \theta$ ...for an inclined plate

$= \rho a V^2 (1 + \cos \theta)$ ...for a curved plate and jet strikes at the centre

$= 2\rho a V^2 \cos \theta$ ...for a curved plate and jet strikes at one of the tips of the jet.

where V = Velocity of the jet,

θ = Angle between the jet and the plate for inclined plate,

= Angle made by the jet with the direction of motion for curved plates.

2. When a jet of water strikes a vertical hinged plate, the angle of swing about the hinge is given by

$$\sin \theta = \frac{\rho a V^2}{W}$$

where V = Velocity of the jet of water, W = Weight of the hinged plate.

3. The force exerted by a jet of water on a moving plate, in the direction of the motion of the plate, is given by

$F_x = \rho a (V - u)^2$ ...for a moving vertical plate,

$= \rho a (V - u)^2 \sin^2 \theta$ ...for an inclined moving plate,

$= \rho a (V - u)^2 (1 + \cos \theta)$ ...when jet strikes the curved plate at the centre

4. When a jet of water strikes a curved moving plate at one of its tips and comes out at the other tip, the force exerted and work done are obtained from velocity triangles at inlet and outlet. The expression for force and work done are

$$F_x = \rho a V_{r_1} \left[V_{w_1} \pm V_{w_2} \right]$$

$$\text{Work done per second} = \rho a V_{r_1} \left[V_{w_1} \pm V_{w_2} \right] \times u$$

+ve sign is taken when β is an acute angle. If β is an obtuse angle then –ve sign is taken. If β is 90°,

$$V_{w_2} = 0.$$

$$\text{Work done per second per kg of fluid} = \frac{1}{g} \left[V_{w_1} \pm V_{w_2} \right].$$

5. For a series of vanes, the force and work done are given as $F_x = \rho a V_{r_1} \left[V_{w_1} \pm V_{w_2} \right]$

$$\text{Work done/sec} = \rho a V_{r_1} \left[V_{w_1} \pm V_{w_2} \right] \times u$$

$$\text{Work done/sec per kg} = \frac{1}{g} \left[V_{w_1} \pm V_{w_2} \right] \times u.$$

6. Efficiency of a series of vanes is given as $\eta = \dfrac{2u(V - u)}{V^2}$

and condition of max. η is $u = \dfrac{V}{2}$

Max. $\eta = 50\%$.

7. For a curved radial vane, the work done per second $= \rho a V_1 \left[V_{w_1} u_1 \pm V_{w_2} u_2 \right]$

where V_1 = Absolute velocity of jet at inlet, V_{w_1} = Velocity of whirl at inlet

u_1 = Tangential velocity of vane at inlet, V_{w_2} = Velocity of whirl at outlet

u_2 = Tangential velocity of vane at outlet.

8. For a curved radial vane the efficiency is given by

$$\eta = \frac{\rho a V_1 \left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{\frac{1}{2}(\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{V_1^2}.$$

9. Jet propulsion means the propulsion of a vessel with the help of the jet. The reaction of the jet is used for propelling the vessel. The propelling force exerted on a tank with a orifice is given by

$$F_x = \rho a (V + u) \times V$$

where V = Absolute velocity of the jet of water, u = Velocity of the tank.

10. The efficiency of propulsion is given by $\eta = \frac{2Vu}{(V+u)^2}$ and u = V for maximum efficiency

∴ Maximum $\eta = 50\%$.

11. Ships are also propelled by jets. The intake water by the centrifugal pump is taken by two ways. In one case, the water is taken from orifices which are at right angles to the direction of the motion of the ship and in the other case the water is taken through orifices which are facing the direction of motion of the ship.

EXERCISE

(A) THEORETICAL PROBLEMS

1. Define the terms : (*a*) Impact of jets, and (*b*) Jet propulsion.
2. Obtain an expression for the force exerted by a jet of water on a fixed vertical plate in the direction of the jet.
3. Show that the force exerted by a jet of water on an inclined fixed plate in the direction of the jet is given by,

$$F_x = \rho a V^2 \sin^2 \theta$$

where a = Area of the jet, V = Velocity of the jet

and θ = Inclination of the plate with the jet.

4. Prove that the force exerted by a jet of water on a fixed semi-circular plate in the direction of the jet when the jet strikes at the centre of the semi-circular plate is two times the force exerted by the jet on an fixed vertical plate.
5. Show that the angle of swing of a vertical hinged plate is given by $\sin \theta = \frac{\rho a V^2}{W}$

where V = Velocity of the jet striking the plate, a = Area of the jet, and W = Weight of the plate.

6. Differentiate between : (*i*) the force exerted by a jet of water on a fixed vertical plate and moving vertical plate, and (*ii*) the force exerted by a jet on a single curved moving plate and a series of curved moving plate.
7. Prove that the work done per second on a series of moving curved vanes by a jet of water striking at one of the tips of the vane is given by,

$$\text{Work done/sec} = \rho a V_1 \left[V_{w_1} \pm V_{w_2}\right] \times u.$$

8. Find an expression for the efficiency of a series of moving curved vanes when a jet of water strikes the vanes at one of its tips. Prove that maximum efficiency is when $u = V$ and the value of maximum efficiency is 50%.

9. Show that for a curved radial vane, the work done per second is given by, $\rho a V_1 \left[V_{w_1} u_1 \pm V_{w_2} u_2\right]$.

10. Find an expression for the propelling force and the work done per second on a tank which is provided with an orifice through which jet of water is coming out and tank is free to move.

11. Show that the efficiency of a free jet striking normally on a series of flat plates mounted on the periphery of a wheel can never exceed 50%.

12. Show that the force exerted by a jet of water on moving inclined plate in the direction of jet is given by

$$F_x = \rho a(V - u)^2 \sin^2\theta$$

where a = area of jet,
θ = inclination of the plate with the jet, and
V = velocity of jet. (*J.N.T.U., Hyderabad, S 2002*)

(B) NUMERICAL PROBLEMS

1. Find the force exerted by a jet of water of diameter 100 mm on a stationary flat plate, when the jet strikes the plate normally with a velocity of 30 m/s. [**Ans.** 7068.6 N]

2. A jet of water of diameter 50 mm moving with a velocity of 20 m/s strikes a fixed plate in such a way that the angle between the jet and the plate is 60°. Find the force exerted by the jet on the pate (*i*) in the direction normal to the plate, and (*ii*) in the direction of the jet. [**Ans.** (*i*) 680.13 N, (*ii*) 589 N]

3. A jet of water of diameter 100 mm moving with a velocity of 30 m/s strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate. [**Ans.** 10602.7 N]

4. A jet of water of the diameter 100 mm moving with a velocity of 20 m/s strikes a curved fixed plate tangentially at one end at an angle of 30° to the horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal and vertical directions. [**Ans.** 5672.34 N, 496.3 N]

5. A jet of water of 30 mm diameter, moving with a velocity of 15 m/s, strikes a hinged square plate of weight 245.25 N at the centre of the plate. The plate is of uniform thickness. Find the angle through which the plate will swing. [**Ans.** $\theta = 40° 25.6'$]

6. A plate is acted upon at its centre by a jet of water of diameter 20 mm with a velocity of 20 m/s. The plate is hinged and is deflected through an angle of 15°. Find the weight of the plate. If the plate is not allowed to swing, what will be the force required at the lower edge of the plate to keep the plate in vertical position. [**Ans.** 485.5 N, 62.8 N]

7. A jet of water of diameter 150 mm strikes a flat plate normally with a velocity of 12 m/s. The plate is moving with a velocity of 6 m/s in the direction of the jet and away from the jet. Find : (*i*) the force exerted by the jet on the plate, (*ii*) work done by the jet on the plate per second, (*iii*) power of the jet, and (*iv*) efficiency of the jet. [**Ans.** (*i*) 636.3 N, (*ii*) 3817.6 Nm/s, (*iii*) 3.82 kW, (*iv*) 25%]

8. If in the problem 7, the jet strikes the plate in such a way that the normal on the plate makes an angle of 30° to the axis of the jet, find : (*i*) The normal force exerted on the plate, (*ii*) power, and (*iii*) efficiency of the jet. [**Ans.** (*i*) 551 N, (*ii*) 2.86 kW, (*iii*) 18.74%]

9. A jet of water of diameter 100 mm strikes a curved plate at its centre with a velocity of 15 m/s. The curved plate is moving with a velocity of 7 m/s in the direction of the jet. The jet is deflected through an angle of 150°. Assuming the plate smooth find : (*i*) force exerted on the plate in the direction of the jet, (*ii*) power of the jet, and (*iii*) efficiency. [**Ans.** (*i*) 938 N (*ii*) 6.56 kW, (*iii*) 49.53%]

10. A jet of water having a velocity of 30 m/s strikes a curved vane, which is moving with a velocity of 15 m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves at an angle of 120° to the direction of motion of vane at outlet. Calculate : (*i*) Vane angles, if the water enters and leaves the vane without shock, (*ii*) Work done per second per unit weight of water striking the vanes per second. [**Ans.** (*i*) 53° 47.7′, 15° 41′, (*ii*) 44.15 Nm/N]

11. A jet of water of diameter 50 mm, having a velocity of 30 m/s strikes a curved vane which is moving with a velocity of 15 m/s in the direction of the jet. The jet leaves the vane at an angle of 60° to the direction of motion of vanes at outlet. Determine : (*i*) the force exerted by the jet on the vane in the direction of motion, (*ii*) work done per second by the jet. [**Ans.** (*i*) 662.5 N, (*ii*) 9937.5 Nm/s]

12. A jet of water having a velocity of 20 m/s strikes a curved vane which is moving with a velocity of 9 m/s. The vane is symmetrical and is so shaped that the jet is deflected through 120°. Find the angle of the jet at inlet of the vane so that there is no shock. What is the absolute velocity of the jet at outlet in magnitude and direction and the work done per second per unit weight of water strikings ? Assume the vane to be smooth. [**Ans.** 17°, 5.95 m/s, $\beta = 79° 6'$, 18.57 Nm/N]

13. A jet of water, having a velocity of 15 m/s, strikes a curved vane which is moving with a velocity of 6 m/s in the same direction as that of the jet at inlet. The vane is so shaped that the jet is deflected through 135°. The diameter of the jet is 150 mm. Assuming the vane to be smooth, find : (*i*) the force exerted by the jet on the vane in the direction of motion, (*ii*) power of the vane, and (*iii*) efficiency of the vane. [**Ans.** (*i*) 2443.5 N, (*ii*) 14.65 kW, (*iii*) 49.16%]

14. If in the above problem, the jet of water instead of striking a single plate, strikes a series of curved vanes, find : (*i*) force exerted by the jet on the vanes in the direction the motion, (*ii*) power of the vane, and (*iii*) efficiency of the vane. [**Ans.** (*i*) 4072.5 N, (*ii*) 24.43 kW, (*iii*) 81.9%]

15. A jet of water having a velocity of 30 m/s, strikes a series of radial curved vanes mounted on a wheel which is rotating at 300 r.p.m. The jet makes an angle of 30° with the tangent to wheel at inlet and leaves the wheel with a velocity of 4 m/s at an angle of 120° to the tangent to the wheel at outlet. Water is flowing from outward in a radial direction. The outer and inner radii of the wheel are 0.6 m and 0.3 m respectively. Determine : (*i*) vane angles at inlet and outlet, (*ii*) work done per second per kg of water, and (*iii*) efficiency of the wheel. [**Ans.** (*i*) 42° 10.7′, 27°17.8′, (*ii*) 52.92, (*iii*) 56.5%]

16. The head of water from the centre of the orifice fitted to a tank is maintained at 6 m of water. The diameter of the orifice is 150 mm. The tank is fitted with frictionless wheels at the bottom and the tank is moving with a velocity of 4 m/s due to the reaction of the jet coming out from the orifice. Determine : (*i*) propelling force on the tank, (*ii*) work done per second, and (*iii*) efficiency of propulsion [**Ans.** (*i*) 2847.3 N, (*ii*) 11389 Nm/s, (*iii*) $\eta = 39.36\%$]

17. The water in a jet propelled boat is drawn mid-ship and is discharged at the back with an absolute velocity of 30 m/s. The cross-sectional area of the jet at the back is 0.04 m^2 and the boat is moving in sea-water with a speed of 30 km/hour. Determine : (*i*) propelling force of the boat, (*ii*) power, and (*iii*) efficiency of the jet propulsion. [**Ans.** (*i*) 45995.6 N, (*ii*) 383.14 kW, (*iii*) 34.02%]

18. The water in a jet propelled boat is drawn through inlet openings facing the direction of motion of the ship. The boat is moving in sea-water with a speed of 40 km/hr. The absolute velocity of the jet of the water discharged at the back is 40 m/s and the area of the jet of water is 0.04 m^2. Find the propelling force and efficiency of propulsion. [**Ans.** 81775.3 N, $\eta = 35.71\%$]