9 Bathtub hydraulics

Having a bath is a good practical way to learn about hydraulics. Here are just some of the things to look out for.

Just filling the bath can be an experience in itself. The hot and cold water taps are running but then you notice that the two are not mixing well. One side of the bath is hot and the other cold. There is some mixing at the interface due to turbulence but this is all. The only way to get an even temperature is to stir the water vigorously with your hand. The reason for this mixing problem is that water density varies slightly with temperature and this density difference is enough to inhibit mixing. This is a major problem at most power generating stations that use vast quantities of water from rivers and the sea to cool their systems. High towers help to cool water before it is returned to the river or sea but any slight difference in temperature will stop it mixing fully with the receiving flow. Swimming downstream of a power station can be a pleasant, warm experience. The challenge for the engineers is to find ways of mixing the water thoroughly so that the receiving water returns to its original temperature as quickly as possible so as not to affect local aquatic plant and fish life. It also stops hot water short-circuiting the system and finding its way into the intake and back into the power station as can happen with coastal stations.

As water flows from the taps across the bottom of the bath the flow is usually super-critical – fast and shallow. But this state soon changes as a hydraulic jump forms at the far end and then quickly makes its way back towards the tap end as a travelling surge wave (Section 6.4). For a while a stable circular hydraulic jump can be seen just where the water plunges into the bath but eventually this is drowned out as the level in the bath rises. The incoming flow is still super-critical and this now shoots under the slow flow. The energy is not dispersed in the hydraulic jump, it is gradually absorbed as friction along the base of the bath slows the water down. When this occurs in natural channels it can cause severe erosion of the bed and sides (Section 7.2.1).

Once your bath is full to the right level and is at the right temperature, there is now that 'Eureka moment' that Archimedes experienced when he stepped into the water. Archimedes first discovered the significance of this some 2000 years ago when he realised that the water displaced when you get into the bath has the same volume as your body. He also noticed that if you float on the water instead of sitting on the bottom then the amount of water you displace is equal to your weight. From this he was able to solve the problem that the King of Syracuse had over what materials had been used to make his crown – was it gold or was it really lead (Section 2.12)? This was the beginning of our understanding of hydrostatics (water which is not moving) and led to formulae for the design and construction of water tanks, dams and submarines that we still use today. It is an almost perfect theory. It was unfortunate that the Greeks also tried their hand at hydrodynamics – water which is moving – but they got this bit wrong and sent science off in the wrong direction for almost 2000 years. We had to wait until the likes of Sir Isaac Newton came along to put things right.

Sitting still in a bath and soaking up the warmth is a good experience – but for children this is almost impossible. Sliding up and down quickly is much more fun as you can make waves and even make the water flow over the sides of the bath. This is because water is a real fluid with viscous properties. As you start to move up and down the bath you transfer your body momentum to the water by surface and form drag (Section 3.10) – the larger you are the more form drag you can create and the more water you can move. When you stop sliding about, the waves seem to continue for a while before they stop. This is because the only force available to absorb the wave energy is friction and as there is very little of this in a bath it takes some time to suppress the waves. Water can also slosh about in harbours in much the same way as the bath tub and it can cause lots of problems for ships. The wave energy comes from the sea, it enters the harbour and it is difficult to get rid of because, like the bathtub, the walls of the harbour reflect the energy rather than absorb it. The movement of the water can move ships back and forth on their moorings which can be a major problem if the ship happens to be a supertanker and you are trying to keep it still while loading it with oil. This is why harbour entrances are narrow and specially angled to stop wave energy from entering. You may have noticed that the sea is much calmer inside a harbour than outside. Some harbours though have been known to behave in quite the opposite way. When the entrance is narrow the waves inside seem to get larger. This is a resonance effect that harbour designers must guard against.

As you relax in the bath you decide to have a drink. A glass of whisky will do the trick but you want water with it. So here is a little puzzle to while away the time. Do you put the water in the whisky or the whisky in the water? Start with a glass of water and a glass of whisky. One spoonful of whisky is put into the water and mixed and then one spoonful of the mixture is put back into the whisky. Is there more whisky in the water or more water in the whisky? If you cannot work it out then have a look at the solution in the box.

It is now time to get out of your bath and take out the plug. Notice how this sets up a nice whirlpool around the plug hole and it seems to be hollow down the middle. This is a boundary layer effect similar to those described in Section 3.9.3. The boundary layer close to the outlet slows the flow velocity which makes the water swirl and the vortex then forms. We say that the boundary layer curls up. All water intakes at reservoirs and control gates along rivers suffer from vortices like this which draw in air and reduce the water discharge. If you put some floating object over the vortex, such as your plastic duck, it stops the swirling and the discharge down the plughole increases. This is what engineers do in practice to stop vortices from forming at off-takes and also pump station suction inlets. Setting the outlet or the pump intake deep below the water surface will also suppress the vortices.

Finally there is the inevitable question that I am sure will keep you awake at night – which way does the vortex go? Some say that the vortex goes in a clockwise direction in the northern hemisphere and anticlockwise in the southern hemisphere. There is a very practical demonstration of this by an enterprising young man who lives on the equator in Kenya and puts on demonstrations for the tourists. He has a large can filled with water which he sets up 20 m north of the equator. When he pulls a plug out of the bottom of the container the water slowly starts to swirl in a clockwise direction. He then does the same test 20 m south of the equator and the swirl starts in an anticlockwise direction. It works every time. Convinced? The reality is that no one is absolutely sure. Experiments to test the theory have been tried but they are very difficult to do in a laboratory. The problem is that the force which causes the movement – the Coriolis force which comes from the earth's rotation – is very small in comparison to other forces around such as minor vibrations due to traffic outside the laboratory or temperature changes in the room setting up convection currents in the tank. All these can significantly influence which way the water will begin to swirl and override the effects of the Coriolis force. A large tank of water is needed to get an appreciable Coriolis force but arranging this under laboratory conditions is not very practicable.

If you have an aversion to baths and you prefer a shower there is always something to learn here though it may not be as much fun. When you switch on the shower and draw the plastic curtain around you for a bit of privacy, have you noticed how it tends to cling to your body – it is cold and uncomfortable. This is because the fast downward flow of water from the shower causes a slight drop in air pressure within the curtained space (remember the energy equation – energy changes from pressure energy to kinetic energy). The pressure outside the curtain is still at atmospheric pressure – slightly greater than the air pressure inside – and so the pressure difference causes the curtain to move towards the water and to cling to you.

Once you start to appreciate what is going on in your bath, bath-times will never be the same again!

EXAMPLE: A MIXING PROBLEM

Take one glass of water and a equal glass of whisky. One spoonful of whisky is put into the water and mixed. One spoonful of the mixture is put back into the whisky. Is there more whisky in the water or more water in the whisky?

Start by assuming that each glass holds 10 spoonfuls – so the water glass holds 10 spoonfuls of water and the whisky glass holds 10 spoonfuls of whisky. Now follow the argument below under the water and whisky headings as liquid is moved from one to the other:

Water	*Whisky*
10_{water}	10_{whisky}
One 'spoonful' (one part) of water is taken from the water and added to the whisky	
9_{water}	$10_{whisky} + 1_{water}$
The whisky glass now holds 11 spoonfuls of the mix. Each spoonful of the mix comprises 10 parts water and one part whisky i.e. $(10_{whisky} + 1_{water})/11$. Now take one spoonful of the mix and return this to the water:	
$9_{water} + (10_{whisky} + 1_{water})/11$ $9_{water} + 1/11\ _{water} + 10/11\ _{whisky}$ $9\ 1/11_{water} + 10/11_{whisky}$	$10_{whisky} + 1_{water} - (10_{whisky} + 1_{water})/11$ $10_{whisky} + 1_{water} - 10/11\ _{whisky} - 1/11\ _{water}$ $9\ 1/11_{whisky} + 10/11_{water}$

So the result is the amount of water in the whisky is the same as the amount of whisky in the water. So whichever way you mix your drinks it make no difference.

References and further reading

BS3680 *Measurement of Liquid Flow in Open Channels*. British Standards Institution, London (also ISO 748 1979)

(1981) Part 4A *Thin Plate Weirs*

(1981) Part 4C *Flumes*

Part 4E *Rectangular Broad Crested Weirs*

These are the standards which are the basis for the design, installation and operation of flow measuring structures.

Chadwick, A., Morfett, J. and Borthwick, M. (2004) *Hydraulics in Civil and Environmental Engineering*. 4th edition E & FN Spon, London.

Similar in some ways to Webber, this is another excellent undergraduate text but its coverage of hydraulics is much wider and includes sediment transport, river and canal engineering, coastal engineering and hydrology.

Chow, V.T. (1981) *Open Channel Hydraulics*. International Student Edition. McGraw Hill.

Although an 'old' book it is still the bible of open channel hydraulics. It is a handbook with lots of information on all aspects of channel flow and design – it is not an elementary text.

Douglas, J.F. (1996) *Solving Problems in Fluid Mechanics Parts I and II*. Longman, Harlow, UK.

This is a good problem-solving-oriented text. It approaches the principles of hydraulics by showing how to solve numerical problems.

Hamill, L. (2001) *Understanding Hydraulics*. MacMillan Press Ltd, Basingstoke, UK.

This is a book written by a teacher of hydraulics who clearly has a lot of experience of teaching a difficult subject. It is written in a conversational style with the student asking questions and the teacher providing the answers. It is sympathetic to the needs of young engineers but makes no allowances for those with limited mathematical skills.

Hydraulics Research (1990) *Charts for the Hydraulic Design of Channels and Pipes*. 6th Edition. Thomas Telford Ltd.

An excellent and widely used book of design charts based on the Colebrook-White equation.

King, H.W. and Brater, E.F. (1996) *Handbook of Hydraulics for the Solution of Hydrostatic and Fluid Flow Problems*. 7th edition. McGraw Hill.

Very useful for the solution of Manning's equation for channel flow. His method is used in this text.

Portland Cement Association (1964) *Handbook of Concrete Culvert Pipe Hydraulics*. Chicago, IL.

Very comprehensive book on culvert hydraulics and their construction. Although it is primarily supporting concrete culverts they can be made out of other materials as well.

Rouse, H. and Ince, S. (1957) *History of Hydraulics*. Dover Publications, New York.

A comprehensive, well-researched and very readable history of hydraulics from the early work of Greek scholars to our modern notions of fluid behaviour. Rouse is a well-known authority on hydraulics.

USBR (1974) *Design of Small Dams*. United States Dept of the Interior, Bureau of Reclamation.
Lots of useful construction details of hydraulic structures and particularly weirs and spillways.
Vallentine, H.R. (1967) *Water in the Service of Man*. Penguin Books Ltd, Harmondsworth, UK.
Sadly this book is now out of print but copies are still available on the internet. It is an excellent, easy to read, introduction to the fundamentals of water flow in pipes, channels and pumps as well as providing a broader appreciation of water and its uses including its history. The text is very descriptive, anecdotal and entertaining in style with lots of good explanations and very little mathematics. Vallentine is clearly an engineer who knows how to communicate his ideas in a practical and interesting way. Penguin really ought to reprint this book.
Webber, N.B. (1971) *Fluid Mechanics for Civil Engineers*. Chapman Hall, London.
An excellent, comprehensive undergraduate civil engineering textbook covering both basic principles and practical applications. Now out of print but copies still available on the internet.

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