5 Channels

5.1 Introduction

Natural rivers and man-made canals are open channels. They have many advantages over pipes and have been used for many centuries for water supply, for transport and for agriculture. The Romans made extensive use of channels and built aqueducts for their sophisticated water supply schemes. Barge canals are still an important means of transporting heavy bulk materials in Europe and irrigation canals bring life and prosperity to the arid lands of North Africa, Middle East, India and Australia as they have done for thousands of years.

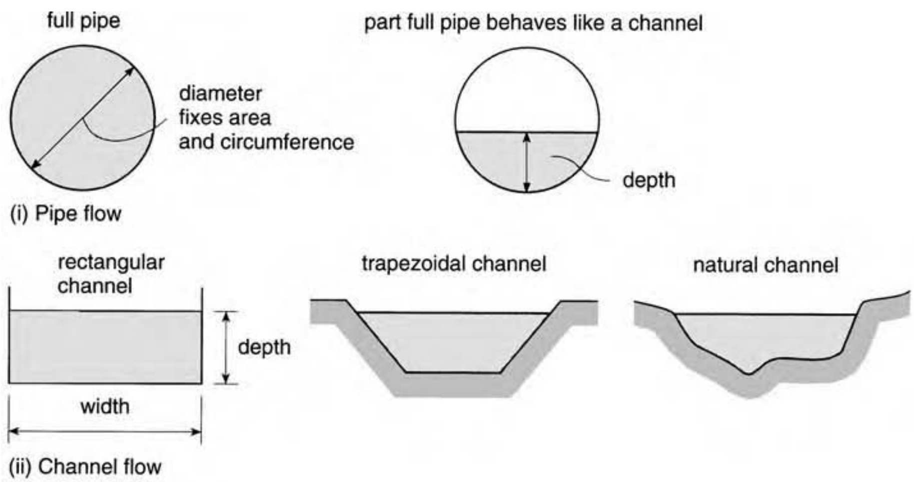
5.2 Pipes or channels?

The choice between pipes and open channels is most likely to depend on which provides the cheapest solution both in terms of capital expenditure and the recurrent costs of operation and maintenance. However, there are advantages and disadvantages with each which may influence or restrict the final choice.

Channels, for example, are very convenient and economical for conveying large quantities of water over relatively flat land such as in large irrigation systems on river flood plains. It is hard to imagine some of the large irrigation canals in India and Pakistan being put into pipes, although 5 m diameter pipes are used in Libya to transport water across the desert to coastal cities and irrigation schemes. In hilly areas the cost of open channels can rise significantly because the alignment must follow the land contours to create a gentle downward slope for the flow. A more direct route would be too steep causing erosion and serious damage to channels. Pipes would be more suitable in such conditions. They can be used in any kind of terrain and can take a more direct route. Water velocities too can be much higher in pipes because there is no risk of erosion.

Although there are obvious physical differences between channels and pipes, there are several important hydraulic differences between them:

- Open channels have a free or open water surface whereas pipes are enclosed and always flow full.
- Water can only flow downhill in channels but in pipes it can flow both uphill and downhill. Flow in pipes depends on a pressure difference between the inlet and outlet. As long as the pressure is higher at the inlet than at the outlet then water will flow even though the



(a)

(b) A lined irrigation canal in Nigeria

5.1 Channels can have many different shapes and sizes.

pipeline route may be undulating. Channels depend entirely on the force of gravity to make water move and so they can only flow downhill.

- Man-made channels can have many different shapes (circular, rectangular or trapezoidal) and sizes (different depths, widths and velocities). Natural river channels are irregular in shape (Figure 5.1). Pipes in contrast are circular in section and their shape is characterised by one simple dimension – the diameter. This fixes the area of the water way and the friction from the pipe circumference.

- Water velocities are usually lower in channels than in pipes. This is because channels are often in natural soils which erode easily. So channels are usually much larger than pipes for the same flow.
- Channels need much more attention than pipes. They tend to erode and weeds grow in waterways and so regular cleaning is required. Water losses from seepage and evaporation can also be a problem.

These differences make channels a little more complicated to deal with than pipes but most open channel problems can be solved using the basic tools of hydraulics; discharge and continuity, energy and momentum.

The study of open channels is not just confined to channel shapes and sizes. It can also include waves; the problem of handling varying flood flows down rivers and sediment transport associated with the scouring and silting of rivers and canals. Some of these issues are touched on in this chapter but waves are discussed more fully in Chapter 6.

5.3 Laminar and turbulent flow

Laminar and turbulent flow both occur in channels as well as in pipes. But for all practical purposes, flow in rivers and canals is turbulent and, like pipes, laminar flow is unlikely to occur except for very special conditions. For example, laminar flow only occurs in channels when the depth is less than 25 mm and the velocity is less than 0.025 m/s. This is not a very practical size and velocity and so laminar flow in channels can safely be ignored. The only time it does become important is in laboratory studies where physical hydraulic models are used to simulate large and complex channel flow problems. Scaling down the size to fit in the laboratory often means that the flow in the model becomes laminar. This change in flow regime will affect the results and care is needed when using them to assess what will happen in practice.

5.4 Using the hydraulic tools

Continuity and energy are particularly useful tools for solving open channel flow problems. Momentum is also helpful for problems in which there are energy losses and where there are forces involved.

5.4.1 Continuity

Continuity is used for open channels in much the same way as it is used for pipes (Figure 5.2a). The discharge Q_1 passing point 1 in a channel must be equal to the discharge Q_2 passing point 2.

$$Q_1 = Q_2$$

Writing this in terms of velocity and area:

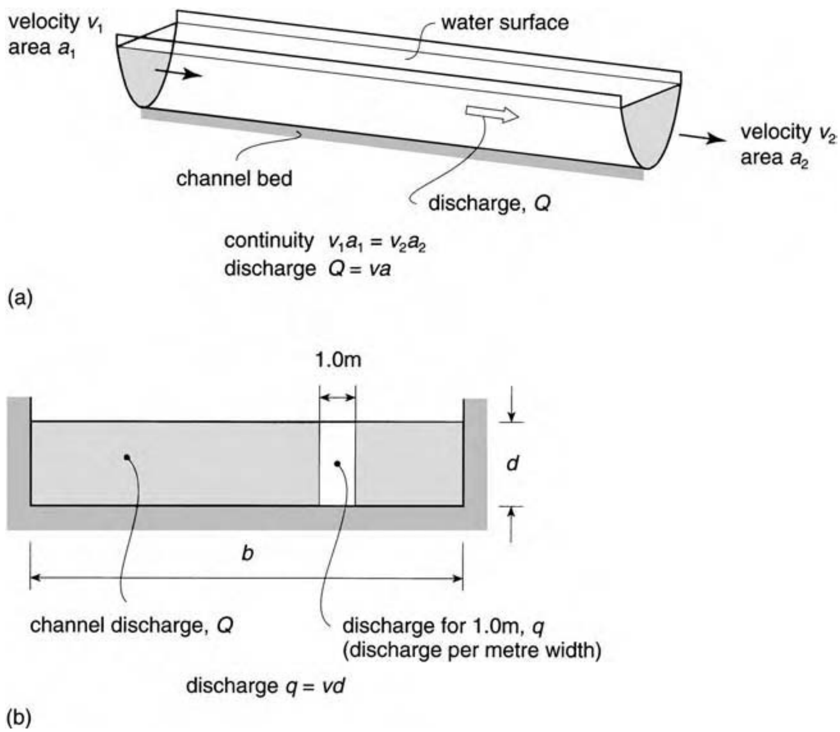
$$v_1 a_1 = v_2 a_2$$

The term discharge per unit width (q) is often used to describe channel flow rather than the total discharge (Q). This is the flow in a 1.0 m wide portion of a channel (Figure 5.2b).

To calculate the discharge per unit width (q) for a rectangular channel:

Use the continuity equation $Q = va$

where $a = bd$



5.2 Continuity in channels.

To calculate q assume $b = 1.0$ m. Therefore:

$$a = 1.0 \times d = d$$

And so:

$$q = vd$$

So when a channel width (b) is 7 m and it carries a discharge (Q) of 10 m^3/s, the flow per unit width is calculated as follows:

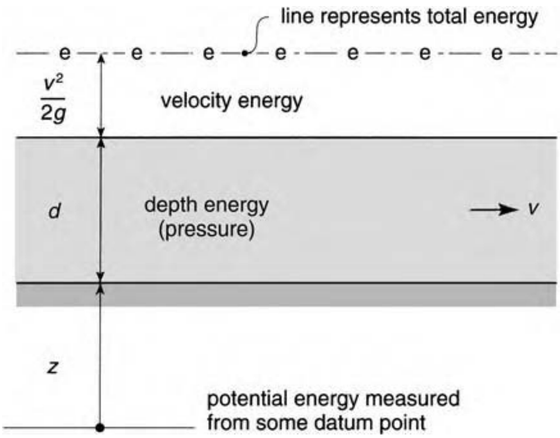
$$q = \frac{Q}{b}$$
$$= \frac{10}{7} = 1.43 \text{ m}^3\text{/s/m width of channel}$$

The discharge and continuity equations for channels are often written in terms of discharge per unit width as follows:

$$q = vd$$
$$v_1d_1 = v_2d_2$$

5.4.2 Energy

The idea of total energy being the same at different points in a system is also useful for channel flow. So when water flows between two points 1 and 2 in a channel, the total energy at 1



5.3 Energy in channels.

will be the same as the total energy at 2. As in the case of pipe flow this is only true when there is no energy loss. For some channel problems this is a reasonable assumption to make but for others an additional energy loss term is needed.

The energy equation for channel flow is a little different to the equation for pipe flow (Figure 5.3). For pipe flow the pressure energy is $p/\rho g$. For channel flow this term is replaced by the depth of water d. Remember that $p/\rho g$ is a pressure head and is already measured in metres and so for channels this is the same as the pressure on the channel bed resulting from the depth of water d. The potential energy z is measured from some datum point to the bed of the channel. The velocity energy $v^2/2g$ remains the same. Note that all the terms are measured in metres and so they can all be added together to determine the total energy in a channel.

Writing the total energy equation for an open channel:

$$\text{total energy} = d + \frac{v^2}{2g} + z$$

Sometimes the velocity v is written in terms of the discharge per unit width q and depth d. This is done using the discharge equation:

$$q = vd$$

Rearrange this for velocity:

$$v = \frac{q}{d}$$

So the velocity energy becomes:

$$\frac{v^2}{2g} = \frac{q^2}{2gd^2}$$

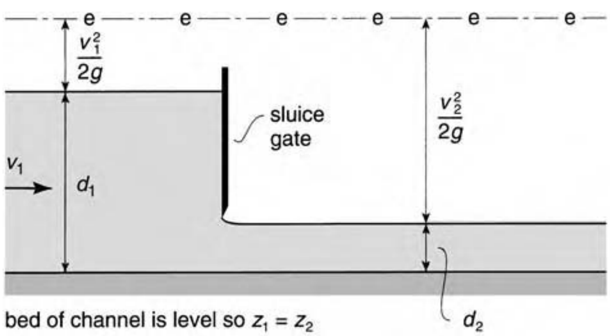
Substitute this in the energy equation:

$$\text{total energy} = d + \frac{q^2}{2gd^2} + z$$

Total energy can be represented diagrammatically by the *total energy line* e—e—e (Figure 5.3). This provides a visual indication of the total energy available and how it is changing. Notice that the line only slopes downwards in the direction of the flow to show the gradual loss of energy from friction. The water surface is the channel equivalent of the hydraulic gradient for pipes; it represents the pressure on the bed of the channel.

5.4.3 Using energy and continuity

One example of the use of the continuity and energy equations in an open channel is to calculate the discharge under a sluice gate (Figure 5.4). The sluice gate is a common structure for controlling flows in channels and it can also be used to measure flow if the water depths upstream



(a)

(b) Sluice gates controlling discharge into irrigation canal, Iraq

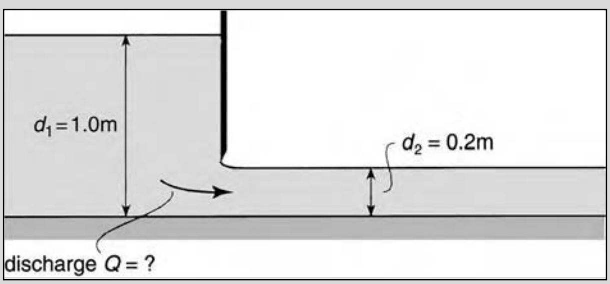
5.4 Using energy and continuity.

and downstream of the gate are measured. The approach is very similar to the venturi problem in pipe flow but in this case a gate is used to change the energy conditions in the channel. Notice how the energy line has been drawn to indicate the level of total energy. Firstly it shows there is no energy loss as water flows under the gate. This is reasonable because the flow is converging under the gate and this tends to suppress turbulence which means little or no energy loss. Secondly it shows that there is a significant change in the components of the total energy across the gate even though the total is the same. Upstream the flow is slow and deep whereas downstream the flow is very shallow and fast. The discharge is the same on both sides of the gate but it is clear that the two flows are quite different. In fact they behave quite differently too – but more about this later in Section 5.7.

The example in the box illustrates how to calculate the discharge under a sluice gate when the upstream and downstream water depths are known.

EXAMPLE: CALCULATING DISCHARGE UNDER A SLUICE GATE

A sluice gate is used to control and measure the discharge in an open channel. Calculate the discharge in the channel when the upstream and downstream water depths are 1.0 m and 0.2 m respectively.



5.5 Calculating discharge under a sluice gate.

When the flow is contracting as it does under a sluice gate, turbulence is suppressed and the flow transition occurs smoothly. Very little energy is lost and so the energy equation can be applied as follows:

total energy at point 1 = total energy at point 2

$$d_1 + \frac{v_1^2}{2g} + z_1 = d_2 + \frac{v_2^2}{2g} + z_1$$

As the channel is horizontal:

$$z_1 = z_2$$

And so:

$$d_1 + \frac{v_1^2}{2g} = d_2 + \frac{v_2^2}{2g}$$

Bring the d terms and v terms together and put in the values for depth:

$$1.0 - 0.2 = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

Both velocities v_1 and v_2 are unknown and so the continuity equation is needed to solve the problem:

$$v_1 d_1 = v_2 d_2$$

Put in the depths:

$$v_1 \times 1.0 = v_2 \times 0.2$$

And so:

$$v_1 = 0.2 v_2$$

Substitute for v_1 in the energy equation:

$$0.8 = \frac{v_2^2}{2g} - 0.04\frac{v_2^2}{2g}$$

Rearrange this to find v_2:

$$v_2^2 = \frac{0.8 \times 2 \times 9.81}{1 - 0.04} = 16.35$$

$$v_2 = 4 \text{ m/s}$$

Calculate the discharge:

$$q = v_2 d_2$$
$$q = 4 \times 0.2$$
$$q = 3.27 \text{ m}^3\text{/s/m width of channel.}$$

5.4.4 Taking account of energy losses

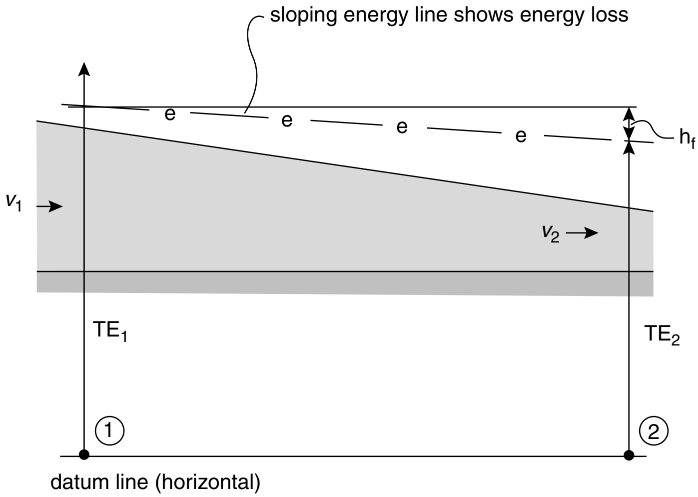
Energy loss occurs in channels due to friction. In short lengths of channel such as in the sluice gate example this is very small and so it is not taken into account in any calculations. But energy loss in long channels must be taken into account in the energy equation to avoid serious errors (Figure 5.6). Writing the energy equation for two points in a channel:

total energy at 1 (TE_1) = total energy at 2 (TE_2) + h_f

$$d_1 + \frac{v_1^2}{2g} + z_1 = d_2 + \frac{v_2^2}{2g} + z_2 + h_f$$

where h_f is energy loss due to friction (m).

This equation is very similar to that for pipe flow and in that case the Darcy-Weisbach formula was used to calculate the energy loss term h_f. This was the link between energy losses and pipe size. Similar formulae have been developed to calculate h_f for open channels and these link energy loss both to the size and shape of channels needed to carry a given discharge. But as with pipe flow there are a few important steps to take before getting to the formula. The first of these steps is the concept of uniform flow.



5.6 Energy losses in channels.

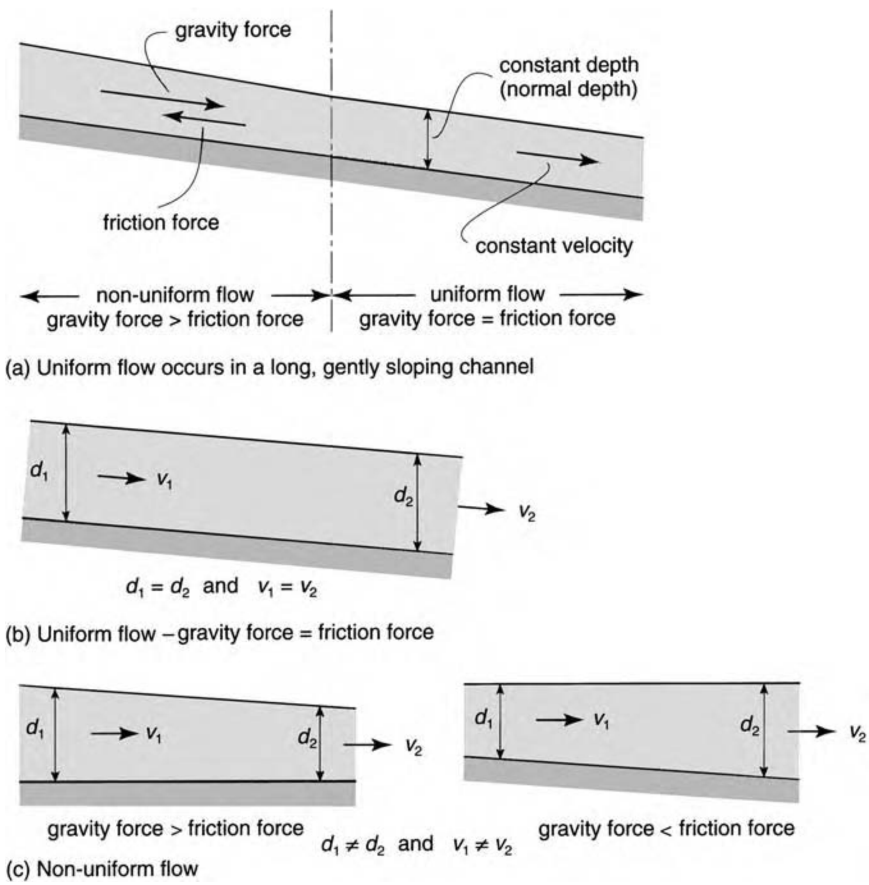
5.5 Uniform flow

All the energy loss formulae for open channels are based on *uniform flow*. This is a special condition that only occurs when water flows down a long, straight, gently sloping channel (Figure 5.7). The flow is pulled down the slope by the force of gravity but there is friction from the bed and sides of the channel slowing it down. When the friction force is larger than the gravity force it slows down the flow. When the friction force is smaller than the gravity force the flow moves faster down the slope. But the friction force is not constant, it depends on velocity and so as the velocity increases so does the friction force. At some point the two forces become equal. Here the forces are in balance and as the flow continues down the channel the depth and velocity remain constant. This flow condition is called *uniform flow* and the water depth is called the *normal depth*.

Unfortunately, in most channels this balance of forces rarely occurs and so the depth and velocity are usually changing gradually even though the discharge is constant. Even in long channels where uniform flow has a chance of occurring there is usually some variation in channel shape or slope or a hydraulic structure which changes the depth and the velocity (Figure 5.7c). So most channels have *non-uniform flow*. It is also called *gradually varied flow* because the changes take place gradually along the channel.

So if non-uniform flow is the most common and uniform flow rarely occurs, why are all the formulae based on uniform flow? Why not accept that non-uniform flow is the norm and develop formulae for this condition? The answer is quite simple. Uniform flow is much easier to deal with from a calculation point of view and engineers are always looking for ways of simplifying problems but without losing accuracy. There are methods of designing channels for non-uniform flow but they are much more cumbersome to use and usually they produce the same shape and size of channel as uniform flow methods. So it has become accepted practice to assume that channel flow is uniform for all practical purposes. For all gently sloping channels on flood plains (i.e. 99% of all channels including rivers, canals and drainage ditches) this assumption is a good one. Only in steep sloping channels in the mountains does it cause problems.

Remember too that great accuracy may not be necessary for designing channels and dimensions do not need to be calculated to the nearest millimetre for construction purposes. The nearest 0.05 m is accurate enough for concrete channels and probably 0.1 m for earth channels is more than enough. From a practical construction point of view it will be difficult to



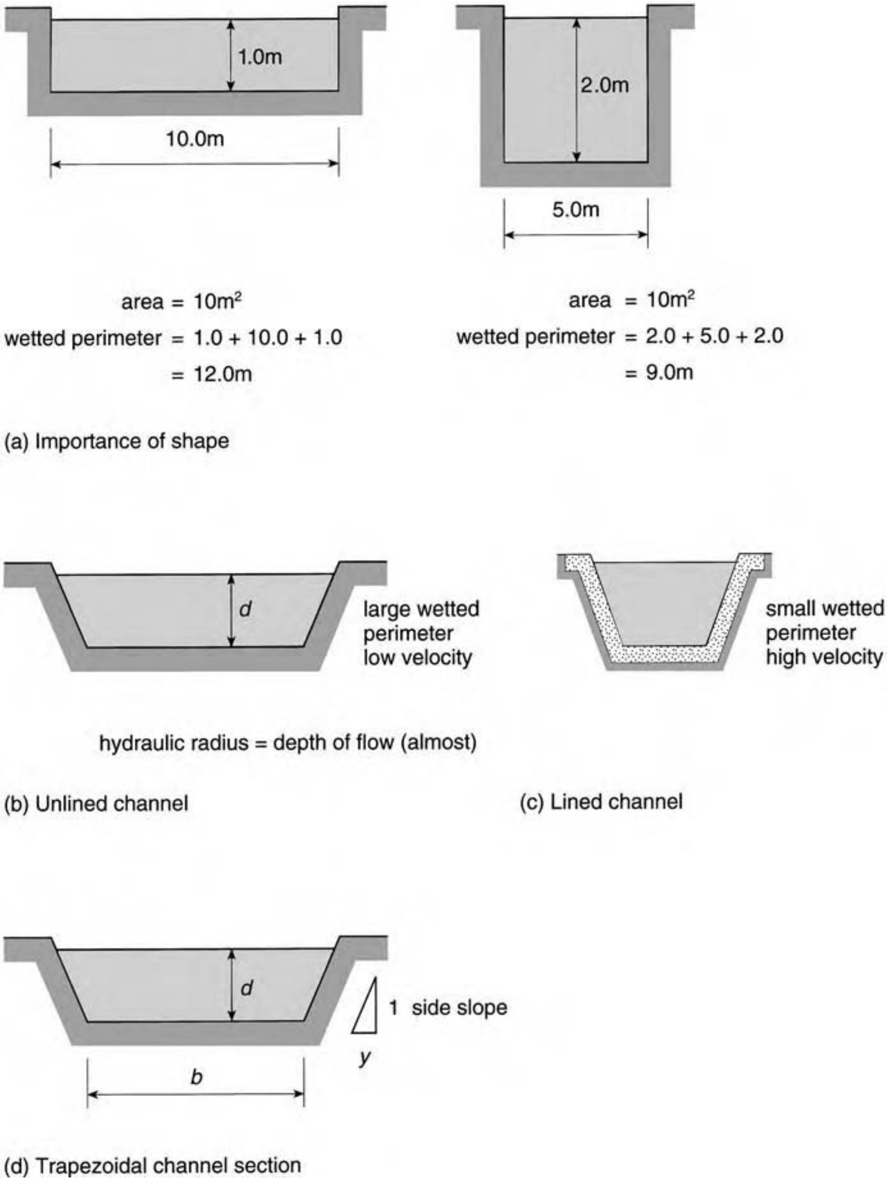
5.7 Uniform and non-uniform flow.

find a hydraulic excavator operator who can (or who would want to) trim channel shapes to an accuracy greater than this.

5.5.1 *Channel shapes*

For pipe flow the issue of shape does not arise. Pipes are all circular in section and so their hydraulic shape is determined by one dimension – the pipe diameter – and so both shape and size are taken into account in the Darcy-Weisbach formula. But channels come in a variety of shapes, the more common ones being rectangular, trapezoidal or semi-circular (Figure 5.8). They also come in different sizes with different depths and widths. So any formula for channels must take into account both shape and size.

To demonstrate the importance of shape consider two rectangular channels each with the same area of flow but one is narrow and deep and the other is shallow and wide (Figure 5.8a). Both channels have the same flow area of 10 m^2 and so they might be expected to carry the same discharge. But this is not the case. Friction controls the velocity in channels and when the friction changes the velocity will also change. The channel boundary in contact with the water (called the *wetted perimeter*) is the main source of friction. In the narrow channel the length of the boundary in contact with the water is 9 m whereas in the wide channel it is 12 m. So the



5.8 Channel shapes.

wide channel produces more friction than the narrow one and as a result the velocity (and hence the discharge) in the wide channel will be less than in the narrow one. So channels with the same flow area have different carrying capacities depending on their shape.

Understanding this can be very useful when deciding on the general shape of channels. Suppose you are constructing a channel in natural soil and there are worries about erosion of the bed and sides from a high water velocity. By making the channel wide and shallow the increased friction will slow down the water and avoid the problem (Figure 5.8b). Many natural river channels have a shallow, wide profile as they have adapted over many years to the erosion of the natural soils in which they flow.

Table 5.1 Minimum wetted perimeters for different channel shapes.

Channel shape	*Wetted perimeter*	*Hydraulic radius*
Rectangle	$4d$	$0.5d$
Trapezoid (half a hexagon)	$3.463d$	$0.5d$
Semi-circle	πd	$0.5d$

Note
d is the depth of flow.

For lined channels (e.g. concrete) the main issue is one of cost and not erosion. Lined canals are very expensive and so it is important to minimise the amount of lining needed. This is done by using a channel shape that has a small wetted perimeter (Figure 5.8c). Friction will be low which means the velocity will be high but this is not a problem as the lining will resist erosion. Minimum wetted perimeters for selected channel shapes are shown in Table 5.1.

5.5.2 Factors affecting flow

Channel flow is influenced not just by the channel shape and size but also by slope and roughness.

5.5.2.1 Area and wetted perimeter

The cross-sectional area of a channel (a) defines the flow area and the wetted perimeter (p) defines the boundary between the water and the channel. This boundary in contact with the water is the source of frictional resistance to the flow of water. The greater the wetted perimeter, the greater is the frictional resistance of the channel.

The area and wetted perimeter for rectangular and circular channels are easily calculated but trapezoidal channels are a bit more difficult. Unfortunately they are the most common and so given below are formulae for area and wetted perimeter (Figure 5.8d).

Area of waterway (a) = $(b + yd)d$
Wetted perimeter (p) = $b + 2d\,(1 + y^2)^{1/2}$

Where b is bed width (m); d is depth of flow (m); y is side slope.

5.5.2.2 Hydraulic radius

As the wetted perimeter can vary considerably for the same area, some measure of the hydraulic shape of a channel is needed. This is called the *hydraulic radius* and it is determined from the area and the wetted perimeter as follows:

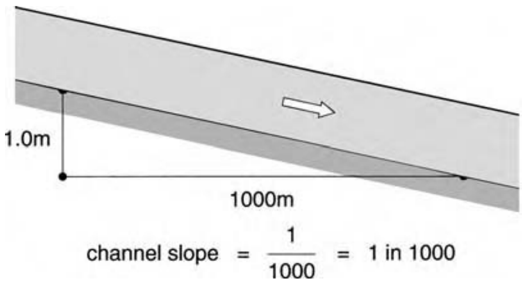
$$\text{Hydraulic radius (m)} = \frac{\text{area (m}^2\text{)}}{\text{wetted perimeter (m)}}$$

In the two channels in Figure 5.8, the hydraulic radius would be 1.11 m and 0.83 m respectively and this shows numerically just how hydraulically different are the two channels.

5.5.2.3 Slope

Water only flows downhill in channels and the steepness of the slope affects the velocity and hence the discharge. As the slope gets steeper the velocity increases and so does the discharge.

(remember $Q = va$)



5.9 Channel slope.

Slope is measured as a gradient rather than an angle in degrees. So a channel slope is expressed as 1 in 1000, that is, 1.0 m drop in 1000 m of channel length (Figure 5.9).

Slopes need not be very steep for water to flow. Many of the irrigation canals in the Nile valley in Egypt have slopes of only 1 in 10 000. This is the same as 1.0 m in 10 km or 100 mm per kilometre. This is a very gentle slope and you would not be able to detect it by just looking at the landscape, but it is sufficient to make water flow as the evidence of the Nile shows.

A question which sometimes arises about channel slopes is – does slope refer to the water surface slope, the channel bed or the slope of the energy line? For uniform flow the question is irrelevant because the depth and velocity remain the same along the entire channel and so the water surface and the bed are parallel and have the same slope. For non-uniform flow it is the slope of the energy line that is important as is the driving force for the flow. Even when the bed is flat, water will still flow provided there is an energy gradient.

5.5.2.4 Roughness

The *roughness* of the bed and sides of a channel also contribute to friction. The rougher they are the slower will be the water velocity. Channels tend to have much rougher surfaces than pipes. They may be relatively smooth when lined with concrete but they can be very rough when excavated in the natural soil or infested with weeds. Roughness is taken into account in channel design formula and this is demonstrated in the next section.

5.5.3 Channel design formulae

There are two commonly used formulae which link energy loss in channels to their size, shape, slope and roughness: the *Chezy formula* and the *Manning formula*. Both are widely used and were developed on the assumption that the flow is uniform.

5.5.3.1 Chezy formula

This formula was developed by Antoine Chezy, a French engineer who, in 1768, was asked to design a canal for the Paris water supply.

The Chezy formula, as it is now known, is usually written as follows:

$$v = C\sqrt{RS}$$

where v is velocity (m/s); R is hydraulic radius (m); S is channel slope (m/m); C is the Chezy coefficient describing channel roughness.

This may not look much like a formula for friction loss in a channel but it is derived from the energy equation allowing for energy loss in the same way as was done for pipe flow. For pipe flow the outcome was the Darcy-Weisbach formula, for channel flow the outcome is the Chezy formula. For those interested in the origins of this formula, which are interesting both from a mathematical and historical point of view, a derivation is shown in the box. The formula shown above is the more familiar way of presenting the Chezy formula in hydraulic textbooks.

Once the velocity has been calculated the discharge can be determined using the discharge equation:

$$Q = va$$

DERIVATION: CHEZY FORMULA

To see how Chezy developed his formula look first at the energy equation describing the changes which take place between two points in a long channel (Figure 5.7b) but taking into account energy loss in the channel h_f.

$$d_1 + \frac{v_1^2}{2g} + z_1 = d_2 + \frac{v_2^2}{2g} + z_2 + h_f$$

He suggested that the energy loss h_f could be determined by:

$$h_f = \frac{Lv^2}{C^2R}$$

where L is the length of channel over which the energy loss occurs (m); v is velocity (m/s); R is hydraulic radius (m); C is Chezy coefficient describing roughness.

Notice how similar this equation is to the Darcy-Weisbach equation. Friction depends on the length and the square of the velocity. The Chezy coefficient C describes the friction in the channel and is similar to λ in the Darcy-Weisbach formula. Like λ it does not have a constant value. C depends on the Reynolds Number and also on the dimensions of the channel.

Put this into the energy equation:

$$d_1 + \frac{v_1^2}{2g} + z_1 = d_2 + \frac{v_2^2}{2g} + z_2 + \frac{Lv^2}{C^2R}$$

Now uniform flow is defined by the depths and velocities remaining the same along the whole length of a channel and so:

$$d_1 = d_2$$

And:

$$v_1 = v_2$$

This reduces the energy equation to:

$$\frac{Lv^2}{C^2R} = z_1 - z_2$$

Now divide both sides of the equation by L:

$$\frac{v^2}{C^2R} = \frac{z_1 - z_2}{L}$$

But $\frac{z_1 - z_2}{L}$ is the slope of the channel bed *S.* It is also the slope of the water surface. Remember that the two are parallel for uniform flow. Hence:

$$\frac{z_1 - z_2}{L} = S$$

And so:

$$\frac{v^2}{C^2 R} = S$$

Rearrange this to calculate the velocity:

$$v = C\sqrt{RS}$$

This is the familiar form of the Chezy equation that is quoted in hydraulic text books.

5.5.3.2 Manning formula

The Manning formula is an alternative to Chezy and is one of the most commonly used formulae for designing channels. This was developed by Robert Manning (1816–1897) an Irish civil engineer. It is an empirical formula developed from many observations made on natural channels.

$$v = \frac{R^{2/3} S^{1/2}}{n}$$

where v is velocity (m/s); R is hydraulic radius (m); S is channel bed slope (m/m); n is Manning's roughness coefficient.

Manning's n values depend on the surface roughness of a channel. Typical values are listed in Table 5.2.

The value of Manning's n is not just determined by the material from which the channel is made but it is also affected by vegetation growth. This can make it difficult to determine with any accuracy. The n value can also change over time as weeds grow and it can also change with changes in flow. At low discharges weeds and grasses will be upright and so cause great roughness but at higher discharges they may be flattened by the flow and so the channel becomes much smoother. There is an excellent book, *Open Channel Flow* by Ven Te Chow (see references), which has a series of pictures of channels with different weed growths and suggested n values. These pictures can be compared with existing channels to get some indication of n. But

Table 5.2 Values of Manning's n.

Channel type	*Manning's n values*
Concrete lined canals	0.012–0.017
Rough masonry	0.017–0.030
Roughly dug earth canals	0.025–0.033
Smooth earth canals	0.017–0.025
Natural river in gravel	0.040–0.070

how is Manning's n selected for a natural, winding channel with varying flow areas; with trees and grasses along its banks (perhaps also including the odd bicycle or super-market trolley) and flowing under bridges and over weirs? Clearly, in this situation, choosing n is more of an art than a science. It may well be that several values are needed to describe the roughness along different sections of the river.

5.5.4 Using Manning's formula

Manning's formula is not the easiest of formula to work with. It is quite straightforward to use when calculating discharge for a given shape and size of channel but it is not so easy to use the other way round, that is, to calculate channel dimensions for a given discharge. Unfortunately this is by far the most common use of Manning. One approach is to use a trial and error technique to obtain the channel dimensions. This means guessing suitable values for depth and width and then putting them into the formula to see if they meet the discharge requirements. If they do not then the values are changed until the right dimensions are found. Usually there can be a lot of trials and a lot of errors. Modern computer spreadsheets can speed up this painful process.

Another approach for those who do not have spreadsheet skills is the method developed by HW King in his *Handbook for the Solution of Hydrostatic and Fluid Flow Problems* (see references). This is a very simple and useful method and is ideally suited to designing trapezoidal channels, which are the most common. He modified Manning's formula to look like this:

$$Q = \frac{1}{n} jkd^{8/3} S^{1/2}$$

where d is depth of flow (m); S is channel slope; j and k are constants.

The values of j and k depend on the ratio of the bed width to depth and the channel side slope. This is the slope of the side embankments and not the longitudinal slope of a channel S. King's book has a very comprehensive range of j and k values. A selection of the most common values is shown in Table 5.3.

To use the method, values of the ratio of channel bed width to depth and side slope are first chosen. Values of j and k are then obtained from Table 5.3 and put into the formula from which a value of depth can be calculated. As the bed width to depth ratio is known, the bed width can now be calculated. If the resulting channel shape or its dimensions appear to be unsuitable for any reason (e.g. the velocity may be too high) then another ratio of bed width to depth ratio can be chosen and the calculation repeated. As well as providing j and k values, King also

Table 5.3 Values of j and k for Manning's formula.

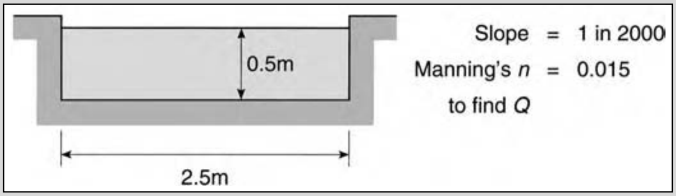
	Ratios of bed width (b) to water depth (d)					
	Values of j			*Values of k*		
Side slope	*b = d*	*b = 2d*	*b = 3d*	*b = d*	*b = 2d*	*b = 3d*
vertical	1	2	3	0.48	0.63	0.71
1 in 1	2	3	4	0.64	0.73	0.77
1 in 1.5	2.5	3.5	4.5	0.66	0.73	0.77
1 in 2	3	4	5	0.66	0.72	0.76
1 in 3	4	5	6	0.67	0.71	0.74

supplies values of the power function 8/3 so that it is easy to calculate the depth of flow. Remember that his book was written some years ago before everyone had a calculator.

Examples of the use of Manning's formula are shown in the boxes.

EXAMPLE: CALCULATING DISCHARGE USING MANNING'S FORMULA

Calculate the discharge in a rectangular concrete lined channel of width 2.5 m and depth 0.5 m with a slope of 1 in 2000 and a Manning's *n* value is 0.015.



5.10 Calculating discharge using Manning's equation.

The first step is to calculate the velocity but before this can be done the area, wetted perimeter and hydraulic radius must be determined:

Area (a) = depth × width
$= 0.5 \times 2.5 = 1.25\ m^2$
Wetted perimeter $(p) = 0.5 + 2.5 + 0.5 = 3.5$ m

And so:

$$\text{Hydraulic radius } (R) = \frac{a}{p}$$

$$= \frac{1.25}{3.5} = 0.36\ \text{m}$$

Next calculate velocity:

$$v = \frac{0.36^{2/3} \times \left(\frac{1}{2000}\right)^{1/2}}{0.015}$$

$$= 0.75\ \text{m/s}$$

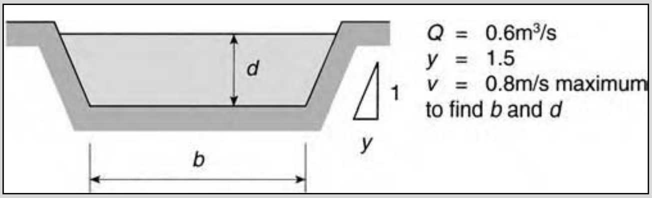
Now calculate discharge:

$Q = va$

$Q = 1.25 \times 0.75 = 0.94\ m^3/s$

EXAMPLE: CALCULATING DEPTH OF FLOW AND BED WIDTH USING MANNING'S FORMULA (KING'S METHOD)

Calculate a suitable bed width and depth of flow for an unlined trapezoidal channel to carry a discharge of 0.6 m^3/s on a land slope of 1 in 1000. The soil is a clay loam and so the side slope will be stable at 1.5:1 and the maximum permissible velocity is 0.8 m/s (Table 5.4).



5.11 Calculating depth of flow and bed width using King's method.

The first step is to select a suitable value for Manning's n ($n = 0.025$ for natural soil) and then select a bed width to depth ratio (try $b = d$).

Now obtain values for j and k from Table 5.3.

$j = 2.5$ and $k = 0.66$

Calculate d using the Manning formula:

$$Q = \frac{1}{n} jkd^{8/3}s^{1/2}$$

$$0.6 = \frac{1}{0.025} \times 2.5 \times 0.66 \times d^{8/3} \times 0.001^{1/2}$$

Rearrange this for d:

$d^{8/3} = 0.286$
$d = 0.63$ m

As the ratio of bed width to depth is known calculate b. In this case:

$b = d$

And so:

$b = 0.63$ m

All the channel dimensions are now known but do they comply with the velocity limit? Check the velocity using the discharge equation:

$Q = av$

For a trapezoidal channel:

$$a = (b + yd)d$$
$$= (0.63 + 1.5 \times 0.63)\ 0.63$$
$$= 0.993 \text{ m}^2$$

Substitute this and the value for discharge into the discharge equation:

$0.6 = 0.993 \times v$

Calculate velocity:

$v = 0.6$ m/s

This is less than the maximum permissible velocity of 0.8 m/s and so these channel dimensions are acceptable.

Note that there are many different channel dimensions that could be chosen to meet the design criteria. This is just one answer. Choosing another *b*:*d* ratio would produce different dimensions but they would be acceptable provided they met the criteria. Increasing the *b*:*d* ratio would reduce the velocity whereas decreasing the *b*:*d* ratio would increase the velocity. The latter would not be an option in this example as the velocity is close to the maximum permissible already.

A freeboard would normally be added to this to ensure that the channel is not over-topped.

5.5.5 Practical design

In engineering practice the usual design problem is to determine the size, shape and slope of a channel to carry a given discharge. There are many ways to approach this problem but here are some guidelines.

Whenever possible the channel slope should follow the natural land slope. This is done for cost as it helps to reduce the amount of soil excavation and embankment construction needed. But when the land slope is steep, high water velocities may occur and cause erosion in unlined channels. The most effective way to avoid erosion is to limit the velocity. Maximum non-scouring velocities for different soil types are shown in Table 5.4. Channels can also be lined for protection. Slope can also be reduced to lower the velocity to an acceptable level by using drop structures to take the flow down the slope in a series of steps – like a staircase.

It is important to understand that there is no single correct answer to the size and shape of a channel, but a range of possibilities. If three people were each asked to design a trapezoidal channel for a given discharge, it is likely that they would come up with three different answers, and all could be acceptable. It is the designer's job to select the most appropriate one. Usually the selection is made simpler because of the limited range of values that are practicable. For example, land slope will limit the choice of slope and the construction materials will limit the velocity. But even within these boundaries there are still many possibilities.

One of the problems of channel design is that of choosing suitable values of depth and bed width. King's method gets around this problem by asking the designer to select a ratio between them rather than the values themselves. Another way to simplify the problem is to assume that the hydraulic radius *R* is equal to the depth of the water *d*. This is a reasonable assumption to make when the channel is shallow and wide. Referring to the example in Figure 5.8a, the hydraulic radius was 1.11 m in the wider channel when the depth was 1.0 m. This is close enough for channel design purposes. The depth can then be calculated using the Manning formula and the bed width determined using the area and discharge.

Table 5.4 Maximum permissible velocities.

Material	*Maximum velocity (m/s)*
Silty sand	0.30
Sandy loam	0.50
Silt loam	0.60
Clay loam	0.8
Stiff clay	1.10

The depth and width of a channel also influence velocity. For lined channels, which are expensive, it is important to keep the wetted perimeter (*p*) as small as possible as this keeps the cost down. This results in channels which are narrow and deep (Figure 5.8c). For unlined channels the velocity must be kept well within the limits set in Table 5.4. Making channels wide and shallow increases the wetted perimeter and channel resistance and this slows down the flow (Figure 5.8b). Look at any stream or river flowing in natural soil. Unless it is constrained by rocks or special training works it will naturally flow wide and shallow. So new channels which are to be constructed in similar material should also follow this trend.

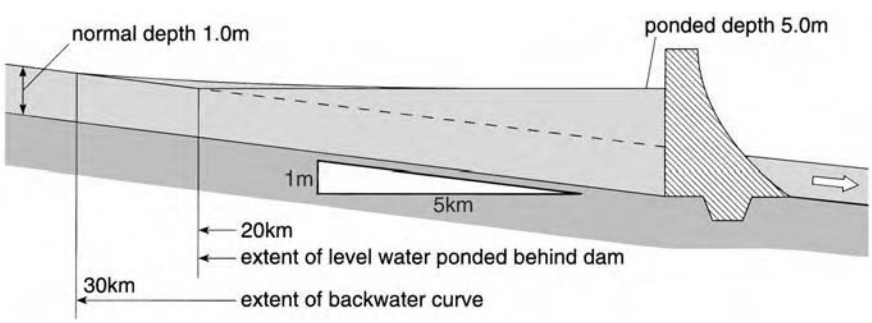
5.6 Non-uniform flow: gradually varied

There are two kinds of non-uniform flow. The first is *gradually varied flow*. This is the most common type of flow and has already been described earlier in this chapter. It occurs when there are gradual changes taking place in the depth and velocity due to an imbalance of the force of gravity trying to make the flow go faster down a slope and the channel friction slowing it down. The gradual changes in depth take place over long distances and the water surface follows a gradual curve.

Engineers recognise 12 different surface water curves depending on the different gradually varied flow conditions that can occur in channels but the most common is the *backwater curve*. This occurs when a channel is dammed (Figure 5.12). For example, a river flowing at a normal depth of 1.0 m down a gradient of 1 m in 5 km is dammed so that the water level rises to a depth of 5.0 m. For a level water surface behind the dam its influence extends 20 km upstream. But because the river is flowing there is a backwater curve which extends the influence of the dam up to 30 km. This effect can be important for river engineers who wish to ensure that a river's embankments are high enough to contain flows and for landowners along a river whose land may be flooded by the dam construction. The backwater curve can be predicted using the basic tools of hydraulics but they go beyond the scope of this book. One problem is that they depend largely for their accuracy on predicting the value of Manning's *n* which can be very difficult in natural channels.

5.7 Non-uniform flow: rapidly varied

The second type of non-uniform flow is *rapidly varied flow*. As its name implies, sudden changes in depth and velocity occur and this is the result of sudden changes in either the shape or size of channels. The change usually takes place over a few metres, unlike gradually varied flow where changes take place slowly over many kilometres. Hydraulic structures are often the cause of rapidly varied flow and the sluice gate in Section 5.4.3 is a good example of this. In this case



5.12 Backwater curve.

the gate changed the flow suddenly from a deep, slow flow upstream to a fast, shallow flow downstream. Building a weir or widening (or deepening) a channel will also cause sudden changes to occur. But unfortunately, all flows do not behave in the same way. For example, a weir in a channel will have quite a different effect on the deep, slow flow than on the shallow, fast flow. So a further classification of channel flow is needed, this time in terms of how flow behaves when channel size or shape is changed suddenly.

The two contrasting types of flow described above are now well recognised by engineers. The more scientific name for deep, slow flow is *sub-critical flow* and for shallow, fast flow is *super-critical flow*. This implies that there is some *critical point* when the flow changes from one to the other and that this point defines the difference between the two flow types. This is indeed the case. At the critical point the velocity becomes the *critical velocity* and the depth becomes the *critical depth*. The critical point is important not just to classify the two flow types. It plays an important role in the measuring discharges in channels. This is discussed later in Chapter 8.

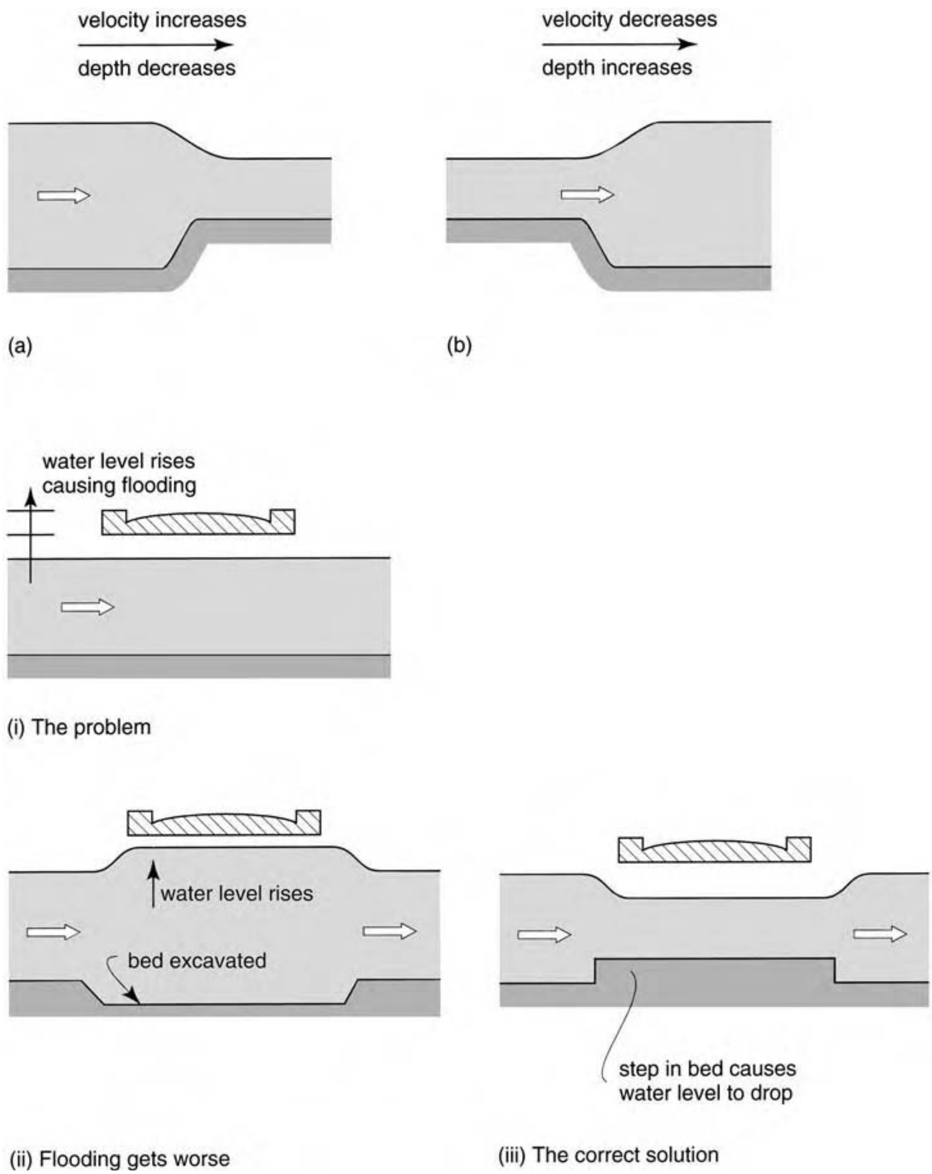
5.7.1 Flow behaviour

Just how do sub-critical and super-critical flows behave when there are sudden changes in the channel?

5.7.1.1 Sub-critical flow

This is by far the most common flow type and is associated with all natural and gently flowing rivers and canals. The effect of a sluice gate on this kind of flow has already been described. The effect of a weir is very similar (Figure 5.13a). A weir is like a step up on the bed of a channel. Such a step causes the water level to drop and the velocity to increase. The step up reduces the flow area in a channel but the water does not slow down because of this. Its velocity increases (remember the way flow behaves in constrictions – Section 3.7.1). This increases the kinetic energy, but as there is no change in the total energy this is at the expense of the depth (pressure) energy. So the depth is reduced causing a drop in the water level. Weirs are a common sight on rivers and most people will have seen this sudden but smooth drop in water level over a weir. A similar, though not so dramatic, drop in water level occurs when water flows under a bridge. The water level drops because the reduced width of the river increases its velocity.

Interestingly, the converse is true. When a channel is made larger by increasing its width or depth, the water level rises. This is not so easy to believe. But it follows from the energy equation and it actually happens in practice (Figure 5.13b). This was highlighted by a problem facing engineers who were troubled by flooding from a river flowing through a town and under the town bridge (Figure 5.13c). During stormy weather, the river level rises and reaches the underside of the bridge. The extra friction from the bridge slows the flow causing the water level upstream of the bridge to rise even further and flood the town. The problem was how to increase the carrying capacity of the river through the town, and particularly under the bridge, to avoid the flooding. The engineers decided that the most obvious solution was to make the channel deeper – but this made flooding worse, not better. Clearly the engineers did not understand the hydraulic tools of continuity and energy. The increase in channel depth reduced the velocity and hence the kinetic energy. As the total energy remained the same, the depth energy increased causing the river level to rise and not fall as expected. They eventually opted for the correct solution which was to reduce the flow area under the bridge by constructing a step on the bed of the river. This increased the velocity energy and reduced the depth of flow. So even when the river was in flood, the flow was able to pass safely under the bridge. The local engineers did not believe the solution at first and insisted on building a hydraulic model in the laboratory to test it. Seeing is believing and this convinced them!

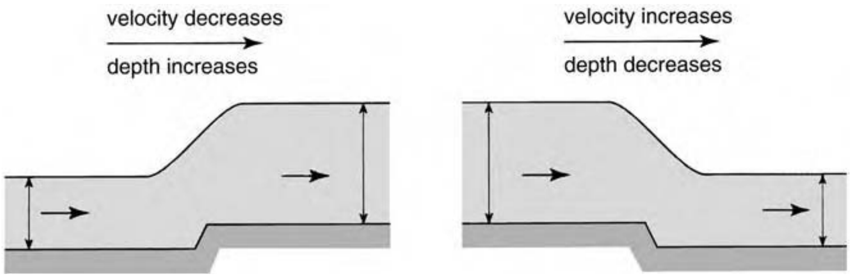


5.13 Rapidly varied flow – sub-critical.

Canoeists are well aware of the way in which rapid changes in water surface levels are a direct result of changes on the river bed. They are wary of those parts of a river where the current looks swifter. It may be tempting to steer your canoe into the faster moving water but it is a sign of shallow water and there may be rocks just below the surface which can damage a canoe. The slower moving water may not be so attractive but at least it will be deep and safe.

5.7.1.2 Super-critical flow

This type of flow behaves in completely the opposite way to sub-critical flow. A step up on the bed of a channel in super-critical flow causes the water depth to rise as it passes over it and a



5.14 Rapidly varied flow – super-critical.

channel which is excavated deeper causes the water depth to drop (Figure 5.14). Super-critical flow is very difficult to deal with in practice. Not only does the faster moving water cause severe erosion in unprotected channels, it is also difficult to control with hydraulic structures. Trying to guide a super-critical flow around a bend in a channel, for example, is like trying to drive a car at high speed around a sharp road bend. It has a tendency to overshoot and to leave the channel. Fortunately super-critical flows rarely occur and are confined to steep rocky streams and just downstream of sluice gates and dam spillways where water can reach speeds of 20 m/s and more. When they do occur engineers have developed ways of quickly turning them back into sub-critical flows so they can be dealt with more easily (see Chapter 6, Section 6.7.6).

5.7.1.3 General rules

So another way of classifying channel flow is in terms of how a flow behaves when the size or shape of a channel is changed suddenly:

For sub-critical flow the water depth decreases when a channel flow area is reduced either by raising its bed or reducing its width. Conversely, the depth increases when the flow area is increased.

For super-critical flow the water depth increases when the flow area is reduced and decreases when it is increased.

5.7.1.4 Spotting the difference

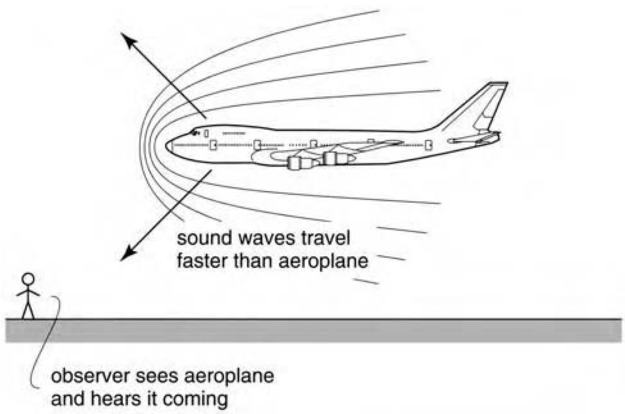
Sometimes the difference between sub- and super-critical flow is obvious. The sluice gate example demonstrates both sub-critical and super-critical flow in the same channel at the same discharge and the same total energy. In this situation there is a clear visual difference between them. But if the two flows occurred in separate straight channels it would be more difficult to tell them apart just by looking. However, if some obstruction is put into the two flows such as a bridge pier or a sharp bend the difference between them would be immediately obvious.

A more scientific way of distinguishing between the two flows is to establish the point of change from sub-critical to super-critical flow – the critical point. There are several very practical ways of doing this but before describing these it might be helpful to look first at another critical point which is similar to, but perhaps more familiar than channel flow.

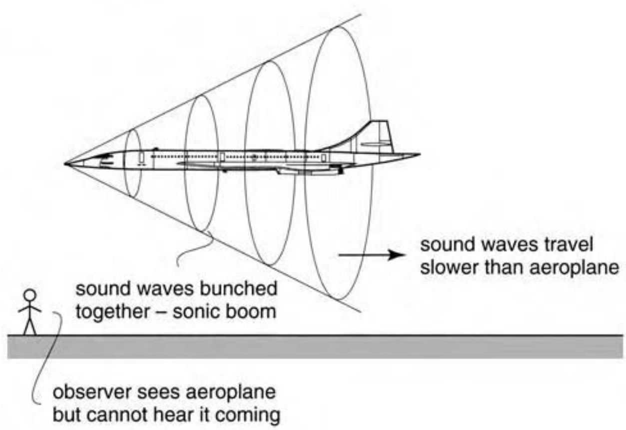
5.7.1.5 An airflow analogy

Aeroplanes are now an everyday part of our lives. Although you may not be aware of it, and you certainly cannot see it, the airflow around an aeroplane in flight is, in fact very similar to water flow in a channel. They are both fluids and so a look at air flow may help to understand some of the complexities of water flow, and in particular the critical point.

Most people will have noticed that jumbo jets and Concorde have very different shapes (Figure 5.15). This is because the two aeroplanes are designed to travel at very different speeds. Jumbo jets are relatively slow and travel at only 800 km/h whereas Concorde travels at much higher speeds of 2000 km/h and more. But the change in aircraft shape is not a gradual one; a sudden



(a) Subsonic flight



(b) Supersonic flight

(c) Aircraft going through the sound barrier from subsonic to supersonic flight

5.15 Using waves to determine flow type.

change is needed when aeroplanes fly over 1200 km/h. This is the speed at which sound waves travel through still air. Sound waves move through air in much the same way as waves travel across a water surface and although they cannot be seen, they can be heard. When someone fires a gun, say 1 km away it takes 3 seconds before you hear the bang. This is the time it takes for sound waves to travel through the air from the gun to your ear at a velocity of 1200 km/h. Notice how you see the gun flash immediately. This is because light waves travel much faster than sound waves at a velocity of 300 000 km/s. This is the reason why lightening in a storm is seen long before the thunder is heard even when the storm is several kilometres away.

When an aeroplane is flying the noise from its engines travels outwards in all directions in the form of sound waves. When it is travelling below the speed of sound, the sound travels faster than the aeroplane and so an observer hears the aeroplane coming before it reaches him (Figure 5.15a). This is known as subsonic flight and aeroplanes which fly below the speed of sound have large rounded shapes like the jumbo jets. When an aeroplane is flying faster than the speed of sound, the sound travels slower than the aeroplane and is left far behind. An observer will see the aeroplane approaching before hearing it (Figure 5.15b). This is known as supersonic flight and aeroplanes travelling at such speeds have slim, dart-like shapes. When the observer does eventually hear it, there is usually a very loud bang. This is the result of a pressure wave, known as the sonic boom, which comes from all the sound waves being bunched up together behind the aeroplane.

So there are two types of airflow, subsonic and supersonic, and there is also a clear point at which the flow changes from one to the other – the speed of sound in still air.

5.7.1.6 Back to water

The purpose of this lengthy explanation about aeroplanes in flight is to demonstrate the close similarity between air flow and water flow. Sub-critical and super-critical flow are very similar to subsonic and supersonic flight. The majority of aeroplanes travel at subsonic speeds and there are very few design problems. In contrast, very few aeroplanes travel at supersonic speeds and their design problems are much greater and more expensive to solve. The same is true for water. Most flows are sub-critical and are easily dealt with. But in a few special cases the flow is super-critical and this is much more difficult to deal with.

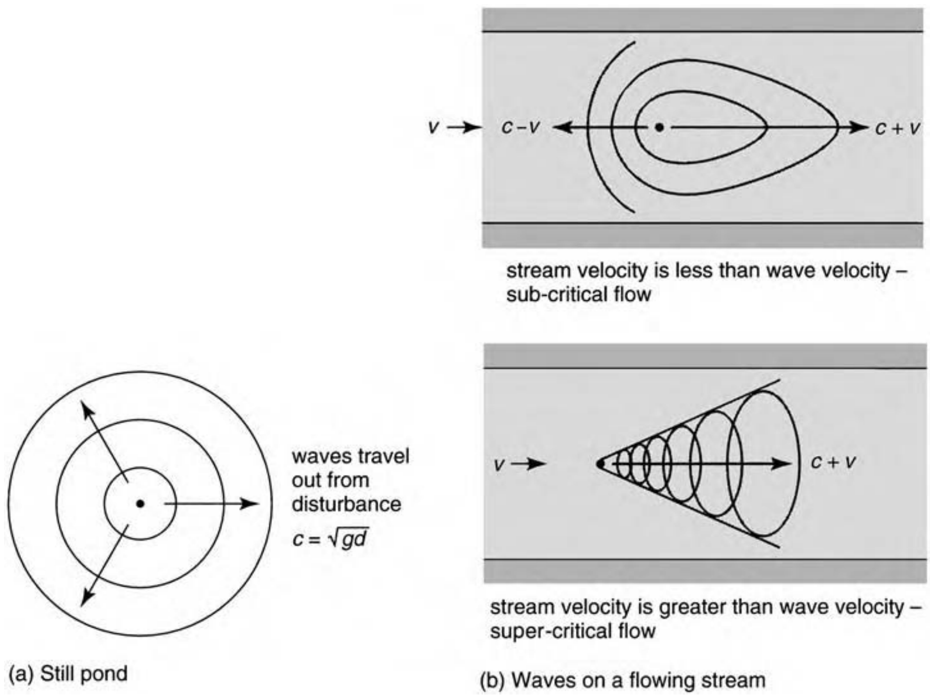
The change from sub- to supersonic occurs when the aeroplane reaches the speed of sound waves in still air. But this is where water is a little different. The change does not occur at the speed of sound in water, although sound does travel through water very effectively. It occurs when the flow reaches the same velocity as waves on the water surface. To avoid confusion between wave velocity and water velocity, waves velocity is often referred to as *wave celerity* (see Section 6.2).

Waves occur on the surface of water when it is disturbed. When you throw a stone into a still pond of water, waves travel out across the surface towards the bank (Figure 5.16a). Although there seems to be a definite movement towards the bank, it is only the waves that are moving outwards and not the water. The water only moves up and down as the waves pass. A duck floating on the pond would only bob up and down with the wave motion and would not be washed up on the bank! But the wave celerity is not a fixed value like the speed of sound. It depends on the depth of water and it can be calculated using the equation (see Section 6.4):

$$c = \sqrt{gd}$$

where c is wave celerity (m/s); g is gravity constant (9.81 m/s^2); d is depth of water (m).

So wave celerity sets the boundary between sub-critical and super-critical flow for a given flow. When the flow velocity is less than the wave velocity the flow is sub-critical. When it is greater, the flow is super-critical.



5.16 Using waves to determine flow type.

As water waves are easily seen they provide a good visual way of determining the type of flow. When you disturb there is a disturbance in a stream, waves move out in all directions from the point of disturbance but the pattern is distorted by the velocity of the water *v* (Figure 5.16b). Some waves move upstream but struggle against the flow and so appear to move more slowly than on the still pond. Others move downstream and are assisted by the flow and so they move faster. Because waves can still move upstream of the disturbance it means that the stream velocity *v* is less than the wave celerity *c* and so the flow must be sub-critical. When the stream velocity *v* is increased and becomes greater than the wave celerity *c*, then waves can no longer travel upstream against the flow. They are all swept downstream and form a vee pattern. This means the flow must be super-critical (Figure 5.16b).

When the stream velocity *v* is equal to the wave celerity *c* then the flow is at the change over point – the *critical point*. At this point the depth of the flow is the critical depth and the velocity is the critical velocity.

Notice the similarity between the sound waves around an aeroplane and the water wave patterns around the disturbance in a stream. There is even an equivalent of the sonic boom in water although it is much less noisy. This is the hydraulic jump which is described in more detail in Section 5.7.6.

5.7.1.7 The finger test

Dropping stones into streams is one way of deciding if the flow is sub-critical or super-critical. Another way is to dip your finger into the water. If the waves you produce travel upstream then the flow is sub-critical. If the waves are swept downstream and the water runs up your arm (you have created a stagnation point in the flow and your wet sleeve is a sign of the high velocity energy) then you are seeing super-critical flow.

5.7.2 *Froude Number*

Another way of determining whether a flow is sub- or super-critical is to use the *Froude Number* (*F*). Returning to the airflow analogy for a moment, aircraft designers use the *Mach Number* or *Mach speed* to describe subsonic and supersonic flight. This is a dimensionless number and is the ratio of two velocities:

$$\text{Mach No.} = \frac{\text{velocity of aircraft}}{\text{velocity of sound in still air}}$$

This dimensionless number was developed by an Austrian Physicist Ernst Mach (1838–1916). A Mach No. less than 1 indicates subsonic flight and a Mach No. greater than 1 indicates supersonic flight. It follows that a Mach No. of 1 means that the aircraft is travelling at the speed of sound.

A dimensionless number similar to the Mach No. was developed by William Froude (1810–1879) to describe sub- and super-critical flow in channels and is now referred to as the Froude Number. It is a ratio of the stream velocity to the wave celerity and is calculated as follows:

$$\text{Froude No. (F)} = \frac{\text{stream velocity (m/s)}}{\text{wave celerity (m/s)}}$$

Note that Froude Number is dimensionless. Now wave celerity:

$$c = \sqrt{gd}$$

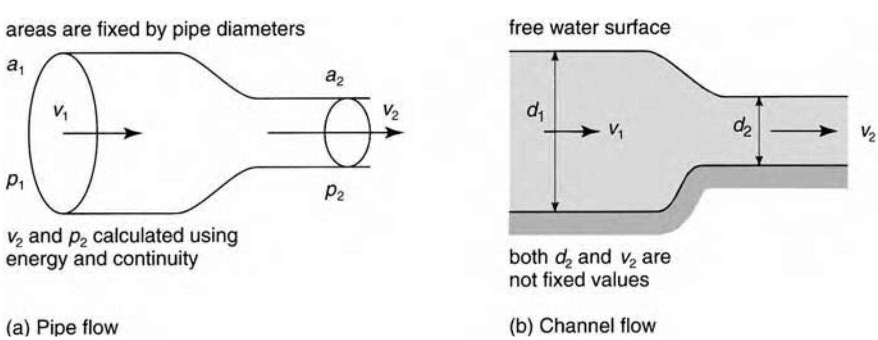
And so:

$$F = \frac{v}{\sqrt{gd}}$$

A Froude Number of less than 1 indicates sub-critical flow and a Froude Number greater than 1 indicates super-critical flow. It follows that a Froude Number of 1 means that the channel is flowing at critical depth and velocity. So calculating the Froude Number is another way of determining when the flow is sub- or super-critical.

5.7.3 *Specific energy*

It should be possible to quantify the changes in depth and velocity resulting from sudden changes in a channel by making use of the energy and continuity equations. Remember that a similar problem occurred in pipes when a venturi meter was inserted to measure discharge (see Section 4.10). The equations of energy and continuity were used to work out the pressure and velocity changes as a result of the sudden changes in the size of the pipe (Figure 5.17a). The solution was



5.17 Predicting changes in depth and velocity in a channel.

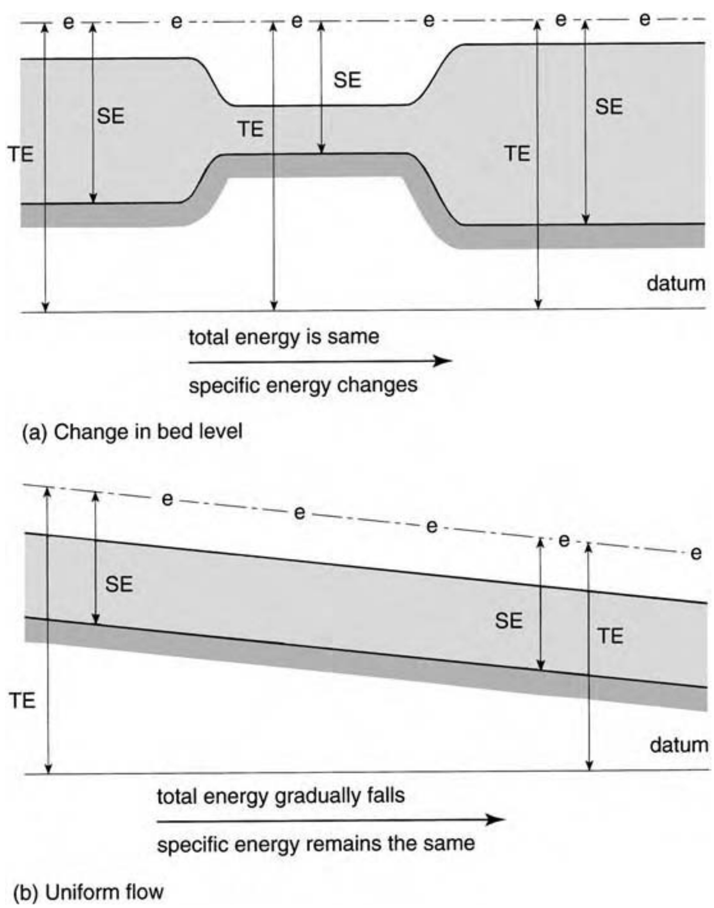
straight forward because the area of flow was fixed by the pipe diameter and so only the velocity needed to be calculated. But for a channel the flow area is not fixed. It is open to the atmosphere and so it can flow at many different depths (Figure 5.17b). So both the flow area and the velocity are unknown. If the energy and continuity equations are applied to this problem the result is a cubic equation which means there are three possible answers for the downstream depth and velocity. One answer is negative and this can be dismissed immediately as impracticable. But the two remaining answers are both possibilities, but which one is the right one?

To help solve this problem Boris Bakhmateff (1880–1951) introduced a very helpful concept which he called *specific energy* (*E*). Simply stated: *Specific energy is the energy in a channel measured from the bed of a channel*.

Writing this as an equation:

$$E = d + \frac{q^2}{2gd^2}$$

It is important at this point to draw a clear distinction between total energy and specific energy. They are linked but they are quite different (Figure 5.18a). Total energy is measured



5.18 Specific energy.

from some fixed datum and its value can only reduce as energy is lost through friction. When there is a change in the bed level of a channel (e.g. when water flows over a weir) there are also changes in the energy components but the total energy remains the same. Specific energy, in contrast, is measured from the bed of a channel and so when the bed level changes the specific energy also changes. It also means that specific energy can rise as well as fall depending on what is happening to the channel bed. When the flow moves from the channel over a weir the specific energy falls and when it comes off the weir it rises again.

The difference between total and specific energy is highlighted by uniform flow (Figure 5.18b). Total energy falls gradually as energy is lost through friction. But specific energy remains constant along the channel because there are no changes in depth and velocity.

The physical significance of specific energy beyond its simple definition is not so obvious and many engineers still struggle with it. However, it is a very easy and very practical concept to use. Rather than be too concerned about what it means, it is better to think of it as a simple mechanism for solving a problem. It is like a key for opening a lock. You do not need to know how the lock works in order to use it. You just put the key in and turn it. In the same way, specific energy unlocks the problem of quantifying the effects that sudden changes in a channel have on depth and velocity. It also helps to establish whether the flow is sub-critical or super-critical and it also unlocks the problem of how to measure discharges in channels (see Section 7.5). So on the whole it is a pretty effective key.

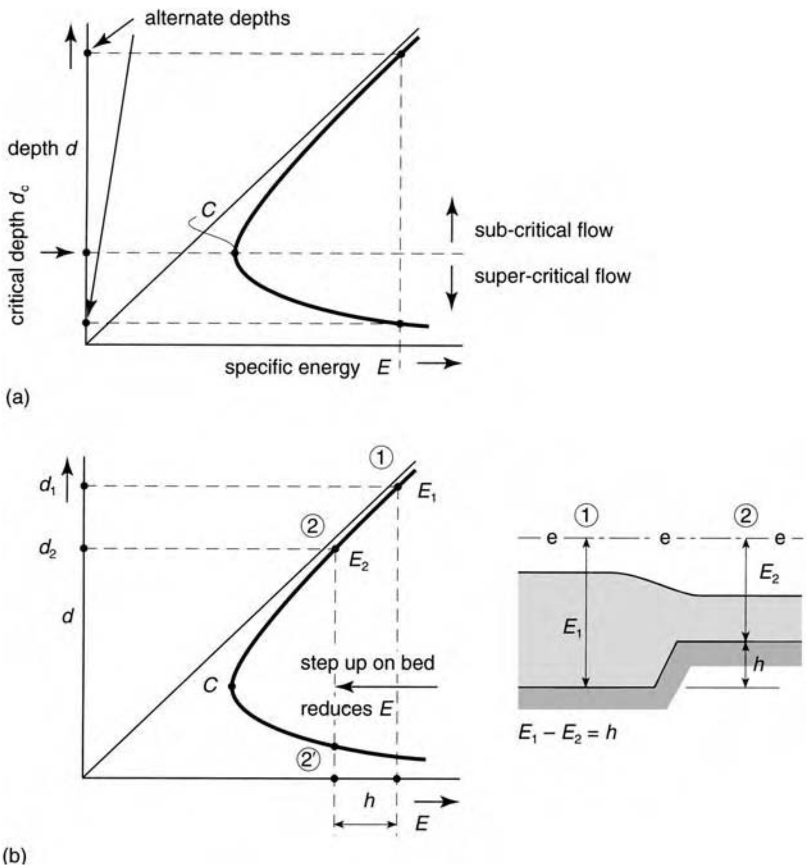
The best way to show how this works is with an example. A channel is carrying a discharge per unit width q. From the specific energy equation it is possible to calculate a range of values for specific energy by putting in different values of d. When the results of the calculations are plotted on a graph (Figure 5.19a) the result is the *specific energy diagram*. There are two limbs to the specific energy diagram and this shows the two possible solutions for depth for any given value of E. These are called the *alternate depths* and are the sub- and super-critical depths described earlier. The upper limb of the curve describes sub-critical flow and the lower limb describes super-critical flow. The change over between the two types of flow occurs at the point C on the graph and this is the *critical point.* It is the only place on the graph where there is only one depth of flow for a given value of E and not two.

So the specific energy diagram defines sub-critical and super-critical flow by defining the critical depth. It also confirms the earlier descriptions of the effects on depth and velocity of sudden changes in a channel. Take any point E_1 on the sub-critical part of the curve and look what happens when the value of E changes as a result of raising or lowering the channel bed. A step up on the bed reduces E and the graph shows that d decreases also (remember E is measured from the bed of the channel). A step down on the bed increases E and the graph shows that d increases. A similar example can be applied to the super-critical part of the diagram to show the effects of changing E on the depth of flow. In this case we see the opposite effect. A step up reduces E and increases d.

So the depth and velocity change, but by how much? This is where the specific energy diagram becomes very useful for quantifying these changes. To see how this is done consider what happens when there is a step up of height h on the bed of a channel from point 1 to point 2 (Figure 5.19b). This reduces the specific energy E_2 by an amount h. So:

$$E_1 - E_2 = h \quad \text{and} \quad E_2 = E_1 - h$$

E_2 on the curve can be found by subtracting h from the value of E_1 and this represents the condition on the step from which it is possible to determine the depth d_2 and velocity v_2. The same logic can also be applied to super-critical flow. An example in the box shows how this is done in practice for both sub-critical and super-critical flows.



5.19 Specific energy diagram.

One question this raises is: why does the flow stay sub-critical when it moves from point 1 to point 2? Why does it not go to point 2′, which has the same value of specific energy, and so become super-critical? The answer is in the specific energy diagram. Remember the diagram is for a given value of discharge. So suddenly 'leaping' across the diagram from 2 to 2′ would mean a change in discharge and this is not possible. The only way to get from 2 to 2′ is down the specific energy curve and through the critical point. This can only be done by increasing the value of *h* (i.e. raising the step even further) so that E_2 gets smaller until it reaches the critical point. At this point the flow can go super-critical. As *h* in this case is not high enough to create critical conditions, the flow moves from point 1 to point 2 and stays sub-critical.

The same argument can be applied to changes in super-critical flow also. The flow can only go sub-critical by going around the specific energy curve and through the critical depth.

To summarise some points about specific energy:

- Specific energy is used to quantify the changes in depth and velocity in a channel as a result of sudden changes in the size and shape of a channel. It also helps to determine which of the two possible answers for depth and velocity is the right one.
- Specific energy is different from total energy. It can increase as well as decrease. It depends on what happens to the bed of the channel. In contrast, total energy can only decrease as energy is lost through friction or sudden changes in flow.

- The specific energy diagram in Figure 5.19 is for one value of discharge. When the discharge changes a new diagram is needed. So for any channel there will be a whole family of specific energy diagrams representing a range of discharges.
- Specific energy is the principle on which many channel flow measuring devices such as weirs and flumes are based. They depend on changing the specific energy enough to make the flow go critical. At this point there is only one value of depth for one value of specific energy and from this is possible to develop a formula for discharge (see Section 7.5). Such devices are sometimes referred to as *critical depth structures*.
- For uniform flow the value of specific energy remains constant along the entire channel. This is because the depth and velocity are the same. In contrast, total energy gradually reduces as energy is lost along the channel.

5.7.3.1 Is the flow sub-critical or super-critical?

The answer comes from calculating the normal depth of flow, using a formula such as Manning's, and then comparing it with the critical depth. If the depth is greater than the critical depth the flow will be sub-critical and if it is less it will be super-critical.

EXAMPLE: CALCULATING THE EFFECT ON DEPTH OF CHANGING THE CHANNEL BED LEVEL

A 0.3 m step is to be built in a rectangular channel carrying a discharge per unit width of 0.5 m^3/s (Figure 5.20). Calculate the effect of this change on the water level in both sub-critical and super-critical flow conditions assuming the specific energy in the upstream channel $E_1 = 1.0$ m. Calculate the effect in the same channel of lowering a section of the channel bed by 0.3 m.

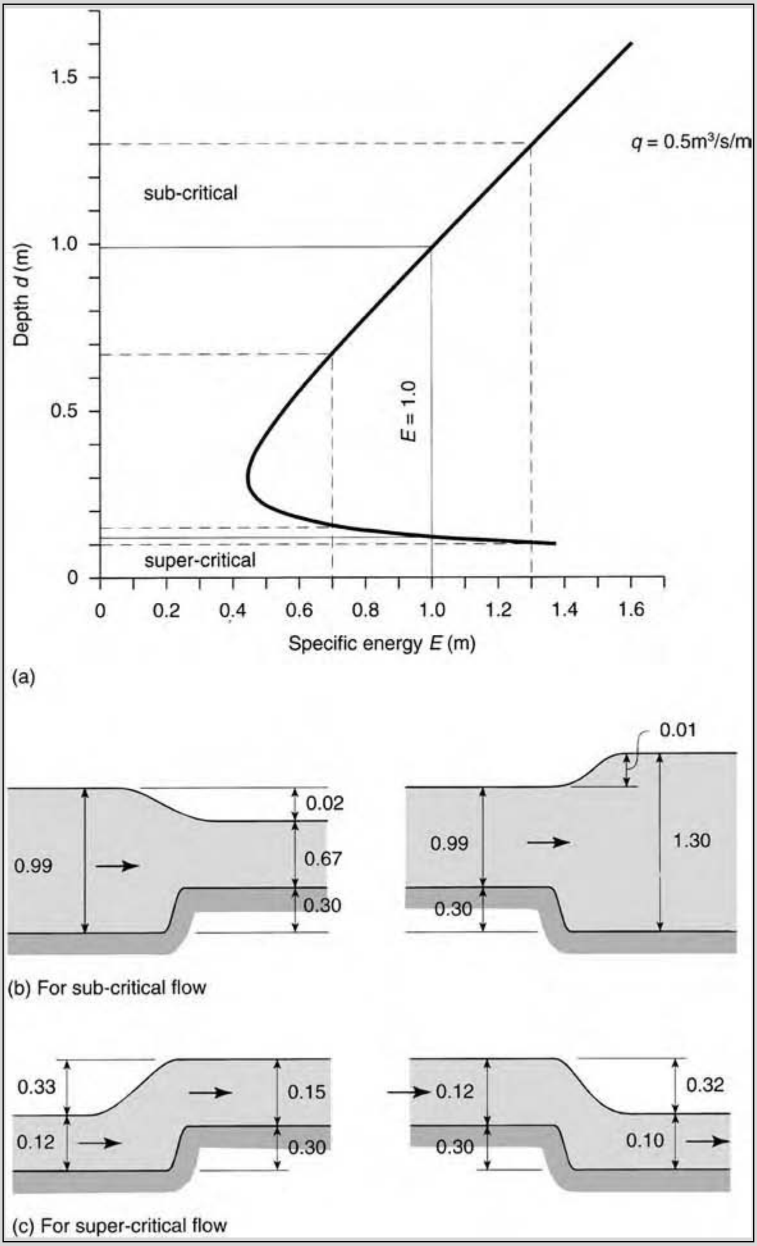
The first step is to calculate specific energy E for a range of depths for a discharge of 0.5 m^3/s:

Depth of flow (m)	*Specific energy (m)*
Sub-critical flow curve	
0.4	0.480
0.5	0.551
0.6	0.635
0.7	0.726
0.8	0.820
0.9	0.916
1	1.013
1.2	1.209
1.4	1.407
1.6	1.605
Super-critical flow curve	
0.1	1.374
0.12	1.005
0.15	0.716
0.2	0.519
0.3	0.442
0.25	0.454
0.35	0.454

Plot the results on a graph (See Figure 5.20a).

From the graph the alternate depths for $E_1 = 1$ m are $d_1 = 0.99$ m (sub-critical) and 0.12 m (super-critical). Consider the sub-critical flow case first.

For sub-critical flow – calculate the effect on water level of a 0.3 m high step up on the bed of the channel:



5.20 Calculating specific energy.

Specific energy in the channel is:

$E_1 = 1.0$ m

And:

$d_1 = 0.99$ m

Next calculate the specific energy on the step which is 0.3 m higher than the channel bed:

$E_2 = E_1 - 0.3 = 0.7$ m

Now locate E_2 on the graph and read off the value for d_2:

$d_2 = 0.67$ m

Depth d_2 is measured from the step and so the water level is 0.97 m above the original channel bed. This means that the water level has dropped 0.02 m as a result of raising the channel bed.

For sub-critical flow – calculate the effect on water level of a 0.3 m step down on the bed of the channel:

Specific energy in the channel is:

$E_1 = 1.0$ m

And:

$d_1 = 0.99$ m

Next calculate the specific energy where the channel has been excavated by 0.3 m:

$E_2 = E_1 + 0.3 = 1.3$ m

Now locate E_2 on the graph and read off the value for d_2:

$d_2 = 1.30$ m

Depth d_2 is measured from the bed of the excavated section and so the water level is now 1.30 m above the channel bed. This means that the water level rises by 0.01 m as a result of the step down in the channel bed.

For super-critical flow – calculate the effect on water level of a 0.3 m step up on the bed of the channel:

Specific energy in the channel is:

$E_1 = 1.0$ m

And:

$d_1 = 0.12$ m

Next calculate the specific energy on the step which is 0.3 m higher than the channel bed:

$E_2 = E_1 - 0.3 = 0.7$ m

Now locate E_2 on the graph and read off the value for d_2:

$$d_2 = 0.15 \text{ m}$$

Now depth d_2 is measured from the top of the raised bed and so the water level is now 0.45 m above the original channel bed. This means that the water level rises 0.33 m as a result of raising the channel bed.

For super-critical flow – calculate the effect on water level of a 0.3 m step down on the bed of the channel:

Specific energy in the channel is:

$$E_1 = 1.0 \text{ m}$$

And:

$$d_1 = 0.12 \text{ m}$$

Next calculate the specific energy where the channel has been excavated by 0.3 m:

$$E_2 = E_1 + 0.3 = 1.3 \text{ m}$$

Now locate E_2 on the graph and read off the value for d_2:

$$d_2 = 0.10 \text{ m}$$

Now depth d_2 is measured from the bed of the excavated bed and so the water level is now 0.20 m below the original channel bed. This means that the water level drops 0.32 m as a result of the step down in the channel bed.

5.7.4 Critical depth

The critical depth d_c can be determined directly from the specific energy diagram but it can be difficult to locate its exact position because of the rounded shape of the curve close to the critical point. To overcome this problem it can be calculated using a formula derived from the specific energy equation:

$$d_c = \sqrt[3]{\frac{q^2}{g}}$$

This formula shows that the critical depth is influenced only by the discharge per unit width q. It has nothing to do with the slope or the normal depth. For the mathematically minded a proof of this is given in the box.

DERIVATION: CRITICAL DEPTH EQUATION

Derive a formula for the critical depth and its location on the specific energy diagram (Figure 5.20).

Use the specific energy equation:

$$E = d + \frac{q^2}{2gd^2}$$

At the critical point the specific energy E is at its lowest value and so the gradient of the curve dE/dd is equal to zero. The equation for the gradient can be found using calculus and differentiating the above equation for the curve. If you are not familiar with calculus you will have to accept this step as given:

$$\frac{dE}{dd} = 1 - \frac{q^2}{gd^3} = 0$$

Depth d now becomes the critical depth d_c and so:

$$\frac{q^2}{gd_c^3} = 1$$

Rearrange this for d_c:

$$d_c^3 = \frac{q^2}{g}$$

$$d_c = \sqrt[3]{\frac{q^2}{g}}$$

So the critical depth d_c depends only on the discharge per unit width q.
To calculate the specific energy at the critical point first write down the specific energy equation for critical conditions:

$$E_c = d_c + \frac{q^2}{2gd_c^2}$$

But at the critical depth (see equation earlier in this proof):

$$\frac{q^2}{gd_c^3} = 1$$

Divide both sides by 2 and multiplying by d_c:

$$\frac{q^2}{2gd_c^2} = \frac{d_c}{2}$$

The left-hand side is now equivalent to the kinetic energy term so put this into the specific energy equation:

$$E_c = d_c + \frac{d_c}{2}$$

$$= \frac{3d_c}{2}$$

So for any given discharge the critical depth can be calculated as well as the specific energy at the critical point. These two values exactly locate the critical point on the specific energy diagram.
Note also from the above analysis that when:

$$\frac{q^2}{gd_c^3} = 1$$

$$v_c^2 = gd_c$$

And so:

$$v_c = \sqrt{gd_c}$$

This is the equation for the celerity of surface water waves and it shows that at the critical point the water velocity v_c equals the wave celerity $\sqrt{gd_c}$. Remember that the velocity of waves across water following a disturbance is often used to determine whether flow is sub-critical or super-critical.

EXAMPLE: CALCULATING CRITICAL DEPTH

Using information in the previous example calculate the critical depth and the step up in the bed level required to ensure that the flow will reach the critical depth. Assume the initial specific energy $E_1 = 1.0$ m.

First calculate the critical depth:

$$d_c = \sqrt[3]{\frac{q^2}{g}}$$

$$d_c = \sqrt[3]{\frac{0.5^2}{9.81}} = 0.29 \text{ m}$$

Now calculate the specific energy on the step up:

$$E_2 = d_2 + \frac{q^2}{2gd_2^2}$$

When this is critical flow:

$$d_2 = d_c$$

And so:

$$E_2 = 0.29 + \frac{0.5^2}{2 \times 9.81 \times 0.29^2}$$
$$= 0.44 \text{ m}$$

But:

$$E_1 - E_2 = h$$

that is, the change in specific energy is a direct result of the height of the step up h: And so:

$$h = 1.0 - 0.44 = 0.56 \text{ m}$$

The bed level of the channel must be raised by 0.56 m to ensure that the flow goes critical.

5.7.5 Critical flow

Although critical flow is important, it is a flow condition best avoided in uniform flow. There is no problem when flow goes quickly through the critical point on its way from sub- to super-critical or from super to sub but there are problems when the normal flow depth is near to the critical depth. This is a very unstable condition as the flow tends to oscillate from sub to super and back to sub-critical again resulting in surface waves which can travel for many kilometres eroding and damaging channel banks. The explanation for the instability is in the shape of the specific energy diagram close to the critical point (Figure 5.20a). Small changes in specific energy E, possibly as a result of small channel irregularities, can cause large changes in depth as the flow oscillates between sub- and super-critical flow. As the flow *hunts* back and forth it sets up waves. So although critical flow is very useful in some respects it can cause serious problems in others.

5.7.6 Flow transitions

Changes in a channel which result in changes in flow from sub-critical to super-critical and vice-versa are referred to as *transitions*. The following are examples of some common transitions.

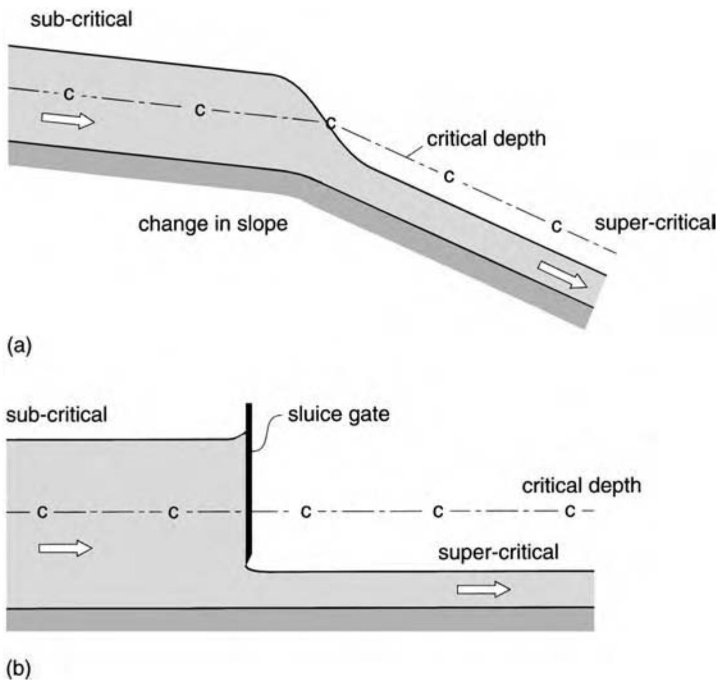
5.7.6.1 Sub- to super-critical flow

When flow goes from sub- to super-critical it does so smoothly. In Figure 5.21a the channel gradient is increased which changes the flow from sub- to super-critical. The water surface curves rapidly but smoothly as the flow goes through the critical point. There is no energy loss as the flow is contracting. Notice how the critical depth is shown as c—c—c so that the two types of flow are clearly distinguishable. Remember, the critical depth is the same in both sections of the channel because it depends only on the discharge and not on slope.

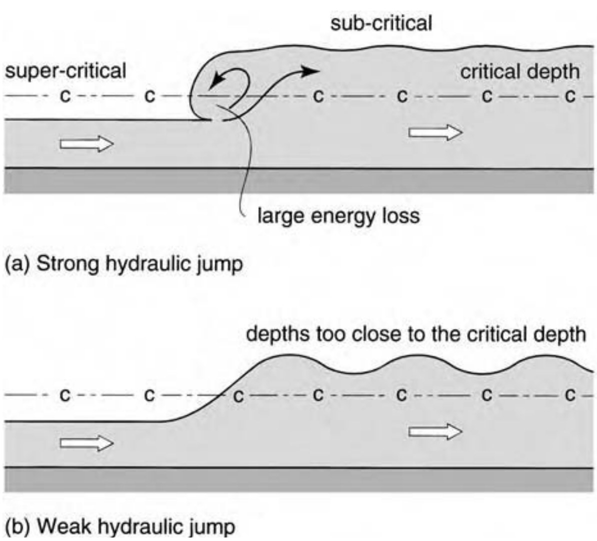
Another example of this type of transition is the sluice gate (Figure 5.21b). In this case a gate is used to force the change in flow. Again the transition occurs smoothly with no energy loss.

5.7.6.2 Super- to sub-critical flow (hydraulic jump)

The change from super- to sub-critical is not so smooth. In fact a vigorous turbulent mixing action occurs as the flow jumps abruptly from super- to sub-critical flow (Figure 5.22). It is aptly called a *hydraulic jump* and as the flow expands there is a significant loss of energy due to the turbulence.



5.21 Flow transitions – sub- to super-critical.



5.22 Flow transitions – super- to sub-critical.

Hydraulic jumps are very useful for many purposes:

- Getting rid of unwanted energy, such as at the base of dam spillways.
- Mixing chemicals in water. The vigorous turbulence ensures that any added chemical is thoroughly dispersed throughout the flow.

- Converting super-critical flow downstream of hydraulic structures into sub-critical flow to avoid erosion damage in unprotected channels.

Hydraulic jumps are usually described by their strength and the Froude Number of the super-critical flow. A *strong jump* is the most desirable. It is very vigorous, has a high Froude Number, well above one and the turbulent mixing is confined to a short length of channel. A *weak jump*, on the other hand, is not so violent. It has a low Froude No. approaching one, which means the depth of flow is close to the critical depth. This kind of jump is not confined and appears as waves which can travel downstream for many kilometres. This is undesirable because the waves can do a great deal of damage to channel banks.

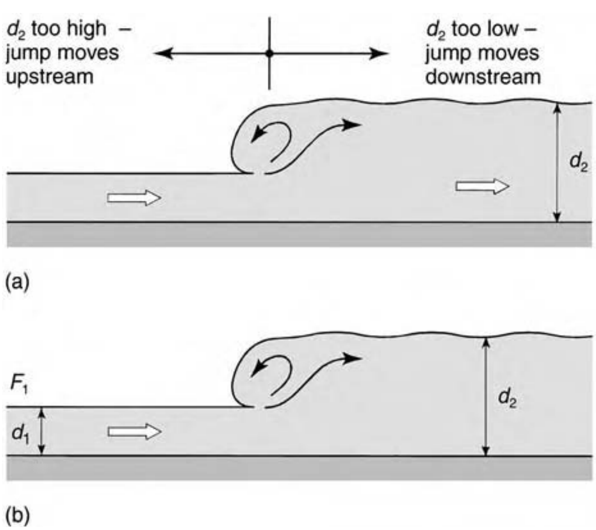
5.7.6.3 Creating a hydraulic jump

There are two conditions required to form a hydraulic jump:

- Flow upstream of the jump must be super-critical, that is, the Froude Number must be greater than 1.
- Downstream flow must be sub-critical and deep enough for the jump to form.

Once there is super-critical flow in a channel it is the downstream depth of flow that determines if a jump will occur. To create a jump the downstream depth must be just right. If the depth is too shallow a jump will not form and the super-critical will continue down the channel (Figure 5.23). Conversely, if the flow is too deep the jump will move upstream and if it reaches a sluice gate it may drown it out. This can be a problem as the high speed super-critical flow is not dispersed as it would be in a full jump and it can cause erosion downstream (see Section 7.32.1 for drowned flow from a sluice gate).

When the upstream depth and velocity are known it is possible to calculate the downstream depth and velocity which will create a jump by using the momentum equation. The energy equation cannot be used at this stage because of the large and unknown energy loss at a jump.



5.23 Forming a hydraulic jump.

The formula is derived from the momentum equation which links the two depths of flow d_1 and d_2:

$$\frac{d_2}{d_1} = \frac{1}{2}\left(\sqrt{1+8F_1^2} - 1\right)$$

Where d_1 is upstream depth; d_2 is downstream depth; F_1 is upstream Froude No.

Most hydraulic jump problems involve calculating the downstream depth d_2 in order to determine the conditions under which a jump will occur. But determining a value for d_2 is fine in theory, but in practice the formation of a jump is very sensitive to the downstream depth. This means that when the downstream depth is a little more or less than the calculated value, the jump does not stabilise at the selected location but moves up and down the channel *hunting* for the right depth of flow. In practice it is very difficult to control water depths with great accuracy and so some method is needed to remove the sensitivity of the jump to downstream water level and so stabilise it. This is the job of a *stilling basin* which is a concrete apron located downstream of a weir or sluice gate. Its primary job is to dissipate unwanted energy to protect the downstream channel and it does this by making sure the hydraulic jump forms on the concrete slab even though the downstream water level may vary considerably. This is discussed further in Section 7.10.

5.7.6.4 Calculating energy losses

Once the downstream depth and velocity have been calculated using the momentum equation, the loss of energy at a jump can be determined using the total energy equation as follows:

Total energy upstream = total energy downstream + energy loss at jump

That is:

$$d_1 + \frac{v_1^2}{2g} + z_1 = d_2 + \frac{v_2^2}{2g} + z_2 + \text{losses}$$

EXAMPLE: CALCULATING THE DOWNSTREAM DEPTH TO FORM A HYDRAULIC JUMP

Calculate the depth required downstream to create the jump in a channel carrying a discharge of 0.8 m^3/s per m width at a depth of 0.25 m.

The downstream depth can be calculated using the formula:

$$\frac{d_2}{d_1} = \frac{1}{2}\left(\sqrt{1+8F_1^2} - 1\right)$$

First calculate the velocity using the discharge equation:

$$q = v_1 d_1$$

$$v_1 = \frac{0.8}{0.25} = 3.2 \text{ m/s}$$

Next calculate the Froude No:

$$F_i = \frac{v_1}{\sqrt{gd_1}}$$

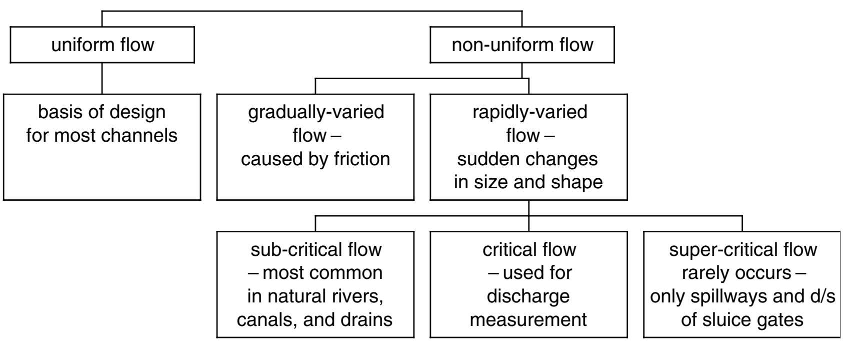
$$= \frac{3.2}{\sqrt{9.81 \times 0.25}} = 2.04$$

Substitute the values into the above formula:

$$\frac{d_2}{0.25} = \frac{1}{2}\left(\sqrt{1+(8\times 2.04^2)}-1\right)$$

$$d_2 = 0.25 \times 2.43 = 0.61 \text{ m}$$

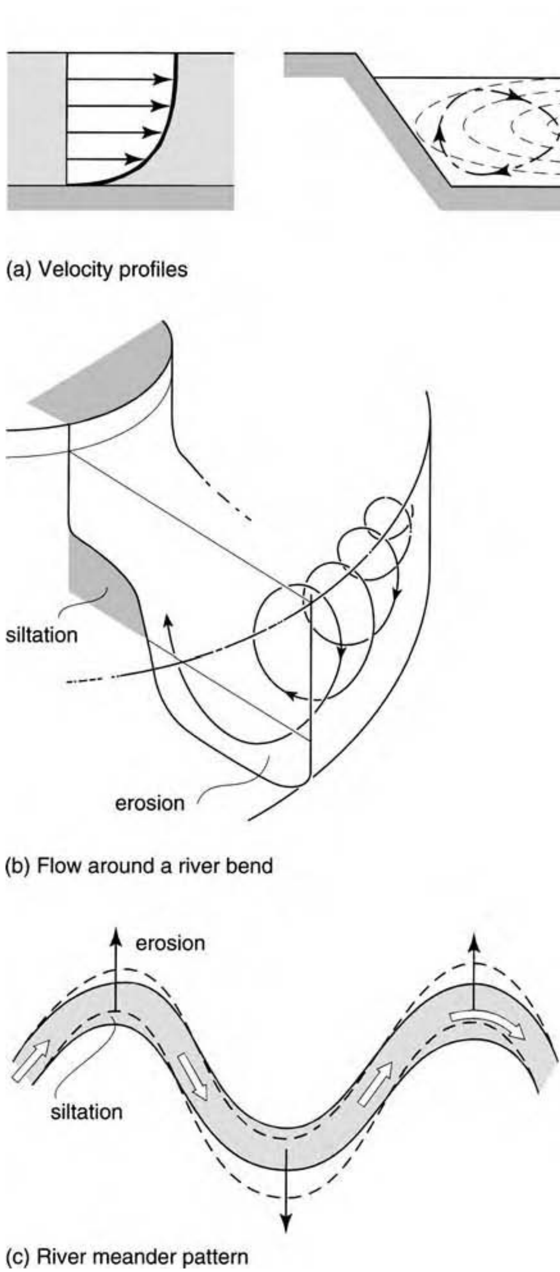
To summarise all the various types of flow that can occur in channels:



5.8 Secondary flows

One interesting aspect of channel flow (which all anglers know about) is *secondary flows*. These are small but important currents that occur in flowing water and explain many important phenomena.

In ideal channels the velocity is assumed to be the same across the entire channel. But in real channels the velocity is usually much higher in the middle than at the sides and bed. Figure 5.24 shows the velocity profile in a typical channel and in cross section, isotachs (lines of equal velocity) have been drawn. The changes in velocity across the channel cause small changes in pressure (remember the energy equation) and these are responsible for setting up cross currents which flow from the sides of the channel to the centre. As the water does not pile up in the middle of a channel there must be an equivalent flow from the centre to the edge. These circulating cross flows are called *secondary flows.* In very wide channels several currents can be set up in this way. So water does not just flow straight down a channel, it flows along a spiral path.



5.24 Secondary flows in channels.

5.8.1 Channel bends

An important secondary flow occurs at channel bends (Figure 5.24). This is created by a combination of the difference in velocity between the surface and the bed and the centrifugal forces as the water moves around the bend. A secondary current is set up which moves across the bed from the outside of the bend to the inside and across the surface from the inside to the outside. The secondary flow can erode loose material on the outside of a bend and carry it across a river bed and deposit it on the inside of the bend. This is contrary to the common belief that sediment on the bed of the river is thrown to the outside of a bend by the strong centrifugal forces.

One consequence of this in natural erodible river channels is the process known as *meandering.* Very few natural rivers are straight. They tend to form a snake-like pattern of curves which are called *meanders*. The outside of bends are progressively scoured and the inside silts up causing the river cross section to change shape. The continual erosion gradually alters the course of the river. Sometimes the meanders become so acute that parts of the river are eventually cut off and form what are called *ox-bow lakes.*

Note that it is not a good idea to go swimming on the outside bend of a river. The downward current can be very strong and pull the unwary swimmer down into the mud on the river bed!

5.8.2 Siting river offtakes

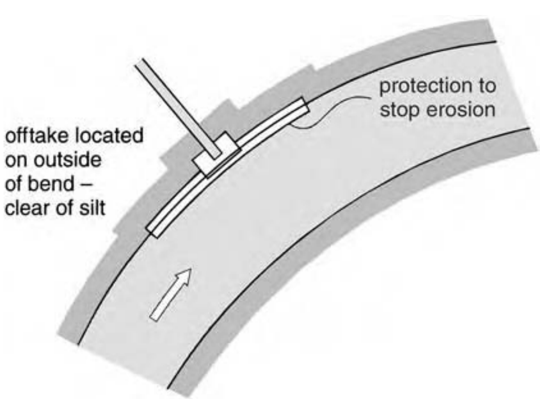
A common engineering problem is to select a site for abstracting water from a river for domestic use or irrigation. This may be a pump or some gated structure. The best location is on the outside of a river bend so that it will be free from (Figure 5.25). If located on the inside of a bend it would be continually silting up as a result of the actions of the secondary flows. The outside of a bend may need protecting with stone pitching to stop any further erosion which might destroy the offtake.

5.8.3 Bridge piers

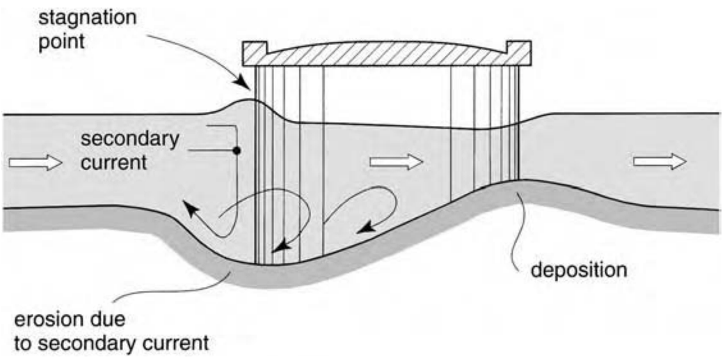
Scouring around bridge piers can be a serious problem. This is the result of a secondary flow set up by the stagnation pressure on the nose of a pier (Figure 5.26). The rise in water level as the flow is stopped causes a secondary downward current towards the bed. This is pushed around the pier by the main flow into a spiral current which can cause severe scouring both in front and around the sides of piers. Heavy stone protection can reduce the problem but a study of the secondary flows has shown that the construction of low walls upstream can also help by upsetting the pattern of the destructive secondary currents.

5.8.4 Vortices at sluice gates

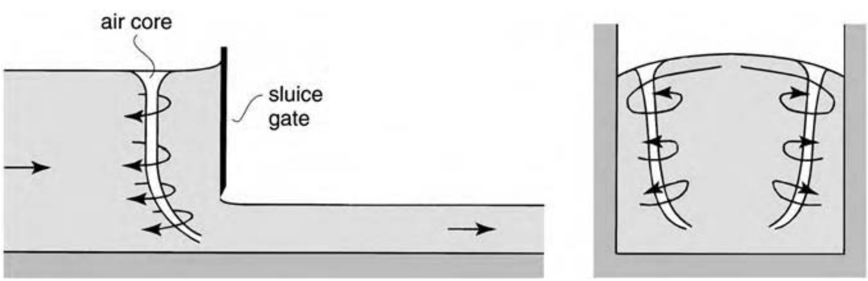
Vortices can develop upstream of sluice gates and may be so strong that they extend from the water surface right underneath the gate drawing air down into its core (Figure 5.27). They can



5.25 Siting river offtakes.



5.26 Flow around bridge piers.



5.27 Vortices at sluice gates.

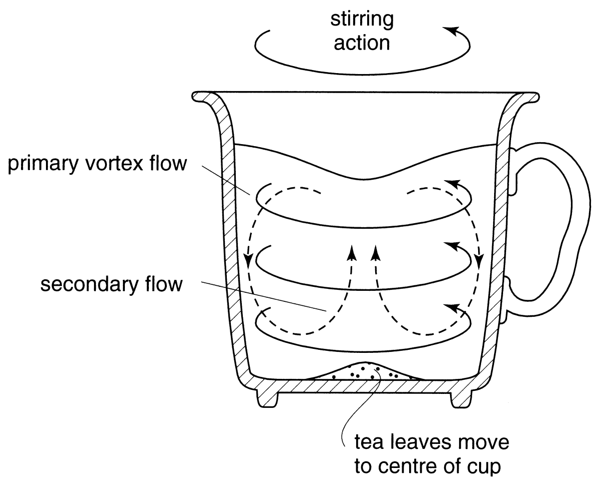
become so severe that they reduce the flow through the gate. They look like small tornadoes and are the result of secondary currents. They form when the surface flow reaches a sluice gate the water stops, that is, it stagnates. But the flow in the middle of the stream was moving faster than near the sides and so the rise in water level due to the stagnation is higher in the middle than at the sides. This causes a secondary flow from the middle of the stream to the sides and sets up two circulating vortices.

5.8.5 Tea cups

An interesting secondary flow occurs when stirring a cup of tea (Figure 5.28). First make sure there is no milk in the tea to obscure the view, then notice how the tea leaves at the bottom of the cup move in towards the centre when the tea is stirred. The stirring action sets up a vortex which is similar to the flow round a bend in a river. The water surface drops in the centre of the flow causing a pressure difference which results in a downward current on the outside of the cup which moves across the bottom and up the middle. Any tea leaves lying on the bottom of the cup are swept along a spiral path towards the centre of the cup.

5.9 Sediment transport

Sediment movement in channels is usually referred to as *sediment transport* and it is not a subject normally covered in texts on basic hydraulics. But a study of channels would not be complete without mentioning it. Most channels are either naturally occurring or excavated in



5.28 Stirring the tea.

the natural soil and so are prone to scouring and silting. Very few have the luxury of a lining to protect them against erosion. Natural rivers often carry silt and sand washed from their catchments, some even carry large boulders in their upper reaches. Man-made canals not only have to resist erosion but need to avoid becoming blocked with silt and sand. Indeed the sediment is often described as the sediment load, indicating the burden that channels must carry.

Channels carrying sediment are designed in a different way to those carrying clear water. Normally Manning or Chezy formulae are used for clear water but they are not well suited to sediment laden water. There is no simple accepted theory which provides a thorough understanding of sediment movement on which to base channel design. When engineers meet a problem for which there is no acceptable theory they do not wait for the scientists to find one. They try to find a way round the problem and develop alternative design methods. This is what happened in India and Pakistan at the turn of the century when British engineers were faced with designing and operating large canals which carried both water and silt from the Indus and Ganges rivers for the irrigation of vast tracts of land. They observed and measured the hydraulic parameters of existing canals that seemed to have worked well under similar conditions over a long period of time. From these data they developed equations which linked together sediment, velocity, hydraulic radius, slope and width and used them to design new canals. They came to be known as the regime equations. The word regime described the conditions in the canals where, over a period of time, the canals neither silted nor scoured and all the silt that entered the canals at the head of the systems was transported through to the fields. In fact an added benefit of this was the silt brought with it natural fertilisers for the farmers. But this is not claimed to be a perfect solution. Often the canals silted up during heavy floods and often they would scour when canal velocities were excessive. But on balance over a period of time (which may be several years) the canals did not change much and were said to be *in regime*. In spite of all the research over the past century, engineers still use these equations because they are still the best available today.

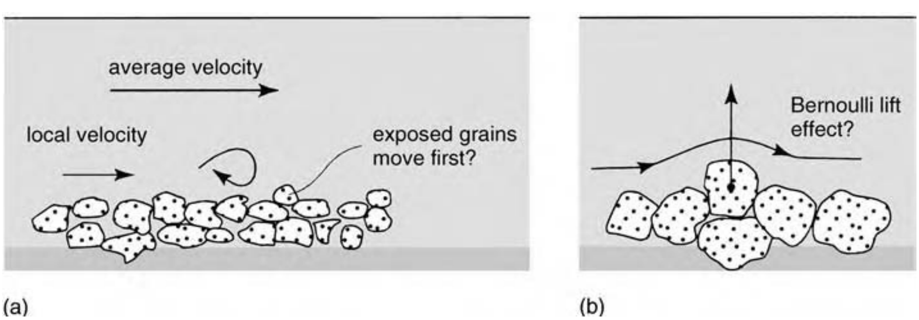
The lack of progress in our understanding of sediment transport comes from the nature of the problem. In other aspects of hydraulics there is a reasonably clear path to solving a problem. In channels one single force dominates channel design – the gravity force. All other factors such as viscosity can be safely ignored without it causing much error. But in sediment transport there are several factors which control what happens and no single one dominates. Gravity influences both water and sediment, but viscosity and density also appear to be important. This makes any fundamental analysis of the problem complex.

At the heart of the sediment transport problem is how to determine when sediment begins to move – the *threshold of movement*. Some investigators, particularly engineers, have tried to link this with average stream velocity because it is simple to measure and it is obviously linked to the erosive power of a channel. But research has shown that it is not as simple as this. Look at the particles of sand on the bed of a channel (Figure 5.29). Is it the average velocity that is important or the local velocity and turbulence close to the sand grains? Some grains are more exposed than others. Do they move first? Does grain size and density matter? Do the rounded grains move more easily than the angular ones? Is there a Bernoulli (energy change) lifting effect when flow moves around exposed grains which will encourage them to move? How important is the apparent loss in weight of sand grains when they are submerged in water? There are no clear answers to these questions but what is clear is that many factors influence the threshold of movement and this has made it a very difficult subject to study from an analytical point of view.

There is, however, some good news which has come from experimental work. It is in fact very easy to observe the threshold of movement and to say when it has been reached. Imagine a channel with a sandy bed with water flowing over it. When the flow is increased the sand will at some point begin to move. But the interesting point is that this begins quite suddenly and not gradually as might be expected. When the threshold is reached, the whole channel bed comes alive suddenly as all the sand begins to move at the same time. So if several observers, watching the channel, are asked to say when they think the threshold has been reached, they will have no problem in agreeing the point at which it occurs. What they will not be able to say for certain is what caused it.

Because of this clear observation of the threshold much of the progress had been made by scientists using experimental methods. Shields in 1936 successfully establish the conditions for the threshold of movement on an experimental basis for a wide range of sediments and these are the data that are still used today for designing channels to avoid erosion. It provides a much sounder basis for design than simply using some limiting velocity.

Working out the amount of sediment being transported once it begins to move is fraught with difficulties. The reason for this is that once movement begins the amount of sediment on the move is very sensitive to small changes in the factors which caused the movement in the first place. This means that small changes in what could be called the erosive power of a channel can result in very large changes in sediment transport. Even if it was possible to calculate such changes, which some experimenters have tried to do, it is even more difficult to verify this by measuring sediment transport in the laboratory and almost impossible to measure it with any



5.29 Threshold of movement.

accuracy in the field. For these reasons it seems unlikely that there will be any significant improvements in the predictions of sediment transport and that engineers will have to rely on the regime equations for some time to come.

5.10 Some examples to test your understanding

1 An open channel of rectangular section has a bed width of 1.0 m. If the channel carries a discharge 1.0 m^3/s calculate the depth of flow when the Manning's roughness coefficient is 0.015 and the bed slope is 1 in 1000. Calculate the Froude Number in the channel and the critical depth (0.5 m; 0.45 m; 0.29 m).

2 A rectangular channel of bed width 2.5 m carries a discharge of 1.75 m^3/s. Calculate the normal depth of flow when the Chezy coefficient is 60 and the slope is 1 in 2000. Calculate the critical depth and say whether the flow is sub-critical or super-critical (0.75 m; 0.37 m; flow is sub-critical).

3 A trapezoidal channel is to be designed and constructed in a sandy loam with a longitudinal slope of 1 in 5000 to carry a discharge of 2.3 m^3/s. Calculate suitable dimensions for the depth and bed width assuming Manning's n is 0.022 and the side slope is 1 in 2 ($d = 0.98$ m; $b = 2.94$ m ($b = 3d$)).

4 A trapezoidal channel carrying a discharge of 0.75 m^3/s is to be lined with concrete to avoid seepage problems. Calculate the channels dimensions which will minimise the amount of concrete when Manning's n for concrete is 0.015 and the channel slope is 1 in 1250 (for a hexagonal channel $d = 0.37$ m; $b = 0.8$ m. Note that other answers are possible depending on choice of side slope).

5 A hydraulic jump occurs in a rectangular channel 2.3 m wide when the discharge is 1.5 m^3/s. If the upstream depth is 0.25 m calculate the upstream Froude Number, the depth of flow downstream of the jump and the energy loss in the jump (2.78 m; 0.87 m; 0.3 m).