14 CHAPTER

FORCES ON SUB-MERGED BODIES

▶ 14.1 INTRODUCTION

When a fluid is flowing over a stationary body, a force is exerted by the fluid on the body. Similarly, when a body is moving in a stationary fluid, a force is exerted by the fluid on the body. Also when the body and fluid both are moving at different velocities, a force is exerted by the fluid on the body. Some of the examples of the fluids flowing over stationary bodies or bodies moving in a stationary fluid are :

1. Flow of air over buildings,
2. Flow of water over bridges,
3. Submarines, ships, airplanes and automobiles moving through water or air.

▶ 14.2 FORCE EXERTED BY A FLOWING FLUID ON A STATIONARY BODY

Consider a body held stationary in a real fluid, which is flowing at a uniform velocity U as shown in Fig. 14.1.

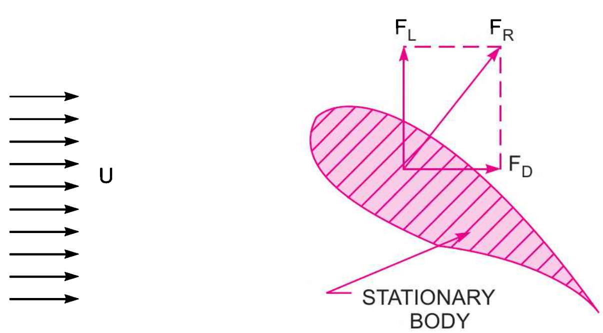

Fig. 14.1 *Force on a stationary body.*

The fluid will exert a force on the stationary body. The total force (F_R) exerted by the fluid on the body is perpendicular to the surface of the body. Thus the total force is inclined to the direction of motion. The total force can be resolved in two components, one in the direction of motion and other perpendicular to the direction of motion.

14.2.1 Drag. The component of the total force (F_R) in the direction of motion is called 'drag'. This component is denoted by F_D. Thus drag is the force exerted by the fluid in the direction of motion.

14.2.2 Lift. The component of the total force (F_R) in the direction perpendicular to the direction of motion is known as 'lift'. This is denoted by F_L. Thus lift is the force exerted by the fluid in the direction perpendicular to the direction of motion. Lift force occurs only when the axis of the body is inclined to the direction of fluid flow. If the axis of the body is parallel to the direction of fluid flow, lift force is zero. In that case only drag force acts.

If the fluid is assumed ideal and the body is symmetrical such as a sphere or cylinder, both the drag and lift will be zero.

▶ 14.3 EXPRESSION FOR DRAG AND LIFT

Consider an arbitrary shaped solid body placed in a real fluid, which is flowing with a uniform velocity U in a horizontal direction as shown in Fig. 14.2. Consider a small elemental area dA on the surface of the body. The forces acting on the surface area dA are :

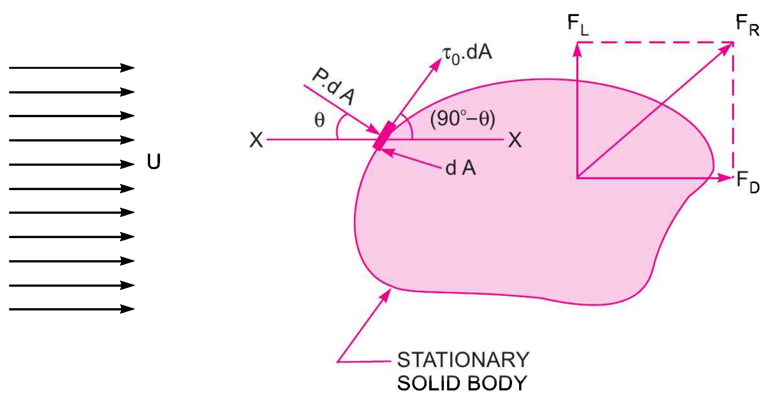

Fig. 14.2 *Drag and lift.*

1. Pressure force equal to $p \times dA$, acting perpendicular to the surface and
2. Shear force equal to $\tau_0 \times dA$, acting along the tangential direction to the surface.

Let θ = Angle made by pressure force with horizontal direction.

(*a*) **Drag Force (F_D).** The drag force on elemental area

= Force due to pressure in the direction of fluid motion
+ Force due to shear stress in the direction of fluid motion.

$= pdA \cos \theta + \tau_0 dA \cos (90° - \theta) = pdA \cos \theta + \tau_0 dA \sin \theta$

∴ Total drag, F_D = Summation of $pdA \cos \theta$ + Summation of $\tau_0 dA \sin \theta$

$$= \int p \cos \theta dA + \int \tau_0 \sin \theta dA. \quad ...(14.1)$$

The term $\int p \cos \theta dA$ is called the pressure drag or form drag while the term $\int \tau_0 \sin \theta dA$ is called the friction drag or skin drag or shear drag.

(*b*) **Lift Force (F_L).** The lift force on elemental area

= Force due to pressure in the direction perpendicular to the direction of motion
+ Force due to shear stress in the direction perpendicular to the direction of motion.

$$= -pdA \sin\theta + \tau_0 dA \sin(90° - \theta) = -pdA \sin\theta + \tau_0 dA \cos\theta$$

Negative sign is taken with pressure force as it is acting in the downward direction while shear force is acting vertically up.

$\therefore$ Total lift,

$$F_L = \int \tau_0 \, dA \cos\theta - \int pdA \sin\theta$$

$$= \int \tau_0 \cos\theta dA - \int p \sin\theta dA \quad \text{...(14.2)}$$

The drag and lift for a body moving in a fluid of density ρ, at a uniform velocity U are calculated mathematically, as

$$F_D = C_D A \frac{\rho U^2}{2} \quad \text{...(14.3)}$$

$$F_L = C_L A \frac{\rho U^2}{2} \quad \text{...(14.4)}$$

where C_D = Co-efficient of drag,

C_L = Co-efficient of lift,

A = Area of the body which is the projected area of the body perpendicular to the direction of flow

Or

= Largest projected area of the immersed body.

Then resultant force on the body, $F_R = \sqrt{F_D^2 + F_L^2}$ ...(14.5)

The equations (14.3) and (14.4) which give the mathematical expression for drag and lift are derived by the method of dimensional analysis.

14.3.1 Dimensional Analysis of Drag and Lift. In the chapter of Dimensional and Model Analysis it is shown in problem 12.6 that the force exerted by a fluid on a supersonic plane is given by :

$$F = \rho L^2 U^2 \phi\left[\frac{\mu}{\rho U L} \cdot \frac{K}{\rho U^2}\right] \quad \text{...}(i)$$

Also in problem 12.7, it is shown that the force exerted by a fluid on a partially sub-merged body is given by :

$$F = \rho L^2 U^2 \phi\left[\frac{\mu}{\rho U L} \cdot \frac{Lg}{U^2}\right] \quad \text{...}(ii)$$

Thus the general expression for the force exerted by a fluid (air or water) on a body (completely sub-merged or partially sub-merged) is given as

$$\text{F} = \rho L^2 U^2 \phi\left[\frac{\mu}{\rho U L} \cdot \frac{Lg}{U^2} \cdot \frac{K}{\rho U^2}\right] \quad \text{...(14.6)}$$

where L = Length of body, U = Velocity of body,

μ = Viscosity of fluid, ρ = Density of fluid

F = Force exerted, k = Bulk modulus of fluid,

g = Acceleration due to gravity.

If the body is completely sub-merged in the fluid, the force exerted by the fluid on the body due to gravitational effect is negligible. Hence the non-dimensional term containing 'g' in equation (14.6), *i.e.*, $\frac{Lg}{U^2}$ is neglected. If the velocity of the body is comparable with velocity of sound, the effect due to compressibility is to be considered. But if the ratio of the velocity of the body to the velocity of the sound is less than 0.3, the force exerted by the fluid on the body due to compressibility is negligible. Hence the non-dimensional term in equation (14.6) containing K can be neglected. Then the force exerted by fluid on the body is given as

$$F = \rho L^2 U^2 \phi \left[\frac{\mu}{\rho UL}\right] = \rho L^2 U^2 \phi \left[\frac{\rho UL}{\mu}\right]$$

where $\frac{\rho UL}{\mu}$ = Reynolds number = R_e

$$F = \rho L^2 U^2 \phi \, [R_e]. \quad \text{...(14.7)}$$

Now F is the total force exerted by the fluid on the body. The total force is having two components, one in the direction of motion called drag force and other component in the direction perpendicular to the direction of motion, called lift force.

The two components of F are expressed as

$$F_D = \frac{\rho L^2 U^2}{2} \times C_D$$

where C_D is a function of R_e and is called co-efficient of drag

$$= C_D A \frac{\rho U^2}{2} \qquad \{\because \ L^2 = \text{Area} = A\}$$

And

$$F_L = \frac{\rho L^2 U^2}{2} \times C_L$$

where C_L is a function of R_e and is called co-efficient of lift

$$= C_L \cdot A \frac{\rho U^2}{2}.$$

Problem 14.1 *A flat plate 1.5 m × 1.5 m moves at 50 km/hour in stationary air of density 1.15 kg/m^3. If the co-efficients of drag and lift are 0.15 and 0.75 respectively, determine :*

(i) The lift force, *(ii) The drag force,*
(iii) The resultant force, and
(iv) The power required to keep the plate in motion.

Solution. Given :

Area of the plate, $A = 1.5 \times 1.5 = 2.25 \text{ m}^2$

Velocity of the plate, $U = 50 \text{ km/hr} = \frac{50 \times 1000}{60 \times 60} \text{ m/s} = 13.89 \text{ m/s}$

Density of air $\rho = 1.15 \text{ kg/m}^3$

Co-efficient of drag, $C_D = 0.15$

Co-efficient of lift, $C_L = 0.75$

(*i*) **Lift Force (F_L).** Using equation (14.4),

$$F_L = C_L A \times \frac{\rho U^2}{2} = 0.75 \times 2.25 \times \frac{1.15 \times 13.89^2}{2} \text{ N} = \textbf{187.20 N. Ans.}$$

(*ii*) **Drag Force (F_D).** Using equation (14.3),

$$F_D = C_D \times A \times \frac{\rho U^2}{2} = 0.15 \times 2.25 \times \frac{1.15 \times 13.89^2}{2} \text{ N} = \textbf{37.44. N. Ans.}$$

(*iii*) **Resultant Force (F_R).** Using equation (14.5),

$$F_R = \sqrt{F_D^2 + F_L^2} = \sqrt{37.44^2 + 187.20^2} \text{ N}$$

$$= \sqrt{1400 + 35025} = \textbf{190.85 N. Ans.}$$

(*iv*) **Power Required to keep the Plate in Motion**

$$P = \frac{\text{Force in the direction of motion} \times \text{Velocity}}{1000} \text{ kW}$$

$$= \frac{F_D \times U}{1000} = \frac{37.425 \times 13.89}{1000} \text{ kW} = \textbf{0.519 kW. Ans.}$$

Problem 14.2 *Experiments were conducted in a wind tunnel with a wind speed of 50 km/hour on a flat plate of size 2 m long and 1 m wide. The density of air is 1.15 kg/m³. The co-efficients of lift and drag are 0.75 and 0.15 respectively. Determine :*

(*i*) *the lift force,* (*ii*) *the drag force,*
(*iii*) *the resultant force,* (*iv*) *direction of resultant force and*
(*v*) *power exerted by air on the plate.*

Solution. Given :

Area of plate, $A = 2 \times 1 = 2 \text{ m}^2$

Velocity of air, $U = 50 \text{ km/hr} = \frac{50 \times 1000}{60 \times 60} \text{ m/s} = 13.89 \text{ m/s}$

Density of air, $\rho = 1.15 \text{ kg/m}^3$

Value of $C_D = 0.15$ and $C_L = 0.75$

(*i*) *Lift force* (F_L)

Using equation (14.4), $F_L = C_L \times A \times \rho \times U^2/2$

$= 0.75 \times 2 \times 1.15 \times 13.89^2/2 =$ **166.404 N. Ans.**

(*ii*) *Drag force* (F_D)

Using equation (14.3), $F_D = C_D \times A \times \rho \times U^2/2$

$= 0.15 \times 2 \times 1.15 \times 13.89^2/2 =$ **33.28 N. Ans.**

(*iii*) *Resultant force* (F_R)

Using equation (14.5), $F_R = \sqrt{F_D^2 + F_L^2} = \sqrt{33.28^2 + 166.404^2} =$ **169.67 N. Ans.**

(*iv*) *The direction of resultant force* (θ)

The direction of resultant force is given by,

$$\tan\theta = \frac{F_L}{F_D} = \frac{166.38}{33.275} = 5$$

$\therefore$ $\theta = \tan^{-1} 5 =$ **78.69°. Ans.**

(*v*) *Power exerted by air on the plate*

Power = Force in the direction of motion × Velocity

$= F_D \times U$ N m/s $= 33.280 \times 13.89$ W ($\because$ Watt = N m/s)

= **462.26 W. Ans.**

Problem 14.3 *Find the difference in drag force exerted on a flat plate of size 2 m × 2 m when the plate is moving at a speed of 4 m/s normal to its plane in : (i) water, (ii) air of density 1.24 kg/m³. Co-efficient of drag is given as 1.15.*

Solution. Given :

Area of plate, $A = 2 \times 2 = 4 \text{ m}^2$

Velocity of plate, $U = 4$ m/s

Co-efficient of drag, $C_D = 1.15$

(*i*) Drag force when the plate is moving in water.

Using equation (14.3), $F_D = C_D \times A \times \dfrac{\rho U^2}{2}$, where ρ for water = 1000

$$= 1.15 \times 4 \times 1000 \times \frac{4^2}{2} \text{ N} = 36800 \text{ N.} \quad \text{...}(i)$$

(*ii*) Drag force when the plate is moving in air,

$$F_D = C_D \times A \times \frac{\rho U^2}{2}, \quad \text{where } \rho \text{ for air} = 1.24$$

$$\therefore \quad F_D = 1.15 \times 4.0 \times 1.24 \times \frac{4.0^2}{2.0} \text{ N} = 45.6 \text{ N} \quad \text{...}(ii)$$

∴ Difference in drag force = (*i*) – (*ii*)

= 36800 – 45.6 = **36754. 4 N. Ans.**

Problem 14.4 *A truck having a projected area of 6.5 square metres travelling at 70 km/hour has a total resistance of 2000 N. Of this 20 per cent is due to rolling friction and 10 per cent is due to surface friction. The rest is due to form drag. Calculate the co-efficient of form drag. Take density of air = 1.25 kg/m³.*

Solution. Given :

Area of truck, $A = 6.5 \text{ m}^2$

Speed of truck, $U = 70 \text{ km/hr} = \dfrac{70 \times 100}{60 \times 60} = 19.44 \text{ m/s}$

Total resistance, $F_T = 2000$ N

Rolling friction resistance, $F_C = 20\%$ of total resistance $= \dfrac{20}{100} \times 2000 = 400$ N

Surface friction resistance, $F_S = 10\%$ of total resistance $= \dfrac{10}{100} \times 2000 = 200$ N

∴ Form drag, $F_D = 2000 - F_C - F_S = 2000 - 400 - 200 = 1400$ N

Using equation (14.3), $F_D = C_D \times A \times \dfrac{\rho U^2}{2}$

where if F_D = Form drag then C_D = Co-efficient of form drag

$$\therefore \quad 1400 = C_D \times 6.5 \times 1.25 \times \frac{19.44^2}{2} \quad (\rho = \text{Density of air} = 1.25 \text{ kg/m}^3)$$

$$\therefore \quad C_D = \frac{1400 \times 2}{6.5 \times 1.25 \times 19.44 \times 19.44} = \mathbf{0.912.\ Ans.}$$

Problem 14.5 *A circular disc 3 m in diameter is held normal to a 26.4 m/s wind of density 0.0012 gm/cc. What force is required to hold it at rest ? Assume co-efficient of drag of disc = 1.1.*

Solution. Given :

Diameter of disc = 3 m

$\therefore$ Area, $A = \frac{\pi}{4} \times (3)^2 = 7.0685 \text{ m}^2$

Velocity of wind, $U = 26.4$ m/s

Density of wind, $\rho = 0.0012 \text{ gm/cm}^3 = \frac{.0012}{1000} \text{ kg/cm}^3$

$$= \frac{0.0012}{1000} \times 10^6 \frac{\text{kg}}{\text{m}^3} = 1.2 \text{ kg/m}^3$$

Co-efficient of drag, $C_D = 1.1$

The force required to hold the disc at rest is equal to the drag exerted by wind on the disc.

Drag (F_D) is given by equation (14.3) as

$$F_D = C_D \times A \times \frac{\rho U^2}{2} = \frac{1.1 \times 7.0685 \times 1.2 \times 26.4^2}{2.0} = \textbf{3251.4 N. Ans.}$$

Problem 14.6 *A man weighing 90 kgf descends to the ground from an aeroplane with the help of a parachute against the resistance of air. The velocity with which the parachute, which is hemispherical in shape, comes down is 20 m/s. Find the diameter of the parachute. Assume C_D = 0.5 and density of air = 1.25 kg/m^3.*

Solution. Given :

Weight of man, $W = 90 \text{ kgf} = 90 \times 9.81 \text{ N} = 882.9 \text{ N}$ ($\because$ 1 kgf = 9.81 N)

Velocity of parachute, $U = 20$ m/s

Co-efficient of drag, $C_D = 0.5$

Density of air, $\rho = 1.25 \text{ kg/m}^3$

Let the diameter of parachute $= D$

$\therefore$ Area, $A = \frac{\pi}{4} D^2 \text{ m}^2$.

When the parachute with the man comes down with a uniform velocity, U = 20 m/s, the drag resistance will be equal to the weight of man, neglecting the weight of parachute. And projected area of the hemispherical parachute will be equal to $\frac{\pi}{4} D^2$.

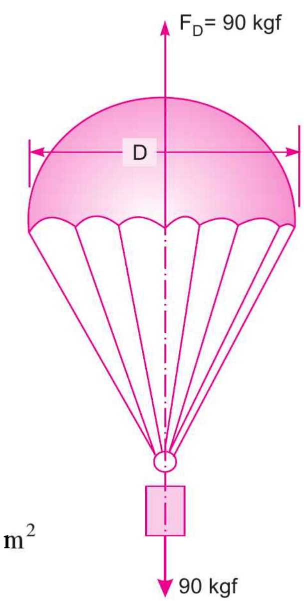

Fig. 14.3

$\therefore$ Drag, $F_D = 90 \text{ kgf} = 90 \times 9.81 = 882.9 \text{ N}$

Using equation (14.3), $F_D = C_D \times A \times \frac{\rho U^2}{2}$

$$\therefore \quad 882.9 = 0.5 \times \frac{\pi}{4} D^2 \times \frac{1.25 \times 20^2}{2}$$

$$\therefore \quad D^2 = \frac{882.9 \times 4 \times 2.0}{0.5 \times \pi \times 1.25 \times 20 \times 20} = 8.9946 \text{ m}^2$$

or $D = \sqrt{8.9946} = \textbf{2.999 m. Ans.}$

Problem 14.7 *A man weighing 981 N descends to the ground from an aeroplane with the help of a parachute against the resistance of air. The shape of the parachute is hemispherical of 2 m diameter. Find the velocity of the parachute with which it comes down. Assume C_d = 0.5 and ρ for air = 0.00125 gm/cc and ν = 0.015 stoke.*

Solution. Given :

Weight of the man, $W = 981$ N

∴ Drag force, $F_D = W = 981$ N

Diameter of the parachute, $D = 2$ m

∴ Projected area, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 2^2 = \pi \text{ m}^2$

Co-efficient of drag, $C_d = 0.5$

Density for air, $\rho = 0.00125 \text{ gm/cm}^3 = \frac{.00125}{1000} \text{ kg/cm}^3$

$$= \frac{.00125}{1000} \times 10^6 \frac{\text{kg}}{\text{m}^3} = 1.25 \text{ kg/m}^3$$

Let the velocity of parachute $= U$

Using equation (14.3), $F_D = C_D \times A \times \frac{\rho U^2}{2}$ or $981 = 0.5 \times \pi \times \frac{1.25 \times U^2}{2.0}$

∴ $U = \sqrt{\frac{981 \times 2.0}{0.5 \times \pi \times 1.25}}$ = **31.61 m/s. Ans.**

Problem 14.8 *A man descends to the ground from an aeroplane with the help of a parachute which is hemispherical having a diameter of 4 m against the resistance of air with a uniform velocity of 25 m/s. Find the weight of the man if the weight of parachute is 9.81 N. Take C_D = 0.6 and density of air = 1.25 kg/m³.*

Solution. Given :

Diameter of parachute, $D = 4$ m

∴ Projected area, $A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 4^2 = 4\pi \text{ m}^2$

Velocity of parachute, $U = 25$ m/s

Weight of parachute, $W_1 = 9.81$ N

Co-efficient of drag, $C_D = 0.6$

Density of air, $\rho = 1.25 \text{ kg/m}^3$

Let the weight of man $= W_2$

Then weight of man + Weight of parachute = $W_2 + W_1 = (W_2 + 9.81)$

Hence drag force will be equal to the weight of man plus weight of parachute.

∴ Drag force, $F_D = (W_2 + 9.81)$

Using equation (14.3), we have $F_D = C_D \times A \times \frac{\rho U^2}{2}$

or $$(W_2 + 9.81) = 0.6 \times 4\pi \times \frac{1.25 \times 25^2}{2.0} = 2945.24 \text{ N}$$

∴ $W_2 = 2945.24 - 9.81$ = **2935.43 N. Ans.**

Problem 14.9 *Calculate the diameter of a parachute to be used for dropping an object of mass 100 kg so that the maximum terminal velocity of dropping is 5 m/s. The drag co-efficient for the parachute, which may be treated as hemispherical is 1.3. The density of air is 1.216 kg/m³.*

Solution. Given :

Mass of object, $M = 100$ kg

Weight of object, $W = 100 \times 9.81 = 981$ N

∴ Drag force, $F_D = 981$ N

Velocity of object, $U = 5$ m/s

Drag co-efficient, $C_D = 1.3$

Density of air, $\rho = 1.216$ kg/m^3

Let the diameter of parachute $= D$ m

∴ Projected area, $A = \frac{\pi}{4} D^2$ m^2

Using equation (14.3), $F_D = C_D \times A \times \frac{\rho U^2}{2}$

or $$981 = 1.3 \times \frac{\pi}{4} D^2 \times \frac{1.216 \times 5^2}{2}$$

or $$D^2 = \frac{981 \times 4 \times 2}{1.3 \times \pi \times 1.216 \times 5 \times 5} = 63.21$$

∴ $D = \sqrt{63.21}$ = **7.95 m. Ans.**

Problem 14.10 *A kite 0.8 m × 0.8 m weighing 0.4 kgf (3.924 N) assumes an angle of 12° to the horizontal. The string attached to the kite makes an angle of 45° to the horizontal. The pull on the string is 2.5 kgf (24.525 N) when the wind is flowing at a speed of 30 km/hour. Find the corresponding co-efficient of drag and lift. Density of air is given as 1.25 kg/m³.*

Solution. Given :

Projected area of kite, $A = 0.8 \times 0.8 = 0.64$ m^2

Weight of kite, $W = 0.4$ kgf $= 0.4 \times 9.81 = 3.924$ N

Angle made by kite with horizontal, $\theta_1 = 12°$

Angle made by string with horizontal, $\theta_2 = 45°$

Pull on the string, $P = 2.5$ kgf $= 2.5 \times 9.81 = 24.525$ N

Speed of wind, $U = 30$ km/hr $= \frac{30 \times 1000}{60 \times 60}$ m/s $= 8.333$ m/s

Density of air, $\rho = 1.25$ kg/m^3

Drag force, F_D = Force exerted by wind in the direction of motion (*i.e.*, in the X-X direction)
= Component of pull, P along X-X
= $P \cos 45° = 24.525 \cos 45° = 17.34$ N

And lift force, F_L = Force exerted by wind on the kite perpendicular to the direction of motion (*i.e.*, along Y-Y direction)
= Component of P in vertically downward direction + Weight of kite (W)

$= P \sin 45° + W = 24.525 \sin 45° + 3.924$ N
$= 17.34 + 3.924 = 21.264$ N.

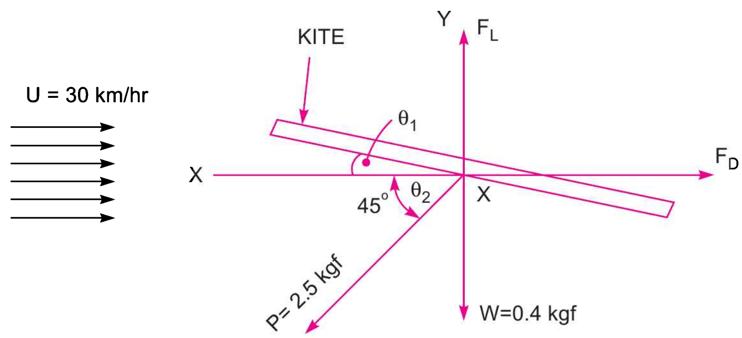

Fig. 14.4

(*i*) **Drag Co-efficient (C_D).** Using equation (14.3), we have

$$F_D = C_D \times A \times \frac{\rho U^2}{2}$$

or $$C_D = \frac{2 \times F_D}{A\rho U^2} = \frac{2 \times 17.34}{0.64 \times 1.25 \times 8.333^2} = \textbf{0.624. Ans.}$$

(*ii*) **Lift Co-efficient (C_L).** Using equation (14.4), we have

$$F_L = C_L \times A \times \frac{\rho U^2}{2}$$

or $$C_L = \frac{2 \times F_L}{A \times \rho \times U^2} = \frac{2 \times 21.264}{0.64 \times 1.25 \times 8.333^2} = \textbf{0.765. Ans.}$$

Problem 14.11 *A kite weighing 0.8 kgf (7.848 N) has an effective area of 0.8 m². It is maintained in air at an angle of 10° to the horizontal. The string attached to the kite makes an angle of 45° to the horizontal and at this position the value of co-efficient of drag and lift are 0.6 and 0.8 respectively. Find the speed of the wind and the tension in the string. Take the density of air as 1.25 kg/m³.*

Solution. Given :

Weight of kite, $W = 0.8 \text{ kgf} = 0.8 \times 9.81 = 7.848$ N

Effective area, $A = 0.8 \text{ m}^2$

Angle made by kite with horizontal = 10°

Angle made by string with horizontal = 45°

$C_D = 0.6$

$C_L = 0.8$

∴ Density of air, $\rho = 1.25 \text{ kg/m}^3$

Let the speed of the wind $= U$ m/s

and tension in string $= T$

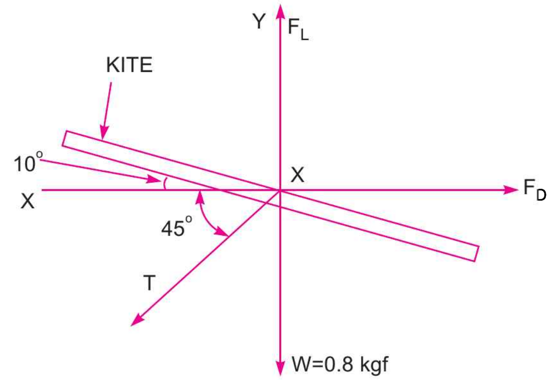

Fig. 14.5

The free body diagram for the kite is shown in Fig. 14.5.

Drag force, F_D = Component of T along X-X = $T \cos 45°$

But drag force F_D is also $= C_D \times A \times \dfrac{\rho U^2}{2} = \dfrac{0.6 \times 0.8 \times 1.25 \times U^2}{2} = 0.3\ U^2$

Equating the two values of F_D, $T \cos 45° = 0.3\ U^2$ ...(*i*)

Now lift force from Fig. 14.5, F_L = Component of T vertically downward + Weight of kite

$= T \sin 45° + 7.848$

Also lift force, $F_L = \dfrac{C_L \times A \times \rho U^2}{2} = \dfrac{0.8 \times 0.8 \times 1.25 \times U^2}{2.0} = 0.4\ U^2$

Equating the two values of F_L, $T \sin 45° + 7.848 = 0.4\ U^2$

or $T \sin 45° = .04076\ U^2 - 7.848$ ...(*ii*)

From equations (*i*) and (*ii*), as $T \cos 45° = T \sin 45°$,

$$0.3\ U^2 = 0.4\ U^2 - 7.848$$

or $7.848 = 0.4\ U^2 - 0.3\ U^2 = 0.1\ U^2$

$\therefore$ $U^2 = \dfrac{7.848}{0.1} = 78.48$ ...(*iii*)

$\therefore$ $U = \sqrt{78.48} = 8.86$ m/s $= \dfrac{8.86 \times 60 \times 60}{1000}$ km/hr = **31.89 km/hr. Ans.**

Substituting the value of $U^2 = 78.48$ given by equation (*iii*) into equation (*i*), we get

$$T \cos 45° = 0.3 \times 78.48 = 23.544$$

$\therefore$ $T = \dfrac{23.544}{\cos 45°}$ = **33.3 N. Ans.**

Problem 14.12 *The air is flowing over a cylinder of diameter 50 mm and infinite length with a velocity of 0.1 m/s. Find the total drag, shear drag and pressure drag on 1 m length of the cylinder if the total drag co-efficient is equal to 1.5 and shear drag co-efficient equal to 0.2. Take density of air = 1.25 kg/cm*^{3}.

Solution. Given :

Diameter of cylinder, $D = 50 \text{ mm} = 0.05 \text{ m}$

Length of cylinder, $L = 1.0 \text{ m}$

∴ Projected Area, $A = L \times D = 1 \times .05 = 0.05 \text{ m}^2$

Velocity of air, $U = 0.1 \text{ m/s}$

Total drag co-efficient, $C_{DT} = 1.5$

Shear drag co-efficient, $C_{DB} = 0.2$

Density of air, $\rho = 1.25 \text{ kg/m}^3$

Total drag is given by, $F_{DT} = C_{DT} \times A \times \dfrac{\rho U^2}{2}$

$$= 1.5 \times .05 \times \frac{1.25 \times (0.1)^2}{2} = \mathbf{0.000468 \text{ N. Ans.}}$$

Shear drag is given by, $F_{DS} = C_{DS} \times A \times \dfrac{\rho U^2}{2}$

$$= 0.2 \times .05 \times \frac{1.25 \times (0.1)^2}{2} = \mathbf{0.0000625 \text{ N. Ans.}}$$

From equation (14.1), Total drag, F_{DT} = Pressure drag + Shear drag

∴ Pressure drag, = Total drag – Shear drag

= 0.000468 – 0.0000625 = **0.0004055 N. Ans.**

Problem 14.13 *A body of length 2.0 m has a projected area 1.5 m*^{2} *normal to the direction of its motion. The body is moving through water, which is having viscosity = 0.01 poise. Find the drag on the body if it has a drag co-efficient 0.5 for a Reynold number of* 8×10^6.

Solution. Given :

Length of body, $L = 2.0 \text{ m}$

Projected Area, $A = 1.5 \text{ m}^2$

Viscosity of water, $\mu = 0.01 \text{ poise} = \dfrac{0.01}{10} = 0.001 \dfrac{\text{Ns}}{\text{m}^2}$

Drag co-efficient, $C_d = 0.5$

Reynold number, $R_e = 8 \times 10^6$

Let the drag force on body $= F_D$

First find the velocity with which body is moving in water. It is calculated from the given Reynold number.

$$R_e = \frac{\rho U L}{\mu}, \text{ where } \rho \text{ for water} = 1000$$

or $$8 \times 10^6 = \frac{1000 \times U \times 2.0}{0.001} = 2 \times 10^6\, U$$

∴ $$U = \frac{8 \times 10^6}{2 \times 10^6} = 4.0 \text{ m/s}$$

Using equation (14.3), $$F_D = C_d \times A \times \frac{\rho U^2}{2} = 0.5 \times 1.5 \times 1000 \times \frac{4.0^2}{2} = \mathbf{6049\ N.\ Ans.}$$

Problem 14.14 *A sub-marine which may be supposed to approximate a cylinder 4 m in diameter and 20 m long travels sub-merged at 1.3 m/s in sea-water. Find the drag exerted on it, if the drag co-efficient for Reynold number greater than 10^5 may be taken as 0.75. The density of sea-water is given as 1035 kg/m³ and kinematic viscosity as .015 stokes.*

Solution. Given :

Dia. of cylinder, $D = 4$ m

Length of cylinder, $L = 20$ m

Velocity of cylinder, $U = 1.3$ m/s

Density of sea-water, $\rho = 1035$ kg/m³

Kinematic viscosity $\nu = 0.015$ stokes $= .015$ cm²/s $= .015 \times 10^{-4}$ m²/s

Let the drag force $= F_D$

Reynold number, $$R_e = \frac{U \times D}{\nu} = \frac{1.3 \times 4.0}{.015 \times 10^{-4}} = 3.466 \times 10^6$$

Since $R_e > 10^5$, hence $C_D = 0.75$

Drag force is given by equation (14.3) as

$$F_D = C_D \times A \times \frac{\rho U^2}{2}$$

where A = projected area of cylinder $= L \times D = 20 \times 4.0 = 80.0$ m²

∴ $$F_D = 0.75 \times 80 \times \frac{1035 \times 1.3^2}{2.0} \text{ kgf} = \mathbf{52472.2\ N.\ Ans.}$$

Problem 14.15 *A jet plane which weighs 29.43 kN and having a wing area of 20 m² flies at a velocity of 950 km/hour, when the engine delivers 7357.5 kW power. 65% of the power is used to overcome the drag resistance of the wing. Calculate the co-efficients of lift and drag for the wing. The density of the atmospheric air is 1.21 kg/m³.*

Solution. Given :

Weight of plane, $W = 29.43$ kN $= 29.43 \times 1000$ N $= 29430$ N

Wing area, $A = 20$ m²

Speed of plane, $$U = 950 \text{ km/hr} = \frac{950 \times 1000}{60 \times 60} = 263.88 \text{ m/s}$$

Engine power, $P = 7357.5$ kW

Power used to overcome drag resistance $= 65\%$ of $7357.5 = \frac{65}{100} \times 7357.5 = 4782.375$ kW

$\therefore$ Density of air, $\rho = 1.21 \text{ kg/m}^3$

Let C_D = Co-efficient of drag and C_L = Co-efficient of lift.

Now power used in kW to overcome drag resistance $= \dfrac{F_D \times U}{1000}$ or $4782.375 = \dfrac{F_D \times 263.88}{1000}$

$$\therefore \quad F_D = \frac{4782.375 \times 1000}{263.88}$$

But from equation (14.3), we have $F_D = C_D \cdot A \cdot \dfrac{\rho U^2}{2}$

$$\therefore \quad \frac{4782.375 \times 1000}{263.88} = C_D \times 20 \times 1.21 \times \frac{263.88^2}{2}$$

$$\therefore \quad C_D = \frac{4782.375 \times 1000 \times 2}{20 \times 1.21 \times 263.88^3} = \mathbf{0.0215.\ Ans.}$$

The lift force should be equal to weight of the plane

$$\therefore \quad F_L = W = 29430 \text{ N}$$

But $F_L = C_L \cdot A \cdot \dfrac{\rho U^2}{2}$ or $29430 = C_L \times 20 \times 1.21 \times \dfrac{263.88^2}{2}$

$$\therefore \quad C_L = \frac{29430 \times 2}{20 \times 1.21 \times 263.88^2} = \mathbf{0.0349.\ Ans.}$$

14.3.2 Pressure Drag and Friction Drag. The total drag on a body is given by equation (14.1) as

Total drag, $F_D = \int p \cos \theta dA + \int \tau_0 \sin \theta dA$ ...(*i*)

where $\int p \cos \theta dA$ = Pressure drag or form drag, and

$\int \tau_0 \sin \theta dA$ = Friction drag or skin drag or shear drag

The relative contribution of the pressure drag and friction drag to the total drag depends on :

(*i*) Shape of the immersed body,

(*ii*) Position of the body immersed in the fluid, and

(*iii*) Fluid characteristics.

Consider the flow of a fluid over a flat plate when the plate is placed parallel to the direction of the flow as shown in Fig. 14.6. In this cos θ, which is the angle made by pressure with the direction motion, will be 90°. Thus the term $\int p \cos \theta dA$ will be zero and hence total drag will be equal to friction drag (or shear drag). If the plate is placed perpendicular to the flow as shown in Fig. 14.7, the angle θ, made by the pressure with the direction of motion will be zero. Hence the term $\int \tau_0 \sin \theta dA$ will become equal to zero and hence total drag will be due to the pressure difference between the upstream and downstream side of the plate. If the plate is held at an angle with the direction of flow, both the terms $\int p \cos \theta dA$ and $\int \tau_0 \sin \theta dA$ will exist and total drag will be equal to the sum of pressure drag and friction drag.

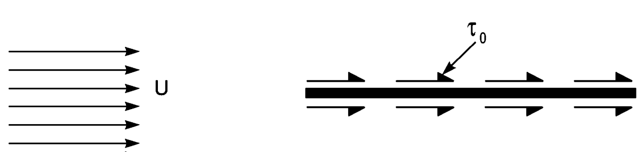

Fig. 14.6 *Flat plate parallel to flow.*

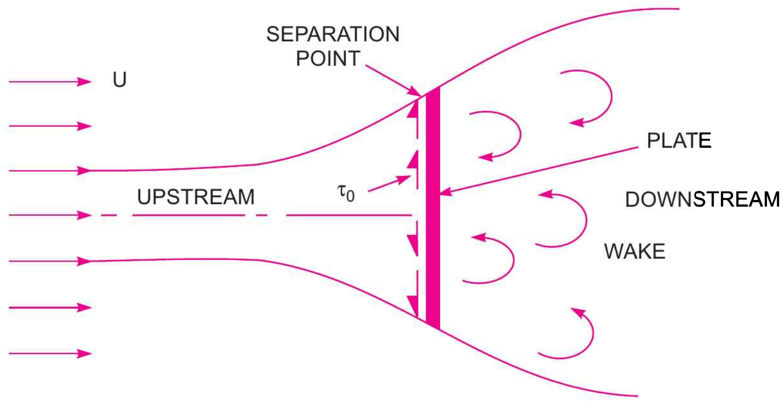

Fig. 14.7 *Flat plate perpendicular to flow.*

14.3.3 Stream-lined Body. A stream-lined body is defined as that body whose surface coincides with the stream-lines, when the body is placed in a flow. In that case the separation of flow will take place only at the trailing edge (or rearmost part of the body). Though the boundary layer will start at the leading edge, will become turbulent from laminar, yet it does not separate upto the rearmost part of the body in the case of stream-lined body. Thus behind a stream-lined body, wake formation zone will be very small and consequently the pressure drag will be very small. Then the total drag on the stream-lined body will be due to friction (shear) only. A body may be stream-lined :

1. at low velocities but may not be so at higher velocities.
2. when placed in a particular position in the flow but may not be so when placed in another position.

14.3.4 Bluff Body. A bluff body is defined as that body whose surface does not coincide with the streamlines, when placed in a flow. Then the flow is separated from the surface of the body much ahead of its trailing edge with the result of a very large wake formation zone. Then the drag due to pressure will be very large as compared to the drag due to friction on the body. Thus the bodies of such a shape in which the pressure drag is very large as compared to friction drag are called bluff bodies.

▶ 14.4 DRAG ON A SPHERE

Consider the flow of a real fluid past a sphere.

Let U = Velocity of the flow of fluid over sphere,
D = Diameter of sphere,
ρ = Mass density of fluid, and
μ = Viscosity of fluid.

If the Reynolds number of the flow is very small less than 0.2 $\left(i.e., R_e = \frac{UD\rho}{\mu} < 0.2\right)$, the viscous forces are much more important than the inertial forces as in this case viscous forces are much more predominate than the inertial forces, which may be assumed negligible. G.G. Stokes, developed a mathematical equation for the total drag on a sphere immersed in a flowing fluid for which Reynolds number is upto 0.2, so that inertia forces may be assumed negligible. According to his solution, total drag is

$$F_D = 3\pi\mu DU. \quad ...(14.8)$$

He further observed that out of the total drag given by equation (14.8), two-third is contributed by skin friction and the remaining one-third by pressure difference. Thus

Skin friction drag, $F_{D_f} = \frac{2}{3} F_D = \frac{2}{3} \times 3\pi\mu DU = 2\pi\mu DU$

and pressure drag, $F_{D_p} = \frac{1}{3} F_D = \frac{1}{3} \times 3\pi\mu DU = \pi\mu DU.$

(*i*) **Expression of C_d for Sphere when Reynolds Number is less than 0.2.** From equation (14.3), the total drag is given by

$$F_D = C_D \times A \times \frac{\rho U^2}{2}$$

For sphere, $F_D = 3\pi\mu DU$

$$A = \text{Projected area of the sphere} = \frac{\pi}{4} D^2$$

$$\therefore \quad 3\pi\mu DU = C_D \times \frac{\pi}{4} D^2 \frac{\rho U^2}{2}$$

$$C_D = \frac{3\pi\mu DU}{\frac{\pi}{4} D^2 \times \frac{\rho U^2}{2}} = \frac{24\mu}{\rho UD} = \frac{24}{R_e} \quad ...(14.9) \quad \left(\because \frac{\mu}{\rho DU} = R_e\right)$$

Equation (14.9) is called **'Stoke's law'.**

(*ii*) **Value of C_D for Sphere when R_e is between 0.2 and 5.** With the increase of Reynolds number, the inertia forces increase and must be taken into account. When R_e lies between 0.2 and 5, Oseen, a Swedish physicist, improved Stoke's law as

$$C_D = \frac{24}{R_e}\left[1 + \frac{3}{16R_e}\right] \quad ...(14.10)$$

Equation (14.10) is called Oseen formulae and is valid for R_e between 0.2 and 5.

(*iii*) **Value of C_D for R_e from 5.0 to 1000.** The drag co-efficient for the Reynolds number from 5 to 1000 is equal to 0.4.

(*iv*) **Value of C_D for R_e from 1000 to 100,000.** In this range, C_D is independent of the Reynolds number and its value is approximately equal to 0.5.

(*v*) **Value of C_D for R_e more than 10^5.** The value of C_D is approximately equal to 0.2 for the Reynolds number more than 10^5.

Problem 14.16 *Calculate the weight of a ball of diameter 80 mm which is just supported in a vertical air stream which is flowing at a velocity of 7 m/s. The density of air is given as 1.25 kg/m³. The kinematic viscosity of air = 1.5 stokes.*

Solution. Given :

Dia. of ball, $D = 80 \text{ mm} = 0.08 \text{ m}$

Velocity of air, $U = 7 \text{ m/s}$

Density of air, $\rho = 1.25 \text{ kg/m}^3$

Kinematic viscosity, $\nu = 1.5 \text{ stokes} = 1.5 \times 10^{-4} \text{ m}^2/\text{s}$

Reynold number, $R_e = \frac{U \times D}{\nu} = \frac{7 \times .08}{1.5 \times 10^{-4}} = 0.373 \times 10^4 = 3730.$

Thus the value of R_e lies between 1000 and 100,000 and hence $C_D = 0.5$.

When the ball is supported in a vertical air stream, the weight of ball is equal to the drag force as shown in Fig. 14.8.

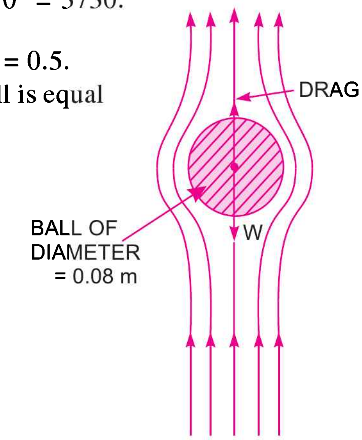

Fig. 14.8

But drag force, $F_D = C_D \times A \times \frac{\rho U^2}{2}$

where A = projected area of ball

$$= \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.08)^2 = 0.005026 \text{ m}^2$$

∴ Drag force, $F_D = 0.5 \times 0.005026 \times \frac{1.25 \times 7^2}{2}$ N

$= 0.07696$ N

∴ Weight of ball $= F_D =$ **0.07696 N. Ans.**

▶ 14.5 TERMINAL VELOCITY OF A BODY

Terminal velocity is defined as the maximum constant velocity of a falling body (such as sphere or a composite body such as parachute together with man) with which the body will be travelling. When the body is allowed to fall from rest in the atmosphere, the velocity of the body increases due to acceleration of gravity. With the increase of the velocity, the drag force, opposing the motion of body also increases. A stage is reached when the upward drag force acting on the body will be equal to the weight of the body. Then the net external force acting on the body will be zero and the body will be travelling at constant speed. This constant speed is called terminal velocity of the falling body.

If the body drops in a fluid, at the instant it has acquired terminal velocity, the net force acting on the body will be zero. The forces acting on the body at this state will be :

1. Weight of body (W), acting downward,
2. Drag force (F_D), acting vertically upward, and
3. Buoyant force (F_B), acting vertically up.

The net force on the body should be zero, *i.e.*, $W = F_D + F_B$ ...(14.11)

Problem 14.17 *A metallic sphere of sp. gr. 7.0 falls in an oil of density 800 kg/m³. The diameter of the sphere is 8 mm and it attains a terminal velocity of 40 mm/s. Find the viscosity of the oil in poise.*

Solution. Given :

Sp. gr. of metallic sphere = 7.0

∴ Density of metallic sphere, $\rho_s = 7 \times 1000 = 7000$ kg/m³

Density of oil, $\rho_o = 800$ kg/m³

Dia. of sphere, $D = 8 \text{ mm} = 8 \times 10^{-3}$ m

Terminal velocity, $U = 40$ mm/s = .04 m/s

Let the viscosity of oil $= \mu$

Weight of sphere, $W = \rho_s \times g \times$ Volume of sphere

$$= 7000 \times 9.81 \times \frac{\pi}{6} D^3$$

$$= 7000 \times 9.81 \times \frac{\pi}{6} \times (8 \times 10^{-3})^3 = 18.4 \times 10^{-3} = 0.0184 \text{ N}$$

Buoyant force on sphere, F_B = Density of oil × g × Volume of sphere

$$= 800 \times 9.81 \times \frac{\pi}{6} (8 \times 10^{-3})^3 \text{ N} = 0.002103 \text{ N.}$$

Drag force, F_D on the sphere is given by equation (14.8) as

$$F_D = 3\pi\mu DU = 3\pi\mu \times 8 \times 10^{-3} \times .04 = .003015\ \mu$$

Using equation (14.11), $W = F_D + F_B$

or $0.0184 = .003015\ \mu + 0.002103$

or $.003015\ \mu = 0.0184 - 0.002103 = 0.016297$

or $\mu = \frac{0.016297}{.003015} = 5.4 \frac{\text{Ns}}{\text{m}^2} = 5.4 \times 10 =$ **54.0 poise. Ans.**

The expression for drag given by equation (14.11) is valid only upto Reynolds number less than 0.2. Hence it is necessary to calculate Reynold number for the flow.

∴ Reynold number, $R_e = \frac{\rho UD}{\mu}$, where ρ for oil = 800 kg/m³

∴ $R_e = 800 \times \frac{.04 \times 8 \times 10^{-3}}{5.4} = 0.0474$

Hence $R_e < 0.2$ and so the expression $F_D = 3\pi\mu DU$ is valid.

Problem 14.18 *A spherical steel ball of diameter 40 mm and of density 8500 kg/m³ is dropped in large mass of water. The co-efficient of drag of the ball in water is given as 0.45. Find the terminal velocity of the ball in water. If the ball is dropped in air, find the increase in terminal velocity of ball. Take the density of air = 1.25 kg/m³ and C_D = 0.1.*

Solution. Given :

Diameter of steel ball, D = 40 mm = 0.04 m

Density of ball, ρ_s = 8500 kg/m³

C_D for ball in water = 0.45

Let the terminal velocity in water = U_1

The forces acting on the spherical ball are :

1. Weight, W = Density of ball × g × Volume of spherical ball

$$= \rho_s \times g \times \frac{\pi}{6} D^3 = 8500 \times 9.81 \times \frac{\pi}{6} (.04)^3 = \mathbf{2.794\ N.}$$

2. Buoyant force, F_B = Density of water × g × Volume of ball

$$= 1000 \times 9.81 \times \frac{\pi}{6} (0.04)^3 = 0.3286 \text{ N.}$$

3. Drag force, $F_D = C_D \times A \times \frac{\rho U^2}{2}$

where A = projected area = $\frac{\pi}{4} D^2 = \frac{\pi}{4} (.04)^2$, ρ = 1000 for water

$$F_D = 0.45 \times \frac{\pi}{4} \times (.04)^2 \times 1000 \times \frac{U_1^2}{2} = 0.2825\ U_1^2$$

Using equation (14.11), we get $W = F_D + F_B$

or $$2.794 = 0.2825\ U_1^2 + 0.3286$$

or $$U_1^2 = \frac{2.794 - 0.3286}{0.2825} = 8.725$$

$$\therefore \quad U_1 = \sqrt{8.725} = \mathbf{2.953\ m/s.\ Ans.}$$

When ball is dropped in air. Let the terminal velocity = U_2

Weight, $W = 2.794$

Buoyant force, F_B = Density of air × g × Volume of ball

$$= 1.25 \times 9.81 \times \frac{\pi}{6}(.04)^3 = 0.000411 \text{ N}$$

Drag force, $$F_D = C_D \times A \times \frac{\rho U^2}{2}, \quad \text{where } \rho \text{ for air} = 1.25$$

$$F_D = 0.1 \times \frac{\pi}{4}(.04)^2 \times 1.25\ \frac{U_2^2}{2} = 0.0000785\ U_2^2.$$

The buoyant force in air is 0.000411, while weight of the ball is 2.794 N. Hence buoyant force is negligible.

∴ For equilibrium of the ball in air, F_D = Weight of ball

or $$0.0000785\ U_2^2 = 2.794 \text{ or } U_2 = \sqrt{\frac{2.794}{0.0000785}} = 188.67 \text{ m/s}$$

∴ Increase in terminal velocity in air = $U_2 - U_1$ = 188.67 – 2.9533 = **185.717 m/s. Ans.**

Problem 14.19 *A metallic ball of diameter 2×10^{-3} m drops in a fluid of sp. gr. 0.95 and viscosity 15 poise. The density of the metallic ball is 12000 kg/m^3. Find :*

(i) The drag force exerted by fluid on metallic ball,

(ii) The pressure drag and skin friction drag,

(iii) The terminal velocity of ball in fluid.

Solution. Given :

Diameter of metallic ball, $D = 2 \times 10^{-3}$ m

Sp. gr. of fluid, $S_0 = 0.95$

∴ Density of fluid, $\rho_0 = 0.95 \times 1000 = 950$ kg/m^3

Viscosity of fluid, $$\mu = 15 \text{ poise} = \frac{15}{10} = 1.5 \frac{\text{Ns}}{\text{m}^2}$$

Density of ball, $\rho_s = 12000$ kg/m^3

The forces acting on the ball are :

Weight of ball, W = Density of ball × g × Volume of ball

$$= 12000 \times 9.81 \times \frac{\pi}{6} D^3$$

$$= 12000 \times 9.81 \times \frac{\pi}{6} \times (2 \times 10^{-3})^3 \text{ N} = 0.000493 \text{ N}$$

Buoyant force, F_B = Density of fluid × g × Volume of ball

$$= 950 \times 9.81 \times \frac{\pi}{6}(2 \times 10^{-3})^3 \text{ N} = 0.000039 \text{ N}$$

When the metallic ball reaches the terminal velocity, equation (14.11) is applicable.

$\therefore$ $W = F_D + F_B$ or $F_D = W - F_B = 0.000493 - 0.000039$
$= \mathbf{0.000454\ N.\ Ans.}$

(*i*) Drag force, $F_D = 0.000454$ N

(*ii*) Pressure drag $= \frac{1}{3} F_D = \frac{1}{3} \times 0.000454 = \mathbf{0.0001513\ N.\ Ans.}$

Skin friction drag $= \frac{2}{3} \times F_D = \frac{2}{3} \times 0.000454 = \mathbf{0.0003028\ N.\ Ans.}$

(*iii*) Let the terminal velocity $= U$

Then drag force (F_D) is given by equation (14.8) as $F_D = 3\pi\mu DU$

But $F_D = 0.000454$ N

Equating the two values of F_D, we have

$$3\pi\mu DU = 0.000454 \quad \text{or} \quad 3\pi \times \frac{15}{10} \times 2 \times 10^{-3} \times U = 0.000454$$

or
$$U = \frac{10 \times 0.000454}{3\pi \times 15 \times 2 \times 10^{-3}} = \mathbf{0.016\ m/s.\ Ans.}$$

Let us check for Reynold number, R_e

$$R_e = \frac{\rho UD}{\mu}, \quad \text{where } \rho = 950 \text{ kg/m}^3$$

$$= 950 \times \frac{0.016 \times 2 \times 10^{-3}}{1.5} = 0.02$$

Hence the Reynolds number is less than 0.2 and so the expression $F_D = 3\pi\mu DU$ for calculating terminal velocity is valid.

Problem 14.20 *Determine the velocity of fall of rain drops of a 30 × 10^{-3} cm diameter, density 0.0012 gm/cm^3 and kinematic viscosity 0.15 cm^2/s.*

Solution. Given :

Diameter of rain drops, $D = 30 \times 10^{-3}$ cm

Density of rain drops, $\rho = 0.0012$ gm/cm^3

Kinematic viscosity, $\nu = 0.15$ cm^2/s

Using the relation, $\nu = \frac{\mu}{\rho}$ or $0.15 = \frac{\mu}{.0012}$

$\therefore$ $\mu = 0.15 \times .0012 = 0.00018 \frac{\text{gm}}{\text{cm sec}}$

Now weight of rain drop $= \rho \times g \times$ Volume of rain drop

$= \rho \times g \times \frac{\pi}{6} D^3$ ($\because$ Rain drop is a sphere)

Drag force, F_D, on rain drop is given by equation (14.8) as $F_D = 3\pi\mu DU$

When rain drop is falling with a uniform velocity U, the drag force must be equal to the weight of rain drop. Hence equating these two values, we get

Weight of rain drop = Drag force

or
$$\rho \times g \times \frac{\pi}{6} D^3 = 3\pi\mu DU \quad \text{or} \quad U = \frac{\rho \times g \times \frac{\pi}{6} \times D^3}{3\pi\mu D} = \frac{\rho g D^2}{18\mu}$$

$$= \frac{0.0012 \times 981 \times (30 \times 10^{-3})^2}{18 \times .00018} = \mathbf{0.327\ cm/s.\ Ans.}$$

Let us check for Reynolds number, R_e

$$R_e = \frac{\rho UD}{\mu} = \frac{UD}{\nu} = \frac{0.327 \times 30 \times 10^{-3}}{0.15} = 0.0654$$

As the Reynolds number is less than 0.2, the expression $F_D = 3\pi\mu DU$ is valid.

▶ 14.6 DRAG ON A CYLINDER

Consider a real fluid flowing over a circular cylinder of diameter D and length L, when the cylinder is placed in the fluid such that its length is perpendicular to the direction of flow. If the Reynolds number of the flow is less than 0.2 $\left(i.e., \frac{U \times d}{\nu} < 0.2\right)$, the inertia force is negligibly small as compared to viscous force and hence the flow pattern about the cylinder will be symmetrical. As the Reynolds number is increased, inertia forces increase and hence they must be taken into consideration for analysis of flow over cylinder. With the increase of the Reynolds number, the flow pattern becomes unsymmetrical with respect to an axis perpendicular to the direction of flow. The drag force, *i.e.*, the force exerted by the flowing fluid on the cylinder in the direction of flow depends upon the Reynolds number of the flow. From experiments, it has been observed that :

(*i*) When Reynolds number (R_e) < 1, the drag force is directly proportional to velocity and hence the drag co-efficient (C_D) is inversely proportional to Reynolds number.

(*ii*) With the increase of the Reynolds number from 1 to 2000, the drag co-efficient decreases and reaches a minimum value of 0.95 at $R_e = 2000$.

(*iii*) With the further increase of the Reynolds number from 2000 to 3×10^4, the co-efficient of drag increases and attains maximum value of 1.2 at $R_e = 3 \times 10^4$.

(*iv*) The value of co-efficient of drag decreases if the Reynolds number is increased from 3×10^4 to 3×10^5. At $R_e = 3 \times 10^5$, the value of $C_D = 0.3$.

(*v*) If the Reynolds number is increased beyond 3×10^6, the value of C_D increases and it becomes equal to 0.7 in the end.

▶ 14.7 DEVELOPMENT OF LIFT ON A CIRCULAR CYLINDER

When a body is placed in a fluid in such a way that its axis is parallel to the direction of fluid flow and body is symmetrical, the resultant force acting on the body is in the direction of flow. There is no force component on the body perpendicular to the direction of flow. But the component to the force on the body perpendicular to the direction of flow, is known as '*Lift*'. Hence in this case lift will be zero.

The lift will be acting on the body when the axis of the symmetrical body is inclined to the direction of flow or body is unsymmetrical. In the case of circular cylinder, the body is symmetrical and the axis is parallel to the direction of flow when cylinder is stationary. Hence the lift will be zero. But if the cylinder is rotated, the axis of the cylinder is no longer parallel to the direction of flow and hence lift will be acting on the rotating cylinder. This is explained by considering the following cases :

14.7.1 Flow of Ideal Fluid over Stationary Cylinder. Consider the flow of an ideal fluid over a cylinder, which is stationary as shown in Fig. 14.9.

Let U = Free stream velocity of fluid

R = Radius of the cylinder

θ = Angle made by any point say C on the circumference of the cylinder with the direction of flow.

The flow pattern will be symmetrical and the velocity at any point say C on the surface of the cylinder is given by $u_\theta = 2U \sin \theta$ ...(14.12)

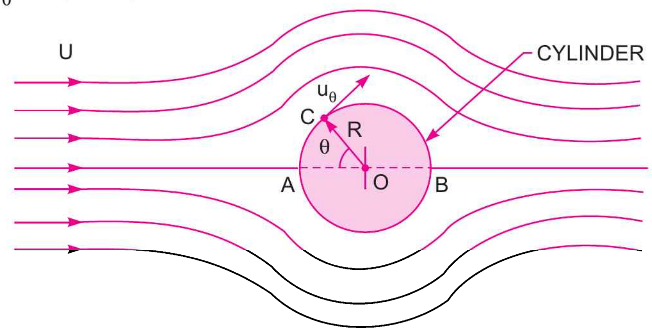

Fig. 14.9 *Flow of ideal fluid over stationary cylinder.*

The velocity distribution over the upper half and lower half of the cylinder from the axis AB of the cylinder are identical and hence the pressure distributions will also be same. Hence the lift acting on the cylinder will be zero.

14.7.2 Flow Pattern Around the Cylinder when a Constant Circulation Γ is Imparted to the Cylinder. Circulation is defined as the flow along a closed curve. Mathematically, the circulation is obtained if the product of the velocity component along the curve at any point and the length of the small element containing that point is integrated around the curve.

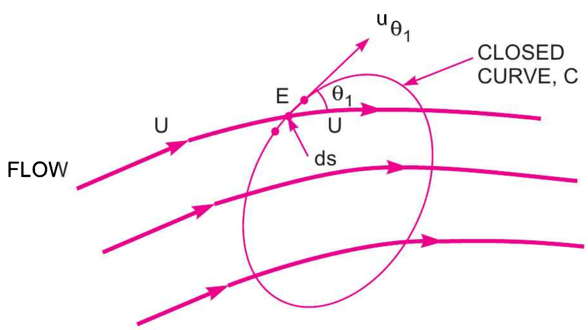

Fig. 14.10 *Circulation.*

Consider a fluid flowing with a free stream velocity equal to U. Within the fluid consider a closed curve as shown in Fig. 14.10. Let E is any point on the closed curve and 'dS' is a small length of the closed curve containing point E.

Let θ_1 = Angle made by the tangent at E with the direction of flow,

u_{θ_1} = Component of free stream velocity along the tangent at E and is given as

$= U \cos \theta_1$

∴ By definition, circulation along the closed curve is

$$\Gamma = \oint \text{velocity component along curve} \times \text{Length of element}$$

$$= \oint U \cos \theta_1 \times dS \qquad \text{...(14.13)}$$

where $\oint$ = Integral for the complete closed curve.

Circulation for the Flow-field in a Free-Vortex. The equation for the free vortex flow is given by

$$u_{\theta_1} \times r = \text{Constant say} = k \qquad \text{...}(i)$$

where u_{θ_1} = velocity of fluid in a free-vortex flow

r = Radius, where velocity is u_{θ_1}.

The flow-pattern for a free-vortex flow consists of streamlines which are series of concentric circles as shown in Fig. 14.11 (*a*).

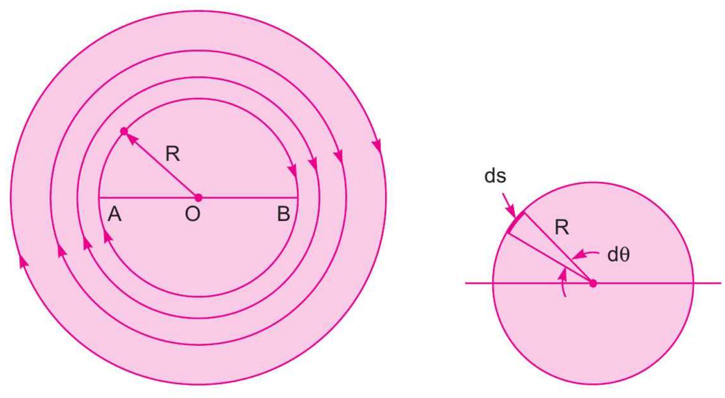

Fig. 14.11 *Stream-lines for free vortex.*

In case of free-vortex flow, the stream velocity at any point on a circle of radius R is equal to the tangential velocity at that point. This means that angle between the stream-lines and tangent on the stream is zero. Also from Fig. 14.11 (*b*), the length of the element 'ds' is given as $ds = Rd\theta$

∴ For a free-vortex flow, $U = u_{\theta_1}$; $\cos \theta_1 = 1$; $ds = Rd\theta$

Substituting these values in equation (14.13), we get the circulation for a free vortex as

$$\Gamma = \oint u_{\theta_1} \times 1 \times Rd\theta$$

But from equation (*i*), for a radius R, we have

$$u_{\theta_1} \times R = K$$

$$\therefore \quad \Gamma = \oint Kd\theta = 2\pi K \qquad \left(\because \oint d\theta = 2\pi\right)$$

$$= 2\pi u_{\theta_1} \times R \qquad (\because K = u_{\theta_1} \times R)$$

$$\therefore \quad u_{\theta_1} = \frac{\Gamma}{2\pi R}. \qquad \text{...(14.14)}$$

Flow over Cylinder due to Constant Circulation. The flow pattern over a cylinder to which a constant circulation (Γ) is imparted is obtained by combining the flow patterns shown in Figs. 14.9 and Fig. 14.11 (*a*). The resultant flow pattern is shown in Fig. 14.12. The velocity at any point on the surface of the cylinder is obtained by adding equations (14.12) and (14.14) as

$$u = u_\theta + u_{\theta_1} = 2U \sin \theta + \frac{\Gamma}{2\pi R}. \qquad ...(14.15)$$

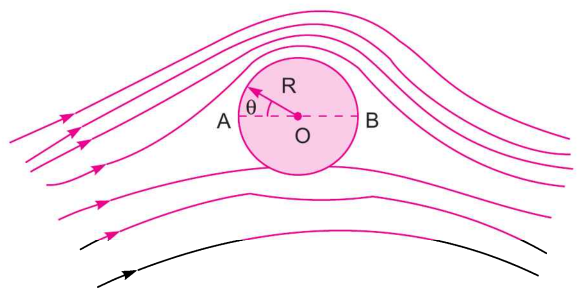

Fig. 14.12 *Flow pattern over a rotating cylinder.*

For the upper half portion of the cylinder, θ varies from 0° to 180° and hence component of velocity, $2U \sin \theta$ is positive. But for the lower half portion of the cylinder, θ varies from 180° to 360°. As sin θ for the values of θ more than 180° and less than 360° is negative and hence component of velocity $2U \sin \theta$ will be negative. This means, the velocity on the upper half portion of the cylinder will be more than the velocity on the lower half portion of the cylinder. But from Bernoulli's theorem we know that at a surface where velocity is less, pressure will be more there and *vice-versa*. Hence on the lower half portion of cylinder, where velocity is less, pressure will be more than the pressure on the upper half portion of the cylinder. Due to this difference of pressure on the two portions of the cylinder, a force will be acting of the cylinder in a direction perpendicular to the direction of flow. This force is nothing but a lift force. Thus by rotating a cylinder at constant velocity in a uniform flow field, a lift force can be developed.

14.7.3 Expression for Lift Force Acting on Rotating Cylinder.

Let a cylinder is rotating in a uniform flow field. The resultant flow pattern will be as shown in Fig. 14.12. Consider a small length of the element on the surface of the cylinder.

Let p_s = Pressure on the surface of the element on cylinder

ds = Length of element

R = Radius of cylinder

$d\theta$ = Angle made by the length ds at the centre of the cylinder as shown in Fig. 14.13.

p = Pressure of the fluid far away from the cylinder

U = Velocity of fluid far away from the cylinder

u_s = Velocity of fluid on the surface of the cylinder.

Applying Bernoulli's equation to a point far away from cylinder and to a point lying on the surface of cylinder such that both the points are on the same horizontal line, we have

$$\frac{p}{\rho g}+\frac{U^2}{2g}=\frac{p_s}{\rho g}+\frac{u_s^2}{2g}$$

$$\therefore \qquad \frac{p_s}{\rho g}=\frac{p}{\rho g}+\frac{U^2}{2g}-\frac{u_s^2}{2g}$$

$$=\frac{p}{\rho g}+\frac{U^2}{2g}\left[1-\frac{u_s^2}{U^2}\right] \qquad \ldots(i)$$

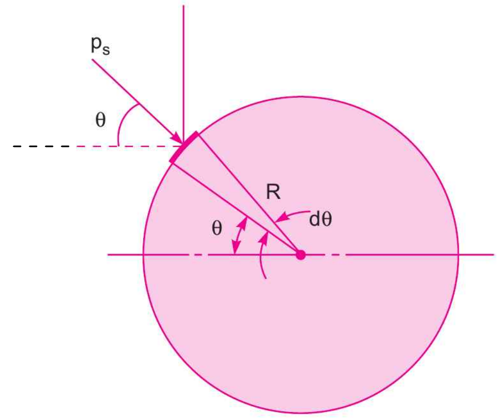

Fig. 14.13 *Lift on a rotating cylinder.*

But the velocity on the surface of the cylinder is given by equation (14.15). Hence

$$u_s=u=2U\sin\theta+\frac{\Gamma}{2\pi R}$$

Substituting this value of u_s in (i), we get

$$\frac{p_s}{\rho g}=\frac{p}{\rho g}+\frac{U^2}{2g}\left[1-\frac{\left(2U\sin\theta+\frac{\Gamma}{2\pi R}\right)^2}{U^2}\right]$$

$$=\frac{p}{\rho g}+\frac{U^2}{2g}\left[1-\frac{\left(4U^2\sin^2\theta+\frac{\Gamma^2}{4\pi^2R^2}+4U\sin\theta\,\frac{\Gamma}{2\pi R}\right)}{U^2}\right]$$

$$=\frac{p}{\rho g}+\frac{U^2}{2g}\left[1-\left(4\sin^2\theta+\frac{\Gamma^2}{4\pi^2R^2U^2}+\frac{4\sin\theta\,\Gamma}{U\times 2\pi R}\right)\right]$$

or

$$p_s=p+\frac{\rho gU^2}{2g}\left[1-4\sin^2\theta-\frac{\Gamma^2}{4\pi^2R^2U^2}-\frac{4\sin\theta\,\Gamma}{U\times 2\pi R}\right] \qquad \ldots(ii)$$

From Fig. 14.13, we have the lift force acting on the small length ds on the element, due to pressure p_s as

= Component of p_s in the direction perpendicular to flow × Area of the element

= $(-p_s\sin\theta)\times(ds\times L)$

Negative sign is taken, as the component of p_s perpendicular to the flow is acting in the downward direction.

Now L = length of the cylinder

$ds=R\times d\theta$

∴ Lift force on the element $=-p_s\sin\theta\times Rd\theta\times L$ $(\because\ ds=Rd\theta)$ ...(iii)

The total force is obtained by integrating equation (iii) over the centre surface of the cylinder.

∴ Total lift, $F_L=\int_0^{2\pi}-p_s\sin\theta\times Rd\theta\times L=\int_0^{2\pi}-p_s\times R\times L\times\sin\theta\,d\theta$

Substituting the value of p_s from equation (ii), we get

$$F_L=\int_0^{2\pi}-\left[p+\frac{\rho gU^2}{2g}\left(1-4\sin^2\theta-\frac{\Gamma}{4\pi^2R^2U^2}-\frac{4\sin\theta\,\Gamma}{U\times 2\pi R}\right)\right]RL\times\sin\theta\,d\theta$$

$$= -RL\int_0^{2\pi}\left[p\sin\theta + \frac{\rho g U^2}{2g}\left(\sin\theta - 4\sin^3\theta - \frac{\Gamma\sin\theta}{4\pi^2R^2U^2} - \frac{4\sin^2\theta\,\Gamma}{U\times 2\pi R}\right)\right]d\theta$$

But $\int_0^{2\pi}\sin\theta\,d\theta = \int_0^{2\pi}\sin^3\theta\,d\theta = 0$

$$\therefore \quad F_L = -R\times L\int_0^{2\pi}\frac{\rho g U^2}{2g}\left(-\frac{4\sin^2\theta\,\Gamma}{U\times 2\pi R}\right)d\theta$$

$$= R\times L\times\frac{\rho g U^2}{2g}\times\frac{4\,\Gamma}{U\times 2\pi R}\int_0^{2\pi}\sin^2\theta\,d\theta = \frac{L}{g}\frac{\rho g U\Gamma}{\pi}\int_0^{2\pi}\sin^2\theta\,d\theta$$

But $\int_0^{2\pi}\sin^2\theta\,d\theta = \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi} = \left(\frac{2\pi}{2} - \frac{\sin 4\pi}{4}\right) = \pi$

$$\therefore \quad F_L = \frac{L}{g}\frac{\rho g}{\pi}U\Gamma\times\pi = \frac{L}{g}\rho g U\Gamma = \frac{\rho g}{g}LU\Gamma = \rho LU\Gamma \qquad ...(14.16)$$

Equation (14.16) is known as Kutta-Joukowski equation.

14.7.4 Drag Force Acting on a Rotating Cylinder. The resultant flow pattern for a rotating cylinder in a uniform flow field is shown in Fig. 14.12. The resultant flow pattern is symmetrical about the vertical axis of the cylinder. Hence the velocity distribution and also pressure distribution is symmetrical about the vertical axis and as such there will be no drag on the cylinder.

14.7.5 Expression for Lift Co-efficient for Rotating Cylinder. The lift co-efficient is defined by the equation (14.4) as

$$F_L = C_L A\,\frac{\rho U^2}{2} \qquad ...(i)$$

where C_L = Lift co-efficient, A = Projected area

U = Free stream velocity or uniform velocity of flow.

For a rotating cylinder, the lift force is given by equation (14.16)

$F_L = \rho LU\Gamma$

A = Projected area of cylinder = $2RL$

$\therefore$ Substituting these values in equation (*i*), we get

$$\rho LU\Gamma = C_L\times 2RL\times\frac{\rho U^2}{2} \quad\text{or}\quad C_L = \frac{\rho LU\Gamma}{RL\rho U^2} = \frac{\Gamma}{RU} \qquad ...(14.17)$$

From equation (14.14), we have $u_{\theta_1} = \frac{\Gamma}{2\pi R}$ or $\frac{\Gamma}{R} = 2\pi u_{\theta_1}$

Substituting this value of $\frac{\Gamma}{R}$ in equation (14.17), C_L is also expressed

$$C_L = \frac{2\pi u_{\theta_1}}{U} \qquad ...(14.18)$$

where u_{θ_1} = Velocity of rotation of the cylinder in the tangential direction.

14.7.6 Location of Stagnation Points for a Rotating Cylinder in a Uniform Flow-field. Stagnation points are those points on the surface of the cylinder, where velocity is zero. For a rotating cylinder as shown in Fig. 14.12, the resultant velocity is given by equation (14.15) as

$$u = 2U \sin\theta + \frac{\Gamma}{2\pi R}.$$

For stagnation point, $u = 0$

$$\therefore \quad 2U \sin\theta + \frac{\Gamma}{2\pi R} = 0 \text{ or } 2U \sin\theta = -\frac{\Gamma}{2\pi R}$$

or

$$\sin\theta = -\frac{\Gamma}{4\pi UR}. \qquad ...(14.19)$$

The solution of equation (14.19) gives the location of stagnation points on the surface of the cylinder. There are two values of θ, which satisfy equation (14.19). As sin θ is negative in equation (14.19), it means θ is more than 180° but less than 360°. The two values of θ are such that one value is between 180° and 270° and other value is between 270° and 360°.

For a single stagnation point, θ = 270° and then equation (14.19) becomes as

$$\sin 270° = -\frac{\Gamma}{4\pi UR} \text{ or } -1 = -\frac{\Gamma}{4\pi UR} \qquad (\because \sin 270° = -1)$$

$$\therefore \quad \Gamma = 4\pi UR. \qquad ...(14.20)$$

14.7.7 Magnus Effect. When a cylinder is rotated in a uniform flow, a lift force is produced on the cylinder. This phenomenon of the lift force produced by a rotating cylinder in a uniform flow is known as Magnus Effect. This fact was investigated by a German physicist H.G. Magnus and hence the name is given as Magnus Effect.

Problem 14.21 *A cylinder rotates at 150 r.p.m. with its axis perpendicular in an air stream which is having uniform velocity of 25 m/s. The cylinder is 1.5 m in diameter and 10 m long. Assuming ideal fluid theory, find (i) the circulation, (ii) lift force, and (iii) position of stagnation points. Take density of air as 1.25 kg/m*^{3}.

Solution. Given :

Speed of cylinder, $N = 150$ r.p.m.

Velocity of air, $U = 25$ m/s

Diameter of cylinder, $D = 1.5$ m

∴ Radius of cylinder, $R = \frac{D}{2} = \frac{1.5}{2} = 0.75$ m

Length of cylinder, $L = 10$ m

Density of air, $\rho = 1.25$ kg/m^3

Tangential velocity of cylinder is given as $u_\theta = \frac{\pi DN}{60} = \frac{\pi \times 1.5 \times 150}{60} = 11.78$ m/s.

(*i*) Circulation (Γ) is obtained from equation (14.14), as $u_\theta = \frac{\Gamma}{2\pi R}$

$$\therefore \quad \Gamma = 2\pi R \times u_\theta = 2\pi \times 0.75 \times 11.78 = \mathbf{55.51\ m^2/s}. \quad \textbf{Ans.}$$

(*ii*) Lift force, F_L is given by equation (14.16) as

$$F_L = \rho L U \Gamma = 1.25 \times 10 \times 25 \times 55.51$$

$$= \mathbf{17344\ N}. \quad \textbf{Ans.}$$

(*iii*) Position of stagnation points are given by equation (14.19) as

$$\sin\theta = -\frac{\Gamma}{4\pi UR} = -\frac{55.51}{4\pi \times 25 \times 0.75} = 0.2356$$

$$= -\sin(13.62°)$$

$$= \sin[180° + 13.62°] \text{ and } \sin[360° - 13.62°]$$

$$\therefore \quad \theta = (180° + 13.62°) \text{ and } (360° - 13.62°)$$

$$= \mathbf{193.62° \text{ and } 346.38°.} \quad \textbf{Ans.}$$

The location of stagnation points are shown in Fig. 14.14.

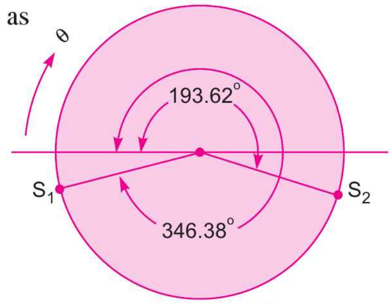

Fig. 14.14

Problem 14.22 *A cylinder whose axis is perpendicular to the stream of air having a velocity of 20 m/s, rotates at 300 r.p.m. The cylinder is 2 m in diameter and 10 m long. (a) Find : (i) the circulation, (ii) theoretical lift force per unit length, (iii) position of stagnation points, and (iv) the actual lift, drag and direction of resultant force. Take density of air 1.24 kg/m³. For actual drag and lift, take C_L = 3.4, C_D = 0.65 and $\frac{u_\theta}{U}$ = 1.57. (b) Find the speed of rotation of the cylinder which will give only a single stagnation point.*

Solution. Given :

Velocity of air,	$U = 20$ m/s
Speed of rotation,	$N = 300$ r.p.m.
Diameter of cylinder,	$D = 2$ m
Length of cylinder,	$L = 10$ m
Density of air,	$\rho = 1.24$ kg/m³

Tangential velocity of cylinder is given as $u_\theta = \frac{\pi DN}{60} = \frac{\pi}{60} \times 2.0 \times 300 = 31.42$ m/s.

(*a*) (*i*) Now the circulation (Γ) is given by equation (14.14) as $u_\theta = \frac{\Gamma}{2\pi R}$

$$\therefore \quad \Gamma = 2\pi R u_\theta = 2\pi \times \frac{D}{2} \times 31.42$$

$$= 2\pi \times \frac{3}{2} \times 31.42 = \mathbf{197.41\ m^2/s.\ Ans.}$$

(*ii*) The theoretical lift (F_L) is given by equation (14.16) as

$$F_L = \rho UL\Gamma = 1.24 \times 20 \times 10 \times 197.41 = 48957.7 \text{ N}$$

$$\therefore \quad \text{Theoretical lift per unit length} = \frac{F_L}{L} = \frac{48957.7}{10} = \mathbf{4895.77\ N/m.\ Ans.}$$

(*iii*) Position of stagnation points are obtained from equation (14.19) as

$$\sin\theta = -\frac{\Gamma}{4\pi UR} = -\frac{197.41}{4\pi \times 20 \times D/2} = \frac{197.41}{4\pi \times 20 \times 1} = -0.7854$$

$$= \sin[180° + 51.75°] \text{ and } \sin[360° - 51.75°]$$

$$= \sin[231.75°] \text{ and } \sin[308.25°]$$

$$\therefore \quad \theta = \mathbf{231.75° \text{ and } 308.25°.\ Ans.}$$

Stagnation points will be at an angle of 231.75° and 308.25°.

(*iv*) *Actual Lift, Drag and Direction of Resultant Force*

For actual lift and drag, given $\frac{u_\theta}{U} = 1.57$, $C_L = 3.4$ and $C_D = 0.65$.

The ratio of $\frac{u_\theta}{U}$ from theoretical consideration is given as $\frac{u_\theta}{U} = \frac{31.42}{20} = 1.57$

Now actual lift force is given by $F_L = \frac{1}{2}\,\rho A U^2 \times C_L$

$$= \frac{1}{2} \times 1.24 \times (2 \times 10) \times 20^2 \times 3.4 = 16864 \text{ N}$$

where A = projected area of cylinder = 2×10 m^2

$\therefore$ F_L = **16864 N. Ans.**

Actual drag force, $F_D = \frac{1}{2}\,\rho A U^2 \times C_D = \frac{1}{2} \times 1.24 \times (2 \times 10) \times 20^2 \times 0.65 =$ **3224 N. Ans.**

$\therefore$ Resultant force $= \sqrt{F_L^2 + F_D^2} = \sqrt{16864^2 + 3224^2}$

$= \sqrt{284394496 + 10394176}$ = **17169.4 N. Ans.**

The direction of the resultant force with the horizontal is given by

$$\tan\theta = \frac{F_L}{F_D} = \frac{16864}{3224} = 5.23$$

$\therefore$ $\theta = \tan^{-1} 5.23$ = **79.1°. Ans.**

(*b*) Speed of rotation of the cylinder for single stagnation point.

For a single point stagnation, the equation (14.20) is used.

$\therefore$ $\Gamma = 4\pi UR = 4\pi \times 20 \times 1 = 251.32$ m^2/s $\left(\because\ R = \frac{D}{2} = \frac{2}{2} = 1 \text{ m}\right)$

The speed of rotation, corresponding to the circulation = 251.32 is given by equation (14.14) as

$$u_\theta = \frac{\Gamma}{2\pi R} = \frac{251.32}{2\pi \times 1} = 40.0$$

But $u_\theta = \frac{\pi D N}{60}$

$\therefore$ $N = \frac{60 \times u_\theta}{\pi \times D} = \frac{60 \times 40}{\pi \times 2.0} = 381.97$ r.p.m. $\simeq$ **382.0 (say) r.p.m. Ans.**

Problem 14.23 *The air having a velocity of 40 m/s is flowing over a cylinder of diameter 1.5 m and length 10 m, when the axis of the cylinder is perpendicular to the air stream. The cylinder is rotated about its axis and a lift of 6867 N per metre length of the cylinder is developed. Find the speed of rotation and location of the stagnation points. The density of air is given as 1.25 kg/m*^{3}.

Solution. Given :

Velocity of air, $U = 40$ m/s

Diameter of cylinder, $D = 1.5$ m

Length of the cylinder, $L = 10$ m

Lift/metre length, $\frac{F_L}{L} = 6867$ N

Density of air, $\rho = 1.25 \text{ kg/m}^2$

From equation (14.16), we have $F_L = \rho LU\Gamma$ or $\frac{F_L}{L} = \rho U\Gamma$

$$\therefore \quad 6867 = 1.25 \times 40 \times \Gamma$$

$$\therefore \quad \Gamma = \frac{6867}{1.25 \times 40} = 137.36 \text{ m}^2\text{/s.}$$

Let the speed of rotation corresponding to circulation 137.36 = u_θ. Using equation (14.14),

$$u_\theta = \frac{\Gamma}{2\pi R} = \frac{137.36}{2\pi \times \frac{D}{2}} = \frac{137.36 \times 2}{2\pi \times 1.5} = 29.15 = \frac{\pi DN}{60}$$

$$\therefore \quad N = \frac{60 \times 29.15}{\pi \times D} = \frac{60 \times 29.15}{\pi \times 1.5} = \textbf{371.15 r.p.m. Ans.}$$

Position of stagnation points are given by equation (14.19)

$$\sin\theta = -\frac{\Gamma}{4\pi UR} = -\frac{137.36}{4\pi \times 40 \times \frac{D}{2}} = -\frac{137.36 \times 2}{4\pi \times 53 \times 1.5}$$

$$= -0.3643 = -\sin 21.36^\circ$$

$$= \sin(180^\circ + 21.36^\circ) \text{ and } \sin[360^\circ - 21.36^\circ]$$

$$= \sin 201.36^\circ \text{ and } \sin 338.64^\circ$$

$$\therefore \quad \theta = \textbf{201.36° and 338.64°. Ans.}$$

▶ 14.8 DEVELOPMENT OF LIFT ON AN AIRFOIL

Fig. 14.15 shows the two shapes of the airfoils, which are stream-lines bodies which may be symmetrical or unsymmetrical in shapes. The airfoil is characterized by its chord length C, angle of attack α (which is the angle between the direction of the fluid flowing and chord line) and span L of the airfoil. The lift on the airfoil is due to negative pressure created on the upper part of the airfoil. The drag force on the airfoil is always small due to the design of the shape of the body, which is stream-lined.

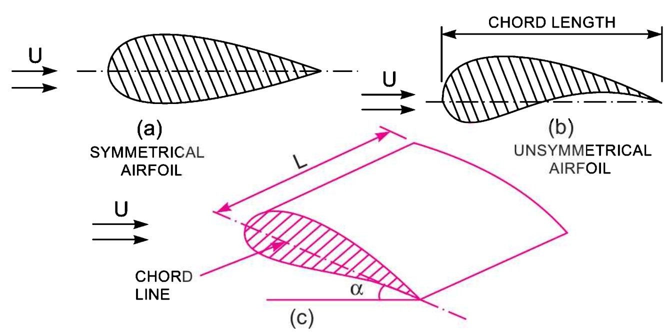

Fig. 14.15 *Shapes of airfoils.*

From the theoretical analysis, the circulation Γ developed on the airfoil so that the stream-line at the trailing edge of the airfoil is tangential to the airfoil, is given as

$$\Gamma = \pi C U \sin \alpha \qquad \text{...(14.21)}$$

where C = Chord length, U = Free stream velocity of airfoil, α = Angle of attack

Lift force F_L is given by equation (14.16) as

$$F_L = \rho U L \Gamma = \rho U L \times \pi C U \sin \alpha \qquad (\because \Gamma = \pi C U \sin \alpha)$$

$$= \pi \rho C U^2 L \sin \alpha \qquad \text{...(14.22)}$$

The lift force is also given by equation (14.4), as

$$F_L = C_L \times A \times \frac{\rho U^2}{2}$$

where C_L = Co-efficient of lift

A = Projected area = $C \times L$ for airfoil

$$\therefore \quad F_L = C_L \times C \times L \times \frac{\rho U^2}{2} \qquad \text{...(14.23)}$$

Equating the two value of lift force given by equations (14.22) and (14.23), we get

$$\pi \rho C U^2 L \sin \alpha = C_L \times C \times L \times \frac{\rho U^2}{2}$$

$$\therefore \quad C_L = \frac{2\pi \rho C U^2 L \sin \alpha}{C \times L \times \rho U^2} = 2\pi \sin \alpha \qquad \text{...(14.24)}$$

Thus it is clear from equation (14.23), that co-efficient of lift depends upon the angle of attack.

14.8.1 Steady-state of a Flying Object. When a flying object for example airplane is in a steady-state, the weight of the airplane is equal to the lift force and thrust developed by the engine is equal to the drag force. Hence

$$W = \text{Lift force} = C_L \frac{\rho A U^2}{2} \qquad \text{...(14.25)}$$

where W = Weight of the airplane and $C_L \frac{\rho A U^2}{2}$ = Lift force.

Problem 14.24 *An airfoil of chord length 2 m and of span 15 m has an angle of attack as 6°. The airfoil is moving with a velocity of 80 m/s in air whose density is 1.25 kg/m³. Find the weight of the airfoil and the power required to drive it. The values of co-efficient of drag and lift corresponding to angle of attack are given as 0.03 and 0.5 respectively.*

Solution. Given :

Chord length, $C = 2$ m

Span of airfoil, $L = 15$ m

Angle of attack, $\alpha = 6°$

Velocity of airfoil, $U = 80$ m/s

Density of air, $\rho = 1.25$ kg/m^3

Co-efficient of drag, $C_D = 0.03$

Co-efficient of lift, $C_L = 0.50$

From equation (14.25), we know that

$$\text{Weight of airfoil} = \text{Lift force} = C_L \frac{\rho A U^2}{2} = 0.50 \times 1.25 \times (C \times L) \times \frac{80^2}{2}$$

$$= 0.50 \times 1.25 \times (2 \times 15) \times \frac{80^2}{2} = \mathbf{60000\ N.\ Ans.}$$

Now drag force, $$F_D = C_D \times \rho \times \frac{AU^2}{2}$$

$$= 0.03 \times 1.25 \times \frac{(2 \times 15) \times 80^2}{2} = \mathbf{3600\ N.\ Ans.}$$

∴ Power required in kW $$= \frac{F_D \times U}{1000} = \frac{3600 \times 80}{1000} = \mathbf{288\ kW.\ Ans.}$$

Problem 14.25 *A jet plane which weighs 29430 N and has a wing area of 20 m² flies at a velocity of 250 km/hr. When the engine delivers 7357.5 kW. 65% of the power is used to overcome the drag resistance of the wing. Calculate the co-efficient of lift and co-efficient of drag for the wing. Take density of air equal to 1.21 kg/m³.*

Solution. Given :

Weight of plane, $W = 29430$ N

Wing area, $A = 20\ m^2$

Velocity of plane, $$U = 250 \text{ km/hr} = \frac{250 \times 1000}{60 \times 60} \text{ m/s} = 69.44 \text{ m/s}$$

Power delivered by engine = 7357.5 kW

Power required to overcome drag resistance

= 65% of 7357.5 = 0.65 × 7357.5 = 4782.375 kW.

Density of air, $\rho = 1.21\ kg/m^3$

Now, weight of plane $$= \text{Lift force} = C_L \times A \times \frac{\rho U^2}{2}$$

∴ $$29430 = C_L \times 20 \times 1.21 \times \frac{69.44^2}{2}$$

∴ $$C_L = \frac{29430 \times 2}{20 \times 1.21 \times 69.44^2} = \mathbf{0.5046.\ Ans.}$$

Let F_D = Drag force

Power required to overcome drag resistance $$= \frac{F_D \times U}{1000} \text{ kW}$$

∴ $$4782.375 = \frac{F_D \times 69.44}{1000}$$

∴ $$F_D = \frac{4782.375 \times 1000}{69.44} = 68870.6 \text{ N}$$

Now drag force, $$F_D = C_D \times A \times \frac{\rho U^2}{2}$$

∴ $$68870.6 = C_D \times 20 \times \frac{1.21 \times 69.44^2}{2}$$

∴ $$C_D = \frac{68870.6 \times 2}{20 \times 1.21 \times 69.44^2} = \mathbf{1.18.\ Ans.}$$

HIGHLIGHTS

1. The force exerted by a fluid on a solid body immersed in the fluid in the direction of motion is called drag force while the force perpendicular to the direction of motion, on the body is known as lift force.

2. The mathematical expression for the drag and lift force are,

$$F_L = C_D A \frac{\rho U^2}{2} \; ; F_L = C_L A \frac{\rho U^2}{2}$$

where C_D = Co-efficient of drag, C_L = Co-efficient of lift,
A = Projected area of the body, ρ = Density of fluid,
U = Free-stream velocity of fluid.

3. The resultant force exerted by fluid on solid body is $F_R = \sqrt{F_D^2 + F_L^2}$.

4. Total drag on a body is the sum of pressure drag and friction drag.

5. A body whose surface coincides with the stream-lines, when the body is placed in a flow is called stream-lined body. If the surface of the body does not coincide with the stream-lines, the body is called bluff body.

6. The drag on a sphere for Reynolds number less than 0.2 is given by $F_D = 3\pi\mu DU$

Out of this total drag, Skin friction drag $= \frac{2}{3} \times 3\pi\mu DU = 2\pi\mu DU$

and pressure drag $= \frac{1}{3} \times 3\pi\mu DU = \pi\mu DU$.

7. Values of C_D for sphere for different Reynold number is

$$C_D = \frac{24}{R_e} \text{ ... when } R_e < 0.2$$

$$= \frac{24}{R_e}\left[1 + \frac{3}{16R_e}\right] \text{ ..... when } 0.2 < R_e < 5.0$$

$= 0.4$ ..... when R_e lies between 5 and 1000

$= 0.5$ ..... when R_e lies between 100 and 100000

$= 0.2$ ..... when $R_e > 10^5$.

8. Terminal velocity is defined as the maximum constant velocity of a falling body with which it will travel. At the terminal velocity, the weight of the body is equal to the drag force plus the buoyant force. Hence $W = F_D + F_B$.

9. The velocity of ideal fluid at any point on the surface of the cylinder is given by

$$u_\theta = 2U \sin\theta$$

where u_θ = Tangential velocity on the surface of the cylinder
U = Uniform velocity or free stream velocity
θ = Angle made by any point on the surface of the cylinder with the direction of flow.

10. Circulation is the flow along a closed curve and is obtained when the product of velocity along the closed curve and length of the small element is integrated around the curve. Circulation for free vortex at any radius R is given by

$$\Gamma = 2\pi R \times u_\theta.$$

11. The resultant velocity on a circular cylinder which is rotated at constant speed in uniform flow-field is

$$u = 2U \sin\theta + \frac{\Gamma}{2\pi R}.$$

12. When a circular cylinder is rotated in a uniform flow, a lift force is produced on the cylinder. The magnitude of the lift force F_L is given by

$$F_L = \rho L U \Gamma$$

where L = Length of cylinder, U = Free stream velocity, Γ = Circulation.

13. The expression for lift co-efficients for a rotating cylinder in a uniform flow is given by

$$C_L = \frac{\Gamma}{RU} \quad \text{...(in term of circulation)}$$

$$= \frac{2\pi u_\theta}{U} \quad \text{...(in term of tangential speed)}$$

14. The location of stagnation points is given by $\sin\theta = -\dfrac{\Gamma}{4\pi UR}$

where R = Radius of cylinder, U = Free stream velocity.

15. For a single stagnation point, the condition is

$$\Gamma = 4\pi UR \quad \text{...(in terms of circulation)}$$

or $$u_\theta = 2U \quad \text{...(in terms of tangential velocity)}$$

16. Circulation developed on the airfoil is given by

$$\Gamma = \pi CU \sin\alpha$$

where C = Chord length, U = Velocity of airfoil, α = Angle of attack.

17. The expression for co-efficient of lift for an airfoil is $C_L = 2\pi \sin\alpha$.

18. When an airplane is in steady-state,

Weight of plane = Lift force

Thrust by engine = Drag force.

EXERCISE

(A) THEORETICAL PROBLEMS

1. Define the terms : drag and lift.
2. What do you understand by : Total drag on a body, resultant force on a body, co-efficient of drag and co-efficient of lift.
3. Differentiate between (*i*) stream-lines body and bluff body, (*ii*) Friction drag and pressure drag.
4. (*a*) What is the expression for the drag on a sphere, when the Reynolds number of the flow is upto 0.2 ? Hence prove that the co-efficient of drag for sphere for this range of the Reynolds number is given by

$$C_D = \frac{24}{R_e}, \text{ where } R_e = \text{Reynolds number.}$$

(*b*) Draw C_D versus R_e diagram for a sphere and explain why C_D suddenly drops at $R_e = 3 \times 10^3$.

(*c*) Draw pressure distribution diagrams in dimensionless form for flow past sphere when fluid has no viscosity, when $R_e = 10^4$ and when $R_e = 10^6$.
5. What do you mean by 'Terminal velocity of a body' ? What is the relation between the weight of the body, drag force on the body and buoyant force when the body has acquired terminal velocity ?
6. What is circulation ? Find an expression for circulation for a free-vortex of radius R.
7. Obtain an expression for the lift produced on a rotating cylinder placed in a uniform flow field such that the axis of the cylinder is perpendicular to the direction of flow.
8. What is Magnus effect ? Why is it known as Magnus effect ?

9. Prove that the co-efficient of lift for a rotating cylinder placed in a uniform flow is given by

$$C_L = \frac{\Gamma}{RU}$$

where Γ = Circulation, R = Radius of cylinder, U = Free-stream velocity.
10. Define stagnation points. How the position of the stagnation points for a rotating cylinder in a uniform flow is determined ? What is the condition for single stagnation point ?
11. Define the terms : Airfoil, chord length, angle of attack, span of an airfoil.
12. If the circulation developed on an airfoil is equal to $\pi CU \sin \alpha$, then prove that co-efficient of lift for airfoil is given by $C_L = 2\pi \sin \alpha$, where α = angle of attack.
13. Explain the terms : (*i*) Friction drag, (*ii*) Pressure drag and profile drag.
14. (*a*) How are drag and lift forces caused on a body immersed in a moving fluid ?
(*b*) What is the drag force on a sphere in the stoke range ?

(B) NUMERICAL PROBLEMS

1. A flat plate 2 m × 2 m moves at 40 km/hour in stationary air of density 1.25 kg/m^3. If the co-efficient of drag and lift are 0.2 and 0.8 respectively, find : (*i*) the lift force, (*ii*) the drag force, (*iii*) the resultant force, and (*iv*) the power required to keep the plate in motion.
[**Ans.** (*i*) 246.86 N, (*ii*) 61.715 N, (*iii*) 254.4 N, (*iv*) 0.684 kW]
2. Find the drag force difference on a flat plate of size 1.5 m × 1.5 m when the plate is moving at a speed of 5 m/s normal to its plate first in water and second in air of density 1.24 kg/m^3. Co-efficient of drag is given as 1.10. [**Ans.** 30899 N]
3. A truck having a projected area of 12 square metres travelling at 60 km/hr has a total resistance of 2943 N. Of this 25% is due to rolling friction and 15% is due to surface fraction. The rest is due to form drag. Calculate the coefficient of form drag if the density of air = 1.25 kg/m^3. [**Ans.** 0.847]
4. A circular disc 4 m in diameter is held normal to 30 m/s wind of density 1.25 kg/m^3. If the co-efficient of drag of disc = 1.1, what force is required to hold the disc at rest ? [**Ans.** 7775.4 N]
5. Find the diameter of a parachute with which a man of mass 80 kg descends to the ground from an aeroplane against the resistance of air, with a velocity of 25 m/s. Take C_d = 0.5 and density of air = 1.25 kg/m^3. [**Ans.** 2.26 m]
6. A man descends to the ground with the help of a parachute from an aeroplane against the resistance of air with a uniform velocity of 10 m/s. The parachute is hemispherical in shape and is having diameter of 5 m. Find the weight of man if C_d = 0.5 and density of air = 1.25 kg/m^3. [**Ans.** 613.52 N]
7. A kite 60 cm × 60 cm weighing 2.943 N assumes an angle of 10° to the horizontal. The string attached to the kite makes an angle of 45° to the horizontal. If the pull on the string is 29.43 N when the wind is flowing at a speed of 40 km/hr. Find the corresponding co-efficient to drag and lift. Density of air is given as 1.25 kg/m^3. [**Ans.** C_D = 0.7489, C_L = .8548]
8. The air is flowing over a cylinder of diameter 100 mm and of infinite length with a velocity of 150 mm/s. Find the total drag, shear drag and pressure drag on 1 m length of the cylinder if the total drag co-efficient = 1.5 m and shear drag co-efficient = 0.25. The density of air is given as = 1.25 kg/m^3.
[**Ans.** 0.00211 N, 0.000351 N, 0.001756 N]
9. A body of length 2.5 m has a projected area 1.8 m^2 normal to the direction of its motion. The body is moving through water with a velocity such that the Reynold number = 6×10^6 and the drag co-efficient = 0.5. Find the drag on the body. Take viscosity of water = 0.01 poise. [**Ans.** 2592 N]
10. Calculate the weight of a ball of diameter 50 mm which is just supported in a vertical air stream which is flowing at a velocity of 10 m/s. The density of air = 1.25 kg/m^3 and kinematic viscosity = 1.5 stokes.
[**Ans.** 0.0613 N]

11. A metallic sphere of sp. gr. 8.0 falls in an oil of density 800 kg/m^3. The diameter of the sphere is 10 mm and it attains a terminal velocity of 50 mm/s. Find the viscosity of the oil in poise. [**Ans.** 78.48 poise]

12. A metallic ball of diameter 5 mm drops in a fluid of sp. gr. 0.8 and viscosity 30 poise. The specific gravity of the metallic ball, is 9.0. Find : (*i*) the drag force exerted by fluid on metallic ball, (*ii*) the pressure drag and skin friction drag, and (*iii*) terminal velocity of ball in fluid.
[**Ans.** (*i*) 0.005264 N, (*ii*) 0.001754 N, 0.003527 N, (*iii*) 3.7 cm/s]

13. A cylinder rotates at 200 r.p.m. with its axis perpendicular in an air stream which is having uniform velocity of 20 m/s. The cylinder is 2 m in diameter and 8 m long. Assuming ideal fluid theory, find (*i*) the circulation, (*ii*) lift force, and (*iii*) position of stagnation points. Take density of air as 1.25 kg/m^3.
[**Ans.** (*i*) 131.57 m^2/s, (*ii*) 26309.4 N, (*iii*) $\theta = 211.56°$ and 328.44°]

14. For the problem 13, find the speed of rotation of the cylinder which will give only a single stagnation point. [**Ans.** 381.97 r.p.m.]

15. The air having a velocity of 30 m/s is flowing over a cylinder of diameter 1.4 m and length 10 m, when the axis of the cylinder is perpendicular to the air stream. The cylinder is rotated about its axis and a total lift of 58860 N is produced. Find the speed of rotation and location of the stagnation points. The density of air is given as 1.25 kg/m^3. [**Ans.** $N = 486.87$ r.p.m., $\theta = 216.5°$ and 323.5°]

16. A jet plane which weighs 19620 N has a wing area of 25 m^2. It is flying at a speed of 200 km per hour. When the engine develops 588.6 kW, 70% of this power is used to overcome the drag resistance of the wing. Calculate the co-efficient of lift and co-efficient of drag for the wing. Taken density of air as 1.25 kg/m^3. [**Ans.** $C_L = .407$, $C_D = .114$]

17. Experiments were conducted in a wind tunnel with a wind speed of 50 km/hour on a flat plate of size 2 m long and 1 m wide. The density of air is 1.15 kg/m^3. The plate is kept at such an angle that co-efficients of lift and drag are 0.75 and 0.15 respectively. Determine : (*i*) lift force, (*ii*), drag force, (*iii*) resultant force, (*iv*) its direction, and (*v*) power exerted by the air stream on the plate.
[**Ans.** (*i*) 166.4 N, (*ii*) 33.28 N, (*iii*) 169.64 N, (*iv*) 78.7°, (*v*) 0.461 kW]