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Free labor mobility and indeterminacy in models of neoclassical growth

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Abstract
This paper establishes the conditions under which indeterminacy can occur in a Neoclassical growth model with international labor mobility. In the model, workers are supposed to move freely across countries without restrictions, and according to a Harris–Todaro mechanism that makes migration flows sensitive to differences among labor markets conditions. The paper shows that indeterminacy requires the marginal returns to immigrant labor to be diminishing, and no need for productivity externalities at a social level. It also shows that immigration quotas can serve it well to eliminate indeterminacy and stabilize final output.

Keywords Indeterminacy · Free labor mobility · Temporary migration · Ramsey-like growth

JEL Classification E1 · F1 · O4

1 Introduction

The effects of immigration on labor market performance has been extensively studied by economic literature (Borjas 1995, 1999, 2001; Card 2001, 2005, 2009; Ottaviano and Peri 2012). Recently, economists' interest has been attracted to the

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long-run implications of migration on macroeconomic dynamics and economic growth. This literature comprises, among others, contributions by Ben Gad (2004, 2008), Klein and Ventura (2007, 2009) and Mandelman and Zlate (2012). In this set of papers, workers' movements are supposed to be sluggish and migration is taken to be a permanent change of locations, with the result that local indeterminacy of the equilibrium, i.e. a continuum of equilibrium paths converging towards a unique steady state, never appears because of international labor mobility. In this vein, the unique exception is my own paper (Parello 2019), in which I show that free-labor mobility and temporary migration[1] can be responsible for the emergence of indeterminacy of the equilibrium in a standard one-sector Neoclassical growth model of a small open economy.

This paper takes a step forward in this direction and shows that indeterminacy may also emerge in a one-sector Ramsey economy with closed capital account and even in the absence of external effects. With respect to my previous study, the main innovation of this paper is in the determination of the interest rate. Indeed, while in small open-economy models the interest rate is totally delinked from the marginal returns of domestic capital and fixed by world capital markets, in this paper it equates the rate of return on capital and is set so as to balance the internal supply and demand of funds for capital accumulation. As is known, changes in the composition of the domestic workforce can affects macroeconomic equilibria in different ways depending on whether the capital account of the economy is supposed to be closed or open to international capital markets. It is then interesting to investigate whether the indeterminacy result of Parello (2019) can be extended to a more standard Ramsey–Cass–Koopmans economy, in which the host economy is no longer a price-taker in financial markets.

To do this, in this paper I construct a one-sector perfectly-competitive economy based on the following assumptions: (i) labor is freely mobile across countries; (ii) the host economy, i.e. the economy that hosts immigrants, is closed to international trade and characterized by only one domestically-created asset: physical capital; (iii) technology is described by a strictly concave production function of three inputs: capital, native labor and immigrant labor; (iv) migration is assumed to be temporary and determined by an Harris–Todaro migration function (cf. Harris and Todaro 1970). Under these assumptions, I find that the unique steady-state equilibrium is locally indeterminate and also that migration can serve as an impulse for fluctuations in final output. In particular, I find that, for indeterminacy to arise,

[1] Free labor movements, together with free movement of capital, goods and services, is one the pillars of the European Economic Area (EEA). However, as many commentors point out, adopting international free labor mobility might incentivise the use of "temporary" migration, i.e. lawful migration to a country without having neither citizenship nor permanent residence permits, but with full work rights. According to Forte and Portes (2017), free movement within the EEA has been one of the major drivers for immigration in the UK, and its removal, due to Brexit, is likely to have significant negative impact on the UK economy overall. Constant and Zimmermann (2011) report that around two thirds of the so-called "guestworker" program generation left Germany and returned home, while Hugo (2008) finds a complex migration system India, China and Australia, involving circularity and remigration. As for the US, Massey (1987) and Massey and Espinosa (1997) establish that Mexicans moving into the U.S. are essentially "circular" (or "repeated") migrants (i.e. migrants who tends to go back and forth from a country to another in search of temporary jobs).

the diminishing marginal returns to private inputs have to be strong enough to make the trace of the Jacobian show the right sign. When this happens, first-order and transversality conditions are not sufficient to select a unique converging path, with the result that the host economy is characterized by extrinsic uncertainty and macroeconomic instability. The policy implication of this finding is that immigration quotas can serve it well to smooth belief-driven output fluctuations.

My results are in line with the empirical literature on migration and the business cycle, finding that migration is procyclical and that the response of output and income to country-specific shocks becomes stronger in the presence of migration. For instance, in the SVAR literature, Furlanetto and Robstad (2019) find that immigration shocks are non-negligible drivers of the Norwegian business cycle, explaining on average around 15–20 percent of the output fluctuations. Using similar econometric techniques, Kiguchi and Mountford (2019) and Smith and Thoenissen (2019) find that an unexpected positive migration shock improves GDP per capita in US and New Zealand, albeit the short-run effects on unemployment, consumption and capital intensity are found to be negative.

As for the European countries, estimates show that migration matters for the business cycle and *vice versa*. Huart and Tchakpalla (2015) report that differences in business cycles—measured through cross-country differences in the unemployment rates—are the main drivers for intra-EU migration, while D'Albis et al. (2016) report that immigration positively affects GDP per capita in France, but also that optimistic expectations about France's macroeconomic perspectives have a sizable impact on migration. The latter result is also confirmed by Lozej (2019), who finds evidence, for many EEA, of large and positive correlations between net migration and output dynamics on the one hand, and the tendency for migration to amplify host economies' cyclical fluctuations on the other.

This paper provides a possible economic explanation for how migration might amplify business cycle fluctuations based on equilibrium indeterminacy. In summary, when agents come to believe that migration will increase tomorrow, they act as if the marginal product of capital shifted up. In turn, the expected rise in the rental rate of capital will drive up both the pace of capital accumulation, through an increase in agents' investment rate, and the level of the wage rates, through an increase in firms' demand for labor (including immigrant labor). However, the increase in immigrants' wage rate will incentivize foreign workers to migrate to the host economy, thereby making the initial guess of agents about a future increase in migration self-fulfilled. Eventually, the economy reaches a new steady-state equilibrium in which both income per capita and immigration ratio have increased because of the belief-driven increase in immigration.

This paper can be seen as unifying two different branches of economic literature. On the one hand, the paper extends and challenges previous results of the literature on the macroeconomic effects of international migration. For example, in the Neoclassical growth settings with legal migration of Ben Gad (2004, 2008), Klein and Ventura (2007, 2009), Khraiche (2015), Mandelman and Zlate (2012) and Ikhenaode and Parello (2020), the long-run equilibrium of the receiving economy is always determinate, implying that migration never behaves as a possible source of macroeconomic instability. In contrast, this paper shows under what conditions free movement of workers across countries can be responsible for the emergence of equilibrium indeterminacy. In particular, because free-labor mobility causes migration to act as an extrinsic random variable, the paper shows how swings in agents' sentiment about future migration can cause the host economy to either "select" low performing equilibrium paths or lead to events of macroeconomic instability not directly driven by macroeconomic fundamentals.

On the other hand, an important branch of literature explores the sources of indeterminacy in one-sector models of Neoclassical growth. In this research area, indeterminacy emerges when: (*i*) the production activity is able to generate both negative factor externalities and increasing social marginal products (Boldrin and Rustichini 1994; Xie 1994; Hintermaier 2003; Wirl 2011; *ii*) the magnitude of labor externalities are large enough to guarantee that the demand for labor is upward-sloping (Benhabib and Farmer 1994, 1996; Benhabib and Nishimura 1998; Garnier et al. 2013; *iii*) the rate of time preference of consumers is endogenously related to some social variables (Drugeon 1998; Meng 2006; Yanase and Karasawa-Ohtashiro 2019; *iv*) income tax rates are determined under a balanced-budget rule with a pre-set level of "wasteful" government spending (Schmitt-Grohé and Uribe 1997; Guo and Harrison 2004, 2008).

My paper contributes to this literature as follows. First, it introduces an endogenously-determined mechanism of labor migration in a naïve Ramsey–Cass–Koopmans model of growth and shows that migration can be taken as a further source of indeterminacy. Second, it shows that indeterminacy does not require the existence of any form of production externalities or market imperfection to emerge. Third, the paper provides a detailed analysis about the mechanics leading to local indeterminacy and demonstrates that the emergence of a multiplicity of equilibria can be attributed to the presence of diminishing marginal returns to immigrant labor. In this context, it is also shown how immigration quotas can be used as a stabilizing device to select among different convergent paths.

The outline of the paper is the following. Section 2 presents the theoretical framework. Section 3 characterizes the dynamic equilibrium of the model and analyzes the conditions for indeterminacy. Section 4 discusses the economic implications of indeterminacy and provides some policy considerations. Finally, Sect. 5 concludes.

2 The model

2.1 Migration

I consider a closed economy populated by a continuum N of native consumers/workers, and a continuum M of immigrant consumers/workers, such that $m := M/N$ is the immigration ratio of the host economy at time t.

The growth rate of the native population is exogenous and equal to $v > 0$, while the growth rate of the immigrant population is assumed to be endogenous and dependant upon the existence of a positive differential between the expected wage rate paid in the receiving economy, denoted by w_M^e, and the level of a reference income offered in the country of origin, denoted by $\bar{w}_0$. Following Parello (2019), I focus on a Harris–Todaro migration function in the form

$$\dot{M} = \varphi\left(w_M^e - \bar{w}_0\right)M, \tag{1}$$

where φ is an exogenous parameter that captures the adjustment speed of migration in case of mismatch between the expected wage rate of immigrants w_M^e and their reference income $\bar{w}_0$.

I assume that there is no fundamental uncertainty present in the economy, so that expectations of factor prices conform to the values that they ultimately obtain. This implies that $w_M^e = w_M$ holds at any $t > 0$, and hence that (1) can be written in terms of m to obtain

$$\dot{m} = \varphi(w_M - \bar{w}_0)m - vm. \tag{2}$$

Equation (2) is one of the key equations of the model. Since foreign workers are free to enter and exit the host economy as much as they like without restrictions or costs, the variable m can make discontinuous jumps in response to new information about the future values of w_M. Consequently, in the rest of this section, m will be treated as a non-predetermined, jump variable.

2.2 Production, consumption and capital accumulation

Production in the host economy is carried out by a continuum of identical competitive firms, with the total number normalized to one. The aggregate production function is Cobb-Douglas, given by

$$Y = AK^{\alpha}L^{1-\alpha}, A > 0, \alpha \in (0,1), \tag{3}$$

where Y is final output, A is a constant parameter measuring TFP, K is physical capital and L is an aggregator over labor inputs. Specifically, I assume assume a CES aggregator of the form[2]

$$L := \left[(1-\theta)N^{1-1/\phi} + \theta M^{1-1/\phi}\right]^{1/(1-1/\phi)}, \theta \in (0,1), \phi > 1,$$

where θ is the share parameter of the CES aggregator and ϕ is the elasticity of substitution between labor types.[3]

[2] Production function (3) has the virtue of encompassing both cases in which the two types of labor are either substitute or complement in production depending on the value of the parameter ϕ. Specifically, if $\phi = 0$, then domestic and foreign workers are used in fixed proportions and the CES aggregator becomes a "Leontief-like" function leading to an aggregate production function of the type: $Y = AK^{\alpha}[\min(N,M)]^{1-\alpha}$. Likewise, if $\phi = \infty$, domestic and foreign workers are perfect substitutes in production and (3) modifies to $Y = AK^{\alpha}[(1-\theta)N + \theta M]^{1-\alpha}$. Finally, if $\phi = 1$, then domestic and foreign workers are Edgeworth complements in production with unitary elasticity of substitution and (3) boils down to a standard Cobb-Douglas production technology in the form: $Y = AK^{1-\beta-\eta}N^{\beta}M^{\eta}$, where $\beta := (1-\theta)(1-\alpha)$ and $\eta := \theta(1-\alpha)$.

[3] For the sake of truth, the range of variation of the parameter ϕ should be between 0 and $+\infty$. However, two influential contributions by Manacorda et al. (2012) and Ottaviano and Peri (2012) find significant high values for ϕ (imperfect substitutability) ranging from 7 (Manacorda et al. 2012) to 20 (Ottaviano and Peri 2012). As result, in the rest of the paper I will assume $\phi > 1$ and restrict my attention to the special case in which natives and immigrants are imperfect substitutes in production.

For simplicity, physical capital is assumed not to depreciate. Consequently, under the assumption that input markets are perfectly competitive, the firm's necessary and sufficient conditions for profit maximization are given by

$$r = \alpha A k^{\alpha-1}\left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha)/(1-1/\phi)} \tag{4a}$$

$$w_N = (1-\alpha)(1-\theta)Ak^{\alpha}\left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha\phi)/(\phi-1)} \tag{4b}$$

$$w_M = (1-\alpha)\theta A k^{\alpha}\left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha\phi)/(\phi-1)} m^{-1/\phi}, \tag{4c}$$

where r is the rental rate of capital, w_ℓ (with $\ell = \{N, M\}$) is the wage rate of each type of workers and $k := K/N$ is the stock of capital per native individual at time t.

Individuals are assumed to be infinitely-lived agents, to have perfect-foresight and to face perfect capital markets. They are also supposed to be endowed with one unit of labor each, which they supply inelastically. In this environment, agents are postulated to derive utility from consumption according to the lifetime utility function

$$U_\ell(t) = \int_0^\infty e^{-\rho t}\ln c_\ell \mathrm{d}t, \rho > v, \tag{5}$$

where ρ is the rate of time preference of all of the individuals residing in the host economy and $c_\ell := C_\ell/\ell$, with $\ell = \{N, M\}$, is the level of per capita consumption referred to the individual of type ℓ.

To simplify the model, I suppose that immigrants are financially-constrained agents devoting all of their wage income to consumption.[4] This implies that $c_M = w_M$ holds at each time t. As for natives, their objective is to choose the optimal consumption path $\{c_N(t)\}_{t\in(0,\infty)}$ to maximize (5) subject to the flow budget constraint

$$\dot{k} = (r-v)k + w_N - c_N, with k(0) = k_0 \text{ given.} \tag{6a}$$

The current-value Hamiltonian for this problem is $\mathcal{H} = \ln c_N + \lambda[(r-v)k + w_N - c_N]$, where λ is the costate variable associated with k, also acting as the shadow price of investment. The first-order and transversality conditions are $\partial\mathcal{H}/\partial c_N = 0$, $\partial\mathcal{H}/\partial k = \rho\lambda - \dot{\lambda}$ and $\lim_{t\to\infty}\left\{e^{-\int_0^t[r(z)-v]\mathrm{d}t}\lambda k\right\} = 0$. Combining these conditions yields

$$\lambda = \frac{1}{c_N} \tag{7}$$

[4] It can be shown that extending the model to fully-optimizing immigrants does not change the main findings of the paper. The results of such a extension are available upon request.

$$\dot{\lambda} = -(r - v - \rho)\lambda \tag{8}$$

$$\lim_{t\to\infty}\left\{e^{-\int_0^t [r(z)-v]\mathrm{d}t}\frac{k}{c_N}\right\} = 0. \tag{9a}$$

Thus, from (7) and (8), I can obtain the following Euler condition

$$\dot{c}_N = (r - \rho - v)c_N, \tag{10}$$

according to which the per capita consumption of natives grows over time if and only if the rental rate of capital, r, is larger than the sum of the rate of time preference and the growth rate of the native population, $\rho + v$.

3 Equilibrium analysis

3.1 Characterization of the equilibrium

For any given $\bar{w}_0$ and k_0 a perfect-foresight equilibrium for the competitive economy is characterized by a triplet of paths for consumption, physical capital and immigration ratio, $\{c_N(t), k(t), m(t)\}_{t\in(0,\infty)}$, a pair of paths for the wage rates, $\{w_N(t), w_M(t)\}_{t\in(0,\infty)}$, and a path for the rental rate of capital $\{r(t)\}_{t\in(0,\infty)}$, such that: (i) natives maximize their discounted utility function (5) subject to the accumulation constraint (6a) and the no-Ponzi condition (9a); (ii) firms maximize profits subject to the technology constraint (3); (iii) people migrate either in or out of the host economy according to (2); (iv) all markets clear.

The intertemporal behavior of the competitive economy can be characterized through a system of six equations—namely (2), (10), (4a), (4b), (4c) and (6a)-, in six unknowns: c_N, k, m, r, w_N and w_M. Using (4a), (4b) and (4c) to substitute for r, w_N and w_M in (2), (6a) and (10), the equilibrium system of the model can be reduced to a 3×3 differential system in the form

$$\dot{k} = Ak^{\alpha}\frac{\alpha\left(1-\theta+\theta m^{1-1/\phi}\right)+(1-\alpha)(1-\theta)}{\left(1-\theta+\theta m^{1-1/\phi}\right)^{(\phi\alpha-1)/(\phi-1)}} - c_N - vk \tag{11a}$$

$$\dot{c}_N = \alpha Ak^{\alpha-1}\left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha)/(1-1/\phi)}c_N - (\rho+v)c_N \tag{11b}$$

$$\dot{m} = \varphi(1-\alpha)\theta Ak^{\alpha}\left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\phi\alpha)/(\phi-1)}m^{1-1/\phi} - (\varphi\bar{w}_0+v)m. \tag{11c}$$

Setting $\dot{k} = \dot{c}_N = \dot{m} = 0$ in (11a)-(11c), it implies that there exists a unique triplet of steady-state values for k, c_N and m given by

$$\hat{k} = \frac{(1-\theta)^{1/(1-1/\phi)}\left(\frac{\alpha A}{\rho+v}\right)^{1/(1-\alpha)}}{\left[1-\theta\zeta(\bar{w}_0)^{\phi-1}\right]^{1/(1-1/\phi)}} \tag{12a}$$

$$\hat{c}_N = \frac{(1-\theta)^{1/(1-1/\phi)}\left(\frac{\alpha A}{\rho+v}\right)^{1/(1-\alpha)}\left\{\frac{\rho+v}{\alpha}\left[1-\theta(1-\alpha)\zeta(\bar{w}_0)^{\phi-1}\right]-v\right\}}{\left[1-\theta\zeta(\bar{w}_0)^{\phi-1}\right]^{1/(1-1/\phi)}} \tag{12b}$$

$$\hat{m} = \frac{(1-\theta)^{1/(1-1/\phi)}}{\left[\zeta(\bar{w}_0)^{1-\phi}-\theta\right]^{1/(1-1/\phi)}}, \tag{12c}$$

where "ˆ" denotes steady-states values and $\zeta(\bar{w}_0) > 0$ is a collection of given parameters including $\bar{w}_0$ (see Appendix A for details).

The steady state (12a)–(12c) is said to be locally indeterminate if there exists a continuum of converging paths to $\langle \hat{k}, \hat{c}_N, \hat{m} \rangle$ such that first-order and transversality conditions hold simultaneously. The following proposition establishes the conditions under this result holds.

Proposition 1 *If the level of the reference wage $\bar{w}_0$ is such that $\zeta(\bar{w}_0)^{1-\phi}-\theta > 0$ holds, then the steady-state equilibrium is unique and locally indeterminate.*

Proof See Appendix 1 & 2. □

The indeterminacy result showed by Proposition 1 implies that first-order and transversality conditions are not sufficient to select the optimal converging path to the steady state. To put it differently, what emerges from Proposition 1 is the existence of a continuum of dynamic equilibria (i.e., a continuum of converging paths to the unique steady state), each of which can be indexed by an initial value of the immigration ratio m. In order to understand under what conditions indeterminacy takes place, in the next subsection I shall compare the dynamic properties of the model with those of the standard model of Neoclassical growth without migration. Then, in a dedicated discussion section, I shall discuss about the economic factors underlying the emergence of indeterminacy.

3.2 Indeterminacy conditions

Without migration, $m = 0$ and the model reduces to the standard decentralized version of the Neoclassical growth model of Cass (1965) and Koopmans (1965), with equilibrium system given by

$$\dot{k} = f(k) - c - vk$$
$$\dot{c} = c(f'(k) - \rho - v),$$

where, for the sake of simplicity, I set $\theta = 0$ so that the intensive-form production function can be written as $f(k) := Ak^{\alpha}$. In the steady state, it must be that $f'(\hat{k}) = \rho + v$ and $\hat{c} = f(\hat{k}) - v\hat{k}$. Accordingly, log-linearizing the equilibrium system about $\langle \hat{k}, \hat{c} \rangle$ yields[5]

$$\begin{bmatrix} \dot{k} \\ \dot{c} \end{bmatrix} = \hat{Z} \begin{bmatrix} k - \hat{k} \\ c - \hat{c} \end{bmatrix},$$

where

$$\hat{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \rho & -1 \\ \hat{c}f''(\hat{k}) & 0 \end{bmatrix},$$

is the Jacobian matrix associated with the linearized system. Straightforward computations give

$$\begin{aligned} \det(\hat{Z}) &= -z_{12}z_{21} = \hat{c}f''(\hat{k}) = -A(1-\alpha)[\rho + (1-\alpha)v] < 0 \\ tr(\hat{Z}) &= z_{11} + z_{22} = \rho > 0. \end{aligned}$$

Since the determinant is negative, the steady-state equilibrium is saddle-path stable. As is well known, such a property of Neoclassical growth models is due to the diminishing marginal product of capital—captured by the term $f''(\hat{k}) < 0$ located at position $(2, 1)$ of matrix $\hat{Z}$-, which in turn causes the real rental rate of capital to fall gradually with capital accumulation. Now, allowing for free labor mobility across countries modifies the intensive-form production function, which becomes $f(k, m) := Ak^{\alpha}\left(1 - \theta + \theta m^{1-1/\phi}\right)^{(1-\alpha)/(1-1/\phi)}$, and adds one more endogenous variable (a jump variable) to the equilibrium system. Thus, by linearizing system (11a)-(11c) about the steady state $\langle \hat{k}, \hat{c}_N, \hat{m} \rangle$, I have (see Appendix B for details)

$$\begin{bmatrix} \dot{k} \\ \dot{c}_N \\ \dot{m} \end{bmatrix} = \hat{J} \begin{bmatrix} k - \hat{k} \\ c_N - \hat{c}_N \\ m - \hat{m} \end{bmatrix},$$

where the matrix of coefficient is given by

$$\hat{J} = \begin{pmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{pmatrix} = \begin{pmatrix} \rho - \hat{m}\dfrac{\partial^2 f(\hat{k}, \hat{m})}{\partial k \partial m} & -1 & -\hat{m}\dfrac{\partial^2 f(\hat{k}, \hat{m})}{\partial m^2} \\ \hat{c}_N \dfrac{\partial^2 f(\hat{k}, \hat{m})}{\partial k^2} & 0 & \hat{c}_N \dfrac{\partial^2 f(\hat{k}, \hat{m})}{\partial k \partial m} \\ \eta\hat{m}\dfrac{\partial^2 f(\hat{k}, \hat{m})}{\partial k \partial m} & 0 & \eta\hat{m}\dfrac{\partial^2 f(\hat{k}, \hat{m})}{\partial m^2} \end{pmatrix}.$$

A straightforward comparison between $\hat{Z}$ and $\hat{J}$ reveals two main differences

[5] Formally, the steady-state equilibrium is given by the following pair of stationary values: $\hat{k}_N = [\alpha\mathcal{A}/(\rho + v)]^{1/(1-\alpha)}$ and $\hat{c}_N = \mathcal{A}[\rho + (1-\alpha)v][\alpha\mathcal{A}/(\rho + v)]^{1/(1-\alpha)}/(\rho + v)$.

between the two models. First, the existence of credit-constraints for immigrants makes it impossible for this type of agents to smooth consumption relative to income. This feature of the model is captured by the negative term $-\hat{m}\frac{\partial w_m}{\partial k} = -\hat{m}\frac{\partial^2 f(\hat{k},\hat{m})}{\partial k \partial m} < 0$ appearing at position $(1,1)$ of $\hat{J}$. Second, migration enlarges the dimension of the Jacobian matrix by including a new row and column, whose entries are linked to the capital-labor complementarity and the degree of diminishing returns of immigrant labor. These further features of the model are respectively captured by the mixed partial derivative $\partial^2 f(\hat{k},\hat{m})/(\partial k \partial m)$ appearing at position $(2,3)$ and $(3,1)$ of $\hat{J}$, and the second partial derivative $\partial^2 f(\hat{k},\hat{m})/\partial m^2$ appearing at position $(1,3)$ and $(3,3)$ of the Jacobian matrix.

Unsmoothed life-cycle consumption profile for immigrants, capital-labor complementarity and diminishing marginal returns of immigrant labor are thus at the roots of the emergence of indeterminacy in my model.[6] To see this, I calculate the determinant and trace of $\hat{J}$ to obtain

$$\det(\hat{J}) = j_{12}(-1)^3(j_{21}j_{33} - j_{23}j_{31}) = \eta\hat{c}\hat{m}\left[\frac{\partial^2 f(\hat{k},\hat{m})}{\partial k^2}\frac{\partial^2 f(\hat{k},\hat{m})}{\partial m^2} - \left(\frac{\partial^2 f(\hat{k},\hat{m})}{\partial k \partial m}\right)^2\right] \quad (13)$$

$$tr(\hat{J}) = j_{11} + j_{33} = \rho - \hat{m}\left[\frac{\partial^2 f(\hat{k},\hat{m})}{\partial k \partial m} - \eta\frac{\partial^2 f(\hat{k},\hat{m})}{\partial m^2}\right]. \quad (14)$$

Because the production function is strictly concave in capital and immigrant labor, the term in squared brackets of (13) - which, in turn, represents the Hessian matrix of the intensive form production function $f(\hat{k},\hat{m})$—is positive, implying that the determinant is always positive. The system might therefore have either 0 or 2 eigenvalues with negative real part depending on the sign of the trace. However, from (14) it follows that the sign of $tr(\hat{J})$ is ambiguous. Indeed, for indeterminacy to emerge, it must be that the term $\hat{m}\left[\frac{\partial^2 f(\hat{k},\hat{m})}{\partial k \partial m} - \eta\frac{\partial^2 f(\hat{k},\hat{m})}{\partial m^2}\right]$ on the second equality of (14) dominates ρ. This result occurs if, for any given value of $\hat{m}$, ρ is relatively smaller (in absolute value) than $\hat{m}\left[\frac{\partial^2 f(\hat{k},\hat{m})}{\partial k \partial m} - \eta\frac{\partial^2 f(\hat{k},\hat{m})}{\partial m^2}\right]$; that is, if the degree of complementarity between capital and labor (captured by $\frac{\partial^2 f(\hat{k},\hat{m})}{\partial k \partial m}$) and the curvature of the production function with respect to immigrant labor (captured by $\frac{\partial^2 f(\hat{k},\hat{m})}{\partial m^2}$) are high enough to guarantee that the trace is negative. Appendix B shows that this is the case and hence that the characteristic polynomial of the Jacobian matrix has only one positive root out of three, implying that the steady-state equilibrium of the host economy is locally indeterminate.

[6] As a matter of facts, it can be demonstrated that the presence of credit constraints for immigrants is not key for the indeterminacy result. For this reason, in the rest of this section I shall focus on only diminishing returns as a source of indeterminacy. The details about the formal derivation of this result are in a complementary appendix available upon request.

4 Discussion

In this section, I shall provide the economic intuition for how migration can lead to indeterminacy and for how the government can intervene in the economy to preserve macroeconomic stability. In doing this, I shall follow the literature on belief-driven business cycle fluctuations, specifically the studies of Harrison (2001) and Guo and Harrison (2004, 2008). The section begins by explaining the economic forces driving equilibrium indeterminacy. It then turns to the policy implications of this result.

4.1 Explaining indeterminacy

Consider an economy in its own steady-state equilibrium and assume that, at time t, agents come to believe that the rate of return on capital will increase. As a result of this belief, the shadow price of investment will increase, thereby inducing agents to increase their investment rate through a reduction in present consumption.[7]

If there were no migration, physical capital would accelerate its accumulation pace because of the increase in investment, but the marginal product of capital would fall as a result of diminishing returns to capital. Yet, for agents to remain onto an accumulation trajectory, the decline in the marginal product of capital ought to be offset by an increase in the shadow price of investment. That would in fact validate agents' initial belief that a greater stock of capital will eventually yield higher rental rates. However, since this does not occurs, after few periods of overaccumulation agents will realize that their initial belief was wrong, and then that they are actually laying onto an explosive path characterized by too much capital accumulation and oversaving. This will eventually induce them to divert income from investment to consumption, thereby shifting the economy from an explosive path to the unique converging path that guarantees the convergence to the steady state.

In contrast, in the presence of migration, the above adjustment dynamics do not hold and any conjecture about the existence of an alternative equilibrium path can be self-fulfilled. Indeed, when agents are allowed to move internationally, the initial rise in capital stock due to the increase in investment incentivizes firms to increase their demand for labor, with the result that the wage rate of immigrant workers w_m will temporarily exceeds their reference income $\bar{w}$. This, in turn, will stir up

[7] As Harrison (2001) points out, the shadow price of investment can be thought as the discounted sum of the future values of the marginal product of capital. To see this more formally, consider the consumer maximization problem of Sect. 2.2. Combining (7) and (), integrating the resulting expression and then using transversality condition (9a), it can be shown that the current value of the shadow price of investment is equal to the discounted sum of the future values of the marginal product of capital adjusted for the marginal utility of consumption; that is:

$$\lambda(0) = \int_0^\infty e^{-\rho t} \frac{r - v}{c_N} \mathrm{d}t.$$

If agents start believing that the rate of return on capital will increase in the next future, $\lambda(0)$ will increase accordingly, implying that native consumers will find it maximizing to forgo present consumption in exchange of future consumption.

migration and will cause the marginal product schedule of capital (see equation (4a)) to shift rightward, thereby offsetting the initial decline in the marginal returns to capital. Since the shift in the marginal product schedule (4a) is initially larger than the decline in the marginal returns to capital, agents will find it convenient to keep accumulating physical capital and keep hiring immigrant labor, thereby strengthening their initial prophecy that the rental rate of capital would increased in the near future. However, as additional capital is accumulated and new immigrant workers have entered the host economy's labor market, the diminishing returns to immigrant labor causes w_m to get closer and closer to $\bar{w}$, until they equalize each other. When this happens, migration ceases and agents have no further incentive to invest in capital accumulation. The host economy is thus in a new steady-state equilibrium characterized by a higher level of capital per worker and a higher ratio between immigrants and natives.

It is therefore the presence of diminishing returns to labor inputs that drives indeterminacy. Notice that self-fulfilled beliefs about a future increase in cross-country movements of workers can make migration procyclical with respect to income, in a fashion similar to that reported by the empirical literature on migration and the business cycle. Yet, here the drawback of having a continuum of self-fulfilled equilibria is that uncontrolled cross-country labor mobility can be seen as a source of extrinsic uncertainty for the aggregate economy, meaning that any arbitrary change in expectations about tomorrow's migration flows might disturb the dynamics of final output. Indeed, as is easy to check from (4a), the marginal product schedule of capital is a positive function of the immigration rate m. Consequently, if agents come to believe that the rental rate of capital will increase tomorrow because of an increase in m, the impact on the macroeconomy of the host economy of such an optimistic prophecy will be the same as those described earlier, implying that migration can indeed be seen as a source of business cycle fluctuations.

Evidently, indeterminacies of this sort indicate that rational expectations equilibria involve random variables that are unrelated to the economy's fundamentals. Indeed, similarly to Farmer and Guo (1994), in a stochastic version of the model it can be shown that business cycle fluctuations can be solely driven by *i.i.e.* sunspot shocks on future migration. As I shall discuss in the next subsection, this is an important by-product of the paper because it provides the government with an opportunity to intervene in the economic system to restore macroeconomic stability.

4.2 Policy implications

In the proposed model, extrinsic uncertainty comes from the presence of a continuum of a converging paths to the steady state, each of which can be parametrized by the initial value for the immigration ratio m. This causes the realization of a specific equilibrium to be dependant upon agents' expectations and the initial position of the host economy. In this subsection, I shall show how introducing immigration quotas can help agents to select the right converging path and hence to prevent the economy from experiencing prolonged periods of belief-driven macroeconomic instability.

To see this, suppose the host economy is in its own steady state, such that $m = \hat{m}$, and also that, at time $t = t_0$, the government announces his intention to introduce an immigration quota at m^*, with $m^* > \hat{m}$.[8] Because the steady-state immigration rate is lower than quota, if firms become optimistic about a future increase in migration, the most they can expect is to see m to meet m^*, and then to see all the marginal product schedules to adjust to accommodate new migration. In particular, from (4a) it can be shown that the rental rate of capital changes because of the quota to read

$$r^* = \alpha \mathcal{A} k^{\alpha-1}, \mathcal{A} := A\left[1-\theta+\theta(m^*)^{1-1/\phi}\right]^{(1-\alpha)/(1-1/\phi)},$$

where $\mathcal{A}$ is a new constant parameter measuring aggregate productivity amended to account for the presence of an upper bound to immigration. Moreover, with $m = m^*$, (11c) disappears from the dynamic system of the model, while the capital accumulation equation () and Euler condition (11b) change accordingly to become

$$\dot{k} = \psi \mathcal{A} k^{\alpha} - c_N - vk, \psi := \frac{1-(1-\alpha)\theta}{(1-\theta)(m^*)^{1/\phi-1}+\theta},$$
$$\dot{c}_N = \alpha \mathcal{A} k^{\alpha-1} c_N - (\rho + v) c_N,$$

where the term $\psi \mathcal{A} k^{\alpha}$ on the right-hand of the capital accumulation equation is the amount of income left over after immigrant's consumption (i.e., $\psi \mathcal{A} k^{\alpha} = \mathcal{A} k^{\alpha} - c_M m^* = \mathcal{A} k^{\alpha} - w_m^* m^*$).

As is easy to verify, the above 2×2 dynamic system admits only one steady-state solution and also that the resulting long-run equilibrium is saddle-path stable. The latter result does not come as a surprise, since the reduced-form system of the model with immigration quotas of this section shares the same dynamic properties of the standard Ramsey–Cass–Koopmans model without migration of Sect. 3.2. Indeed, in the scenario with the quota, the expected rental rate of capital r^* is no longer uncertain because of migration, implying that first-order and transversality conditions are now sufficient to select a unique converging path to the steady state. This settles the extrinsic uncertainty problem generated by free labor mobility and prevents the economy from embarking in periods of business cycle fluctuations.

Summing-up, regulating free labor mobility through immigration quotas can indeed provide agents with a clear-cut guideline to predict the future trends of migration and, simultaneously, provide the government with a simple *automatic stabilizer* capable of eliminating the extrinsic uncertainty due to the unpredictability of future migration flows.

5 Conclusions

In this paper, I have presented an extension of the Ramsey–Cass–Koopmans model of economic growth to international migration to study the short- and long-run effects of free labor mobility across countries. The analysis focused on the so-called

[8] Notice that an equilibrium with $\hat{m} > m^*$ is not sustainable in the long run. In fact, if, at a given $t > 0$, $\hat{m}$ would exceed m^*, then the government would enforce the quota by reducing the working permits.

temporary (or “circular”) migration and postulated that native and immigrant workers are imperfect substitutes in production.

I find that the diminishing marginal returns to immigrant labor causes the steady state to be locally undetermined. More specifically, compared with the standard Ramsey–Cass–Koopmans model without migration, the reduced system of the variant model with free-labor mobility is found to have one additional equation, based on the evolution of the immigrant-to-native ratio, which can create indeterminacy in so far as the marginal contribution of an extra unit of immigrant labor is decreasing. In this context, I find that there exists a coordination problem among agents, in that optimistic expectations about future migration (i.e., beliefs that immigration can increase tomorrow) could prove self-fulfilling and stir the economy towards a high-income/high-migration equilibrium, while pessimistic expectations about migration (i.e., beliefs that immigration can decrease tomorrow) can work in the opposite direction and drive the economy towards a low-income/low-migration equilibrium.

This result is in line with empirical literature finding that migration can amplify the output fluctuations of the hosting economies and can help to provide a possible explanation of the reason why per capita income and net migration are positively correlated variables. Moreover, because free-labor mobility causes expectations about migration not to depend upon macroeconomic fundaments, my results theoretically justify the use of immigration quotas as a selection device capable of solving the agents’ coordination problem generated by the emergence of local indeterminacy.

The paper presents at least two caveats. First, the model relies on a particular type of labor migration (temporary/circular migration) which makes the supply of immigrant labor particularly volatile and dependent upon the contingent conditions of the labor market of the host country. Clearly, considering other motivations for migrating than exploiting the temporary wage differentials between countries would enrich the model considerably and would also challenge the results found here in this paper. However, modelling migration as both a temporary and permanent phenomenon would require discussion and analysis of other migration-related issues such as, for instance, social integration and networking that go beyond the scope of this study.

Second, the fact that the diminishing marginal returns to immigrant labor emerged as the main source of indeterminacy opens up the door for another possible research line involving endogenous growth and migration. Indeed, because in this paper strict concavity of the production technology turns out to be key for the generation of indeterminacy, a natural step forward in this research line would be to allow for productivity externalities at a social level, and hence to allow for the existence of a dichotomy between private and social production function. Yet again, such an extension goes beyond the aims of the paper and is left as avenue of future research.

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Appendix

In this appendix, I prove that the steady-state equilibrium is unique and locally indeterminate. The first part of the appendix (Appendix 1) provides the necessary and sufficient conditions for the steady-state equilibrium to exist and to be unique. Then (Appendix 2), it characterizes the conditions for indeterminacy.

Appendix 1: Existence and unicity of the steady state

Setting the zero growth condition to the dynamic system (11a)–(11c) yields

$$A\hat{k}^{\alpha}\frac{\alpha\left(1-\theta+\theta\hat{m}^{1-1/\phi}\right)+(1-\alpha)(1-\theta)}{\left(1-\theta+\theta\hat{m}^{1-1/\phi}\right)^{(\phi\alpha-1)/(\phi-1)}}=\hat{c}_N+v\hat{k} \tag{15a}$$

$$\alpha A\hat{k}^{\alpha-1}\left(1-\theta+\theta\hat{m}^{1-1/\phi}\right)^{(1-\alpha)/(1-1/\phi)}=\rho+v \tag{15b}$$

$$(1-\alpha)A\hat{k}^{\alpha}\left(1-\theta+\theta\hat{m}^{1-1/\phi}\right)^{(1-\phi\alpha)/(\phi-1)}\hat{m}^{-1/\phi}=\frac{(1-\alpha)}{\zeta(\bar{w}_0)}\left(\frac{\alpha A^{1/\alpha}}{\rho+v}\right)^{\alpha/(1-\alpha)}, \tag{15c}$$

System (15a)–(15c) presents a bloc-recursive structure. Using (15a) to substitute for $\hat{k}_N$ in (15c) yields

$$\zeta(\bar{w}_0)\left[(1-\theta)\hat{m}^{-(1-1/\phi)}+\theta\right]^{1/(\phi-1)}=1, \tag{16}$$

where

$$\zeta(\bar{w}_0) := \frac{(1-\alpha)\theta\varphi}{v+\bar{w}_0\varphi}\left(\frac{\alpha A^{1/\alpha}}{\rho+v}\right)^{\alpha/(1-\alpha)} > 0, \text{ with } \zeta'(\bar{w}_0) < 0,$$

is a collection of exogenous parameters.

A steady state exists if and only if (16) has at least one solution for $\hat{m}$, with $\hat{m} \in \mathbb{R}_+$. The following Lemma establishes the necessary and sufficient conditions under which the steady-state equilibrium exists and is unique.

Lemma 1 *If the level of the reference wage* $\bar{w}_0$ *is such that* $\zeta(\bar{w}_0) < \theta^{1/(1-\phi)}$ *holds, then there exists a unique (positive) steady-state equilibrium for the immigration ratio* $\hat{m}$.

Proof Proving the existence and uniqueness of a stationary point for the immigration ratio reduces to proving that the intersection between the left-hand side of (16) and 1 occurs in the positive orthant. Rewrite the left-hand side of (16) as $\zeta(\bar{w}_0)f(m)$, where

$$f(m) := \left[(1-\theta)m^{-(1-1/\phi)} + \theta\right]^{1/(\phi-1)}.$$

with $f(0) \to \infty$ and $f(\infty) \to \theta^{1/(\phi-1)} > 0$. Since

$$f'(m) = -\frac{1-\theta}{\phi}\left[(1-\theta)m^{-(1-1/\phi)} + \theta\right]^{1/(\phi-1)-1} m^{-(1-1/\phi)-1} < 0,$$

the left-hand side of (16) is monotonically decreasing for any $m > 0$. As a result, for $\zeta(\bar{w}_0)f(m)$ to cross 1 in the positive orthant, it must be that $\zeta(\bar{w}_0)\theta^{1/(\phi-1)} < 1$, and thus that $\zeta(\bar{w}_0) < \theta^{1/(1-\phi)}$ holds (with $\theta^{1/(1-\phi)} > 1$). Because $\partial\zeta(\bar{w}_0)/\partial\bar{w}_0 < 0$, $\zeta(\bar{w}_0) < \theta^{1/(1-\phi)}$ requires $\bar{w}_0$ to be not lower than the threshold

$$\tilde{\omega}_0 := \theta^{1/(1-1/\phi)}(1-\alpha)\left(\frac{\alpha A^{1/\alpha}}{\rho+v}\right)^{\alpha/(1-\alpha)} - \frac{v}{\varphi}.$$

This provides the necessary and sufficient condition for the existence and uniqueness of a positive steady-state value for the immigration ratio, $\hat{m}$. □

From the steady-state value of m, the steady-state value of k_N and c_N can be obtained recursively from (15a) and (15c). And this concludes the demonstration of the first statement of Proposition 1.

Appendix 2: Indeterminacy conditions

Suppose the assumptions of Lemma 1 hold. Log-linearizing around the steady state $\langle \hat{c}_N, \hat{k}_N, \hat{m}\rangle'$, the dynamics of k_N, m and c_N can be approximated by

$$\begin{pmatrix} \dot{\hat{k}}_N \\ \dot{c}_N \\ \dot{m} \end{pmatrix} = \hat{J}\begin{pmatrix} \hat{k}_N - \hat{k}_N \\ c_N - \hat{c}_N \\ m - \hat{m} \end{pmatrix},$$

where the Jacobian matrix is given by

$$\hat{J} = \begin{pmatrix} \rho - \theta(1-\alpha)(\rho+v)\zeta(\bar{w}_0)^{\phi-1} & -1 & \dfrac{(1-\alpha)\theta\left[1-\theta(1-\alpha\phi)\zeta(\bar{w}_0)^{\phi-1}\right]}{\phi\zeta(\bar{w}_0)(\rho+v)^{\alpha/(1-\alpha)}(\alpha A^{1/\alpha})^{\alpha/(\alpha-1)}} \\ -\dfrac{(\rho+v)(1-\alpha)\left\{(\rho+v)\left[1-\theta(1-\alpha)\zeta(\bar{w}_0)^{\phi-1}\right]-v\alpha\right\}}{\alpha} & 0 & \dfrac{(1-\alpha)\theta\left\{(\rho+v)\left[1-(1-\alpha)\theta\zeta(\bar{w}_0)^{\phi-1}\right]-\alpha v\right\}}{\zeta(\bar{w}_0)(\rho+v)^{\alpha/(1-\alpha)}(\alpha A^{1/\alpha})^{\alpha/(\alpha-1)}} \\ \theta(1-\alpha)(\rho+v)\varphi\zeta(\bar{w}_0)^{\phi-1} & 0 & -\dfrac{(1-\alpha)\theta\varphi\left[1-\theta(1-\alpha\phi)\zeta(\bar{w}_0)^{\phi-1}\right]}{\phi\zeta(\bar{w}_0)(\rho+v)^{\alpha/(1-\alpha)}(\alpha A^{1/\alpha})^{\alpha/(\alpha-1)}} \end{pmatrix}, \tag{17}$$

and $\zeta(\bar{w}_0)$ is the same collection of exogenous parameters reported in the main text. Variable k_N is the predetermined variable of the model, while c_N and m are non-predetermined variables. Therefore, determinacy occurs if one eigenvalue of $\hat{J}$ has negative real part and the other two have positive real part, whereas indeterminacy arises if two eigenvalues of $\hat{J}$ have negative real part and the other one has positive real part.

The coefficients of the characteristic polynomial of $\hat{J}$ are given by the sequence

$$1 - tr(\hat{J})\hat{J}_{11} + \hat{J}_{22} + \hat{J}_{33} - \det(\hat{J}), \tag{18}$$

where

$$tr(\hat{J}) = \rho - \theta(1-\alpha)(\rho+v)\zeta(\bar{w}_0)^{\phi-1} - \frac{(1-\alpha)\theta\varphi\left[1-\theta(1-\alpha\phi)\zeta(\bar{w}_0)^{\phi-1}\right]}{\phi\zeta(\bar{w}_0)(\rho+v)^{\alpha/(1-\alpha)}(\alpha A^{1/\alpha})^{\alpha/(\alpha-1)}} \tag{19}$$

$$\det(\hat{J}) = \frac{(\rho+v)\theta\varphi(1-\alpha)^2\left(1-\theta\zeta(\bar{w}_0)^{\phi-1}\right)\left\{(\rho+v)\left[1-\theta(1-\alpha)\zeta(\bar{w}_0)^{\phi-1}\right]-v\alpha\right\}}{\alpha\phi\zeta(\bar{w}_0)(\rho+v)^{\alpha/(1-\alpha)}(\alpha A^{1/\alpha})^{\alpha/(\alpha-1)}}, \tag{20}$$

are the trace and the determinant of $\hat{J}$, and $\hat{J}_{ii}$, with $i = 1, 2, 3$, are the cofactors of the diagonal elements of $\hat{J}$. From Descartes' rule of signs, it follows that the emergence of a continuum of converging paths around the steady state depends on whether the number of sign changes in (18) is 1.

The sign of the trace is ambiguous, while the sign of the determinant and cofactors of (17) are established by following Lemma

Lemma 2 *If restriction* $\zeta(\bar{w}_0) < \theta^{1/(1-\phi)}$ *holds, then the following equivalences hold*:

$$\det > 0$$
$$\hat{J}_{11} = 0,\ \hat{J}_{22} < 0, \hat{J}_{33} < 0.$$

Proof Recall that $\zeta(\bar{w}_0) < \theta^{1/(1-\phi)}$ guarantees the existence of a unique steady-state equilibrium with positive immigration ratio (by Lemma 1). From (20), it follows that $\det(\hat{J}) \leq 0$if and only if the following hold

$$\theta^{1/(1-\phi)} \leq \zeta(\bar{w}_0) \leq \left[\frac{(\rho+v)(1-\alpha)}{\rho+(1-\alpha)v}\right]^{1/(1-\phi)} \theta^{1/(1-\phi)}.$$

However, under restriction $\zeta(\bar{w}_0) < \theta^{1/(1-\phi)}$, it follows that $\zeta(\bar{w}_0)$ never fall in the above range and hence that $\det(\hat{J})$ is always positive. As regards the three cofactors of the elements on the principal diagonal of (17), straightforward calculations give

$$\hat{J}_{11} = 0$$
$$\hat{J}_{22} = -\frac{(\rho+v)(1-\alpha)\left\{(\rho+v)\left[1-\theta(1-\alpha)\zeta(\bar{w}_0)^{\phi-1}\right]-v\alpha\right\}}{\alpha} \tag{21}$$

$$\hat{J}_{33} = -\frac{(1-\alpha)\theta\varphi\rho\left[1-\theta(1-\alpha\phi)\zeta(\bar{w}_0)^{\phi-1}\right]}{\phi\zeta(\bar{w}_0)(\rho+v)^{\alpha/(1-\alpha)}\left(\alpha A^{1/\alpha}\right)^{\alpha/(\alpha-1)}}. \tag{22}$$

The signs of the two nonzero cofactors, $\hat{J}_{22}$ and $\hat{J}_{33}$, are as follows. From (B.5), it follows that $\hat{J}_{22} \leq 0$ if and only if the following restriction is met

$$\zeta(\bar{w}_0) \leq \left[\frac{(\rho+v)(1-\alpha)}{\rho+(1-\alpha)v}\right]^{1/(1-\phi)} \theta^{1/(1-\phi)}.$$

Because $\{[(\rho+v)(1-\alpha)]/[\rho+(1-\alpha)v]\}^{1/(1-\phi)} > 1$ holds for $\alpha > 0$ and $\rho > 0$, the previous inequality is always met under the assumption of Lemma 1 of $\zeta(\bar{w}_0) < \theta^{1/(1-\phi)}$, and this establishes that the sign of the cofactor $\hat{J}_{22}$, evaluated at the steady state, is always negative. As far as the sign of $\hat{J}_{33}$ is concerned, from (22), it can be established that $\hat{J}_{33} < 0$ if and only if

$$\zeta(\bar{w}_0) \leq (1-\alpha\phi)^{1/(1-\phi)} \theta^{1/(1-\phi)}.$$

Since $\alpha \in (0,1)$, it follows that $(1-\alpha\phi)^{1/(1-\phi)} > 1$ and hence that $(1-\alpha\phi)^{1/(1-\phi)}\theta^{1/(1-\phi)} > \theta^{1/(1-\phi)}$ if $\phi > 1$. I can therefore conclude that the sign of the cofactor $\hat{J}_{33}$, evaluated at the steady state, is always negative, and this concludes the proof of Lemma 2.

From Lemma 2, it follows $\det(\hat{J}) > 0$ and $\hat{J}_{11} + \hat{J}_{22} + \hat{J}_{33} < 0$, and hence that the signs of the coefficients of (18) are as follows

$$\begin{matrix} 1 & -tr(\hat{J}) & \hat{J}_{11} + \hat{J}_{22} + \hat{J}_{33} & -\det(\hat{J}) \\ (+) & (?) & (-) & (-) \end{matrix}.$$

Therefore, it has the number of sign changes in polynomial's coefficients is independent of the sign of the trace and is equal to one. Based on the Descartes' rule of signs, I can thus conclude that $\hat{J}$ has 1 eigenvalue with positive real part and 2 eigenvalues with negative real part, and hence that, under the assumptions of Lemma 1, the unique steady-state equilibrium is locally indeterminate. This concludes the proof of Proposition 1.

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