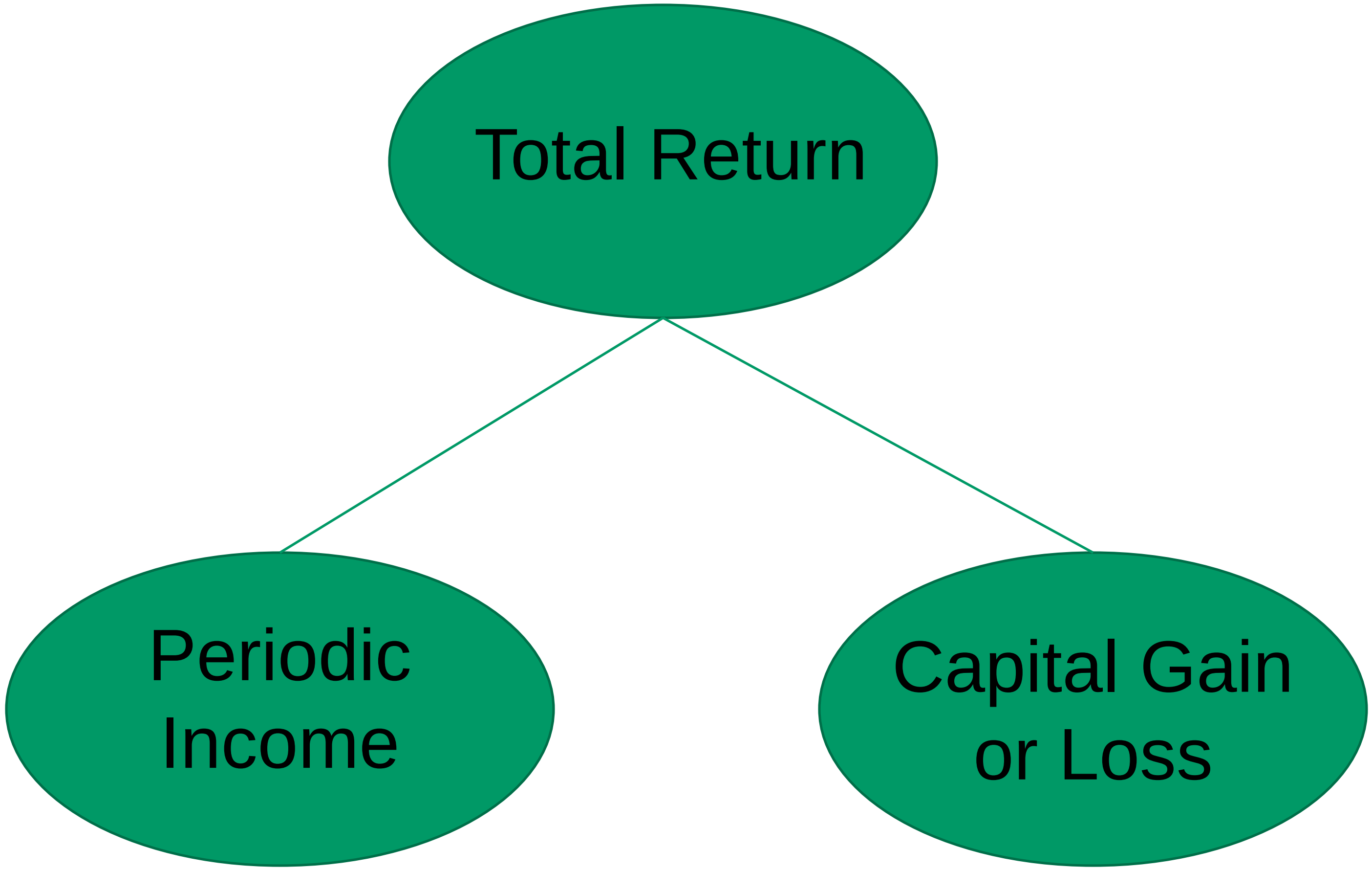
CHAPTER 5
PORTFOLIO RISK AND RETURN: PART I

Presenter
Venue
Date

CFA Institute

RETURN ON FINANCIAL ASSETS



HOLDING PERIOD RETURN

A *holding period return* is the return from holding an asset for a single specified period of time.

$$R = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t}{P_{t-1}}$$

$$= \text{Capital gain} + \text{Dividend yield}$$

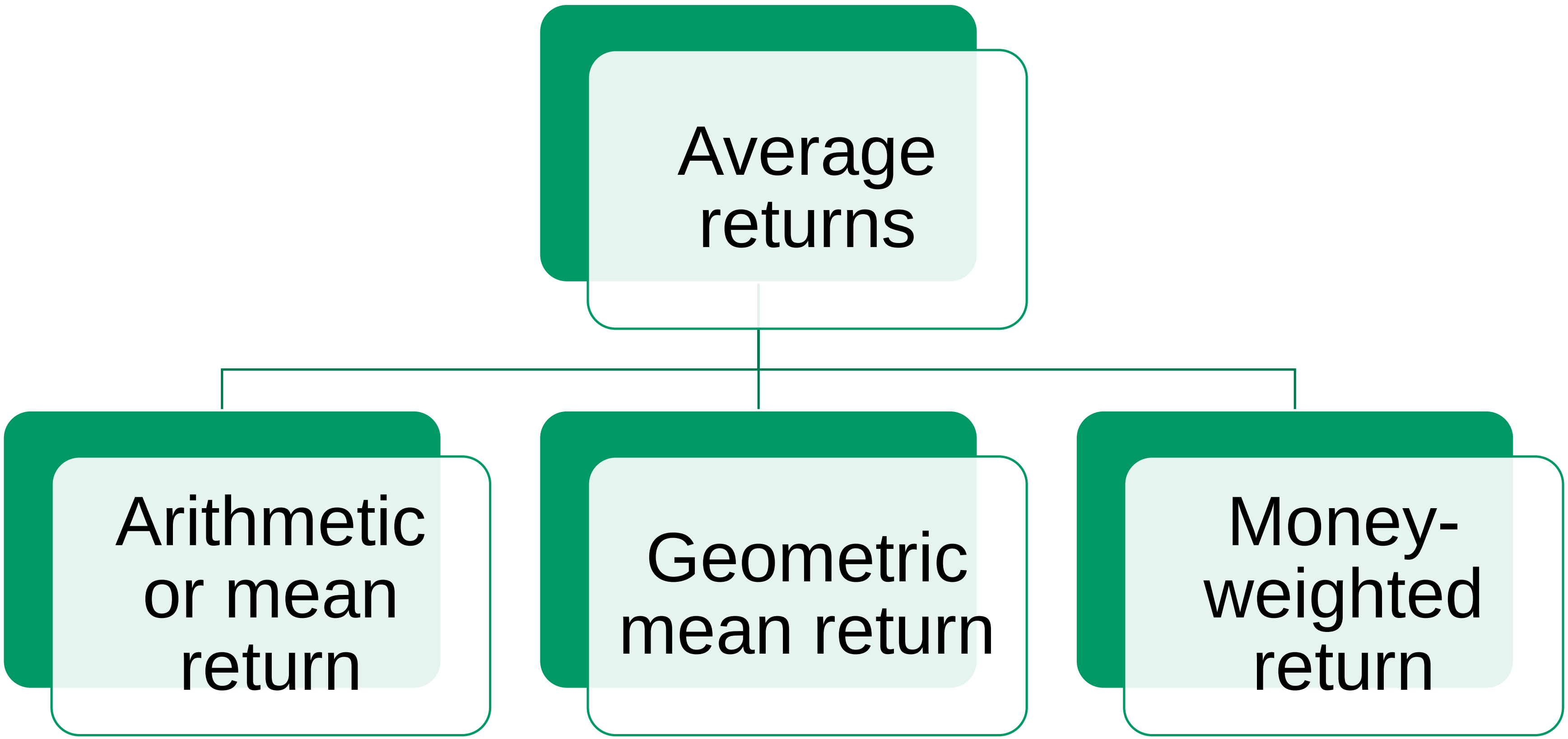
$$R = \frac{105 - 100}{100} + \frac{2}{100} = 5\% + 2\% = 7\%$$

HOLDING PERIOD RETURNS

What is the 3-year holding period return if the annual returns are 7%, 9%, and –5%?

$$R = \left[\left(1+R_1\right)\times\left(1+R_2\right)\times\left(1+R_3\right)\right]-1$$
$$= \left[\left(1+.07\right)(1+.09)(1+-.05)\right]-1 \approx .1080 = 10.80\%$$

AVERAGE RETURNS



ARITHMETIC OR MEAN RETURN

The *arithmetic* or *mean return* is the simple average of all holding period returns.

$$\overline{R}_i = \frac{R_{i1} + R_{i2} + \cdots + R_{iT-1} + R_{iT}}{T} = \frac{1}{T}\sum_{t=1}^{T} R_{it}$$

$$\overline{R}_i = \frac{-50\% + 35\% + 27\%}{3} = 4\%$$

GEOMETRIC MEAN RETURN

The *geometric mean return* accounts for the compounding of returns.

$$\overline{R}_{Gi} = \sqrt[T]{(1+R_{i1})\times(1+R_{i2})\times\cdots\times(1+R_{iT-1})\times(1+R_{iT})} - 1$$

$$= \sqrt[T]{\prod_{t=1}^{T}(1+R_{it})} - 1$$

$$R_{Gi} = \sqrt[3]{(1-.50)\times(1+.35)\times(1+.27)} - 1 \approx -5.0\%$$

MONEY-WEIGHTED RETURN

Year	**1**	**2**	**3**
Balance from previous year	€0	€50	€1,000
New investment by the investor (cash inflow for the mutual fund) at the start of the year	100	950	0
Net balance at the beginning of year	100	1,000	1,000
Investment return for the year	–50%	35%	27%
Investment gain (loss)	–50	350	270
Withdrawal by the investor (cash outflow for the mutual fund) at the end of the year	0	–350	0
Balance at the end of year	€50	€1,000	€1,270

$$\frac{CF_0}{(1+IRR)^0}+\frac{CF_1}{(1+IRR)^1}+\frac{CF_2}{(1+IRR)^2}+\frac{CF_3}{(1+IRR)^3}=0$$

$$\frac{-100}{1}+\frac{-950}{(1+IRR)^1}+\frac{+350}{(1+IRR)^2}+\frac{+1270}{(1+IRR)^3}=0$$

$$IRR = 26.11\%$$

ANNUALIZED RETURN

$$r_{annual} = \left(1 + r_{period}\right)^{c} - 1$$

$c:$ number of periods in a year

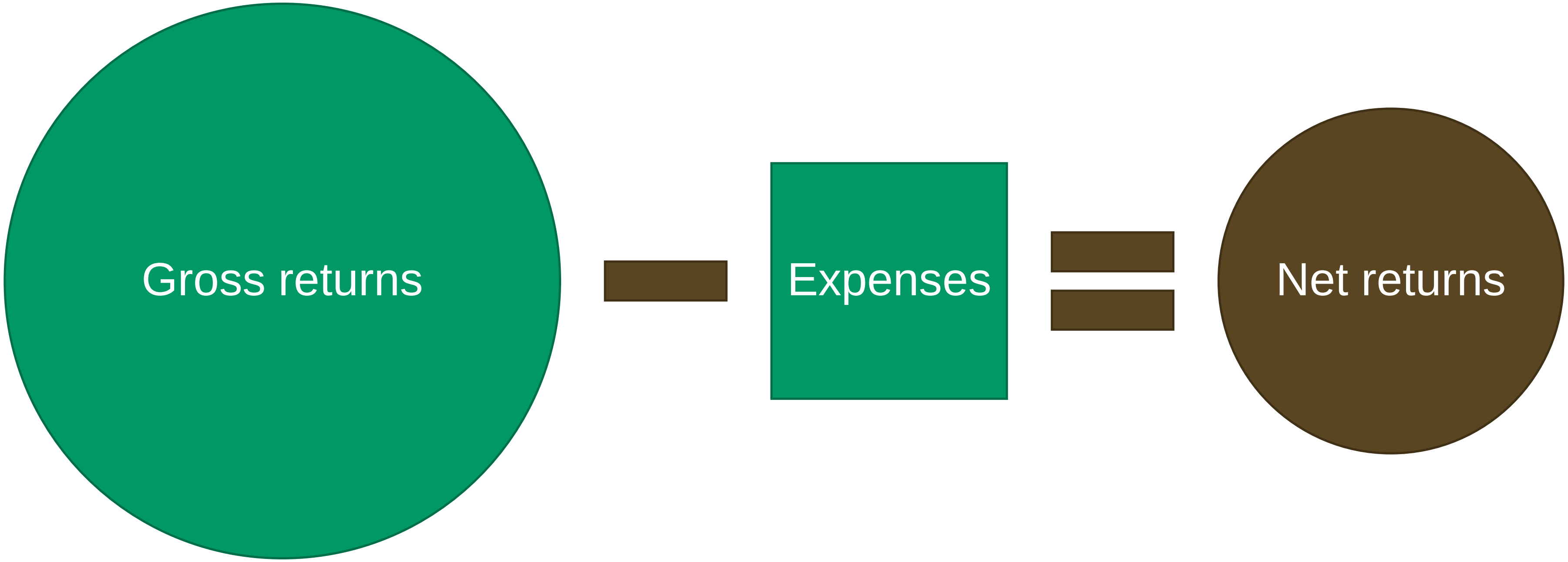
Weekly return of 0.20%:

$$r_{annual} = (1 + 0.002)^{52} - 1 = .1095 = 10.95\%$$

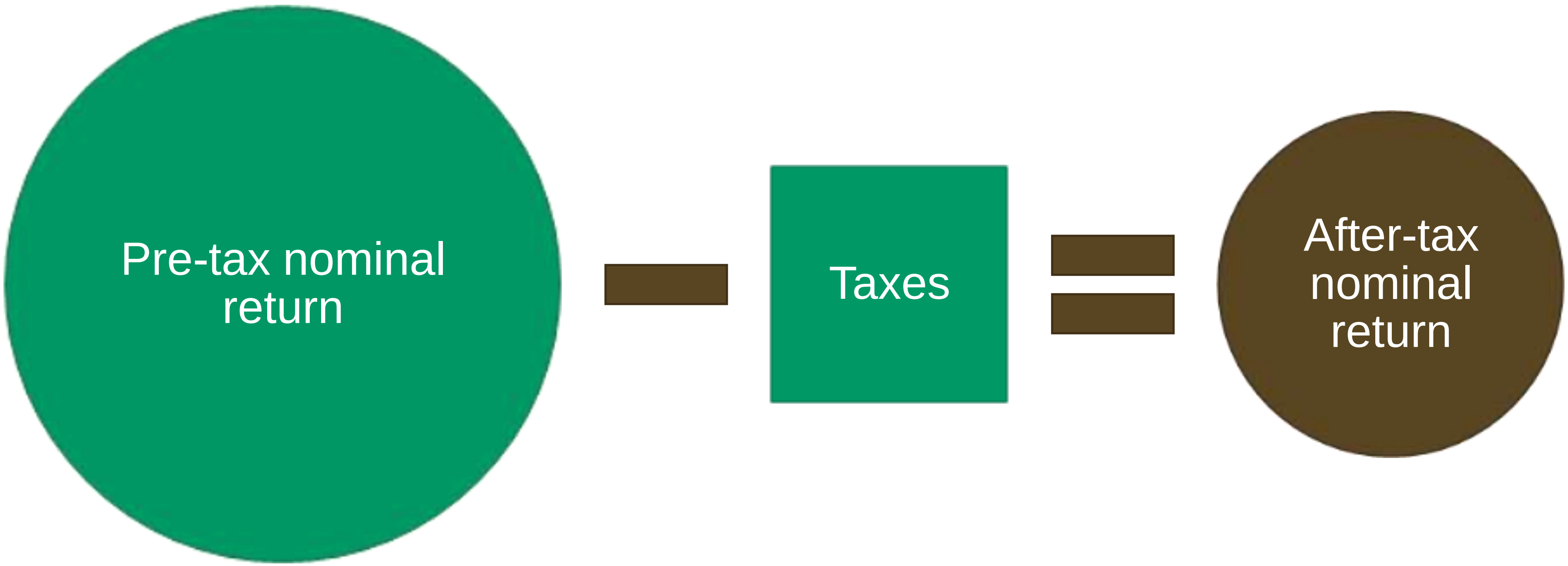
18-month return of 20%:

$$r_{annual} = (1 + 0.20)^{2/3} - 1 = 0.1292 = 12.92\%$$

GROSS AND NET RETURNS



PRE-TAX AND AFTER-TAX NOMINAL RETURN



NOMINAL RETURNS AND REAL RETURNS

$$(1+r)=\left(1+r_{rF}\right)\times(1+\pi)\times(1+RP)=(1+0.03)\times(1+0.02)\times(1+0.05)$$

$$r=10.313\%$$

$$\left(1+r_{real}\right)=\left(1+r_{rF}\right)\times(1+RP)=(1+0.03)\times(1+0.05)$$

$$r_{real}=8.15\%$$

$$\left(1+r_{real}\right)=(1+r)\div(1+\pi)=(1+0.10313)\div(1+0.02)$$

$$r_{real}=8.15\%$$

VARIANCE AND STANDARD DEVIATION OF A SINGLE ASSET

Population

Sample

$$\sigma^2 = \frac{\sum_{t=1}^{T} (R_t - \mu)^2}{T}$$

$$\sigma = \sqrt{\sigma^2}$$

$$s^2 = \frac{\sum_{t=1}^{T} (R_t - \overline{R})^2}{T - 1}$$

$$s = \sqrt{s^2}$$

VARIANCE OF A PORTFOLIO OF ASSETS

Variance can be determined for *N* securities in a portfolio using the formulas below. $Cov(R_i, R_j)$ is the covariance of returns between security *i* and security *j* and can be expressed as the product of the correlation between the two returns ($\rho_{i,j}$) and the standard deviations of the two assets, $Cov(R_i, R_j) = \rho_{i,j}\,\sigma_i\sigma_j$.

$$\sigma_P^2 = Var(R_P) = Var\left(\sum_{i=1}^{N} w_i R_i\right)$$

$$= \sum_{i,j=1}^{N} w_i w_j Cov(R_i, R_j)$$

$$= \sum_{i=1}^{N} w_i^2 Var(R_i) + \sum_{i,j=1,\, i\neq j}^{N} w_i w_j Cov(R_i, R_j)$$

EXAMPLE 5-4 RETURN AND RISK OF A TWO-ASSET PORTFOLIO

Assume that as a U.S. investor, you decide to hold a portfolio with 80 percent invested in the S&P 500 U.S. stock index and the remaining 20 percent in the MSCI Emerging Markets index. The expected return is 9.93 percent for the S&P 500 and 18.20 percent for the Emerging Markets index. The risk (standard deviation) is 16.21 percent for the S&P 500 and 33.11 percent for the Emerging Markets index. What will be the portfolio’s expected return and risk given that the covariance between the S&P 500 and the Emerging Markets index is 0.0050?

EXAMPLE 5-4 RETURN AND RISK OF A TWO-ASSET PORTFOLIO (CONTINUED)

$$R_P = w_1 R_1 + w_2 R_2 = (0.80 \times 0.0993) + (0.20 \times 0.1820)$$
$$= 0.1158 = 11.58\%$$

$$\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 Cov(R_1, R_2)$$
$$= (0.80^2 \times 0.1621^2) + (0.20^2 \times 0.3311^2) + (2 \times 0.80 \times 0.20 \times 0.0050)$$
$$= 0.02281$$

$$\sigma_P = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 Cov(R_1, R_2)}$$
$$= \sqrt{0.02281} = 0.1510 = 15.10\%$$

EXAMPLE 5-4 RETURN AND RISK OF A TWO-ASSET PORTFOLIO (CONTINUED)

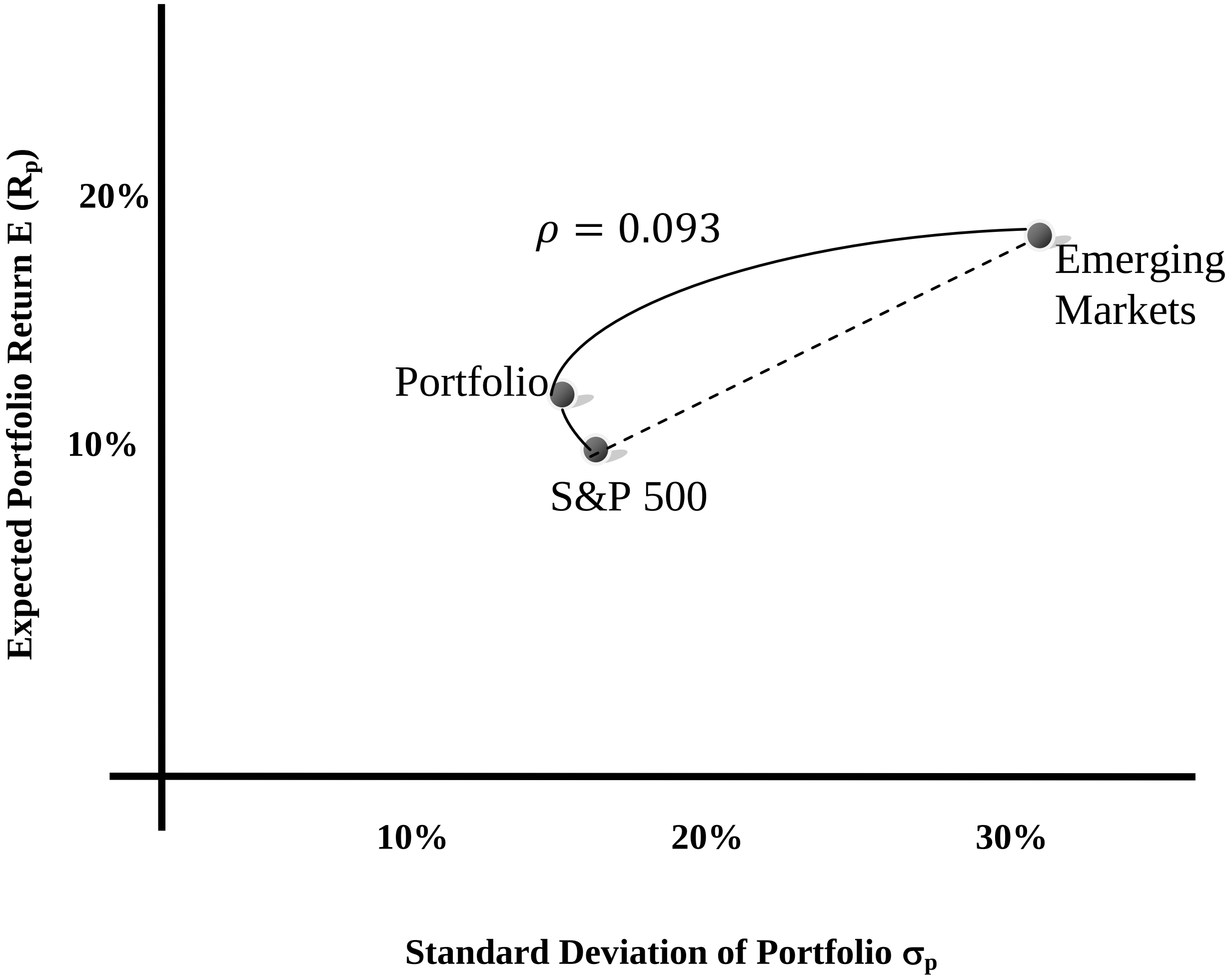


EXHIBIT 5-5 RISK AND RETURN FOR U.S. ASSET CLASSES BY DECADE (%)

		1930s	1940s	1950s	1960s	1970s	1980s	1990s	2000s*	1926–2008
Large company stocks	Return	–0.1	9.2	19.4	7.8	5.9	17.6	18.2	–3.6	9.6
	Risk	41.6	17.5	14.1	13.1	17.2	19.4	15.9	15.0	20.6
Small company stocks	Return	1.4	20.7	16.9	15.5	11.5	15.8	15.1	4.1	11.7
	Risk	78.6	34.5	14.4	21.5	30.8	22.5	20.2	24.5	33.0
Long-term corporate bonds	Return	6.9	2.7	1.0	1.7	6.2	13.0	8.4	8.2	5.9
	Risk	5.3	1.8	4.4	4.9	8.7	14.1	6.9	11.3	8.4
Long-term government bonds	Return	4.9	3.2	–0.1	1.4	5.5	12.6	8.8	10.5	5.7
	Risk	5.3	2.8	4.6	6.0	8.7	16.0	8.9	11.7	9.4
Treasury bills	Return	0.6	0.4	1.9	3.9	6.3	8.9	4.9	3.1	3.7
	Risk	0.2	0.1	0.2	0.4	0.6	0.9	0.4	0.5	3.1
Inflation	Return	–2.0	5.4	2.2	2.5	7.4	5.1	2.9	2.5	3.0
	Risk	2.5	3.1	1.2	0.7	1.2	1.3	0.7	1.6	4.2

Returns are measured as annualized geometric mean returns.
Risk is measured by annualizing monthly standard deviations.
* Through 31 December 2008.
Source: 2009 Ibbotson SBBI Classic Yearbook (Tables 2-1, 6-1, C-1 to C-7).

EXHIBIT 5-7 NOMINAL RETURNS, REAL RETURNS, AND RISK PREMIUMS FOR ASSET CLASSES (1900–2008)

		United States			**World**			**World excluding U.S.**		
	Asset	*GM*	*AM*	*SD*	*GM*	*AM*	*SD*	*GM*	*AM*	*SD*
Nominal Returns	Equities	9.2%	11.1%	20.2%	8.4%	9.8%	17.3%	7.9%	9.7%	20.1%
	Bonds	5.2%	5.5%	8.3%	4.8%	5.2%	8.6%	4.2%	5.0%	13.0%
	Bills	4.0%	4.0%	2.8%	–	–	–	–	–	–
	Inflation	3.0%	3.1%	4.9%	–	–	–	–	–	–
Real Returns	Equities	6.0%	8.0%	20.4%	5.2%	6.7%	17.6%	4.8%	6.7%	20.2%
	Bonds	2.2%	2.6%	10.0%	1.8%	2.3%	10.3%	1.2%	2.2%	14.1%
	Bills	1.0%	1.1%	4.7%	–	–	–	–	–	–
Premiums	Equities vs. bills	5.0%	7.0%	19.9%	–	–	–	–	–	–
	Equities vs. bonds	3.8%	5.9%	20.6%	3.4%	4.6%	15.6%	3.5%	4.7%	15.9%
	Bonds vs. bills	1.1%	1.4%	7.9%	–	–	–	–	–	–

All returns are in percent per annum measured in US$. GM = geometric mean, AM = arithmetic mean, SD = standard deviation.

"World" consists of 17 developed countries: Australia, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, South Africa, Spain, Sweden, Switzerland, United Kingdom, and the United States. Weighting is by each country's relative market capitalization size.

Sources: Credit Suisse Global Investment Returns Sourcebook, 2009. Compiled from tables 62, 65, and 68. T-bills and inflation rates are not available for the world and world excluding the United States.

IMPORTANT ASSUMPTIONS OF MEAN-VARIANCE ANALYSIS

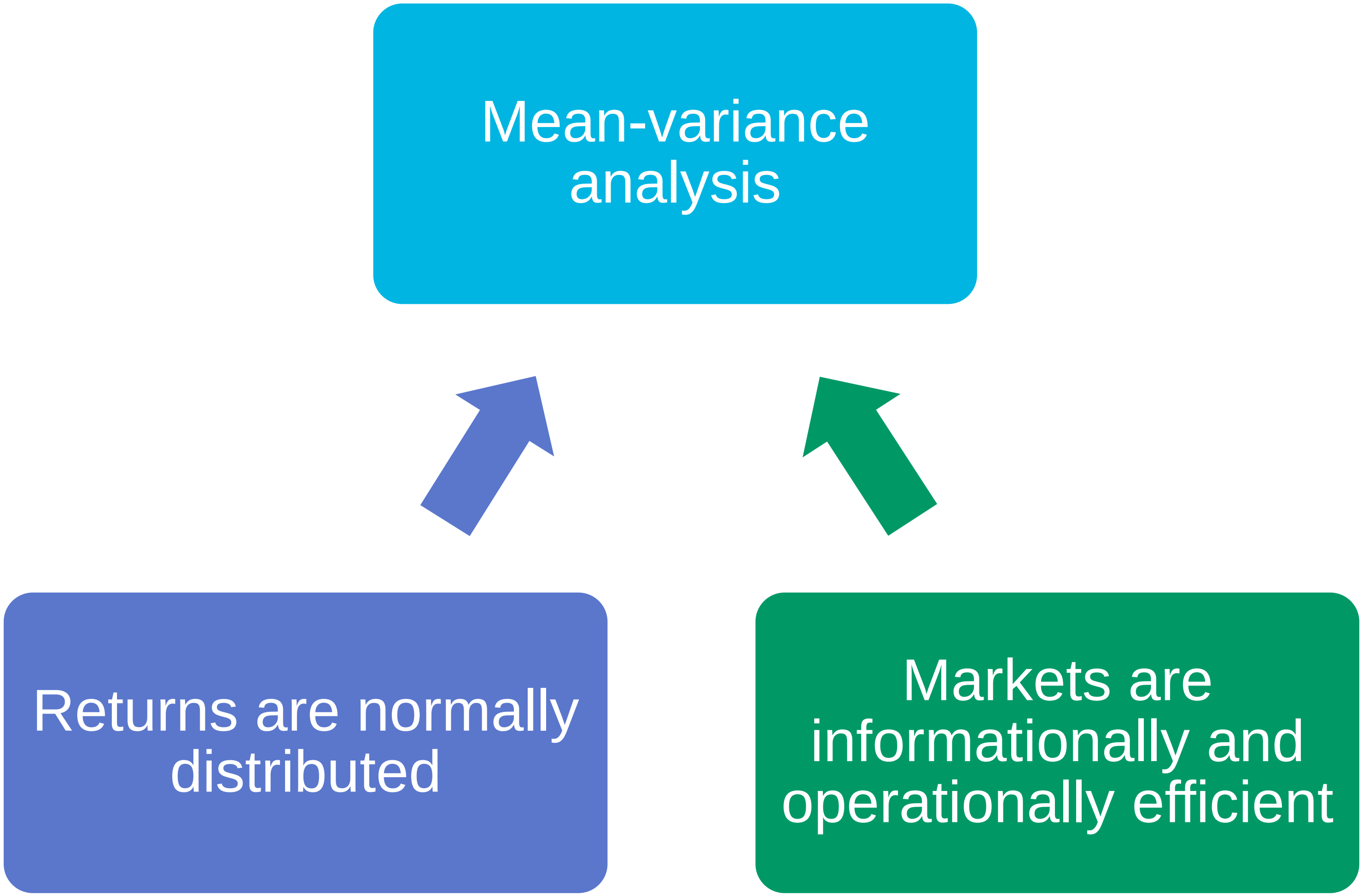
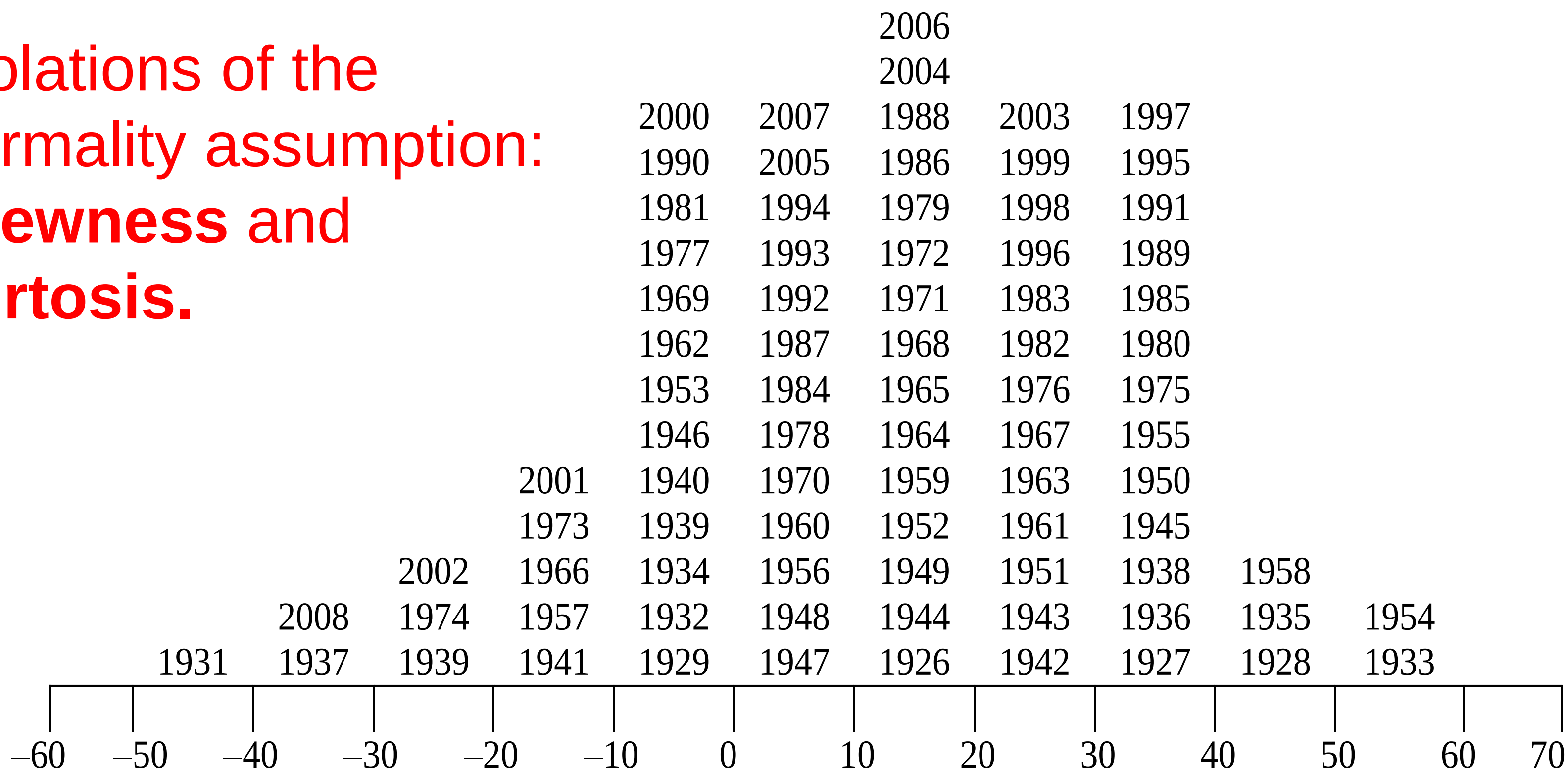


EXHIBIT 5-9 HISTOGRAM OF U.S. LARGE COMPANY STOCK RETURNS, 1926-2008

Violations of the normality assumption: **skewness** and **kurtosis.**



UTILITY THEORY

Expected return

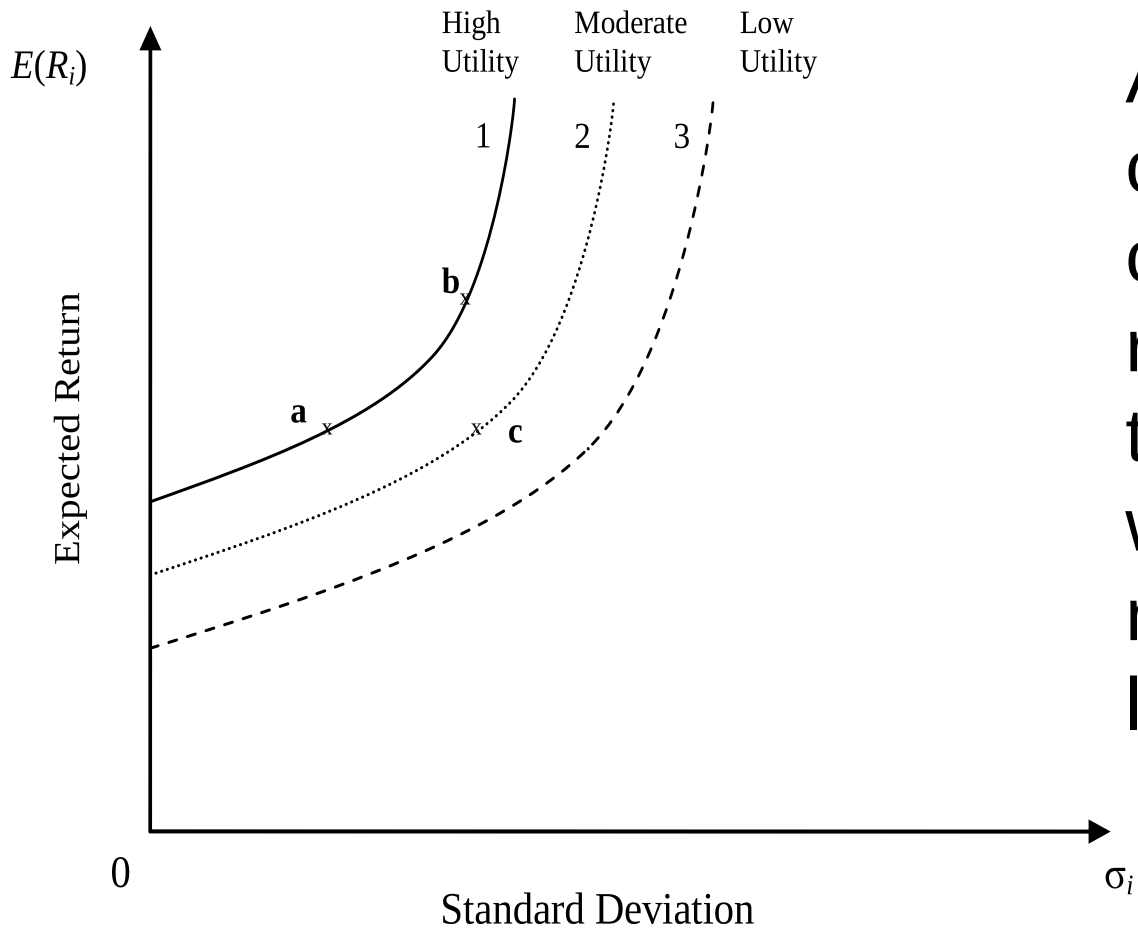
Variance or risk

$$U = E(r) - \frac{1}{2} A\sigma^2$$

Utility of an investment

Measure of risk tolerance or risk aversion

INDIFFERENCE CURVES



An indifference curve plots the combination of risk-return pairs that an investor would accept to maintain a given level of utility.

PORTFOLIO EXPECTED RETURN AND RISK ASSUMING A RISK-FREE ASSET

Assume a portfolio of two assets, a risk-free asset and a risky asset. Expected return and risk for that portfolio can be determined using the following formulas:

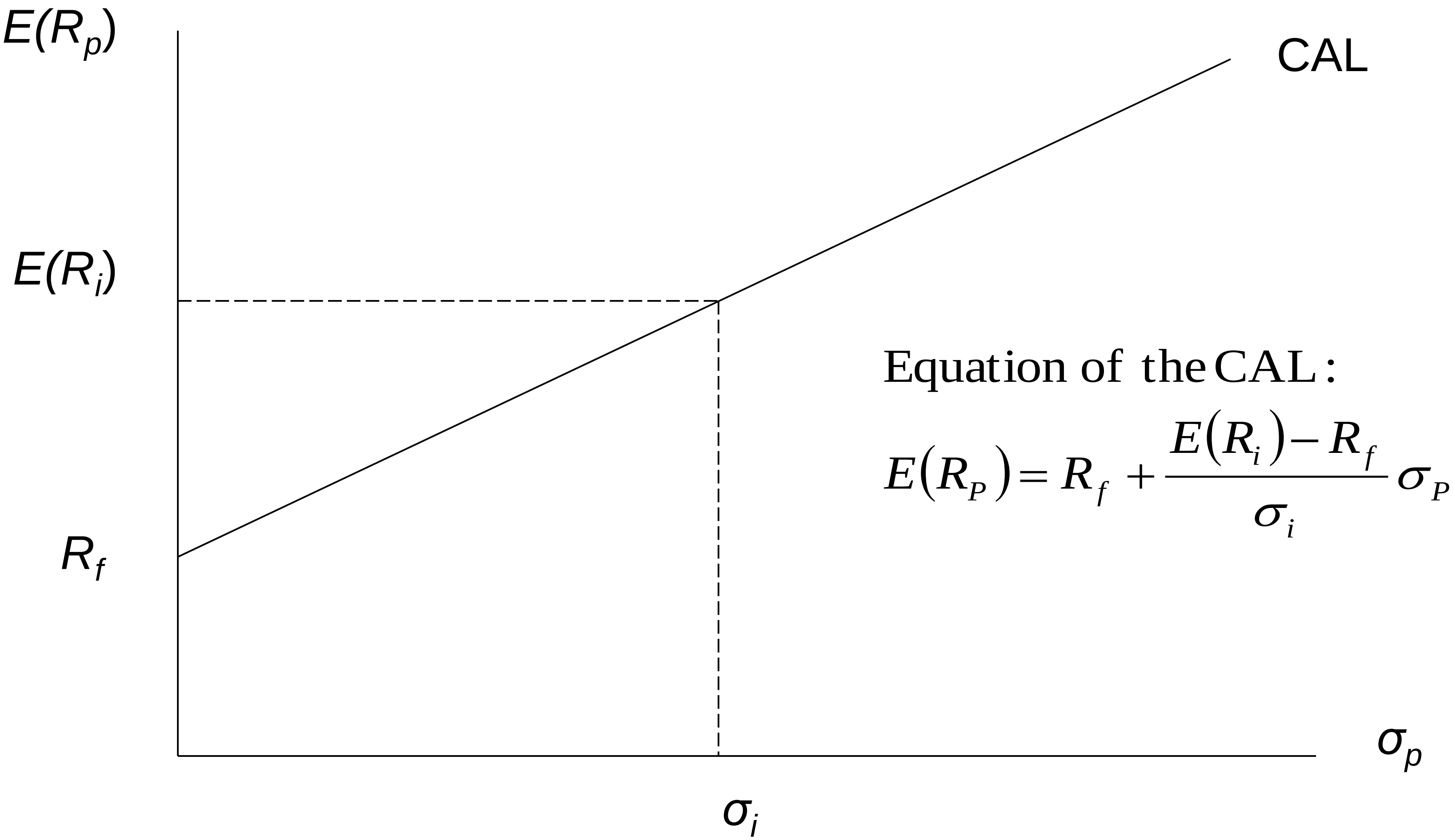
$$E(R_P) = w_1 R_f + (1 - w_1) E(R_i)$$

$$\sigma_P^2 = w_1^2 \sigma_f^2 + (1 - w_1)^2 \sigma_i^2 + 2 w_1 (1 - w_1) \rho_{fi} \sigma_f \sigma_i$$

$$= (1 - w_1)^2 \sigma_i^2$$

$$\sigma_P = \sqrt{(1 - w_1)^2 \sigma_i^2} = (1 - w_1) \sigma_i$$

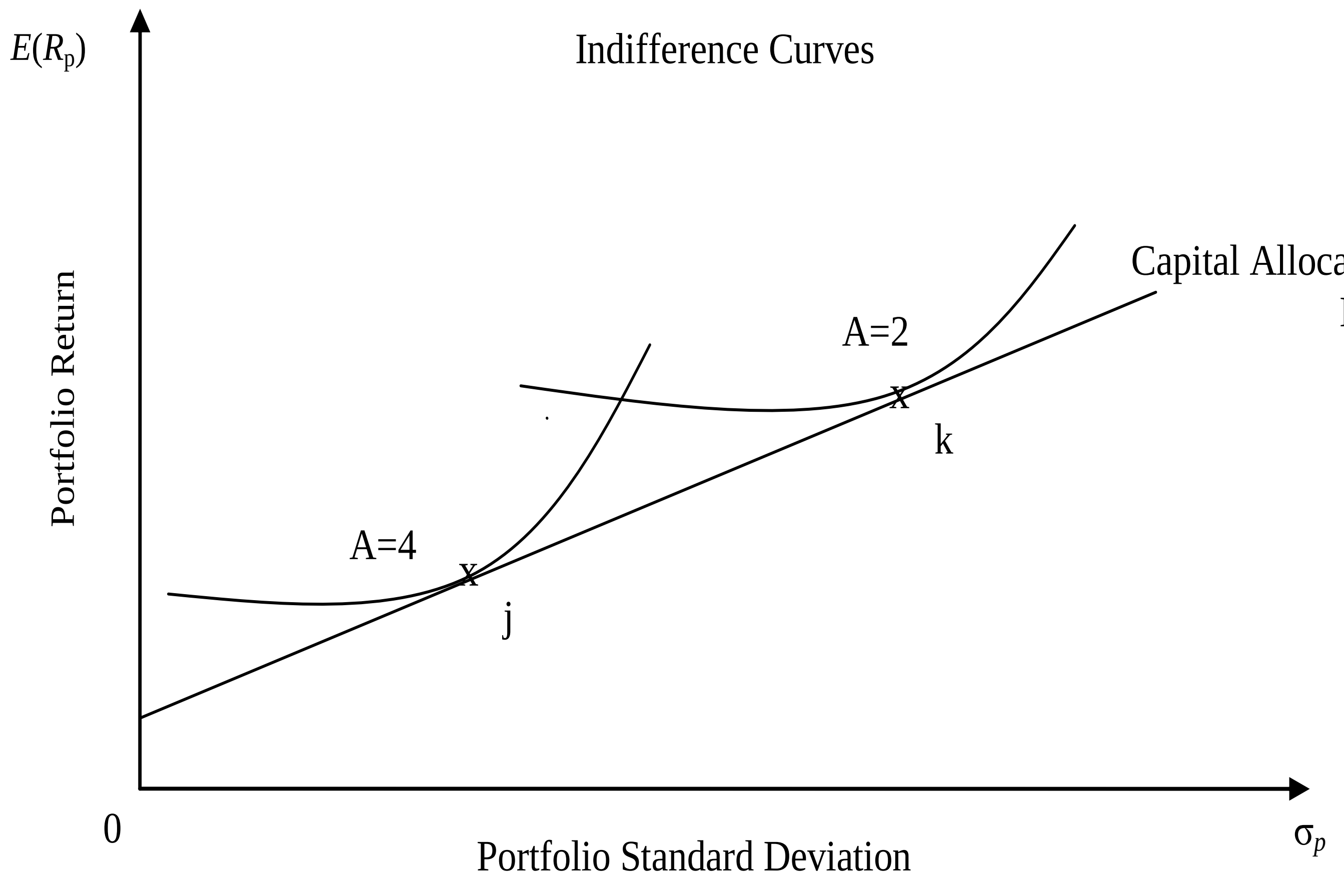
THE CAPITAL ALLOCATION LINE (CAL)



Equation of the CAL:

$$E(R_P) = R_f + \frac{E(R_i) - R_f}{\sigma_i}\sigma_P$$

EXHIBIT 5-15 PORTFOLIO SELECTION FOR TWO INVESTORS WITH VARIOUS LEVELS OF RISK AVERSION



CORRELATION AND PORTFOLIO RISK

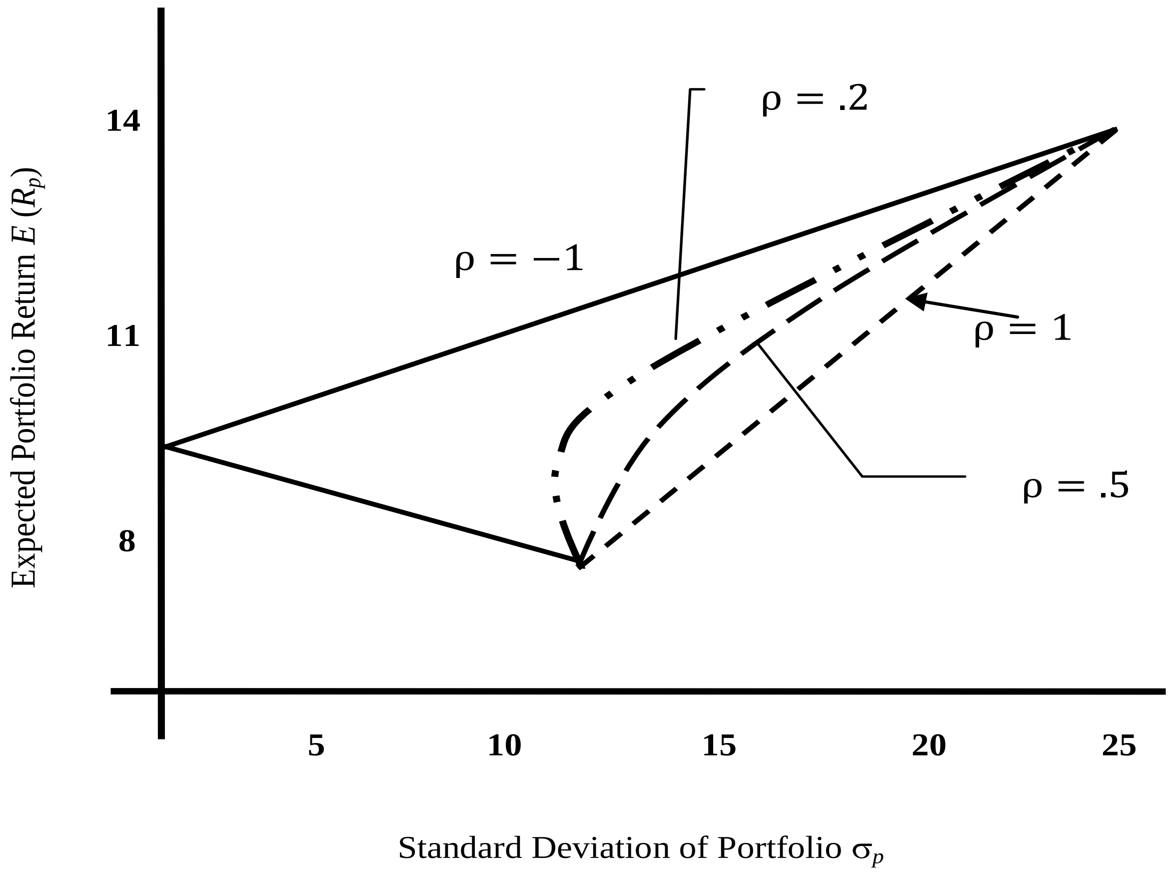
Correlation between assets in the portfolio

Portfolio risk

EXHIBIT 5-16 RELATIONSHIP BETWEEN RISK AND RETURN

Weight in Asset 1	Portfolio Return	Portfolio Risk with Correlation of			
		1.0	0.5	0.2	–1.0
0%	15.0	25.0	25.0	25.0	25.0
10%	14.2	23.7	23.1	22.8	21.3
20%	13.4	22.4	21.3	20.6	17.6
30%	12.6	21.1	19.6	18.6	13.9
40%	11.8	19.8	17.9	16.6	10.2
50%	11.0	18.5	16.3	14.9	6.5
60%	10.2	17.2	15.0	13.4	2.8
70%	9.4	15.9	13.8	12.3	0.9
80%	8.6	14.6	12.9	11.7	4.6
90%	7.8	13.3	12.2	11.6	8.3
100%	7.0	12.0	12.0	12.0	12.0

EXHIBIT 5-17 RELATIONSHIP BETWEEN RISK AND RETURN



AVENUES FOR DIVERSIFICATION

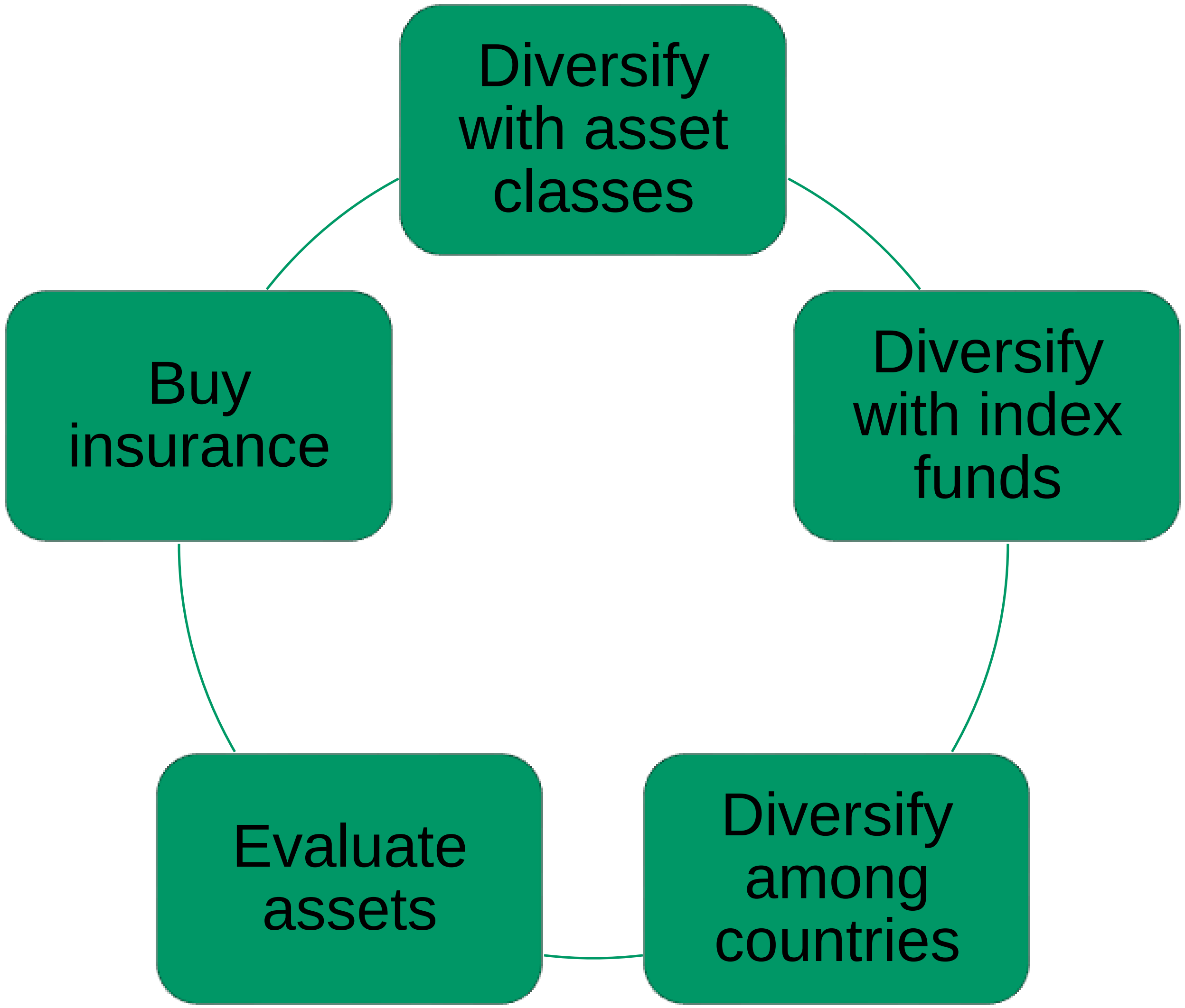


EXHIBIT 5-22 MINIMUM-VARIANCE FRONTIER

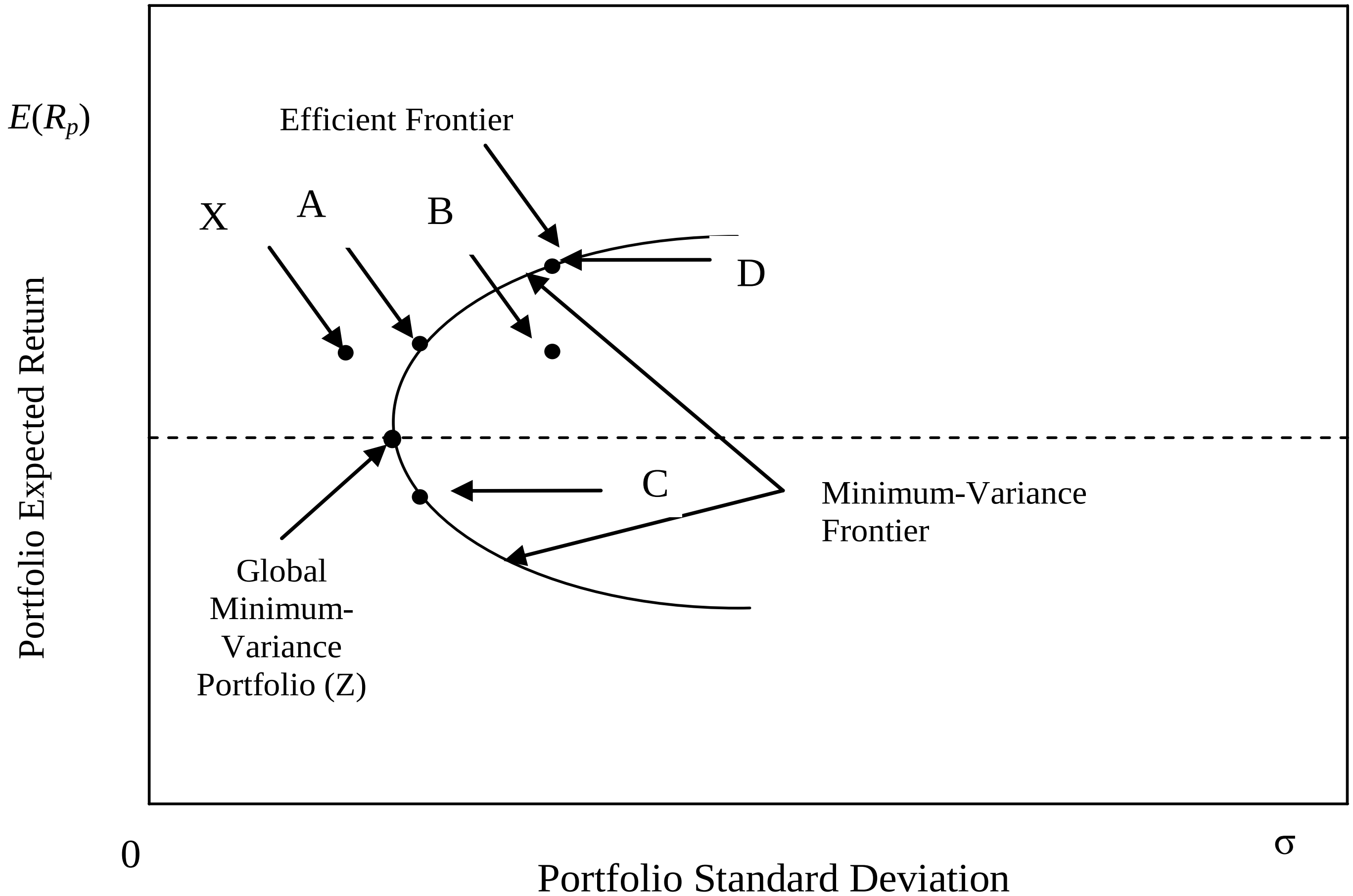
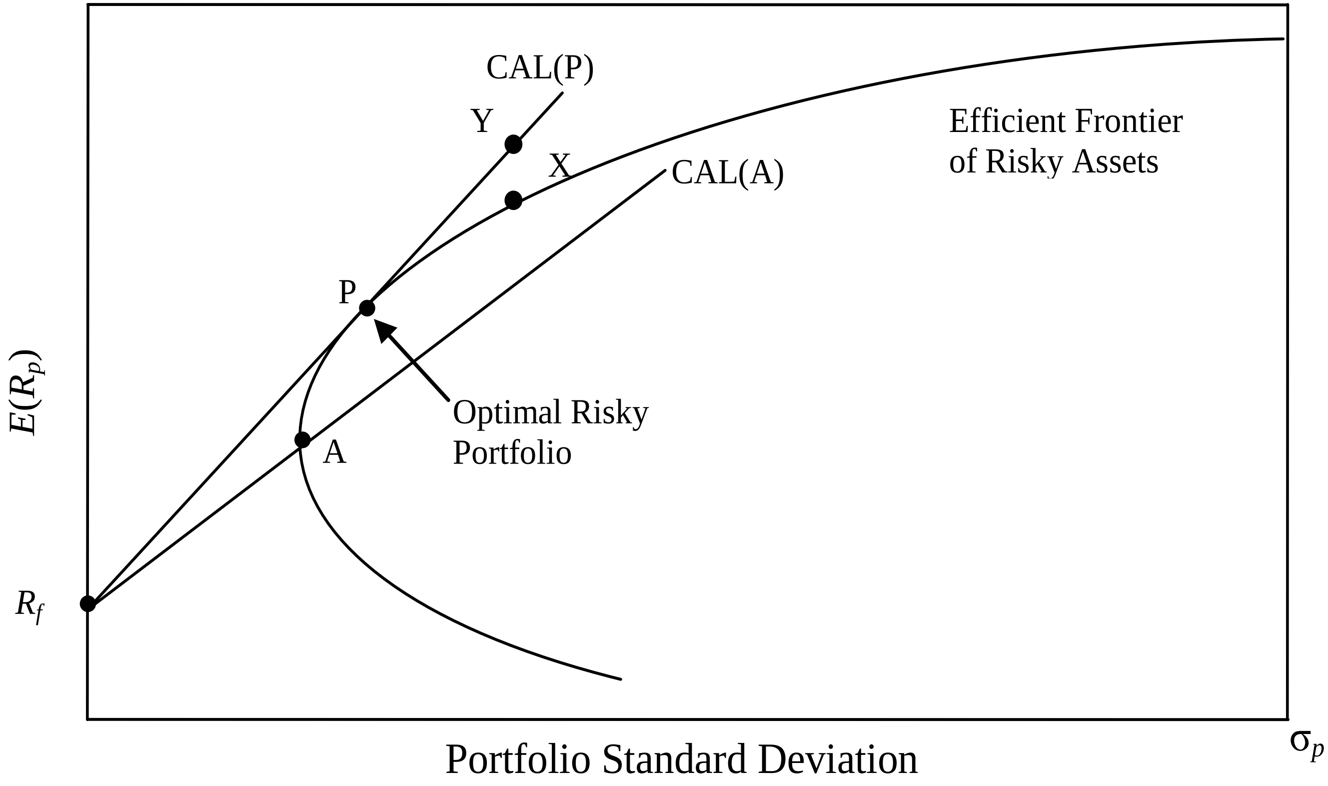


EXHIBIT 5-23 CAPITAL ALLOCATION LINE AND OPTIMAL RISKY PORTFOLIO



CAL(P) is the optimal capital allocation line and portfolio P is the optimal risky portfolio.

THE TWO-FUND SEPARATION THEOREM

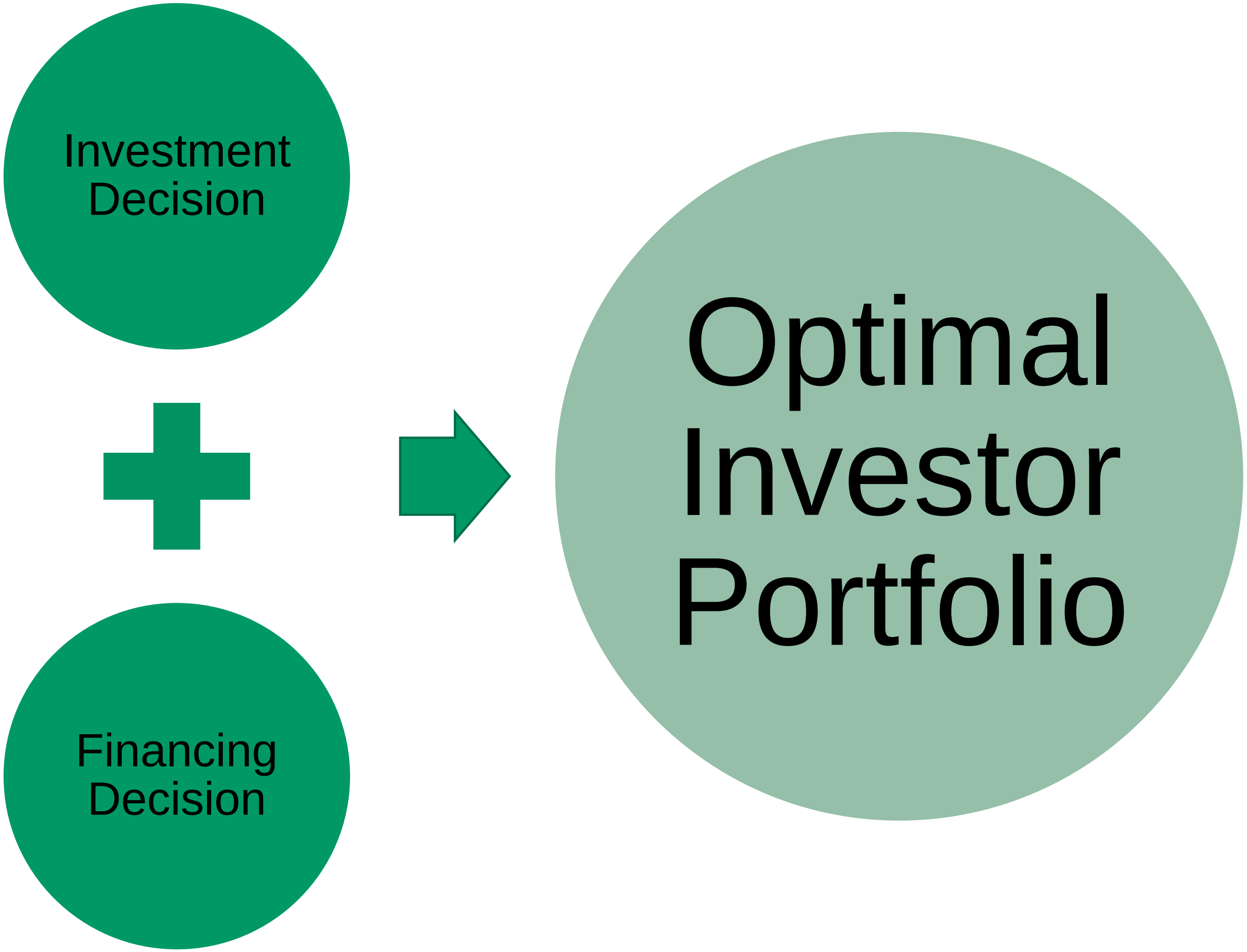
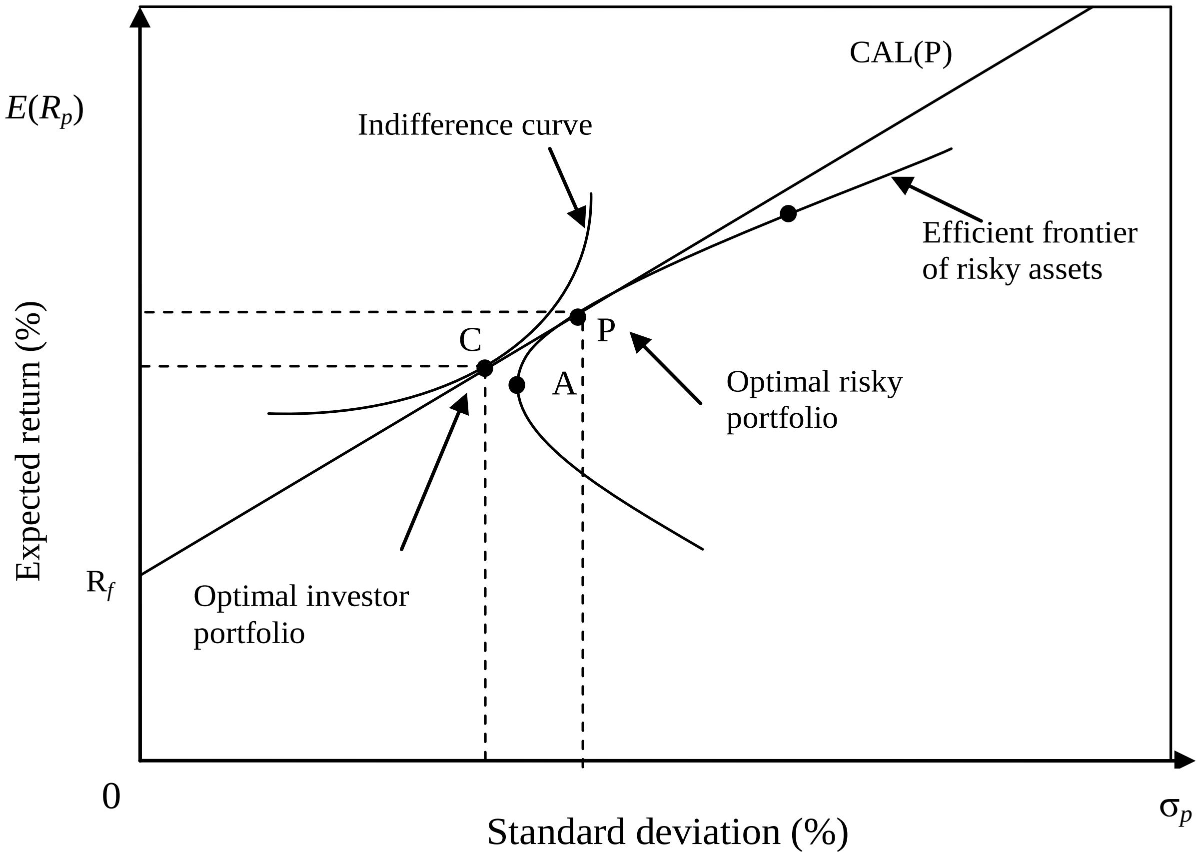


EXHIBIT 5-25 OPTIMAL INVESTOR PORTFOLIO



Given the investor's indifference curve, portfolio C on CAL(P) is the optimal portfolio.

SUMMARY

- Different approaches for determining return
- Risk measures for individual assets and portfolios
- Market evidence on the risk-return tradeoff
- Correlation and portfolio risk
- The risk-free asset and the optimal risky portfolio
- Utility theory and the optimal investor portfolio