CHAPTER 6
PORTFOLIO RISK AND RETURN: PART II

Presenter
Venue
Date

CFA Institute

FORMULAS FOR PORTFOLIO RISK AND RETURN

$$E\left(R_p\right)=\sum_{i=1}^{N} w_i E\left(R_i\right)$$

$$\sigma_p^2 = \sum_{i=1,\ j=1}^{N} w_i w_j \mathrm{Cov}(i, j)$$

$$\sum_{i=1}^{N} w_i = 1$$

$$\text{Given: } \mathrm{Cov}(i, j) = \rho_{ij}\sigma_i\sigma_j \text{ and } \mathrm{Cov}(i,i) = \sigma_i^2$$

$$\text{Then: } \sigma_p^2 = \sum_{i=1}^{N} w_i^2\sigma_i^2 + \sum_{i,\ j=1, i\neq j}^{N} w_i w_j \rho_{ij}\sigma_i\sigma_j$$

$$\sigma_p = \sqrt{\sigma_p^2}$$

EXHIBIT 6-1 PORTFOLIO RISK AND RETURN

Portfolio	Weight in Asset 1	Weight in Asset 2	Portfolio Return	Portfolio Standard Deviation
X	25.0%	75.0%	6.25%	9.01%
Y	50.0	50.0	7.50	11.18
Z	75.0	25.0	8.75	15.21
Return	10.0%	5.0%		
Standard deviation	20.0%	10.0%		
Correlation between Assets 1 and 2		0.0		

$$\sigma_X = \sqrt{(.25^2)(.20^2) + (.75^2)(.10)^2 + (.25)(0)(.20)(.10) + (.75)(0)(.10)(.20)} \approx 9.01\%$$

PORTFOLIO OF RISK-FREE AND RISKY ASSETS

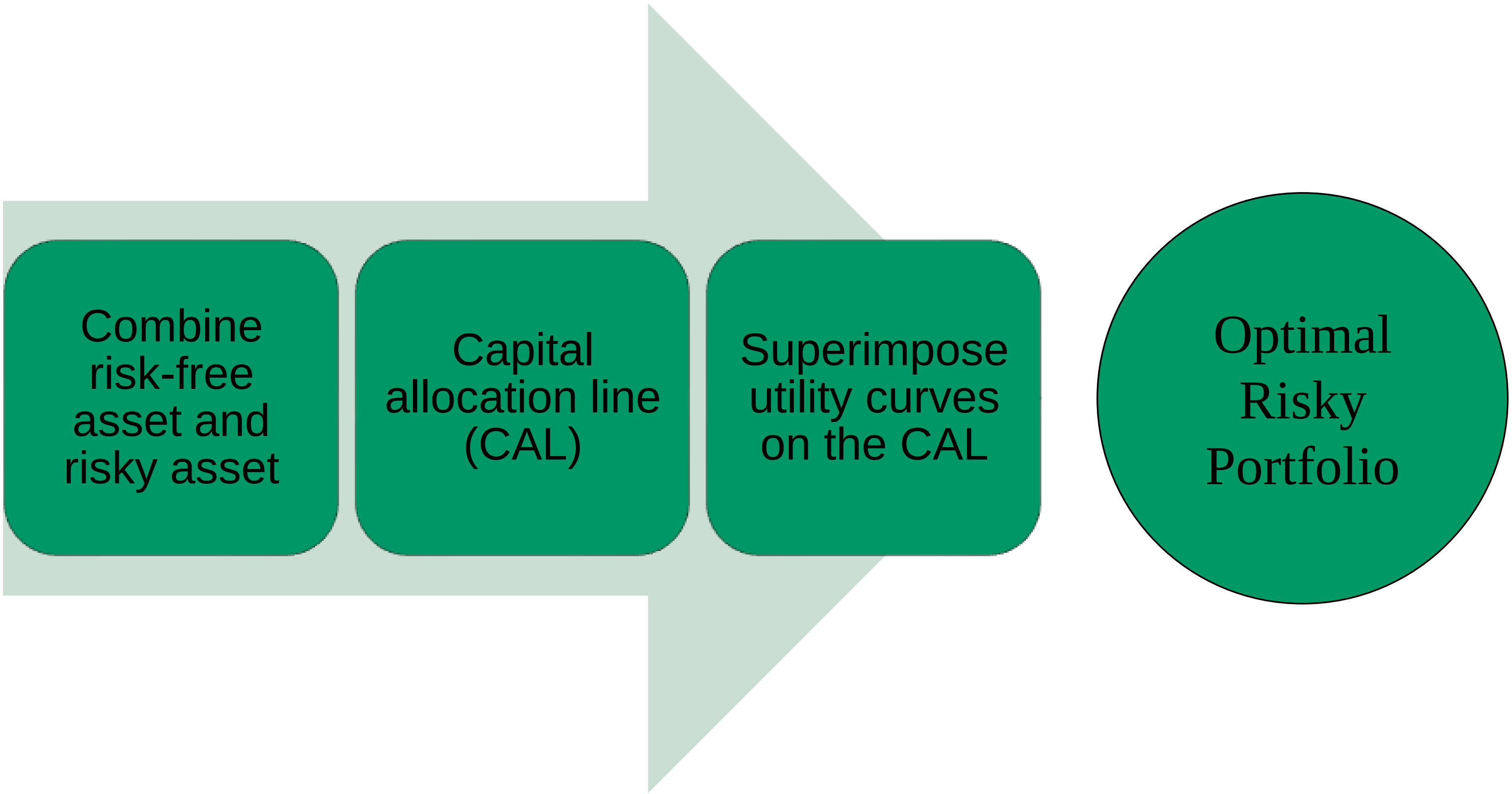
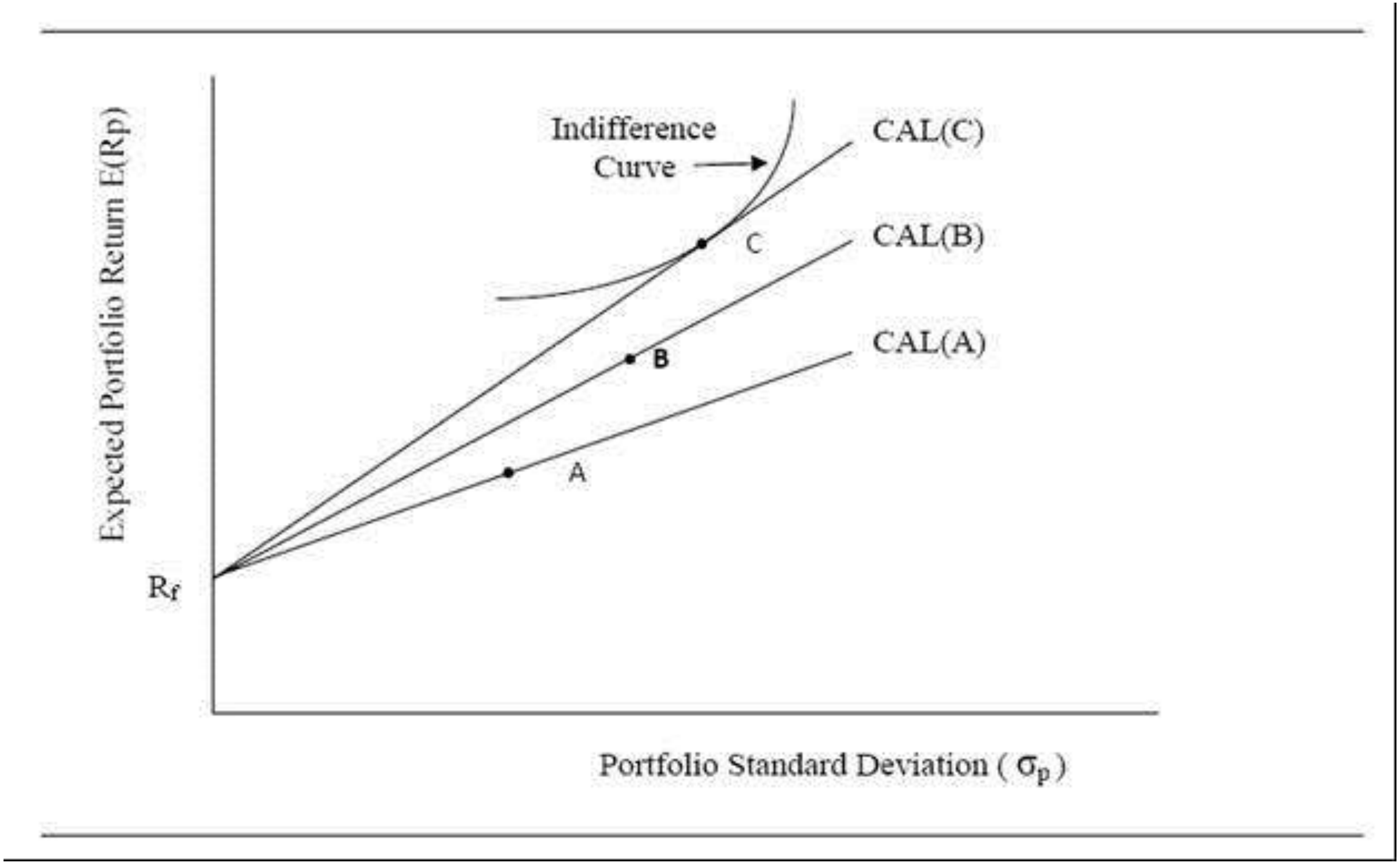


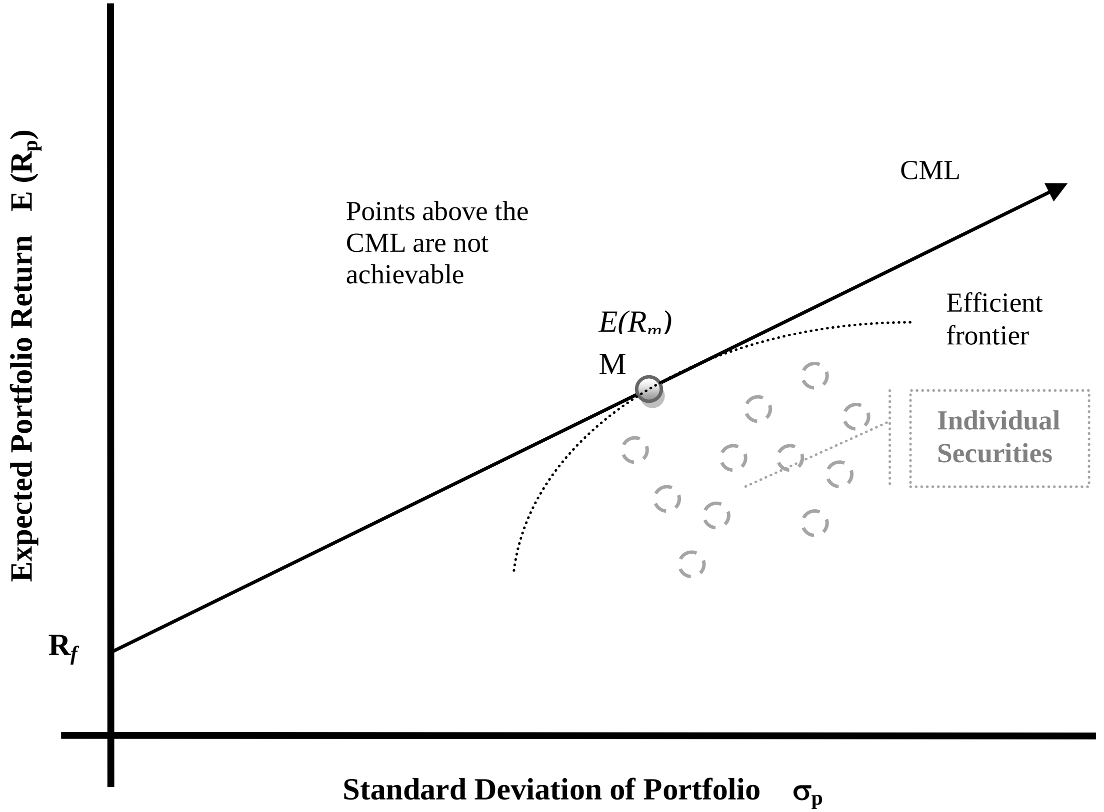
EXHIBIT 6-2 RISK-FREE ASSET AND PORTFOLIO OF RISKY ASSETS



DOES A UNIQUE OPTIMAL RISKY PORTFOLIO EXIST?

CAPITAL MARKET LINE (CML)

EXHIBIT 6-3 CAPITAL MARKET LINE



CML: RISK AND RETURN

$$E\left(R_p\right) = w_1 R_f + \left(1 - w_1\right) E\left(R_m\right)$$

$$\sigma_p = \left(1 - w_1\right) \sigma_m$$

By substitution, $E(R_p)$ can be expressed in terms of σ_p, and this yields the equation for the CML:

$$E\left(R_p\right) = R_f + \left(\frac{E\left(R_m\right) - R_f}{\sigma_m}\right) \times \sigma_p$$

EXAMPLE 6-1 RISK AND RETURN ON THE CML

Mr. Miles is a first time investor and wants to build a portfolio using only U.S. T-bills and an index fund that closely tracks the S&P 500 Index. The T-bills have a return of 5%. The S&P 500 has a standard deviation of 20% and an expected return of 15%.

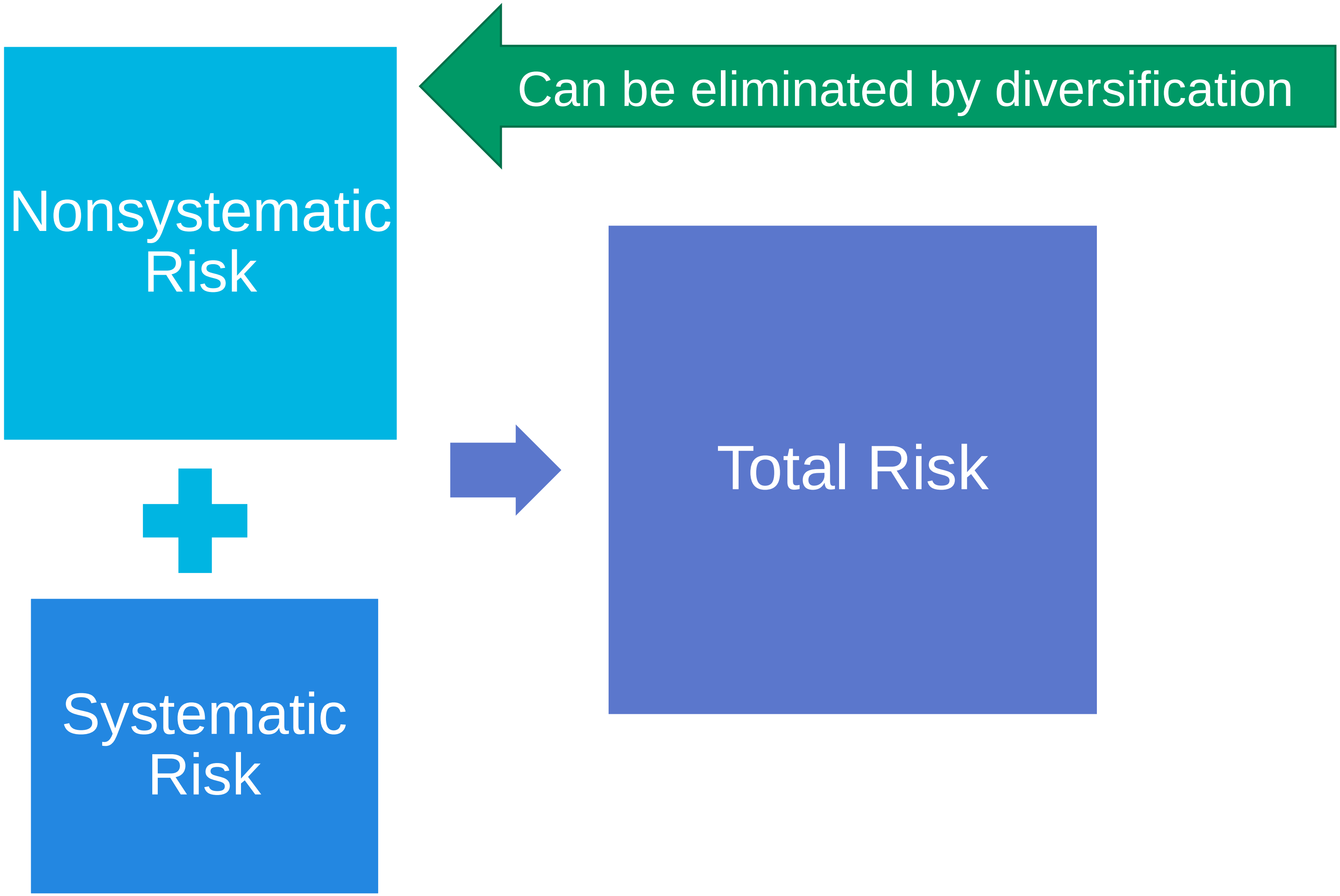
1. Draw the CML and mark the points where the investment in the market is 0%, 25%, 75%, and 100%.

2. Mr. Miles is also interested in determining the exact risk and return at each point.

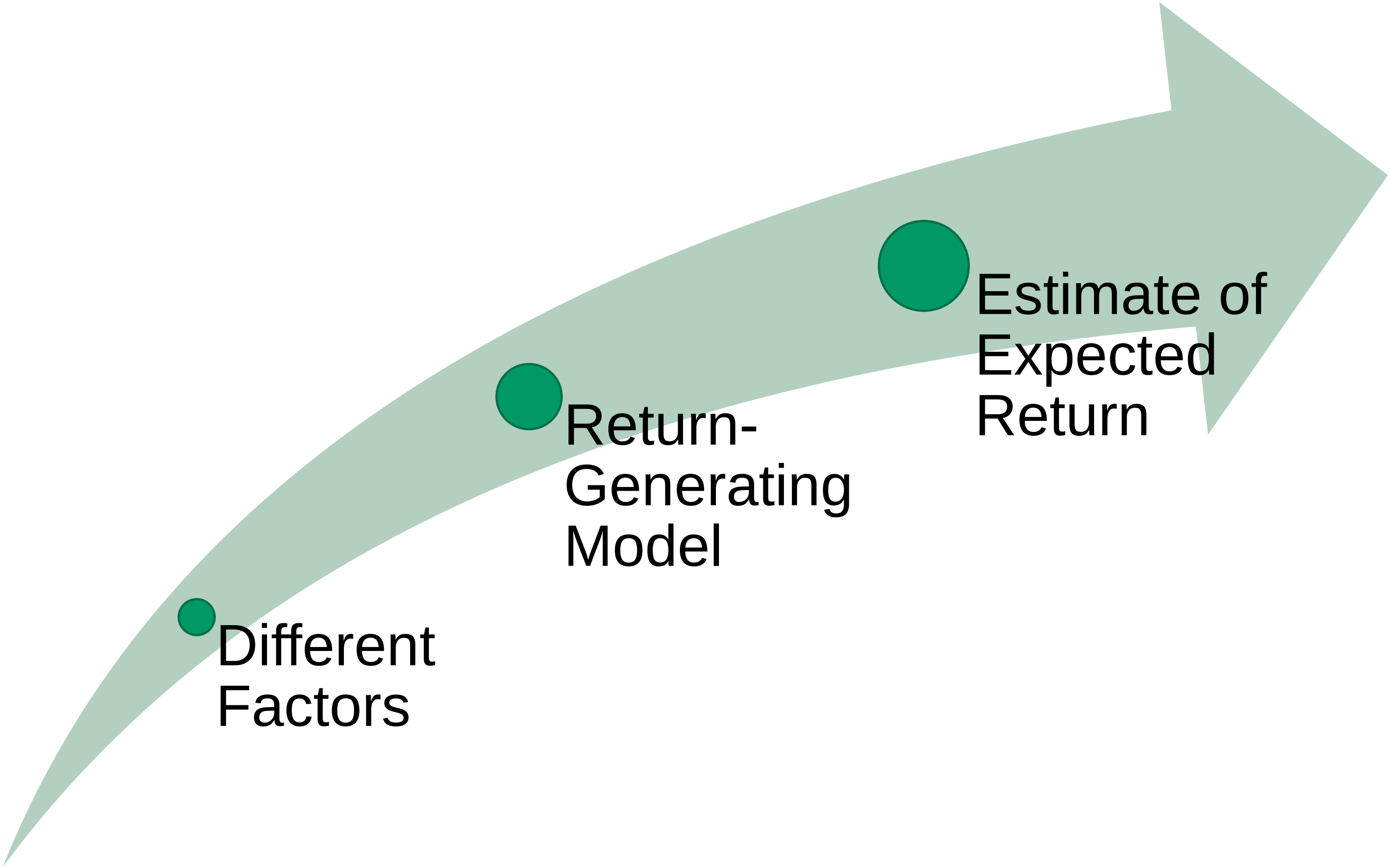
EXAMPLE 6-2 RISK AND RETURN OF A LEVERAGED PORTFOLIO WITH EQUAL LENDING AND BORROWING RATES

Mr. Miles decides to set aside a small part of his wealth for investment in a portfolio that has greater risk than his previous investments because he anticipates that the overall market will generate attractive returns in the future. He assumes that he can borrow money at 5% and achieve the same return on the S&P 500 as before: an expected return of 15% with a standard deviation of 20%. Calculate his expected risk and return if he borrows 25%, 50%, and 100% of his initial investment amount.

SYSTEMATIC AND NONSYSTEMATIC RISK



RETURN-GENERATING MODELS



GENERAL FORMULA FOR RETURN-GENERATING MODELS

All models contain return on the market portfolio as a key factor

Factor weights or factor loadings

$$E(R_i) - R_f = \sum_{j=1}^{k} \beta_{ij} E(F_j) = \beta_{i1}\left[E(R_m) - R_f\right] + \sum_{j=2}^{k} \beta_{ij} E(F_j)$$

Risk factors

THE MARKET MODEL

$$E(R_i) - R_f = \beta_i \left[E(R_m) - R_f \right]$$

Single-index model

$$R_i - R_f = \beta_i \left(R_m - R_f \right) + e_i$$

The difference between expected returns and realized returns is attributable to an error term, e_i.

$$R_i = \alpha_i + \beta_i R_m + e_i$$

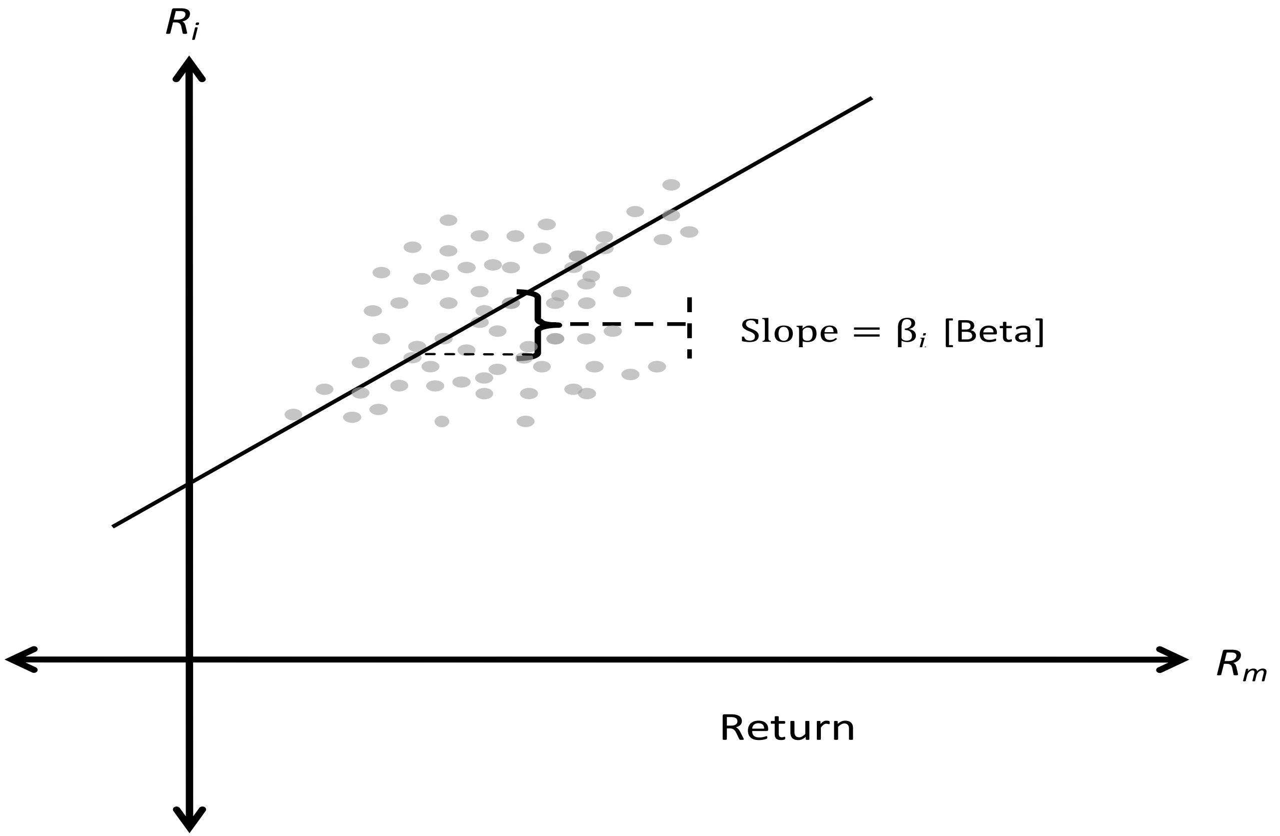
The market model: the intercept, α_i, and slope coefficient, β_i, can be estimated by using historical security and market returns. Note $\alpha_i = R_f(1 - \beta_i)$.

CALCULATION AND INTERPRETATION OF BETA

$$\beta_i = \frac{\text{Cov}\left(R_i, R_m\right)}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i\sigma_m}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$$

$$\beta_i = \frac{0.026250}{0.02250} = \frac{0.70 \times 0.25 \times 0.15}{0.02250} = \frac{0.70 \times 0.25}{0.15} = 1.17$$

EXHIBIT 6-6 BETA ESTIMATION USING A PLOT OF SECURITY AND MARKET RETURNS



CAPITAL ASSET PRICING MODEL (CAPM)

Beta is the primary determinant of expected return

$$E(R_i) = R_f + \beta_i \left[E(R_m) - R_f \right]$$

$$E(R_i) = 3\% + 1.5[9\% - 3\%] = 12.0\%$$

$$E(R_i) = 3\% + 1.0[9\% - 3\%] = 9.0\%$$

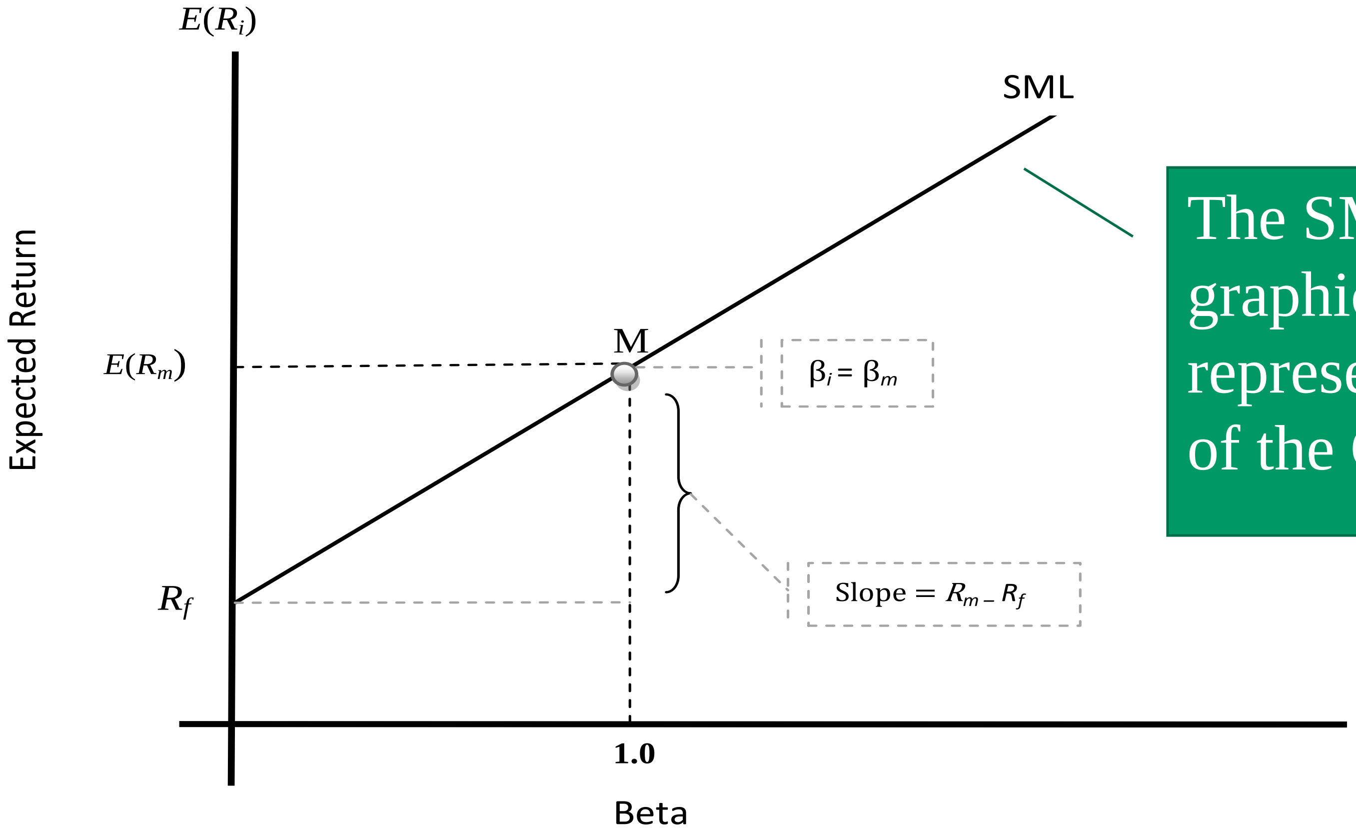
$$E(R_i) = 3\% + 0.5[9\% - 3\%] = 6.0\%$$

$$E(R_i) = 3\% + 0.0[9\% - 3\%] = 3.0\%$$

ASSUMPTIONS OF THE CAPM

- Investors are risk-averse, utility-maximizing, rational individuals.
- Markets are frictionless, including no transaction costs or taxes.
- Investors plan for the same single holding period.
- Investors have homogeneous expectations or beliefs.
- All investments are infinitely divisible.
- Investors are price takers.

EXHIBIT 6-7 THE SECURITY MARKET LINE (SML)



The SML is a graphical representation of the CAPM.

PORTFOLIO BETA

Portfolio beta is the weighted sum of the betas of the component securities:

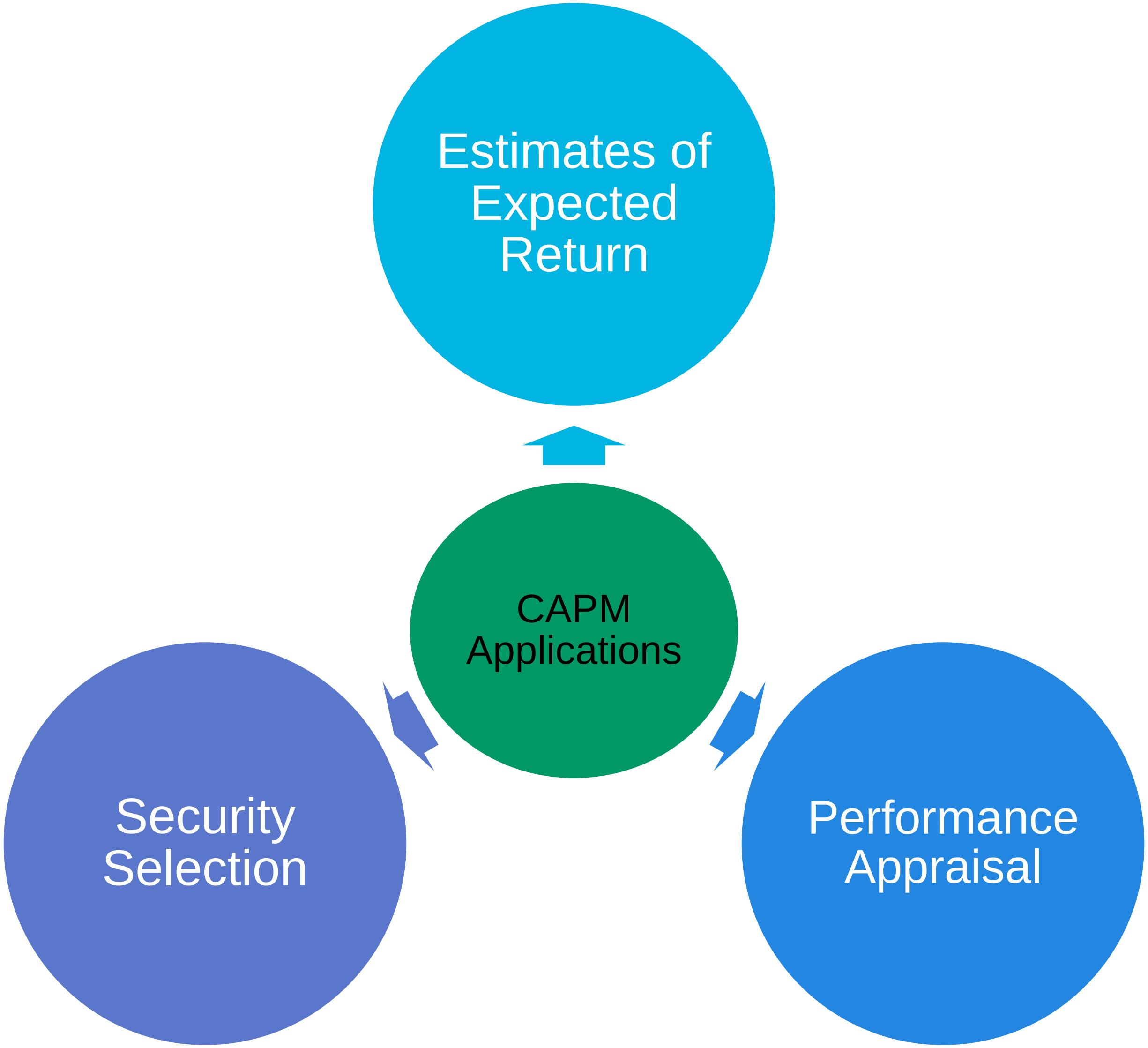
$$\beta_p = \sum_{i=1}^{N} w_i \beta_i = (0.40 \times 1.50) + (0.60 \times 1.20) = 1.32$$

The portfolio's expected return given by the CAPM is:

$$E\left(R_p\right) = R_f + \beta_p \left[E\left(R_m\right) - R_f \right]$$

$$E\left(R_p\right) = 3\% + 1.32\left[9\% - 3\%\right] = 10.92\%$$

APPLICATIONS OF THE CAPM



PERFORMANCE EVALUATION: SHARPE RATIO AND TREYNOR RATIO

Sharpe Ratio

- Focus on total risk

$$\text{Sharpe ratio} = \frac{R_p - R_f}{\sigma_p}$$

Treynor Ratio

- Focus on systematic risk

$$\text{Treynor ratio} = \frac{R_p - R_f}{\beta_p}$$

PERFORMANCE EVALUATION: *M*-SQUARED (M^2)

Sharpe Ratio

- Identical rankings
- Expressed in percentage terms

$$M^2 = \left(R_p - R_f\right)\frac{\sigma_m}{\sigma_p} - \left(R_m - R_f\right)$$

PERFORMANCE EVALUATION: JENSEN'S ALPHA

Estimate portfolio beta → Determine risk-adjusted return → Subtract risk-adjusted return from actual return

$$\alpha_p = R_p - \left[R_f + \beta_p \left(R_m - R_f \right) \right]$$

EXHIBIT 6-8 MEASURES OF PORTFOLIO PERFORMANCE EVALUATION

Manager	R_i	σ_i	β_i	$E(R_i)$	Sharpe Ratio	Treynor Ratio	M^2	α_i
X	10.0%	20.0%	1.10	9.6%	0.35	0.064	0.65%	0.40%
Y	11.0	10.0	0.70	7.2	0.80	0.114	9.20	3.80
Z	12.0	25.0	0.60	6.6	0.36	0.150	0.84	5.40
M	9.0	19.0	1.00	9.0	0.32	0.060	0.00	0.00
R_f	3.0	0.0	0.00	3.0	–	–	–	0.00

EXHIBIT 6-11 THE SECURITY CHARACTERISTIC LINE (SCL)

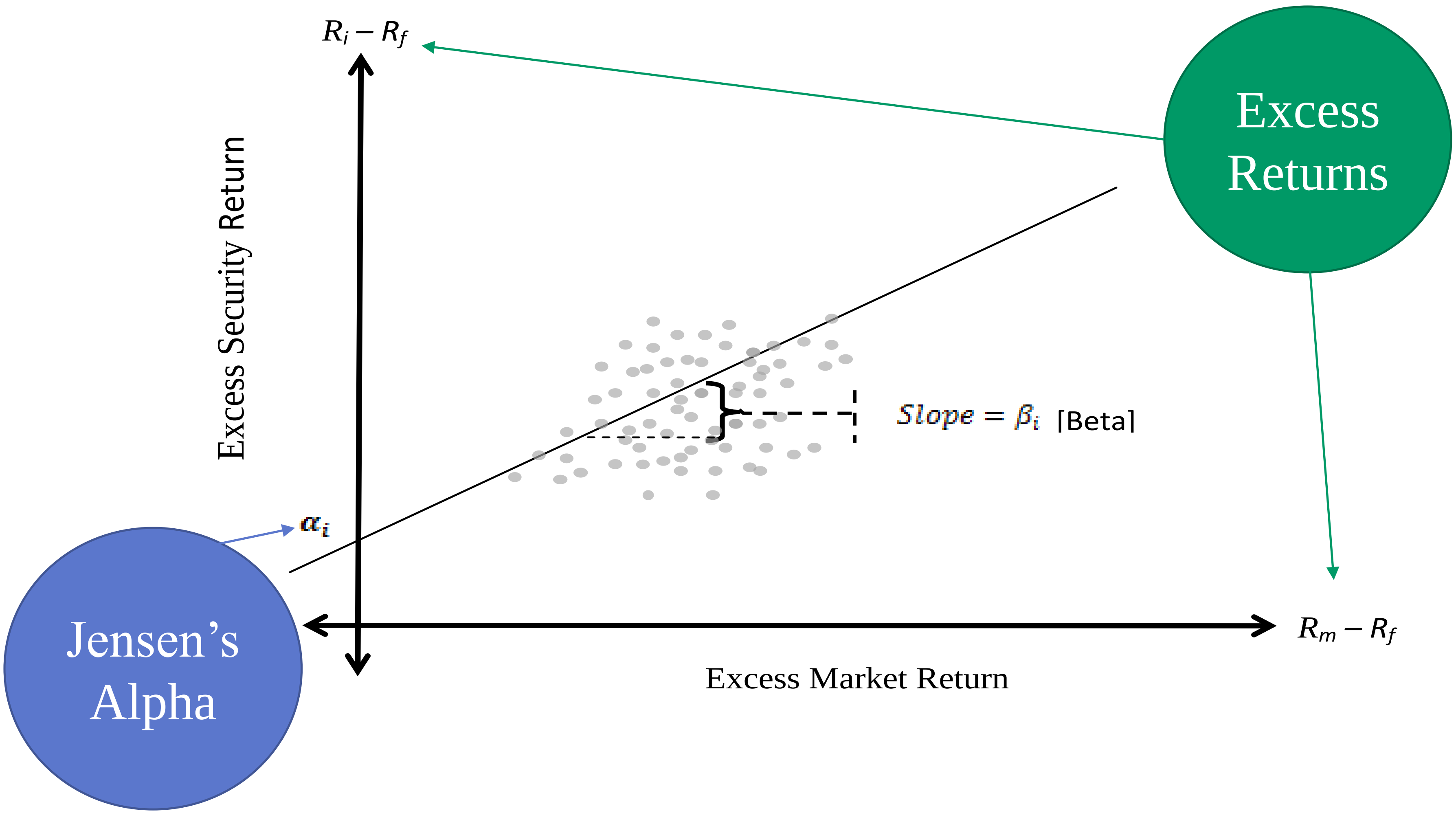
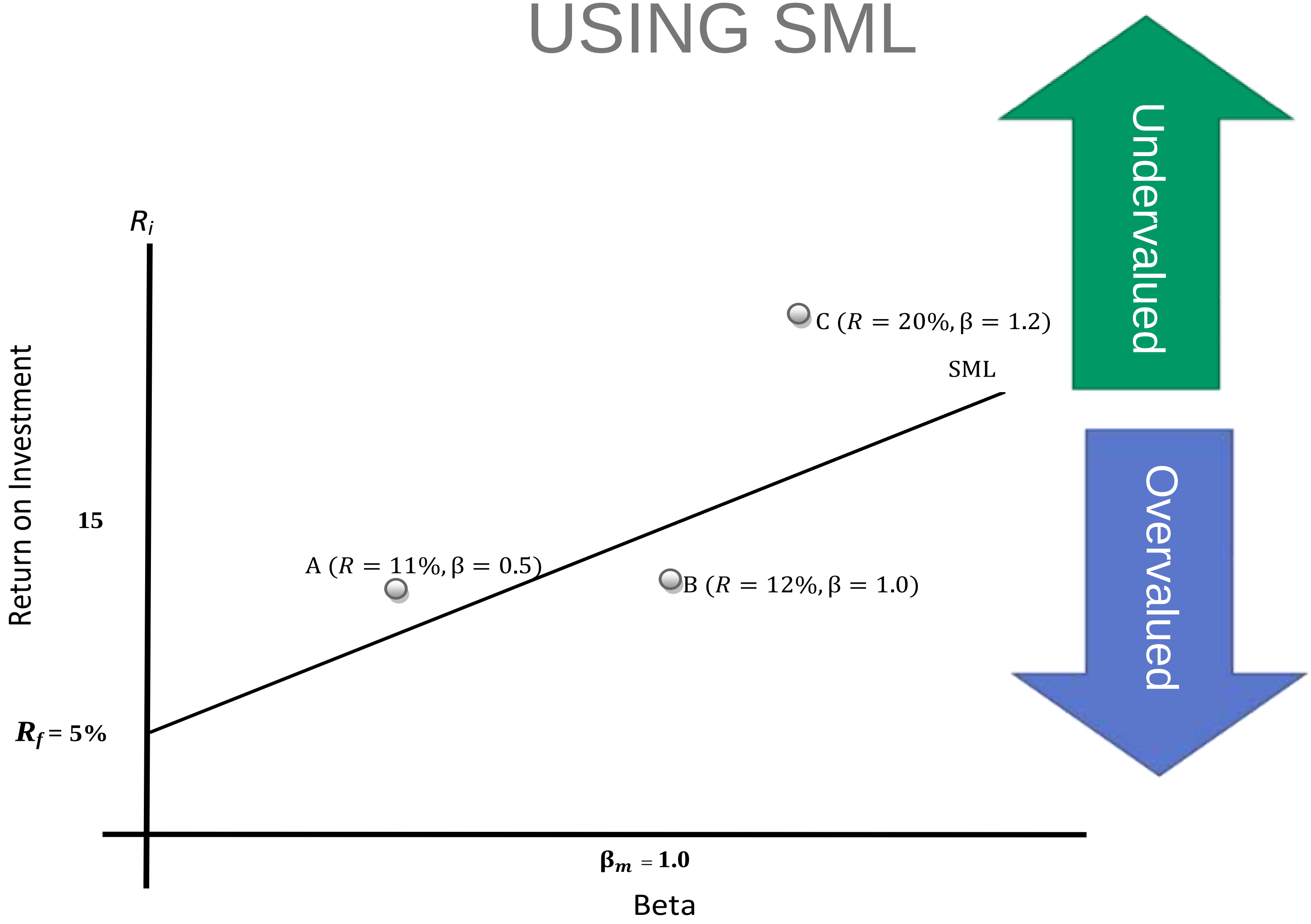


EXHIBIT 6-12 SECURITY SELECTION USING SML



DECOMPOSITION OF TOTAL RISK FOR A SINGLE-INDEX MODEL

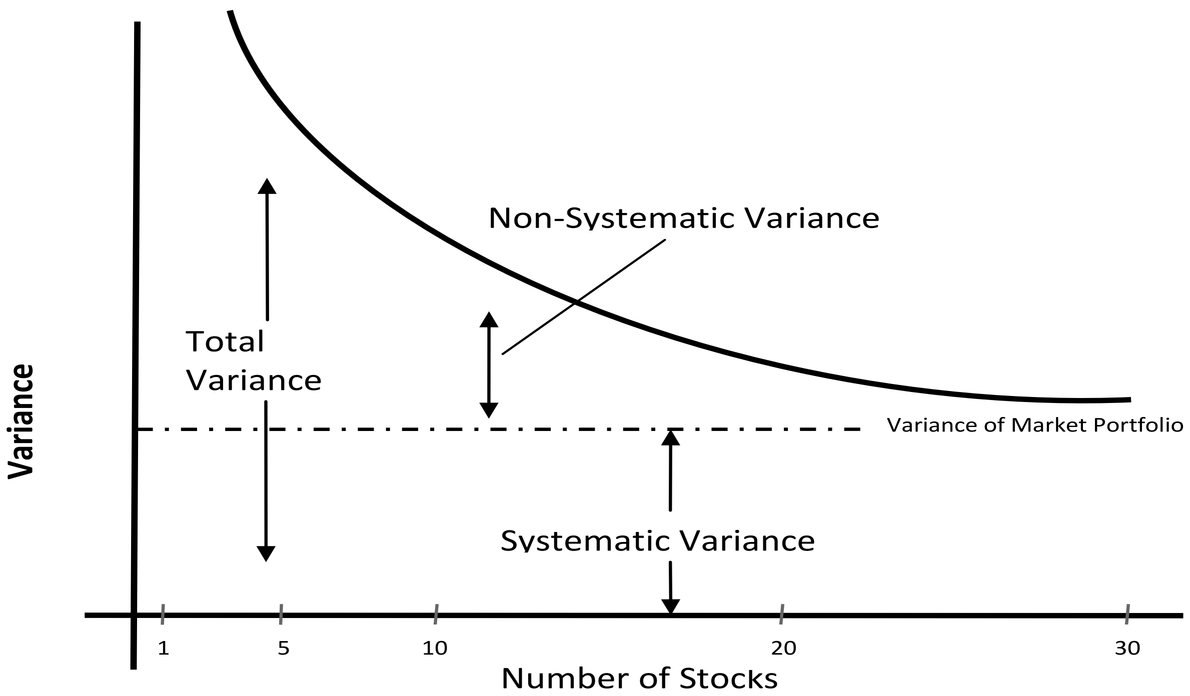
$$R_i - R_f = \beta_i \left(R_m - R_f\right) + e_i$$

Total variance = Systematic variance + Nonsystematic variance

$$\sigma_{\mathrm{i}}^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2 + 2\mathrm{Cov}\left(R_m, e_i\right) = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2$$

Zero

EXHIBIT 6-13 DIVERSIFICATION WITH NUMBER OF STOCKS



WHAT SHOULD THE RELATIVE WEIGHT OF SECURITIES IN THE PORTFOLIO BE?

Higher Alpha →Higher Weight

Greater Non-Systematic Risk →Lower Weight

$$\text{Information ratio} = \frac{\alpha_i}{\sigma^2_{ei}}$$

LIMITATIONS OF THE CAPM

Theoretical

- Single-factor model
- Single-period model

Practical

- Market portfolio
- Proxy for a market portfolio
- Estimation of beta
- Poor predictor of returns
- Homogeneity in investor expectations

EXTENSIONS TO THE CAPM: ARBITRAGE PRICING THEORY (APT)

Risk Premium for Factor 1

$$E\left(R_p\right) = R_F + \lambda_1 \beta_{p,1} + \cdots + \lambda_K \beta_{p,K}$$

Sensitivity of the Portfolio to Factor 1

FOUR-FACTOR MODEL

Systematic Risk

Value Anomaly

$$E\left(R_{it}\right) = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \beta_{i,UMD} UMD_t$$

Size Anomaly

Momentum Anomaly

SUMMARY

- Portfolio risk and return
- Optimal risky portfolio and the capital market line (CML)
- Return-generating models and the market model
- Systematic and non-systematic risk
- Capital asset pricing model (CAPM) and the security market line (SML)
- Performance measures
- Arbitrage pricing theory (APT) and factor models