2 CHAPTER

PRESSURE AND ITS MEASUREMENT

▶ 2.1 FLUID PRESSURE AT A POINT

Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure or simply pressure and this ratio is represented by p. Hence mathematically the pressure at a point in a fluid at rest is

$$p = \frac{dF}{dA}.$$

If the force (F) is uniformly distributed over the area (A), then pressure at any point is given by

$$p = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}.$$

$\therefore$ Force or pressure force, $F = p \times A$.

The units of pressure are : (*i*) kgf/m^2 and kgf/cm^2 in MKS units, (*ii*) Newton/m^2 or N/m^2 and N/mm^2 in SI units. N/m^2 is known as Pascal and is represented by Pa. Other commonly used units of pressure are :

$$\text{kPa} = \text{kilo pascal} = 1000 \text{ N/m}^2$$
$$\text{bar} = 100 \text{ kPa} = 10^5 \text{ N/m}^2.$$

▶ 2.2 PASCAL'S LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. This is proved as :

The fluid element is of very small dimensions *i.e.*, dx, dy and ds.

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in Fig. 2.1. Let the width of the element perpendicular to the plane of paper is unity and p_x,

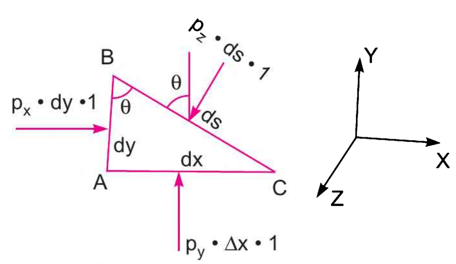


Fig. 2.1 *Forces on a fluid element.*

p_y and p_z are the pressures or intensity of pressure acting on the face AB, AC and BC respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are :

1. Pressure forces normal to the surfaces, and
2. Weight of element in the vertical direction.

The forces on the faces are :

Force on the face AB $= p_x \times$ Area of face AB

$= p_x \times dy \times 1$

Similarly force on the face $AC = p_y \times dx \times 1$

Force on the face BC $= p_z \times ds \times 1$

Weight of element $=$ (Mass of element) $\times g$

$= (\text{Volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g,$

where ρ = density of fluid.

Resolving the forces in x-direction, we have

$$p_x \times dy \times 1 - p\,(ds \times 1) \sin (90° - \theta) = 0$$

or
$$p_x \times dy \times 1 - p_z\, ds \times 1 \cos \theta = 0.$$

But from Fig. 2.1, $ds \cos \theta = AB = dy$

$\therefore$
$$p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

or
$$p_x = p_z \qquad \text{...(2.1)}$$

Similarly, resolving the forces in y-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos (90° - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

or
$$p_y \times dx - p_z\, ds \sin \theta - \frac{dxdy}{2} \times \rho \times g = 0.$$

But $ds \sin \theta = dx$ and also the element is very small and hence weight is negligible.

$\therefore$
$$p_y dx - p_z \times dx = 0$$

or
$$p_y = p_z \qquad \text{...(2.2)}$$

From equations (2.1) and (2.2), we have

$$p_x = p_y = p_z \qquad \text{...(2.3)}$$

The above equation shows that the pressure at any point in x, y and z directions is equal.

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

▶ 2.3 PRESSURE VARIATION IN A FLUID AT REST

The pressure at any point in a fluid at rest is obtained by the Hydrostatic Law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point. This is proved as :

Consider a small fluid element as shown in Fig. 2.2

Let ΔA = Cross-sectional area of element

ΔZ = Height of fluid element

p = Pressure on face AB

Z = Distance of fluid element from free surface.

The forces acting on the fluid element are :

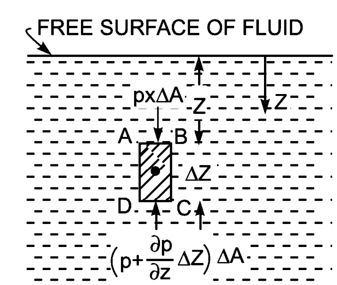


Fig. 2.2 *Forces on a fluid element.*

1. Pressure force on $AB = p \times \Delta A$ and acting perpendicular to face AB in the downward direction.

2. Pressure force on $CD = \left(p + \frac{\partial p}{\partial Z}\Delta Z\right) \times \Delta A$, acting perpendicular to face CD, vertically upward direction.

3. Weight of fluid element = Density $\times g \times$ Volume $= \rho \times g \times (\Delta A \times \Delta Z)$.

4. Pressure forces on surfaces BC and AD are equal and opposite. For equilibrium of fluid element, we have

$$p\Delta A - \left(p + \frac{\partial p}{\partial Z}\Delta Z\right) \Delta A + \rho \times g \times (\Delta A \times \Delta Z) = 0$$

or
$$p\Delta A - p\Delta A - \frac{\partial p}{\partial Z} \Delta Z\Delta A + \rho \times g \times \Delta A \times Z = 0$$

or
$$-\frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A\Delta Z = 0$$

or
$$\frac{\partial p}{\partial Z}\Delta Z\Delta A = \rho \times g \times \Delta A\Delta Z \quad \text{or} \quad \frac{\partial p}{\partial Z} = \rho \times g \text{ [cancelling } \Delta A\Delta Z \text{ on both sides]}$$

$\therefore$
$$\frac{\partial p}{\partial Z} = \rho \times g = w \qquad (\because \rho \times g = w) \qquad \text{...(2.4)}$$

where w = Weight density of fluid.

Equation (2.4) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is **Hydrostatic Law.**

By integrating the above equation (2.4) for liquids, we get

$$\int dp = \int \rho g dZ$$

or
$$p = \rho g Z \qquad \text{...(2.5)}$$

where p is the pressure above atmospheric pressure and Z is the height of the point from free surfaces.

From equation (2.5), we have
$$Z = \frac{p}{\rho \times g} \qquad \text{...(2.6)}$$

Here Z is called **pressure head.**

Problem 2.1 *A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.*

Solution. Given :

Dia. of ram, $D = 30 \text{ cm} = 0.3 \text{ m}$

Dia. of plunger, $d = 4.5 \text{ cm} = 0.045 \text{ m}$

Force on plunger, $F = 500 \text{ N}$

Find weight lifted $= W$

Area of ram,
$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.3)^2 = 0.07068 \text{ m}^2$$

Area of plunger,
$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.045)^2 = .00159 \text{ m}^2$$

Pressure intensity due to plunger

$$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{.00159} \text{ N/m}^2.$$

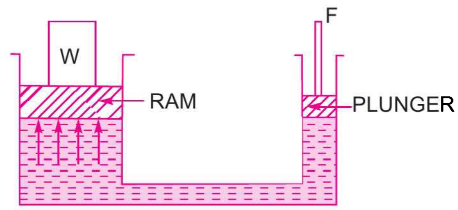


Fig. 2.3

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram

$$= \frac{500}{.00159} = 314465.4 \text{ N/m}^2$$

But pressure intensity at ram $= \dfrac{\text{Weight}}{\text{Area of ram}} = \dfrac{W}{A} = \dfrac{W}{.07068} \text{ N/m}^2$

$$\frac{W}{.07068} = 314465.4$$

∴ Weight $= 314465.4 \times .07068 = 22222$ N = **22.222 kN. Ans.**

Problem 2.2 *A hydraulic press has a ram of 20 cm diameter and a plunger of 3 cm diameter. It is used for lifting a weight of 30 kN. Find the force required at the plunger.*

Solution. Given :

Dia. of ram, $D = 20$ cm $= 0.2$ m

∴ Area of ram, $A = \dfrac{\pi}{4} D^2 = \dfrac{\pi}{4}(.2)^2 = 0.0314 \text{ m}^2$

Dia. of plunger $d = 3$ cm $= 0.03$ m

∴ Area of plunger, $a = \dfrac{\pi}{4}(.03)^2 = 7.068 \times 10^{-4} \text{ m}^2$

Weight lifted, $W = 30$ kN $= 30 \times 1000$ N $= 30000$ N.

See Fig. 2.3.

Pressure intensity developed due to plunger $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{a}$.

By Pascal's Law, this pressure is transmitted equally in all directions

Hence pressure transmitted at the ram $= \dfrac{F}{a}$

∴ Force acting on ram = Pressure intensity × Area of ram

$$= \frac{F}{a} \times A = \frac{F \times .0314}{7.068 \times 10^{-4}} \text{ N}$$

But force acting on ram = Weight lifted = 30000 N

$$\therefore \quad 30000 = \frac{F \times .0314}{7.068 \times 10^{-4}}$$

$$\therefore \quad F = \frac{30000 \times 7.068 \times 10^{-4}}{.0314} = \textbf{675.2 N. Ans.}$$

Problem 2.3 *Calculate the pressure due to a column of 0.3 of (a) water, (b) an oil of sp. gr. 0.8, and (c) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000$ kg/m^3.*

Solution. Given :

Height of liquid column, $Z = 0.3$ m.

The pressure at any point in a liquid is given by equation (2.5) as

$$p = \rho g Z$$

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$\therefore$ $p = \rho g Z = 1000 \times 9.81 \times 0.3 = 2943 \text{ N/m}^2$

$$= \frac{2943}{10^4} \text{ N/cm}^2 = \mathbf{0.2943 \text{ N/cm}^2. \text{ Ans.}}$$

(*b*) For oil of sp. gr. 0.8,

From equation (1.1*A*), we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water.

$\therefore$ Density of oil, $\rho_0 = \text{Sp. gr. of oil} \times \text{Density of water}$ (ρ_0 = Density of oil)

$$= 0.8 \times \rho = 0.8 \times 1000 = 800 \text{ kg/m}^3$$

Now pressure, $p = \rho_0 \times g \times Z$

$$= 800 \times 9.81 \times 0.3 = 2354.4 \frac{\text{N}}{\text{m}^2} = \frac{2354.4}{10^4} \frac{\text{N}}{\text{cm}^2}.$$

$$= \mathbf{0.2354 \frac{N}{cm^2}. \text{ Ans.}}$$

(*c*) For mercury, sp. gr. = 13.6

From equation (1.1*A*) we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water

$\therefore$ Density of mercury, $\rho_s = \text{Specific gravity of mercury} \times \text{Density of water}$

$$= 13.6 \times 1000 = 13600 \text{ kg/m}^3$$

$\therefore$ $p = \rho_s \times g \times Z$

$$= 13600 \times 9.81 \times 0.3 = 40025 \frac{\text{N}}{\text{m}^2}$$

$$= \frac{40025}{10^4} = \mathbf{4.002 \frac{N}{cm^2}. \text{ Ans.}}$$

Problem 2.4 *The pressure intensity at a point in a fluid is given 3.924 N/cm². Find the corresponding height of fluid when the fluid is : (a) water, and (b) oil of sp. gr. 0.9.*

Solution. Given :

Pressure intensity, $p = 3.924 \frac{\text{N}}{\text{cm}^2} = 3.924 \times 10^4 \frac{\text{N}}{\text{m}^2}.$

The corresponding height, Z, of the fluid is given by equation (2.6) as

$$Z = \frac{p}{\rho \times g}$$

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$$\therefore \quad Z = \frac{p}{\rho \times g} = \frac{3.924 \times 10^4}{1000 \times 9.81} = \mathbf{4 \text{ m of water. Ans.}}$$

(*b*) For oil, sp. gr. = 0.9

$\therefore$ Density of oil $\rho_0 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

$$\therefore \quad Z = \frac{p}{\rho_0 \times g} = \frac{3.924 \times 10^4}{900 \times 9.81} = \mathbf{4.44 \text{ m of oil. Ans.}}$$

Problem 2.5 *An oil of sp. gr. 0.9 is contained in a vessel. At a point the height of oil is 40 m. Find the corresponding height of water at the point.*

Solution. Given :

Sp. gr. of oil, $S_0 = 0.9$

Height of oil, $Z_0 = 40$ m

Density of oil, $\rho_0 = \text{Sp. gr. of oil} \times \text{Density of water} = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Intensity of pressure, $p = \rho_0 \times g \times Z_0 = 900 \times 9.81 \times 40 \dfrac{\text{N}}{\text{m}^2}$

$$\therefore \text{ Corresponding height of water} = \frac{p}{\text{Density of water} \times g}$$

$$= \frac{900 \times 9.81 \times 40}{1000 \times 9.81} = 0.9 \times 40 = \mathbf{36 \text{ m of water. Ans.}}$$

Problem 2.6 *An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1 m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.*

Solution. Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000 \text{ kg/m}^3$

Density of oil, $\rho_2 = \text{Sp. gr. of oil} \times \text{Density of water}$
$= 0.9 \times 1000 = 900 \text{ kg/m}^3$

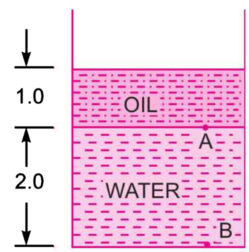


Fig. 2.4

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$

(*i*) At interface, *i.e.*, at *A*

$$p = \rho_2 \times g \times 1.0$$
$$= 900 \times 9.81 \times 1.0$$
$$= 8829 \frac{\text{N}}{\text{m}^2} = \frac{8829}{10^4} = \mathbf{0.8829 \text{ N/cm}^2. \text{ Ans.}}$$

(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times gZ_2 + \rho_1 \times g \times Z_1 = 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$
$$= 8829 + 19620 = 28449 \text{ N/m}^2 = \frac{28449}{10^4} \text{ N/cm}^2 = \mathbf{2.8449 \text{ N/cm}^2. \text{ Ans.}}$$

Problem 2.7 *The diameters of a small piston and a large piston of a hydraulic jack are 3 cm and 10 cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when :*

(a) the pistons are at the same level.

(b) small piston is 40 cm above the large piston.

The density of the liquid in the jack is given as 1000 kg/m³.

Solution. Given :

Dia. of small piston, $d = 3$ cm

$\therefore$ Area of small piston, $a = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4} \times (3)^2 = 7.068 \text{ cm}^2$

Dia. of large piston, $D = 10$ cm

$\therefore$ Area of larger piston, $A = \frac{p}{4} \times (10)^2 = 78.54$ cm^2

Force on small piston, $F = 80$ N

Let the load lifted $= W$.

(*a*) **When the pistons are at the same level**

Pressure intensity on small piston

$$\frac{F}{a} = \frac{80}{7.068} \text{ N/cm}^2$$

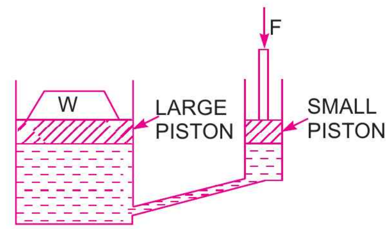


Fig. 2.5

This is transmitted equally on the large piston.

$\therefore$ Pressure intensity on the large piston

$$= \frac{80}{7.068}$$

$\therefore$ Force on the large piston = Pressure × Area

$$= \frac{80}{7.068} \times 78.54 \text{ N} = \mathbf{888.96 \text{ N. Ans.}}$$

(*b*) **When the small piston is 40 cm above the large piston**

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} \frac{\text{N}}{\text{cm}^2}$$

$\therefore$ Pressure intensity at section *A-A*

$$= \frac{F}{a} + \text{Pressure intensity due to height of 40 cm of liquid.}$$

But pressure intensity due to 40 cm of liquid

$$= \rho \times g \times h = 1000 \times 9.81 \times 0.4 \text{ N/m}^2$$

$$= \frac{1000 \times 9.81 \times .40}{10^4} \text{ N/cm}^2 = 0.3924 \text{ N/cm}^2$$

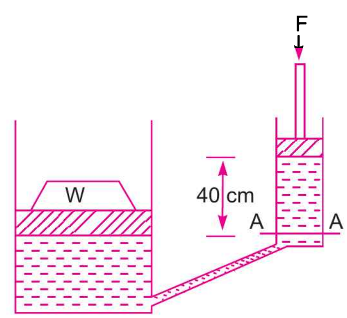


Fig. 2.6

$\therefore$ Pressure intensity at section *A-A*

$$= \frac{80}{7.068} + 0.3924$$

$$= 11.32 + 0.3924 = 11.71 \text{ N/cm}^2$$

$\therefore$ Pressure intensity transmitted to the large piston = 11.71 N/cm^2

$\therefore$ Force on the large piston = Pressure × Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54 = 919.7 \text{ N.}$$

▶ 2.4 ABSOLUTE, GAUGE, ATMOSPHERIC AND VACUUM PRESSURES

The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure. Thus :

1. Absolute pressure is defined as the pressure which is measured with reference to absolute vacuum pressure.

2. Gauge pressure is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

3. Vacuum pressure is defined as the pressure below the atmospheric pressure.

The relationship between the absolute pressure, gauge pressure and vacuum pressure are shown in Fig. 2.7.

Mathematically :

(*i*) Absolute pressure

= Atmospheric pressure + Gauge pressure

or $$p_{ab} = p_{atm} + p_{gauge}$$

(*ii*) Vacuum pressure

= Atmospheric pressure – Absolute pressure.

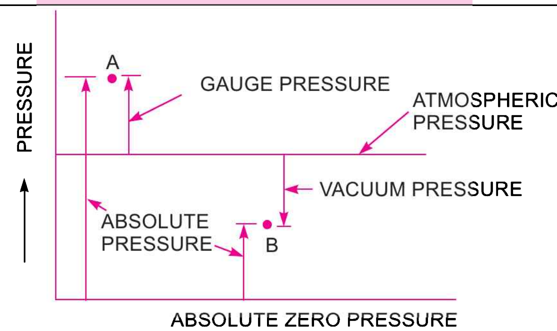


Fig. 2.7 *Relationship between pressures.*

Note. (*i*) The atmospheric pressure at sea level at 15°C is 101.3 kN/m^2 or 10.13 N/cm^2 in SI unit. In case of MKS units, it is equal to 1.033 kgf/cm^2.

(*ii*) The atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

Problem 2.8 *What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of 1.53 × 10^3 kg/m^3 if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m^3.*

Solution. Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

∴ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = Density of Hg = Sp. gr. of mercury × Density of water = 13.6 × 1000 kg/m^3

and Z_0 = Pressure head in terms of mercury.

∴ $p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75$ N/m^2 (∵ $Z_0 = 0.75$)

$= 100062$ N/m^2

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$$p = \rho_1 \times g \times Z_1$$
$$= (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

∴ Gauge pressure, p = **45028 N/m^2. Ans.**

Now absolute pressure = Gauge pressure + Atmospheric pressure

= 45028 + 100062 = **145090 N/m^2. Ans.**

▶ 2.5 MEASUREMENT OF PRESSURE

The pressure of a fluid is measured by the following devices :

1. Manometers 2. Mechanical Gauges.

2.5.1 Manometers. Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(*a*) Simple Manometers, (*b*) Differential Manometers.

2.5.2 Mechanical Gauges. Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are :

(*a*) Diaphragm pressure gauge,
(*b*) Bourdon tube pressure gauge,
(*c*) Dead-weight pressure gauge, and
(*d*) Bellows pressure gauge.

▶ 2.6 SIMPLE MANOMETERS

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer,`
2. U-tube Manometer, and
3. Single Column Manometer.

2.6.1 Piezometer. It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. 2.8. The rise of liquid gives the pressure head at that point. If at a point *A*, the height of liquid say water is *h* in piezometer tube, then pressure at *A*

$$= \rho \times g \times h \ \frac{\text{N}}{\text{m}^2}.$$

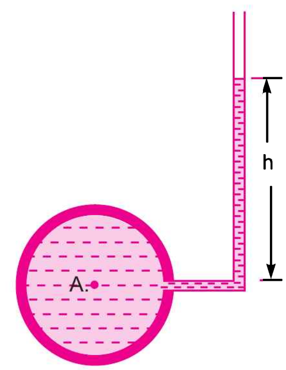
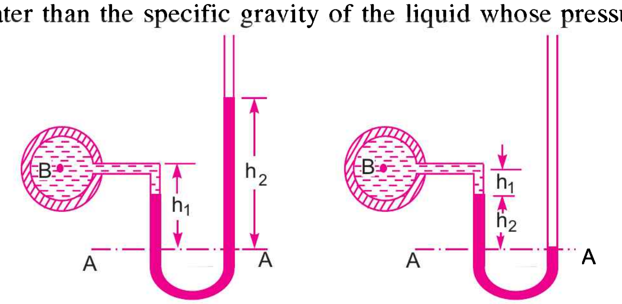


Fig. 2.8 *Piezometer.*

2.6.2 U-tube Manometer. It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.9. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



(a) For gauge pressure (b) For vacuum pressure

Fig. 2.9 *U-tube Manometer.*

(*a*) **For Gauge Pressure.** Let *B* is the point at which pressure is to be measured, whose value is *p*. The datum line is *A-A*.

Let h_1 = Height of light liquid above the datum line
h_2 = Height of heavy liquid above the datum line
S_1 = Sp. gr. of light liquid
ρ_1 = Density of light liquid = $1000 \times S_1$
S_2 = Sp. gr. of heavy liquid
ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line *A-A* in the left column and in the right column of U-tube manometer should be same.

Pressure above *A-A* in the left column $= p + \rho_1 \times g \times h_1$

Pressure above *A-A* in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$$\therefore \quad p = (\rho_2 g h_2 - \rho_1 \times g \times h_1). \quad ...(2.7)$$

(*b*) **For Vacuum Pressure.** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (*b*). Then

Pressure above *A-A* in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above *A-A* $= 0$

$$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1). \quad ...(2.8)$$

Problem 2.9 *The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Sp. gr. of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$

Difference of mercury level, $h_2 = 20 \text{ cm} = 0.2 \text{ m}$

Height of fluid from *A-A*, $h_1 = 20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$

Let p = Pressure of fluid in pipe

Equating the pressure above *A-A*, we get

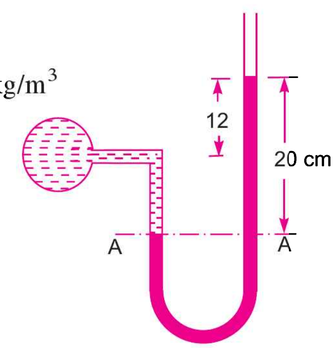


Fig. 2.10

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or $$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$$= 26683 - 706 = 25977 \text{ N/m}^2 = \mathbf{2.597 \text{ N/cm}^2}. \textbf{ Ans.}$$

Problem 2.10 *A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.8$

Sp. gr. of mercury, $S_2 = 13.6$

Density of fluid, $\rho_1 = 800$

Density of mercury, $\rho_2 = 13.6 \times 1000$

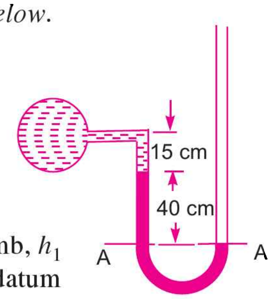


Fig. 2.11

Difference of mercury level, $h_2 = 40 \text{ cm} = 0.4 \text{ m}$. Height of liquid in left limb, $h_1 = 15 \text{ cm} = 0.15 \text{ m}$. Let the pressure in pipe $= p$. Equating pressure above datum line *A-A*, we get

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$\therefore \quad p = -[\rho_2 g h_2 + \rho_1 g h_1]$

$= -[13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$

$= -[53366.4 + 1177.2] = -54543.6\ \text{N/m}^2 = \mathbf{-5.454\ N/cm^2}$. **Ans.**

Problem 2.11 *A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m², calculate the new difference in the level of mercury. Sketch the arrangements in both cases.*

Solution. Given :

Difference of mercury = 10 cm = 0.1 m

The arrangement is shown in Fig. 2.11 (*a*)

Ist Part

Let p_A = (pressure of water in pipe line (*i.e.,* at point *A*)

The points *B* and *C* lie on the same horizontal line. Hence pressure at *B* should be equal to pressure at *C*. But pressure at *B*

= Pressure at *A* + Pressure due to 10 cm (or 0.1 m) of water

$= p_A + \rho \times g \times h$

where $\rho = 1000\ \text{kg/m}^3$ and $h = 0.1$ m

$= p_A + 1000 \times 9.81 \times 0.1$

$= p_A + 981\ \text{N/m}^2 \quad \ldots(i)$

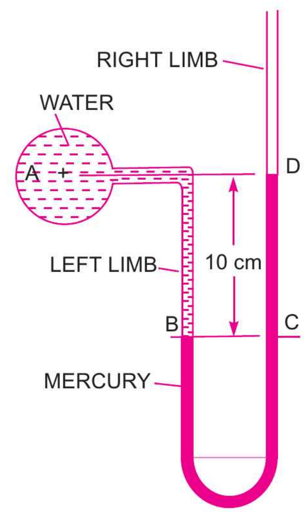


Fig. 2.11 (*a*)

Pressure at *C* = Pressure at *D* + Pressure due to 10 cm of mercury

$= 0 + \rho_0 \times g \times h_0$

where ρ_0 for mercury $= 13.6 \times 1000\ \text{kg/m}^3$

and $h_0 = 10\ \text{cm} = 0.1\ \text{m}$

$\therefore$ Pressure at $C = 0 + (13.6 \times 1000) \times 9.81 \times 0.1$

$= 13341.6\ \text{N} \quad \ldots(ii)$

But pressure at *B* is equal to pressure at *C*. Hence equating the equations (*i*) and (*ii*), we get

$p_A + 981 = 13341.6$

$\therefore \quad p_A = 13341.6 - 981$

$= \mathbf{12360.6\ \frac{N}{m^2}}$. **Ans.**

IInd Part

Given, $p_A = 9810\ \text{N/m}^2$

Find new difference of mercury level. The arrangement is shown in Fig. 2.11 (*b*). In this case the pressure at *A* is 9810 N/m² which is less than the 12360.6 N/m². Hence mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let *x* = Rise of mercury in left limb in cm

Then fall of mercury in right limb = *x* cm

The points *B*, *C* and *D* show the initial conditions whereas points *B**, *C** and *D** show the final conditions.

The pressure at B^* = Pressure at C^*

or Pressure at A + Pressure due to $(10 - x)$ cm of water

= Pressure at D^* + Pressure due to $(10 - 2x)$ cm of mercury

or $p_A + \rho_1 \times g \times h_1 = p_D{}^* + \rho_2 \times g \times h_2$

or $1910 + 1000 \times 9.81 \times \left(\frac{10 - x}{100}\right)$

$= 0 + (13.6 \times 1000) \times 9.81 \times \left(\frac{10 - 2x}{100}\right)$

Dividing by 9.81, we get

or $1000 + 100 - 10x = 1360 - 272x$

or $272x - 10x = 1360 - 1100$

or $262x = 260$

$\therefore$ $x = \frac{260}{262} = 0.992$ cm

$\therefore$ New difference of mercury = $10 - 2x$ cm $= 10 - 2 \times 0.992$

= **8.016 cm. Ans.**

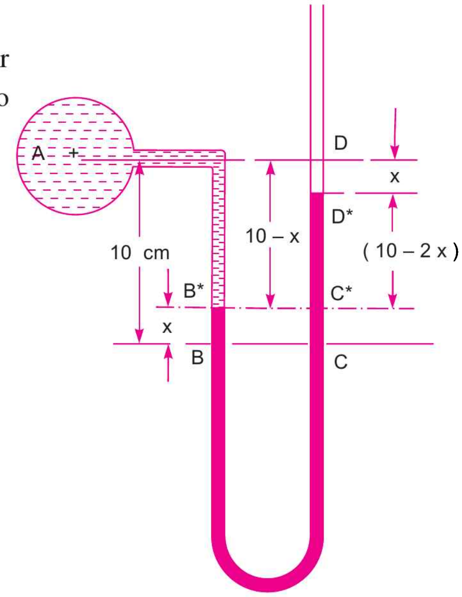


Fig. 2.11 (*b*)

Problem 2.12 *Fig. 2.12 shows a conical vessel having its outlet at A to which a U-tube manometer is connected. The reading of the manometer given in the figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.*

Solution. Vessel is empty. Given :

Difference of mercury level $h_2 = 20$ cm

Let h_1 = Height of water above X-X

Sp. gr. of mercury, $S_2 = 13.6$

Sp. gr. of water, $S_1 = 1.0$

Density of mercury, $\rho_2 = 13.6 \times 1000$

Density of water, $\rho_1 = 1000$

Equating the pressure above datum line X-X, we have

$$\rho_2 \times g \times h_2 = \rho_1 \times g \times h_1$$

or $13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$

$h_1 = 2.72$ m of water.

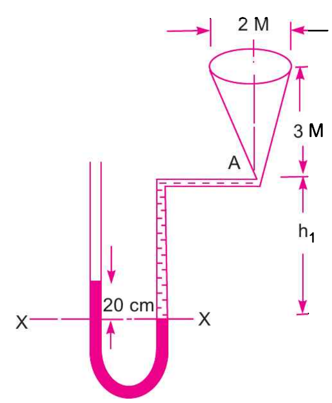


Fig. 2.12

Vessel is full of water. When vessel is full of water, the pressure in the right limb will increase and mercury level in the right limb will go down. Let the distance through which mercury goes down in the right limb be, y cm as shown in Fig. 2.13. The mercury will rise in the left by a distance of y cm. Now the datum line is Z-Z. Equating the pressure above the datum line Z-Z.

Pressure in left limb = Pressure in right limb

$13.6 \times 1000 \times 9.81 \times (0.2 + 2y/100)$

$= 1000 \times 9.81 \times (3 + h_1 + y/100)$

or $\quad 13.6 \times (0.2 + 2y/100) = (3 + 2.72 + y/100) \quad (\because \; h_1 = 2.72 \text{ cm})$

or $\quad 2.72 + 27.2y/100 = 3 + 2.72 + y/100$

or $\quad (27.2y - y)/100 = 3.0$

or $\quad 26.2y = 3 \times 100 = 300$

$\therefore \quad y = \dfrac{300}{26.2} = 11.45 \text{ cm}$

The difference of mercury level in two limbs

$= (20 + 2y)$ cm of mercury

$= 20 + 2 \times 11.45 = 20 + 22.90$

$= 42.90$ cm of mercury

$\therefore$ Reading of manometer = **42.90 cm. Ans.**

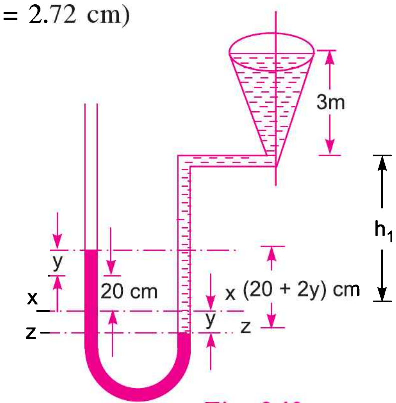


Fig. 2.13

Problem 2.13 *A pressure gauge consists of two cylindrical bulbs B and C each of 10 sq. cm cross-sectional area, which are connected by a U-tube with vertical limbs each of 0.25 sq. cm cross-sectional area. A red liquid of specific gravity 0.9 is filled into C and clear water is filled into B, the surface of separation being in the limb attached to C. Find the displacement of the surface of separation when the pressure on the surface in C is greater than that in B by an amount equal to 1 cm head of water.*

Solution. Given :

Area of each bulb B and C, $\quad A = 10 \text{ cm}^2$

Area of each vertical limb, $\quad a = 0.25 \text{ cm}^2$

Sp. gr. of red liquid $\quad = 0.9 \quad \therefore$ Its density = 900 kg/m^3

Let $\quad X\text{-}X$ = Initial separation level

h_C = Height of red liquid above $X\text{-}X$

h_B = Height of water above $X\text{-}X$

Pressure above $X\text{-}X$ in the left limb $= 1000 \times 9.81 \times h_B$

Pressure above $X\text{-}X$ in the right limb $= 900 \times 9.81 \times h_C$

Equating the two pressure, we get

$$1000 \times 9.81 \times h_B = 900 \times 9.81 \times h_C$$

$\therefore \quad h_B = 0.9\, h_C \quad \ldots(i)$

When the pressure head over the surface in C is increased by 1 cm of water, let the separation level falls by an amount equal to Z. Then $Y\text{-}Y$ becomes the final separation level.

Now fall in surface level of C multiplied by cross-sectional area of bulb C must be equal to the fall in separation level multiplied by cross-sectional area of limb.

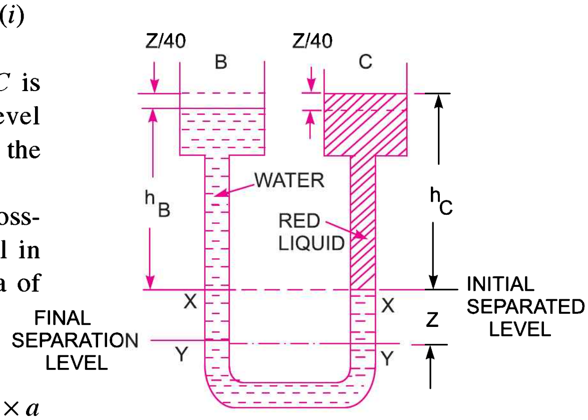


Fig. 2.14

$\therefore$ Fall in surface level of C

$$= \frac{\text{Fall in separation level} \times a}{A}$$

$$= \frac{Z \times a}{A} = \frac{Z \times 0.25}{10} = \frac{Z}{40}.$$

Also fall in surface level of C

$$= \text{Rise in surface level of } B$$

$$= \frac{Z}{40}$$

The pressure of 1 cm (or 0.01 m) of water = ρgh = 1000 × 9.81 × 0.01= 98.1 N/m^2

Consider final separation level Y-Y

Pressure above Y-Y in the left limb = $1000 \times 9.81 \left(Z + h_B + \frac{Z}{40}\right)$

Pressure above Y-Y in the right limb = $900 \times 9.81 \left(Z + h_C - \frac{Z}{40}\right) + 98.1$

Equating the two pressure, we get

$$1000 \times 9.81 \left(Z + h_B + \frac{Z}{40}\right) = \left(Z + h_C - \frac{Z}{40}\right) 900 \times 9.81 + 98.1$$

Dividing by 9.81, we get

$$1000 \left(Z + h_B + \frac{Z}{40}\right) = 900 \left(Z + h_C - \frac{Z}{40}\right) + 10$$

Dividing by 1000, we get $Z + h_B + \frac{Z}{40} = 0.9 \left(Z + h_C - \frac{Z}{40}\right) + 0.01$

But from equation (i), $h_B = 0.9\, h_C$

$$\therefore \quad Z + 0.9\, h_C + \frac{Z}{40} = \frac{39Z}{40} \times 0.9 + 0.9\, h_C + 0.01$$

or
$$\frac{41Z}{40} = \frac{39}{40} \times .9Z + .01$$

or
$$Z\left(\frac{41}{40} - \frac{39 \times .9}{40}\right) = .01 \quad \text{or} \quad Z\left(\frac{41 - 35.1}{40}\right) = .01$$

$$\therefore \quad Z = \frac{40 \times 0.01}{5.9} = \mathbf{0.0678\ m = 6.78\ cm.\ Ans.}$$

2.6.3 Single Column Manometer. Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in Fig. 2.15. Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as :

1. Vertical Single Column Manometer.
2. Inclined Single Column Manometer.

1. Vertical Single Column Manometer

Fig. 2.15 shows the vertical single column manometer. Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let Δh = Fall of heavy liquid in reservoir

h_2 = Rise of heavy liquid in right limb

h_1 = Height of centre of pipe above X-X

p_A = Pressure at A, which is to be measured

A = Cross-sectional area of the reservoir

a = Cross-sectional area of the right limb

S_1 = Sp. gr. of liquid in pipe

S_2 = Sp. gr. of heavy liquid in reservoir and right limb

ρ_1 = Density of liquid in pipe

ρ_2 = Density of liquid in reservoir

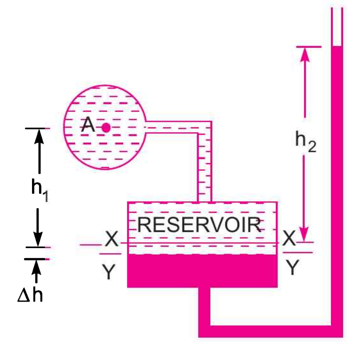


Fig. 2.15 *Vertical single column manometer.*

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore \quad A \times \Delta h = a \times h_2$$

$$\therefore \quad \Delta h = \frac{a \times h_2}{A} \quad \ldots(i)$$

Now consider the datum line Y-Y as shown in Fig. 2.15. Then pressure in the right limb above Y-Y.

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb above Y-Y = $\rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating these pressures, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + p_A$$

or

$$p_A = \rho_2 g\,(\Delta h + h_2) - \rho_1 g(\Delta h + h_1)$$

$$= \Delta h[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

But from equation (i), $\Delta h = \dfrac{a \times h_2}{A}$

$$\therefore \quad p_A = \frac{a \times h_2}{A}\,[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g \quad \ldots(2.9)$$

As the area A is very large as compared to a, hence ratio $\dfrac{a}{A}$ becomes very small and can be neglected.

Then $p_A = h_2\rho_2 g - h_1\rho_1 g$ ...(2.10)

From equation (2.10), it is clear that as h_1 is known and hence by knowing h_2 or rise of heavy liquid in the right limb, the pressure at A can be calculated.

2. Inclined Single Column Manometer

Fig. 2.16 shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.

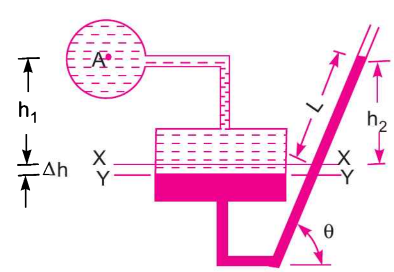


Fig. 2.16 *Inclined single column manometer.*

Let L = Length of heavy liquid moved in right limb from X-X

θ = Inclination of right limb with horizontal

h_2 = Vertical rise of heavy liquid in right limb from X-X = $L \times \sin\theta$

From equation (2.10), the pressure at A is

$$p_A = h_2\rho_2 g - h_1\rho_1 g.$$

Substituting the value of h_2, we get

$$p_A = \sin\theta \times \rho_2 g - h_1\rho_1 g. \quad \text{...(2.11)}$$

Problem 2.14 *A single column manometer is connected to a pipe containing a liquid of sp. gr. 0.9 as shown in Fig. 2.17. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading shown in Fig. 2.17. The specific gravity of mercury is 13.6.*

Solution. Given :

Sp. gr. of liquid in pipe, $S_1 = 0.9$

$\therefore$ Density $\rho_1 = 900 \text{ kg/m}^3$

Sp. gr. of heavy liquid, $S_2 = 13.6$

Density, $\rho_2 = 13.6 \times 1000$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of liquid, $h_1 = 20 \text{ cm} = 0.2 \text{ m}$

Rise of mercury in right limb,

$h_2 = 40 \text{ cm} = 0.4 \text{ m}$

Let p_A = Pressure in pipe

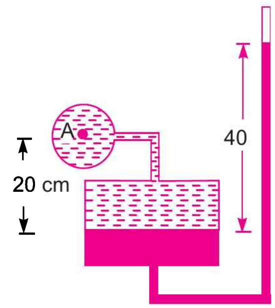


Fig. 2.17

Using equation (2.9), we get

$$p_A = \frac{a}{A} h_2[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

$$= \frac{1}{100} \times 0.4[13.6 \times 1000 \times 9.81 - 900 \times 9.81] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100}[133416 - 8829] + 53366.4 - 1765.8$$

$= 533.664 + 53366.4 - 1765.8 \text{ N/m}^2 = 52134 \text{ N/m}^2 =$ **5.21 N/cm². Ans.**

▶ 2.7 DIFFERENTIAL MANOMETERS

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are :

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

2.7.1 U-tube Differential Manometer.

Fig. 2.18 shows the differential manometers of U-tube type.

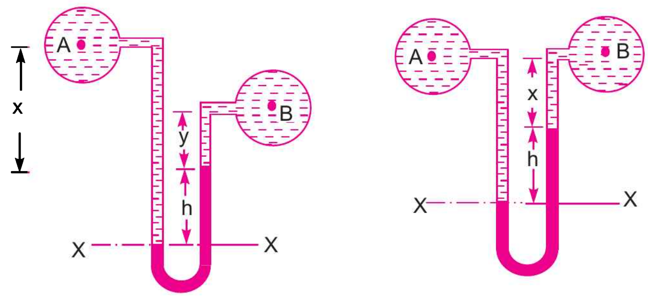


Fig. 2.18 *U-tube differential manometers.*

In Fig. 2.18 (*a*), the two points A and B are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B are p_A and p_B.

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of B, from the mercury level in the right limb.

x = Distance of the centre of A, from the mercury level in the right limb.

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X.

Pressure above X-X in the left limb $= \rho_1 g(h + x) + p_A$

where p_A = pressure at A.

Pressure above X-X in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at B.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 gy + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 gy - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx \qquad ...(2.12)$$

$\therefore$ Difference of pressure at A and $B = h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

In Fig. 2.18 (*b*), the two points A and B are at the same level and contains the same liquid of density ρ_1. Then

Pressure above X-X in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above X-X in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 gx + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 gx - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1). \qquad ...(2.13)$$

Problem 2.15 *A pipe contains an oil of sp. gr. 0.9. A differential manometer connected at the two points A and B shows a difference in mercury level as 15 cm. Find the difference of pressure at the two points.*

Solution. Given :

Sp. gr. of oil, $S_1 = 0.9$ ∴ Density, $\rho_1 = 0.9 \times 1000 = 900$ kg/m^3

Difference in mercury level, $h = 15$ cm $= 0.15$ m

Sp. gr. of mercury, $S_g = 13.6$ ∴ Density, $\rho_g = 13.6 \times 1000$ kg/m^3

The difference of pressure is given by equation (2.13)

or $$p_A - p_B = g \times h(\rho_g - \rho_1)$$
$$= 9.81 \times 0.15\ (13600 - 900) = \mathbf{18688\ N/m^2}.\ \textbf{Ans.}$$

Problem 2.16 *A differential manometer is connected at the two points A and B of two pipes as shown in Fig. 2.19. The pipe A contains a liquid of sp. gr. = 1.5 while pipe B contains a liquid of sp. gr. = 0.9. The pressures at A and B are 1 kgf/cm^2 and 1.80 kgf/cm^2 respectively. Find the difference in mercury level in the differential manometer.*

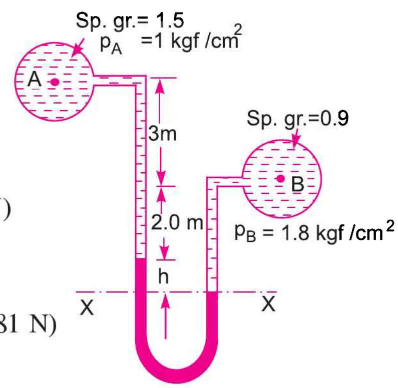


Fig. 2.19

Solution. Given :

Sp. gr. of liquid at A, $S_1 = 1.5$ ∴ $\rho_1 = 1500$

Sp. gr. of liquid at B, $S_2 = 0.9$ ∴ $\rho_2 = 900$

Pressure at A, $p_A = 1$ kgf/cm^2 $= 1 \times 10^4$ kgf/m^2
$= 10^4 \times 9.81$ N/m^2 (∵ 1 kgf = 9.81 N)

Pressure at B, $p_B = 1.8$ kgf/cm^2
$= 1.8 \times 10^4$ kgf/m^2
$= 1.8 \times 10^4 \times 9.81$ N/m^2 (∵ 1 kgf = 9.81 N)

Density of mercury $= 13.6 \times 1000$ kg/m^3

Taking X-X as datum line.

Pressure above X-X in the left limb
$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + p_A$$
$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above X-X in the right limb $= 900 \times 9.81 \times (h + 2) + p_B$
$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Equating the two pressure, we get
$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4$$
$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Dividing by 1000×9.81, we get
$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$
or $$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$
or $$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.7h = 2.3$$
∴ $$h = \frac{2.3}{12.7} = 0.181 \text{ m} = \mathbf{18.1\ cm.\ Ans.}$$

Problem 2.17 *A differential manometer is connected at the two points A and B as shown in Fig. 2.20. At B air pressure is 9.81 N/cm^2 (abs), find the absolute pressure at A.*

Solution. Given :

Air pressure at $B = 9.81$ N/cm^2

or $$p_B = 9.81 \times 10^4 \text{ N/m}^2$$

Density of oil $= 0.9 \times 1000 = 900$ kg/m^3
Density of mercury $= 13.6 \times 1000$ kg/m^3
Let the pressure at A is p_A
Taking datum line at X-X
Pressure above X-X in the right limb

$= 1000 \times 9.81 \times 0.6 + p_B$
$= 5886 + 98100 = 103986$

Pressure above X-X in the left limb

$= 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + p_A$
$= 13341.6 + 1765.8 + p_A$

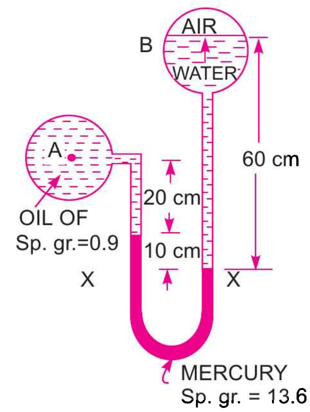


Fig. 2.20

Equating the two pressure heads

$$103986 = 13341.6 + 1765.8 + p_A$$

$\therefore$ $p_A = 103986 - 15107.4 = 88876.8$

$\therefore$ $p_A = 88876.8 \text{ N/m}^2 = \dfrac{88876.8\text{N}}{10000 \text{ cm}^2} = 8.887 \dfrac{\text{N}}{\text{cm}^2}.$

$\therefore$ Absolute pressure at A = **8.887 N/cm². Ans.**

2.7.2 Inverted U-tube Differential Manometer. It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig. 2.21 shows an inverted U-tube differential manometer connected to the two points A and B. Let the pressure at A is more than the pressure at B.

Let h_1 = Height of liquid in left limb below the datum line X-X
h_2 = Height of liquid in right limb
h = Difference of light liquid
ρ_1 = Density of liquid at A
ρ_2 = Density of liquid at B
ρ_s = Density of light liquid
p_A = Pressure at A
p_B = Pressure at B.

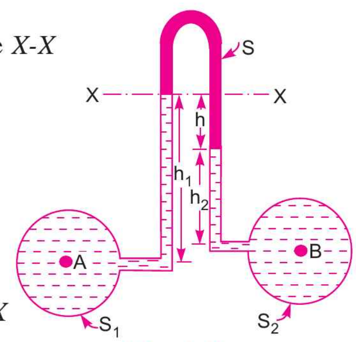


Fig. 2.21

Taking X-X as datum line. Then pressure in the left limb below X-X

$= p_A - \rho_1 \times g \times h_1.$

Pressure in the right limb below X-X

$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$

Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or $$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h. \quad ...(2.14)$$

Problem 2.18 *Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings as shown in Fig. 2.22.*

Solution. Given :

Pressure head at $A = \dfrac{p_A}{\rho g} = 2$ m of water

$\therefore$ $p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620$ N/m^2

Fig. 2.22 shows the arrangement. Taking X-X as datum line.

Pressure below X-X in the left limb $= p_A - \rho_1 \times g \times h_1$

$= 19620 - 1000 \times 9.81 \times 0.3 = 16677$ N/m^2.

Pressure below *X-X* in the right limb

$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$

$= p_B - 981 - 941.76 = p_B - 1922.76$

Equating the two pressure, we get

$16677 = p_B - 1922.76$

or $p_B = 16677 + 1922.76 = 18599.76$ N/m^2

or $p_B =$ **1.8599 N/cm^2. Ans.**

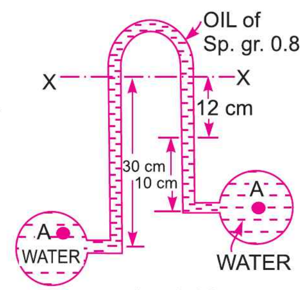


Fig. 2.22

Problem 2.19 *In Fig. 2.23, an inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the figure, find the pressure difference between A and B.*

Solution. Given :

Sp. gr. of oil $= 0.8 \quad \therefore \quad \rho_s = 800$ kg/m^3

Difference of oil in the two limbs

$= (30 + 20) - 30 = 20$ cm

Taking datum line at *X-X*

Pressure in the left limb below *X-X*

$= p_A - 1000 \times 9.81 \times 0$

$= p_A - 2943$

Pressure in the right limb below *X-X*

$= p_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$

$= p_B - 2943 - 1569.6 = p_B - 4512.6$

Equating the two pressure $p_A - 2943 = p_B - 4512.6$

$\therefore \quad p_B - p_A = 4512.6 - 2943 =$ **1569.6 N/m^2. Ans.**

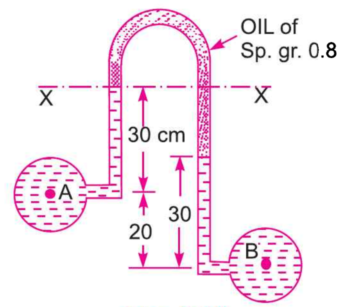


Fig. 2.23

Problem 2.20 *Find out the differential reading 'h' of an inverted U-tube manometer containing oil of specific gravity 0.7 as the manometric fluid when connected across pipes A and B as shown in Fig. 2.24 below, conveying liquids of specific gravities 1.2 and 1.0 and immiscible with manometric fluid. Pipes A and B are located at the same level and assume the pressures at A and B to be equal.*

Solution. Given :

Fig. 2.24 shows the arrangement. Taking *X-X* as datum line.

Let $p_A =$ Pressure at *A*

$p_B =$ Pressure at *B*

Density of liquid in pipe *A* = Sp. gr. × 1000

$= 1.2 \times 1000$

$= 1200$ kg/m^2

Density of liquid in pipe *B* $= 1 \times 1000 = 1000$ kg/m^3

Density of oil $= 0.7 \times 1000 = 700$ kg/m^3

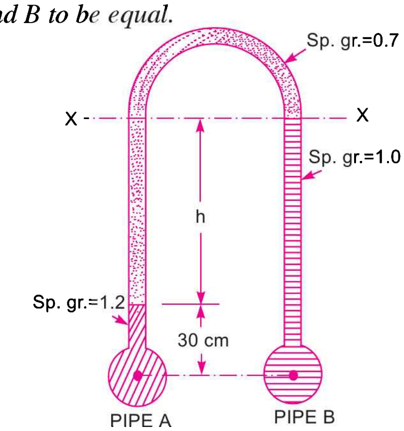


Fig. 2.24

Now pressure below X-X in the left limb

$$= p_A - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h$$

Pressure below X-X in the right limb

$$= p_B - 1000 \times 9.81 \times (h + 0.3)$$

Equating the two pressure, we get

$$p_A - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h = p_B - 1000 \times 9.81\ (h + 0.3)$$

But $\qquad p_A = p_B$ (given)

$$\therefore \quad - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h = - 1000 \times 9.81\ (h + 0.3)$$

Dividing by 1000×9.81

$$- 1.2 \times 0.3 - 0.7h = - (h + 0.3)$$

or $\qquad 0.3 \times 1.2 + 0.7h = h + 0.3$ or $0.36 - 0.3 = h - 0.7h = 0.3h$

$$\therefore \quad h = \frac{0.36 - 0.30}{0.30} = \frac{0.06}{0.30} \text{ m}$$

$$= \frac{1}{5} \text{ m} = \frac{1}{5} \times 100 = \mathbf{20\ cm.\ Ans.}$$

Problem 2.21 *An inverted U-tube manometer is connected to two horizontal pipes A and B through which water is flowing. The vertical distance between the axes of these pipes is 30 cm. When an oil of specific gravity 0.8 is used as a gauge fluid, the vertical heights of water columns in the two limbs of the inverted manometer (when measured from the respective centre lines of the pipes) are found to be same and equal to 35 cm. Determine the difference of pressure between the pipes.*

Solution. Given :

Specific gravity of measuring liquid = 0.8

The arrangement is shown in Fig. 2.24 (a).

Let $\quad p_A$ = pressure at A

p_B = pressure at B.

The points C and D lie on the same horizontal line.

Hence pressure at C should be equal to pressure at D.

But pressure at C $\quad = p_A - \rho g\ h$

$$= p_A - 1000 \times 9.81 \times (0.35)$$

And pressure at D $\quad = p_B - \rho_1 g h_1 - \rho_2 g h_2$

$$= p_B - 1000 \times 9.81 \times (0.35) - 800 \times 9.81 \times 0.3$$

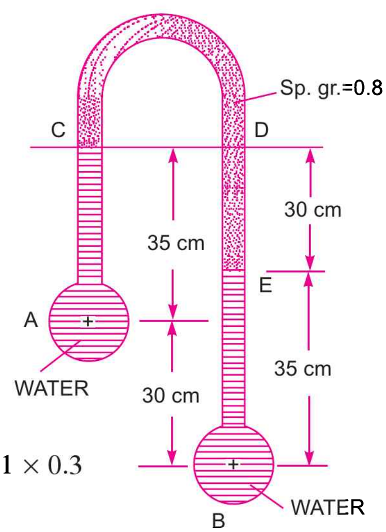


Fig. 2.24 *(a)*

But pressure at C = pressure at D

$$\therefore \quad p_A - 1000 \times 9.81 \times .35$$

$$= p_B - 1000 \times 9.81 \times 0.35 - 800 \times 9.81 \times 0.3$$

or $\qquad 800 \times 9.81 \times 0.3 = p_B - p_A$

or $$p_B - p_A = 800 \times 9.81 \times 0.3 = \mathbf{2354.4\ \frac{N}{m^2}.\ Ans.}$$

2.8 PRESSURE AT A POINT IN COMPRESSIBLE FLUID

For compressible fluids, density (ρ) changes with the change of pressure and temperature. Such problems are encountered in aeronautics, oceanography and meteorology where we are concerned with atmospheric* air where density, pressure and temperature changes with elevation. Thus for fluids with variable density, equation (2.4) cannot be integrated, unless the relationship between p and ρ is known. For gases the equation of state is

$$\frac{p}{\rho} = RT \qquad \text{...(2.15)}$$

or

$$\rho = \frac{p}{RT}$$

Now equation (2.4) is

$$\frac{dp}{dZ} = w = \rho g = \frac{p}{RT} \times g$$

$$\therefore \quad \frac{dp}{p} = \frac{g}{RT} dZ \qquad \text{...(2.16)}$$

In equation (2.4), Z is measured vertically downward. But if Z is measured vertically up, then $\frac{dp}{dZ} = -\rho g$ and hence equation (2.16) becomes

$$\frac{dp}{p} = \frac{-g}{RT} dZ \qquad \text{...(2.17)}$$

2.8.1 Isothermal Process. **Case I.** If temperature T is constant which is true for **isothermal process**, equation (2.17) can be integrated as

$$\int_{p_0}^{p} \frac{dp}{p} = -\int_{Z_0}^{Z} \frac{g}{RT} dz = -\frac{g}{RT} \int_{Z_0}^{Z} dz$$

or

$$\log \frac{p}{p_0} = \frac{-g}{RT} [Z - Z_0]$$

where p_0 is the pressure where height is Z_0. If the datum line is taken at Z_0, then $Z_0 = 0$ and p_0 becomes the pressure at datum line.

$$\therefore \quad \log \frac{p}{p_0} = \frac{-g}{RT} Z$$

$$\frac{p}{p_0} = e^{-gZ/RT}$$

or pressure at a height Z is given by $p = p_0 e^{-gZ/RT}$ ...(2.18)

2.8.2 Adiabatic Process. If temperature T is not constant but the process follows adiabatic law then the relation between pressure and density is given by

$$\frac{p}{\rho^k} = \text{Constant} = C \qquad \text{...}(i)$$

* The standard atmospheric pressure, temperature and density referred to STP at the sea-level are : Pressure = 101.325 kN/m^2 ; Temperature = 15°C and Density = 1.225 kg/m^3.

where k is ratio of specific constant.

$$\therefore \qquad \rho^k = \frac{p}{C}$$

$$\text{or} \qquad \rho = \left(\frac{p}{C}\right)^{1/k} \qquad \ldots(ii)$$

Then equation (2.4) for Z measured vertically up becomes,

$$\frac{dp}{dZ} = -\rho g = -\left(\frac{p}{C}\right)^{1/k} g$$

$$\text{or} \qquad \frac{dp}{\left(\frac{p}{C}\right)^{1/k}} = -gdZ \text{ or } C^{1/k}\frac{dp}{\frac{1}{p^k}} = -gdZ$$

Integrating, we get $\int_{p_0}^{p} C^{1/k}\, p^{-1/k}\, dp = \int_{Z_0}^{Z} -gdZ$

$$\text{or} \qquad C^{1/k}\left[\frac{p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -g\,[Z]_{Z_0}^{Z}$$

$$\text{or} \qquad \left[\frac{C^{1/k}\, p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -g\,[Z]_{Z_0}^{Z} \qquad [C \text{ is a constant, can be taken inside}]$$

But from equation (i), $C^{1/k} = \left(\frac{p}{\rho^k}\right)^{1/k} = \frac{p^{1/k}}{\rho}$

Substituting this value of $C^{1/k}$ above, we get

$$\left[\frac{p^{1/k}}{\rho}\frac{p^{-1/k+1}}{-\frac{1}{k}+1}\right]_{p_0}^{p} = -g[Z - Z_0]$$

$$\text{or} \qquad \left[\frac{p^{\frac{1-1+k}{k}}}{\rho^{\frac{k-1}{k}}}\right]_{p_0}^{p} = -g[Z - Z_0] \text{ or } \left[\frac{k}{k-1}\frac{p}{\rho}\right]_{p_0}^{p} = -g[Z - Z_0]$$

$$\text{or} \qquad \frac{k}{k-1}\left[\frac{p}{\rho} - \frac{p_0}{\rho_0}\right] = -g[Z - Z_0]$$

If datum line is taken at Z_0, where pressure, temperature and density are p_0, T_0 and ρ_0, then $Z_0 = 0$.

$$\therefore \qquad \frac{k}{k-1}\left[\frac{p}{\rho} - \frac{p_0}{\rho_0}\right] = -gZ \quad \text{or} \quad \frac{p}{\rho} - \frac{p_0}{\rho_0} = -gZ\frac{(k-1)}{k}$$

$$\text{or} \qquad \frac{p}{\rho} = \frac{p_0}{\rho_0} - gZ\frac{(k-1)}{k} = \frac{p_0}{\rho_0}\left[1 - \frac{k-1}{k}gZ\frac{\rho_0}{p_0}\right]$$

or $$\frac{p}{\rho} \times \frac{\rho_0}{p_0} = \left[1 + \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right] \qquad ...(iii)$$

But from equation (*i*), $$\frac{p}{\rho^k} = \frac{p_0}{\rho_0^k} \quad \text{or} \quad \left(\frac{\rho_0}{\rho}\right)^k = \frac{p_0}{p} \text{ or } \frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{1/k}$$

Substituting the value of $\frac{\rho_0}{\rho}$ in equation (*iii*), we get

$$\frac{p}{p_0} \times \left(\frac{p_0}{p}\right)^{1/k} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]$$

or $$\frac{p}{p_0} \times \left(\frac{p}{p_0}\right)^{-1/k} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]$$

or $$\left(\frac{p}{p_0}\right)^{1-\frac{1}{k}} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]$$

∴ $$\frac{p}{p_0} = \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{\rho_0}\right]^{\frac{k}{k-1}}$$

∴ Pressure at a height Z from ground level is given by

$$p = p_0 \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}} \qquad ...(2.19)$$

In equation (2.19), p_0 = pressure at ground level, where $Z_0 = 0$

ρ_0 = density of air at ground level

Equation of state is $$\frac{p_0}{\rho_0} = RT_0 \text{ or } \frac{\rho_0}{p_0} = \frac{1}{RT_0}$$

Substituting the values of $\frac{\rho_0}{p_0}$ in equation (2.19), we get

$$p = p_0 \left[1 - \frac{k-1}{k} \frac{gZ}{RT_0}\right]^{\frac{k}{k-1}} \qquad ...(2.20)$$

2.8.3 Temperature at any Point in Compressible Fluid. For the adiabatic process, the temperature at any height in air is calculated as :

Equation of state at ground level and at a height Z from ground level is written as

$$\frac{p_0}{\rho_0} = RT_0 \text{ and } \frac{p}{\rho} = RT$$

Dividing these equations, we get

$$\left(\frac{p_0}{\rho_0}\right) \div \frac{p}{\rho} = \frac{RT_0}{RT} = \frac{T_0}{T} \quad \text{or} \quad \frac{p_0}{\rho_0} \times \frac{\rho}{p} = \frac{T_0}{T}$$

or $$\frac{T}{T_0} = \frac{\rho_0}{p_0} \times \frac{p}{\rho} = \frac{p}{p_0} \times \frac{\rho_0}{\rho} \qquad ...(i)$$

But $\frac{p}{p_0}$ from equation (2.20) is given by

$$\frac{p}{p_0} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}}$$

Also for adiabatic process $\frac{p}{\rho^k} = \frac{p_0}{\rho_0^k}$ or $\left(\frac{\rho_0}{\rho}\right)^k = \frac{p_0}{p}$

or
$$\frac{\rho_0}{\rho} = \left(\frac{p_0}{p}\right)^{\frac{1}{k}} = \left(\frac{p}{p_0}\right)^{-\frac{1}{k}}$$

$$= \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\left(\frac{k}{k-1}\right)\times\left(-\frac{1}{k}\right)} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{-\frac{1}{k-1}}$$

Substituting the values of $\frac{p}{p_0}$ and $\frac{\rho_0}{\rho}$ in equation (*i*), we get

$$\frac{T}{T_0} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}} \times \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{-\frac{1}{k-1}}$$

$$= \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}-\frac{1}{k-1}} = \left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]$$

$$\therefore \qquad T = T_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right] \qquad \text{...(2.21)}$$

2.8.4 Temperature Lapse-Rate (L). It is defined as the rate at which the temperature changes with elevation. To obtain an expression for the temperature lapse-rate, the temperature given by equation (2.21) is differentiated with respect to Z as

$$\frac{dT}{dZ} = \frac{d}{dZ}\left[T_0\left(1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right)\right]$$

where T_0, K, g and R are constant

$$\therefore \qquad \frac{dT}{dZ} = -\frac{k-1}{k}\times\frac{g}{RT_0}\times T_0 = \frac{-g}{R}\left(\frac{k-1}{k}\right)$$

The temperature lapse-rate is denoted by L and hence

$$L = \frac{dT}{dZ} = \frac{-g}{R}\left(\frac{k-1}{k}\right) \qquad \text{...(2.22)}$$

In equation (2.22), if (*i*) $k = 1$ which means isothermal process, $\frac{dT}{dZ} = 0$, which means temperature is constant with height.

(*ii*) If $k > 1$, the lapse-rate is negative which means temperature decreases with the increase in height.

In atmosphere, the value of k varies with height and hence the value of temperature lapse-rate also varies. From the sea-level upto an elevation of about 11000 m (or 11 km), the temperature of air decreases uniformly at the rate of 0.0065°C/m. from 11000 m to 32000 m, the temperature remains constant at – 56.5°C and hence in this range lapse-rate is zero. Temperature rises again after 32000 m in air.

Problem 2.22 (SI Units) *If the atmosphere pressure at sea level is 10.143 N/cm², determine the pressure at a height of 2500 m assuming the pressure variation follows (i) Hydrostatic law, and (ii) isothermal law. The density of air is given as 1.208 kg/m³.*

Solution. Given :

Pressure at sea-level, $p_0 = 10.143 \text{ N/cm}^2 = 10.143 \times 10^4 \text{ N/m}^2$

Height, $Z = 2500$ m

Density of air, $\rho_0 = 1.208 \text{ kg/m}^3$

(*i*) **Pressure by hydrostatic law.** For hydrostatic law, ρ is assumed constant and hence p is given by equation $\frac{dp}{dZ} = -\rho g$

Integrating, we get $$\int_{p_0}^{p} dp = \int -\rho g\, dZ = -\rho g \int_{Z_0}^{Z} dZ$$

or $$p - p_0 = -\rho g\,[Z - Z_0]$$

For datum line at sea-level, $Z_0 = 0$

∴ $$p - p_0 = -\rho g Z \quad \text{or} \quad p = p_0 - \rho g Z$$

$$= 10.143 \times 10^4 - 1.208 \times 9.81 \times 2500 \quad [\because \rho = \rho_0 = 1.208]$$

$$= 101430 - 29626 = 71804 \frac{\text{N}}{\text{m}^2} \text{ or } \frac{71804}{10^4} \text{ N/cm}^2$$

$$= \mathbf{7.18\ N/cm^2.\ Ans.}$$

(*ii*) **Pressure by Isothermal Law.** Pressure at any height Z by isothermal law is given by equation (2.18) as

$$p = p_0\, e^{-gZ/RT}$$

$$= 10.143 \times 10^4\, e^{-\frac{gZ \times \rho_0}{p_0}} \qquad \left[\because \frac{p_0}{\rho_0} = RT \text{ and } \rho_0 g = w_0\right]$$

$$= 10.143 \times 10^4\, e^{-\frac{Z\rho_0 \times g}{p_0}}$$

$$= 10.143 \times 10^4\, e^{(-2500 \times 1.208 \times 9.81)/10.143 \times 10^4}$$

$$= 101430 \times e^{-.292} = 101430 \times \frac{1}{1.3391} = 75743 \text{ N/m}^2$$

$$= \frac{75743}{10^4} \text{ N/cm}^2 = \mathbf{7.574\ N/cm^2.\ Ans.}$$

Problem 2.23 *The barometric pressure at sea level is 760 mm of mercury while that on a mountain top is 735 mm. If the density of air is assumed constant at 1.2 kg/m³, what is the elevation of the mountain top?*

Solution. Given :

Pressure* at sea, $p_0 = 760$ mm of Hg

$$= \frac{760}{1000} \times 13.6 \times 1000 \times 9.81 \text{ N/m}^2 = 101396 \text{ N/m}^2$$

---

* Here pressure head (Z) is given as 760 mm of Hg. Hence $(p/\rho g)$ = 760 mm of Hg. The density (ρ) for mercury $= 13.6 \times 1000 \text{ kg/m}^3$. Hence pressure ($p$) will be equal to $\rho \times g \times Z$ *i.e.*, $13.6 \times 1000 \times 9.81 \times \frac{760}{1000}$ N/m².

Pressure at mountain, $p = 735$ mm of Hg

$$= \frac{735}{1000} \times 13.6 \times 1000 \times 9.81 = 98060 \text{ N/m}^2$$

Density of air, $\rho = 1.2 \text{ kg/m}^3$

Let h = Height of the mountain from sea-level.

We know that as the elevation above the sea-level increases, the atmospheric pressure decreases. Here the density of air is given constant, hence the pressure at any height 'h' above the sea-level is given by the equation,

$$p = p_0 - \rho \times g \times h$$

or

$$h = \frac{p_0 - p}{\rho \times g} = \frac{101396 - 98060}{1.2 \times 9.81} = \mathbf{283.33 \text{ m. Ans.}}$$

Problem 2.24 *Calculate the pressure at a height of 7500 m above sea level if the atmospheric pressure is 10.143 N/cm² and temperature is 15°C at the sea-level, assuming (i) air is incompressible, (ii) pressure variation follows isothermal law, and (iii) pressure variation follows adiabatic law. Take the density of air at the sea-level as equal to 1.285 kg/m³. Neglect variation of g with altitude.*

Solution. Given :

Height above sea-level, $Z = 7500$ m

Pressure at sea-level, $p_0 = 10.143 \text{ N/cm}^2 = 10.143 \times 10^4 \text{ N/m}^2$

Temperature at sea-level, $t_0 = 15°C$

$\therefore$ $T_0 = 273 + 15 = 288°K$

Density of air, $\rho = \rho_0 = 1.285 \text{ kg/m}^3$

(*i*) *Pressure when air is incompressible :*

$$\frac{dp}{dZ} = -\rho g$$

$$\therefore \quad \int_{p_0}^{p} dp = -\int_{Z_0}^{Z} \rho g dz \quad \text{or} \quad p - p_0 = -\rho g[Z - Z_0]$$

or

$$p = p_0 - \rho g Z \qquad \{\because \ Z_0 = \text{datum line} = 0\}$$

$$= 10.143 \times 10^4 - 1.285 \times 9.81 \times 7500$$

$$= 101430 - 94543 = 6887 \text{ N/m}^2 = \mathbf{0.688 \ \frac{N}{cm^2}. \ Ans.}$$

(*ii*) *Pressure variation follows isothermal law :*

Using equation (2.18), we have $p = p_0 e^{-gZ/RT}$

$$= p_0 \, e^{-gZ\rho_0/p_0} \qquad \left\{\because \frac{p_0}{\rho_0} = RT \ \therefore \ \frac{\rho_0}{p_0} = \frac{1}{RT}\right\}$$

$$= 101430 \, e^{-gZ\rho_0/p_0} = 101430 \, e^{-7500 \times 1.285 \times 9.81/101430}$$

$$= 101430 \, e^{-.9320} = 101430 \times .39376$$

$$= \mathbf{39939 \text{ N/m}^2 \text{ or } 3.993 \text{ N/cm}^2. \text{ Ans.}}$$

(*iii*) *Pressure variation follows adiabatic law :* [$k = 1.4$]

Using equation (2.19), we have $p = p_0 \left[1 - \frac{k-1}{k} gZ \frac{\rho_0}{p_0}\right]^{k/(k-1)}$, where $\rho_0 = 1.285 \text{ kg/m}^3$

$$\therefore \qquad p = 101430\left[1-\frac{(1.4-1.0)}{1.4}\times 9.81\times\frac{(7500\times 1.285)}{101430}\right]^{\frac{1.4}{1.4-1.0}}$$

$$= 101430\,[1-.2662]^{1.4/.4} = 101430\times(.7337)^{3.5}$$

$$= \mathbf{34310\ N/m^2\ or\ 3.431\frac{N}{cm^2}.\ Ans.}$$

Problem 2.25 *Calculate the pressure and density of air at a height of 4000 m from sea-level where pressure and temperature of the air are 10.143 N/cm² and 15°C respectively. The temperature lapse rate is given as 0.0065°C/m. Take density of air at sea-level equal to 1.285 kg/m³.*

Solution. Given :

Height, $Z = 4000$ m

Pressure at sea-level, $p_0 = 10.143\ \text{N/cm}^2 = 10.143\times 10^4 = 101430\ \frac{\text{N}}{\text{m}^2}$

Temperature at sea-level, $t_0 = 15°\text{C}$

$\therefore$ $T_0 = 273 + 15 = 288°\text{K}$

Temperature lapse-rate, $L = \frac{dT}{dZ} = -\,0.0065°\text{K/m}$

$\rho_0 = 1.285\ \text{kg/m}^3$

Using equation (2.22), we have $L = \frac{dT}{dZ} = -\frac{g}{R}\left(\frac{k-1}{k}\right)$

or $$-\,0.0065 = -\frac{9.81}{R}\left(\frac{k-1}{k}\right),\ \text{where } R = \frac{p_0}{\rho_0 T_0} = \frac{101430}{1.285\times 288} = 274.09$$

$$\therefore \qquad -\,0.0065 = \frac{-9.81}{274.09}\times\left(\frac{k-1}{k}\right)$$

$$\therefore \qquad \frac{k-1}{k} = \frac{0.0065\times 274.09}{9.81} = 0.1815$$

$$\therefore \qquad k[1-.1815] = 1$$

$$\therefore \qquad k = \frac{1}{1-.1815} = \frac{1}{.8184} = 1.222$$

This means that the value of power index $k = 1.222$.

(*i*) **Pressure** at 4000 m height is given by equation (2.19) as

$$p = p_0\left[1-\frac{k-1}{k}\,gZ\,\frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}},\ \text{where } k = 1.222 \text{ and } \rho_0 = 1.285$$

$$\therefore \qquad p = 101430\left[1-\left(\frac{1.222-1.0}{1.222}\right)\times 9.81\times\frac{4000\times 1.285}{101430}\right]^{\frac{1.222}{1.222-1.0}}$$

$$= 101430\,[1-0.09]^{5.50} = 101430\times .595$$

$$= 60350\ \text{N/m}^2 = \mathbf{6.035\ \frac{N}{cm^2}.\ Ans.}$$

(*ii*) **Density.** Using equation of state, we get

$$\frac{p}{\rho} = RT$$

where p = Pressure at 4000 m height

ρ = Density at 4000 m height

T = Temperature at 4000 m height

Now T is calculated from temperature lapse-rate as

$$t \text{ at } 4000 \text{ m} = t_0 + \frac{dT}{dZ} \times 4000 = 15 - .0065 \times 4000 = 15 - 26 = -11°\text{C}$$

$$\therefore \quad T = 273 + t = 273 - 11 = 262°\text{K}$$

$\therefore$ Density is given by $\rho = \dfrac{p}{RT} = \dfrac{60350}{274.09 \times 262}$ kg/m^3 = **0.84 kg/m^3. Ans.**

Problem 2.26 *An aeroplane is flying at an altitude of 5000 m. Calculate the pressure around the aeroplane, given the lapse-rate in the atmosphere as 0.0065°K/m. Neglect variation of g with altitude. Take pressure and temperature at ground level as 10.143 N/cm^2 and 15°C and density of air as 1.285 kg/cm^3.*

Solution. Given :

Height, $Z = 5000$ m

Lapse-rate, $L = \dfrac{dT}{dZ} = -.0065°\text{K/m}$

Pressure at ground level, $p_0 = 10.143 \times 10^4 \text{ N/m}^2$

$t_0 = 15°\text{C}$

$\therefore \quad T_0 = 273 + 15 = 288°\text{K}$

Density, $\rho_0 = 1.285 \text{ kg/m}^3$

$\therefore$ Temperature at 5000 m height $= T_0 + \dfrac{dT}{dZ} \times \text{Height} = 288 - .0065 \times 5000$

$= 288 - 32.5 = 255.5°\text{K}$

First find the value of power index k as

From equation (2.22), we have $L = \dfrac{dT}{dZ} = -\dfrac{g}{R}\left(\dfrac{k-1}{k}\right)$

or

$$-.0065 = -\frac{9.81}{R}\left(\frac{k-1}{k}\right)$$

where $R = \dfrac{p_0}{\rho_0 T_0} = \dfrac{101430}{1.285 \times 288} = 274.09$

$$\therefore \quad -.0065 = -\frac{9.81}{274.09}\left(\frac{k-1}{k}\right)$$

$$\therefore \quad k = 1.222$$

The pressure is given by equation (2.19) as

$$p = p_0\left[1 - \frac{k-1}{k}\, gZ \frac{\rho_0}{p_0}\right]^{\left(\frac{k}{k-1}\right)}$$

$$= 101430\left[1-\left(\frac{1.222-1.0}{1.222}\right)\times 9.81\times\frac{5000\times 1.285}{101430}\right]^{\frac{1.222}{1.222-1.0}}$$

$$= 101430\left[1-\frac{.222}{1.222}\times 9.81\times\frac{5000\times 1.285}{101430}\right]^{\frac{1.222}{.222}}$$

$$= 101430\,[1-0.11288]^{5.50} = 101430\times 0.5175 = 52490\ \text{N/m}^3$$

= 5.249 N/cm². Ans.

HIGHLIGHTS

1. The pressure at any point in a fluid is defined as the force per unit area.
2. The Pascal's law states that intensity of pressure for a fluid at rest is equal in all directions.
3. Pressure variation at a point in a fluid at rest is given by the hydrostatic law which states that the rate of increase of pressure in the vertically downward direction is equal to the specific weight of the fluid, $\frac{dp}{dZ} = w = \rho \times g$.
4. The pressure at any point in a incompressible fluid (liquid) is equal to the product of density of fluid at that point, acceleration due to gravity and vertical height from free surface of fluid, $p = \rho \times g \times Z$.
5. Absolute pressure is the pressure in which absolute vacuum pressure is taken as datum while gauge pressure is the pressure in which the atmospheric pressure is taken as datum,
$$p_{abs.} = p_{atm} + p_{gauge.}$$
6. Manometer is a device used for measuring pressure at a point in a fluid.
7. Manometers are classified as (*a*) Simple manometers and (*b*) Differential manometers.
8. Simple manometers are used for measuring pressure at a point while differential manometers are used for measuring the difference of pressures between the two points in a pipe, or two different pipes.
9. A single column manometer (or micrometer) is used for measuring small pressures, where accuracy is required.
10. The pressure at a point in static compressible fluid is obtained by combining two equations, *i.e.*, equation of state for a gas and equation given by hydrostatic law.
11. The pressure at a height Z in a static compressible fluid (gas) under going isothermal compression $\left(\frac{p}{\rho} = \text{const.}\right)$
$$p = p_0\, e^{-gZ/RT}$$
where p_0 = Absolute pressure at sea-level or at ground level
Z = Height from sea or ground level
R = Gas constant
T = Absolute temperature.
12. The pressure and temperature at a height Z in a static compressible fluid (gas) undergoing adiabatic compression (p/ρ^k = const.)
$$p = p_0\left[1-\frac{k-1}{k}\,gZ\,\frac{\rho_0}{p_0}\right]^{\frac{k}{k-1}} = p_0\left[1-\frac{k-1}{k}\,\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}}$$

and temperature, $$T = T_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]$$

where p_0, T_0 are pressure and temperature at sea-level $k = 1.4$ for air.

13. The rate at which the temperature changes with elevation is known as Temperature Lapse-Rate. It is given by

$$L = \frac{-g}{R}\left(\frac{k-1}{k}\right)$$

if (*i*) $k = 1$, temperature is zero.

(*ii*) $k > 1$, temperature decreases with the increase of height.

EXERCISE

(A) THEORETICAL PROBLEMS

1. Define pressure. Obtain an expression for the pressure intensity at a point in a fluid.
2. State and prove the Pascal's law.
3. What do you understand by Hydrostatic Law ?
4. Differentiate between : (*i*) Absolute and gauge pressure, (*ii*) Simple manometer and differential manometer, and (*iii*) Piezometer and pressure gauges.
5. What do you mean by vacuum pressure ?
6. What is a manometer ? How are they classified ?
7. What do you mean by single column manometers ? How are they used for the measurement of pressure ?
8. What is the difference between U-tube differential manometers and inverted U-tube differential manometers ? Where are they used ?
9. Distinguish between manometers and mechanical gauges. What are the different types of mechanical pressure gauges ?
10. Derive an expression for the pressure at a height Z from sea-level for a static air when the compression of the air is assumed isothermal. The pressure and temperature at sea-levels are p_0 and T_0 respectively.
11. Prove that the pressure and temperature for an adiabatic process at a height Z from sea-level for a static air are :

$$p_0 = p_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]^{\frac{k}{k-1}} \text{ and } T = T_0\left[1 - \frac{k-1}{k}\frac{gZ}{RT_0}\right]$$

where p_0 and T_0 are the pressure and temperature at sea-level.

12. What do you understand by the term, 'Temperature Lapse-Rate'? Obtain an expression for the temperature Lapse-Rate.
13. What is hydrostatic pressure distribution? Give one example where pressure distribution is non-hydrostatic.
14. Explain briefly the working principle of Bourdon Pressure Gauge with a neat sketch.
(J.N.T.U., Hyderabad, S 2002)

(B) NUMERICAL PROBLEMS

1. A hydraulic press has a ram of 30 cm diameter and a plunger of 5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 400 N. [**Ans.** 14.4 kN]
2. A hydraulic press has a ram of 20 cm diameter and a plunger of 4 cm diameter. It is used for lifting a weight of 20 kN. Find the force required at the plunger. [**Ans.** 800 N]

3. Calculate the pressure due to a column of 0.4 m of (*a*) water, (*b*) an oil of sp. gr. 0.9, and (*c*) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000 \frac{kg}{m^3}$. **[Ans.** (*a*) 0.3924 N/cm^2, (*b*) 0.353 N/cm^2, (*c*) 5.33 N/cm^2]

4. The pressure intensity at a point in a fluid is given 4.9 N/cm^2. Find the corresponding height of fluid when it is : (*a*) water, and (*b*) an oil of sp. gr. 0.8. **[Ans.** (*a*) 5 m of water, (*b*) 6.25 m of oil]

5. An oil of sp. gr. 0.8 is contained in a vessel. At a point the height of oil is 20 m. Find the corresponding height of water at that point. **[Ans.** 16 m]

6. An open tank contains water upto a depth of 1.5 m and above it an oil of sp. gr. 0.8 for a depth of 2 m. Find the pressure intensity : (*i*) at the interface of the two liquids, and (*ii*) at the bottom of the tank. **[Ans.** (*i*) 1.57 N/cm^2, (*ii*) 3.04 N/cm^2]

7. The diameters of a small piston and a large piston of a hydraulic jack are 2 cm and 10 cm respectively. A force of 60 N is applied on the small piston. Find the load lifted by the large piston, when : (*a*) the pistons are at the same level, and (*b*) small piston is 20 cm above the large piston. The density of the liquid in the jack is given as $1000 \frac{kg}{m^2}$. **[Ans.** (*a*) 1500 N, (*b*) 1520.5 N]

8. Determine the gauge and absolute pressure at a point which is 2.0 m below the free surface of water. Take atmospheric pressure as 10.1043 N/cm^2. **[Ans.** 1.962 N/cm^2 (gauge), 12.066 N/cm^2 (abs.)]

9. A simple manometer is used to measure the pressure of oil (sp. gr. = 0.8) flowing in a pipe line. Its right limb is open to the atmosphere and left limb is connected to the pipe. The centre of the pipe is 9 cm below the level of mercury (sp. gr. 13.6) in the right limb. If the difference of mercury level in the two limbs is 15 cm, determine the absolute pressure of the oil in the pipe in N/cm^2. **[Ans.** 12.058 N/cm^2]

10. A simple manometer (U-tube) containing mercury is connected to a pipe in which an oil of sp. gr. 0.8 is flowing. The pressure in the pipe is vacuum. The other end of the manometer is open to the atmosphere. Find the vacuum, pressure in pipe, if the difference of mercury level in the two limbs is 20 cm and height of oil in the left limb from the centre of the pipe is 15 cm below. **[Ans.** – 27.86 N/cm^2]

11. A single column vertical manometer (*i.e.,* micrometer) is connected to a pipe containing oil of sp. gr. 0.9. The area of the reservoir is 80 times the area of the manometer tube. The reservoir contains mercury of sp. gr. 13.6. The level of mercury in the reservoir is at a height of 30 cm below the centre of the pipe and difference of mercury levels in the reservoir and right limb is 50 cm. Find the pressure in the pipe. **[Ans.** 6.474 N/cm^2]

12. A pipe contains an oil of sp. gr. 0.8. A differential manometer connected at the two points A and B of the pipe shows a difference in mercury level as 20 cm. Find the difference of pressure at the two points. **[Ans.** 25113.6 N/m^2]

13. A U-tube differential manometer connects two pressure pipes *A* and *B*. Pipe *A* contains carbon tetrachloride having a specific gravity 1.594 under a pressure of 11.772 N/cm^2 and pipe *B* contains oil of sp. gr. 0.8 under a pressure of 11.772 N/cm^2. The pipe *A* lies 2.5 m above pipe *B*. Find the difference of pressure measured by mercury as fluid filling U-tube. **[Ans.** 31.36 cm of mercury]

14. A differential manometer is connected at the two points *A* and *B* as shown in Fig. 2.25. At *B* air pressure is 7.848 N/cm^2 (abs.), find the absolute pressure at *A*. **[Ans.** 6.91 N/cm^2]

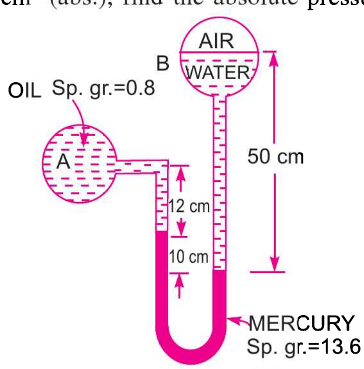


Fig. 2.25

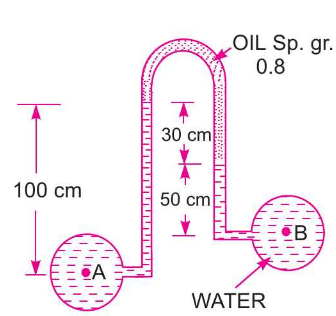


Fig. 2.26

15. An inverted differential manometer containing an oil of sp. gr. 0.9 is connected to find the difference of pressures at two points of a pipe containing water. If the manometer reading is 40 cm, find the difference of pressures. [**Ans**. 392.4 N/m^2]

16. In above Fig. 2.26 shows an inverted differential manometer connected to two pipes *A* and *B* containing water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the figure, find the difference of pressure head between *A* and *B*. [**Ans.** 0.26 m of water]

17. If the atmospheric pressure at sea-level is 10.143 N/cm^2, determine the pressure at a height of 2000 m assuming that the pressure variation follows : (*i*) Hydrostatic law, and (*ii*) Isothermal law. The density of air is given as 1.208 kg/m^3. [**Ans.** (*i*) 7.77 N/cm^2, (*ii*) 8.03 N/cm^3]

18. Calculate the pressure at a height of 8000 m above sea-level if the atmospheric pressure is 101.3 kN/m^2 and temperature is 15°C at the sea-level assuming (*i*) air is incompressible, (*ii*) pressure variation follows adiabatic law, and (*iii*) pressure variation follows isothermal law. Take the density of air at the sea-level as equal to 1.285 kg/m^3. Neglect variation of g with altitude.

[**Ans.** (*i*) 607.5 N/m^2, (*ii*) 31.5 kN/m^2 (*iii*) 37.45 kN/m^2]

19. Calculate the pressure and density of air at a height of 3000 m above sea-level where pressure and temperature of the air are 10.143 N/cm^2 and 15°C respectively. The temperature lapse-rate is given as 0.0065° K/m. Take density of air at sea-level equal to 1.285 kg/m^3. [**Ans.** 6.896 N/cm^2, 0.937 kg/m^3]

20. An aeroplane is flying at an altitude of 4000 m. Calculate the pressure around the aeroplane, given the lapse-rate in the atmosphere as 0.0065°K/m. Neglect variation of g with altitude. Take pressure and temperature at ground level as 10.143 N/cm^2 and 15°C respectively. The density of air at ground level is given as 1.285 kg/m^3. [**Ans.** 6.038 N/cm^2]

21. The atmospheric pressure at the sea-level is 101.3 kN/m^2 and the temperature is 15°C. Calculate the pressure 8000 m above sea-level, assuming (*i*) air is incompressible, (*ii*) isothermal variation of pressure and density, and (*iii*) adiabatic variation of pressure and density. Assume density of air at sea-level as 1.285 kg/m^3. Neglect variation of 'g' with altitude.

[**Ans.** (*i*) 501.3 N/m^2, (*ii*) 37.45 kN/m^2, (*iii*) 31.5 kN/m^2]

22. An oil of sp. gr. is 0.8 under a pressure of 137.2 kN/m^2

(*i*) What is the pressure head expressed in metre of water ?

(*ii*) What is the pressure head expressed in metre of oil ? [**Ans.** (*i*) 14 m, (*ii*) 17.5 m]

23. The atmospheric pressure at the sea-level is 101.3 kN/m^2 and temperature is 15°C. Calculate the pressure 8000 m above sea-level, assuming : (*i*) isothermal variation of pressure and density, and (*ii*) adiabatic variation of pressure and density. Assume density of air at sea-level as 1.285 kg/m^3. Neglect variation of 'g' with altitude.

Derive the formula that you may use. [**Ans.** (*i*) 37.45 kN/m^2, (*ii*) 31.5 kN/m^2]

24. What are the gauge pressure and absolute pressure at a point 4 m below the free surface of a liquid of specific gravity 1.53, if atmospheric pressure is equivalent to 750 mm of mercury.

[**Ans.** 60037 N/m^2 and 160099 N/m^2]

25. Find the gauge pressure and absolute pressure in N/m^2 at a point 4 m below the free surface of a liquid of sp. gr. 1.2, if the atmospheric pressure is equivalent to 750 mm of mercury.

[**Ans.** 47088 N/m^2 ; 147150 N/m^2]

26. A tank contains a liquid of specific gravity 0.8. Find the absolute pressure and gauge pressure at a point, which is 2 m below the free surface of the liquid. The atmospheric pressure head is equivalent to 760 mm of mercury. [**Ans.** 117092 N/m^2 ; 15696 N/m^2]