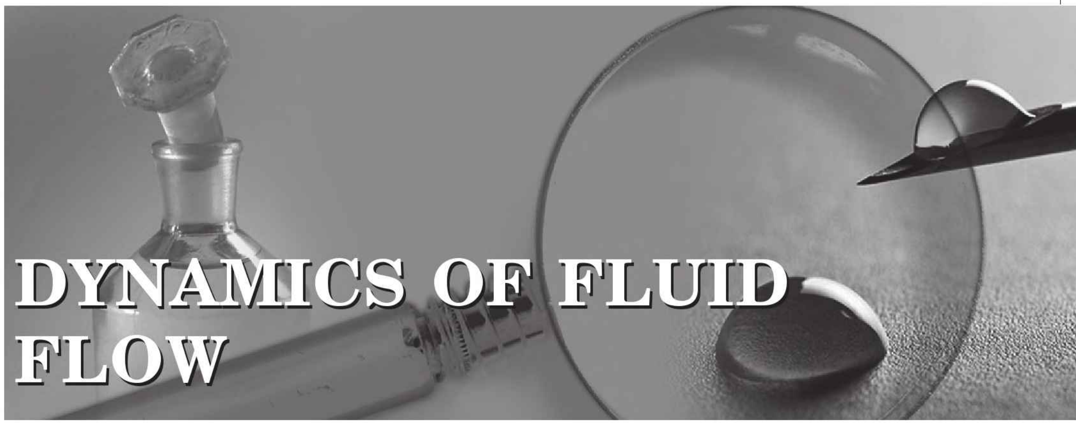

CHAPTER 6 DYNAMICS OF FLUID FLOW

▶ 6.1 INTRODUCTION

In the previous chapter, we studied the velocity and acceleration at a point in a fluid flow, without taking into consideration the forces causing the flow. This chapter includes the study of forces causing fluid flow. Thus dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton's second law of motion, which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

▶ 6.2 EQUATIONS OF MOTION

According to Newton's second law of motion, the net force F_x acting on a fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a_x in the x-direction. Thus mathematically,

$$F_x = m.a_x \qquad \text{...(6.1)}$$

In the fluid flow, the following forces are present :

(*i*) F_g, gravity force.

(*ii*) F_p, the pressure force.

(*iii*) F_v, force due to viscosity.

(*iv*) F_t, force due to turbulence.

(*v*) F_c, force due to compressibility.

Thus in equation (6.1), the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x.$$

(*i*) If the force due to compressibility, F_c is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

and equation of motions are called **Reynold's equations of motion.**

(*ii*) For flow, where (F_t) is negligible, the resulting equations of motion are known as **Navier-Stokes Equation.**

(*iii*) If the flow is assumed to be ideal, viscous force (F_v) is zero and equation of motions are known as **Euler's equation of motion.**

▶ 6.3 EULER'S EQUATION OF MOTION

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as :

Consider a stream-line in which flow is taking place in s-direction as shown in Fig. 6.1. Consider a cylindrical element of cross-section dA and length ds. The forces acting on the cylindrical element are:

1. Pressure force pdA in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element × acceleration in the direction s.

$$\therefore \quad pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta$$

$$= \rho dA ds \times a_s \quad \text{...(6.2)}$$

where a_s is the acceleration in the direction of s.

Now $$a_s = \frac{dv}{dt}, \text{ where } v \text{ is a function of } s \text{ and } t.$$

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{ds}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \quad a_s = \frac{v\partial v}{\partial s}$$

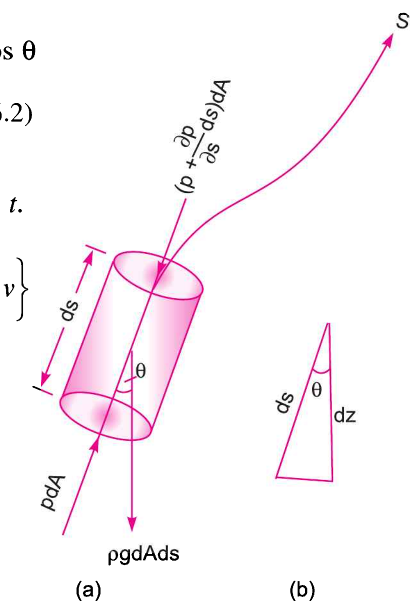

Fig. 6.1 *Forces on a fluid element.*

Substituting the value of a_s in equation (6.2) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} ds dA - \rho g\, dA ds \cos\theta = \rho dA ds \times \frac{\partial v}{\partial s}$$

Dividing by $\rho ds dA$, $-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$

or $$\frac{\partial p}{\rho \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

But from Fig. 6.1 (b), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \quad \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{vdv}{ds} = 0 \quad \text{or} \quad \frac{dp}{\rho} + gdz + vdv = 0$$

or $$\frac{dp}{\rho} + gdz + vdv = 0 \quad \text{...(6.3)}$$

Equation (6.3) is known as Euler's equation of motion.

▶ 6.4 BERNOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \quad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

or

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

or

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \qquad \text{...(6.4)}$$

Equation (6.4) is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

▶ 6.5 ASSUMPTIONS

The following are the assumptions made in the derivation of Bernoulli's equation :

(*i*) The fluid is ideal, *i.e.*, viscosity is zero (*ii*) The flow is steady

(*iii*) The flow is incompressible (*iv*) The flow is irrotational.

Problem 6.1 *Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.*

Solution. Given :

Diameter of pipe = 5 cm = 0.5 m

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0$ m/s

Datum head, $z = 5$ m

Total head = pressure head + kinetic head + datum head

Pressure head $= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{ \rho \text{ for water} = 1000 \frac{\text{kg}}{\text{m}^3} \right\}$

Kinetic head $= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

∴ Total head $= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

Problem 6.2 *A pipe, through which water is flowing, is having diameters, 20 cm and 10 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.*

Solution. Given :

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

∴ Area,
$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_1 = 4.0 \text{ m/s}$$

$$D_2 = 0.1 \text{ m}$$

∴
$$A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$$

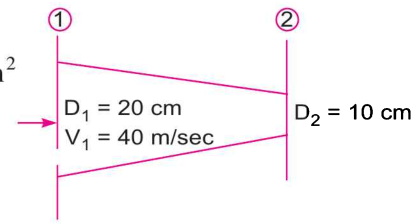

Fig. 6.2

(*i*) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \textbf{0.815 m. Ans.}$$

(*ii*) Velocity head at section 2 = $V_2^2/2g$

To find V_2, apply continuity equation at 1 and 2

∴
$$A_1V_1 = A_2V_2 \quad \text{or} \quad V_2 = \frac{A_1V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

∴ Velocity head at section 2
$$= \frac{V_2^2}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} = \textbf{83.047 m. Ans.}$$

(*iii*) Rate of discharge
$$= A_1V_1 \quad \text{or} \quad A_2V_2$$
$$= 0.0314 \times 4.0 = 0.1256 \text{ m}^3\text{/s}$$
$$= \textbf{125.6 litres/s. Ans.} \qquad \{\because 1 \text{ m}^3 = 1000 \text{ litres}\}$$

Problem 6.3 *State Bernoulli's theorem for steady flow of an incompressible fluid. Derive an expression for Bernoulli's equation from first principle and state the assumptions made for such a derivation.*

Solution. Statement of Bernoulli's Theorem. It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are :

$$\text{Pressure energy} = \frac{p}{\rho g}$$

$$\text{Kinetic energy} = \frac{v^2}{2g}$$

$$\text{Datum energy} = z$$

Thus mathematically, Bernoulli's theorem is written as

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{Constant.}$$

Derivation of Bernoulli's theorem. For derivation of Bernoulli's theorem, Articles 6.3 and 6.4 should be written.

Assumptions are given in Article 6.5.

Problem 6.4 *The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.*

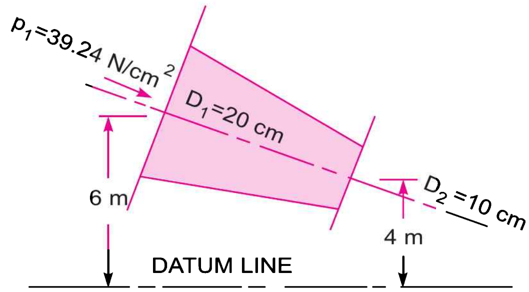

Fig. 6.3

Solution. Given :

At section 1, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

$$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0 \text{ m}$$

At section 2, $D_2 = 0.10 \text{ m}$

$$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$

Rate of flow, $Q = 35 \text{ lit/s} = \dfrac{35}{1000} = .035 \text{ m}^3\text{/s}$

Now $Q = A_1V_1 = A_2V_2$

$\therefore$ $V_1 = \dfrac{Q}{A_1} = \dfrac{.035}{.0314} = 1.114 \text{ m/s}$

and $V_2 = \dfrac{Q}{A_2} = \dfrac{.035}{.00785} = 4.456 \text{ m/s}$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or $$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

or $$46.063 = \frac{p_2}{9810} + 5.012$$

$\therefore$ $$\frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$\therefore$ $$p_2 = 41.051 \times 9810 \text{ N/m}^2$$

$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2. \text{ Ans.}}$$

Problem 6.5 *Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm² and the pressure at the upper end is 9.81 N/cm². Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.*

Solution. Given :

Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow = 40 lit/s

or $$Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$$

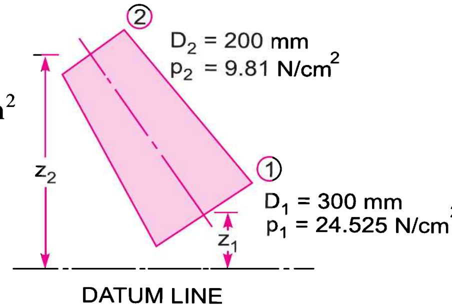

Fig. 6.4

Now $A_1V_1 = A_2V_2 = \text{rate of flow} = 0.04$

$$\therefore \quad V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4}D_1^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658 \text{ m/s}$$

$$\simeq 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(0.2)^2} = 1.274 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

or $$25 + .32 + z_1 = 10 + 1.623 + z_2$$

or $$25.32 + z_1 = 11.623 + z_2$$

$$\therefore \quad z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$$

∴ Difference in datum head $= z_2 - z_1 =$ **13.70 m. Ans.**

Problem 6.6 *The water is flowing through a taper pipe of length 100 m having diameters 600 mm at the upper end and 300 mm at the lower end, at the rate of 50 litres/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm².*

Solution. Given :

Length of pipe, $L = 100 \text{ m}$

Dia. at the upper end, $D_1 = 600 \text{ mm} = 0.6 \text{ m}$

∴ Area, $$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (.6)^2$$

$$= 0.2827 \text{ m}^2$$

p_1 = pressure at upper end

$= 19.62 \text{ N/cm}^2$

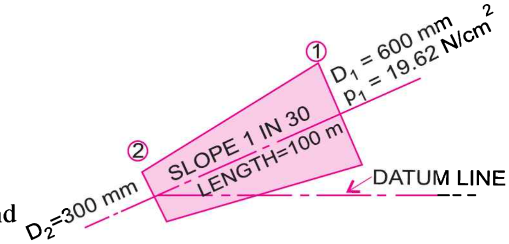

Fig. 6.5

$= 19.62 \times 10^4 \text{ N/m}^2$

Dia. at lower end, $D_2 = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (.3)^2 = 0.07068 \text{ m}$

$Q = \text{rate of flow} = 50 \text{ litres/s} = \frac{50}{1000} = 0.05 \text{ m}^3\text{/s}$

Let the datum line passes through the centre of the lower end.

Then $z_2 = 0$

As slope is 1 in 30 means $z_1 = \frac{1}{30} \times 100 = \frac{10}{3} \text{ m}$

Also we know $Q = A_1V_1 = A_2V_2$

$\therefore$ $V_1 = \frac{Q}{A} = \frac{0.05}{.2827} = 0.1768 \text{ m/sec} = 0.177 \text{ m/s}$

and $V_2 = \frac{Q}{A_2} = \frac{0.5}{.07068} = 0.7074 \text{ m/sec} = 0.707 \text{ m/s}$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{.177^2}{2 \times 9.81} + \frac{10}{3} = \frac{p_2}{\rho g} + \frac{.707^2}{2 \times 9.81} + 0$$

or $$20 + 0.001596 + 3.334 = \frac{p_2}{\rho g} + 0.0254$$

or $$23.335 - 0.0254 = \frac{p_2}{1000 \times 9.81}$$

or $p_2 = 23.3 \times 9810 \text{ N/m}^2 = 228573 \text{ N/m}^2 = \mathbf{22.857 \text{ N/cm}^2}$. **Ans.**

▶ 6.6 BERNOULLI'S EQUATION FOR REAL FLUID

The Bernoulli's equation was derived on the assumption that fluid is inviscid (non-viscous) and therefore frictionless. But all the real fluids are viscous and hence offer resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoulli's equation, these losses have to be taken into consideration. Thus the Bernoulli's equation for real fluids between points 1 and 2 is given as

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L \quad \text{...(6.5)}$$

where h_L is loss of energy between points 1 and 2.

Problem 6.7 *A pipe of diameter 400 mm carries water at a velocity of 25 m/s. The pressures at the points A and B are given as 29.43 N/cm² and 22.563 N/cm² respectively while the datum head at A and B are 28 m and 30 m. Find the loss of head between A and B.*

Solution. Given :

Dia. of pipe, $D = 400 \text{ mm} = 0.4 \text{ m}$

Velocity, $V = 25 \text{ m/s}$

At point *A*, $p_A = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

$z_A = 28 \text{ m}$

$v_A = v = 25 \text{ m/s}$

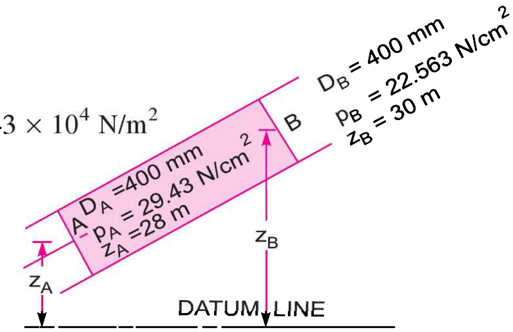

Fig. 6.6

$\therefore$ Total energy at *A*,
$$E_A = \frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A$$
$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28$$
$$= 30 + 31.85 + 28 = 89.85 \text{ m}$$

At point *B*, $p_B = 22.563 \text{ N/cm}^2 = 22.563 \times 10^4 \text{ N/m}^2$

$z_B = 30 \text{ m}$

$v_B = v = v_A = 25 \text{ m/s}$

$\therefore$ Total energy at *B*,
$$E_B = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B$$
$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 = 23 + 31.85 + 30 = 84.85 \text{ m}$$

$\therefore$ Loss of energy $= E_A - E_B = 89.85 - 84.85 =$ **5.0 m. Ans.**

Problem 6.8 *A conical tube of length 2.0 m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 5 m/s while at the lower end it is 2 m/s. The pressure head at the smaller end is 2.5 m of liquid. The loss of head in the tube is* $\frac{0.35(v_1 - v_2)^2}{2g}$, *where* v_1 *is the velocity at the smaller end and* v_2 *at the lower end respectively. Determine the pressure head at the lower end. Flow takes place in the downward direction.*

Solution. Let the smaller end is represented by (1) and lower end by (2)

Given :

Length of tube, $L = 2.0 \text{ m}$

$v_1 = 5 \text{ m/s}$

$p_1/\rho g = 2.5 \text{ m of liquid}$

$v_2 = 2 \text{ m/s}$

Loss of head
$$= h_L = \frac{0.35(v_1 - v_2)^2}{2g}$$

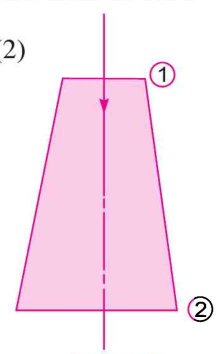

Fig. 6.7

$$= \frac{0.35\,[5-2]^2}{2g} = \frac{0.35 \times 9}{2 \times 9.81} = 0.16 \text{ m}$$

Pressure head, $\frac{p_2}{\rho g} = ?$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

Let the datum line passes through section (2). Then $z_2 = 0$, $z_1 = 2.0$

$$\therefore \quad 2.5 + \frac{5^2}{2 \times 9.81} + 2.0 = \frac{p_2}{\rho g} + \frac{2^2}{2 \times 9.81} + 0 + 0.16$$

$$2.5 + 1.27 + 2.0 = \frac{p_2}{\rho g} + 0.203 + .16$$

or $$\frac{p_2}{\rho g} = (2.5 + 1.27 + 2.0) - (.203 + .16)$$

$$= 5.77 - .363 = \mathbf{5.407 \text{ m of fluid. Ans.}}$$

Problem 6.9 *A pipeline carrying oil of specific gravity 0.87, changes in diameter from 200 mm diameter at a position A to 500 mm diameter at a position B which is 4 metres at a higher level. If the pressures at A and B are 9.81 N/cm² and 5.886 N/cm² respectively and the discharge is 200 litres/s determine the loss of head and direction of flow.*

Solution. Discharge, $Q = 200 \text{ lit/s} = 0.2 \text{ m}^3/\text{s}$

Sp. gr. of oil $= 0.87$

$\therefore$ ρ for oil $= .87 \times 1000 = 870 \frac{\text{kg}}{\text{m}^3}$

Given : **At section *A*,** $D_A = 200 \text{ mm} = 0.2 \text{ m}$

Area, $A_A = \frac{\pi}{4} (D_A)^2 = \frac{\pi}{4} (.2)^2$

$= 0.0314 \text{ m}^2$

$p_A = 9.81 \text{ N/cm}^2$

$= 9.81 \times 10^4 \text{ N/m}^2$

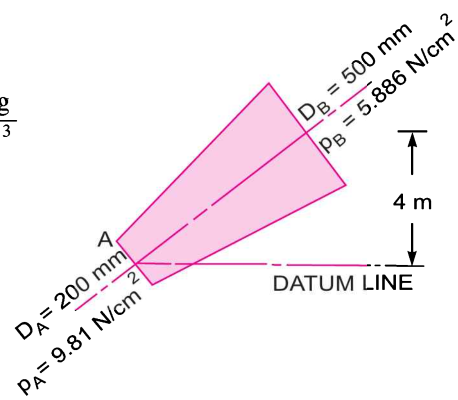

Fig. 6.8

If datum line is passing through *A*, then

$Z_A = 0$

$$V_A = \frac{Q}{A_A} = \frac{0.2}{0.0314} = 6.369 \text{ m/s}$$

At section *B*, $D_B = 500 \text{ mm} = 0.50 \text{ m}$

Area, $A_B = \frac{\pi}{4} D_B^2 = \frac{\pi}{4} (.5)^2 = 0.1963 \text{ m}^2$

$p_B = 5.886 \text{ N/cm}^2 = 5.886 \times 10^4 \text{ N/m}^2$

$$Z_B = 4.0 \text{ m}$$

$$V_B = \frac{Q}{\text{Area}} = \frac{0.2}{.1963} = 1.018 \text{ m/s}$$

Total energy at A $= E_A = \frac{p_A}{\rho g} + \frac{V_A^2}{2g} + Z_A$

$$= \frac{9.81 \times 10^4}{870 \times 9.81} + \frac{(6.369)^2}{2 \times 9.81} + 0 = 11.49 + 2.067 = 13.557 \text{ m}$$

Total energy at B $= E_B = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$

$$= \frac{5.886 \times 10^4}{870 \times 9.81} + \frac{(1.018)^2}{2 \times 9.81} + 4.0 = 6.896 + 0.052 + 4.0 = 10.948 \text{ m}$$

(*i*) **Direction of flow.** As E_A is more than E_B and hence flow is taking place from A to B. **Ans.**

(*ii*) Loss of head = $h_L = E_A - E_B = 13.557 - 10.948 =$ **2.609 m. Ans.**

▶ 6.7 PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices :

1. Venturimeter.
2. Orifice meter.
3. Pitot-tube.

6.7.1 Venturimeter. A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(*i*) A short converging part, (*ii*) Throat, and (*iii*) Diverging part. It is based on the Principle of Bernoulli's equation.

Expression for rate of flow through venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) = $\frac{\pi}{4} d_1^{\,2}$

and d_2, p_2, v_2, a_2 are corresponding values at section (2).

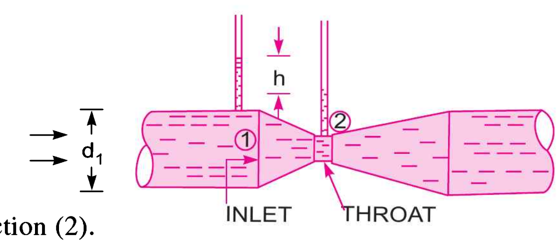

Fig. 6.9 *Venturimeter.*

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1 - p_2}{\rho g} = h$

Substituting this value of $\frac{p_1 - p_2}{\rho g}$ in the above equation, we get

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{...(6.6)}$$

Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or
$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$\therefore$
$$v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$\therefore$ Discharge,
$$Q = a_2 v_2$$
$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.7)}$$

Equation (6.7) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$\therefore$
$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.8)}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

Value of 'h' given by differential U-tube manometer

Case I. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = Sp. gravity of the heavier liquid
S_o = Sp. gravity of the liquid flowing through pipe
x = Difference of the heavier liquid column in U-tube

Then
$$h = x\left[\frac{S_h}{S_o} - 1\right] \quad \text{...(6.9)}$$

Case II. If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_l}{S_o}\right] \qquad ...(6.10)$$

where S_l = Sp. gr. of lighter liquid in U-tube

S_o = Sp. gr. of fluid flowing through pipe

x = Difference of the lighter liquid columns in U-tube.

Case III. Inclined Venturimeter with Differential U-tube manometer. The above two cases are given for a horizontal venturimeter. This case is related to inclined venturimeter having differential U-tube manometer. Let the differential manometer contains heavier liquid then h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right] \qquad ...(6.11)$$

Case IV. Similarly, for inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right] \qquad ...(6.12)$$

Problem 6.10 *A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take C_d = 0.98.*

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

∴ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

∴ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer = x = 20 cm of mercury.

∴ Difference of pressure head is given by (6.9)

or $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_o = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s. Ans.}}$$

Problem 6.11 *An oil of sp. gr. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$.*

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

∴ Difference of pressure head, $h = x \left[\frac{S_h}{S_o} - 1\right]$

$$= 25 \left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25 \, [17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

∴ $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

∴ $a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

∴ The discharge Q is given by equation (6.8)

or
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s}$$

$$= 70465 \text{ cm}^3/\text{s} = \mathbf{70.465 \text{ litres/s. Ans.}}$$

Problem 6.12 *A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of oil of sp. gr. 0.8. The discharge of oil through venturimeter is 60 litres/s. Find the reading of the oil-mercury differential manometer. Take $C_d = 0.98$.*

Solution. Given : $d_1 = 20$ cm

∴ $a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3\text{/s}$

Using the equation (6.8), $\quad Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

or $$60 \times 1000 = 9.81 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h} = \frac{1071068.78\sqrt{h}}{304}$$

or $$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$\therefore \quad h = (17.029)^2 = 289.98 \text{ cm of oil}$

But $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gr. of mercury = 13.6

S_o = Sp. gr. of oil = 0.8

x = Reading of manometer

$$\therefore \quad 289.98 = x\left[\frac{13.6}{0.8} - 1\right] = 16x$$

$$\therefore \quad x = \frac{289.98}{16} = 18.12 \text{ cm.}$$

$\therefore$ Reading of oil-mercury differential manometer = **18.12 cm. Ans.**

Problem 6.13 *A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm² and the vacuum pressure at the throat is 30 cm of mercury. Find the discharge of water through venturimeter. Take C_d = 0.98.*

Solution. Given :

Dia. at inlet, $\quad d_1 = 20 \text{ cm}$

$\therefore \quad a_1 = \frac{\pi}{4} \times (20)^2 = 314.16 \text{ cm}^2$

Dia. at throat, $\quad d_2 = 10 \text{ cm}$

$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.74 \text{ cm}^2$

$p_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$

ρ for water $\quad = 1000 \frac{\text{kg}}{\text{m}^3}$ and $\therefore \frac{p_1}{\rho g} = \frac{17.658 \times 10^4}{9.81 \times 1000} = 18 \text{ m of water}$

$$\frac{p_2}{\rho g} = -\ 30 \text{ cm of mercury}$$

$$= -\ 0.30 \text{ m of mercury} = -\ 0.30 \times 13.6 = -\ 4.08 \text{ m of water}$$

∴ Differential head $= h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 18 - (-4.08)$

$= 18 + 4.08 = 22.08$ m of water = 2208 cm of water

The discharge Q is given by equation (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.74)^2}} \times \sqrt{2 \times 981 \times 2208}$$

$$= \frac{50328837.21}{304} \times 165555 \text{ cm}^3/\text{s} = \textbf{165.555 lit/s. Ans.}$$

Problem 6.14 *The inlet and throat diameters of a horizontal venturimeter are 30 cm and 10 cm respectively. The liquid flowing through the meter is water. The pressure intensity at inlet is 13.734 N/cm² while the vacuum pressure head at the throat is 37 cm of mercury. Find the rate of flow. Assume that 4% of the differential head is lost between the inlet and throat. Find also the value of C_d for the venturimeter.*

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

∴ $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

∴ $a_2 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Pressure, $p_1 = 13.734 \text{ N/cm}^2 = 13.734 \times 10^4 \text{ N/m}^2$

∴ Pressure head, $\frac{p_1}{\rho g} = \frac{13.734 \times 10^4}{1000 \times 9.81} = 14$ m of water

$\frac{p_2}{\rho g} = -37$ cm of mercury

$= \frac{-37 \times 13.6}{100}$ m of water $= -5.032$ m of water

Differential head, $h = p_1/\rho g - p_2/\rho g$

$= 14.0 - (-5.032) = 14.0 + 5.032$

$= 19.032$ m of water = 1903.2 cm

Head lost, $h_f = 4\%$ of $h = \frac{4}{100} \times 19.032 = 0.7613$ m

∴ $C_d = \sqrt{\frac{h - h_f}{h}} = \sqrt{\frac{19.032 - .7613}{19.032}} = 0.98$

$\therefore$ Discharge $= C_d \dfrac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$

$$= \frac{0.98 \times 706.85 \times 78.54 \times \sqrt{2 \times 981 \times 1903.2}}{\sqrt{(706.85)^2 - (78.54)^2}}$$

$$= \frac{105132247.8}{\sqrt{499636.9 - 6168}} = 149692.8 \text{ cm}^3\text{/s} = \mathbf{0.14969 \text{ m}^3\text{/s. Ans.}}$$

PROBLEMS ON INCLINED VENTURIMETER

Problem 6.15 *A 30 cm × 15 cm venturimeter is inserted in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 20 cm. Find the discharge. Take C_d = 0.98.*

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ $a_1 = \dfrac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \dfrac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

$$h = x\left[\frac{S_h}{S_o} - 1\right] = 20\left[\frac{13.6}{1.0} - 1.0\right] = 20 \times 12.6 = 252.0 \text{ cm of water}$$

$C_d = 0.98$

Discharge, $Q = C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.3 - 31222.9}} = \frac{86067593.36}{684.4}$$

$= 125756 \text{ cm}^3\text{/s} =$ **125.756 lit/s. Ans.**

Problem 6.16 *A 20 cm × 10 cm venturimeter is inserted in a vertical pipe carrying oil of sp. gr. 0.8, the flow of oil is in upward direction. The difference of levels between the throat and inlet section is 50 cm. The oil mercury differential manometer gives a reading of 30 cm of mercury. Find the discharge of oil. Neglect losses.*

Solution. Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \dfrac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$\therefore \quad a_2 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_g = 13.6$

Differential manometer reading, $x = 30$ cm

$$\therefore \quad h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_g}{S_o} - 1\right]$$

$$= 30\left[\frac{13.6}{0.8} - 1\right] = 30\,[17 - 1] = 30 \times 16 = 480 \text{ cm of oil}$$

$C_d = 1.0$

The discharge, $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= \frac{1.0 \times 314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 480} \text{ cm}^3\text{/s}$$

$$= \frac{23932630.7}{304} = 78725.75 \text{ cm}^3\text{/s} = \mathbf{78.725 \text{ litres/s. Ans.}}$$

Problem 6.17 *In a vertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16 cm and 8 cm respectively. A is 2 metres above B. The pressure gauge readings have shown that the pressure at B is greater than at A by 0.981 N/cm². Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled with the same liquid and connected to a U-tube containing mercury, calculate the difference of level of mercury in the two limbs of the U-tube.*

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

$\therefore$ Density, $\rho = 0.8 \times 1000 = 800 \frac{\text{kg}}{\text{m}^3}$

Dia. at A, $D_A = 16 \text{ cm} = 0.16 \text{ m}$

$\therefore$ Area at A, $A_1 = \frac{\pi}{4}(.16)^2 = 0.0201 \text{ m}^2$

Dia. at B, $D_B = 8 \text{ cm} = 0.08 \text{ m}$

$\therefore$ Area at B, $A_2 = \frac{\pi}{4}(.08)^2 = 0.005026 \text{ m}^2$

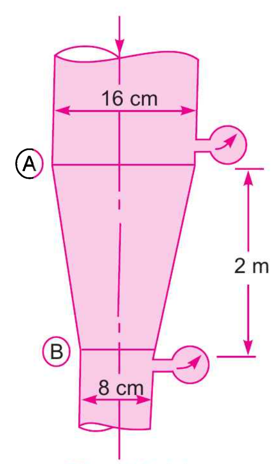

Fig. 6.9 (*a*)

(*i*) Difference of pressures, $p_B - p_A = 0.981 \text{ N/cm}^2$

$$= 0.981 \times 10^4 \text{ N/m}^2 = \frac{9810\text{N}}{\text{m}^2}$$

Difference of pressure head

$$\therefore \quad \frac{p_B - p_A}{\rho g} = \frac{9810}{800 \times 9.81} = 1.25 \qquad (\because \rho = 800 \text{ kg/m}^3)$$

Applying Bernoulli's theorem at A and B and taking the reference line passing through section B, we get

$$\frac{p_A}{\rho g}+\frac{V_A^2}{2g}+Z_A=\frac{p_B}{\rho g}+\frac{V_B^2}{2g}+Z_B$$

or $$\frac{p_A}{\rho g}-\frac{p_B}{p_g}-Z_A-Z_B=\frac{V_B^2}{2g}-\frac{V_A^2}{2g}$$

or $$\left(\frac{p_A-p_B}{\rho g}\right)+2.0-0.0=\frac{V_B^2}{2g}-\frac{V_A^2}{2g}$$

or $$-1.25+2.0=\frac{V_B^2}{2g}-\frac{V_A^2}{2g} \qquad \left(\because \frac{p_B-p_A}{\rho g}=1.25\right)$$

$$0.75=\frac{V_B^2}{2g}-\frac{V_A^2}{2g} \qquad ...(i)$$

Now applying continuity equation at A and B, we get

$$V_A \times A_1 = V_B \times A_2$$

or $$V_B=\frac{V_A\times A_1}{A_2}=\frac{V_A\times\frac{\pi}{4}(.16)^2}{\frac{\pi}{4}(.08)^2}=4V_A$$

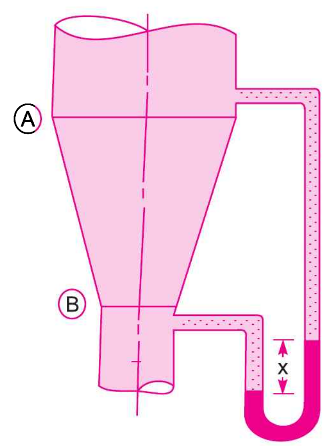

Fig. 6.9 (*b*)

Substituting the value of V_B in equation (*i*), we get

$$0.75=\frac{16V_A^2}{2g}-\frac{V_A^2}{2g}=\frac{15V_A^2}{2g}$$

$\therefore$ $$V_A=\sqrt{\frac{0.75\times 2\times 9.81}{15}}=0.99\text{ m/s}$$

$\therefore$ Rate of flow, $$Q=V_A\times A_1$$
$$=0.99\times 0.0201=\mathbf{0.01989\ m^3/s.\ Ans.}$$

(*ii*) Difference of level of mercury in the *U*-tube.

Let h = Difference of mercury level.

Then $$h=x\left(\frac{S_g}{S_o}-1\right)$$

where $$h=\left(\frac{p_A}{\rho g}+Z_A\right)-\left(\frac{p_B}{\rho g}+Z_B\right)=\frac{p_A-p_B}{\rho g}+Z_A-Z_B$$
$$=-1.25+2.0-0$$
$$=0.75 \qquad \left(\because \frac{p_B-p_A}{\rho g}=1.25\right)$$

$\therefore$ $$0.75=x\left[\frac{13.6}{0.8}-1\right]=x\times 16$$

$\therefore$ $$x=\frac{0.75}{16}=0.04687\text{ m}=\mathbf{4.687\ cm.\ Ans.}$$

Problem 6.18 *Find the discharge of water flowing through a pipe 30 cm diameter placed in an inclined position where a venturimeter is inserted, having a throat diameter of 15 cm. The difference of pressure between the main and throat is measured by a liquid of sp. gr. 0.6 in an inverted U-tube which gives a reading of 30 cm. The loss of head between the main and throat is 0.2 times the kinetic head of the pipe.*

Solution. Dia. at inlet, $d_1 = 30$ cm

$\therefore$ $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Reading of differential manometer, $x = 30$ cm

Difference of pressure head, h is given by

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h$$

Also $$h = x\left[1 - \frac{S_l}{S_o}\right]$$

where $S_l = 0.6$ and $S_o = 1.0$

$$= 30\left[1 - \frac{0.6}{1.0}\right] = 30 \times .4 = 12.0 \text{ cm of water}$$

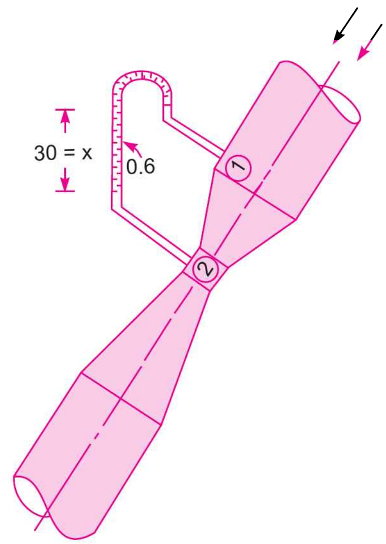

Fig. 6.10

Loss of head, $h_L = 0.2 \times$ kinetic head of pipe $= 0.2 \times \frac{v_1^2}{2g}$

Now applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_L$$

or $$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = h_L$$

But $$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = 12.0 \text{ cm of water}$$

and $$h_L = 0.2 \times v_1^2/2g$$

$\therefore$ $$12.0 + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = 0.2 \times \frac{v_1^2}{2g}$$

$\therefore$ $$12.0 + 0.8\frac{v_1^2}{2g} - \frac{v_2^2}{2g} = 0 \quad \text{...(1)}$$

Applying continuity equation at sections (1) and (2), we get

$$a_1 v_1 = a_2 v_2$$

$$\therefore \qquad v_1 = \frac{a_2}{a_1} v_2 = \frac{\frac{\pi}{4}(15)^2 v_2}{\frac{\pi}{4}(30)^2} = \frac{v_2}{4}$$

Substituting this value of v_1 in equation (1), we get

$$12.0 + \frac{0.8}{2g}\left(\frac{v_2}{4}\right)^2 - \frac{v_2^2}{2g} = 0 \text{ or } 12.0 + \frac{v_2^2}{2g}\left[\frac{0.8}{16} - 1\right] = 0$$

or
$$\frac{v_2^2}{2g}[.05 - 1] = -12.0 \text{ or } \frac{0.95\, v_2^2}{2g} = 12.0$$

$$\therefore \qquad v_2 = \sqrt{\frac{2 \times 981 \times 12.0}{0.95}} = 157.4 \text{ cm/s}$$

$\therefore$ Discharge $= a_2 v_2$

$= 176.7 \times 157.4 \text{ cm}^3\text{/s} = 27800 \text{ cm}^3\text{/s} =$ **27.8 litres/s. Ans.**

Problem 6.19 *A 30 cm × 15 cm venturimeter is provided in a vertical pipe line carrying oil of specific gravity 0.9, the flow being upwards. The difference in elevation of the throat section and entrance section of the venturimeter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Calculate :*

(i) the discharge of oil, and

(ii) the pressure difference between the entrance section and the throat section. Take the co-efficient of discharge as 0.98 and specific gravity of mercury as 13.6.

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ Area, $a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Let section (1) represents inlet and section (2) represents throat. Then $z_2 - z_1 = 30$ cm

Sp. gr. of oil, $S_o = 0.9$

Sp. gr. of mercury, $S_g = 13.6$

Reading of diff. manometer, $x = 25$ cm

The differential head, h is given by

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$$

$$= x\left[\frac{S_g}{S_o} - 1\right] = 25\left[\frac{13.6}{0.9} - 1\right] = 352.77 \text{ cm of oil}$$

(*i*) The discharge, Q of oil $= C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= \frac{0.98 \times 706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} = \sqrt{2 \times 981 \times 352.77}$$

$$= \frac{101832219.9}{684.4} = 148790.5 \text{ cm}^3/\text{s}$$

$$= \textbf{148.79 litres/s. Ans.}$$

(*ii*) Pressure difference between entrance and throat section

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 352.77$$

or $$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2 = 352.77$$

But $$z_2 - z_1 = 30 \text{ cm}$$

$\therefore$ $$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) - 30 = 352.77$$

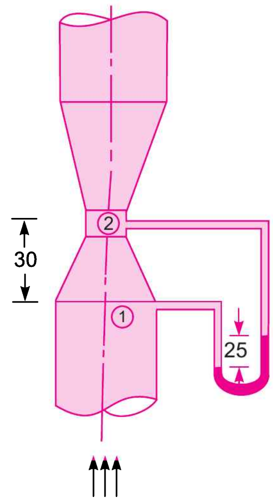

Fig. 6.11

$\therefore$ $$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 352.77 + 30 = 382.77 \text{ cm of oil} = \textbf{3.8277 m of oil. Ans.}$$

or $$(p_1 - p_2) = 3.8277 \times \rho g$$

But density of oil $= \text{Sp. gr. of oil} \times 1000 \text{ kg/m}^3$

$= 0.9 \times 1000 = 900 \text{ kg/cm}^3$

$\therefore$ $$(p_1 - p_2) = 3.8277 \times 900 \times 9.81 \frac{\text{N}}{\text{m}^2}$$

$$= \frac{33795}{10^4} \text{ N/cm}^2 = \textbf{3.3795 N/cm}^2\textbf{. Ans.}$$

Problem 6.20 *Crude oil of specific gravity 0.85 flows upwards at a volume rate of flow of 60 litre per second through a vertical venturimeter with an inlet diameter of 200 mm and a throat diameter of 100 mm. The co-efficient of discharge of the venturimeter is 0.98. The vertical distance between the pressure tappings is 300 mm.*

(i) If two pressure gauges are connected at the tappings such that they are positioned at the levels of their corresponding tapping points, determine the difference of readings in N/cm^2 *of the two pressure gauges.*

(ii) If a mercury differential manometer is connected, in place of pressure gauges, to the tappings such that the connecting tube upto mercury are filled with oil, determine the difference in the level of the mercury column.

Solution. Given :

Specific gravity of oil, $S_o = 0.85$

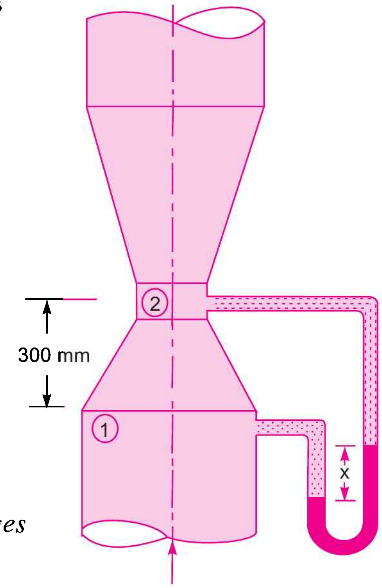

Fig. 6.11 (*a*)

$\therefore$ Density, $\rho = 0.85 \times 1000 = 850 \text{ kg/m}^3$

Discharge, $Q = 60$ litre/s

$$= \frac{60}{1000} = 0.06 \text{ m}^3/\text{s}$$

Inlet dia, $d_1 = 200 \text{ mm} = 0.2 \text{ m}$

$\therefore$ Area, $a_1 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$

Throat dia., $d_2 = 100 \text{ mm} = 0.1 \text{ m}$

$\therefore$ Area, $a_2 = \frac{\pi}{4} (0.1)^2 = 0.00785 \text{ m}^2$

Value of C_d $= 0.98$

Let section (1) represents inlet and section (2) represents throat. Then

$$z_2 - z_1 = 300 \text{ mm} = 0.3 \text{ m}$$

(*i*) *Difference of readings in N/cm² of the two pressure gauges*

The discharge Q is given by,

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

or
$$0.06 = \frac{0.98 \times 0.0314 \times 0.00785}{\sqrt{0.0314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times h}$$

$$= \frac{0.98 \times 0.00024649}{0.0304} \times 4.429 \sqrt{h}$$

$$\therefore \quad \sqrt{h} = \frac{0.06 \times 0.0304}{0.98 \times 0.00024649 \times 4.429} = 1.705$$

$$\therefore \quad h = 1.705^2 = 2.908 \text{ m}$$

But for a vertical venturimeter, $h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$

$$\therefore \quad 2.908 = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2$$

$$\frac{p_1 - p_2}{\rho g} = 2.908 + z_2 - z_1 = 2.908 + 0.3 \qquad (\because \quad z_2 - z_1 = 0.3 \text{ m})$$

$$= 3.208 \text{ m of oil}$$

$$\therefore \quad p_1 - p_2 = \rho g \times 3.208$$

$$= 850 \times 9.81 \times 3.208 \text{ N/m}^2 = \frac{850 \times 9.81 \times 3.208}{10^4} \text{ N/cm}^2$$

$$= \mathbf{2.675 \text{ N/cm}^2}.$$ **Ans.**

(*ii*) *Difference in the levels of mercury columns* (*i.e., x*)

The value of h is given by, $h = x\left[\frac{S_g}{S_o} - 1\right]$

$$\therefore \quad 2.908 = x\left[\frac{13.6}{0.85} - 1\right] = x\,[16 - 1] = 15\,x$$

$$\therefore \quad x = \frac{2.908}{15} = 0.1938 \text{ m} = \textbf{19.38 cm of oil. Ans.}$$

Problem 6.21 *In a 100 mm diameter horizontal pipe a venturimeter of 0.5 contraction ratio has been fixed. The head of water on the metre when there is no flow is 3 m (gauge). Find the rate of flow for which the throat pressure will be 2 metres of water absolute. The co-efficient of discharge is 0.97. Take atmospheric pressure head = 10.3 m of water.*

Solution. Given :

Dia. of pipe, $d_1 = 100 \text{ mm} = 10 \text{ cm}$

$\therefore$ Area, $a_1 = \frac{\pi}{4} d_1^{\,2} = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Dia. at throat, $d_2 = 0.5 \times d_1 = 0.5 \times 10 = 5 \text{ cm}$

$\therefore$ Area, $a_2 = \frac{\pi}{4}(5)^2 = 19.635 \text{ cm}^2$

Head of water for no flow $= \frac{p_1}{\rho g} = 3 \text{ m (gauge)} = 3 + 10.3 = 13.3 \text{ m (abs.)}$

Throat pressure head $= \frac{p_2}{\rho g} = 2$ m of water absolute.

$\therefore$ Difference of pressure head, $h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 13.3 - 2.0 = 11.3 \text{ m} = 1130 \text{ cm}$

$\therefore$ Rate of flow, Q is given by $Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= 0.97 \times \frac{78.54 \times 19.635}{\sqrt{(78.54)^2 - (19.635)^2}} \times \sqrt{2 \times 981 \times 1130}$$

$$= \frac{2227318.17}{76} = 29306.8 \text{ cm}^3\text{/s} = \textbf{29.306 litres/s. Ans.}$$

6.7.2 Orifice Meter or Orifice Plate. It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.

A differential manometer is connected at section (1), which is at a distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, and at section (2), which is at a distance of about half the diameter of the orifice on the downstream side from the orifice plate.

Let p_1 = pressure at section (1),
v_1 = velocity at section (1),
a_1 = area of pipe at section (1), and

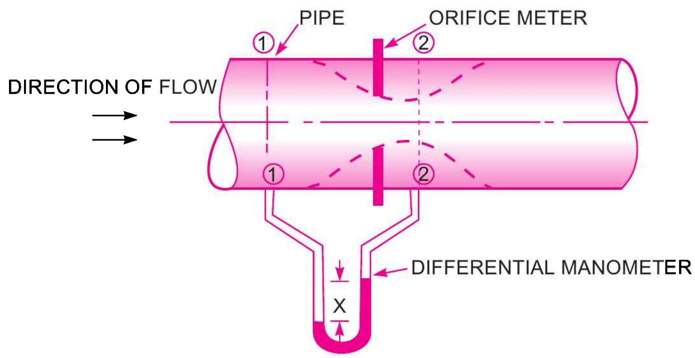

Fig. 6.12. *Orifice meter.*

p_2, v_2, a_2 are corresponding values at section (2). Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

or
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = \text{Differential head}$$

$$\therefore \quad h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{or} \quad 2gh = {v_2}^2 - {v_1}^2$$

or
$$v_2 = \sqrt{2gh + v_1^2} \qquad ...(i)$$

Now section (2) is at the vena-contracta and a_2 represents the area at the vena-contracta. If a_0 is the area of orifice then, we have

$$C_c = \frac{a_2}{a_0}$$

where C_c = Co-efficient of contraction

$$\therefore \quad a_2 = a_0 \times C_c \qquad ...(ii)$$

By continuity equation, we have

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2}{a_1} v_2 = \frac{a_0\, C_c}{a_1} v_2 \qquad ...(iii)$$

Substituting the value of v_1 in equation (*i*), we get

$$v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1^2}}$$

or $$v_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2 \text{ or } v_2^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2gh$$

$\therefore$ $$v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ The discharge $Q = v_2 \times a_2 = v_2 \times a_0\, C_c$ $\{\because a_2 = a_0 C_c$ from (*ii*)$\}$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}} \qquad ...(iv)$$

The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ $$C_c = C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

Substituting this value of C_c in equation (*iv*), we get

$$Q = a_0 \times C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}. \qquad ...(6.13)$$

where C_d = Co-efficient of discharge for orifice meter.

The co-efficient of discharge for orifice meter is much smaller than that for a venturimeter.

Problem 6.22 *An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm² and 9.81 N/cm² respectively. Co-efficient of discharge for the orifice meter is given as 0.6. Find the discharge of water through pipe.*

Solution. Given :

Dia. of orifice, $d_0 = 10$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Dia. of pipe, $d_1 = 20$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

$p_1 = 19.62 \text{ N/cm}^2 = 19.62 \times 10^4 \text{ N/m}^2$

$$\therefore \quad \frac{p_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = 20 \text{ m of water}$$

Similarly
$$\frac{p_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10 \text{ m of water}$$

$$\therefore \quad h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 20.0 - 10.0 = 10 \text{ m of water} = 1000 \text{ cm of water}$$

$C_d = 0.6$

The discharge, Q is given by equation (6.13)

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.6 \times \frac{78.54 \times 314.16}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 1000}$$

$$= \frac{20736838.09}{304} = 68213.28 \text{ cm}^3/\text{s} = \textbf{68.21 litres/s. Ans.}$$

Problem 6.23 *An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of sp. gr. 0.9 when the co-efficient of discharge of the orifice meter = 0.64.*

Solution. Given :

Dia. of orifice, $d_0 = 15$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Dia. of pipe, $d_1 = 30$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Sp. gr. of oil, $S_o = 0.9$

Reading of diff. manometer, $x = 50$ cm of mercury

$$\therefore \text{ Differential head, } \quad h = x\left[\frac{S_g}{S_o} - 1\right] = 50\left[\frac{13.6}{0.9} - 1\right] \text{ cm of oil}$$

$= 50 \times 14.11 = 705.5$ cm of oil

$C_d = 0.64$

∴ The rate of the flow, Q is given by equation (6.13)

$$Q = C_d \cdot \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.64 \times \frac{176.7 \times 706.85}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 705.5}$$

$$= \frac{94046317.78}{684.4} = 137414.25 \text{ cm}^3\text{/s} = \textbf{137.414 litres/s. Ans.}$$

6.7.3 Pitot-tube. It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot-tube consists of a glass tube, bent at right angles as shown in Fig. 6.13.

The lower end, which is bent through 90° is directed in the upstream direction as shown in Fig. 6.13. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

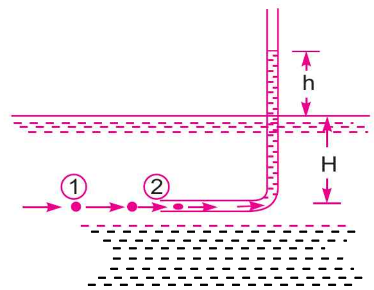

Fig. 6.13 *Pitot-tube.*

Consider two points (1) and (2) at the same level in such a way that point (2) is just as the inlet of the pitot-tube and point (1) is far away from the tube.

Let p_1 = intensity of pressure at point (1)

v_1 = velocity of flow at (1)

p_2 = pressure at point (2)

v_2 = velocity at point (2), which is zero

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equation at points (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$ as points (1) and (2) are on the same line and $v_2 = 0$.

$$\frac{p_1}{\rho g} = \text{pressure head at (1)} = H$$

$$\frac{p_2}{\rho g} = \text{pressure head at (2)} = (h + H)$$

Substituting these values, we get

$$\therefore \quad H + \frac{v_1^2}{2g} = (h + H) \quad \therefore \quad h = \frac{v_1^2}{2g} \quad \text{or} \quad v_1 = \sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

$$(v_1)_{act} = C_v \sqrt{2gh}$$

where C_v = Co-efficient of pitot-tube

$\therefore$ Velocity at any point $\quad v = C_v \sqrt{2gh}$ ...(6.14)

Velocity of flow in a pipe by pitot-tube. For finding the velocity at any point in a pipe by pitot-tube, the following arrangements are adopted :

1. Pitot-tube along with a vertical piezometer tube as shown in Fig. 6.14.
2. Pitot-tube connected with piezometer tube as shown in Fig. 6.15.
3. Pitot-tube and vertical piezometer tube connected with a differential U-tube manometer as shown in Fig. 6.16.

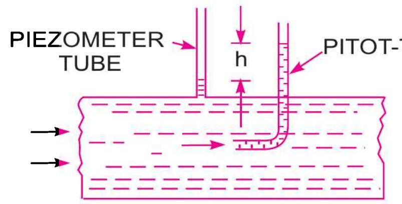

Fig. 6.14

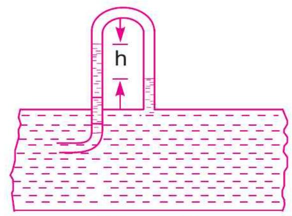

Fig. 6.15

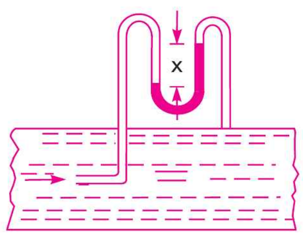

Fig. 6.16

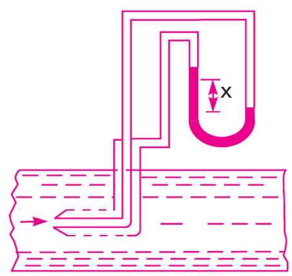

Fig. 6.17

4. Pitot-static tube, which consists of two circular concentric tubes one inside the other with some annular space in between as shown in Fig. 6.17. The outlet of these two tubes are connected to the differential manometer where the difference of pressure head 'h' is measured by knowing the difference of the levels of the manometer liquid say x. Then $h = x\left[\frac{S_g}{S_o} - 1\right]$.

Problem 6.24 *A pitot-static tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60 mm of water. Take the co-efficient of pitot tube as C_v = 0.98.*

Solution. Given :

Dia. of pipe, $\quad d$ = 300 mm = 0.30 m

Diff. of pressure head, $\quad h$ = 60 mm of water = .06 m of water

$C_v = 0.98$

Mean velocity, $\quad \overline{V} = 0.80 \times$ Central velocity

Central velocity is given by equation (6.14)

$$= C_v \sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063 \text{ m/s}$$

$\therefore \quad \overline{V} = 0.80 \times 1.063 = 0.8504$ m/s

Discharge, $Q = \text{Area of pipe} \times \overline{V}$

$$= \frac{\pi}{4} d^2 \times \overline{V} = \frac{\pi}{4} (.30)^2 \times 0.8504 = \mathbf{0.06\ m^3/s.\ Ans.}$$

Problem 6.25 *Find the velocity of the flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot-tube is 100 mm. Take co-efficient of pitot-tube 0.98 and sp. gr. of oil = 0.8.*

Solution. Given :

Diff. of mercury level, $x = 100$ mm $= 0.1$ m

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_g = 13.6$

$C_v = 0.98$

Diff. of pressure head, $$h = x\left[\frac{S_g}{S_o} - 1\right] = .1\left[\frac{13.6}{0.8} - 1\right] = 1.6 \text{ m of oil}$$

$\therefore$ Velocity of flow $= C_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1.6} = \mathbf{5.49\ m/s.\ Ans.}$

Problem 6.26 *A pitot-static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6 m and static pressure head is 5 m. Calculate the velocity of flow assuming the co-efficient of tube equal to 0.98.*

Solution. Given :

Stagnation pressure head, $h_s = 6$ m

Static pressure head, $h_t = 5$ m

$\therefore \quad h = 6 - 5 = 1$ m

Velocity of flow, $V = C_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1} = \mathbf{4.34\ m/s.\ Ans.}$

Problem 6.27 *A sub-marine moves horizontally in sea and has its axis 15 m below the surface of water. A pitot-tube properly placed just in front of the sub-marine and along its axis is connected to the two limbs of a U-tube containing mercury. The difference of mercury level is found to be 170 mm. Find the speed of the sub-marine knowing that the sp. gr. of mercury is 13.6 and that of sea-water is 1.026 with respect of fresh water.*

Solution. Given :

Diff. of mercury level, $x = 170$ mm $= 0.17$ m

Sp. gr. of mercury, $S_g = 13.6$

Sp. gr. of sea-water, $S_o = 1.026$

$$\therefore \quad h = x\left[\frac{S_g}{S_o} - 1\right] = 0.17\left[\frac{13.6}{1.026} - 1\right] = 2.0834 \text{ m}$$

$$\therefore \quad V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.0834} = 6.393 \text{ m/s}$$

$$= \frac{6.393 \times 60 \times 60}{1000} \text{ km/hr} = \mathbf{23.01\ km/hr.\ Ans.}$$

Problem 6.28 *A pitot-tube is inserted in a pipe of 300 mm diameter. The static pressure in pipe is 100 mm of mercury (vacuum). The stagnation pressure at the centre of the pipe, recorded by the* *pitot-tube is 0.981 N/cm^2. Calculate the rate of flow of water through pipe, if the mean velocity of flow is 0.85 times the central velocity. Take C_v = 0.98.*

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.3)^2 = 0.07068 \text{ m}^2$

Static pressure head $= 100$ mm of mercury (vacuum)

$= -\frac{100}{1000} \times 13.6 = -1.36$ m of water

Stagnation pressure $= .981 \text{ N/cm}^2 = .981 \times 10^4 \text{ N/m}^2$

$\therefore$ Stagnation pressure head $= \frac{.981 \times 10^4}{\rho g} = \frac{.981 \times 10^4}{1000 \times 9.81} = 1 \text{ m}$

$\therefore$ h = Stagnation pressure head – Static pressure head

$= 1.0 - (-1.36) = 1.0 + 1.36 = 2.36$ m of water

$\therefore$ Velocity at centre $= C_v \sqrt{2gh}$

$= 0.98 \times \sqrt{2 \times 9.81 \times 2.36} = 6.668$ m/s

Mean velocity, $\overline{V} = 0.85 \times 6.668 = 5.6678$ m/s

$\therefore$ Rate of flow of water $= \overline{V} \times$ area of pipe

$= 5.6678 \times 0.07068 \text{ m}^3/\text{s} =$ **0.4006 m^3/s. Ans.**

▶ 6.8 THE MOMENTUM EQUATION

It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction. The force acting on a fluid mass 'm' is given by the Newton's second law of motion,

$$F = m \times a$$

where a is the acceleration acting in the same direction as force F.

But $a = \frac{dv}{dt}$

$\therefore$ $F = m \frac{dv}{dt}$

$= \frac{d(mv)}{dt}$ {m is constant and can be taken inside the differential}

$$\therefore \quad F = \frac{d(mv)}{dt} \qquad \text{...(6.15)}$$

Equation (6.15) is known as the momentum principle.

Equation (6.15) can be written as $F.dt = d(mv)$ ...(6.16)

which is known as the *impulse-momentum equation* and states that the impulse of a force F acting on a fluid of mass m in a short interval of time dt is equal to the change of momentum $d(mv)$ in the direction of force.

Force exerted by a flowing fluid on a pipe bend

The impulse-momentum equation (6.16) is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

Consider two sections (1) and (2), as shown in Fig. 6.18.

Let v_1 = velocity of flow at section (1),

p_1 = pressure intensity at section (1),

A_1 = area of cross-section of pipe at section (1) and

v_2, p_2, A_2 = corresponding values of velocity, pressure and area at section (2).

Let F_x and F_y be the components of the forces exerted by the flowing fluid on the bend in x-and y-directions respectively. Then the force exerted by the bend on the fluid in the directions of x and y will be equal to F_x and F_y but in the opposite directions. Hence component of the force exerted by bend on the fluid in the x-direction = $-F_x$ and in the direction of $y = -F_y$. The other external forces acting on the fluid are p_1A_1 and p_2A_2 on the sections (1) and (2) respectively. Then momentum equation in x-direction is given by

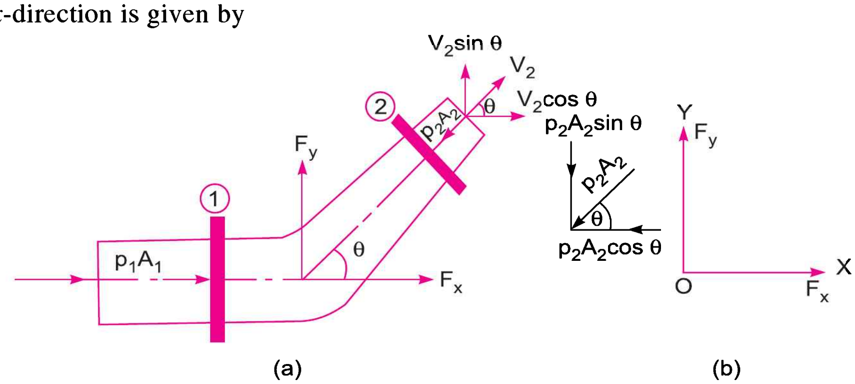

Fig. 6.18 *Forces on bend.*

Net force acting on fluid in the direction of x = Rate of change of momentum in x-direction

$$\therefore \quad p_1A_1 - p_2A_2 \cos\theta - F_x = \text{(Mass per sec) (change of velocity)}$$

$$= \rho Q \text{ (Final velocity in the direction of } x - \text{Initial velocity in the direction of } x)$$

$$= \rho Q (V_2 \cos\theta - V_1) \quad \text{...(6.17)}$$

$$\therefore \quad F_x = \rho Q (V_1 - V_2 \cos\theta) + p_1A_1 - p_2A_2 \cos\theta \quad \text{...(6.18)}$$

Similarly the momentum equation in y-direction gives

$$0 - p_2A_2 \sin\theta - F_y = \rho Q (V_2 \sin\theta - 0) \quad \text{...(6.19)}$$

$$\therefore \quad F_y = \rho Q (-V_2 \sin\theta) - p_2A_2 \sin\theta \quad \text{...(6.20)}$$

Now the resultant force (F_R) acting on the bend

$$= \sqrt{F_x^2 + F_y^2} \quad \text{...(6.21)}$$

And the angle made by the resultant force with horizontal direction is given by

$$\tan\theta = \frac{F_y}{F_x} \quad \text{...(6.22)}$$

Problem 6.29 *A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600 mm and 300 mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 8.829 N/cm² and rate of flow of water is 600 litres/s.*

Solution. Given :

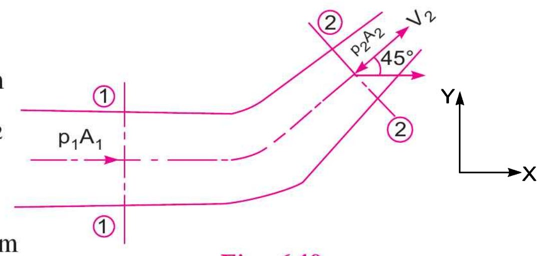

Fig. 6.19

Angle of bend, $\theta = 45°$

Dia. at inlet, $D_1 = 600$ mm $= 0.6$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.6)^2$

$= 0.2827$ m^2

Dia. at outlet, $D_2 = 300$ mm $= 0.30$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4} (.3)^2 = 0.07068$ m^2

Pressure at inlet, $p_1 = 8.829$ N/cm^2 $= 8.829 \times 10^4$ N/m^2

$Q = 600$ lit/s $= 0.6$ m^3/s

$$V_1 = \frac{Q}{A_1} = \frac{0.6}{.2827} = 2.122 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{.07068} = 8.488 \text{ m/s.}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \quad \text{or} \quad \frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{2.122^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{8.488^2}{2 \times 9.81}$$

$$9 + .2295 = p_2/\rho g + 3.672$$

$$\therefore \quad \frac{p_2}{\rho g} = 9.2295 - 3.672 = 5.5575 \text{ m of water}$$

$$\therefore \quad p_2 = 5.5575 \times 1000 \times 9.81 \text{ N/m}^2 = 5.45 \times 10^4 \text{ N/m}^2$$

Forces on the bend in x- and y-directions are given by equations (6.18) and (6.20) as

$$F_x = \rho Q [V_1 - V_2 \cos \theta] + p_1A_1 - p_2A_2 \cos \theta$$

$$= 1000 \times 0.6 [2.122 - 8.488 \cos 45°]$$

$$+ 8.829 \times 10^4 \times .2827 - 5.45 \times 10^4 \times .07068 \times \cos 45°$$

$$= -2327.9 + 24959.6 - 2720.3 = 24959.6 - 5048.2$$

$$= 19911.4 \text{ N}$$

and

$$F_y = \rho Q [- V_2 \sin \theta] - p_2A_2 \sin \theta$$

$$= 1000 \times 0.6 [- 8.488 \sin 45°] - 5.45 \times 10^4 \times .07068 \times \sin 45°$$

$$= - 3601.1 - 2721.1 = - 6322.2 \text{ N}$$

–ve sign means F_y is acting in the downward direction

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{(19911.4)^2 + (-6322.2)^2}$$

$$= \mathbf{20890.9\ N.\ Ans.}$$

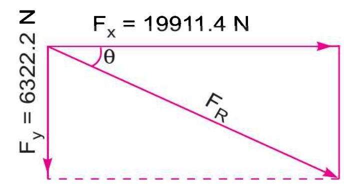

Fig. 6.20

The angle made by resultant force with x-axis is given by equation (6.22) or

$$\tan\theta = \frac{F_y}{F_x} = \frac{6322.2}{19911.4} = 0.3175$$

$$\therefore \qquad \theta = \tan^{-1} .3175 = \mathbf{17° \ 36'.\ Ans.}$$

Problem 6.30 *250 litres/s of water is flowing in a pipe having a diameter of 300 mm. If the pipe is bent by 135° (that is change from initial to final direction is 135°), find the magnitude and direction of the resultant force on the bend. The pressure of water flowing is 39.24 N/cm².*

Solution. Given :

Pressure, $p_1 = p_2 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$

Discharge, $Q = 250 \text{ litres/s} = 0.25 \text{ m}^3\text{/s}$

Dia. of bend at inlet and outlet, $D_1 = D_2 = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Area, $A_1 = A_2 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$

Velocity of water at sections (1) and (2), $V = V_1 = V_2 = \frac{Q}{\text{Area}} = \frac{0.25}{.07068} = 3.537 \text{ m/s}$.

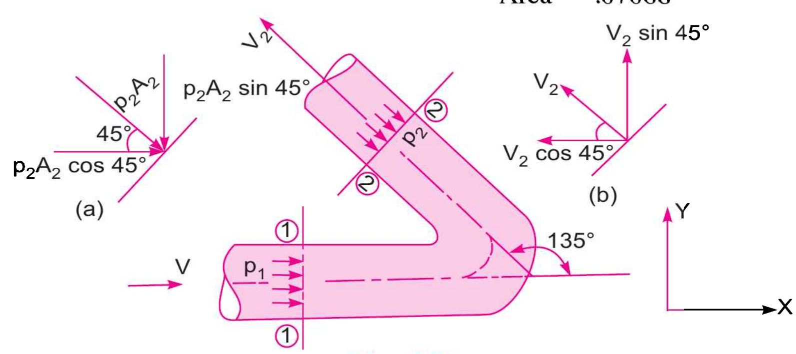

Fig. 6.21

Force along x-axis

$$= F_x = \rho Q[V_{1x} - V_{2x}] + p_{1x}A_1 + p_{2x}A_2$$

where, V_{1x} = initial velocity in the direction of x = 3.537 m/s

V_{2x} = final velocity in the direction of $x = -V_2 \cos 45° = -3.537 \times .7071$

p_{1x} = pressure at section (1) in x-direction
$= 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$

p_{2x} = pressure at section (2) in x-direction
$= p_2 \cos 45° = 39.24 \times 10^4 \times .7071$

$$\therefore \quad F_x = 1000 \times .25[3.537 - (-3.537 \times .7071)] + 39.24 \times 10^4 \times .07068 + 39.24 \times 10^4 \times .07068 \times .7071$$

$$= 1000 \times .25[3.537 + 3.537 \times .7071] + 39.24 \times 10^4 \times .07068\ [1 + .7071]$$

$= 1509.4 + 47346 = 48855.4$ N

Force along y-axis

$= F_y = \rho Q[V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$

where V_{1y} = initial velocity in y-direction = 0

V_{2y} = final velocity in y-direction = $- V_2 \sin 45° = 3.537 \times .7071$

$(p_1A_1)_y$ = pressure force in y-direction = 0

$(p_2A_2)_y$ = pressure force at (2) in y-direction

$= - p_2A_2 \sin 45° = - 39.24 \times 10^4 \times .07068 \times .7071$

∴ $F_y = 1000 \times .25[0 - 3.537 \times .7071] + 0 + (- 39.24 \times 10^4 \times .07068 \times .7071)$

$= - 625.2 - 19611.1 = - 20236.3$ N

–ve sign means F_y is acting in the downward direction

∴ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{48855.4^2 + 20236.3^2}$

= 52880.6 N. Ans.

The direction of the resultant force F_R, with the x-axis is given as

$$\tan \theta = \frac{F_y}{F_x} = \frac{20236.3}{48855.4} = 0.4142$$

∴ θ = **22° 30′. Ans.**

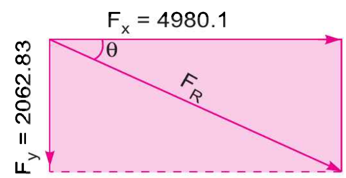

Fig. 6.22

Problem 6.31 *A 300 mm diameter pipe carries water under a head of 20 metres with a velocity of 3.5 m/s. If the axis of the pipe turns through 45°, find the magnitude and direction of the resultant force at the bend.*

Solution. Given :

Dia. of bend, $D = D_1 = D_2 = 300$ mm $= 0.30$ m

∴ Area, $A = A_1 = A_2 = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times .3^2 = 0.07068$ m^2

Velocity, $V = V_1 = V_2 = 3.5$ m/s

$\theta = 45°$

Discharge, $Q = A \times V = 0.07068 \times 3.5 = 0.2475$ m^3/s

Pressure head $= 20$ m of water or $\frac{p}{\rho g} = 20$ m of water

∴ $p = 20 \times \rho g = 20 \times 1000 \times 9.81$ N/m^2 $= 196200$ N/m^2

∴ Pressure intensity, $p = p_1 = p_2 = 196200$ N/m^2

Now $V_{1x} = 3.5$ m/s, $V_{2x} = V_2 \cos 45° = 3.5 \times .7071$

$V_{1y} = 0$, $V_{2y} = V_2 \sin 45° = 3.5 \times .7071$

$(p_1A_1)_x = p_1A_1 = 196200 \times .07068$, $(p_1A_1)_y = 0$

$(p_2A_2)_x = - p_2A_2 \cos 45°$, $(p_2A_2)_y = - p_2A_2 \sin 45°$

Force along x-axis, $F_x = \rho Q[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

$= 1000 \times .2475[3.5 - 3.5 \times .7071] + 196200 \times .07068 - p_2A_2 \times \cos 45°$

$= 253.68 + 196200 \times .07068 - 196200 \times .07068 \times 0.7071$

$= 253.68 + 13871.34 - 9808.04 = 4316.98$ N

Force along y-axis, $F_y = \rho Q\, [V_1y - V_2y] + (p_1A_1)_y + (p_2A_2)_y$

$= 1000 \times .2475[0 - 3.5 \times .7071] + 0 + [- p_2A_2 \sin 45°]$

$= - 612.44 - 196200 \times .07068 \times .7071$

$= - 612.44 - 9808 = - 10420.44$ N

∴ Resultant force $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(4316.98)^2 + (10420.44)^2}$ = **11279 N. Ans.**

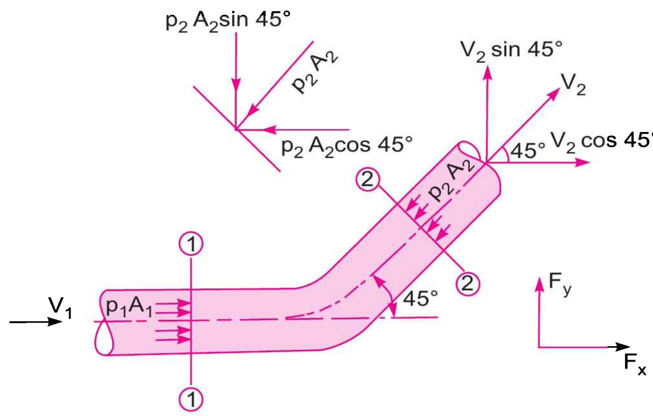

Fig. 6.23

The angle made by F_R with x-axis

$$\tan \theta = \frac{F_y}{F_x} = \frac{10420.44}{4316.98} = 2.411$$

∴ $\theta = \tan^{-1} 2.411$ = **67° 28′. Ans.**

Problem 6.32 *In a 45° bend a rectangular air duct of 1 m² cross-sectional area is gradually reduced to 0.5 m² area. Find the magnitude and direction of the force required to hold the duct in position if the velocity of flow at the 1 m² section is 10 m/s, and pressure is 2.943 N/cm². Take density of air as 1.16 kg/m³.*

Solution. Given :

Area at section (1), $A_1 = 1$ m²

Area at section (2), $A_2 = 0.5$ m²

Velocity at section (1), $V_1 = 10$ m/s

Pressure at section (1), $p_1 = 2.943$ N/cm² $= 2.943 \times 10^4$ N/m² $= 29430$ N/m²

Density of air, $\rho = 1.16$ kg/m³

Applying continuity equation at sections (1) and (2)

$$A_1V_1 = A_2V_2$$

∴ $$V_2 = \frac{A_1V_1}{A_2} = \frac{1}{0.5} \times 10 = 20 \text{ m/s}$$

Discharge $Q = A_1V_1 = 1 \times 10 = 10 \text{ m}^3/\text{s}$

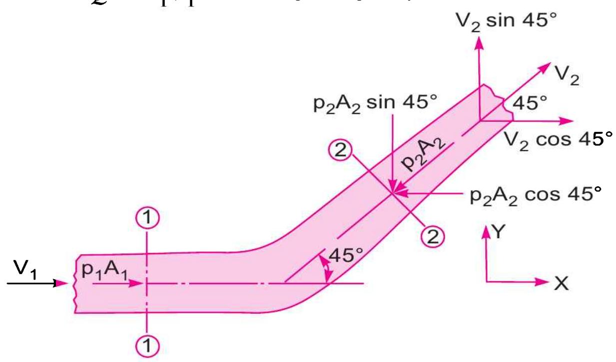

Fig. 6.24

Applying Bernoulli's equation at sections (1) and (2)

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \qquad \{\because \quad Z_1 = Z_2\}$$

or
$$\frac{2.943 \times 10^4}{1.16 \times 9.81} + \frac{10^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{20^2}{2 \times 9.81}$$

$$\therefore \quad \frac{p_2}{\rho g} = \frac{2.943 \times 10^4}{1.16 \times 9.81} + \frac{10^2}{2 \times 9.81} - \frac{20^2}{2 \times 9.81}$$

$$= 2586.2 + 5.0968 - 20.387 = 2570.90 \text{ m}$$

$$\therefore \quad p_2 = 2570.90 \times 1.16 \times 9.81 = 29255.8 \text{ N}$$

Force along x-axis, $F_x = \rho Q\,[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

where $A_{1x} = 10$ m/s, $V_{2x} = V_2 \cos 45° = 20 \times .7071$,

$(p_1A_1)_x = p_1A_1 = 29430 \times 1 = 29430$ N

and $(p_2A_2)_x = -\,p_2A_2 \cos 45° = -\,29255.8 \times 0.5 \times .7071$

$$\therefore \quad F_x = 1.16 \times 10[10 - 20 \times .7071] + 29430 \times 1 - 29255.8 \times .5 \times .7071$$

$$= -\,48.04 + 29430 - 10343.37 = 0 - 19038.59 \text{ N}$$

Similarly *force along y-axis,* $F_y = \rho Q[V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$

where $V_{1y} = 0$, $V_{2y} = V_2 \sin 45° = 20 \times .7071 = 14.142$

$(p_1A_1)_y = 0$ and $(p_2A_2)_y = -\,p_2A_2 \sin 45° = -\,29255.8 \times .5 \times .7071 = -\,10343.37$

$$F_y = 1.16 \times 10\,[0 - 14.142] + 0 - 10343.37$$

$$= -\,164.05 - 10343.37 = -\,10507.42 \text{ N}$$

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(19038.6)^2 + (10507.42)^2}$ = **21746.6 N. Ans.**

The direction of F_R with x-axis is given as

$$\tan\theta = \frac{F_y}{F_x} = \frac{10507.42}{19038.6} = 0.5519$$

$\therefore \quad \theta = \tan^{-1} .5519 =$ **28° 53′. Ans.**

F_R is the force exerted on bend. Hence the force required to hold the duct in position is equal to 21746.6 N but it is acting in the opposite direction of F_R. **Ans.**

Problem 6.33 *A pipe of 300 mm diameter conveying 0.30 m³/s of water has a right angled bend in a horizontal plane. Find the resultant force exerted on the bend if the pressure at inlet and outlet of the bend are 24.525 N/cm² and 23.544 N/cm².*

Solution. Given :

Dia. of bend, $D = 300 \text{ mm} = 0.3 \text{ m}$

∴ Area, $A = A_1 = A_2 = \frac{\pi}{4}(.3)^2 = 0.07068 \text{ m}^2$

∴ Discharge, $Q = 0.30 \text{ m}^3/\text{s}$

∴ Velocity, $V = V_1 = V_2 = \frac{Q}{A} = \frac{0.30}{.07068} = 4.244 \text{ m/s}$

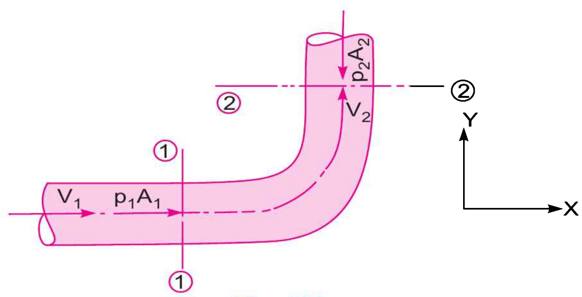

Fig. 6.25

Angle of bend, $\theta = 90°$

$$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2 = 245250 \text{ N/m}^2$$

$$p_2 = 23.544 \text{ N/cm}^2 = 23.544 \times 10^4 \text{ N/m}^2 = 235440 \text{ N/m}^2$$

Force on bend along x-axis $F_x = \rho Q\,[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

where $\rho = 1000,\ V_{1x} = V_1 = 4.244 \text{ m/s},\ V_{2x} = 0$

$(p_1A_1)_x = p_1A_1 = 245250 \times .07068$

$(p_2A_2)_x = 0$

∴ $F_x = 1000 \times 0.30\,[4.244 - 0] + 245250 \times .07068 + 0$

$= 1273.2 + 17334.3 = 18607.5 \text{ N}$

Force on bend along y-axis, $F_y = \rho Q\,[V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$

where $V_{1y} = 0,\ V_{2y} = V_2 = 4.244 \text{ m/s}$

$(p_1A_1)_y = 0,\ (p_2A_2)_y = -\,p_2A_2 = -\,235440 \times .07068 = -\,16640.9$

∴ $F_y = 1000 \times 0.30[0 - 4.244] + 0 - 16640.9$

$= -\,1273.2 - 16640.9 = -\,17914.1 \text{ N}$

∴ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(18607.5)^2 + (17914.1)^2} = -\,25829.3 \text{ N}$

and $\tan\theta = \frac{F_y}{F_x} = \frac{17914.1}{18607.5} = 0.9627$

∴ $\theta =$ **43° 54′. Ans.**

Problem 6.34 *A nozzle of diameter 20 mm is fitted to a pipe of diameter 40 mm. Find the force exerted by the nozzle on the water which is flowing through the pipe at the rate of 1.2 m³/minute.*

Solution. Given :

Dia. of pipe, $D_1 = 40 \text{ mm} = 40 \times 10^{-3} \text{ m} = .04 \text{ m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.04)^2 = 0.001256 \text{ m}^2$

Dia. of nozzle, $D_2 = 20 \text{ mm} = 0.02 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} (.02)^2 = .000314 \text{ m}^2$

Discharge, $Q = 1.2 \text{ m}^3\text{/minute} = \frac{1.2}{60} \text{ m}^3\text{/s} = 0.02 \text{ m}^3\text{/s}$

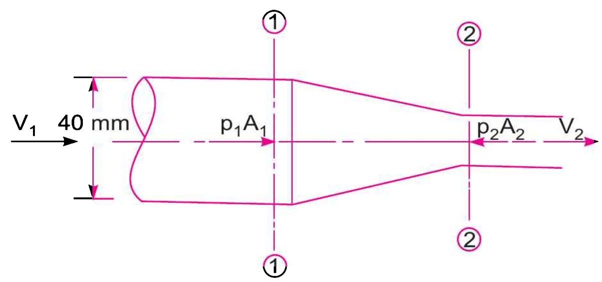

Fig. 6.26

Applying continuity equation at sections (1) and (2),

$$A_1 V_1 = A_2 V_2 = Q$$

$\therefore$ $$V_1 = \frac{Q}{A_1} = \frac{0.2}{.001256} = 15.92 \text{ m/s}$$

and $$V_2 = \frac{Q}{A_2} = \frac{0.2}{.000314} = 63.69 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Now $z_1 = z_2$, $\frac{p_2}{\rho g}$ = atmospheric pressure = 0

$\therefore$ $$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$\therefore$ $$\frac{p_1}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{(63.69^2)}{2 \times 9.81} - \frac{(15.92^2)}{2 \times 9.81} = 206.749 - 12.917$$

$$= 193.83 \text{ m of water}$$

$\therefore$ $$p_1 = 193.83 \times 1000 \times 9.81 \frac{\text{N}}{\text{m}^2} = 1901472 \frac{\text{N}}{\text{m}^2}$$

Let the force exerted by the nozzle on water = F_x

Net force in the direction of x = rate of change of momentum in the direction of x

$\therefore$ $$p_1A_1 - p_2A_2 + F_x = \rho Q(V_2 - V_1)$$

where p_2 = atmospheric pressure = 0 and ρ = 1000

$\therefore$ $1901472 \times .001256 - 0 + F_x = 1000 \times 0.02(63.69 - 15.92)$ or $2388.24 + F_x = 916.15$

$\therefore$ $F_x = -2388.24 + 916.15 = \mathbf{-1472.09}$. **Ans.**

–ve sign indicates that the force exerted by the nozzle on water is acting from right to left.

Problem 6.35 *The diameter of a pipe gradually reduces from 1 m to 0.7 m as shown in Fig. 6.27. The pressure intensity at the centre-line of 1 m section 7.848 kN/m² and rate of flow of water through the pipe is 600 litres/s. Find the intensity of pressure at the centre-line of 0.7 m section. Also determine the force exerted by flowing water on transition of the pipe.*

Solution. Given :

Dia. of pipe at section 1, $D_1 = 1$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4}(1)^2 = 0.7854 \text{ m}^2$

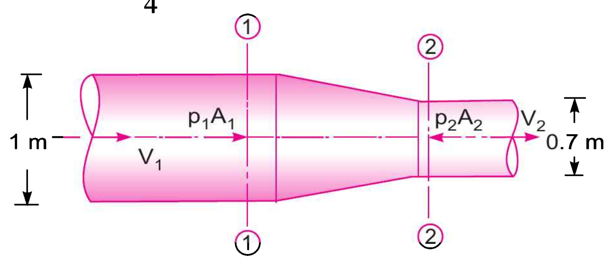

Fig. 6.27

Dia. of pipe at section 2, $D_2 = 0.7$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4}(0.7)^2 = 0.3848 \text{ m}^2$

Pressure at section 1, $p_1 = 7.848 \text{ kN/m}^2 = 7848 \text{ N/m}^2$

Discharge, $Q = 600 \text{ litres/s} = \frac{600}{1000} = 0.6 \text{ m}^3\text{/s}$

Applying continuity equation,

$$A_1V_1 = A_2V_2 = Q$$

$\therefore$ $$V_1 = \frac{Q}{A_2} = \frac{0.6}{0.7854} = 0.764 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{.3854} = 1.55 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2),

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \qquad \{\because \text{ pipe is horizontal, } \therefore z_1 = z_2\}$$

or $$\frac{7848}{1000 \times 9.81} + \frac{(.764)^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{(1.55)^2}{2 \times 9.81}$$

$$\therefore \quad \frac{p_2}{\rho g} = 0.8 + \frac{(.764)^2}{2 \times 9.81} - \frac{(1.55)^2}{2 \times 9.81}$$

$$= 0.8 + 0.0297 - 0.122 = 0.7077 \text{ m of water}$$

$$\therefore \quad p_2 = 0.7077 \times 9.81 \times 1000$$

$$= \mathbf{6942.54\ N/m^2\ or\ 6.942\ kN/m^2.\ Ans.}$$

Let F_x = the force exerted by pipe transition on the flowing water in the direction of flow

Then net force in the direction of flow = rate of change of momentum in the direction of flow

or $$p_1A_1 - p_2A_2 + F_x = \rho(V_2 - V_1)$$

$$\therefore \quad 7848 \times .7854 - 6942.54 \times .3848 + F_x = 1000 \times 0.6[1.55 - .764]$$

or $$6163.8 - 2671.5 + F_x = 471.56$$

$$\therefore \quad F_x = 471.56 - 6163.8 + 2671.5 = -3020.74 \text{ N}$$

∴ The force exerted by water on pipe transition

$$= -F_x = -(-3020.74) = \mathbf{3020.74\ N.\ Ans.}$$

▶ 6.9 MOMENT OF MOMENTUM EQUATION

Moment of momentum equation is derived from moment of momentum principle which states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.

Let V_1 = velocity of fluid at section 1,

r_1 = radius of curvature at section 1,

Q = rate of flow of fluid,

ρ = density of fluid,

and V_2 and r_2 = velocity and radius of curvature at section 2

Momentum of fluid at section 1 = mass × velocity = $\rho Q \times V_1/s$

∴ Moment of momentum per second at section 1,

$$= \rho Q \times V_1 \times r_1$$

Similarly moment of momentum per second of fluid at section 2

$$= \rho Q \times V_2 \times r_2$$

∴ Rate of change of moment of momentum

$$= \rho Q V_2 r_2 - \rho Q V_1 r_1 = \rho Q[V_2 r_2 - V_1 r_1]$$

According to moment of momentum principle

Resultant torque = rate of change of moment of momentum

or $$T = \rho Q[V_2 r_2 - V_1 r_1] \quad \text{...(6.23)}$$

Equation (6.23) is known as moment of momentum equation. This equation is applied :

1. For analysis of flow problems in turbines and centrifugal pumps.
2. For finding torque exerted by water on sprinkler.

Problem 6.36 *A lawn sprinkler with two nozzles of diameter 4 mm each is connected across a tap of water as shown in Fig. 6.28. The nozzles are at a distance of 30 cm and 20 cm from the centre of the tap. The rate of flow of water through tap is 120 cm^3/s. The nozzles discharge water in the downward direction. Determine the angular speed at which the sprinkler will rotate free.*

Solution. Given :

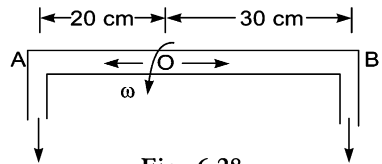

Fig. 6.28

Dia. of nozzles A and B,

$$D = D_A = D_B = 4 \text{ mm} = .004 \text{ m}$$

∴ Area,
$$A = \frac{\pi}{4}(.004)^2 = .00001256 \text{ m}^2$$

Discharge
$$Q = 120 \text{ cm}^3/\text{s}$$

Assuming the discharge to be equally divided between the two nozzles, we have

$$Q_A = Q_B = \frac{Q}{2} = \frac{120}{2} = 60 \text{ cm}^3/\text{s} = 60 \times 10^{-6} \text{ m}^3/\text{s}$$

∴ Velocity of water at the outlet of each nozzle,

$$V_A = V_B = \frac{Q_A}{A} = \frac{60 \times 10^{-6}}{.00001256} = 4.777 \text{ m/s.}$$

The jet of water coming out from nozzles A and B is having velocity 4.777 m/s. These jets of water will exert force in the opposite direction, *i.e.*, force exerted by the jets will be in the upward direction. The torque exerted will also be in the opposite direction. Hence torque at B will be in the anti-clockwise direction and at A in the clockwise direction. But torque at B is more than the torque at A and hence sprinkle, if free, will rotate in the anti-clockwise direction as shown in Fig. 6.28.

Let ω = angular velocity of the sprinkler.

Then absolute velocity of water at A,

$$V_1 = V_A + \omega \times r_A$$

where r_A = distance of nozzle A from the centre of tap

= 20 cm = 0.2 m $\qquad$ {$\omega \times r_A$ = tangential velocity due to rotation}

$$V_1 = (4.777 + \omega \times 0.2) \text{ m/s}$$

Here $\omega \times r_A$ is added to V_A as V_A and tangential velocity due to rotation ($\omega \times r_A$) are in the same direction as shown in Fig. 6.28.

Similarly, absolute velocity of water at B,

$$V_2 = V_B - \text{tangential velocity due to rotation}$$
$$= 4.777 - \omega \times r_B \qquad \{\text{where } r_B = 30 \text{ cm} = 0.3 \text{ m}\}$$
$$= (4.777 - \omega \times 0.3)$$

Now applying equation (6.23), we get

$$T = \rho Q[V_2 r_2 - V_1 r_1] \qquad \Big|\ \text{Here } r_2 = r_B,\ r_1 = r_A$$
$$= \rho Q_A[V_2 r_B - V_1 r_A] \qquad \Big|\ Q = Q_A = Q_B$$
$$= 1000 \times 60 \times 10^{-6} [(4.777 \times 0.3\ \omega) \times .3 - (4.777 + 0.2\ \omega) \times .2]$$

The moment of momentum of the fluid entering sprinkler is given zero and also there is no external torque applied on the sprinkler. Hence resultant external torque is zero, *i.e.*, $T = 0$

∴
$$1000 \times 60 \times 10^{-6} [(4.777 - 0.3\ \omega) \times .3 - (4.777 + 0.2\ \omega) \times .2] = 0$$

or
$$(4.777 - 0.3\ \omega) \times 0.3 - (4.777 + 0.2\ \omega) \times .2 = 0$$

or
$$4.777 \times .3 - .09\ \omega - 4.777 \times .2 - .04\ \omega = 0$$

or
$$0.1 \times 4.777 = (.09 + .04)\omega = .13\ \omega$$

∴
$$\omega = \frac{.4777}{0.13} = \textbf{3.6746 rad/s. Ans.}$$

Problem 6.37 *A lawn sprinkler shown in Fig. 6.29 has 0.8 cm diameter nozzle at the end of a rotating arm and discharges water at the rate of 10 m/s velocity. Determine the torque required to hold the rotating arm stationary. Also determine the constant speed of rotation of the arm, if free to rotate.*

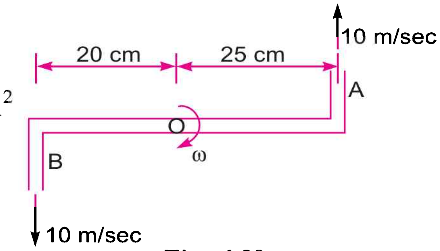

Fig. 6.29

Solution. Dia. of each nozzle = 0.8 cm = .008 m

∴ Area of each nozzle $= \frac{\pi}{4}(.008)^2 = .00005026 \text{ m}^2$

Velocity of flow at each nozzle = 10 m/s.

∴ Discharge through each nozzle,

$$Q = \text{Area} \times \text{Velocity}$$
$$= .00005026 \times 10 = .0005026 \text{ m}^3\text{/s}$$

Torque exerted by water coming through nozzle A on the sprinkler = moment of momentum of water through A

$$= r_A \times \rho \times Q \times V_A = 0.25 \times 1000 \times .0005026 \times 10 \text{ clockwise}$$

Torque exerted by water coming through nozzle B on the sprinkler

$$= r_B \times \rho \times Q \times V_B = 0.20 \times 1000 \times .0005026 \times 10 \text{ clockwise}$$

∴ Total torque exerted by water on sprinkler

$$= .25 \times 1000 \times .0005026 \times 10 + .20 \times 1000 \times .0005026 \times 10$$
$$= 1.2565 + 1.0052 = 2.26 \text{ Nm}$$

∴ Torque required to hold the rotating arm stationary = Torque exerted by water on sprinkler

$$= \mathbf{2.26 \text{ Nm. Ans.}}$$

Speed of rotation of arm, if free to rotate

Let ω = speed of rotation of the sprinkler

The absolute velocity of flow of water at the nozzles A and B are

$$V_1 = 10.0 - 0.25 \times \omega \text{ and } V_2 = 10.0 - 0.20 \times \omega$$

Torque exerted by water coming out at A, on sprinkler

$$= r_A \times \rho \times Q \times V_1 = 0.25 \times 1000 \times .0005026 \times (10 - 0.25\ \omega)$$
$$= 0.12565\ (10 - 0.25\ \omega)$$

Torque exerted by water coming out at B, on sprinkler

$$= r_B \times \rho \times Q \times V_2 = 0.20 \times 1000 \times .0005026 \times (10.0 - 0.2\ \omega)$$
$$= 0.10052\ (10.0 - 0.2\ \omega)$$

∴ Total torque exerted by water = $0.12565\ (10.0 - 0.25\ \omega) + 0.10052\ (10.0 - 0.2\ \omega)$

Since moment of momentum of the flow entering is zero and no external torque is applied on sprinkler, so the resultant torque on the sprinkler must be zero.

$$\therefore\ 0.12565\ (10.0 - 0.25\ \omega) + 0.10052(10.0 - 0.2\ \omega) = 0$$
$$1.2565 - 0.0314\ \omega + 1.0052 - 0.0201\ \omega = 0$$
$$1.2565 + 1.0052 = \omega\ (0.0314 + 0.0201)$$
$$2.2617 = 0.0515\ \omega$$

$$\therefore\ \omega = \frac{2.2617}{0.0515} = \mathbf{43.9 \text{ rad/s. Ans.}}$$

and $$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 43.9}{2\pi} = \textbf{419.2 r.p.m. Ans.}$$

▶ 6.10 FREE LIQUID JETS

Free liquid jet is defined as the jet of water coming out from the nozzle in atmosphere. The path travelled by the free jet is parabolic.

Consider a jet coming from the nozzle as shown in Fig. 6.30. Let the jet at A, makes an angle θ with the horizontal direction. If U is the velocity of jet of water, then the horizontal component and vertical component of this velocity at A are $U \cos \theta$ and $U \sin \theta$.

Consider another point $P(x, y)$ on the centre line of the jet. The co-ordinates of P from A are x and y. Let the velocity of jet at P in the x- and y-directions are u and v. Let a liquid particle takes time 't' to reach from A to P. Then the horizontal and vertical distances travelled by the liquid particle in time 't' are :

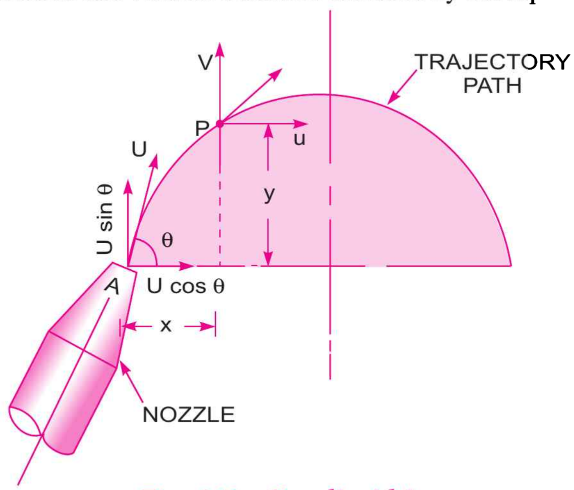

Fig. 6.30 *Free liquid jet.*

$$x = \text{velocity component in } x\text{-direction} \times t$$
$$= U \cos \theta \times t \qquad ...(i)$$

and $$y = (\text{vertical component in } y\text{-direction} \times \text{time} - \frac{1}{2} gt^2)$$
$$= U \sin \theta \times t - \frac{1}{2} gt^2 \qquad ...(ii)$$

{∵ Horizontal component of velocity is constant while the vertical distance is affected by gravity}

From equation (*i*), the value of t is given as $t = \frac{x}{U \cos \theta}$

Substituting this value in equation (*ii*)

$$y = U \sin \theta \times \frac{x}{U \cos \theta} - \frac{1}{2} \times g \times \left(\frac{x}{U \cos \theta}\right)^2 = x \frac{\sin \theta}{\cos \theta} - \frac{gx^2}{2U^2 \cos^2 \theta}$$
$$= x \tan \theta - \frac{gx^2}{2U^2} \sec^2 \theta \qquad \left\{\because \frac{1}{\cos^2 \theta} = \sec^2 \theta\right\} \quad ...(6.24)$$

Equation (6.24) gives the variation of y with the square of x. Hence this is the equation of a parabola. Thus the path travelled by the free jet in atmosphere is parabolic.

(*i*) **Maximum height attained by the jet.** Using the relation $V_2^2 - V_1^2 = -2gS$, we get in this case $V_1 = 0$ at the highest point

$$V_1 = \text{Initial vertical component}$$
$$= U \sin \theta$$

–ve sign on right hand side is taken as g is acting in the downward direction but particles is moving up.

$$\therefore \quad 0 - (U \sin \theta)^2 = -2g \times S$$

where S is the maximum vertical height attained by the particle.

or $$-U^2 \sin^2\theta = -2gS$$

$$\therefore \quad S = \frac{U^2 \sin^2 \theta}{2g} \quad ...(6.25)$$

(*ii*) **Time of flight.** It is the time taken by the fluid particle in reaching from A to B as shown in Fig. 6.30. Let T is the time of flight.

Using equation (*ii*), we have $y = U \sin \theta \times t - \frac{1}{2} gt^2$

when the particle reaches at B, $y = 0$ and $t = T$

$\therefore$ Above equation becomes as $0 = U \sin \theta \times T - \frac{1}{2} g \times T^2$

or $$0 = U \sin \theta - \frac{1}{2} gT \quad \{\text{Cancelling } T\}$$

or $$T = \frac{2U \sin \theta}{g} \quad ...(6.26)$$

(*iii*) **Time to reach highest point.** The time to reach highest point is half the time of flight. Let T^* is the time to reach highest point, then

$$T^* = \frac{T}{2} = \frac{2U \sin \theta}{g \times 2} = \frac{U \sin \theta}{g} \quad ...(6.27)$$

(*iv*) **Horizontal range of the jet.** The total horizontal distance travelled by the fluid particle is called horizontal range of the jet, *i.e.*, the horizontal distance AB in Fig. 6.30 is called horizontal range of the jet. Let this range is denoted by x^*.

Then x^* = velocity component in x-direction × time taken by the particle to reach from A to B

$$= U \cos \theta \times \text{Time of flight}$$

$$= U \cos \theta \times \frac{2U \sin \theta}{g} \quad \left\{\because T = \frac{2U \sin \theta}{g}\right\}$$

$$= \frac{U^2}{g} 2 \cos \theta \sin \theta = \frac{U^2}{g} \sin 2\theta \quad ...(6.28)$$

(*v*) **Value of θ for maximum range.** The range x^* will be maximum for a given velocity of projection (U), when $\sin 2\theta$ is maximum

or when $\sin 2\theta = 1$ or $\sin 2\theta = \sin 90° = 1$

$$\therefore \quad 2\theta = 90° \text{ or } \theta = 45°$$

Then maximum range, $$x^*_{max} = \frac{U^2}{g} \sin^2 \theta = \frac{U^2}{g} \quad \{\because \sin 90° = 1\} \quad ...(6.29)$$

Problem 6.38 *A vertical wall is of 8 m in height. A jet of water is coming out from a nozzle with a velocity of 20 m/s. The nozzle is situated at a distance of 20 m from the vertical wall. Find the angle of projection of the nozzle to the horizontal so that the jet of water just clears the top of the wall.*

Solution. Given :

Height of wall = 8 m

Velocity of jet, $U = 20$ m/s

Distance of jet from wall, $x = 20$ m

Let the required angle $= \theta$

Using equation (6.24), we have

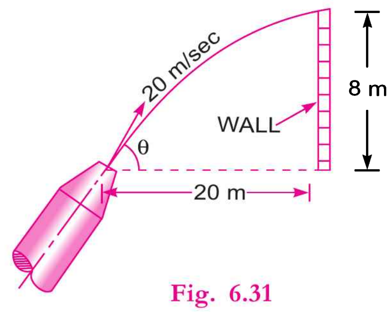

Fig. 6.31

$$y = x \tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$$

where $y = 8$ m, $x = 20$ m, $U = 20$ m/s

$$8 = 20\tan\theta - \frac{9.81\times 20^2}{2\times 20^2}\sec^2\theta$$

$$= 20\tan\theta - 4.905\sec^2\theta$$

$$= 20\tan\theta - 4.905\,[1+\tan^2\theta] \qquad \{\because \sec^2\theta = 1+\tan^2\theta\}$$

$$= 20\tan\theta - 4.905 - 4.905\tan^2\theta$$

or $\qquad 4.905\tan^2\theta - 20\tan\theta + 8 + 4.905 = 0$

or $\qquad 4.905\tan^2\theta - 20\tan\theta + 12.905 = 0$

$$\therefore \qquad \tan\theta = \frac{20\pm\sqrt{20^2 - 4\times 12.905\times 4.905}}{2\times 4.905} = \frac{20\pm\sqrt{400-253.19}}{9.81}$$

$$= \frac{20\pm\sqrt{146.81}}{9.81} = \frac{20\pm 12.116}{9.81} = \frac{32.116}{9.81} \text{ or } \frac{7.889}{9.81}$$

$\therefore \qquad = 3.273$ or 0.8036

$\therefore \qquad$ **$\theta = 73°\ 0.8'$ or $38°\ 37'$. Ans.**

Problem 6.39 *A fire-brigade man is holding a fire stream nozzle of 50 mm diameter as shown in Fig. 6.32. The jet issues out with a velocity of 13 m/s and strikes the window. Find the angle or angles of inclination with which the jet issues from the nozzle. What will be the amount of water falling on the window ?*

Solution. Given :

Dia. of nozzle, $d = 50$ mm $= .05$ m

$\therefore$ Area, $A = \dfrac{\pi}{4}(.05)^2 = 0.001963\ \text{m}^2$

Velocity of jet, $U = 13$ m/s.

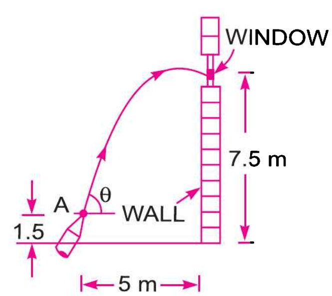

Fig. 6.32

The jet is coming out from nozzle at A. It strikes the window and let the angle made by the jet at A with horizontal is equal to θ.

The co-ordinates of window, with respect to origin at A.

$$x = 5 \text{ m}, y = 7.5 - 1.5 = 6.0 \text{ m}$$

The equation of the jet is given by (6.24) as

$$y = x \tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$$

or $$6.0 = 5 \times \tan\theta - \frac{9.81 \times 5}{2 \times 13^2}[1 + \tan^2\theta] \qquad \{\because \quad \sec^2\theta = 1 + \tan^2\theta\}$$

or $$6.0 = 5\tan\theta - .7256\,(1 + \tan^2\theta)$$

$$= 5\tan\theta - .7256 - .7256\tan^2\theta$$

or $$0.7256\tan^2\theta - 5\tan\theta + 6 + .7256 = 0$$

or $$0.7256\tan^2\theta - 5\tan\theta + 6.7256 = 0$$

This is a quadratic equation in $\tan\theta$. Hence solution is

$$\tan\theta = \frac{5 \pm \sqrt{5^2 - 4 \times .7256 \times 6.7256}}{2 \times .7256}$$

$$= \frac{5 \pm \sqrt{25 - 19.52}}{1.4512} = \frac{5 + 2.341}{1.4512} = 5.058 \text{ or } 1.8322$$

$\therefore$ $$\theta = \tan^{-1} 5.058 \text{ or } \tan^{-1} 1.8322 = \mathbf{78.8° \text{ or } 61.37°.\ Ans.}$$

Amount of water falling on window = Discharge from nozzle

= Area of nozzle × Velocity of jet at nozzle

= 0.001963 × U = 0.001963 × 13.0 = **0.0255 m^3/s. Ans.**

Problem 6.40 *A nozzle is situated at a distance of 1 m above the ground level and is inclined at an angle of 45° to the horizontal. The diameter of the nozzle is 50 mm and the jet of water from the nozzle strikes the ground at a horizontal distance of 4 m. Find the rate of flow of water.*

Solution. Given :

Distance of nozzle above ground = 1 m

Angle of inclination, $\theta = 45°$

Dia. of nozzle, d = 50 mm = .05 m

$\therefore$ Area, $A = \frac{\pi}{4}(.05)^2 = .001963 \text{ m}^2$

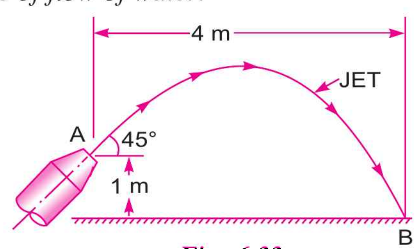

Fig. 6.33

The horizontal distance x = 4 m

The co-ordinates of the point B, which is on the centre-line of the jet of water and is situated on the ground, with respect to A (origin) are

x = 4 m and y = – 1.0 m {From A, point B is vertically down by 1 m}

The equation of the jet is given by (6.24) as $y = x\tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$

Substituting the known values as

$$-1.0 = 4\tan 45° - \frac{9.81 \times 4^2}{2U^2} \times \sec^2 45°$$

$$= 4 - \frac{78.48}{U^2} \times \left(\sqrt{2}\right)^2 \qquad \left\{\sec 45° = \frac{1}{\cos 45°} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}\right\}$$

$$-1.0 = 4 - \frac{78.48 \times 2}{U^2} \quad \text{or} \quad \frac{78.48 \times 2}{U^2} = +4.0 + 1.0 = 5.0$$

$$\therefore \quad U^2 = \frac{78.48 \times 2.0}{5.0} = 31.39$$

$$\therefore \quad U = \sqrt{31.39} = 5.60 \text{ m/s}$$

Now the rate of flow of fluid $= \text{Area} \times \text{Velocity of jet}$

$= A \times U = .001963 \times 5.6 \text{ m}^3\text{/sec}$

$= 0.01099 \approx$ **.011 m³/s. Ans.**

Problem 6.41 *A window, in a vertical wall, is at a distance of 30 m above the ground level. A jet of water, issuing from a nozzle of diameter 50 mm is to strike the window. The rate of flow of water through the nozzle is 3.5 m³/minute and nozzle is situated at a distance of 1 m above ground level. Find the greatest horizontal distance from the wall of the nozzle so that jet of water strikes the window.*

Solution. Given :

Distance of window from ground level = 30 m

Dia. of nozzle, $d = 50 \text{ mm} = 0.05 \text{ m}$

$\therefore$ Area $A = \frac{\pi}{4}(.05)^2 = 0.001963 \text{ m}^2$

The discharge, $Q = 3.5 \text{ m}^3\text{/minute}$

$= \frac{3.5}{60} = 0.0583 \text{ m}^3\text{/s}$

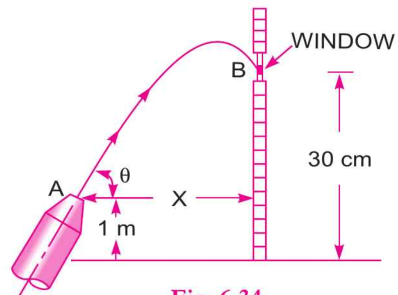

Fig. 6.34

Distance of nozzle from ground = 1 m.

Let the greatest horizontal distance of the nozzle from the wall = x and let angle of inclination = θ. If the jet reaches the window, then the point B on the window is on the centre-line of the jet. The co-ordinates of B with respect to A are

$$x = x, \; y = 30 - 1.0 = 29 \text{ m}$$

The velocity of jet, $U = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{A} = \frac{.0583}{.001963} = 29.69 \text{ m/sec}$

Using the equation (6.34), which is the equation of jet,

$$y = x \tan\theta - \frac{gx^2}{2U^2} \sec^2\theta$$

or

$$29.0 = x \tan\theta - \frac{9.81x^2}{2 \times (29.69)^2} \sec^2\theta$$

$$= x \tan\theta - 0.0055 \sec^2\theta \times x^2$$

$$= x \tan\theta - \frac{.0055\, x^2}{\cos^2\theta}$$

$$x \tan\theta - .0055\, x^2/\cos^2\theta - 29 = 0 \qquad \text{...}(i)$$

The maximum value of x with respect to θ is obtained, by differentiating the above equation w.r.t. θ and substituting the value of $\frac{dx}{d\theta} = 0$. Hence differentiating the equation (*i*) w.r.t. θ, we have

$$\left[x \sec^2\theta + \tan\theta \times \frac{dx}{d\theta}\right] - 0.0055\left[x^2 \times \left(\frac{(-2)}{\cos^3\theta}\right)(-\sin\theta) + \frac{1}{\cos^2\theta} \times 2x\frac{dx}{d\theta}\right]$$

$$\left\{\because \frac{d}{d\theta}(x\tan\theta) = x\sec^2\theta + \tan\theta\frac{dx}{d\theta} \text{ and } \frac{d}{d\theta}\left(\frac{x^2}{\cos^2\theta}\right) = x^2\frac{d}{d\theta}\left(\frac{1}{\cos^2\theta}\right) + \frac{1}{\cos^2\theta}\frac{d}{d\theta}(x^2)\right\}$$

$$\therefore \qquad x\sec^2\theta + \tan\theta\frac{dx}{d\theta} - .0055\left[\frac{2x^2\sin\theta}{\cos^3\theta} + \frac{2x}{\cos^2\theta}\frac{dx}{d\theta}\right] = 0$$

For maximum value of x, w.r.t. θ, we have $\frac{dx}{d\theta} = 0$

Substituting this value in the above equation, we have

$$x\sec^2\theta - .0055\left[\frac{2x^2\sin\theta}{\cos^3\theta}\right] = 0$$

or
$$\frac{x}{\cos^2\theta} - \frac{.0055\times 2x^2\sin\theta}{\cos^3\theta} = 0 \text{ or } x - .011\times x^2\frac{\sin\theta}{\cos\theta} = 0$$

or
$$x - .011\,x^2\tan\theta = 0 \text{ or } 1 - .011\,x\tan\theta = 0$$

or
$$x\tan\theta = \frac{1}{.011} = 90.9 \qquad ...(ii)$$

or
$$x = \frac{90.9}{\tan\theta} \qquad ...(iii)$$

Substituting this value of x in equation (i), we get

$$\frac{90.9}{\tan\theta}\times\tan\theta - .0055\times\frac{(90.9)^2}{\tan^2\theta}\times\frac{1}{\cos^2\theta} - 29 = 0$$

$$90.9 - \frac{45.445}{\sin^2\theta} - 29 = 0 \text{ or } 61.9 - \frac{45.445}{\sin^2\theta} = 0$$

or
$$61.9 = \frac{45.445}{\sin^2\theta} \text{ or } \sin^2\theta = \frac{45.445}{61.90} = 0.7341$$

$$\therefore \qquad \sin\theta = \sqrt{0.7341} = 0.8568$$

$$\therefore \qquad \theta = \tan^{-1}.8568 = 58^\circ\ 57.8'$$

Substituting this value of θ in equation (iii), we get

$$x = \frac{90.9}{\tan\theta} = \frac{90.9}{\tan 58^\circ\ 57.8'} = \frac{90.9}{\tan 58.95} = \frac{90.9}{1.66} = 54.759 \text{ m}$$

= 54.76 m. Ans.

HIGHLIGHTS

1. The study of fluid motion with the forces causing flow is called dynamics of fluid flow, which is analysed by the Newton's second law of motion.
2. Bernoulli's equation is obtained by integrating the Euler's equation of motion. Bernoulli's equation states "For a steady, ideal flow of an incompressible fluid, the total energy which consists of pressure energy, kinetic energy and datum energy, at any point of the fluid is constant". Mathematically,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

where $\frac{p_1}{\rho g}$ = pressure energy per unit weight = pressure head

$\frac{v_1^2}{2g}$ = kinetic energy per unit weight = kinetic head

z_1 = datum energy per unit weight = datum head.

3. Bernoulli's equation for real fluids

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

where h_L = loss of energy between sections 1 and 2.

4. The discharge, Q, through a venturimeter or an orifice meter is given by

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

where a_1 = area at the inlet of venturimeter,
a_2 = area at the throat of venturimeter,
C_d = co-efficient of venturimeter,
h = difference of pressure head in terms of fluid head flowing through venturimeter.

5. The value of h is given by differential U-tube manometer

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$ ... (when differential manometer contains heavier liquid)

$$h = x\left[1 - \frac{S_l}{S_o}\right]$$ ...(when differential manometer contains lighter liquid)

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right]$$ ...(for inclined venturimeter in which differential manometer contains heavier liquid)

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right]$$ ...(for inclined venturimeter in which differential manometer contains lighter liquid)

where x = difference in the readings of differential manometer,
S_h = sp. gr. of heavier liquid
S_o = sp. gr. of fluid flowing through venturimeter
S_l = sp. gr. of lighter liquid.

6. Pitot-tube is used to find the velocity of a flowing fluid at any point in a pipe or a channel. The velocity is given by the relation

$$V = C_v \sqrt{2gh}$$

where C_v = co-efficient of Pitot-tube

h = rise of liquid in the tube above free surface of liquid

$$= x\left[\frac{S_g}{S_o} - 1\right] \text{ (for pipes or channels).}$$

7. The momentum equation states that the net force acting on a fluid mass is equal to the change in momentum per second in that direction. This is given as $F = \frac{d}{dt}(mv)$

The impulse-momentum equation is given by $F \cdot dt = d(mv)$.

8. The force exerted by a fluid on a pipe bend in the directions of x and y are given by

$$F_x = \frac{\text{mass}}{\text{sec}}(\text{Initial velocity in the direction of } x - \text{Final velocity in } x\text{-direction})$$

$$+ \text{ Initial pressure force in } x\text{-direction} + \text{Final pressure force in } x\text{-direction}$$

$$= \rho Q[V_{1x} - V_{2x}] + (p_1A_1)_x + (p_2A_2)_x$$

and $$F_y = \rho Q[V_{1y} - V_{2y}] + (p_1A_1)_y + (p_2A_2)_y$$

Resultant force, $$F_R = \sqrt{F_x^2 + F_y^2}$$

and the direction of the resultant with horizontal is $\tan\theta = \frac{F_y}{F_x}$.

9. The force exerted by the nozzle on the water is given by $F_x = \rho Q[V_{2x} - V_{1x}]$ and force exerted by the water on the nozzle is $= -F_x = \rho Q[V_{1x} - V_{2x}]$.

10. Moment of momentum equation states that the resultant torque acting on a rotating fluid is equal to the rate of change of moment of momentum. Mathematically, it is given by $T = \rho Q[V_2r_2 - V_1r_1]$.

11. Free liquid jet is the jet of water issuing from a nozzle in atmosphere. The path travelled by the free jet is parabolic. The equation of the jet is given by

$$y = x\tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$$

where x, y = co-ordinates of any point on jet w.r.t. to the nozzle

U = velocity of jet of water issuing from nozzle

θ = inclination of jet issuing from nozzle with horizontal.

12. (*i*) Maximum height attained by jet $= \frac{U^2\sin^2\theta}{2g}$

(*ii*) Time of flight, $T = \frac{2U\sin\theta}{g}$

(*iii*) Time to reach highest point, $T^* = \frac{T}{2} = \frac{U\sin\theta}{2g}$

(*iv*) Horizontal range of the jet, $x^* = \frac{U^2}{g}\sin 2\theta$

(*v*) Value of θ for maximum range, $\theta = 45°$

(*vi*) Maximum range, $x^*_{max} = U^2/g$.

EXERCISE

(A) THEORETICAL PROBLEMS

1. Name the different forces present in a fluid flow. For the Euler's equation of motion, which forces are taken into consideration.

2. What is Euler's equation of motion ? How will you obtain Bernoulli's equation from it ?

3. Derive Bernoulli's equation for the flow of an incompressible frictionless fluid from consideration of momentum.

4. State Bernoulli's theorem for steady flow of an incompressible fluid. Derive an expression for Bernoulli's theorem from first principle and state the assumptions made for such a derivation.

5. What is a venturimeter ? Derive an expression for the discharge through a venturimeter.

6. Explain the principle of venturimeter with a neat sketch. Derive the expression for the rate of flow of fluid through it.

7. Discuss the relative merits and demerits of venturimeter with respect to orifice-meter.
(*Delhi University, Dec. 2002*)

8. Define an orifice-meter. Prove that the discharge through an orifice-meter is given by the relation

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

where a_1 = area of pipe in which orifice-meter is fitted
a_0 = area of orifice (*Technical University of M.P., S 2002*)

9. What is a pitot-tube ? How will you determine the velocity at any point with the help of pitot-tube ?
(*Delhi University, Dec. 2002*)

10. What is the difference between pitot-tube and pitot-static tube ?

11. State the momentum equation. How will you apply momentum equation for determining the force exerted by a flowing liquid on a pipe bend ?

12. What is the difference between momentum equation and impulse momentum equation.

13. Define moment of momentum equation. Where this equation is used.

14. What is a free jet of liquid ? Derive an expression for the path travelled by free jet issuing from a nozzle.

15. Prove that the equation of the free jet of liquid is given by the expression,

$$y = x \tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$$

where x, y = co-ordinates of a point on the jet
U = velocity of issuing jet
θ = inclination of the jet with horizontal.

16. Which of the following statement is correct in case of pipe flow :
(*a*) flow takes place from higher pressure to lower pressure ;
(*b*) flow takes place from higher velocity to lower velocity ;
(*c*) flow takes place from higher elevation to lower elevation ;
(*d*) flow takes place from higher energy to lower energy.

17. Derive Euler's equation of motion along a stream line for an ideal fluid stating clearly the assumptions. Explain how this is integrated to get Bernoulli's equation along a stream-line.

18. State Bernoulli's theorem. Mention the assumptions made. How is it modified while applying in practice? List out its engineering applications.

19. Define continuity equation and Bernoulli's equation.

20. What are the different forms of energy in a flowing fluid ? Represent schematically the Bernoulli's equation for flow through a tapering pipe and show the position of total energy line and the datum line.
21. Write Euler's equation of motion along a stream line and integrate it to obtain Bernoulli's equation. State all assumptions made.
22. Describe with the help of sketch the construction, operation and use of Pitot-static tube.
23. Starting with Euler's equation of motion along a stream line, obtain Bernoulli's equation by its integration. List all the assumptions made.
24. State the different devices that one can use to measure the discharge through a pipe and also through an open channel. Describe one of such devices with a neat sketch and explain how one can obtain the actual discharge with its help?
25. Derive Bernoulli's equation from fundamentals.

(B) NUMERICAL PROBLEMS

1. Water is flowing through a pipe of 100 mm diameter under a pressure of 19.62 N/cm^2 (gauge) and with mean velocity of 3.0 m/s. Find the total head of the water at a cross-section, which is 8 m above the datum line. [**Ans.** 28.458 m]
2. A pipe, through which water is flowing is having diameters 40 cm and 20 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 5.0 m/s. Find the velocity head at the sections 1 and 2 and also rate of discharge. [**Ans.** 1.274 m ; 20.387 m ; 0.628 m^3/s]
3. The water is flowing through a pipe having diameters 20 cm and 15 cm at sections 1 and 2 respectively. The rate of flow through pipe is 40 litres/s. The section 1 is 6 m above datum line and section 2 is 3 m above the datum. If the pressure at section 1 is 29.43 N/cm^2, find the intensity of pressure at section 2. [**Ans.** 32.19 N/cm^2]
4. Water is flowing through a pipe having diameters 30 cm and 15 cm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 29.43 N/cm^2 and the pressure at the upper end is 14.715 N/cm^2. Determine the difference in datum head if the rate of flow through pipe is 50 lit/s. [**Ans.** 14.618 m]
5. The water is flowing through a taper pipe of length 50 m having diameters 40 cm at the upper end and 20 cm at the lower end, at the rate of 60 litres/s. The pipe has a slope of 1 in 40. Find the pressure at the lower end if the pressure at the higher level is 24.525 N/cm^2. [**Ans.** 25.58 N/cm^2]
6. A pipe of diameter 30 cm carries water at a velocity of 20 m/sec. The pressures at the points A and B are given as 34.335 N/cm^2 and 29.43 N/cm^2 respectively, while the datum head at A and B are 25 m and 28 m. Find the loss of head between A and B. [**Ans.** 2 m]
7. A conical tube of length 3.0 m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 4 m/s while at the lower end it is 2 m/s. The pressure head at the smaller end is 2.0 m of liquid. The loss of head in the tube is $0.95\ (v_1 - v_2)^2/2g$, where v_1 is the velocity at the smaller end and v_2 at the lower end respectively. Determine the pressure head at the lower end. Flow takes place in downward direction. [**Ans.** 5.56 m of fluid]
8. A pipe line carrying oil of specific gravity 0.8, changes in diameter from 300 mm at a position A to 500 mm diameter to a position B which is 5 m at a higher level. If the pressures at A and B are 19.62 N/cm^2 and 14.91 N/cm^2 respectively, and the discharge is 150 litres/s, determine the loss of head and direction of flow. [**Ans.** 1.45 m, Flow takes place from A to B]
9. A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to inlet and throat is 10 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$. [**Ans.** 88.92 litres/s]

10. An oil of sp. gr. 0.9 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 20 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$. [**Ans.** 59.15 litres/s]
11. A horizontal venturimeter with inlet diameter 30 cm and throat diameter 15 cm is used to measure the flow of oil of sp. gr. 0.8. The discharge of oil through venturimeter is 50 litres/s, find the reading of the oil-mercury differential manometer. Take $C_d = 0.98$. [**Ans.** 2.489 cm]
12. A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 14.715 N/cm^2 and vacuum pressure at the throat is 40 cm of mercury. Find the discharge of water through venturimeter. [**Ans.** 162.539 lit./s]
13. A 30 cm × 15 cm venturimeter is inserted in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 30 cm. Find the discharge. Take $C_d = 0.98$. [**Ans.** 154.02 lit/s]
14. If in the problem 13, instead of water, oil of sp. gr. 0.8 is flowing through the venturimeter, determine the rate of flow of oil in litres/s. [**Ans.** 173.56 lit/s]
15. The water is flowing through a pipe of diameter 30 cm. The pipe is inclined and a venturimeter is inserted in the pipe. The diameter of venturimeter at throat is 15 cm. The difference of pressure between the inlet and throat of the venturimeter is measured by a liquid of sp. gr. 0.8 in an inverted *U*-tube which gives a reading of 40 cm. The loss of head between the inlet and throat is 0.3 times the kinetic head of the pipe. Find the discharge. [**Ans.** 22.64 lit./s]
16. A 20 × 10 cm venturimeter is provided in a vertical pipe line carrying oil of sp. gr. 0.8, the flow being upwards. The difference in elevation of the throat section and entrance section of the venturimeter is 50 cm. The differential *U*-tube mercury manometer shows a gauge deflection of 40 cm. Calculate : (*i*) the discharge of oil, and (*ii*) the pressure difference between the entrance section and the throat section. Take $C_d = 0.98$ and sp. gr. of mercury as 13.6. [**Ans.** (*i*) 89.132 lit/s, (*ii*) 5.415 N/cm^2]
17. In a 200 mm diameter horizontal pipe a venturimeter of 0.5 contraction ratio has been fixed. The head of water on the venturimeter when there is no flow is 4 m (gauge). Find the rate of flow for which the throat pressure will be 4 metres of water absolute. Take $C_d = 0.97$ and atmospheric pressure head = 10.3 m of water. [**Ans.** 111.92 lit/s]
18. An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter give readings of 14.715 N/cm^2 and 9.81 N/cm^2 respectively. Find the rate of flow of water through the pipe in litres/s. Take $C_d = 0.6$. [**Ans.** 108.434 lit/s]
19. If in problem 18, instead of water, oil of sp. gr. 0.8 is flowing through the orifice meter in which the pressure difference is measured by a mercury oil differential manometer on the two sides of the orifice meter, find the rate of flow of oil when the reading of manometer is 40 cm. [**Ans.** 122.68 lit/s]
20. The pressure difference measured by the two tappings of a pitot-static tube, one tapping pointing upstream and other perpendicular to the flow, placed in the centre of a pipe line of diameter 40 cm is 10 cm of water. The mean velocity in the pipe is 0.75 times the central velocity. Find the discharge through the pipe. Take co-efficient of pitot-tube as 0.98. [**Ans.** 0.1293 m^3/s]
21. Find the velocity of flow of an oil through a pipe, when the difference of mercury level in a differential *U*-tube manometer connected to the two tappings of the pitot-tube is 15 cm. Take sp. gr. of oil = 0.8 and co-efficient of pitot-tube as 0.98. [**Ans.** 6.72 m/s]
22. A sub-marine moves horizontally in sea and has its axis 20 m below the surface of water. A pitot-static tube placed in front of sub-marine and along its axis, is connected to the two limbs of a *U*-tube containing mercury. The difference of mercury level is found to be 20 cm. Find the speed of sub-marine. Take sp. gr. of mercury 13.6 and of sea-water 1.026. [**Ans.** 24.958 km/hr.]
23. A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 40 cm and 20 cm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet of bend is 21.58 N/cm^2. The rate of flow of water is 500 litres/s. [**Ans.** 22696.5 N ; 20° 3.5′]

24. The discharge of water through a pipe of diameter 40 cm is 400 litres/s. If the pipe is bend by 135°, find the magnitude and direction of the resultant force on the bend. The pressure of flowing water is 29.43 N/cm^2. [**Ans.** 7063.2 N, $\theta = 22° 29.9'$ with x-axis clockwise]
25. A 30 cm diameter pipe carries water under a head of 15 metres with a velocity of 4 m/s. If the axis of the pipe turns through 45°, find the magnitude and direction of the resultant force at the bend. [**Ans.** 8717.5 N, $\theta = 67° 30'$]
26. A pipe of 20 cm diameter conveying 0.20 m^3/sec of water has a right angled bend in a horizontal plane. Find the resultant force exerted on the bend if the pressure at inlet and outlet of the bend are 22.563 N/cm^2 and 21.582 N/cm^2 respectively. [**Ans.** 11604.7 N, $\theta = 43° 54.2'$]
27. A nozzle of diameter 30 mm is fitted to a pipe of 60 mm diameter. Find the force exerted by the nozzle on the water which is flowing through the pipe at the rate of 4.0 m^3/minute. [**Ans.** 7057.7 N]
28. A lawn sprinkler with two nozzles of diameters 3 mm each is connected across a tap of water. The nozzles are at a distance of 40 cm and 30 cm from the centre of the tap. The rate of water through tap is 100 cm^3/s. The nozzle discharges water in the downward directions. Determine the angular speed at which the sprinkler will rotate free. [**Ans.** 2.83 rad/s]
29. A lawn sprinkler has two nozzles of diameters 8 mm each at the end of a rotating arm and the velocity of flow of water from each nozzle is 12 m/s. One nozzle discharges water in the downward direction, while the other nozzle discharges water vertically up. The nozzles are at a distance of 40 cm from the centre of the rotating arm. Determine the torque required to hold the rotating arm stationary. Also determine the constant speed of rotation of arm, if it is free to rotate. [**Ans.** 5.78 Nm, 30 rad/s]
30. A vertical wall is of 10 m in height. A jet of water is issuing from a nozzle with a velocity of 25 m/s. The nozzle is situated at a horizontal distance of 20 m from the vertical wall. Find the angle of projection of the nozzle to the horizontal so that the jet of water just clears the tap of the wall. [**Ans.** 79° 55′ or 36° 41′]
31. A fire-brigade man is holding a fire stream nozzle of 50 mm diameter at a distance of 1 m above the ground and 6 m from a vertical wall. The jet is coming out with a velocity of 15 m/s. This jet is to strike a window, situated at a distance of 10 m above ground in the vertical wall. Find the angle or angles of inclination with the horizontal made by the jet, coming out from the nozzle. What will be the amount of water falling on the window ? [**Ans.** 79° 16.7′ or 67° 3.7′ ; 0.0294 m^3/s]
32. A window, in a vertical wall, is at a distance of 12 m above the ground level. A jet of water, issuing from a nozzle of diameter 50 mm, is to strike the window. The rate of flow of water through the nozzle is 40 litres/sec. The nozzle is situated at a distance of 1 m above ground level. Find the greatest horizontal distance from the wall of the nozzle so that jet of water strikes the window. [**Ans.** 29.38 m]
33. Explain in brief the working of a pitot-tube. Calculate the velocity of flow of water in a pipe of diameter 300 mm at a point, where the stagnation pressure head is 5 m and static pressure head is 4 m. Given the co-efficient of pitot-tube = 0.97. [**Ans.** 4.3 m/sec]
34. Find the rate of flow of water through a venturimeter fitted in a pipeline of diameter 30 cm. The ratio of diameter of throat and inlet of the venturimeter is *. The pressure at the inlet of the venturimeter is 13.734 N/cm^2 (gauge) and vacuum in the throat is 37.5 cm of mercury. The co-efficient of venturimeter is given as 0.98. [**Ans.** 0.15 m^3/s]
35. A 30 cm × 15 cm venturimeter is inserted in a vertical pipe carrying an oil of sp. gr. 0.8, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 30 cm. The difference in the elevation of the throat section and inlet section is 50 cm. Find the rate of flow of oil.
36. A venturimeter is used for measurement of discharge of water in horizontal pipe line. If the ratio of upstream pipe diameter to that of throat is 2 : 1, upstream diameter is 300 mm, the difference in pressure between the throat and upstream is equal to 3 m head of water and loss of head through meter is one-eighth of the throat velocity head, calculate the discharge in the pipe. [**Ans.** 0.107 m^3/s]
37. A liquid of specific gravity 0.8 is flowing upwards at the rate of 0.08 m^3/s, through a vertical venturimeter with an inlet diameter of 200 mm and throat diameter of 100 mm. The C_d = 0.98 and the vertical distance between pressure tappings is 300 mm. Find :

(*i*) the difference in readings of the two pressure gauges, which are connected to the two pressure tappings, and

(*ii*) the difference in the level of the mercury columns of the differential manometer which is connected to the tappings, in place of pressure gauges. **[Ans.** (*i*) 42.928 kN/m^2, (*ii*) 32.3 cm]

[Hint. $Q = 0.08$ m^3/s, $d_1 = 200$ mm $= 0.2$ m, $d_2 = 100$ mm $= 0.1$ m,

$$C_d = 0.98,\ z_2 - z_1 = 300 \text{ mm} = 0.3 \text{ m},\ a_1 = \frac{\pi}{4}(.2^2) = 0.0314 \text{ m}^2$$

$$a_2 = \frac{\pi}{4}(.1^2) = 0.007854 \text{ m}^2. \text{ Using } Q = C_d \frac{a_1 \times a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Find 'h'. This value of $h = 5.17$ m.

Now use $$h = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + (z_1 - z_2), \text{ where } \rho = 800 \text{ kg/m}^3. \text{ Find } (p_1 - p_2).$$

Now use the formula $$h = x\left[\frac{S_g}{S_f} - 1\right],$$

where $h = 5.17$ m, $S_g = 13.6$ and $S_f = 0.8$. Find the value of x which will be 32.3 cm.]

38. A venturimeter is installed in a 300 mm diameter horizontal pipe line. The throat pipe rates is 1/3. Water flows through the installation. The pressure in the pipe line is 13.783 N/cm^2 (gauge) and vacuum in the throat is 37.5 cm of mercury. Neglecting head loss in the venturimeter, determine the rate of flow in the pipe line. **[Ans.** 0.153 m^3/sec]

[Hint. $d_1 = 300$ mm $= 0.3$ m, $d_2 = \frac{1}{3} \times 300 = 100$ mm $= 0.1$ m, $p_1 = 13.783$ N/cm^2 $= 13.783 \times 10^4$ N/m^2.

Hence $p_1/\rho \times g = 13.783 \times 10^4/1000 \times 9.81$
$= 14.05$ m, $p_2/\rho g = -\,37.5$ cm of Hg $= -\,0.375 \times 13.6$ m of water
$= -\,5.1$ m of water. Hence $h = 14.05 - (-\,5.1) = 19.15$ m of water.

Value of $C_d = 1.0$. Now use the formula $Q = C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$]

39. The maximum flow through a 300 mm diameter horizontal main pipe line is 18200 litre/minute. A venturimeter is introduced at a point of the pipe line where the pressure head is 4.6 m of water. Find the smallest dia. of throat so that the pressure at the throat is never negative. Assume co-efficient of meter as unity. **[Ans.** $d_2 = 192.4$ mm]

[Hint. $d_1 = 300$ mm $= 0.3$ m, $Q = 18200$ litres/minute $= 18200/60 = 303.33$ litres/s $= 0.3033$ m^3/s, $p_1/\rho g = 4.6$ m, $p_2/\rho g = 0$. Hence $h = 4.6$ m, $C_d = 1$. $d_2 =$ dia. at throat. Use formula $Q = C_d \dfrac{a_1 \times a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$ and find the value of a_2. Then $a_2 = \frac{\pi}{4} d_2^2$ and find d_2.]

40. The following are the data given of a change in diameter effected in laying a water supply pipe. The change in diameter is gradual from 20 cm at A to 50 cm at B. Pressures at A and B are 7.848 N/cm^2 and 5.886 N/cm^2 respectively with the end B being 3 m higher than A. If the flow in the pipe line is 200 litre/s, find : (*i*) direction of flow, (*ii*) the head lost in friction between A and B.

[Ans. (*i*) From A to B, (*ii*) 1.015 m]

[Hint. $D_A = 20$ cm $= 0.2$ m, $D_B = 50$ cm $= 0.5$ m, $p_A = 7.848$ N/cm^2 $= 7.848 \times 10^4$ N/m^2
$p_B = 5.886$ N/cm^2 $= 5.886 \times 10^4$ N/m^2, $Z_A = 0$, $Z_B = 3$ m, $Q = 0.2$ m^3/s

$V_A = 0.2/\frac{\pi}{4}(.2^2) = 6.369$ m/s, $V_B = 0.2/\frac{\pi}{4}(.5^2) = 1.018$ m/s

$$E_A = (p_A/\rho \times g) + \frac{V_A^2}{2g} + Z_A = (7.848 \times 10^4/1000 \times 9.81) + (6.369^2/2 \times 9.81) + 0 = 10.067 \text{ m}$$

$$E_B = (p_B/\rho \times g) + \frac{V_B^2}{2g} + Z_B = (5.886 \times 10^4/1000 \times 9.81) + (1.018^2/2 \times 9.81) + 3 = 9.052 \text{ m}]$$

41. A venturimeter of inlet diameter 300 mm and throat diameter 150 mm is fixed in a vertical pipe line. A liquid of sp. gr. 0.8 is flowing upward through the pipe line. A differential manometer containing mercury gives a reading of 100 mm when connected at inlet and throat. The vertical difference between inlet and throat is 500 mm. If $C_d = 0.98$, then find : (*i*) rate of flow of liquid in litre per second and (*ii*) difference of pressure between inlet and throat in N/m^2. [**Ans.** (*i*) 100 litre/s, (*ii*) 15980 N/m^2]

42. A venturimeter with a throat diameter of 7.5 cm is installed in a 15 cm diameter pipe. The pressure at the entrance to the meter is 70 kPa (gauge) and it is desired that the pressure at any point should not fall below 2.5 m of absolute water. Determine the maximum flow rate of water through the meter. Take $C_d = 0.97$ and atmospheric pressure as 100 kPa. (*J.N.T.U., Hyderabad S 2002*)

[**Hint.** The pressure at the throat will be minimum. Hence $\frac{p_2}{\rho g} = 2.5$ m (abs.)

Given : $d_1 = 15$ cm $\therefore A_1 = \frac{\pi}{4}(15^2) = 176.7$ cm^2

$d_2 = 7.5$ cm $\therefore A_2 = \frac{\pi}{4}(7.5^2) = 44.175$ cm^2

$p_1 = 70$ kPa $= 70 \times 10^3$ N/m^2 (gauge), $p_{atm} = 100$ kPa $= 100 \times 10^3$ N/m^2

$\therefore$ p_1 (abs.) $= 70 \times 10^3 + 100 \times 10^3 = 170 \times 10^3$ N/m^2 (abs.)

$$\therefore \quad \frac{p_1}{\rho g} = \frac{170 \times 10^3}{1000 \times 9.81} = 17.33 \text{ m of water (abs.)}$$

$$\therefore \quad h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 17.33 - 2.5 = 14.83 \text{ m of water} = 1483 \text{ cm of water}$$

Now
$$Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} = \frac{0.97 \times 176.7 \times 44.175 \times \sqrt{2 \times 981 \times 1483}}{\sqrt{176.7^2 - 44 \cdot 175^2}} = 75488 \text{ cm}^3\text{/s}$$

= **75.488 litre/s.**]

43. Find the discharge of water flowing through a pipe 20 cm diameter placed in an inclined position, where a venturimeter is inserted, having a throat diameter of 10 cm. The difference of pressure between the main and throat is measured by a liquid of specific gravity 0.4 in an inverted *U*-tube, which gives a reading of 30 cm. The loss of head between the main and throat is 0.2 times the kinetic head of pipe.

(*Delhi University, Dec. 2002*)

[**Hint.** Given : $d_1 = 20$ cm $\therefore a_1 = \frac{\pi}{4}(20^2) = 100\,\pi$ cm^2 ; $d_2 = 10$ cm $\therefore a_2 = \frac{\pi}{4}(10^2) = 25\,\pi$ cm^2.

$$x = 30 \text{ cm}, h = x\left(1 - \frac{S_l}{S_o}\right) = 30\left(1 - \frac{0.4}{1.0}\right) = 18 \text{ cm} = 0.18 \text{ m}$$

But h is also
$$= \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) \therefore \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 18 \text{ cm} = 0.18 \text{ m}$$

$$h_L = 0.2 \times \frac{V_1^2}{2g}$$

From Bernoulli's equation, $\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$

or $\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = h_L$

or $0.18 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{0.2\, V_1^2}{2g}$ $\left(\because \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 0.18 \text{ m and } h_L = \frac{0.2\, V_1^2}{2g}\right)$

or $0.18 + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{0.2\, V_1^2}{2g} = 0$ or $0.18 + \frac{0.8\, V_1^2}{2g} - \frac{V_2^2}{2g} = 0$

From continuity equation, $a_1 V_1 = a_2 V_2$ or $V_2 = \frac{a_1 V_1}{a_2} = \frac{\frac{\pi}{4}(20^2)\, V_1}{\frac{\pi}{4}(10^2)} = 4V_1$

Now $0.18 + \frac{0.8\, V_1^2}{2g} - \frac{V_2^2}{2g} = 0$ or $0.18 + \frac{0.8\, V_1^2}{2g} - \frac{(4V_1)^2}{2g} = 0$

or $0.18 + \frac{0.8\, V_1^2}{2g} - \frac{16V_1^2}{2g} = 0$ or $0.18 = \frac{16V_1^2}{2g} - \frac{0.8\, V_1^2}{2g} = \frac{15.2V_1^2}{2g}$

$\therefore$ $V_1 = \sqrt{\frac{0.18 \times 2 \times 9.81}{15.2}} = 0.48 \text{ m/s} = 48 \text{ cm/s}$

$\therefore$ $Q = A_1 V_1 = \frac{\pi}{4}(20^2) \times 48 = 15140 \text{ cm}^3\text{/s} =$ **15.14 litre/s.]**