16

CHAPTER

FLOW IN OPEN CHANNELS

▶ 16.1 INTRODUCTION

Flow in open channels is defined as the flow of a liquid with a free surface. A free surface is a surface having constant pressure such as atmospheric pressure. Thus a liquid flowing at atmospheric pressure through a passage is known as flow in open channels. In most of cases, the liquid is taken as water. Hence flow of water through a passage under atmospheric pressure is called flow in open channels. The flow of water through pipes at atmospheric pressure or when the level of water in the pipe is below the top of the pipe, is also classified as open channel flow.

In case of open channel flow, as the pressure is atmospheric, the flow takes place under the force of gravity which means the flow takes place due to the slope of the bed of the channel only. The hydraulic gradient line coincides with the free surface of water.

▶ 16.2 CLASSIFICATION OF FLOW IN CHANNELS

The flow in open channel is classified into the following types :

1. Steady flow and unsteady flow,
2. Uniform flow and non-uniform flow,
3. Laminar flow and turbulent flow, and
4. Sub-critical, critical and super critical flow.

16.2.1 Steady Flow and Unsteady Flow. If the flow characteristics such as depth of flow, velocity of flow, rate of flow at any point in open channel flow do not change with respect to time, the flow is said to be steady flow. Mathematically, steady flow is expressed as

$$\frac{\partial V}{\partial t} = 0, \frac{\partial Q}{\partial t} = 0 \quad \text{or} \quad \frac{\partial y}{\partial t} = 0 \qquad \text{...(16.1)}$$

where V = velocity, Q = rate of flow and y = depth of flow.

If at any point in open channel flow, the velocity of flow, depth of flow or rate of flow changes with respect to time, the flow is said to be unsteady flow. Mathematically, unsteady flow means

$$\frac{\partial V}{\partial t} \neq 0 \quad \text{or} \quad \frac{\partial y}{\partial t} \neq 0 \quad \text{or} \quad \frac{\partial Q}{\partial t} \neq 0.$$

16.2.2 Uniform Flow and Non-uniform Flow. If for a given length of the channel, the velocity of flow, depth of flow, slope of the channel and cross-section remain constant, the flow is said to be uniform. On the other hand, if for a given length of the channel, the velocity of flow, depth of flow etc., do not remain constant, the flow is said to be non-uniform flow. Mathematically, uniform and non-uniform flow are written as :

$$\frac{\partial y}{\partial S} = 0, \frac{\partial V}{\partial S} = 0 \text{ for uniform flow}$$

and $$\frac{\partial y}{\partial S} \neq 0, \frac{\partial V}{\partial S} \neq 0 \text{ for non-uniform flow.}$$

Non-uniform flow in open channels is also called varied flow, which is classified in the following two types as :

(*i*) Rapidly Varied Flow (R.V.F.), and

(*ii*) Gradually Varied Flow (G.V.F.).

(*i*) **Rapidly Varied Flow (R.V.F.).** Rapidly varied flow is defined as that flow in which depth of flow changes abruptly over a small length of the channel. As shown in Fig. 16.1 when there is any obstruction in the path of flow of water, the level of water rises above the obstruction and then falls and again rises over a small length of channel. Thus the depth of flow changes rapidly over a short length of the channel. For this short length of the channel the flow is called rapidly varied flow (R.V.F.).

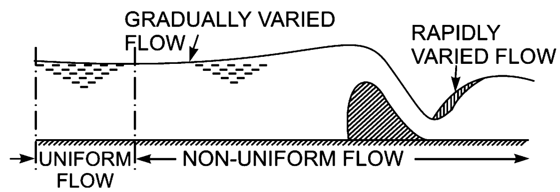

Fig. 16.1 *Uniform and non-uniform flow.*

(*ii*) **Gradually Varied Flow (G.V.F.).** If the depth of flow in a channel changes gradually over a long length of the channel, the flow is said to be gradually varied flow and is denoted by G.V.F.

16.2.3 Laminar Flow and Turbulent Flow.

The flow in open channel is said to be laminar if the Reynold number (R_e) is less than 500 or 600. Reynold number in case of open channels is defined as :

$$R_e = \frac{\rho VR}{\mu} \quad \text{...(16.2)}$$

where V = Mean velocity of flow of water

R = Hydraulic radius or Hydraulic mean depth

$$= \frac{\text{Cross-section area of flow normal to the direction of flow}}{\text{Wetted perimeter}}$$

ρ and μ = Density and viscosity of water.

If the Reynold number is more than 2000, the flow is said to be turbulent in open channel flow. If R_e lies between 500 to 2000, the flow is considered to be in transition state.

16.2.4 Sub-critical, Critical and Super Critical Flow.

The flow in open channel is said to be sub-critical if the Froude number (F_e) is less than 1.0. The Froude number is defined as :

$$F_e = \frac{V}{\sqrt{gD}} \quad \text{...(16.3)}$$

where V = Mean velocity of flow

D = Hydraulic depth of channel and is equal to the ratio of wetted area to the top width of channel

$= \dfrac{A}{T}$, where T = Top width of channel.

Sub-critical flow is also called tranquil or streaming flow. For sub-critical flow, $F_e < 1.0$.

The flow is called critical if $F_e = 1.0$. And if $F_e > 1.0$, the flow is called super critical or shooting or rapid or torrential.

▶ 16.3 DISCHARGE THROUGH OPEN CHANNEL BY CHEZY'S FORMULA

Consider uniform flow of water in a channel as shown in Fig. 16.2. As the flow is uniform, it means the velocity, depth of flow and area of flow will be constant for a given length of the channel. Consider sections 1-1 and 2-2.

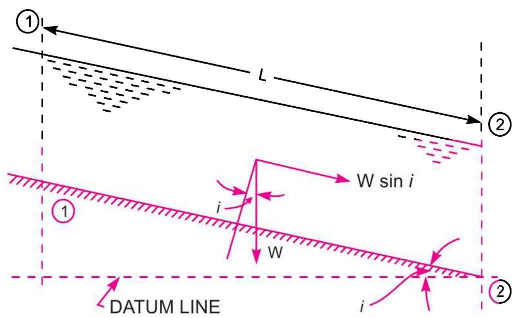

Fig. 16.2 *Uniform flow in open channel.*

Let L = Length of channel,

A = Area of flow of water,

i = Slope of the bed,

V = Mean velocity of flow of water,

P = Wetted perimeter of the cross-section,

f = Frictional resistance per unit velocity per unit area.

The weight of water between sections 1-1 and 2-2.

W = Specific weight of water × volume of water

$= w \times A \times L$

Component of W along direction of flow $= W \times \sin i = wAL \sin i$ ...(*i*)

Frictional resistance against motion of water $= f \times$ surface area $\times$ (velocity $)^n$

The value of n is found experimentally equal to 2 and surface area $= P \times L$

$\therefore$ Frictional resistance against motion $= f \times P \times L \times V^2$ ...(*ii*)

The forces acting on the water between sections 1-1 and 2-2 are:

1. Component of weight of water along the direction of flow,
2. Friction resistance against flow of water,
3. Pressure force at section 1-1,
4. Pressure force at section 2-2.

As the depths of water at the sections 1-1 and 2-2 are the same, the pressure forces on these two sections are same and acting in the opposite direction. Hence they cancel each other. In case of uniform flow, the velocity of flow is constant for the given length of the channel. Hence there is no acceleration acting on the water. Hence the resultant force acting in the direction of flow must be zero.

∴ Resolving all forces in the direction of flow, we get

$$wAL \sin i - f \times P \times L \times V^2 = 0$$

or
$$wAL \sin i = f \times P \times L \times V^2$$

$$V^2 = \frac{wAL \sin i}{f \times P \times L} = \frac{w}{f} \times \frac{A}{P} \times \sin i$$

or
$$V = \sqrt{\frac{w}{f}} \times \sqrt{\frac{A}{P} \times \sin i} \quad \text{...(iii)}$$

But
$$\frac{A}{P} = m$$

= hydraulic mean depth or hydraulic radius,

$$\sqrt{\frac{w}{f}} = C = \text{Chezy's constant}$$

Substituting the values of $\frac{A}{P}$ and $\sqrt{\frac{w}{f}}$ in equation (*iii*), $V = C\sqrt{m \sin i}$

For small values of i, $\sin i \simeq \tan i \approx i \quad \therefore \quad V = C\sqrt{mi}$ ...(16.4)

∴ Discharge, $Q = \text{Area} \times \text{Velocity} = A \times V$

$$= A \times C\sqrt{mi} \quad \text{...(16.5)}$$

Problem 16.1 *Find the velocity of flow and rate of flow of water through a rectangular channel of 6 m wide and 3 m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take Chezy's constant C = 55.*

Solution. Given :

Width of rectangular channel, $b = 6$ m

Depth of channel, $d = 3$ m

∴ Area, $A = 6 \times 3 = 18 \text{ m}^2$

Bed slope, $i = 1 \text{ in } 2000 = \frac{1}{2000}$

Chezy's constant, $C = 55$

Perimeter, $P = b + 2d = 6 + 2 \times 3 = 12$ m

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{18}{12} = 1.5$ m

Velocity of flow is given by equation (16.4) as,

$$V = C\sqrt{mi} = 55\sqrt{1.5 \times \frac{1}{2000}} = \mathbf{1.506 \text{ m/s. Ans.}}$$

Rate of flow, $Q = V \times \text{Area} = V \times A = 1.506 \times 18 =$ **27.108 m³/s. Ans.**

Problem 16.2 *Find the slope of the bed of a rectangular channel of width 5 m when depth of water is 2 m and rate of flow is given as 20 m³/s. Take Chezy's constant, C = 50.*

Solution. Given :

Width of channel, $b = 5$ m

Depth of water, $d = 2$ m

Rate of flow, $Q = 20 \text{ m}^3/\text{s}$

Chezy's constant $C = 50$

Let the bed slope $= i$

Using equation (16.5), we have $Q = AC\sqrt{mi}$

where A = Area = $b \times d = 5 \times 2 = 10$ m^2

$$m = \frac{A}{P} = \frac{10}{b+2d} = \frac{10}{5+2\times 2} = \frac{10}{5+4} = \frac{10}{9} \text{ m}$$

$$\therefore \quad 20.0 = 10 \times 50 \times \sqrt{\frac{10}{9}\times i} \quad \text{or} \quad \sqrt{\frac{10}{9}}\, i = \frac{20.0}{500} = \frac{2}{50}$$

Squaring both sides, we have $\frac{10}{9} i = \frac{4}{2500}$

$$\therefore \quad i = \frac{4}{2500} \times \frac{9}{10} = \frac{36}{25000} = \frac{1}{\frac{25000}{36}} = \frac{\mathbf{1}}{\mathbf{694.44}}. \textbf{ Ans.}$$

∴ Bed slope is 1 in 694.44.

Problem 16.3 *A flow of water of 100 litres per second flows down in a rectangular flume of width 600 mm and having adjustable bottom slope. If Chezy's constant C is 56, find the bottom slope necessary for uniform flow with a depth of flow of 300 mm. Also find the conveyance K of the flume.*

Solution. Given :

Discharge, $Q = 100 \text{ litres/s} = \frac{100}{1000} = 0.10 \text{ m}^3\text{/s}$

Width of channel, $b = 600 \text{ mm} = 0.60 \text{ m}$

Depth of flow, $d = 300 \text{ mm} = 0.30 \text{ m}$

∴ Area of flow, $A = b \times d = 0.6 \times 0.3 = 0.18 \text{ m}^2$

Chezy's constant, $C = 56$

Let the slope of bed $= i$

Hydraulic mean depth, $m = \frac{A}{P} = \frac{0.18}{b+2d} = \frac{0.18}{0.6+2\times 0.30} = \frac{0.18}{1.2} = 0.15 \text{ m}$

Using equation (16.5), we have $Q = AC\sqrt{mi}$

or $$0.10 = 0.18 \times 56 \times \sqrt{0.15\times i} \quad \text{or} \quad \sqrt{0.15 i} = \frac{0.10}{0.18\times 56}$$

Squaring both sides, we have $0.15\, i = \left(\frac{0.10}{0.18\times 56}\right)^2 = .000098418$

$$\therefore \quad i = \frac{.000098418}{0.15} = .0006512 = \frac{1}{\frac{1}{.0006512}} = \frac{\mathbf{1}}{\mathbf{1524}}\textbf{ . Ans.}$$

∴ Slope of the bed is 1 in 1524.

Conveyance K of the channel

Equation (16.5) is given as $Q = AC\sqrt{mi}$

which can be written as $Q = K\sqrt{i}$

where $K = AC\sqrt{m}$ and K is called conveyance of the channel section.

$\therefore$ $K = AC\sqrt{m} = 0.18 \times 56 \times \sqrt{0.15} =$ **3.9039 m³/s. Ans.**

Problem 16.4 *Find the discharge through a trapezoidal channel of width 8 m and side slope of 1 horizontal to 3 vertical. The depth of flow of water is 2.4 m and value of Chezy's constant, C = 50. The slope of the bed of the channel is given 1 in 4000.*

Solution. Given :

Width, $b = 8$ m

Side slope = 1 hor. to 3 vertical

Depth, $d = 2.4$ m

Chezy's constant, $C = 50$

Bed slope, $i = \frac{1}{4000}$

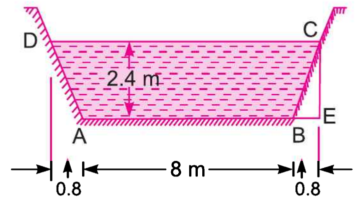

Fig. 16.3

From Fig. 16.3 when depth , $CE = 2.4$,

the horizontal distance $BE = 2.4 \times \frac{1}{3} = 0.8$ m

$\therefore$ Top width of the channel,

$$CD = AB + 2 \times BE = 8.0 + 2 \times 0.8 = 9.6 \text{ m}$$

$\therefore$ Area of trapezoidal channel, $ABCD$ is given as,

$$A = (AB + CD) \times \frac{CE}{2} = (8 + 9.6) \times \frac{2.4}{2} = 17.6 \times 1.2 = 21.12 \text{ m}^2$$

Wetted perimeter , $P = AB + BC + AD = AB + 2BC$ $\quad (\because \; BC = AD)$

But $BC = \sqrt{BE^2 + CE^2} = \sqrt{(0.8)^2 + (2.4)^2} = 2.529$ m

$\therefore$ $P = 8.0 + 2 \times 2.529 = 13.058$ m

Hydraulic mean depth, $m = \frac{A}{P} = \frac{12.12}{13.058} = 1.617$ m

The discharge, Q is given by equation (16.5) as

$$Q = AC\sqrt{mi} = 21.12 \times 50\sqrt{1.617 \times \frac{1}{4000}} = \textbf{21.23 m}^3\textbf{/s. Ans.}$$

Problem 16.5 *Find the bed slope of trapezoidal channel of bed width 6 m, depth of water 3 m and side slope of 3 horizontal to 4 vertical, when the discharge through the channel is 30 m³/s. Take Chezy's constant, C = 70.*

Solution. Given :

Bed width, $b = 6.0$ m

Depth of flow, $d = 3.0$ m

Side slope = 3 horizontal to 4 vertical

Discharge, $Q = 30 \text{ m}^3/\text{s}$

Chezy's constant, $C = 70$

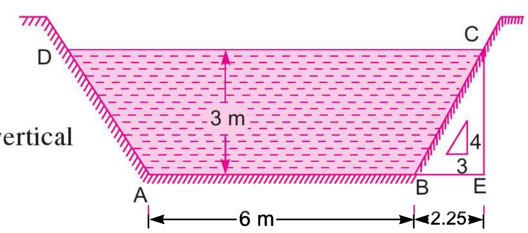

Fig. 16.4

From Fig. 16.4, for depth of flow

$= 3 \text{ m} = CE$

Distance, $BE = 3 \times \frac{3}{4} = \frac{9}{4} = 2.25$ m

∴ Top width, $CD = AB + 2 \times BE = 6.0 + 2 \times 2.25 = 10.50$ m

Wetted perimeter, $P = AD + AB + BC = AP + 2BC$ $(\because BC = AD)$

$$= AB + 2\sqrt{BE^2 + CE^2} = 6.0 + 2\sqrt{(2.25)^2 + (3)^2} = 13.5 \text{ m}$$

Area of flow, A = Area of trapezoidal $ABCD$

$$= \frac{(AB + CD) \times CE}{2} = \frac{(6 + 10.50)}{2} \times 3.0 = 24.75 \text{ m}^2$$

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{24.75}{13.50} = 1.833$

Using equation (16.5), $Q = AC\sqrt{mi}$

or $$30.0 = 24.75 \times 70 \times \sqrt{1.833 \times i} = 2345.6\sqrt{i}$$

$$i = \left(\frac{30}{2345.6}\right)^2 = \frac{1}{\left(\frac{2345.6}{30}\right)^2} = \frac{1}{6133}. \text{ Ans.}$$

Problem 16.6 *Find the discharge of water through the channel shown in Fig. 16.5. Take the value of Chezy's constant = 60 and slope of the bed as 1 in 2000.*

Solution. Given :

Chezy's constant, $C = 60$

Bed slope, $i = \frac{1}{2000}$

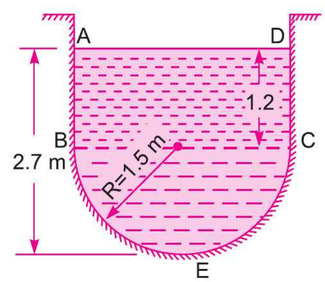

Fig. 16.5

From Fig.16.5, Area, A = Area $ABCD$ + Area BEC

$$= (1.2 \times 3.0) + \frac{\pi R^2}{2}$$

$$= 3.6 + \frac{(1.5)^2}{2} = 7.134 \text{ m}^2$$

Wetted perimeter, $P = AB + BEC + CD$

$$= 1.2 + \pi R + 1.2 = 1.2 + \pi \times 1.5 + 1.2 = 7.1124 \text{ m}$$

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{7.134}{7.1124} = 1.003$

The discharge, Q is given by equation (16.5) as

$$Q = AC\sqrt{mi}$$

$$= 7.134 \times 60 \times \sqrt{1.003 \times \frac{1}{2000}} = \mathbf{9.585 \text{ m}^3\text{/s. Ans.}}$$

Problem 16.7 *Find the rate of flow of water through a V-shaped channel as shown in Fig. 16.6. Take the value of C = 55 and slope of the bed 1 in 2000.*

Solution. Given :

Chezy's constant, $C = 55$

Bed slope, $i = \frac{1}{1000}$

Depth of flow, $d = 4.0$ m

Angle made by each side with vertical,

i.e., $\angle ABD = \angle CBD = 30°$

From Fig. 16.6, we have

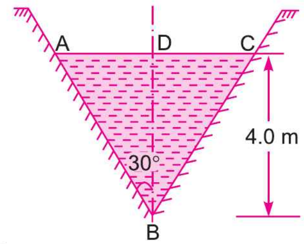

Fig. 16.6

Area, A = Area of ABC

$$= 2 \times \text{Area } ABD = \frac{2 \times AD \times BD}{2} = AD \times BD$$

$$= BD \tan 30° \times BD \qquad \left(\because \tan 30° = \frac{AD}{BD}, AD = BD \tan 30°\right)$$

$$= 4 \tan 30° \times 4 = 9.2376 \text{ m}^2$$

Wetted perimeter, $P = AB + BC = 2AB$ $(\because AB = BC)$

$$= 2\sqrt{BD^2 + AD^2} = 2\sqrt{4.0^2 + (4 \tan 30°)^2}$$

$$= 2\sqrt{16.0 + 5.333} = 9.2375 \text{ m.}$$

$\therefore$ Hydraulic mean depth, $m = \dfrac{A}{P} = \dfrac{9.2376}{9.2375} = 1.0$ m

Using equation (16.5) for discharge,

$$Q = AC\sqrt{mi} = 9.2376 \times 55 \times \sqrt{1 \times \frac{1}{1000}} = \mathbf{16.066 \text{ m}^3/\text{s. Ans.}}$$

▶ 16.4 EMPIRICAL FORMULAE FOR THE VALUE OF CHEZY'S CONSTANT

Equation (16.4) is known Chezy's formula after the name of a French Engineer, Antoine Chezy who developed this formula in 1975. In this equation C is known as Chezy's constant, which is not a dimensionless co-efficient. The dimension of C is

$$= \frac{V}{\sqrt{mi}} = \frac{L/T}{\sqrt{\frac{A}{P}i}} = \frac{L/T}{\sqrt{\frac{L^2}{L}i}} = \frac{L}{T\sqrt{Li}} = \frac{\sqrt{L}}{T}$$

$$= L^{1/2}T^{-1} \qquad \{ i \text{ is dimensionless}\}$$

Hence the value of C depends upon the system of units. The following are the empirical formulae, after the name of their inventors, used to determine the value of C:

1. Bazin formula (In MKS units) : $C = \dfrac{157.6}{1.81 + \dfrac{K}{\sqrt{m}}}$ ...(16.6)

where K = Bazin's constant and depends upon the roughness of the surface of channel, whose values are given in Table 16.1.

m = Hydraulic mean depth or hydraulic radius.

2. Ganguillet-Kutter Formula. The value of C is given in MKS unit as

$$C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{i}\right)\frac{N}{\sqrt{m}}} \qquad ...(16.7)$$

where N = Roughness co-efficient which is known as Kutter's constant, whose value for different surfaces are given in Table 16.2

i = Slope of the bed

m = Hydraulic mean depth.

Table 16.1 Values of K in the Bazin's Formula

S. No.	*Nature of Channel inside surface*	*Value of K*
1.	Smooth cemented or planned wood	0.11
2.	Brick or concrete or unplanned wood	0.21
3.	Rubble masonary or Ashlar or poor brick work	0.83
4.	Earthen channel of very good surface	1.54
5.	Earthen channel of ordinary surface	2.36
6.	Earthen channel of rough surface	3.17

Table 16.2 Value of N in the Ganguillet-Kutter Formula

S. No.	*Nature of Channel inside surface*	*Value of N*
1.	Very smooth surface of glass, plastic or brass	0.010
2.	Smooth surface of concrete	0.012
3.	Rubble masonary or poor brick work	0.017
4.	Earthen channels neatly excavated	0.018
5.	Earthen channels of ordinary surface	0.027
6.	Earthen channels of rough surface	0.030
7.	Natural streams, clean and straight	0.030
8.	Natural streams with weeds, duppools etc.	0.075 to .15

3. Manning's Formula. The value of C according to this formula is given as

$$C = \frac{1}{N} m^{1/6} \qquad ...(16.8)$$

where m = Hydraulic mean depth

N = Manning's constant which is having same value as Kutter's constant for the normal range of slope and hydraulic mean depth. The values of N are given in Table 16.2.

Problem 16.8 *Find the discharge through a rectangular channel 2.5 m wide, having depth of water 1.5 m and bed slope as 1 in 2000. Take the value of k = 2.36 in Bazin's formula.*

Solution. Given :

Width of channel, $b = 2.5$ m

Depth of flow, $d = 1.5$ m

$\therefore$ Area, $A = b \times d = 2.5 \times 1.5 = 3.75 \text{ m}^2$

Wetted Perimeter, $P = d + b + d = 1.5 + 2.5 + 1.5 = 5.5$ m

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{3.75}{5.50} = 0.682$

Bed slope, $i = \frac{1}{2000}$

Bazin's constant, $K = 2.36$

Using Bazin's formula given by equation (16.6), as

$$C = \frac{157.6}{1.81 + \frac{K}{\sqrt{m}}} = \frac{157.6}{1.81 + \frac{2.36}{\sqrt{0.682}}} = 33.76$$

Discharge, Q is given by equation (16.5), as

$$Q = AC\sqrt{mi}$$

$$= 3.75 \times 33.76 \times \sqrt{0.682 \times \frac{1}{2000}} = \mathbf{2.337\ m^3/s.\ Ans.}$$

Problem 16.9 *Find the discharge through a rectangular channel 14 m wide, having depth of water 3 m and bed slope 1 in 1500. Take the value of N = 0.03 in the Kutter's formula.*

Solution. Given :

Width of channel, $b = 4$ m

Depth of water, $d = 3$ m

Bed slope, $i = \frac{1}{1500} = 0.000667$

Kutter's constant, $N = 0.03$

Area of flow, $A = b \times d = 4 \times 3 = 12\ m^2$

Wetted perimeter, $P = d + b + d = 3 + 4 + 3 = 10$ m

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{.12}{10} = 1.2$ m

Using Kutter's formula given by equation (16.7), as

$$C = \frac{23 + \frac{.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{.00155}{i}\right) \times \frac{N}{\sqrt{m}}} = \frac{23 + \frac{.00155}{.000667} + \frac{1}{.03}}{1 + \left(23 + \frac{.00155}{.000667}\right) \times \frac{.03}{\sqrt{1.20}}}$$

$$= \frac{23 + 2.3238 + 33.33}{1 + (23 + 2.3238) \times .03286} = \frac{58.633}{1.832} = 32.01$$

Discharge, Q is given by equation (16.5), as

$$Q = AC\sqrt{mi} = 12 \times 32.01 \times \sqrt{12 \times .000667} = \mathbf{10.867\ m^3/s.\ Ans.}$$

Problem 16.10 *Find the discharge through a rectangular channel of width 2 m, having a bed slope of 4 in 8000. The depth of flow is 1.5 m and take the value of N in Manning's formula as 0.012.*

Solution. Given :

Width of the channel, $b = 2$ m

Depth of the flow, $d = 1.5$ m

$\therefore$ Area of flow, $A = b \times d = 2 \times 1.5 = 3.0\ m^2$

Wetted perimeter, $P = b + d + d = 2 + 1.5 + 1.5 = 5.0$ m

$\therefore$ Hydraulic mean depth, $m = \dfrac{A}{P} = \dfrac{3.0}{5.0} = 0.6$

Bed slope, $i = 4 \text{ in } 8000 = \dfrac{4}{8000} = \dfrac{1}{2000}$

Value of N $= 0.012$

Using Manning's formula, given by equation (16.8), as

$$C = \frac{1}{N} m^{1/6} = \frac{1}{0.012} \times 0.6^{1/6} = 76.54$$

Discharge, Q is given by equation (16.5), as

$$Q = AC\sqrt{mi}$$

$$= 3.0 \times 76.54 \sqrt{0.6 \times \frac{1}{2000}} \text{ m}^2\text{/s} = \mathbf{3.977 \text{ m}^3\text{/s. Ans.}}$$

Problem 16.11 *Find the bed slope of trapezoidal channel of bed width 4 m, depth of water 3 m and side slope of 2 horizontal to 3 vertical, when the discharge through the channel is 20* m^3/s.

Take Manning's $N = 0.03$ *in Manning's formula* $C = \dfrac{1}{N} m^{1/6}$.

Solution. Given :

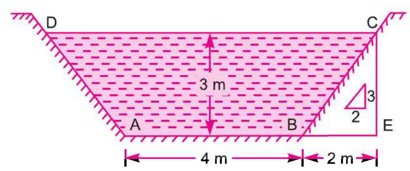

Fig. 16.7

Bed width, $b = 4$ cm

Depth of flow, $d = 3$ m

Side slope $= 2$ hor. to 3 vert.

Discharge, $Q = 20.0 \text{ m}^3\text{/s}$

Manning's, $N = 0.03$

From Fig. 16.7, we have

Distance, $BE = d \times \dfrac{2}{3} = 3 \times \dfrac{2}{3} = 2$ m

$\therefore$ Top width, $CD = AB + 2BE$

$= 4 + 2 \times 2 = 8.0$ m

$\therefore$ Area of flow, A = Area of trapezoidal section $ABCD$

$$= \frac{(AB + CD)}{2} \times d = \frac{(4 + 8)}{2} \times 3 = 18 \text{ m}^2$$

Wetted perimeter, $P = AD + AB + BC = AB + 2BC$ $(\because \; AD = BC)$

$$= 4.0 + 2\sqrt{BE^2 + EC^2} = 4.0 + 2\sqrt{2^2 + 3^2} = 4.0 + 2 \times \sqrt{13} = 11.21 \text{ m}$$

$\therefore$ Hydraulic mean depth, $m = \dfrac{A}{P} = \dfrac{18}{11.21} = 1.6057$

Using Manning's formula, $C = \dfrac{1}{N} m^{1/6} = \dfrac{1}{0.03} \times (1.6057)^{1/6} = 36.07$

Using equation (16.5) for discharge,

$$Q = AC\sqrt{mi} = 18 \times 36.07 \times \sqrt{1.6057 \times i} \quad \text{or } 20.0 = 822.71\sqrt{i}$$

$$\therefore \quad i = \left(\frac{20.0}{822.71}\right)^2 = 0.0005909 = \frac{\mathbf{1}}{\mathbf{1692}}. \textbf{ Ans.}$$

Problem 16.12 *Find the diameter of a circular sewer pipe which is laid at a slope of 1 in 8000 and carries a discharge of 800 litres/s when flowing half full. Take the value of Manning's N = 0.020.*

Solution. Given :

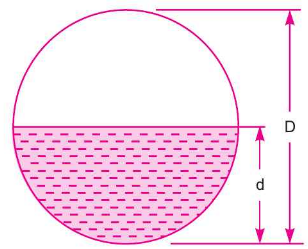

Fig. 16.8

Slope of pipe, $i = \frac{1}{8000}$

Discharge, $Q = 800$ litres/s $= 0.8$ m^3/s

Manning's, $N = 0.020$

Let the dia. of sewer pipe, $= D$

Depth of flow, $d = \frac{D}{2}$

$\therefore$ Area of flow, $A = \frac{\pi}{4}D^2 \times \frac{1}{2} = \frac{\pi D^2}{8}$

Wetted perimeter, $P = \frac{\pi D}{2}$

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{\frac{\pi D^2}{8}}{\pi D / 2} = \frac{D}{4}$

Using Manning's formula given by equation (16.8), $C = \frac{1}{N} m^{1/6}$

The discharge, Q through pipe is given by equation (16.6), as

$$Q = AC\sqrt{mi}$$

$$= \frac{\pi D^2}{8} \times \frac{1}{N} m^{1/6}\sqrt{mi}$$

or

$$0.80 = \frac{\pi}{8}D^2 \times \frac{1}{.020} \times m^{1/6} \times m^{1/2} \times \sqrt{i}$$

$$= \frac{\pi}{8}D^2 \times \frac{1}{.020} m^{(1/6 + 1/2)} \times \sqrt{\frac{1}{8000}} = \frac{\pi}{8}D^2 \times \frac{1}{.020} \times m^{2/3} \times 0.01118$$

$$= 0.2195 \times D^2 \times \left(\frac{D}{4}\right)^{2/3} \qquad \left(\because \; m = \frac{D}{4}\right)$$

$$= \frac{.2195}{4^{2/3}} \times D^2 \times D^{2/3} = 0.0871\ D^{8/3}$$

or

$$D^{8/3} = \frac{0.80}{.0871} = 9.1848$$

$$\therefore \quad D = (9.1848)^{3/8} = (9.1848)^{0.375} = \textbf{2.296 m. Ans.}$$

▶ 16.5 MOST ECONOMICAL SECTION OF CHANNELS

A section of a channel is said to be most economical when the cost of construction of the channel is minimum. But the cost of construction of a channel depends upon the excavation and the lining. To keep the cost down or minimum, the wetted perimeter, for a given discharge, should be minimum. This condition is utilized for determining the dimensions of a economical sections of different form of channels.

Most economical section is also called the best section or most efficient section as the discharge, passing through a most economical section of channel for a given cross-sectional area (A), slope of the bed (i) and a resistance co-efficient, is maximum. But the discharge, Q is given by equation (16.5) as

$$Q = AC\sqrt{mi} = AC\sqrt{\frac{A \times i}{P}} \qquad \left(\because \; m = \frac{A}{P}\right)$$

For a given A, i and resistance co-efficient C, the above equation is written as

$$Q = K\frac{1}{\sqrt{P}}, \quad \text{where } K = AC\sqrt{Ai} = \text{constant}$$

Hence the discharge, Q will be maximum, when the wetted perimeter P is minimum. This condition will be used for determining the best section of a channel *i.e.*, best dimensions of a channel for a given area.

The conditions to be most economical for the following shapes of the channels will be considered :

1. Rectangular Channel, 2. Trapezoidal Channel, and 3. Circular Channel.

16.5.1 Most Economical Rectangular Channel.

The condition for most economical section, is that for a given area, the perimeter should be minimum. Consider a rectangular channel as shown in Fig. 16.9

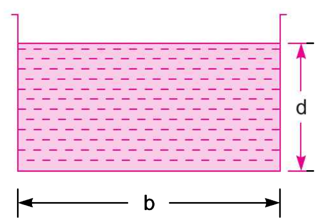

Fig. 16.9 *Rectangular channel.*

Let b = width of channel,

d = depth of the flow,

$\therefore$ Area of flow, $A = b \times d$ ...(*i*)

Wetted perimeter, $P = d + b + d = b + 2d$ ...(*ii*)

From equation (*i*), $b = \frac{A}{d}$

Substituting the value of b in (*ii*),

$$P = b + 2d = \frac{A}{d} + 2d \qquad \text{...(iii)}$$

For most economical section, P should be minimum for a given area.

or $$\frac{dP}{d(d)} = 0$$

Differentiating the equation (*iii*) with respect to d and equating the same to zero, we get

$$\frac{d}{d(d)}\left[\frac{A}{d} + 2d\right] = 0 \quad \text{or} \quad -\frac{A}{d^2} + 2 = 0 \quad \text{or} \quad A = 2d^2$$

But $A = b \times d$, $\therefore$ $b \times d = 2d^2$ or $b = 2d$ ...(16.9)

Now hydraulic mean depth, $m = \frac{A}{P} = \frac{b \times d}{b + 2d}$ ($\because$ $A = bd$, $P = b + 2d$)

$$= \frac{2d \times d}{2d + 2d} \qquad (\because \quad b = 2d)$$

$$= \frac{2d^2}{4d} = \frac{d}{2} \qquad ...(16.10)$$

From equations (16.9) and (16.10), it is clear that rectangular channel will be most economical when:

(*i*) Either $b = 2d$ means width is two times depth of flow.

(*ii*) Or $m = \frac{d}{2}$ means hydraulic depth is half the depth of flow.

Problem 16.13 *A rectangular channel of width, 4 m is having a bed slope of 1 in 1500. Find the maximum discharge through the channel. Take value of C = 50.*

Solution. Given :

Width of channel, $b = 4$ m

Bed slope, $i = \frac{1}{1500}$

Chezy's constant, $C = 50$

Discharge will be maximum, when the channel is most economical. The conditions for most economical rectangular channel are :

(*i*) $b = 2d$ or $d = \frac{b}{2} = \frac{4}{2} = 2.0$ m

(*ii*) $m = \frac{d}{2} = \frac{2}{2} = 1.0$ m

$\therefore$ Area of most economical rectangular channel, $A = b \times d = 4.0 \times 2.0 = 8$ m^2

Using equation (16.5) for discharge as

$$Q = AC\sqrt{mi} = 8.0 \times 50 \times \sqrt{1.0 \times \frac{1}{1500}} = \mathbf{10.328\ m^3/s.\ Ans.}$$

Problem 16.14 *A rectangular channel carries water at the rate of 400 litres/s when bed slope is 1 in 2000. Find the most economical dimensions of the channel if C = 50.*

Solution. Given :

Discharge, $Q = 400$ litres/s $= 0.4$ m^3/s

Bed slope, $i = \frac{1}{2000}$

Chezy's constant, $C = 50$

For the rectangular channel to be most economical,

(*i*) Width, $b = 2d$

(*ii*) Hydraulic mean depth, $m = \frac{d}{2}$

$\therefore$ Area of flow, $A = b \times d = 2d \times d = 2d^2$

Using equation (16.5) for discharge,

$$Q = AC\sqrt{mi}$$

or $$0.4 = 2d^2 \times 50 \times \sqrt{\frac{d}{2} \times \frac{1}{2000}} = 2 \times 50 \times \sqrt{\frac{1}{2 \times 2000}}\, d^{5/2} = 1.581\, d^{5/2}$$

$\therefore$ $$d^{5/2} = \frac{0.4}{1.581} = 0.253$$

$\therefore$ $$d = (.253)^{2/5} = \mathbf{0.577\ m.\ Ans.}$$

$$b = 2d = 2 \times .577 = \mathbf{1.154\ m.\ Ans.}$$

Problem 16.15 *A rectangular channel 4 m wide has depth of water 1.5 m. The slope of the bed of the channel is 1 in 1000 and value of Chezy's constant C = 55. It is desired to increase the discharge to a maximum by changing the dimensions of the section for constant area of cross-section, slope of the bed and roughness of the channel. Find the new dimensions of the channel and increase in discharge.*

Solution. Given :

Width of channel, $b = 4.0$ m

Depth of flow, $d = 1.5$ m

$\therefore$ Area of flow, $A = b \times d = 4 \times 1.5 = 6.0\ m^2$

Slope of bed, $i = \frac{1}{1000}$

Chezy's constant, $C = 55$

Wetted perimeter, $P = d + b + d = 1.5 + 4 + 1.5 = 7.0$ m

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{4.0}{7.0} = 0.857$

The discharge, Q is given by $$Q = AC\sqrt{mi} = 6.0 \times 55\sqrt{0.857 \times \frac{1}{1000}} = 9.66\ m^3/s \quad ...(i)$$

For maximum discharge for a given area, slope of bed and roughness we proceed as :

Let b' = new width of channel

d' = new depth of flow

Then, Area, $A = b' \times d'$, where A = constant = 6.0 m^2

$\therefore$ $$b' \times d' = 6.0 \quad ...(ii)$$

Also for maximum discharge $b' = 2d'$ ...(*iii*)

Substituting the value of b' in equation (*ii*), we have

$$2d' \times d' = 6.0 \text{ or } d'^2 = \frac{6.0}{2} = 3.0$$

$\therefore$ $$d' = \sqrt{3} = 1.732$$

Substituting the value of d' in (*iii*), we get

$$b' = 2 \times 1.732 = 3.464$$

$\therefore$ New dimensions of the channel are

Width, $b' =$ **3.464 m. Ans.**

Depth, $d' =$ **1.732 m. Ans.**

Wetted perimeter, $P' = d' + b' + d' = 1.732 + 3.464 + 1.732 = 6.928$

$\therefore$ Hydraulic mean depth, $m' = \frac{A}{P'} = \frac{6.0}{6.928} = 0.866$ m

(New hydraulic mean depth, m' corresponds to the condition of maximum discharge. And hence also equal to

$$\frac{d'}{2} = \frac{1.732}{2} = 0.866 \text{ m})$$

Max. discharge, Q', is given by $Q' = AC\sqrt{m'i} = 6.0 \times 55 \times \sqrt{0.866 \times \frac{1}{1000}} = 9.71 \text{ m}^3\text{/s}$ ...(*iv*)

$\therefore$ Increase in discharge $= Q' - Q = 9.71 - 9.66 =$ **0.05 m³/s. Ans.**

16.5.2 Most Economical Trapezoidal Channel. The trapezoidal section of a channel will be most economical, when its wetted perimeter is minimum. Consider a trapezoidal section of channel as shown in Fig. 16.10.

Let b = width of channel at bottom,
d = depth of flow,
θ = angle made by the sides with horizontal,

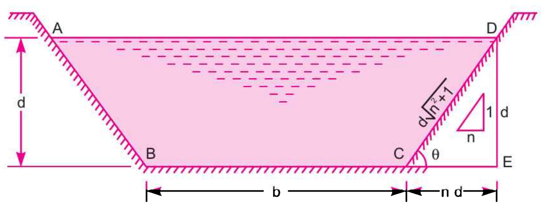

Fig. 16.10 *Trapezoidal section.*

(*i*) The side slope is given as 1 vertical to n horizontal.

$\therefore$ Area of flow, $$A = \frac{(BC + AD)}{2} \times d = \frac{b + (b + 2nd)}{2} \times d \qquad (\because \; AD = b + 2nd)$$

$$= \frac{2b + 2nd}{2} \times d = (b + nd) \times d \qquad \text{...}(i)$$

$$\therefore \quad \frac{A}{d} = b + nd$$

$$\therefore \quad b = \frac{A}{d} - nd \qquad \text{...}(ii)$$

Now wetted perimeter, $P = AB + BC + CD = BC + 2CD \qquad (\because \; AB = CD)$

$$= b + 2\sqrt{CE^2 + DE^2} = b + 2\sqrt{n^2d^2 + d^2} = b + 2d\sqrt{n^2 + 1} \qquad \text{...}(iia)$$

Substituting the value of b from equation (*ii*), we get

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2 + 1} \qquad \text{...}(iii)$$

For most economical section, P should be minimum or $\frac{dP}{d(d)} = 0$

$\therefore$ Differentiating equation (*iii*) with respect to d and equating it equal to zero, we get

$$\frac{d}{d(d)}\left[\frac{A}{d} - nd + 2d\sqrt{n^2 + 1}\right] = 0$$

or $$-\frac{A}{d^2} - n + 2\sqrt{n^2 + 1} = 0 \qquad (\because \; n \text{ is constant})$$

or $$\frac{A}{d^2} + n = 2\sqrt{n^2+1}$$

Substituting the value of A from equation (i) in the above equation,

$$\frac{(b+nd)d}{d^2} + n = 2\sqrt{n^2+1} \quad \text{or} \quad \frac{b+nd}{d} + n = 2\sqrt{n^2+1}$$

or $$\frac{b+nd+nd}{d} = \frac{b+2nd}{d} = 2\sqrt{n^2+1} \quad \text{or} \quad \frac{b+2nd}{2} = d\sqrt{n^2+1} \qquad ...(16.11)$$

But from Fig. 16.10, $\frac{b+2nd}{2}$ = Half of top width

and $d\sqrt{n^2+1} = CD$ = one of the sloping side

Equation (16.11) is the required condition for a trapezoidal section to be most economical, which can be expressed as half of the top width must be equal to one of the sloping sides of the channel.

(ii) **Hydraulic mean depth**

Hydraulic mean depth, $m = \frac{A}{P}$

Value of A from (i), $A = (b + nd) \times d$

Value of P from (iia), $P = b + 2d\sqrt{n^2+1} = b + (b + 2nd)$ ($\because$ From equation (16.11) $b + 2nd = 2d\sqrt{n^2+1}$)

$$= 2b + 2nd = 2(b+ nd)$$

$\therefore$ Hydraulic mean depth, $$m = \frac{A}{P} = \frac{(b+nd)d}{2(b+nd)} = \frac{d}{2} \qquad ...(16.12)$$

Hence for a trapezoidal section to be most economical hydraulic mean depth must be equal to half the depth of flow,

(iii) The three sides of the trapezoidal section of most economical section are tangential to the semi-circle described on the water line. This is proved as :

Let Fig. 16.11 shows the trapezoidal channel of most economical section.

Let θ = angle made by the sloping side with horizontal, and

O = the centre of the top width, AD.

Draw OF perpendicular to the sloping side AB.

ΔOAF is a right-angled triangle and angle $OAF = \theta$

$\therefore$ $\sin\theta = \frac{OF}{OA}$ $\qquad \therefore$ $OF = AO \sin\theta \qquad ...(iv)$

In ΔAEB, $$\sin\theta = \frac{AE}{AB} = \frac{d}{\sqrt{d^2+n^2d^2}}$$

$$= \frac{d}{d\sqrt{1+n^2}} = \frac{1}{\sqrt{1+n^2}}$$

Substituting $\sin\theta = \frac{1}{\sqrt{1+n^2}}$ in equation (iv), we get

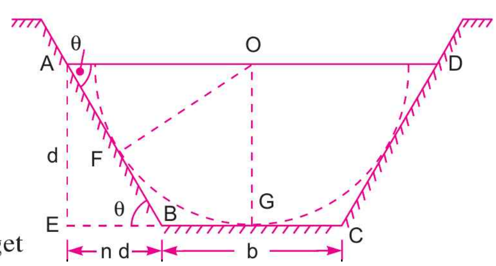

Fig. 16.11

$$OF = AO \times \frac{1}{\sqrt{1+n^2}} \qquad ...(v)$$

But AO = half of top width

$$= \frac{b+2nd}{2} = d\sqrt{n^2+1} \text{ from equation (16.11)}$$

Substituting this value of AO in equation (v),

$$OF = \frac{d\sqrt{n^2+1}}{\sqrt{n^2+1}} = d \text{ depth of flow} \qquad ...(16.13)$$

Thus, if a semi-circle is drawn with O as centre and radius equal to the depth of flow d, the three sides of most economical trapezoidal section will be tangential to the semi-circle.

Hence the conditions for the most economical trapezoidal section are:

1. $\frac{b+2nd}{2} = d\sqrt{n^2+1}$

2. $m = \frac{d}{2}$

3. A semi-circle drawn from O with radius equal to depth of flow will touch the three sides of the channel.

Problem 16.16 *A trapezoidal channel has side slopes of 1 horizontal to 2 vertical and the slope of the bed is 1 in 1500. The area of the section is 40 m². Find the dimensions of the section if it is most economical. Determine the discharge of the most economical section if C = 50.*

Solution. Given :

Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{1}{2}$

Bed slope, $i = \frac{1}{1500}$

Area of section, $A = 40 \text{ m}^2$

Chezy's constant, $C = 50$

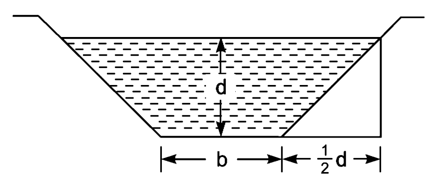

Fig. 16.12

For the most economical section, using equation (16.11)

$$\frac{b+2nd}{2} = d\sqrt{n^2+1} \quad \text{or} \quad \frac{b+2\times\frac{1}{2}\times d}{2} = d\sqrt{\left(\frac{1}{2}\right)^2+1}$$

or $$\frac{b+d}{2} = d\sqrt{\frac{1}{4}+1} = 1.118\,d$$

or $$b = 2\times 1.118d - d = 1.236\,d \qquad ...(i)$$

But area of trapezoidal section, $A = \frac{b+(b+2nd)}{2} \times d = (b+nd)\,d$

$$= (1.236\,d + \tfrac{1}{2}\,d)\,d \qquad (\because \; b = 1.236\,d \text{ and } n = \tfrac{1}{2})$$

$$= 1.736\,d^2$$

But $A = 40 \text{ m}^2$ (given)

$\therefore$ $40 = 1.736\,d^2$

$\therefore$ $$d = \sqrt{\frac{40}{1.736}} = \textbf{4.80 m. Ans.}$$

Substituting the value of d in equation (i), we get

$$b = 1.236 \times 4.80 = \textbf{5.933 m. Ans.}$$

Discharge for most economical section. Hydraulic mean depth for most economical section is

$$m = \frac{d}{2} = \frac{4.80}{2} = 2.40 \text{ m}$$

$\therefore$ Discharge $$Q = AC\sqrt{mi} = 40 \times 50 \times \sqrt{2.40 \times \frac{1}{1500}}$$

$$= \textbf{80 m}^3\textbf{/s. Ans.}$$

Problem 16.17 *A trapezoidal channel has side slopes of 3 horizontal to 4 vertical and slope of its bed is 1 in 2000. Determine the optimum dimensions of the channel, if it is to carry water at 0.5 m³/s. Take Chezy's constant as 80.*

Solution. Given :

Side slopes $$n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{3}{4}$$

Slope of bed, $$i = \frac{1}{2000}$$

Discharge, $Q = 0.5 \text{ m}^3\text{/s}$

Chezy's constant, $C = 80$

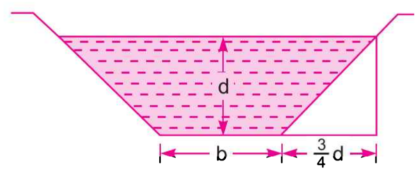

Fig. 16.13

For the most economical section, the condition is given by equation (16.11) as

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}, \text{ where } b = \text{width of section}, d = \text{depth of flow}$$

or $$\frac{b + 2 \times \frac{3}{4}d}{2} = d\sqrt{\left(\frac{3}{4}\right)^2 + 1} = \frac{5}{4}d \quad \text{or} \quad \frac{b + 1.5d}{2} = 1.25\, d$$

or $$b = 2 \times 1.25\, d - 1.5\, d = d \qquad ...(i)$$

For the discharge, Q, using equation (16.5) as

$$Q = AC\sqrt{mi} \qquad ...(ii)$$

But for most economical section, hydraulic mean depth $m = \dfrac{d}{2}$

Substituting the value of m and other known values in equation (ii)

$$0.50 = A \times 80 \times \sqrt{\frac{d}{2} \times \frac{1}{2000}} \qquad ...(iii)$$

But area of trapezoidal section is given as

$$A = (b + nd) \times d = (d + \tfrac{3}{4}d) \times d \quad (\because \text{ From } (i)\ b = d \text{ and } n = \tfrac{3}{4})$$

$$= \frac{7}{4}d^2 = 1.75\, d^2$$

Substituting the value of A in equation (iii), we get

$$0.50 = 1.75\, d^2 \times 80 \times \sqrt{\frac{d}{2} \times \frac{1}{2000}} = 2.2135\, d^{5/2}$$

$\therefore$ $$d = \left(\frac{0.50}{2.2135}\right)^{2/5} = \mathbf{0.55\ m.\ Ans.}$$

From equation (i), $b = d =$ **0.55 m. Ans.**

$\therefore$ Optimum dimensions of the channel are width = depth = 0.55 m.

Problem 16.18 *A trapezoidal channel with side slopes of 1 to 1 has to be designed to convey 10 m^3/s at a velocity of 2 m/s so that the amount of concrete lining for the bed and sides is the minimum. Calculate the area of lining required for one metre length of canal.*

Solution. Given :

Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = 1$

Discharge $Q = 10\ m^3/s$

Velocity, $V = 2.0$ m/s

$\therefore$ Area of flow, $A = \frac{\text{Discharge}}{\text{Velocity}} = \frac{10.0}{2.0} = 5\ m^2$ ...(i)

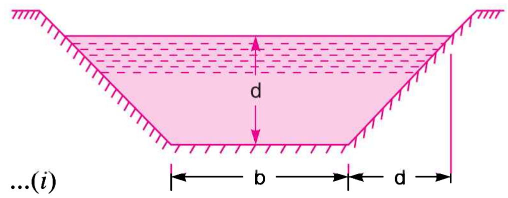

Fig. 16.14

Let b = Width of the channel

d = Depth of flow

For the amount of concrete lining for the bed and sides to be minimum the section should be most economical. But for the most economical trapezoidal section, the condition is from equation (16.11) as

Half of the top width = one of the sloping side

i.e., $$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

For $n = 1$, the condition becomes

$$\frac{b + 2 \times 1d}{2} = d\sqrt{1^2 + 1} = 1.414\, d$$

or $b = 2 \times 1.414d - 2d = 0.828\, d$ ...(ii)

But area, $A = (b + nd)\, d = (0.828d + 1 \times d)\, d$ ($\because$ $b = 0.828\, d$, $n = 1$)

$= 1.828\, d^2$

Also from equation (i), $A = 5\ m^2$

Equating the two values of A, we get

$$5 = 1.828\, d^2 \quad \text{or} \quad d = \sqrt{\frac{5}{1.828}} = 1.6538 \simeq 1.654 \text{ m}$$

From equation (ii), $b = 0.828\, d = 0.828 \times 1.654 = 1.369$ m

Area of lining required for one metre length of canal

= Wetted perimeter × length of canal

$= P \times 1$

where $P = b + 2d\sqrt{n^2 + 1} = 1.369 + 2 \times 1.654\sqrt{1^2 + 1} = 6.047$ m

$\therefore$ Area of lining $= 6.047 \times 1 =$ **6.047 m². Ans.**

Problem 16.19 *A trapezoidal channel has side slopes 1 to 1. It is required to discharge 13.75 m^3/s of water with a bed gradient of 1 in 1000. If unlined the value of Chezy's C is 44. If lined with concrete, its value is 60. The cost per m^3 of excavation is four times the cost per m^2 of lining. The channel is to be the most efficient one. Find whether the lined canal or the unlined canal will be cheaper. What will be the dimensions of that economical canal ?*

Solution. Given :

Side slope, $n = \frac{1}{1} = 1$

Discharge, $Q = 13.75 \text{ m}^3/\text{s}$

Slope of bed, $i = \frac{1}{1000}$

For unlined, $C = 44$

For lined $C = 60$

Cost per m^3 of excavation = 4 × cost per m^2 of lining

Let the cost per m^2 of lining = x

Then cost per m^3 of excavation = $4x$

As the channel is most efficient,

∴ Hydraulic mean depth, $m = \frac{d}{2}$, where d = depth of channel

Let b = width of channel

Also for the most efficient trapezoidal channel, from equation (16.11), we have

Half of top width = length of sloping side

or $$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

or $$\frac{b + 2 \times 1 \times d}{2} = d\sqrt{1^2 + 1} = \sqrt{2}d$$

∴ $$b = 2 \times \sqrt{2}d - 2d = 0.828\, d \quad \text{...}(i)$$

Area, $$A = (b + nd) \times d = (0.828\, d + 1 \times d) \times d$$
$$= 1.828\, d^2 \quad \text{...}(ii)$$

1. **For unlined channel**

Value of $C = 44$

The discharge, Q is given by, $Q = A \times V = A \times C\sqrt{mi}$

or $$13.75 = 1.828\, d^2 \times 44 \times \sqrt{\frac{d}{2} \times \frac{1}{1000}} \qquad \left(\because \; A = 1.828\, d^2,\; m = \frac{d}{2}\right)$$

$$= \frac{1.828 \times 44}{\sqrt{2000}} \times d^{5/2}$$

$$d^{5/2} = \frac{13.75 \times \sqrt{2000}}{1.828 \times 44} = 7.6452$$

∴ $$d = (7.6452)^{2/5} = 2.256 \text{ m}$$

Substituting this value in equation (i), we get

$$b = 0.828 \times 2.256 = 1.868 \text{ m.}$$

Now cost of excavation per running metre length of unlined channel

$$= \text{Volume of channel} \times \text{cost per m}^3 \text{ of excavation}$$
$$= (\text{Area of channel} \times 1) \times 4x = [(b + nd) \times d \times 1] \times 4x$$
$$= (1.868 + 1 \times 2.256) \times 2.256 \times 1 \times 4x = 37.215\,x \quad \ldots(iii)$$

2. **For lined channels**

Value of $C = 60$

The discharge is given by the equation, $Q = A \times C \times \sqrt{mi}$

Substituting the value of A from equation (*ii*) and value of $m = \frac{d}{2}$, we get

$$13.75 = 1.828\, d^2 \times 60 \times \sqrt{\frac{d}{2} \times \frac{1}{1000}} \qquad (\because \quad Q = 13.75)$$

$$= 1.828 \times 60 \times \frac{1}{\sqrt{2000}} \times d^{5/2}$$

$$\therefore \quad d^{5/2} = \frac{13.75 \times \sqrt{2000}}{1.828 \times 60} = 5.606$$

$$\therefore \quad d = (5.606)^{2/5} = 1.992 \text{ m}$$

Substituting this value in equation (*i*), we get

$$b = 0.828 \times 1.992 = 1.649 \text{ m}$$

In case of lined channel, the cost of lining as well as cost of excavation is to be considered.

Now cost of excavation $= (\text{Volume of channel}) \times \text{cost per m}^3 \text{ of excavation}$

$$= (b + nd) \times d \times 1 \times 4x$$
$$= (1.649 + 1 \times 1.992) \times 1.992 \times 1 \times 4x = 29.01\,x$$

Cost of lining $= \text{Area of lining} \times \text{cost per m}^2 \text{ of lining}$

$$= (\text{Perimeter of lining} \times 1) \times x$$
$$= (b + 2d\sqrt{1+n^2}) \times 1 \times x = (1.649 + 2 \times 1.992\sqrt{1+1^2}) \times 1 \times x$$
$$= (1.649 + 2 \times 1.992 \times \sqrt{2})\, x = 7.283\,x$$

$$\therefore \quad \text{Total cost} = 29.01x + 7.283x = 36.293x$$

The total cost of lined channel is $36.293x$ whereas the total cost of unlined channel is $37.215x$. Hence lined channel will be cheaper. The dimensions are $b = 1.649$ m and $d = 1.992$ m. **Ans.**

Problem 16.20 *An open channel of most economical section, having the form of a half hexagon with horizontal bottom is required to give a maximum discharge of 20.2 m*^{3}*/s of water. The slope of the channel bottom is 1 in 2500. Taking Chezy's constant, C = 60 in Chezy's equation, determine the dimensions of the cross-section.*

Solution. Given :

Maximum discharge, $Q = 20.2 \text{ m}^3/\text{s}$

Bed slope, $i = \frac{1}{2500}$

Chezy's constant, $C = 60$

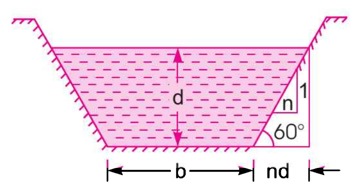

Fig. 16.15

Channel is the form of a half hexagon as shown in Fig. 16.15. This means that the angle made by the sloping side with horizontal will be 60°.

$$\therefore \quad \tan\theta = \tan 60° = \sqrt{3} = \frac{1}{n}$$

$$\therefore \quad n = \frac{1}{\sqrt{3}}$$

Let b = width of the channel, d = depth of the flow.

As the channel given is of most economical section, hence the condition given by equations (16.11) and (16.12) should be satisfied *i.e.*,

Half of the top width = one of the sloping side

And hydraulic mean depth = half of depth of flow

From equation (16.11), $\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$

For $n = \frac{1}{\sqrt{3}}$, $\quad \frac{b + 2 \times \frac{1d}{\sqrt{3}}}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} = \frac{2d}{\sqrt{3}}$

or $\quad \frac{\sqrt{3}b + 2d}{2\sqrt{3}} = \frac{2d}{\sqrt{3}}$ or $\frac{\sqrt{3}b + 2d}{2} = 2d$

$$\therefore \quad b = \frac{2 \times 2d - 2d}{\sqrt{3}} = \frac{2d}{\sqrt{3}} \quad \ldots(i)$$

Area of flow, $\quad A = (b + nd)\, d = \left(\frac{2}{\sqrt{3}}d + \frac{d}{\sqrt{3}}\right)d \qquad \left(\because\ n = \frac{1}{\sqrt{3}}, b = \frac{2d}{\sqrt{3}}\right)$

$$= \frac{3}{\sqrt{3}}d^2 = \sqrt{3}d^2$$

From equation (16.12) $\quad m = \frac{d}{2}$

Using equation (16.5) for discharge Q as

$$Q = AC\sqrt{mi} \quad \text{or} \quad 20.2 = \sqrt{3}\ d^2 \times 60 \times \sqrt{\frac{d}{2} \times \frac{1}{2500}} = 1.4694\ d^{5/2}$$

$$\therefore \quad d^{5/2} = \frac{20.2}{1.4696} = 13.745$$

$$\therefore \quad d = (13.745)^{2/5} = \mathbf{2.852\ m.\ Ans.}$$

Substituting this value in equation (i), we get

$$b = \frac{2d}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times 2.852 = \mathbf{3.293\ m.\ Ans.}$$

Problem 16.21 *A trapezoidal channel to carry 142 m³/minute of water is designed to have a minimum cross-section. Find the bottom width and depth if the bed slope is 1 in 1200, the side slopes at 45° and Chezy's co-efficient = 55.*

Solution. Given : Discharge, $Q = 142 \text{ m}^3\text{/min.} = \frac{142}{60} = 2.367 \text{ m}^3\text{/s}$

Bed slope, $i = 1 \text{ in } 1200 = \frac{1}{1200}$

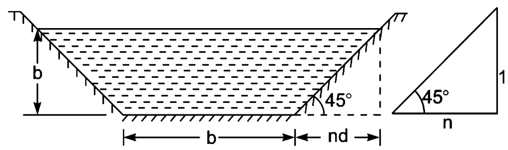

Fig. 16.16

Side slope, $\theta = 45°$

$\therefore \quad \tan\theta = \frac{1}{n} \quad \text{or} \quad \tan 45° = \frac{1}{n}$

$\therefore \quad 1 = \frac{1}{n} \quad \text{or} \quad n = 1$

Chezy's constant, $C = 55$

Let b = Width of the channel, d = Depth of the flow.

As the channel is to be designed for a minimum cross-section (*i.e.*, channel is of most economical section), the conditions given by equations (16.11) and (16.12) should be satisfied *i.e.*,

(*i*) Half of top with = Length of sloping side

(*ii*) Hydraulic mean depth = Half of depth of flow

From equation (16.11), $\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$

or $$\frac{b + 2 \times 1 \times d}{2} = d\sqrt{1^2 + 1} \qquad (\because n = 1)$$

or $$b + 2d = 2\sqrt{2}d = 2 \times 1.414\, d = 2.828\, d$$

$\therefore$ $$b = 2.828\, d - 2d = 0.828\, d \qquad ...(i)$$

Now using equation (16.5) for discharge Q, we get

$$Q = A \, . \, C \, . \sqrt{mi}$$

or $$2.367 = (b + nd)\, d \times 55\sqrt{\frac{d}{2} \times \frac{1}{1200}} \qquad \left(\because A = (b + nd) \times d \text{ and } m = \frac{d}{2}\right)$$

$$= (0.828d \times 1 \times d)\, d \times 55\sqrt{\frac{d}{2400}} \qquad (\because b = 0.828d)$$

$$= (1.828d) \times d \times 55\sqrt{\frac{d}{2400}} = 2.052\, d^{5/2}$$

$\therefore$ $$d = \left(\frac{2.367}{2.052}\right)^{2/5} = 1.058 \simeq \textbf{1.06 m. Ans.}$$

Substituting this value in equation (*i*), we get

$$b = 0.828 \times 1.06 = \textbf{0.877 m. Ans.}$$

Problem 16.22 *A trapezoidal channel with side slopes of 3 horizontal to 2 vertical has to be designed to convey 10 m^3/s at a velocity of 1.5 m/s, so that the amount of concrete lining for the bed and sides is minimum. Find*

(i) the wetted perimeter, and

(ii) slope of the bed if Manning's N = 0.014 in the formula $C = \frac{1}{N} \times m^{1/6}$

Solution. (*i*) Given :

Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{3}{2} = 1.5$

Discharge, $Q = 10 \text{ m}^3/\text{s}$

Velocity, $V = 1.5$ m/s

Manning's constant, $N = .014$

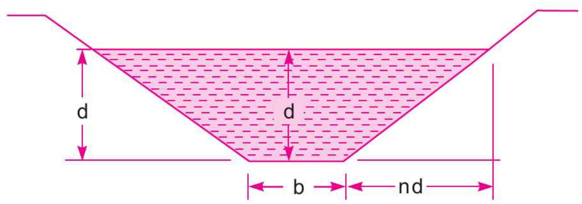

Fig. 16.17

Let b = width of the channel, d = depth of the flow.

The amount of concrete lining for the bed and sides will be minimum, when the section is most economical. For most economical trapezoidal section, the condition is given by equation (16.11) as

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

For $n = 1.5$, $\frac{b + 2 \times 1.5 \times d}{2} = d\sqrt{1.5^2 + 1} = \sqrt{3.25}\, d = 1.8\ d$ or $\frac{b + 3d}{2} = 1.8\ d$

$\therefore$ $b = 2 \times 1.8 - 3d = 0.6\ d$ ...(*i*)

But area of trapezoidal section, $A = (b + nd)d = (0.6d + 1.5d)d$ ($\because$ $b = 0.6d$, $n = 1.5$)

$= 2.1\ d^2$

Also area, $A = \frac{\text{Discharge}}{\text{Velocity}} = \frac{Q}{V} = \frac{10.0}{1.5} = 6.67 \text{ m}^2$

Equating the two values of A, we have $2.1\ d^2 = 6.67$

$\therefore$ $d = \sqrt{\frac{6.67}{2.1}} = 1.78$ m

From equation (*i*), $b = 0.6d = 0.6 \times 1.78 = 1.068 \simeq 1.07$ m

Hence wetted perimeter, $P = b + 2d\sqrt{n^2 + 1} = 1.07 + 2 \times 1.78\sqrt{1.5^2 + 1}$ = **7.48 m. Ans.**

(*ii*) Slope of the bed when $N = 0.014$ in the formula, $C = \frac{1}{N}\ m^{1/6}$

For the most economical trapezoidal section, hydraulic mean depth m, is given by equation (16.12) as

$$m = \frac{d}{2} = \frac{1.78}{2} = 0.89 \text{ m}$$

$$C = \frac{1}{0.014} \times (.89)^{1/6} = 66.09$$

Using equation (16.5), $Q = AC\sqrt{mi}$

or $10.0 = 6.67 \times 66.09\sqrt{0.89} \times i = 415.86\sqrt{i}$

$\therefore$ $i = \left(\frac{10}{415.86}\right)^2 = \mathbf{\frac{1}{1729.4}}$. **Ans.**

Hence slope of the bed is 1 in 1729.4.

16.5.3 Best Side Slope for Most Economical Trapezoidal Section.

Area of trapezoidal section, $A = (b + nd)d$ ...(*i*)

where b = width of trapezoidal channel, d = depth of flow, and

n = slope of the side of the channel

From equation (*i*), $b = \frac{A}{d} - nd$ ...(*ii*)

Perimeter (wetted) of channel, $P = b + 2d\sqrt{n^2 + 1}$

Substituting the value of b from equation (*ii*), perimeter becomes

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2 + 1} \quad \text{...(iii)}$$

For the most economical trapezoidal section, the depth of flow, d and area A are constant. Then n is the only variable. Best side slope will be when section is most economical or in other words, P is minimum. For P to be minimum, we must have $\frac{dP}{dn} = 0$

Hence differentiating equation (*iii*) with respect to n,

$$\frac{d}{dn}\left[\frac{A}{d} - nd + 2d\sqrt{n^2 + 1}\right] = 0$$

or $- d + 2d \times \frac{1}{2} \times (n^2 + 1)^{1/2-1} \times 2n = 0$ or $- d + 2nd \times \frac{1}{\sqrt{n^2 + 1}} = 0$

Cancelling d and re-arranging, we get $2n = \sqrt{n^2 + 1}$

Squaring to both sides,

$$4n^2 = n^2 + 1 \text{ or } 3n^2 = 1 \text{ or } n = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} \quad \text{...(16.14)}$$

If the sloping side makes an angle θ, with the horizontal, then we have

$$\tan \theta = \frac{1}{n} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3} = \tan 60°$$

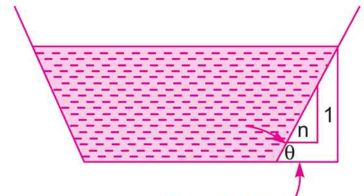

Fig. 16.18

$$\therefore \qquad \theta = 60° \qquad ...(16.15)$$

Hence best side slope is at 60° to the horizontal or the value of n for the best side slope is given by equation (16.14).

For the most economical trapezoidal section, we have

Half of top width = length of one sloping side

or $$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

Substituting the value of n from equation (16.14), we have

$$\frac{b + 2 \times \frac{1}{\sqrt{3}} \times d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} = \frac{2d}{\sqrt{3}} \quad \text{or} \quad \frac{\sqrt{3}b + 2d}{2 \times \sqrt{3}} = \frac{2d}{\sqrt{3}}$$

or $$\sqrt{3}b + 2d = 2 \times \sqrt{3} \times \frac{2d}{\sqrt{3}} = 4d$$

$$\therefore \qquad b = \frac{4d - 2d}{\sqrt{3}} = \frac{2d}{\sqrt{3}} \qquad ...(iv)$$

Now, wetted perimeter, $$P = b + 2d\sqrt{n^2 + 1}$$

$$= \frac{2d}{\sqrt{3}} + 2d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} \qquad \left(\because\ b = \frac{2d}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}\right)$$

$$= \frac{2d}{\sqrt{3}} + 2d \times \frac{2}{\sqrt{3}} = \frac{2d}{\sqrt{3}} + \frac{4d}{\sqrt{3}}$$

or $$P = \frac{6d}{\sqrt{3}} = 3 \times \frac{2d}{\sqrt{3}} = 3 \times b \qquad \left(\because\ \text{From } (iv), \frac{2d}{\sqrt{3}} = b\right)$$

For a slope of 60°, the length of sloping side is equal to the width of the trapezoidal section.

Problem 16.23 *A power canal of trapezoidal section has to be excavated through hard clay at the least cost. Determine the dimensions of the channel given, discharge equal to 14 m³/s, bed slope 1 : 2500 and Manning's N = 0.020.*

Solution. Given :

Discharge, $Q = 14\ m^3/s$

Bed slope, $i = \frac{1}{2500}$

Manning's, $N = 0.020$

For excavation of the canal at the least cost, the trapezoidal section should be most economical. Here side slope (*i.e.*, value of n) is not given. Hence the best side slope for most economical trapezoidal section (*i.e.*, the value of n) is given by equation (16.14) as $n = \frac{1}{\sqrt{3}}$

Let b = width of channel, d = depth of flow

For most economical section,

Half of top width = length of one of sloping side

or
$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

For $n = \frac{1}{\sqrt{3}}$,
$$\frac{b + 2 \times \frac{1}{\sqrt{3}} d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1} = \frac{2d}{\sqrt{3}}$$

or
$$b = \frac{2 \times 2d}{\sqrt{3}} - \frac{2d}{\sqrt{3}} = \frac{2d}{\sqrt{3}} \quad \text{...}(i)$$

Area of trapezoidal section,
$$A = (b + nd) \times d = \left(\frac{2d}{\sqrt{3}} + \frac{1}{\sqrt{3}} d\right) \times d \qquad \left(\because\ b = \frac{2d}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}\right)$$
$$= \sqrt{3} d^2$$

Hydraulic mean depth for most economical section, $m = \frac{d}{2}$

Now discharge, Q is given by $Q = AC\sqrt{mi}$, where $C = \frac{1}{N} m^{1/6}$

$\therefore$
$$Q = \sqrt{3} d^2 \times \frac{1}{N} m^{1/6} \sqrt{m \times \frac{1}{2500}}$$
$$= \sqrt{3} d^2 \times \frac{1}{0.020} \times m^{1/6 + 1/2} \times \sqrt{\frac{1}{2500}} = 1.732\, d^2 \times m^{2/3}$$

or
$$14.0 = 1.732\, d^2 \times \left(\frac{d}{2}\right)^{2/3} = \frac{1.732}{2^{2/3}} d^{8/3} = 1.09\, d^{8/3}$$

$\therefore$
$$d^{8/3} = \frac{14.0}{1.09} = 12.844$$

$\therefore$
$$d = (12.844)^{3/8} = (12.844)^{0.375} = \mathbf{2.605\ m.\ Ans.}$$

From equation (*i*),
$$b = \frac{2d}{\sqrt{3}} = \frac{2 \times 2.605}{1.732} = \mathbf{3.008\ m.\ Ans.}$$

Problem 16.24 *For a trapezoidal channel with bottom width 40 m and side slopes 2H : 1 V, Manning's N is 0.015 and bottom slope is 0.0002. If it carries 60 m^3/s discharge, determine the normal depth.*

Solution. Given :

Bottom width, $b = 40$ m

Side slopes 2 horizontal to 1 vertical *i.e.*, $n = 2$

∴ Manning's constant, $N = 0.015$

Bed slope, $i = 0.0002$

Discharge, $Q = 60\ \text{m}^3/\text{s}$

Let d = Normal depth.

Now $A = (b + nd) \times d = (40 + 2d) \times d$

$$P = b + 2d\sqrt{1+n^2} = 40 + 2d\sqrt{1+2^2} = 40 + 2 \times \sqrt{5}\,d = 40 + 4.472\,d$$

∴ $$m = \frac{A}{P} = \frac{(40+2d)\times d}{40+4.472d}$$

The discharge is given by, Q = Area × Velocity

$$= A \times \frac{1}{N} m^{2/3}\, i^{1/2} = \frac{A}{N} \times m^{2/3} \times i^{1/2}$$

$$60 = \frac{(40+2d)\times d}{0.015} \times \left[\frac{(40+2d)\times d}{40+4.472d}\right]^{2/3} \times 0.0002^{1/2}$$

$$= \frac{[(40+2d)\times d]^{5/3}}{0.015\times(40+4.472d)^{2/3}} \times 0.01414$$

∴ $$\frac{60\times 0.015\times(40+4.472d)^{2/3}}{0.01414} = [(40 + 2d) \times d]^{5/3}$$

$$63.65(40 + 4.472\ d)^{2/3} = (40d + 2d^2)^{5/3}$$

$$(40d + 2d^2)^{5/3} - 63.65\ (40 + 4.472d)^{2/3} = 0 \qquad ...(i)$$

The above equation will be solved by Hit and Trial method.

(*i*) Assume $d = 1$ m, then L.H.S of equation (*i*) will as

$$\text{L.H.S.} = (40 + 2)^{5/3} - 63.65\ (40 + 4.472)^{2/3}$$

$$= 42^{5/3} - 63.65 \times 44.472^{2/3} = 513.838 - 808.4 = -294.56$$

(*ii*) Assume $d = 2$ m, then L.H.S. of equation (*i*) will be as

$$\text{L.H.S.} = (40 \times 2 + 2 \times 2^2)^{5/3} - 63.65(40 + 4.47 \times 2)^{2/3}$$

$$= 88^{5/3} - 63.65 \times 48.944^{2/3} = 1767.2 - 862.77 = 904.43$$

where $d = 1$ m, L.H.S. is – ve. But when $d = 2$ m, L.H.S. is +ve. Hence value of d lies between 1 and 2.

(*iii*) Assume $d = 1.3$ m, then L.H.S. of equation (*i*) will be as

$$\text{L.H.S.} = (40 \times 1.3 + 2 \times 1.3^2)^{5/3} - 63.65(40 + 4.472 \times 1.3)^{2/3}$$

$$= 55.38^{5/3} - 63.65 \times 45.8136^{2/3} = 815.45 - 825.4 = -9.95$$

(*iv*) Assume $d = 1.31$ m, then L.H.S. of equation (*i*) will be

$$\text{L.H.S.} = (40 \times 1.31 + 2 \times 1.31^2)^{5/3} - 63.65(40 + 4.472 \times 1.31)^{2/3}$$

$$= 55.8322^{5/3} - 63.65 \times 45.8583^{2/3} = 826.6 - 825.9 = 0.7$$

The value of L.H.S. = 0.7 is negligible in comparison to the value of 904.43.

∴ Value of d = **1.31 m. Ans.**

16.5.4 Flow Through Circular Channel. The flow of a liquid through a circular pipe, when the level of liquid in the pipe is below the top of the pipe is classified as an open channel flow. The rate of flow through circular channel is determined from the depth of flow and angle subtended by the liquid surface at the centre of the circular channel.

Fig.16.19 shows a circular channel through which water is flowing.

Let d = depth of water,

2θ = angle subtended by water surface AB at the centre in radians,

R = radius of the channel,

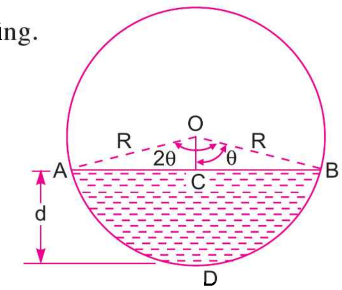

Fig. 16.19 *Circular channel.*

Then the wetted perimeter and wetted area is determine as :

Wetted perimeter, $$P = \frac{2\pi R}{2\pi} \times 2\theta = 2R\theta \qquad ...(16.16)$$

Wetted area, A = Area $ADBA$

= Area of sector $OADBO$ – Area of ΔABO

$$= \frac{\pi R^2}{2\pi} \times 2\theta - \frac{AB \times CO}{2} = R^2\theta - \frac{2BC \times CO}{2} \qquad (\because \ AB = 2BC)$$

$$= R^2\theta - \frac{2 \times R \sin\theta \times R\cos\theta}{2} \qquad (\because \ BC = R\sin\theta,\ CO = R\cos\theta)$$

$$= R^2\theta - \frac{R^2 \times 2\sin\theta\cos\theta}{2} = R^2\theta - \frac{R^2 \sin 2\theta}{2} \qquad (\because \ 2\sin\theta\cos\theta = \sin 2\theta)$$

$$= R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \qquad ...(16.17)$$

Then hydraulic mean depth, $$m = \frac{A}{P} = \frac{R^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2R\theta} = \frac{R}{2\theta}\left(\theta - \frac{\sin 2\theta}{2}\right)$$

And discharge, Q is given by, $Q = AC\sqrt{mi}$.

Problem 16.25 *Find the discharge through a circular pipe of diameter 3.0 m, if the depth of water in the pipe is 1.0 m and the pipe is laid at a slope of 1 in 1000. Take the value of Chezy's constant as 70.*

Solution. Given :

Dia. of pipe, $D = 3.0$

∴ Radius, $R = \frac{D}{2} = \frac{3.0}{2} = 1.50$ m

Depth of water in pipe, $d = 1.0$ m

Bed slope, $i = \frac{1}{1000}$

Chezy's constant, $C = 70$

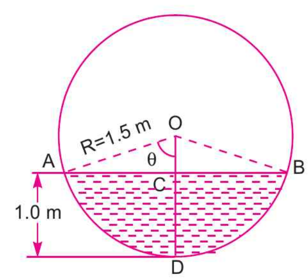

Fig. 16.20

From Fig. 16.20, we have $OC = OD - CD = R - 1.0$

$= 1.5 - 1.0 = 0.5$ m

$AO = R = 1.5$ m

Also $\cos\theta = \dfrac{OC}{AO} = \dfrac{0.5}{1.5} = \dfrac{1}{3}$

∴ $\theta = 70.53° = 70.53 \times \dfrac{\pi}{180} = 1.23$ radians (∵ 180° = π radians)

Wetted perimeter is given by equation (16.16) as

$$P = 2R\theta = 2 \times 1.5 \times 1.23 \quad (\theta \text{ should be in radians})$$
$$= 3.69 \text{ m}$$

Wetted area is given by equation (16.17) as

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 1.5^2\left(1.23 - \frac{\sin(2 \times 70.53°)}{2}\right)$$

$$= 2.25\left[1.23 - \frac{\sin(141.08°)}{2}\right] = 2.25\left[1.23 - \frac{\sin(180° - 141.08°)}{2}\right]$$

$$= 2.25\left[1.23 - \frac{\sin 38.94°}{2}\right] = 2.06 \text{ m}^2$$

∴ Hydraulic mean depth, $m = \dfrac{A}{P} = \dfrac{2.06}{3.69} = .5582$

The discharge is given by, $Q = AC\sqrt{mi} = 2.06 \times 70 \times \sqrt{0.5582 \times \dfrac{1}{1000}}$ = **3.407 m^2/s. Ans.**

Problem 16.26 *If in the problem 16.25, the depth of water in the pipe is 2.5 m, find the rate of flow through the pipe.*

Solution. Given :

Dia. of pipe = 3.0 m

∴ Radius, $R = 1.5$ m

Depth of water, $d = 2.5$ m

$i = \dfrac{1}{1000}$ and $C = 70$

From Fig. 16.21, $OC = CD - OD = 2.5 - R = 2.5 - 1.5 = 1.0$ m

$OA = R = 1.5$ m

From ΔAOC, $\cos\alpha = \dfrac{OC}{OA} = \dfrac{1.0}{1.5} = 0.667$

∴ $\alpha = 48.16°$

$\theta = 180° - \alpha = 180° - 48.16° = 131.84°$

$= 131.84 \times \dfrac{\pi}{180} = 2.30$ radians

Now wetted perimeter is given by equation (16.16) as

$$P = 2R\theta = 2 \times 1.5 \times 2.30 = 6.90 \text{ m}$$

And wetted area is given by equation (16.17) as

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 1.5^2\left(2.30 - \frac{\sin(2\times 131.84°)}{2}\right)$$

$$= 2.25\left(2.30 - \frac{\sin 263.68°}{2}\right)$$

$$= 2.25\left[2.30 - \frac{\sin(180° + 83.68°)}{2}\right]$$

$$= 2.25\left[2.30 - \frac{(-\sin 83.58°)}{2}\right]$$

$$= 2.25\left[2.30 + \frac{\sin 83.68°}{2}\right] = 6.293 \text{ m}^2$$

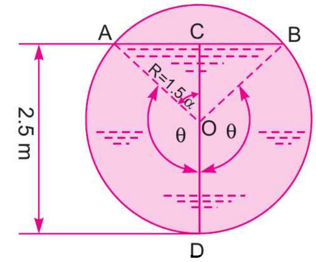

Fig. 16.21

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{6.293}{6.90} = 0.912$ m

Discharge, Q is given by, $Q = AC\sqrt{mi} = 6.293 \times 70 \times \sqrt{0.912 \times \frac{1}{1000}}$ = **13.303 m³/s. Ans.**

Problem 16.27 *Calculate the quantity of water that will be discharged at a uniform depth of 0.9 m in a 1.2 m diameter pipe which is laid at a slope 1 in 1000. Assume Chezy's C = 58.*

Solution. Given :

Dia. of pipe = 1.2 m

∴ Radius, $R = \frac{1.2}{2} = 0.6$ m

Depth of water, $d = 0.9$ m

Slope, $i = \frac{1}{1000}$

Chezy's, $C = 58$

From Fig. 16.22, we have $OC = CD - OD$

$= 0.9 - R = 0.9 - 0.6 = 0.3$ m

$OA = R = 0.6$ m

Now in triangle AOC,

$$\cos\alpha = \frac{OC}{OA} = \frac{0.3}{0.6} = \frac{1}{2}$$

∴ $\alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

∴ θ = Angle $DOA = 180° - \alpha$

$= 180° - 60° = 120° = 120 \times \frac{\pi}{180}$ radians

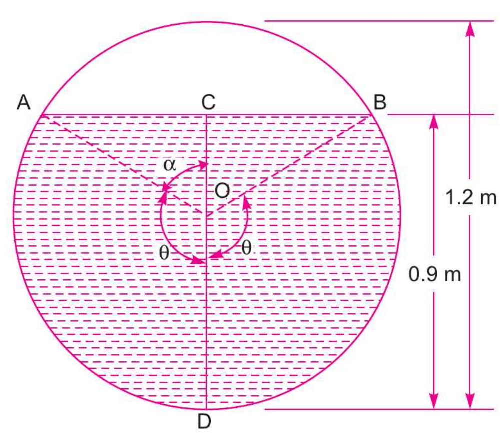

Fig. 16.22

$$= 0.667\ \pi \text{ radians}$$

Now wetted perimeter is given by equation (16.16) as

$$P = 2R\theta = 2 \times 0.6 \times 0.667\ \pi = 2.526 \text{ m}$$

And area of flow is given by equation (16.17) as

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$= 0.6^2\left[0.667\pi - \frac{\sin(2 \times 120°)}{2}\right] = 0.36\left[0.667\pi - \frac{\sin 240°}{2}\right]$$

$$= 0.36\left[0.667\pi - \frac{(-0.866)}{2}\right] = 0.36\ [0.667\ \pi + 0.433] = 0.913 \text{ m}^2$$

Now discharge is given by, $Q = A \times V = A \times C\sqrt{mi} = 0.913 \times 58\sqrt{\frac{A}{P} \times \frac{1}{1000}}$ $\left(\because\ m = \frac{A}{P}\right)$

$$= 0.913 \times 58\sqrt{\frac{0.913}{2.526} \times \frac{1}{1000}} = \mathbf{1.007\ m^3/s.\ Ans.}$$

Problem 16.28 *Water is flowing through a circular channel at the rate of 400 litres/s, when the channel is having a bed slope of 1 in 9000. The depth of water in the channel is 8.0 times the diameter. Find the diameter of the circular channel if the value of Manning's N = 0.015.*

Solution. Given :

Discharge, $Q = 400 \text{ litres/s} = 0.4 \text{ m}^3\text{/s}$

Bed slope, $i = \frac{1}{9000}$

Manning's, $N = 0.015$

Let the diameter of channel $= D$

Then depth of flow, $d = 0.8\ D$

From Fig. 16.23, we have

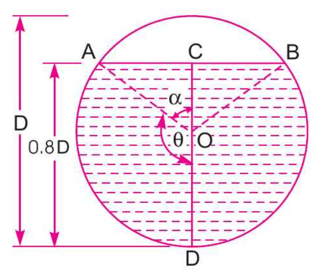

Fig. 16.23

$$OC = CD - OD = 0.8\ D - \frac{D}{2}$$

$$= (0.8 - 0.5)\ D = 0.3\ D$$

And $AO = R = \frac{D}{2} = 0.5\ D$

$\therefore$ $\cos\alpha = \frac{OC}{AO} = \frac{0.3\ D}{0.5\ D} = 0.6$

$\therefore$ $\alpha = 53.13°$

And $\theta = 180° - 53.13 = 126.87° = 126.87 \times \frac{\pi}{180} = 2.214$ radians.

From equation (16.16), wetted perimeter,

$$P = 2R\theta = 2 \times \frac{D}{2} \times 2.214 = 2.214\ D \text{ m.}$$

From equation (16.17), wetted area,

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = \left(\frac{D}{2}\right)^2 \left[2.214 - \frac{\sin(2 \times 126.87°)}{2}\right]$$

$$= \frac{D^2}{4}\left[2.214 - \frac{\sin 253.74°}{2}\right] = \frac{D^2}{4}\left[2.214 - \frac{\sin(180°+73.74°)}{2}\right]$$

$$= \frac{D^2}{4}\left[2.214 - \left(\frac{-\sin 73.74°}{2}\right)\right] = \frac{D^2}{4}\left[2.214 + \frac{\sin 73.74°}{2}\right]$$

$$= \frac{D^2}{4}\ [2.214 + .48] = 0.6735\ D^2$$

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{0.6735\ D^2}{2.214\ D} = 0.3042\ D$

Discharge by Manning's formula is given by,

$$Q = \frac{1}{N} \times A \times m^{2/3} \times i^{1/2}$$

or

$$0.4 = \frac{1}{.015} \times 0.6735\ D^2 \times (.3042\ D)^{2/3} \times \left(\frac{1}{9000}\right)^{1/2}$$

$$= \frac{0.6735}{0.015}\ D^2 \times 34521 \times D^{2/3} \times 0.0105 = 0.213\ D^{8/3}$$

∴ $D^{8/3} = \frac{0.40}{0.213} = 1.8779$

∴ $D = (1.8779)^{3/8} = (1.8779)^{0.375}$ = **1.266 m. Ans.**

Problem 16.29 *A sewer pipe is to be laid at a slope of 1 in 8100 to carry a maximum discharge of 600 litres/s, when the depth of water is 75% of the vertical diameter. Find the diameter of this pipe if the value of Manning's N is 0.025.*

Solution. Given :

Discharge, $Q = 600$ litres/s $= 0.6\ m^3/s$

Bed slope, $i = \frac{1}{8100}$

Manning's, $N = 0.025$

Depth of water = 75% of dia. of pipe = 0.75 dia. of pipe

Let d = depth of water, D = Dia. of pipe

Then $d = 0.75\ D$

From Fig. 16.23 (*a*), we have $OC = CD - OD = 0.75\ D - 0.5\ D = 0.25\ D$

$$AO = R = 0.5\ D$$

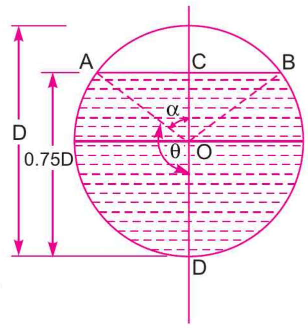

Fig. 16.23 (*a*)

In triangle AOC, $$\cos\alpha = \frac{OC}{AO} = \frac{0.25\ D}{0.5\ D} = 0.5$$

∴ $$\alpha = \cos^{-1} 0.5 = 60°$$

And $$\theta = 180° - \alpha = 180° - 60° = 120°$$

$$= 120 \times \frac{\pi}{180} \text{ radians} = 2.0946 \text{ radians.}$$

From equation (16.16), wetted perimeter

$$P = 2R\theta = 2 \times 0.5\ D \times 2.0496 \qquad (\because\ R = 0.5\ D)$$

$$= 2.0496\ D$$

And from equation (16.17), the area of flow,

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$= (0.5\ D)^2 \left[2.0946 - \frac{\sin(2 \times 120°)}{2}\right]$$

$$= 0.25\ D^2 \left[2.0946 - \left(\frac{-.866}{2}\right)\right] = 0.25\ D^2\ [2.0946 + 0.433]$$

$$= 0.6319\ D^2$$

∴ $$m = \frac{A}{P} = \frac{0.6319\ D^2}{2.0496\ D} = 0.308\ D$$

Discharge by Manning's formula is given by

$$Q = \frac{1}{N} \times A \times m^{2/3} \times i^{1/2}$$

or $$0.6 = \frac{1}{0.025} \times 0.6319\ D^2 \times (0.308\ D)^{2/3} \times \left(\frac{1}{8100}\right)^{1/2} = 0.128 \times D^{8/3}$$

∴ $$D^{8/3} = \frac{0.6}{0.128} = 4.6875$$

∴ $$D = (4.6875)^{3/8} = \mathbf{1.785\ m.\ Ans.}$$

16.5.5 Most Economical Circular Section. We have discussed in Art. 16.5 that for a most economical section the discharge for a constant cross-sectional area, slope of bed and resistance co-efficient, is maximum. But in case of circular channels, the area of flow cannot be maintained constant. With the change of depth of flow in a circular channel of any radius, the wetted area and wetted perimeter changes. Thus in case of circular channels, for most economical section, two separate conditions are obtained. They are :

1. Condition for maximum velocity, and
2. Condition for maximum discharge.

1. Condition for Maximum Velocity for Circular Section. Fig. 16.24 shows a circular channel through which water is flowing.

Let d = depth of water,

2θ = angle subtended at the centre by water surface,

R = radius of channel, and

i = slope of the bed,

The velocity of flow according to Chezy's formula is given as

$$V = C\sqrt{mi} = C\sqrt{\frac{A}{P}\,i} \quad \left(\because \; m = \frac{A}{P}\right)$$

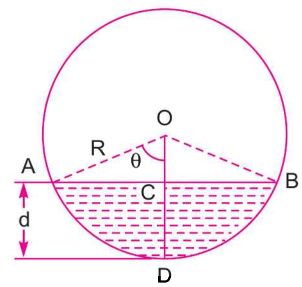

Fig. 16.24

The velocity of flow through a circular channel will be maximum when the hydraulic mean depth m or A/P is maximum for a given value of C and i. In case of circular pipe, the variable is θ only. Hence for maximum value of A/P we have the condition,

$$\frac{d\left(\frac{A}{P}\right)}{d\theta} = 0 \qquad ...(i)$$

where A and P both are functions of θ.

The value of wetted area, A is given by equation (16.17) as

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \qquad ...(ii)$$

The value of wetted perimeter, P is given by equation (16.16) as

$$P = 2R\theta \qquad ...(iii)$$

Differentiating equation (i) with respect to θ, we have

$$\frac{P\frac{dA}{d\theta} - A\frac{dP}{d\theta}}{P^2} = 0 \quad \text{or} \quad P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0 \qquad ...(iv)$$

From equation (ii), $\dfrac{dA}{d\theta} = R^2\left(1 - \dfrac{\cos 2\theta}{2} \times 2\right) = R^2(1 - \cos 2\theta)$

From equation (iii), $\dfrac{dP}{d\theta} = 2R$

Substituting the values of A, $P\dfrac{dA}{d\theta}$ and $\dfrac{dP}{d\theta}$ in equation (iv),

$$2R\theta\left[R^2(1 - \cos 2\theta)\right] - R^2\left(\theta - \frac{\sin 2\theta}{2}\right)(2R) = 0$$

or $$2R^3\theta(1 - \cos 2\theta) - 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

or $$\theta(1 - \cos 2\theta) - \left(\theta - \frac{\sin 2\theta}{2}\right) = 0 \qquad \text{(Cancelling } 2R^3\text{)}$$

$$\theta - \theta \cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

or $$\theta \cos 2\theta = \frac{\sin 2\theta}{2} \quad \text{or} \quad \frac{\sin 2\theta}{\cos 2\theta} = 2\theta$$

$\therefore$ $$\tan 2\theta = 2\theta$$

The solution of this equation by hit and trial, gives

$$2\theta = 257° \; 30' \qquad \text{(approximately)}$$

or $$\theta = 128° \; 45'$$

The depth of flow for maximum velocity from Fig. 16.24, is

$$d = OD - OC = R - R \cos \theta$$
$$= R[1 - \cos \theta] = R[1 - \cos 128° \; 45'] = R[1 - \cos (180° - 51° \; 15')]$$
$$= R[1 - (- \cos 51° \; 15')] = R[1 + \cos 51° \; 15']$$
$$= R[1+ 0.62] = 1.62 \; R = 1.62 \times \frac{D}{2} = 0.81 \; D \qquad \text{...(16.18)}$$

where D = diameter of the circular channel.

Thus for maximum velocity of flow, the depth of water in the circular channel should be equal to 0.81 times the diameter of the channel.

Hydraulic mean depth for maximum velocity is

$$m = \frac{A}{P} = \frac{R^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2R\theta} = \frac{R}{2\theta}\left[\theta - \frac{\sin 2\theta}{2}\right]$$

where $\theta = 128° \; 45' = 128.75°$

$$= 128.75 \times \frac{\pi}{180} = 2.247 \text{ radians}$$

$\therefore$ $$m = \frac{R}{2 \times 2.247}\left[2.247 - \frac{\sin 257° \; 30'}{2}\right] = \frac{R}{4.494}\left[2.247 - \frac{\sin (180° + 87.5°)}{2}\right]$$

$$= \frac{R}{4.494}\left[2.247 + \frac{\sin 87.5°}{2}\right] = 0.611 \; R$$

$$= 0.611 \times \frac{D}{2} = 0.3055 \; D = 0.3 \; D \qquad \text{...(16.19)}$$

Thus for maximum velocity, the hydraulic mean depth is equal to 0.3 times the diameter of circular channel.

2. Condition for Maximum Discharge for Circular Section. The discharge through a channel is given by

$$Q = AC\sqrt{mi} = AC\sqrt{\frac{A}{P}i} \qquad \left(\because \; m = \frac{A}{P}\right)$$

$$= C\sqrt{\frac{A^3}{P}i}$$

The discharge will be maximum for constant values of C and i, when $\frac{A^3}{P}$ is maximum. $\frac{A^3}{P}$ will be maximum when $\frac{d}{d\theta}\left(\frac{A^3}{P}\right) = 0$.

Differentiating this equation with respect to θ and equation the same to zero, we get

$$\frac{P \times 3A^2 \frac{dA}{d\theta} - A^3 \frac{dP}{d\theta}}{P^2} = 0 \quad \text{or} \quad 3PA^2 \frac{dA}{d\theta} - A^3 \frac{dP}{d\theta} = 0$$

Dividing by A^2, $3P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0$ ...(*i*)

But from equation (16.16), $P = 2R\theta$

$$\therefore \quad \frac{dP}{d\theta} = 2R$$

From equation (16.17), $A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$

$$\therefore \quad \frac{dA}{d\theta} = R^2(1 - \cos 2\theta)$$

Substituting the values of P, A, $\frac{dP}{d\theta}$ and $\frac{dA}{d\theta}$ in equation (*i*)

$$3 \times 2R\theta \times R^2(1 - \cos 2\theta) - R^2\left(\theta - \frac{\sin 2\theta}{2}\right) \times 2R = 0$$

$$6R^3\theta(1 - \cos 2\theta) - 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

Dividing by $2R^3$, we get

$$3\theta(1 - \cos 2\theta) - \left(\theta - \frac{\sin 2\theta}{2}\right) = 0 \quad \text{or} \quad 3\theta - 3\theta\cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

or

$$2\theta - 3\theta\cos 2\theta + \frac{\sin 2\theta}{2} = 0 \quad \text{or} \quad 4\theta - 6\theta\cos 2\theta + \sin 2\theta = 0$$

The solution of this equation by hit and trial, gives

$$2\theta = 308^\circ \qquad \text{(approximately)}$$

$$\therefore \quad \theta = \frac{308^\circ}{2} = 154^\circ$$

Depth of flow for maximum discharge [See Fig. 16.24]

$$d = OD - OC = R - R\cos\theta$$
$$= R[1 - \cos\theta] = R[1 - \cos 154^\circ]$$
$$= R[1 - \cos(180^\circ - 26^\circ)] = R[1 + \cos 26^\circ] = 1.898\ R$$
$$= 1.898 \times \frac{D}{2} = 0.948\ D \simeq 0.95\ D \qquad \text{...(16.20)}$$

where D = Diameter of the circular channel.

Thus for maximum discharge through a circular channel the depth of flow is equal to 0.95 times its diameter.

Problem 16.30 *The rate of flow of water through a circular channel of diameter 0.6 m is 150 litres/s. Find the slope of the bed of the channel for maximum velocity. Take C = 60*

Solution. Given :

Discharge, $Q = 150$ litres/s $= 0.15$ m^3/s

Dia. of channel, $D = 0.6$ m

Value of $C = 60$

Let the slope of the bed of channel for maximum velocity $= i$

For maximum velocity through a circular channel, depth of flow is given by equation (16.18) as

$$d = 0.81 \times D = 0.81 \times 0.6 = .486 \text{ m}$$

and $$\theta = 128° \, 45' \quad \text{or} \quad 128.75 \times \frac{\pi}{180} = 2.247 \text{ radians}$$

From equation (16.19), hydraulic mean depth for maximum velocity is given as

$$m = 0.3 \times D = 0.3 \times 0.6 = 0.18 \text{ m}$$

Wetted perimeter for circular pipe is given by equation (16.16),

$$P = 2R\theta = D \times \theta = 0.6 \times 2.247 = 1.3482 \text{ m}$$

But $$m = \frac{A}{P} = 0.18 \text{ m}$$

$\therefore$ Area, $$A = 0.18 \times P = 0.18 \times 1.3482 = 0.2426 \text{ m}^2$$

For discharge, using the relation

$$Q = AC\sqrt{mi} \quad \text{or} \quad 0.15 = 0.2426 \times 60 \times \sqrt{0.81 \times i} = 6.175\sqrt{i}$$

$\therefore$ $$i = \left(\frac{0.15}{6.175}\right)^2 = \frac{\mathbf{1}}{\mathbf{1694.7}}. \textbf{ Ans.}$$

$\therefore$ Bed slope is 1 in 1694.7.

Problem 16.31 *Determine the maximum discharge of water through a circular channel of diameter 1.5 m when the bed slope of the channel is 1 in 1000. Take C = 60.*

Solution. Given :

Dia. of channel, $D = 1.5$ m

$\therefore$ $$R = \frac{1.5}{2} = 0.75 \text{ m}$$

Bed slope, $$i = \frac{1}{1000}$$

Value of $C = 60$

For maximum discharge, $$\theta = 154° \text{ or } \frac{154 \times \pi}{180} = 2.6878 \text{ radians.}$$

Wetted perimeter for a circular channel is given by equation (16.16) as

$$P = 2R\theta = 2 \times \frac{D}{2} \times 2.6878 = 2 \times \frac{1.5}{2} \times 2.6878 = 4.0317 \text{ m}$$

Wetted area A is given by equation (16.17) as

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 0.75^2\left[2.6878 - \frac{\sin(2 \times 154)°}{2}\right]$$

$$= 0.75^2\left[2.6878 - \frac{\sin 308°}{2}\right] = .75^2\left[2.6878 - \frac{\sin(360° - 52°)}{2}\right]$$

$$= 0.75^2\left[2.6878 + \frac{\sin 52°}{2}\right] = 1.7335$$

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{1.7335}{4.0317} = 0.4299$

Maximum discharge is given by $Q = AC\sqrt{mi} = 1.7335 \times 60 \times \sqrt{0.4299 \times \frac{1}{1000}}$

$= \mathbf{2.1565\ m^3/s.}$ **Ans.**

Problem 16.32 *A concrete lined circular channel of diameter 3 m has a bed slope of 1 in 500. Work out the velocity and flow rate for the conditions of (i) maximum velocity and (ii) maximum discharge. Assume Chezy's C = 50.*

Solution. Given :

Dia of channel, $D = 3$ m

Bed slope, $i = \frac{1}{500}$

Value of $C = 50$

(*i*) *Velocity and discharge for maximum velocity*

For maximum velocity, $\theta = 128° \ 45' = 128.75°$

$= 128.75 \times \frac{\pi}{180}$ radians = 2.247 radians

∴ Wetted perimeter, $P = 2 \times R \times \theta$

$= 2 \times 1.5 \times 2.247$ $\left(\because R = \frac{D}{2} = \frac{3}{2} = 1.5\right)$

$= 6.741$ m

Area of flow, $A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right) = 1.5^2\left[2.247 - \frac{\sin(2 \times 128.75°)}{2}\right]$

$= 2.25\ [2.247 - (-0.488)] = 6.1537\ m^2$

∴ Hydraulic mean depth $m^* = \frac{A}{P} = \frac{6.1537}{6.741} = 0.912$

Now velocity, $V = C\sqrt{m \times i} = 50 \times \sqrt{\frac{0.912 \times 1}{500}} = \mathbf{2.135\ m/s.}$ **Ans.**

and discharge, $Q = A \times V = 6.1537 \times 2.135 = \mathbf{13.138\ m^3/s.}$ **Ans.**

(*ii*) *Velocity and discharge for maximum discharge*

For maximum discharge, $\theta = 154° = \frac{154 \times \pi}{180}$ radians = 2.6878 radians

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* From equation (16.19), m is also equal to $0.3055\ D$.
Hence $m = 0.3055 \times 3 = 0.9165$

∴ $$A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right] = 1.5^2\left[2.6878 - \frac{\sin(2\times 154)}{2}\right]$$

$$= 2.25\ [2.6878 - (-0.394)] = 6.934$$

$$P = 2R \times \theta = 2 \times 1.5 \times 2.6878 = 8.0634$$

and $$m = \frac{A}{P} = \frac{6.934}{8.0634} = 0.8599$$

Now velocity $$V = C\sqrt{mi} = 50 \times \sqrt{0.8599 \times \frac{1}{500}} = \mathbf{2.0735\ m/s.\ Ans.}$$

and discharge, $$Q = A \times V = 6.934 \times 2.0735 = \mathbf{14.377\ m^3/s.\ Ans.}$$

▶ 16.6 NON-UNIFORM FLOW THROUGH OPEN CHANNELS

We have defined uniform flow and non-uniform flow in Art. 16.2.2. A flow is said to be uniform if the velocity of flow, depth of flow, slope of the bed of the channel and area of cross-section remain constant for a given length of the channel. On the other hand, if velocity of flow, depth of flow, area of cross-section and slope of the bed of channel do not remain constant for a given length of pipe, the flow is said to be non-uniform.

Non-uniform is further divided into Rapidly Varied Flow (R.V.F.), and Gradually Varied Flow (G.V.F.) depending upon the change of depth of flow over the length of the channel. If the depth of flow changes abruptly over a small length of the channel, the flow is said as rapidly varied flow. And if the depth of flow in a channel changes gradually over a long length of channel, the flow is said to be gradually varied flow.

▶ 16.7 SPECIFIC ENERGY AND SPECIFIC ENERGY CURVE

The total energy of a flowing liquid per unit weight is given by,

Total Energy $$= z + h + \frac{V^2}{2g}$$

where z = Height of the bottom of channel above datum,

h = Depth of liquid, and V = Mean velocity of flow.

If the channel bottom is taken as the datum as shown in Fig. 16.25, then the total energy per unit weight of liquid will be,

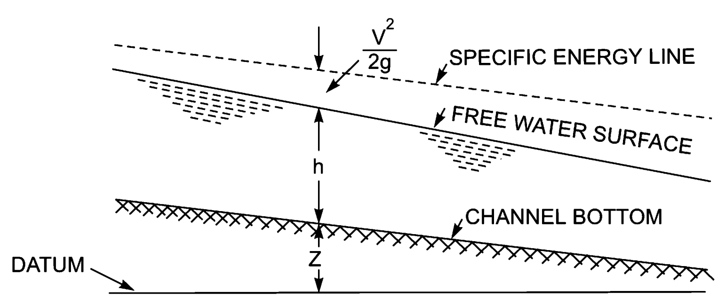

Fig. 16.25 *Specific energy.*

$$E = h + \frac{V^2}{2g} \qquad \text{...(16.21)}$$

The energy given by equation (16.21) is known as **specific energy**. Hence specific energy of a flowing liquid is defined as energy per unit weight of the liquid with respect to the bottom of the channel.

Specific Energy Curve. It is defined as the curve which shows the variation of specific energy with depth of flow. It is obtained as :

From equation (16.21), the specific energy of a flowing liquid

$$E = h + \frac{V^2}{2g} = E_p + E_k$$

where E_p = Potential energy of flow = h

E_k = Kinetic energy of flow $= \frac{V^2}{2g}$

Consider a rectangular channel in which a steady but non-uniform flow is taking place.

Let Q = discharge through the channel,

b = width of the channel,

h = depth of flow, and

q = discharge per unit width.

Then $q = \frac{Q}{\text{width}} = \frac{Q}{b}$ = constant $(\because Q$ and b are constant)

Velocity of flow, $V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{b \times h} = \frac{q}{h}$ $\left(\because \frac{Q}{b} = q\right)$

Substituting the values of V in equation (16.21), we get

$$E = h + \frac{q^2}{2gh^2} = E_p + E_k \qquad \text{...(16.22)}$$

Equation (16.22), gives the variation of specific energy (E) with the depth of flow (h). Hence for a given discharge Q, for different values of depth of flow, the corresponding values of E may be obtained. Then a graph between specific energy (along X-X axis) and depth of flow, h (along Y-Y axis) may be plotted.

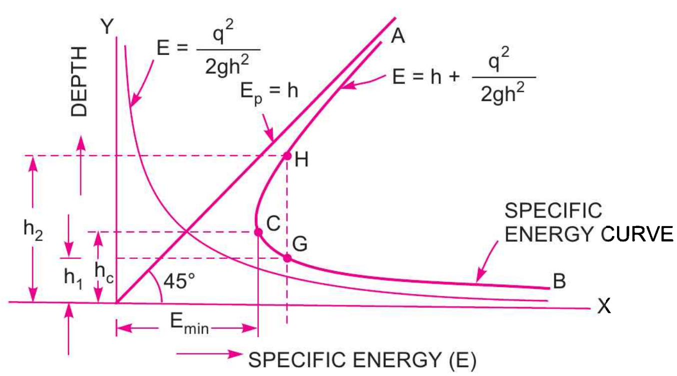

Fig. 16.26 *Specific energy curve.*

The specific energy curve may also be obtained by first drawing a curve for potential energy (*i.e.*, $E_p = h$), which will be a straight line passing through the origin, making an angle of 45° with the X - axis as shown in Fig. 16.26. Then drawing another curve for kinetic energy $\left(i.e., E_k = \frac{q^2}{2gh^2} \text{ or } E_k = \frac{K}{h^2},\right.$ $\left.\text{where } K = \frac{q^2}{2g} = \text{constant}\right)$ which will be a parabola as shown in Fig. 16.26. By combining these two curves, we can obtain the specific energy curve. In Fig. 16.26, curve ACB denotes the specific energy curve.

16.7.1 Critical Depth (h_c). Critical depth is defined as that depth of flow of water at which the specific energy is minimum. This is denoted by 'h_c' . In Fig. 16.26, curve ACB is a specific energy curve and point C corresponds to the minimum specific energy. The depth of flow of water at C is known as critical depth. The mathematical expression for critical depth is obtained by differentiating the specific energy equation (16.22) with respect to depth of flow and equating the same to zero.

or $$\frac{dE}{dh} = 0, \qquad \text{where } E = h + \frac{q^2}{2gh^2} \text{ from equation (16.22)}$$

or $$\frac{d}{dh}\left[h + \frac{q^2}{2gh^2}\right] = 0 \quad \text{or} \quad 1 + \frac{q^2}{2g}\left(\frac{-2}{h^3}\right) = 0 \qquad \left(\because \frac{q^2}{2g} \text{ is constant}\right)$$

or $$1 - \frac{q^2}{gh^3} = 0 \quad \text{or} \quad 1 = \frac{q^2}{gh^3} \quad \text{or} \quad h^3 = \frac{q^2}{g}$$

$\therefore$ $$h = \left(\frac{q^2}{g}\right)^{1/3}$$

But when specific energy is minimum, depth is critical and it is denoted by h_c. Hence critical depth is

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad ...(16.23)$$

16.7.2 Critical Velocity (V_c). The velocity of flow at the critical depth is known as critical velocity. It is denoted by V_c. The mathematical expression for critical velocity is obtained from equation (16.23) as

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Taking cube to both sides, we get $h_c^{\,3} = \frac{q^2}{g}$ or $gh_c^{\,3} = q^2$ ...(*i*)

But $$q = \text{Discharge per unit width} = \frac{Q}{b}$$

$$= \frac{\text{Area} \times V}{b} = \frac{b \times h \times V}{b} = h \times V = h_c \times V_c$$

Substituting this value of q in (i),

$$\therefore \quad gh_c^3 = (h_c \times V_c)^2$$

or $$gh_c^3 = h_c^2 \times V_c^2 \text{ or } gh_c = V_c^2 \qquad \text{[Dividing by } h_c^2\text{]}$$

or $$V_c = \sqrt{g \times h_c} \qquad ...(16.24)$$

16.7.3 Minimum Specific Energy in Terms of Critical Depth. Specific energy equation is given by equation (16.22)

$$E = h + \frac{q^2}{2gh^2}$$

When specific energy is minimum, depth of flow is critical and hence above equation becomes as

$$E_{min} = h_c + \frac{q^2}{2gh_c^2} \qquad ...(ii)$$

But from equation (16.23), $h_c = \left(\frac{q^2}{g}\right)^{1/3}$ or $h_c^3 = \frac{q^2}{g}$

Substituting the value of $\frac{q^2}{g} = h_c^3$ in equation (ii), we get

$$E_{min} = h_c + \frac{h_c^3}{2h_c^2} = h_c + \frac{h_c}{2} = \frac{3h_c}{2}. \qquad ...(16.25)$$

Problem 16.33 *Find the specific energy of flowing water through a rectangular channel of width 5 m when the discharge is 10 m³/s and depth of water is 3 m.*

Solution. Given :

Width of rectangular channel, $b = 5$ m

Discharge, $Q = 10 \text{ m}^3/\text{s}$

Depth of water, $h = 3$ m

Specific energy is given by equation (16.21), as

$$E = h + \frac{V^2}{2g}, \quad \text{where } V = \frac{Q}{\text{Area}} = \frac{10}{b \times h} = \frac{10}{5 \times 3} = \frac{2}{3}$$

$$\therefore \quad E = 3 + \frac{\left(\frac{2}{3}\right)^2}{2 \times 9.81} = 3 + .0226 = \mathbf{3.0226 \text{ m. Ans.}}$$

Problem 16.34 *Find the critical depth and critical velocity of the water flowing through a rectangular channel of width 5 m, when discharge is 15 m³/s.*

Solution. Given :

Width of channel, $b = 5$ m

Discharge, $Q = 15 \text{ m}^3/\text{s}$

$\therefore$ Discharge per unit width, $q = \frac{Q}{b} = \frac{15}{5} = 3 \text{ m}^2/\text{s}$

Critical depth is given by equation (16.23) as

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{3^2}{9.81}\right)^{1/3} = \left(\frac{9}{9.81}\right)^{1/3} = \mathbf{0.972 \text{ m. Ans.}}$$

Critical velocity is given by equation (16.24) as

$$V_c = \sqrt{g \times h_c} = \sqrt{9.81 \times 0.972} = \mathbf{3.088 \text{ m/s. Ans.}}$$

Problem 16.35 *The discharge of water through a rectangular channel of width 8 m, is 15 m^3/s when depth of flow of water is 1.2 m. Calculate :*

(i) Specific energy of the flowing water, *(ii) Critical depth and critical velocity,*

(iii) Value of minimum specific energy.

Solution. Given :

Discharge, $Q = 15 \text{ m}^3/\text{s}$

Width, $b = 8$ m

Depth, $h = 1.2$ m

$\therefore$ Discharge per unit width, $q = \frac{Q}{b} = \frac{15}{8} = 1.875 \text{ m}^2/\text{s}$

Velocity of flow, $V = \frac{Q}{\text{Area}} = \frac{15}{b \times h} = \frac{15.0}{8 \times 1.2} = 1.5625$ m/s

(*i*) Specific energy (E) is given by equation (16.21) as

$$E = h + \frac{V^2}{2g} = 1.2 + \frac{1.5625^2}{8 \times 9.81} = 1.20 + 0.124 = \mathbf{1.324 \text{ m. Ans.}}$$

(*ii*) Critical depth (h_c) is given by equation (16.23) as

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.875^2}{9.81}\right)^{1/3} = \mathbf{0.71 \text{ m. Ans.}}$$

Critical velocity (V_c) is given by equation (16.24) as

$$V_c = \sqrt{g \times h_c} = \sqrt{9.81 \times 0.71} = \mathbf{2.639 \text{ m/s. Ans.}}$$

(*iii*) Minimum specific energy (E_{min}) is given by equation (16.25)

$$E_{min} = \frac{3h_c}{2} = \frac{3 \times 0.71}{2} = \mathbf{1.065 \text{ m. Ans.}}$$

16.7.4 Critical Flow. It is defined as that flow at which the specific energy is minimum or the flow corresponding to critical depth is defined as critical flow. Equation (16.24) gives the relation for critical velocity in terms of critical depth as

$$V_c = \sqrt{g \times h_c} \quad \text{or} \quad \frac{V_c}{\sqrt{gh_c}} = 1 \qquad \left| \text{ where } \frac{V_c}{\sqrt{gh_c}} = \text{Froude number} \right.$$

$\therefore$ Froude number, $F_e = 1.0$ for critical flow.

16.7.5 Streaming Flow or Sub-critical Flow or Tranquil Flow. When the depth of flow in a channel is greater than the critical depth (h_c), the flow is said to be sub-critical flow or streaming flow or tranquil flow. For this type of flow the Froude number is less than one *i.e.*, $F_e < 1.0$.

16.7.6 Super-critical Flow or Shooting Flow or Torrential Flow. When the depth of flow in a channel is less than the critical depth (h_c), the flow is said to be super-critical flow or shooting flow or torrential flow. For this type of flow the Froude number is greater than one *i.e.*, $F_e > 1.0$.

16.7.7 Alternate Depths. In the specific energy curve shown in Fig. 16.26, the point C corresponds to the minimum specific energy and the depth of flow at C is called critical depth. For any other value of the specific energy, there are two depths, one greater than the critical depth and other smaller than the critical depth. These two depths for a given specific energy are called the alternate depths. These depths are shown as h_1 and h_2 in Fig. 16.26. Or the depths corresponding to points G and H in Fig. 16.26 are called alternate depths.

16.7.8 Condition for Maximum Discharge for a Given Value of Specific Energy. The specific energy (E) at any section of a channel is given by equation (16.21) as

$$E = h + \frac{V^2}{2g}, \text{ where } V = \frac{Q}{A} = \frac{Q}{b \times h}$$

$$\therefore \quad E = h + \frac{Q^2}{b^2 \times h^2} \times \frac{1}{2g} = h + \frac{Q^2}{2gb^2h^2}$$

or $$Q^2 = (E - h)\, 2gb^2h^2 \quad \text{or} \quad Q = \sqrt{(E-h)2gb^2h^2} = b\sqrt{2g\left(Eh^2 - h^3\right)}$$

For maximum discharge, Q, the expression $(Eh^2 - h^3)$ should be maximum. Or in other words,

$$\frac{d}{dh}(Eh^2 - h^3) = 0 \quad \text{or} \quad 2Eh - 3h^2 = 0 \qquad (\because \ E \text{ is constant})$$

or $$2E - 3h = 0 \qquad \text{(Dividing by } h\text{)}$$

or $$h = \tfrac{2}{3}E \qquad ...(16.26)$$

or $$E = \frac{3h}{2} \qquad ...(i)$$

But from equation (16.25), specific energy is minimum when it is equal to $\frac{3}{2}$ times the value of depth of critical flow. Here in equation (*i*), the specific energy (E) is equal to $\frac{3}{2}$ times the depth of flow. Thus equation (*i*) represents the minimum specific energy and h is the critical depth. Hence the condition for maximum discharge for given value of specific energy is that the depth of flow should be critical.

Problem 16.36 *The specific energy for a 3 m wide channel is to be 3 kg-m/kg. What would be the maximum possible discharge ?*

Solution. Given :

Width of channel, $b = 3$ m

Specific energy, $E = 3$ kg-m/kg = 3 m

For the given value of specific energy, the discharge will be maximum, when depth of flow is critical. Hence from equation (16.26) for maximum discharge.

$$h_c = h = \tfrac{2}{3}E = \tfrac{2}{3} \times 3.0 = 2.0 \text{ m}$$

∴ Maximum discharge, Q_{max} is given by

$$Q_{max} = \text{Area} \times \text{Velocity} = (b \times \text{depth of flow}) \times \text{Velocity}$$

$$= (b \times h_c) \times V_c \quad (\because \text{ At critical depth, Velocity will be critical})$$

where V_c is critical velocity and it is given by equation (16.24),

$$V_c = \sqrt{g \times h_c} = \sqrt{9.81 \times 2.0} = 4.4249 \text{ m/s}$$

$$\therefore \quad Q_{max} = (b \times h_c) \times V_c = (3 \times 2) \times 4.4249 = \mathbf{26.576 \ m^3/s. \ Ans.}$$

Problem 16.37 *The specific energy for a 5 m wide rectangular channel is to be 4 Nm/N. If the rate of flow of water through the channel is 20 m³/s, determine the alternate depths of flow.*

Solution. Given :

Width of channel, $b = 5$ m

Specific energy, $E = 4$ Nm/N $= 4$ m

Discharge, $Q = 20 \text{ m}^3/\text{s}$

The specific energy (E) is given by equation (16.21) as,

$$E = h + \frac{V^2}{2g}, \quad \text{where } V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{b \times h} = \frac{20}{5 \times h} = \frac{4}{h}$$

∴ Specific energy,

$$E = h + \frac{V^2}{2g} = h + \left(\frac{4}{h}\right)^2 \times \frac{1}{2g} = h + \frac{8}{g \times h^2}$$

But $E = 4.0$

Equating the two values of E, $4 = h + \dfrac{8}{9.81 \times h^2} = h + \dfrac{0.8155}{h^2}$

$$4h^2 = h^3 + .8155 \quad \text{or} \quad h^3 - 4h^2 + .8155 = 0$$

This is a cubic equation. Solving by trial and error, we get

$$h = \mathbf{3.93 \ m \ and \ 0.48 \ m. \ Ans.}$$

▶ 16.8 HYDRAULIC JUMP OR STANDING WAVE

Consider the flow of water over a dam as shown in Fig. 16.27. The height of water at the section 1-1 is small. As we move towards downstream, the height or depth of water increases rapidly over a short length of the channel. This is because at the section 1-1, the flow is a *shooting flow* as the depth of water at section 1-1 is less than critical depth. Shooting flow is an unstable type of flow and does not continue on the downstream side. Then this shooting will convert itself into a streaming or tranquil flow and hence depth of water will increase. This sudden increase of depth of water is called a hydraulic jump or a standing wave. Thus hydraulic jump is defined as :

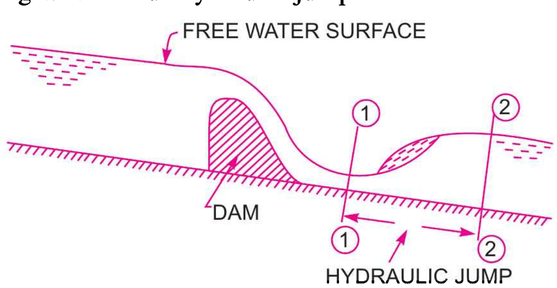

Fig. 16.27 *Hydraulic jump.*

"The rise of water level, which takes place due to the transformation of the unstable shooting flow (Super-critical) to the stable streaming flow (sub-critical flow)."

When hydraulic jump takes place, a loss of energy due to eddy formation and turbulence occurs.

16.8.1 Expression for Depth of Hydraulic Jump. Before deriving an expression for the depth of hydraulic jump, the following assumptions are made :

1. The flow is uniform and pressure distribution is due to hydrostatic before and after the jump.
2. Losses due to friction on the surface of the bed of the channel are small and hence neglected.
3. The slope of the bed of the channel is small, so that the component of the weight of the fluid in the direction of flow is negligibly small.

Consider a hydraulic jump formed in a channel of horizontal bed as shown in Fig. 16.28. Consider two sections 1-1 and 2-2 before and after hydraulic jump.

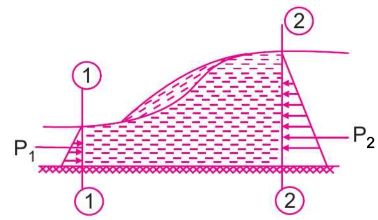

Fig. 16.28 *Hydraulic jump.*

Let d_1 = Depth of flow at section 1-1,

d_2 = Depth of flow at section 2-2,

V_1 = Velocity of flow at section 1-1,

V_2 = Velocity of flow at section 2-2,

$\overline{Z}_1$ = Depth of centroid of area at section 1-1 below free surface,

$\overline{Z}_2$ = Depth of centroid of area at section 2-2 below free surface,

A_1 = Area of cross-section at section 1-1, and

A_2 = Area of cross-section at section 2-2.

Consider unit width of the channel.

The forces acting on the mass of water between sections 1-1 and 2-2 are :

(*i*) Pressure force, P_1 on section 1-1,

(*ii*) Pressure force, P_2 on section 2-2,

(*iii*) Frictional force on the floor of the channel, which is assumed to be negligible.

Let q = discharge per unit width

$$= V_1 d_1 = V_2 d_2 \qquad \ldots(i)$$

Now pressure force P_1 on section 1-1

$$= \rho g A_1 \overline{Z}_1 = \rho g \times d_1 \times 1 \times \frac{d_1}{2}$$

$$= \frac{\rho g d_1^2}{2} \qquad \left(\because \; A_1 = d_1 \times 1, \overline{Z}_1 = \frac{d_1}{2}\right)$$

Similarly pressure force on section 2-2,

$$P_2 = \rho g A_2 \overline{Z}_2$$

$$= \rho g \times d_2 \times 1 \times \frac{d_2}{2} = \frac{\rho g d_2^2}{2}$$

Net force acting on the mass of water between sections 1-1 and 2-2

$$= P_2 - P_1 \qquad | \because \; P_2 \text{ is greater than } P_1 \text{ and } d_2 \text{ is greater than } d_1$$

$$= \frac{\rho g d_2^2}{2} - \frac{\rho g d_1^2}{2} = \frac{\rho g}{2} [d_2^2 - d_1^2] \qquad \ldots(ii)$$

But from momentum principle, the net force acting on a mass of fluid must be equal to the rate of change of momentum in the same section.

∴ Rate of change of momentum in the direction of force

= mass of water per sec × change of velocity in direction of force

Now mass of water per second = $\rho \times$ discharge per unit width × width

$= \rho \times q \times 1 = \rho q \text{ m}^3/\text{s}$

Change of velocity in the direction of force $= (V_1 - V_2)$

[as net force is acting from right to left, the change of velocity should be taken from right to left and hence is equal to $(V_1 - V_2)$]

∴ Rate of change of momentum in the direction of force $= \rho q(V_1 - V_2)$ ...(*iii*)

Hence according to momentum principle, the expression given by equation (*ii*) is equal to the expression given by equation (*iii*)

or $$\frac{\rho g}{2}(d_2^2 - d_1^2) = \rho q(V_1 - V_2)$$

But from equation (*i*), $V_1 = \dfrac{q}{d_1}$ and $V_2 = \dfrac{q}{d_2}$

∴ $$\frac{\rho g}{2}(d_2^2 - d_1^2) = \rho q\left(\frac{q}{d_1} - \frac{q}{d_2}\right)$$

or $$\frac{g}{2}(d_2 + d_1)(d_2 - d_1) = q^2\left(\frac{d_2 - d_1}{d_1 d_2}\right) \quad \text{(Dividing by } \rho)$$

or $$\frac{g}{2}(d_2 + d_1) = \frac{q^2}{d_1 d_2} \quad [\text{Dividing by}(d_2 - d_1)]$$

or $$(d_2 + d_1) = \frac{2q^2}{g d_1 d_2} \quad \text{...(iv)}$$

Multiplying both sides by d_2, we get

$$d_2^2 + d_1 d_2 = \frac{2q^2}{g d_1} \quad \text{or} \quad d_2^2 + d_1 d_2 - \frac{2q^2}{g d_1} = 0 \quad \text{...(v)}$$

Equation (*v*) is a quadratic equation in d_2 and hence its solution is

$$d_2 = \frac{-d_1 \pm \sqrt{d_1^2 - 4 \times 1 \times \left(\dfrac{-2q^2}{g d_1}\right)}}{2 \times 1}$$

$$= \frac{-d_1 \pm \sqrt{d_1^2 + \dfrac{8q^2}{g d_1}}}{2} = -\frac{d_1}{2} \pm \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{g d_1}}$$

The two roots of the equation are $-\dfrac{d_1}{2}-\sqrt{\dfrac{d_1^2}{4}+\dfrac{2q^2}{gd_1}}$ and $-\dfrac{d_1}{2}+\sqrt{\dfrac{d_1^2}{4}+\dfrac{2q^2}{gd_1}}$

First root is not possible as it gives –ve depth. Hence

$$d_2 = -\frac{d_1}{2}+\sqrt{\frac{d_1^2}{4}+\frac{2q^2}{gd_1}} \quad \text{...(16.27)}$$

$$= -\frac{d_1}{2}+\sqrt{\frac{d_1^2}{4}+\frac{2\times(V_1d_1)^2}{gd_1}} \quad \{\because\ q_1 = V_1d_1\}$$

$$= -\frac{d_1}{2}+\sqrt{\frac{d_1^2}{4}+\frac{2V_1^2d_1}{g}} \quad \text{...(16.28)}$$

$\therefore$ Depth of Hydraulic jump $= (d_2 - d_1)$. ...(16.29)

16.8.2 Expression for Loss of Energy Due to Hydraulic Jump. As mentioned in Art. 16.8 that when hydraulic jump takes place, a loss of energy due to eddies formation and turbulence occurs. This loss of energy is equal to the difference of specific energies at sections 1-1 and 2-2.

Or loss of energy due to hydraulic jump,

$$h_L = E_1 - E_2$$

$$= \left(d_1+\frac{V_1^2}{2g}\right)-\left(d_2+\frac{V_2^2}{2g}\right) \quad \left(\because\ E_1 = d_1+\frac{V_1^2}{2g}\text{ and so }E_2\right)$$

$$= \left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right)-(d_2-d_1)$$

$$= \left(\frac{q^2}{2gd_1^2}-\frac{q^2}{2gd_2^2}\right)-(d_2-d_1) \quad \left(\because\ V_1=\frac{q}{d_1}\text{ and }V_2=\frac{q}{d_2}\right)$$

$$= \frac{q^2}{2g}\left[\frac{1}{d_1^2}-\frac{1}{d_2^2}\right]-[d_2-d_1] = \frac{q^2}{2g}\left[\frac{d_2^2-d_1^2}{d_1^2d_2^2}\right]-[d_2-d_1] \quad \text{...}(vi)$$

But from equation (*iv*), $q^2 = gd_1d_2\dfrac{(d_2+d_1)}{2}$

Substituting the value of q^2 in equation (*vi*), we get

Loss of energy, $h_L = gd_1d_2\dfrac{(d_2+d_1)}{2}\times\dfrac{d_2^2-d_1^2}{2gd_1^2d_2^2}-(d_2-d_1)=\dfrac{(d_2+d_1)(d_2^2-d_1^2)}{4d_1d_2}-(d_2-d_1)$

$$= \frac{(d_2+d_1)(d_2+d_1)(d_2-d_1)}{4d_1d_2}-(d_2-d_1)=(d_2-d_1)\left[\frac{(d_2+d_1)^2}{4d_1d_2}-1\right]$$

$$= (d_2 - d_1)\left[\frac{d_2^2 + d_1^2 + 2d_1d_2 - 4d_1d_2}{4d_1d_2}\right] = (d_2 - d_1)\frac{[d_2 - d_1]^2}{4d_1d_2}$$

$\therefore$
$$h_L = \frac{[d_2 - d_1]^3}{4d_1d_2}. \qquad \text{...(16.30)}$$

16.8.3 Expression for Depth of Hydraulic Jump in Terms of Upstream Froude Number.

Let V_1 = Velocity of flow on the upstream side,

and d = Depth of flow on upstream side,

Then Froude Number $(F_e)_1$ on the upstream side of the jump is given by

$$(F_e)_1 = \frac{V_1}{\sqrt{gd_1}} \qquad \text{...(vii)}$$

Now the depth of flow after the hydraulic jump is d_2 and it is given by equation (16.28) as

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2V_1^2d_1}{g}} = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4}\left(1 + \frac{8V_1^2}{gd_1}\right)}$$

$$= -\frac{d_1}{2} + \frac{d_1}{2}\sqrt{1 + \frac{8V_1^2}{gd_1}} \qquad \text{...(viii)}$$

But from equation (*vii*), $(F_e)_1 = \frac{V_1}{\sqrt{gd_1}}$ or $(F_e)_1^2 = \frac{V_1^2}{gd_1}$

Substituting this value in equation (*viii*), we get

$$d_2 = -\frac{d_1}{2} + \frac{d_1}{2}\sqrt{1 + 8(F_e)_1^2} = \frac{d_1}{2}\left(\sqrt{1 + 8(F_e)_1^2} - 1\right) \qquad \text{...(16.31)}$$

16.8.4 Length of Hydraulic Jump. This is defined as the length between the two sections where one section is taken before the hydraulic jump and the second section is taken immediately after the jump. For a rectangular channel from experiments, it has been found equal to 5 to 7 times the height of the hydraulic jump.

Problem 16.38 *The depth of flow of water, at a certain section of a rectangular channel of 4 m wide, is 0.5 m. This discharge through the channel is 16 m^3/s. If a hydraulic jump takes place on the downstream side, find the depth of flow after the jump.*

Solution. Given :

Width of channel, $b = 4$ m

Depth of flow before jump, $d_1 = 0.5$ m

Discharge, $Q = 16$ m^3/s

$\therefore$ Discharge per unit width, $q = \frac{Q}{b} = \frac{16}{4} = 4$ m^2/s

Let the depth of flow after jump = d_2

Depth of flow after the jump is given by equation (16.27), as

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} = -\frac{0.5}{2} + \sqrt{\frac{0.5^2}{4} + \frac{2 \times 4^2}{9.81 \times 0.5}}$$

$$= -0.25 + \sqrt{0.0625 + 6.5239} = -0.25 + 2.566 = \mathbf{2.316\ m.\ Ans.}$$

Problem 16.39 *The depth of flow of water, at a certain section of a rectangular channel of 2 m wide, is 0.3 m. The discharge through the channel is 1.5 m³/s. Determine whether a hydraulic jump will occur, and if so, find its height and loss of energy per kg of water.*

Solution. Given :

Depth of flow, $d_1 = 0.3$ m

Width of channel, $b = 2$ m

Discharge, $Q = 1.5\ m^3/s$

Discharge per unit width, $q = \dfrac{Q}{b} = \dfrac{1.5}{2.0} = 0.75\ m^2/s.$

Hydraulic jump will occur if the depth of flow on the upstream side is less than the critical depth on upstream side or if the Froude number on the upstream side is more than one.

Critical depth (h_c) is given by equation (16.23) as

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{0.75^2}{9.81}\right)^{1/3} = 0.3859$$

Now the depth on the upstream side is 0.3 m. This depth is less than critical depth and hence hydraulic jump will occur.

The depth of flow after hydraulic jump is given by equation (16.27) as

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} = -\frac{0.3}{2} + \sqrt{\frac{0.3^2}{4} + \frac{2\times 0.75^2}{9.81\times 0.3}}$$

$$= -1.5 + \sqrt{.0255 + .3822} = -0.15 + .6362 = 0.4862 \text{ m}$$

$\therefore$ Height of hydraulic jump $= d_2 - d_1 = 0.4862 - 0.30 =$ **0.1862 m. Ans.**

Loss of energy per kg of water due to hydraulic jump is given by equation (16.30) as

$$h_L = \frac{(d_2 - d_1)^3}{4d_1d_2} = \frac{[0.4862 - 0.30]^3}{4\times .4862\times .30}$$

$$= \frac{0.1862^3}{4\times .4862\times .30} = \mathbf{0.01106\ m\text{-}kg/kg.\ Ans.}$$

Problem 16.40 *A sluice gate discharges water into a horizontal rectangular channel with a velocity of 10 m/s and depth of flow of 1 m. Determine the depth of flow after the jump and consequent loss in total head.*

Solution. Given :

Velocity of flow before hydraulic jump, $V_1 = 10$ m/s

Depth of flow before hydraulic jump, $d_1 = 1$ m

Discharge per unit width, $q = V_1 \times d_1 = 10 \times 1 = 10\ m^2/s$

The depth of flow after jump is given by equation (16.27) as

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} = -\frac{1.0}{2} + \sqrt{\frac{1^2}{4} + \frac{2\times 10^2}{9.81\times 1}}$$

$$= -0.50 + \sqrt{0.25 + 20.387} = \mathbf{4.043\ m.\ Ans.}$$

Loss in total head is given by equation (16.30) as

$$h_L = \frac{(d_2 - d_1)^3}{4d_1d_2} = \frac{(4.043 - 1.0)^3}{4\times 1.0\times 4.043} = \mathbf{1.742\ m.\ Ans.}$$

Problem 16.41 *A sluice gate discharges water into a horizontal rectangular channel with a velocity of 6 m/s and depth of flow is 0.4 m. The width of the channel is 8 m. Determine whether a hydraulic jump will occur, and if so, find its height and loss of energy per kg of water. Also determine the power lost in the hydraulic jump.*

Solution. (*i*) Given :

Velocity of flow, $V_1 = 6$ m/s

Depth of flow, $d_1 = 0.4$ m

Width of channel, $b = 8$ m

$\therefore$ Discharge per unit width, $q = \dfrac{Q}{b} = \dfrac{V_1 \times \text{area}}{b} = \dfrac{V_1 \times d_1 \times b}{b}$

$= V_1 \times d_1 = 6 \times 0.4 = 2.4 \text{ m}^2/\text{s}$

Froude number on the upstream side,

$$(F_e)_1 = \frac{V_1}{\sqrt{gd_1}} = \frac{6.0}{\sqrt{9.81 \times 0.4}} = 3.0289 \simeq 3.029.$$

As Froude number is more than one, the flow is shooting on the upstream side. Shooting flow is unstable flow and it will convert itself into streaming flow by raising its height and hence hydraulic jump will take place.

(*ii*) Let the depth of hydraulic jump = d_2

Using equation (16.31), we have

$$d_2 = \frac{d_1}{2}\left(\sqrt{1 + 8(F_e)_1^2} - 1\right) = \frac{0.4}{2}\left(\sqrt{1 + 8 \times 3.029^2} - 1\right) = 1.525 \text{ m}$$

$\therefore$ Height of hydraulic jump $= d_2 - d_1 = 1.525 - 0.4 =$ **1.125 m. Ans.**

(*iii*) Loss of energy per kg of water is given by equation (16.30)

$$h_L = \frac{(d_2 - d_1)^3}{4d_1 d_2} = \frac{(1.525 - 0.4)^3}{4 \times 0.4 \times 1.525} = \textbf{0.5835 m-kg/kg of water. Ans.}$$

(*iv*) Power lost in kW $= \dfrac{\rho g \times Q \times h_L}{1000}$, where $Q = V \times$ area

$= V_1 \times d_1 \times b = 6 \times 0.4 \times 8 = 19.2 \text{ m}^3/\text{s}$

$\therefore$ Power, $P = \dfrac{1000 \times 9.81 \times 19.2 \times 0.5835}{1000} =$ **109.9 kW. Ans.**

Problem 16.42 *A hydraulic jump forms at the downstream end of spillway carrying 17.93 m³/s discharge. If the depth before jump is 0.80 m, determine the depth after the jump and energy loss.*

Solution. Given :

Discharge $Q = 17.93 \text{ m}^3/\text{s}$

Depth before jump, $d_1 = 0.8$ m

Taking width $b = 1$ m, we get

Discharge per unit width, $q = \dfrac{17.93}{1} = 17.93$

Let d_2 = Depth after jump and h_L = Loss of energy.

Using equation (16.27), we get $d_2 = -\dfrac{d_1}{2} + \sqrt{\dfrac{d_1^2}{4} + \dfrac{2q^2}{gd_1}} = -\dfrac{0.8}{2} + \sqrt{\dfrac{0.8^2}{4} + \dfrac{2 \times 17.93^2}{9.81 \times 0.8}}$

$$= -0.4 + \sqrt{0.16 + 81.927} = -0.4 + 9.06 = \mathbf{8.66\ m.\ Ans.}$$

Using equation (16.30) for loss of energy,

$$h_L = \frac{(d_2 - d_1)^3}{4d_1d_2} = \frac{(8.66 - 0.8)^3}{4 \times 0.8 \times 8.66} = \mathbf{17.52\ m.\ Ans.}$$

▶ 16.9 GRADUALLY VARIED FLOW (G.V.F.)

If the depth of flow in a channel changes gradually over a long length of the channel, the flow is said to be gradually varied flow and is denoted by G.V.F.

16.9.1 Equation of Gradually Varied Flow. Before deriving an equation for gradually varied flow, the following assumptions are made :

1. The bed slope of the channel is small,
2. The flow is steady and hence discharge Q is constant,
3. Accelerative effect is negligible and hence hydrostatic pressure distribution prevails over channel cross-section.
4. The energy correction factor, α is unity.
5. The roughness co-efficient is constant for the length of the channel and it does not depend on the depth of flow.
6. The formulae, such as Chezy's formula, Manning's formula, which are applicable, to the uniform flow are also applicable to the gradually varied flow for determining the slope of energy line.
7. The channel is prismatic.

Consider a rectangular channel having gradually varied flow as shown in Fig. 16.29. The depth of flow is gradually decreasing in the direction of flow.

Let Z = height of bottom of channel above datum

h = depth of flow,

V = mean velocity of flow,

i_b = slope of the channel bed,

i_e = slope of the energy line,

b = width of channel, and

Q = discharge through the channel

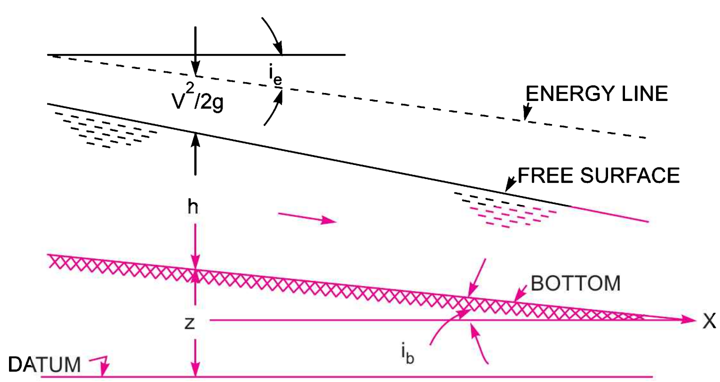

Fig. 16.29 *Equation for gradually varied flow.*

The energy equation at any section is given by Bernoulli's equation,

$$E = Z + h + \frac{V^2}{2g} \qquad ...(i)$$

Differentiating this equation with respect to x, where x is measured along the bottom of the channel in the direction of flow, we get

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dh}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \qquad ...(ii)$$

Now $$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{d}{dx}\left(\frac{Q^2}{A^2 \times 2g}\right) \qquad \left(\because\ V = \frac{Q}{A} = \frac{Q}{b \times h}\right) \text{ (as } A = b \times h)$$

$$= \frac{d}{dx}\left(\frac{Q^2}{b^2h^2 \times 2g}\right) = \frac{Q^2}{b^2 \times 2g}\frac{d}{dx}\left(\frac{1}{h^2}\right) \qquad (\because\ Q, b \text{ and } g \text{ are constant})$$

$$= \frac{Q^2}{b^2 \times 2g}\frac{d}{dh}\left[\frac{1}{h^2}\right]\frac{dh}{dx} = \frac{Q^2}{b^2 \times 2g}\left[\frac{-2}{h^3}\right]\frac{dh}{dx} = \frac{-2Q^2}{b^2 \times 2gh^3}\frac{dh}{dx}$$

$$= -\frac{Q^2}{b^2h^2 \times gh}\frac{dh}{dx} = -\frac{V^2}{gh}\frac{dh}{dx} \qquad \left[\because\ \frac{Q}{bh} = V\right]$$

Substituting the value of $\frac{d}{dx}\left(\frac{V^2}{2g}\right)$ in equation (ii), we get

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dh}{dx} - \frac{V^2}{gh}\frac{dh}{dx} = \frac{dZ}{dx} + \frac{dh}{dx}\left[1 - \frac{V^2}{gh}\right] \qquad ...(iii)$$

Now $$\frac{dE}{dx} = \text{slope of the energy line} = -\,i_e$$

and $$\frac{dZ}{dx} = \text{slope of the bed of the channel} = -\,i_b$$

–ve sign with i_e and i_b is taken as with the increase of x, the value of E and Z decreases.

Substituting the value of $\frac{dE}{dx}$ and $\frac{dZ}{dx}$ in equation (iii), we get

$$-\,i_e = -\,i_b + \frac{dh}{dx}\left[1 - \frac{V^2}{gh}\right] \quad \text{or} \quad i_b - i_e = \frac{dh}{dx}\left[1 - \frac{V^2}{gh}\right]$$

or $$\frac{dh}{dx} = \frac{(i_b - i_e)}{\left[1 - \frac{V^2}{gh}\right]} \qquad ...(16.32)$$

$$= \frac{(i_b - i_e)}{[1 - (F_e)^2]} \qquad \left[\because\ \frac{V}{\sqrt{gh}} = F_e\right] \quad ...(16.33)$$

As h is the depth of flow and x is the distance measured along the bottom of the channel hence $\frac{dh}{dx}$ represents the variation of the water depth along the bottom of the channel.

This is also called the slope of the free water surface. Thus :

(*i*) When $\frac{dh}{dx} = 0$, h is constant or depth of the water above the bottom of channel is constant. It means that free surface of water is parallel to the bed of the channel.

(*ii*) When $\frac{dh}{dx} > 0$ or $\frac{dh}{dx}$ is +ve, it means the depth of water increases in the direction of flow. The profile of the water so obtained is called *back water curve.*

(*iii*) When $\frac{dh}{dx} < 0$ or $\frac{dh}{dx}$ is –ve, it means that the depth of water decreases in the direction of flow. The profile of the water so obtained is called *drop down curve.*

Problem 16.43 *Find the rate of change of depth of water in a rectangular channel of 10 m wide and 1.5 m deep, when the water is flowing with a velocity of 1 m/s. The flow of water through the channel of bed slope 1 in 4000, is regulated in such a way that energy line is having a slope of .00004.*

Solution. Given :

Width of channel, $b = 10$ m

Depth of channel, $h = 3$ m

Velocity of flow, $V = 1$ m/s

Bed slope, $i_b = \frac{1}{4000} = .00025$

Slope of energy line, $i_e = .00004$

Let rate of change of depth of water $= \frac{dh}{dx}$

Using equation (16.32) as $\frac{dh}{dx} = \frac{(i_b - i_e)}{\left(1 - \frac{V^2}{gh}\right)} = \frac{.00025 - .00004}{\left(1 - \frac{1 \times 1}{9.81 \times 3}\right)} = \frac{.00021}{.966} =$ **.000217. Ans.**

Problem 16.44 *Find the slope of the free water surface in a rectangular channel of width 20 m, having depth of flow 5 m. The discharge through the channel is 50 m*^{3}*/s. The bed of the channel is having a slope of 1 in 4000. Take the value of Chezy's constant C = 60.*

Solution. Given :

Width of channel, $b = 20$ m

Depth of flow, $h = 5$ m

Discharge, $Q = 50$ m^3/s

Bed slope, $i_b = \frac{1}{4000} = .00025$

Chezy's constant, $C = 60$

The discharge, Q is given by $Q = V \times \text{Area} = C\sqrt{mi} \times A = AC\sqrt{mi}$

where A = Area of flow $= b \times h = 20 \times 5 = 100$ m^2,

$$m = \text{hydraulic mean depth} = \frac{A}{P} = \frac{100}{b+2h} = \frac{100}{20+2\times 5} = \frac{100}{30} = \frac{10}{3} \text{ m},$$

$i = i_e$ = slope of energy line.

The slope of the energy line* is determined from Chezy's formula

$$50 = 100 \times 60 \times \sqrt{\frac{10}{3} \times i_e} = 10954.45\sqrt{i_e}$$

or
$$i_e = \left(\frac{50}{10954.45}\right)^2 = 0.0000208$$

The slope of free water surface $= \dfrac{dh}{dx}$

Using equation (16.32) as
$$\frac{dh}{dx} = \frac{i_b - i_e}{1 - \dfrac{V^2}{gh}} = \frac{0.00025 - 0.0000208}{1 - \dfrac{V^2}{9.81 \times 5.0}}$$

Now
$$V = \frac{Q}{\text{Area}} = \frac{50}{b \times h} = \frac{50}{20 \times 5} = 0.5$$

$$\frac{dh}{dx} = \frac{0.00025 - 0.0000208}{1 - \dfrac{0.5 \times 0.5}{9.81 \times 5.0}} = \frac{0.0002292}{0.9949} = \mathbf{0.00023.\ Ans.}$$

16.9.2 Back Water Curve and Afflux. Consider the flow over a dam as shown in Fig. 16.30. On the upstream side of the dam, the depth of water will be rising. If there had not been any obstruction (such as dam) in the path of flow of water in the channel, the depth of water would have been constant as shown by dotted line parallel to the bed of the channel in Fig. 16.30. Due to obstruction, the water level rises and it has maximum depth from the bed at some section.

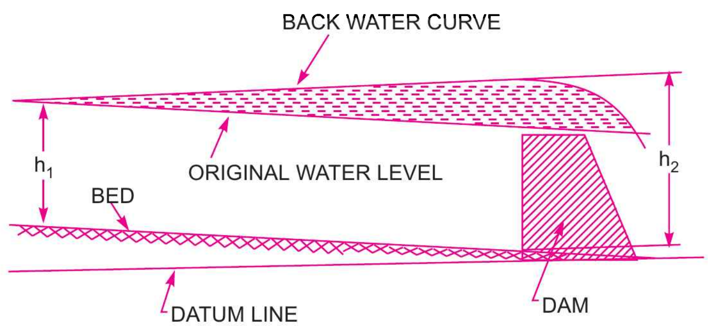

Fig. 16.30 *Back water curve and afflux.*

Let h_1 = depth of water at the point, where the water starts rising up, and

h_2 = maximum height of rising water from bed.

Then $(h_2 - h_1)$ = afflux. Thus *afflux* is defined as the maximum increase in water level due to obstruction in the path of flow of water. The profile of the rising water on the upstream side of the

* Please refer to Art. 16.9.1 point number 6.

dam is called *back water curve*. The distance along the bed of the channel between the section where water starts rising to the section where water is having maximum height is known as *length of back water curve*.

16.9.3 Expression for the Length of Back Water Curve. Consider the flow of water through a channel in which depth of water is rising as shown in Fig. 16.31. Let the two sections 1-1 and 2-2 are at such a distance that the distance between them represents the length of back water curve.

Let h_1 = depth of flow at section 1-1,

V_1 = velocity of flow at section 1-1,

h_2 = depth of flow at section 2-2,

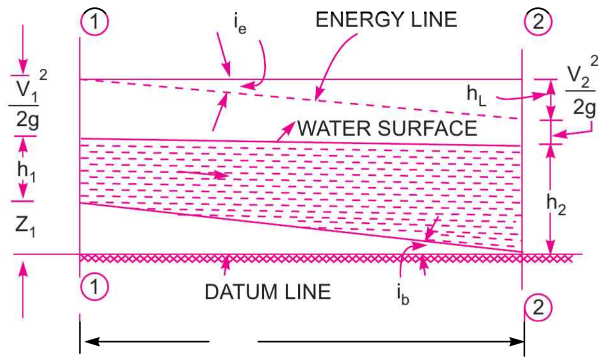

Fig. 16.31 *Length of back water curve.*

V_2 = velocity of flow at section 2-2,

i_b = bed slope,

i_e = energy line slope, and

L = length of back water curve.

Applying Bernoulli's equation at sections 1-1 and 2-2,

$$Z_1 + h_1 + \frac{V_1^2}{2g} = Z_2 + h_2 + \frac{V_2^2}{2g} + h_L \quad \text{...}(i)$$

where h_L = Loss of energy due to friction = $i_e \times L$

Also taking datum line passing through the bed of the channel at section 2-2. Then $Z_2 = 0$

$\therefore$ Equation (*i*) becomes as $Z_1 + h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + i_e \times L$

From Fig. 16.31, $Z_1 = i_b \times L$

$$\therefore \quad i_b \times L + h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + i_e \times L$$

or

$$i_b \times L - i_e \times L = \left(h_2 + \frac{V_2^2}{2g}\right) - \left(h_1 + \frac{V_1^2}{2g}\right)$$

or $$L\,(i_b - i_e) = E_2 - E_1\,, \quad \text{where } E_2 = h_2 + \frac{V_2^2}{2g},\ E_1 = h_1 + \frac{V_1^2}{2g}$$

$\therefore$ $$L = \frac{E_2 - E_1}{i_b - i_e}. \qquad \text{...(16.34)}$$

Equation (16.34) is used to calculate the length of back water curve. The value of i_e (slope of energy line) is calculated either by Manning's formula or by Chezy's formula. The mean values of velocity, depth of flow, hydraulic mean depth etc., are used between sections 1-1 and 2-2 for calculating the value of i_e.

Problem 16.45 *Determine the length of the back water curve caused by an afflux of 2.0 m in a rectangular channel of width 40 m and depth 2.5 m. The slope of the bed is given as 1 in 11000. Take Manning's N = 0.03.*

Solution. Given :

Width of channel, $b = 40$ m

Afflux, $(h_2 - h_1) = 2.0$ m

Depth of channel, $h_1 = 2.5$ m

$\therefore$ $h_2 = 2.0 + 2.5 = 4.5$ m

Bed slope, $$i_b = \frac{1}{11000} = 0.0000909$$

Manning's, $N = 0.03$

Area of flow at section 1, $A_1 = b \times h_1 = 40 \times 2.5 = 100$ m^2

Wetted perimeter, $P_1 = b + 2h_1 = 40 + 2 \times 2.5 = 45$ m

$\therefore$ Hydraulic mean depth, $$m_1 = \frac{A_1}{P_1} = \frac{100}{45} = 2.22 \text{ m}$$

Using Manning's formula, $$V = \frac{1}{N}\,.\,m^{2/3}\,i_b^{1/2}$$

$\therefore$ Velocity at section 1, $$V_1 = \frac{1}{N}\,m_1^{2/3}\,i_b^{1/2} = \frac{1}{0.03} \times 2.22^{2/3} \times 0.0000909^{1/2}$$

$$= \frac{1}{0.03} \times 1.7 \times 0.009534 = 0.54 \text{ m/s}$$

Specific energy at section 1, $$E_1 = \frac{V_1^2}{2g} + h_1 = \frac{0.54^2}{2 \times 9.81} + 2.5 = 2.5148 \text{ m}$$

From continuity, velocity at section 2 is given as

$$V_1 A_1 = V_2 \times A_2$$

$\therefore$ $$V_2 = \frac{V_1 \times A_1}{A_2} = \frac{0.54 \times 100}{b \times h_2} = \frac{0.54 \times 100}{40 \times 4.5} = 0.3 \text{ m/s}$$

where area $A_2 = b \times h_2 = 40 \times 4.5 = 180$ m^2

Wetted perimeter at section 2, $P_2 = b + 2h_2 = 40 + 2 \times 4.5 = 49$ m

$\therefore \qquad m_2 = \dfrac{A_2}{P_2} = \dfrac{180}{49} = 3.673$ m

Specific energy at section 2, $E_2 = h_2 + \dfrac{V_2^2}{2g} = 4.5 + \dfrac{0.3^2}{2 \times 9.81} = 4.504$ m

To find average velocity (V_{av}), first find average depth (h_{av}) as

$$h_{av} = \frac{h_1 + h_2}{2} = \frac{2.5 + 4.5}{2} = 3.5 \text{ m}$$

$$\therefore \qquad V_{av} = \frac{V_1 A_1}{A_{av}} = \frac{V_1 \times b \times h_1}{b \times h_{av}} = \frac{V_1 \times h_1}{h_{av}} = \frac{0.54 \times 2.5}{3.5} = 0.3857 \text{ m/s}$$

Also $\qquad m_{av} = \dfrac{m_1 + m_2}{2} = \dfrac{2.22 + 3.673}{2} = 2.9465$

To find the value of i_e, use Manning's formula as

$$V_{av} = \frac{1}{N} m_{av}^{2/3} \times i_e^{1/2}$$

or $\qquad 0.3857 = \dfrac{1}{0.03} \times 2.9465^{2/3} \times i_e^{1/2} = 68.534\, i_e^{1/2}$

or $\qquad i_e = \left(\dfrac{0.3857}{68.534}\right)^2 = 0.00003167$

The length of back water curve (L) is obtained from equation (16.34)

$$L = \frac{E_2 - E_1}{i_b - i_e} = \frac{4.504 - 2.5148}{0.0000909 - 0.00003167}$$

$$= \frac{1.9892}{0.00005923} = \mathbf{33584.3 \text{ m. Ans.}}$$

HIGHLIGHTS

1. If the depth of flow, velocity of flow, slope of the bed of channel and cross-section remain constant, the flow is called uniform, otherwise it is called non-uniform flow.
2. Non-uniform flow is also called varied flow. If the depth of flow changes abruptly over a small length of channel, the flow is called rapidly varied flow. If the depth of flow changes gradually over a long length of the channel, the flow is said to be gradually varied flow.
3. If Reynold number for open channel flow is less than 500, the flow is said to be laminar and if R_e is more than 2000, the flow is said to be turbulent.
4. If Froude number is less than 1.0, the flow is sub-critical or streaming. If $F_e = 1.0$, the flow is critical. If $F_e > 1.0$, the flow is super-critical or shooting.
5. Velocity of Chezy's formula is given as $V = C\sqrt{mi}$

where C = Chezy's constant, m = Hydraulic mean depth = $\dfrac{\text{Area}}{\text{Wetted perimeter}}$,

i = Slope of the bed.

6. The value of Chezy's constant, C is given by empirical formulae as :

(*i*) $C = \dfrac{157.6}{1.81 + \dfrac{K}{\sqrt{m}}}$ ... Bazin Formula

where K = Bazin's constant, m = Hydraulic mean depth

(*ii*) $C = \dfrac{23 + \dfrac{0.00155}{i} + \dfrac{1}{N}}{1 + \left(23 + \dfrac{0.00155}{i}\right)\dfrac{N}{\sqrt{m}}}$ ... Kutter's Formula

where N = Kutter's constant.

(*iii*) $C = \dfrac{1}{N} m^{1/6}$ ... Manning's Formula

where N = Manning's constant = Kutter's constant.

7. Most economical section is one that gives maximum discharge for a given values of cross-section area, slope of the bed and co-efficient of resistance.

8. Conditions for maximum discharge through :

(*a*) Rectangular section,

(*i*) $b = 2d$ (*ii*) $m = \dfrac{d}{2}$

(*b*) Trapezoidal section,

(*i*) Half of top width = Sloping side or $\dfrac{b + 2nd}{2} = d\sqrt{n^2 + 1}$

(*ii*) $m = \dfrac{d}{2}$

(*iii*) A semi-circle drawn from the mid-point of the top width with radius equal to depth of flow will touch the three sides of the channel.

9. Best side slope for most economical trapezoidal section is,

$$\theta = 60° \text{ or } n = \frac{1}{\sqrt{3}} = \frac{1}{\tan\theta}.$$

10. For circular sections, area cannot be maintained constant and hence there are two different conditions, one is for maximum velocity and other for maximum discharge.

11. Condition for maximum velocity through a circular channel is,

Depth of flow, d = 0.81 diameter of circular channel

Hydraulic mean depth, m = 0.305 diameter of circular channel

12. Condition for maximum discharge through a circular channel is,

Depth, d = 0.95 diameter of circular channel.

13. For a circular channel,

Wetted perimeter, $P = 2R\theta$

Area of flow, $A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$

where R = Radius of circular channel,

θ = Half the angle subtended by the water surface at the centre.

14. Total energy of a flowing liquid per unit weight, Total energy = $z + h + V^2/2g$.

15. Specific energy of a flowing liquid per unit weight,

$$E = h + \frac{V^2}{2g}, \quad \text{where } h = \text{Depth of flow, } V = \text{Velocity of flow.}$$

16. The depth of flow at which specific energy is minimum is called critical depth, which is given by

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}, \quad \text{where } q = \text{discharge per unit width} = \frac{\text{Total discharge}}{b}.$$

17. The velocity of flow at critical depth is known as critical velocity, which is given as $V_c = \sqrt{g \times h_c}$.

18. Minimum specific energy is related with critical depth by the relation, $E_{min} = \frac{3}{2} h_c$.

19. The flow corresponding to critical depth (or when Froude number is equal to 1.0) is known as critical flow.

20. If the depth of flow in a channel is greater than the critical depth (or Froude number is less than 1.0), the flow is said sub-critical or streaming flow.

21. If the depth of flow in a channel is less than the critical depth (or Froude number is more than 1.0), the flow is known as super-critical or shooting flow.

22. The condition for maximum discharge for a given value of specific energy is that the depth of flow should be critical.

23. The rise of water-level which takes place due to the transformation of the shooting to the streaming flow is known as hydraulic jump. The depth of flow after the jump is given by

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} \quad \text{... (In terms of } q)$$

$$= -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2V_1^2 d_1}{g}} \quad \text{... (In terms of } V_1)$$

$$= \frac{d_1}{2}\left(\sqrt{1 + 8(F_e)_1^2} - 1\right) \quad \text{... (In terms of } F_{e_1})$$

Depth of hydraulic jump, $= d_2 - d_1$

where d_1 = depth of flow before hydraulic jump,

V_1 = velocity of flow before hydraulic jump.

24. Energy lost due to hydraulic jump per kg of liquid

$$h_L = (E_1 - E_2) = \left(d_1 + \frac{V_1^2}{2g}\right) - \left(d_2 + \frac{V_2^2}{2g}\right) = \frac{(d_2 - d_1)^3}{4d_1 d_2}.$$

25. Equation of gradually varied flow, $\frac{dh}{dx} = \frac{i_b - i_e}{1 - \frac{V^2}{gh}}$ ... (In terms of V)

$$= \frac{i_b - i_e}{\left(1 - F_e^2\right)}$$ ... (In term of F_e)

where $\frac{dh}{dx}$ = slope of free water surface, i_b = bed slope,

i_e = slope of the energy line, h = depth of flow, and V = velocity of flow.

26. Afflux is the increase in water level due to some obstruction across the flowing liquid, while back water curve is the profile of the rising water on the upstream side of the obstruction.

27. Length of back water curve is given by, $L = \frac{E_2 - E_1}{i_b - i_e}$

where E_1 = Specific energy at the section, where water starts rising = $h_1 + \frac{V_1^2}{2g}$

and E_2 = Specific energy at the end of the water curve = $h_2 + \frac{V_2^2}{2g}$.

EXERCISE

(A) THEORETICAL PROBLEMS

1. What do you understand by 'Flow in open channel' ?

2. Differentiate between : (*i*) Uniform flow and non-uniform flow, (*ii*) Steady and unsteady flow, (*iii*) Laminar and turbulent flow and (*iv*) Critical, sub-critical and super-critical flow in a open channel.

3. Explain the terms : (*i*) Rapidly varied flow and (*ii*) Gradually varied flow.

4. Derive an expression for the discharge through a channel by Chezy's formula.

5. Explain the terms : (*i*) Slope of the bed, (*ii*) Hydraulic mean depth and (*iii*) Wetted perimeter.

6. (*a*) What are the empirical formulae for determining the value of Chezy's constant ?

(*b*) What is the relation between Manning's constant and Chezy's constant.

7. State the following formulae for the values of C :

(*i*) Bazin's formulae, (*ii*) Kutter's formula, and (*iii*) Manning's formula.

8. (*a*) Define the term most economical section of a channel. What are the conditions for the rectangular channel of the best section ?

(*b*) What is meant by an economical section of a channel ?

9. Prove that for the trapezoidal channel of most economical section :

(*i*) Half of top width = Length of one of the sloping sides

(*ii*) Hydraulic mean depth = $\frac{1}{2}$ depth of flow.

10. (*a*) Derive the condition for the best side slope of the most economical trapezoidal channel.

(*b*) Find the side slope in a trapezoidal section of maximum efficiency which will carry the same flow as a half square section of the same area.

11. Prove that for a channel of circular section, the depth of flow,

$d = 0.81\ D$ for maximum velocity, and

$= 0.95\ D$ for maximum discharge,

where D = Diameter of circular channel, d = depth of flow.

12. Explain the terms : Specific energy of a flowing liquid, minimum specific energy, critical depth, critical velocity and alternate depths as applied to non-uniform flow.

13. What is specific energy curve ? Draw specific energy curve, and then derive expressions for critical depth and critical velocity.

14. (*a*) Derive an expression for critical depth and critical velocity.

(*b*) Define critical depth in an open channel in as many ways as you can.

15. Derive the condition for maximum discharge for a given value of specific energy.

16. Explain the term hydraulic jump. Derive an expression for the depth of hydraulic jump in terms of the upstream Froude number.

17. Derive an expression for the variation of depth along the length of the bed of the channel for gradually varied flow in an open channel. State clearly all the assumptions made.

18. Find an expression for loss of energy head for a hydraulic jump.

19. Define the terms : (*i*) Afflux and (*ii*) Back water curve. Prove that the length of the back water curve is given by,

$$L = \frac{(E_2 - E_1)}{i_b - i_e}$$

where L = Length of back water curve,

E_2 = Specific energy at the end of back water curve,

E_1 = Specific energy at the section where water starts rising,

i_b = Slope of bed, and i_e = Slope of the energy line.

20. Find, in terms of specific energy E, an expression for the critical depth in a trapezoidal channel with bottom width B and side slopes of 1 vertical to n horizontal.

21. Show that in a rectangular channel :

(*i*) Critical depth is two-third of specific energy, and

(*ii*) Froude number at critical depth is unity.

22. Obtain the condition for a trapezoidal channel with side slopes 2 H : 1 V to be most efficient for a given area A let B be its bed width.

23. By applying the momentum equation to open channel flow, show that the consequent depths and flow rate are related by $2q^2/g = y_1 y_2 (y_1 + y_2)$.

State the assumptions made in the derivation.

24. Derive the differential equation for steady gradually varied flow in open channels and list all assumptions.

$$\frac{dh}{dx} = \frac{(i_b - i_e)}{(1 - F_e^2)}.$$

25. What is the essential difference between gradually varied flow and rapidly varied flow ? Illustrate with neatly drawn sketches.

26. (*a*) Prove that the loss of energy head in a hydraulic jump is equal to $(d_2 - d_1)^3/4d_1d_2$, where d_1 and d_2 are the conjugate depths.

(*b*) Obtain the relationship between the Froude Numbers of flow before and after the hydraulic jump in a horizontal rectangular channel.

(B) NUMERICAL PROBLEMS

1. Find the velocity of flow and rate of flow of water through a rectangular channel of 5 m wide and 2 m deep, when it is running full. The channel is having bed slope of 1 in 3000. Take Chezy's constant $C = 50$. [**Ans.** 0.962 m/s, 9.62 m^3/s]
2. A flow of water of 150 litres per second flows down in a rectangular flume of width 70 cm and having adjustable bottom slope. If Chezy's constant C is 60, find the bottom slope necessary for uniform flow with a depth of flow of 40 cm. Also find the conveyance K of the flume. [**Ans.** 1 in 2341.5, 7.258]
3. Find the discharge through a trapezoidal channel of width 6 m and side slope of 1 horizontal to 3 vertical. The depth of flow of water is 3 m and Chezy's constant, $C = 60$. The slope of the bed of the channel is given 1 in 5000. [**Ans.** 23.26 m^3/s]
4. Find the rate of flow of water through a V-shaped channel having total angle between the sides as 60°. Take the value of $C = 50$ and slope of the bed 1 in 1500. The depth of flow is 6 m. [**Ans.** 32.864 m^3/s]
5. Find the discharge through a rectangular channel 3 m wide, having depth of water 2 m and bed slope as 1 in 1500. Take the value of $K = 2.36$ in Bazin's formula. [**Ans.** 5.184 m^3/s]
6. Find the discharge through the rectangular channel given in the above question, taking the value of $N = 0.012$ in Manning's formula. [**Ans.** 11.64 m^3/s]
7. Find the bed slope of trapezoidal channel of bed width 3 m, depth of water 2.5 m and side slope of 2 horizontal to 3 vertical, when the discharge through the channel is 10 m^3/s. Taking the value of $N = 0.03$ in Manning's formula

$$C = \frac{1}{N} m^{1/6}.$$

[**Ans.** 1 in 1803]
8. Find the diameter of a circular sewer pipe which is laid at a slope of 1 in 10000 and carries a discharge of 1000 litres/s when flowing half full. Take the value of Manning's $N = 0.02$. [**Ans.** 2.6 m]
9. A rectangular channel carries water at the rate of 500 litres/s when bed slope is 1 in 3000. Find the most economical dimensions of the channel if $C = 60$. [**Ans.** $b = 1.272$ m, $d = 0.636$ m]
10. A trapezoidal channel has side slopes of 1 horizontal to 2 vertical and the slope of the bed is 1 in 2000. The area of the section is 42 m^2. Find the dimensions of the section if it is most economical. Determine the discharge of the most economical section if $C = 60$. [**Ans.** $d = 4.918$ m, $b = 6.08$ m, $Q = 88.36$ m^3/s]
11. A trapezoidal channel with side slopes of 1 to 1 has to be designed to convey 9 m^3/s at a velocity of 1.5 m/s so that the amount of concrete lining for the bed and sides is the minimum. Calculate the area of lining required for one metre length of canal. [**Ans.** 6.62 m^2]
12. A power canal of trapezoidal section has to be excavated through hard clay at the least cost. Determine the dimensions of the channel given, discharge equal to 15 m^3/s, bed slope 1 : 2000 and Manning's, $N = 0.020$. [**Ans.** $b = 2.956$ m, $d = 2.56$ m]
13. Find the discharge through a circular pipe of diameter 4.0 m, if the depth of water in the pipe is 1.33 m and pipe is laid at a slope of 1 in 1500. Take the value of Chezy's constant = 60. [**Ans.** 4.89 m^3/s]
14. Water is flowing through a circular channel at the rate of 500 litres/s. The depth of water in the channel is 0.7 times the diameter and the slope of the bed of the channel is 1 in 8000. Find the diameters of the circular channel if the value of Manning's, $N = 0.015$. [**Ans.** 1.425 m]
15. The rate of flow of water through a circular channel of diameter 0.8 m is 200 litres/s. Find the slope of the bed of the channel for maximum velocity. Take $C = 50$. [**Ans.** 1/2787]
16. Determine the maximum discharge of water through a circular channel of diameter 2.0 m when the bed slope of the channel is 1 in 1500. Take $C = 50$. [**Ans.** 3.02 m^3/s]
17. The discharge of water through a rectangular channel of width 6 m, is 18 m^3/s when depth of flow of water is 2 m. Calculate : (*i*) specific energy of the flowing water, (*ii*) critical depth and critical velocity and (*iii*) value of minimum specific energy. [**Ans.** (*i*) 2.115 m, (*ii*) 0.971 m, 3.087 m/s, (*iii*) 1.457 m]

18. The specific energy for a 6 m wide rectangular channel is to be 5 kg-m/kg. If the rate of flow of water through the channel is 24 m^2/s, determine the alternate depths of flow. [**Ans.** 4.831 m, 0.169 m]

19. The depth of flow of water, at a certain section of a rectangular channel of 5 m wide is 0.6 m. The discharge through the channel is 15 m^3/s. If a hydraulic jump takes place on the downstream side, find the depth of flow after the jump. [**Ans.** 1.474 m]

20. For the Question 19, find the loss of energy per kg of water due to hydraulic jump. [**Ans.** 0.188 m]

21. A sluice gate discharges water into a horizontal rectangular channel with a velocity of 8 m/s and depth of flow is 0.5 m. The width of the channel is 6 m. Determine whether a hydraulic jump will occur, and if so, find its height and loss of energy per kg of water. Also determine the horse power lost in the hydraulic jump. [**Ans.** Yes, 1.816 m, 1.293 m, 413.76 h.p.]

22. Find the rate of change of depth of water in a rectangular channel of 12 m wide and 2 m deep, when the water is flowing with a velocity of 1.5 m/s. The flow of water through the channel of bed slope 1 in 300, is regulated in such a way that energy line is having a slope of 1 in 8000. [**Ans.** 0.000235]

23. Find the slope of the free water surface in a rectangular channel of width 15 m, having depth of flow 4 m. The discharge through the channel is 40 m^3/s. The bed of the channel is having a slope of 1 in 4000. Take the value of Chezy's constant, $C = 50$. [**Ans.** 0.000184]

24. Determine the length of the back water curve caused by an afflux of 1.5 m in a rectangular channel of width 50 m and depth 2.0 m. The slope of the bed is given as 1 in 2000. Take Manning's, $N = 0.03$.

[**Ans.** 4566 m]

25. A trapezoidal channel with bottom slope 0.000169, bottom width 10 m and side slopes 1 : 1 carries 20 m^3/s when Manning's constant = 0.015. Determine the normal depth.

[**Hint.** $i = 0.000169$, $b = 10$ m, $n = 1$, $N = 0.015$, $Q = 20\ m^3/s$

Use
$$Q = \frac{1}{N} m^{2/3} \times i^{1/2} \times A, \text{ where } A = (b + nd) \times d = (10 + d)\, d$$

$$P = b + 2d\sqrt{1+n^2} = 10 + 2 \times \sqrt{2} \times d,\ m = \frac{(10+d)\, d}{(10 + 2\sqrt{2} \times d)}$$

$$\therefore \quad 20 = \frac{1}{0.015} \times \left[\frac{(10+d)\, d}{(10+2\sqrt{2}\, d)}\right]^{2/3} \times 0.000169^{1/2} \times (10+d)\, d$$

or
$$\frac{[(10+d)\, d]^{5/3}}{[10+2\sqrt{2}\, d]^{2/3}} = 23. \text{ Find '}d\text{' by hit and trial.}$$
[$d = 1.65$ m]]