11

CHAPTER

FLOW THROUGH PIPES

▶ 11.1 INTRODUCTION

In chapters 9 and 10, laminar flow and turbulent flow have been discussed. We have seen that when the Reynolds number is less than 2000 for pipe flow, the flow is known as laminar flow whereas when the Reynolds number is more than 4000, the flow is known as turbulent flow. In this chapter, the turbulent flow of fluids through pipes running full will be considered. If the pipes are partially full as in the case of sewer lines, the pressure inside the pipe is same and equal to atmospheric pressure. Then the flow of fluid in the pipe is not under pressure. This case will be taken in the chapter of flow of water through open channels. Here we will consider flow of fluids through pipes under pressure only.

▶ 11.2 LOSS OF ENERGY IN PIPES

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :

Energy Losses

1. Major Energy Losses

   This is due to friction and it is calculated by the following formulae :
   (*a*) Darcy-Weisbach Formula
   (*b*) Chezy's Formula

2. Minor Energy Losses

   This is due to
   (*a*) Sudden expansion of pipe
   (*b*) Sudden contraction of pipe
   (*c*) Bend in pipe
   (*d*) Pipe fittings etc.
   (*e*) An obstruction in pipe.

▶ 11.3 LOSS OF ENERGY (OR HEAD) DUE TO FRICTION

(*a*) **Darcy-Weisbach Formula.** The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation which has been derived in chapter 10 and is given by

$$h_f = \frac{4 . f . L . V^2}{d \times 2g} \qquad ...(11.1)$$

where h_f = loss of head due to friction

f = co-efficient of friction which is a function of Reynolds number

$= \dfrac{16}{R_e}$ for $R_e < 2000$ (viscous flow)

$= \dfrac{0.079}{R_e^{1/4}}$ for R_e varying from 4000 to 10^6

L = length of pipe,

V = mean velocity of flow,

d = diameter of pipe.

(*b*) **Chezy's Formula for loss of head due to friction in pipes.** Refer to chapter 10 article 10.3.1 in which expression for loss of head due to friction in pipes is derived. Equation (*iii*) of article 10.3.1, is

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \qquad \text{...(11.2)}$$

where h_f = loss of head due to friction, P = wetted perimeter of pipe,
A = area of cross-section of pipe, L = length of pipe,
and V = mean velocity of flow.

Now the ratio of $\dfrac{A}{P}\left(= \dfrac{\text{Area of flow}}{\text{Perimeter (wetted)}}\right)$ is called hydraulic mean depth or hydraulic radius and is denoted by m.

$\therefore$ Hydraulic mean depth, $m = \dfrac{A}{P} = \dfrac{\frac{\pi}{4} d^2}{\pi d} = \dfrac{d}{4}$

Substituting $\dfrac{A}{P} = m$ or $\dfrac{P}{A} = \dfrac{1}{m}$ in equation (11.2), we get

$$h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{1}{m} \text{ or } V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L} = \frac{\rho g}{f'} \times m \times \frac{h_f}{L}$$

$$\therefore \quad V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}} = \sqrt{\frac{\rho g}{f'}} \sqrt{m \frac{h_f}{L}} \qquad \text{...(11.3)}$$

Let $\sqrt{\dfrac{\rho g}{f'}} = C$, where C is a constant known as Chezy's constant and $\dfrac{h_f}{L} = i$, where i is loss of head per unit length of pipe.

Substituting the values of $\sqrt{\dfrac{\rho g}{f'}}$ and $\sqrt{\dfrac{h_f}{L}}$ in equation (11.3), we get

$$V = C \sqrt{mi} \qquad \text{...(11.4)}$$

Equation (11.4) is known as Chezy's formula. Thus the loss of head due to friction in pipe from Chezy's formula can be obtained if the velocity of flow through pipe and also the value of C is known. The value of m for pipe is always equal to $d/4$.

Problem 11.1 *Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m, through which water is flowing at a velocity of 3 m/s using (i) Darcy formula, (ii) Chezy's formula for which C = 60.*

Take ν *for water = 0.01 stoke.*

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Length of pipe, $L = 50 \text{ m}$

Velocity of flow, $V = 3 \text{ m/s}$

Chezy's constant, $C = 60$

Kinematic viscosity, $\nu = 0.01 \text{ stoke} = 0.01 \text{ cm}^2/\text{s}$

$= 0.01 \times 10^{-4} \text{ m}^2/\text{s}.$

(*i*) **Darcy Formula** is given by equation (11.1) as

$$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$$

where 'f' = co-efficient of friction is a function of Reynolds number, R_e

But R_e is given by $\quad R_e = \dfrac{V \times d}{\nu} = \dfrac{3.0 \times 0.30}{.01 \times 10^{-4}} = 9 \times 10^5$

$\therefore$ Value of $\quad f = \dfrac{0.079}{R_e^{1/4}} = \dfrac{0.079}{\left(9 \times 10^5\right)^{1/4}} = .00256$

$\therefore$ Head lost, $\quad h_f = \dfrac{4 \times .00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81} =$ **.7828 m. Ans.**

(*ii*) **Chezy's Formula.** Using equation (11.4)

$$V = C\sqrt{mi}$$

where $C = 60$, $m = \dfrac{d}{4} = \dfrac{0.30}{4} = 0.075 \text{ m}$

$\therefore \quad 3 = 60\sqrt{.075 \times i}$ or $i = \left(\dfrac{3}{60}\right)^2 \times \dfrac{1}{.075} = 0.0333$

But $\quad i = \dfrac{h_f}{L} = \dfrac{h_f}{50}$

Equating the two values of i, we have $\dfrac{h_f}{50} = .0333$

$\therefore \quad h_f = 50 \times .0333 =$ **1.665 m. Ans.**

Problem 11.2 *Find the diameter of a pipe of length 2000 m when the rate of flow of water through the pipe is 200 litres/s and the head lost due to friction is 4 m. Take the value of C = 50 in Chezy's formulae.*

Solution. Given :

Length of pipe, $L = 2000$ m

Discharge, $Q = 200$ litre/s = 0.2 m^3/s

Head lost due to friction, $h_f = 4$ m

Value of Chezy's constant, $C = 50$

Let the diameter of pipe = d

Velocity of flow, $$V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{0.2}{\frac{\pi}{4}d^2} = \frac{0.2 \times 4}{\pi d^2}$$

Hydraulic mean depth, $$m = \frac{d}{4}$$

Loss of head per unit length, $$i = \frac{h_f}{L} = \frac{4}{2000} = .002$$

Chezy's formula is given by equation (11.4) as $V = C\sqrt{mi}$

Substituting the values of V, m, i and C, we get

$$\frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times .002} \text{ or } \sqrt{\frac{d}{4} \times .002} = \frac{0.2 \times 4}{\pi d^2 \times 50} = \frac{.00509}{d^2}$$

Squaring both sides, $$\frac{d}{4} \times .002 = \frac{.00509^2}{d^4} = \frac{.0000259}{d^4} \text{ or } d^5 = \frac{4 \times .0000259}{.002} = 0.0518$$

$$\therefore \quad d = \sqrt[5]{0.0518} = (.0518)^{1/5} = 0.553 \text{ m} = \mathbf{553\ mm.\ Ans.}$$

Problem 11.3 *A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300 mm at the rate of 300 litres per sec. Find the head lost due to friction for a length of 50 m of the pipe.*

Solution. Given :

Kinematic viscosity, $\nu = 0.4$ stoke = 0.4 cm^2/s = $.4 \times 10^{-4}\ m^2/s$

Dia. of pipe, $d = 300$ mm = 0.30 m

Discharge, $Q = 300$ litres/s = 0.3 m^3/s

Length of pipe, $L = 50$ m

Velocity of flow, $$V = \frac{Q}{\text{Area}} = \frac{0.3}{\frac{\pi}{4}(0.3)^2} = 4.24 \text{ m/s}$$

$\therefore$ Reynolds number, $$R_e = \frac{V \times d}{\nu} = \frac{4.24 \times 0.30}{0.4 \times 10^{-4}} = 3.18 \times 10^4$$

As R_e lies between 4000 and 100000, the value of f is given by

$$f = \frac{.079}{(R_e)^{1/4}} = \frac{.079}{(3.18 \times 10^4)^{1/4}} = .00591$$

$\therefore$ Head lost due to friction, $h_f = \dfrac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g} = \dfrac{4 \times .00591 \times 50 \times 4.24^2}{0.3 \times 2 \times 9.81}$ = **3.61 m. Ans.**

Problem 11.4 *An oil of sp. gr. 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 litres/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m. Take* ν *= .29 stokes.*

Solution. Given :

Sp. gr. of oil, $S = 0.7$

Dia. of pipe, $d = 300$ mm $= 0.3$ m

Discharge, $Q = 500$ litres/s $= 0.5$ m^3/s

Length of pipe, $L = 1000$ m

Velocity, $$V = \frac{Q}{\text{Area}} = \frac{0.5}{\frac{\pi}{4} d^2} = \frac{0.5 \times 4}{\pi \times 0.3^2} = 7.073 \text{ m/s}$$

$\therefore$ Reynolds number, $$R_e = \frac{V \times d}{\nu} = \frac{7.073 \times 0.3}{0.29 \times 10^{-4}} = 7.316 \times (10)^4$$

$\therefore$ Co-efficient of friction, $$f = \frac{.079}{R_e^{1/4}} = \frac{0.79}{\left(7.316 \times 10^4\right)^{1/4}} = .0048$$

$\therefore$ Head lost due to friction, $$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times .0048 \times 1000 \times 7.073^2}{0.3 \times 2 \times 9.81} = 163.18 \text{ m}$$

Power required $$= \frac{\rho g \,.\, Q \,.\, h_f}{1000} \text{ kW}$$

where ρ = density of oil = 0.7 × 1000 = 700 kg/m^3

$\therefore$ Power required $= \dfrac{700 \times 9.81 \times 0.5 \times 163.18}{1000}$ = **560.28 kW. Ans.**

Problem 11.5 *Calculate the discharge through a pipe of diameter 200 mm when the difference of pressure head between the two ends of a pipe 500 m apart is 4 m of water. Take the value of 'f' = 0.009 in the formula* $h_f = \dfrac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$.

Solution. Given :

Dia. of pipe, $d = 200$ mm $= 0.20$ m

Length of pipe, $L = 500$ m

Difference of pressure head, $h_f = 4$ m of water

$f = .009$

Using equation (11.1), we have $$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$$

or $$4.0 = \frac{4 \times .009 \times 500 \times V^2}{0.2 \times 2 \times 9.81} \text{ or } V^2 = \frac{4.0 \times 0.2 \times 2 \times 9.81}{4.0 \times .009 \times 500} = 0.872$$

$\therefore$ $V = \sqrt{0.872} = 0.9338 \simeq 0.934$ m/s

$\therefore$ Discharge, Q = velocity × area

$= 0.934 \times \frac{\pi}{4} d^2 = 0.934 \times \frac{\pi}{4} (0.2)^2$

$= 0.0293$ m^3/s = **29.3 litres/s. Ans.**

Problem 11.6 *Water is flowing through a pipe of diameter 200 mm with a velocity of 3 m/s. Find the head lost due to friction for a length of 5 m if the co-efficient of friction is given by $f = 0.02 + \frac{.09}{R_e^{0.3}}$, where R_e is Reynolds number. The kinematic viscosity of water = .01 stoke.*

Solution. Given :

Dia. of pipe, $d = 200$ mm $= 0.20$ m

Velocity, $V = 3$ m/s

Length, $L = 5$ m

Kinematic viscosity, $\nu = 0.01$ stoke $= .01 \times 10^{-4}$ m^2/s

$\therefore$ Reynolds number, $R_e = \frac{V \times d}{\nu} = \frac{3 \times 0.20}{.01 \times 10^{-4}} = 6 \times 10^5$

Value of $f = .02 + \frac{0.9}{R_e^{0.3}} = .02 + \frac{.09}{(6 \times 10^5)^{.3}} = .02 + \frac{0.09}{54.13}$

$= .02 + .00166 = 0.02166$

$\therefore$ Head lost due to friction, $h_f = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4.0 \times .02166 \times 5.0 \times 3^2}{0.20 \times 2.0 \times 9.81}$

= 0.993 m of water. Ans.

Problem 11.7 *An oil of sp. gr. 0.9 and viscosity 0.06 poise is flowing through a pipe of diameter 200 mm at the rate of 60 litres/s. Find the head lost due to friction for a 500 m length of pipe. Find the power required to maintain this flow.*

Solution. Given :

Sp. gr. of oil $= 0.9$

Viscosity, $\mu = 0.06$ poise $= \frac{0.06}{10}$ Ns/m^2

Dia. of pipe, $d = 200$ mm $= 0.2$ m

Discharge, $Q = 60$ litres/s $= 0.06$ m^3/s

Length, $L = 500$ m

Density $\rho = 0.9 \times 1000 = 900$ kg/m^3

$\therefore$ Reynolds number, $R_e = \frac{\rho V d}{\mu} = 900 \times \frac{V \times 0.2}{\frac{0.06}{10}}$

where $V = \frac{Q}{\text{Area}} = \frac{0.06}{\frac{\pi}{4} d^2} = \frac{0.06}{\frac{\pi}{4} (.2)^2} = 1.909$ m/s $\simeq 1.91$ m/s

$$\therefore \qquad R_e = 900 \times \frac{1.91 \times 0.2 \times 10}{0.06} = 57300$$

As R_e lies between 4000 and 10^5, the value of co-efficient of friction, f is given by

$$f = \frac{0.079}{R_e^{0.25}} = \frac{0.079}{(57300)^{0.25}} = .0051$$

$$\therefore \text{ Head lost due to friction, } h_f = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times .0051 \times 500 \times 1.91^2}{0.2 \times 2 \times 9.81}$$

$$= \mathbf{9.48 \text{ m of water. Ans.}}$$

$$\therefore \text{ Power required } = \frac{\rho g \cdot Q \cdot h_f}{1000} = \frac{900 \times 9.81 \times 0.06 \times 9.48}{1000} = \mathbf{5.02 \text{ kW. Ans.}}$$

▶ 11.4 MINOR ENERGY (HEAD) LOSSES

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases :

1. Loss of head due to sudden enlargement,
2. Loss of head due to sudden contraction,
3. Loss of head at the entrance of a pipe,
4. Loss of head at the exit of a pipe,
5. Loss of head due to an obstruction in a pipe,
6. Loss of head due to bend in the pipe,
7. Loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in case of a short pipe, these losses are comparable with the loss of head due to friction.

11.4.1 Loss of Head Due to Sudden Enlargement. Consider a liquid flowing through a pipe which has sudden enlargement as shown in Fig. 11.1. Consider two sections (1)-(1) and (2)-(2) before and after the enlargement.

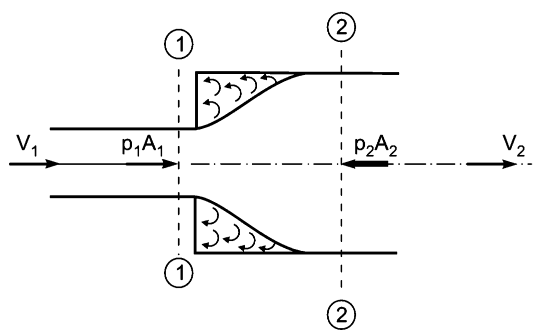


Fig. 11.1 *Sudden enlargement.*

Let p_1 = pressure intensity at section 1-1,
V_1 = velocity of flow at section 1-1,
A_1 = area of pipe at section 1-1,

p_2, V_2 and A_2 = corresponding values at section 2-2.

Due to sudden change of diameter of the pipe from D_1 to D_2, the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary. Thus the flow separates from the boundary and turbulent eddies are formed as shown in Fig. 11.1. The loss of head (or energy) takes place due to the formation of these eddies.

Let p' = pressure intensity of the liquid eddies on the area $(A_2 - A_1)$

h_e = loss of head due to sudden enlargement

Applying Bernoulli's equation at sections 1-1 and 2-2,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{loss of head due to sudden enlargement}$$

But $z_1 = z_2$ as pipe is horizontal

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

or
$$h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \quad ...(i)$$

Consider the control volume of liquid between sections 1-1 and 2-2. Then the force acting on the liquid in the control volume in the direction of flow is given by

$$F_x = p_1A_1 + p'(A_2 - A_1) - p_2A_2$$

But experimentally it is found that $p' = p_1$

$$\therefore \quad F_x = p_1A_1 + p_1(A_2 - A_1) - p_2A_2 = p_1A_2 - p_2A_2$$
$$= (p_1 - p_2)A_2 \quad ...(ii)$$

Momentum of liquid/sec at section 1-1 = mass × velocity

$$= \rho A_1 V_1 \times V_1 = \rho A_1 V_1^2$$

Momentum of liquid/sec at section 2-2 = $\rho A_2V_2 \times V_2 = \rho A_2V_2^2$

∴ Change of momentum/sec = $\rho A_2V_2^2 - \rho A_1V_1^2$

But from continuity equation, we have

$$A_1V_1 = A_2V_2 \text{ or } A_1 = \frac{A_2V_2}{V_1}$$

∴ Change of momentum/sec = $\rho A_2V_2^2 - \rho \times \frac{A_2V_2}{V_1} \times V_1^2 = \rho A_2V_2^2 - \rho A_2V_1V_2$

$$= \rho A_2[V_2^2 - V_1V_2] \quad ...(iii)$$

Now net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum or change of momentum per second. Hence equating (*ii*) and (*iii*)

$$(p_1 - p_2)A_2 = \rho A_2[V_2^2 - V_1V_2]$$

or
$$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1V_2$$

Dividing by g on both sides, we have $\frac{p_1 - p_2}{\rho g} = \frac{V_2^2 - V_1V_2}{g}$ or $\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{V_2^2 - V_1V_2}{g}$

Substituting the value of $\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right)$ in equation (i), we get

$$h_e = \frac{V_2^2 - V_1V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_2^2 + V_1^2 - 2V_1V_2}{2g} = \left(\frac{V_1 - V_2}{2g}\right)^2$$

$$\therefore \quad h_e = \frac{(V_1 - V_2)^2}{2g}. \quad \text{...(11.5)}$$

11.4.2 Loss of Head due to Sudden Contraction. Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in Fig. 11.2. Consider two sections 1-1 and 2-2 before and after contraction. As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at a section *C-C* as shown in Fig. 11.2. This section *C-C* is called Vena-contracta. After section *C-C,* a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from Vena-contracta to smaller pipe.

Let A_c = Area of flow at section *C-C*

V_c = Velocity of flow at section *C-C*

A_2 = Area of flow at section 2-2

V_2 = Velocity of flow at section 2-2

h_c = Loss of head due to sudden contraction.

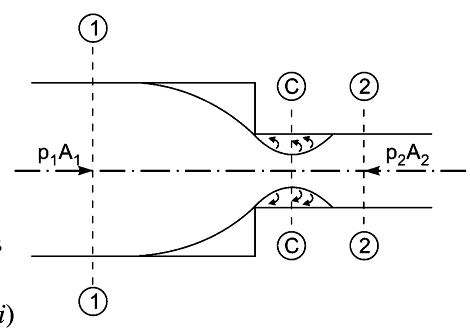


Fig. 11.2 *Sudden contraction.*

Now h_c = actual loss of head due to enlargement from section *C-C* to section 2-2 and is given by equation (11.5) as

$$= \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2 \quad \text{...}(i)$$

From continuity equation, we have

$$A_cV_c = A_2V_2 \quad \text{or} \quad \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c / A_2)} = \frac{1}{C_c} \quad \left[\because C_c = \frac{A_c}{A_2}\right]$$

Substituting the value of $\frac{V_c}{V_2}$ in (i), we get

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 \quad \text{...(11.6)}$$

$$= \frac{kV_2^2}{2g}, \text{ where } k = \left[\frac{1}{C_c} - 1\right]^2$$

If the value of C_c is assumed to be equal to 0.62, then

$$k = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$$

Then h_c becomes as $$h_c = \frac{kV_2^2}{2g} = 0.375\,\frac{V_2^2}{2g}$$

If the value of C_c is not given then the head loss due to contraction is taken as

$$= 0.5\,\frac{V_2^2}{2g} \text{ or } h_c = 0.5\,\frac{V_2^2}{2g}. \qquad ...(11.7)$$

Problem 11.8 *Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 litres/s.*

Solution. Given :

Dia. of smaller pipe, $D_1 = 200$ mm $= 0.20$ m

$\therefore$ Area, $$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.03141 \text{ m}^2$$

Dia. of large pipe, $D_2 = 400$ mm $= 0.4$ m

$\therefore$ Area, $$A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564 \text{ m}^2$$

Discharge, $Q = 250$ litres/s $= 0.25$ m^3/s

Velocity, $$V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96 \text{ m/s}$$

Velocity, $$V_2 = \frac{Q}{A_2} = \frac{0.25}{.12564} = 1.99 \text{ m/s}$$

Loss of head due to enlargement is given by equation (11.5) as

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g} = \textbf{1.816 m of water. Ans.}$$

Problem 11.9 *At a sudden enlargement of a water main from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Estimate the rate of flow.* (J.N.T.U., S 2002)

Solution. Given :

Dia. of smaller pipe, $D_1 = 240$ mm $= 0.24$ m

$\therefore$ Area, $$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.24)^2$$

Dia. of large pipe, $D_2 = 480$ mm $= 0.48$ m

$\therefore$ Area, $$A_2 = \frac{\pi}{4} (0.48)^2$$

Rise of hydraulic gradient*, *i.e.*, $$\left(z_2 + \frac{p_2}{\rho g}\right) - \left(\frac{p_1}{\rho g} + z_1\right) = 10 \text{ mm} = \frac{10}{1000} = \frac{1}{100} \text{ m}$$

Let the rate of flow $= Q$

Applying Bernoulli's equation to both sections, *i.e.*, smaller pipe section, and large pipe section.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Head loss due to enlargement} \qquad ...(i)$$

* Please refer Art. 11.5.1.

But head loss due to enlargement,

$$h_e = \frac{(V_1 - V_2)^2}{2g} \quad \text{...}(ii)$$

From continuity equation, we have $A_1V_1 = A_2V_2$

$$\therefore \quad V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{.48}{.24}\right)^2 \times V_2 = 2^2 \times V_2 = 4V_2$$

Substituting this value in (*ii*), we get

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{(3V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Now substituting the value of h_e and V_1 in equation (*i*),

$$\frac{p_1}{\rho g} + \frac{(4V_2)^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{9V_2^2}{2g}$$

or

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left(\frac{p_2}{\rho g} + z_2\right) - \left(\frac{p_1}{\rho g} + z_1\right)$$

But hydraulic gradient rise

$$= \left(\frac{p_2}{\rho g} + z_2\right) - \left(\frac{p_1}{\rho g} + z_1\right) = \frac{1}{100}$$

$$\therefore \quad \frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \frac{1}{100} \text{ or } \frac{6V_2^2}{2g} = \frac{1}{100}$$

$$\therefore \quad V_2 = \sqrt{\frac{2 \times 9.81}{6 \times 100}} = 0.1808 \simeq 0.181 \text{ m/s}$$

$\therefore$ Discharge, $Q = A_2 \times V_2$

$$= \frac{\pi}{4} D_2^2 \times V_2 = \frac{\pi}{4} (.48)^2 \times .181 = 0.03275 \text{ m}^3/\text{s}$$

= 32.75 litres/s. Ans.

Problem 11.10 *The rate of flow of water through a horizontal pipe is 0.25 m³/s. The diameter of the pipe which is 200 mm is suddenly enlarged to 400 mm. The pressure intensity in the smaller pipe is 11.772 N/cm². Determine :*

(*i*) *loss of head due to sudden enlargement,* (*ii*) *pressure intensity in the large pipe,*
(*iii*) *power lost due to enlargement.*

Solution. Given :

Discharge, $Q = 0.25 \text{ m}^3/\text{s}$

Dia. of smaller pipe, $D_1 = 200 \text{ mm} = 0.20 \text{ m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} (.2)^2 = 0.03141 \text{ m}^2$

Dia. of large pipe, $D_2 = 400 \text{ mm} = 0.40 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} (0.4)^2 = 0.12566 \text{ m}^2$

Pressure in smaller pipe, $p_1 = 11.772 \text{ N/cm}^2 = 11.772 \times 10^4 \text{ N/m}^2$

Now velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96 \text{ m/s}$

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{.12566} = 1.99 \text{ m/s}$

(*i*) Loss of head due to sudden enlargement,

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81} = \textbf{1.816 m. Ans.}$$

(*ii*) Let the pressure intensity in large pipe = p_2.

Then applying Bernoulli's equation before and after the sudden enlargement,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_e$$

But $z_1 = z_2$ (Given horizontal pipe)

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e \text{ or } \frac{p_2}{\rho g} = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$= \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{7.96^2}{2 \times 9.81} - \frac{1.99^2}{2 \times 9.81} - 1.816$$

$$= 12.0 + 3.229 - 0.2018 - 1.8160$$

$$= 15.229 - 2.0178 = 13.21 \text{ m of water}$$

$$\therefore \quad p_2 = 13.21 \times \rho g = 13.21 \times 1000 \times 9.81 \text{ N/m}^2$$

$$= 13.21 \times 1000 \times 9.81 \times 10^{-4} \text{ N/cm}^2 = \textbf{12.96 N/cm}^2\textbf{. Ans.}$$

(*iii*) Power lost due to sudden enlargement,

$$P = \frac{\rho g \cdot Q \cdot h_e}{1000} = \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000} = \textbf{4.453 kW. Ans.}$$

Problem 11.11 *A horizontal pipe of diameter 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm² and 11.772 N/cm² respectively. Find the loss of head due to contraction if $C_c = 0.62$. Also determine the rate of flow of water.*

Solution. Given :

Dia. of large pipe, $D_1 = 500 \text{ mm} = 0.5 \text{ m}$

Area, $A_1 = \frac{\pi}{4} (0.5)^2 = 0.1963 \text{ m}^2$

Dia. of smaller pipe, $D_2 = 250$ mm $= 0.25$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4}(.25)^2 = 0.04908$ m^2

Pressure in large pipe, $p_1 = 13.734$ N/cm^2 $= 13.734 \times 10^4$ N/m^2

Pressure in smaller pipe, $p_2 = 11.772$ N/cm^2 $= 11.772 \times 10^4$ N/m^2

$C_c = 0.62$

Head lost due to contraction $= \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1.0\right]^2 = \frac{V_2^2}{2g}\left[\frac{1}{0.62} - 1.0\right]^2 = 0.375\frac{V_2^2}{2g}$

From continuity equation, we have $A_1V_1 = A_2V_2$

or $$V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.25}{0.50}\right)^2 V_2 = \frac{V_2}{4}$$

Applying Bernoulli's equation before and after contraction,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_c$$

But $z_1 = z_2$ (pipe is horizontal)

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

But $h_c = 0.375\frac{V_2^2}{2g}$ and $V_1 = \frac{V_2}{4}$

Substituting these values in the above equation, we get

$$\frac{13.734 \times 10^4}{9.81 \times 1000} + \frac{(V_2/4)^2}{2g} = \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{V_2^2}{2g} + 0.375\frac{V_2^2}{2g}$$

or $$14.0 + \frac{V_2^2}{16 \times 2g} = 12.0 + 1.375\frac{V_2^2}{2g}$$

or $$14 - 12 = 1.375\frac{V_2^2}{2g} - \frac{1}{16}\frac{V_2^2}{2g} = 1.3125\frac{V_2^2}{2g}$$

or $$2.0 = 1.3125 \times \frac{V_2^2}{2g} \text{ or } V_2 = \sqrt{\frac{2.0 \times 2 \times 9.81}{1.3125}} = 5.467 \text{ m/s.}$$

(*i*) Loss of head due to contraction, $h_c = 0.375\frac{V_2^2}{2g} = \frac{0.375 \times (5.467)^2}{2 \times 9.81}$ = **0.571 m. Ans.**

(*ii*) Rate of flow of water, $Q = A_2V_2 = 0.04908 \times 5.467 = 0.2683$ m^3/s = **268.3 lit/s. Ans.**

Problem 11.12 *If in the problem 11.11, the rate of flow of water is 300 litres/s, other data remaining the same, find the value of co-efficient of contraction, C_c.*

Solution. Given :

$$D_1 = 0.5 \text{ m}, D_2 = 0.25 \text{ m}, p_1 = 13.734 \times 10^4 \text{ N/m}^2,$$
$$p_2 = 11.772 \times 10^4 \text{ N/m}^2, Q = 300 \text{ lit/s} = 0.3 \text{ m}^3\text{/s}$$

Also from Problem 11.11, $V_1 = \frac{V_2}{4}$, where $V_1 = \frac{Q}{A_1} = \frac{0.30}{\frac{\pi}{4}(0.5)^2} = 1.528$ m/s

$\therefore$ $V_2 = 4 \times V_1 = 4 \times 1.528 = 6.112$ m/s

From Bernoulli's equation, we have

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c. \qquad [\because z_1 = z_2]$$

or $$\frac{13.734 \times 10^4}{9.81 \times 1000} + \frac{(1.528)^2}{2 \times 9.81} = \frac{11.772 \times (10)^4}{9.81 \times 1000} + \frac{(6.112)^2}{2 \times 9.81} + h_c$$

or $$14.0 + 0.119 = 12.0 + 1.904 + h_c$$

$$14.119 = 13.904 + h_c$$

$\therefore$ $h_c = 14.119 - 13.904 = 0.215$

But from equation (11.6), $h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$

Hence equating the two values of h_c, we get

$$\frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = 0.215$$

$$V_2 = 6.112, \therefore \frac{6.112^2}{2 \times 9.81}\left[\frac{1}{C_c} - 1\right]^2 = 0.215$$

or $$\left[\frac{1}{C_c} - 1\right]^2 = \frac{0.215 \times 2.0 \times 9.81}{6.112 \times 6.112} = 0.1129$$

or $$\frac{1.0}{C_c} - 1.0 = \sqrt{0.1129} = 0.336 \text{ or } \frac{1.0}{C_c} = 1.0 + 0.336 = 1.336$$

$\therefore$ $C_c = \frac{1.0}{1.336} =$ **0.748. Ans.**

Problem 11.13 *A 150 mm diameter pipe reduces in diameter abruptly to 100 mm diameter. If the pipe carries water at 30 litres per second, calculate the pressure loss across the contraction. Take the co-efficient of contraction as 0.6.*

Solution. Given :

Dia. of large pipe, $D_1 = 150 \text{ mm} = 0.15 \text{ m}$

Area of large pipe, $A_1 = \frac{\pi}{4} (.15)^2 = 0.01767 \text{ m}^2$

Dia. of smaller pipe, $D_2 = 100 \text{ mm} = 1.10 \text{ m}$

Area of smaller pipe, $A_2 = \frac{\pi}{4} (.10)^2 = 0.007854 \text{ m}^2$

Discharge, $Q = 30 \text{ litres/s} = .03 \text{ m}^3\text{/s}$

Co-efficient of contraction, $C_c = 0.6$

From continuity equation, we have $A_1V_1 = A_2V_2 = Q$

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.03}{.01767} = 1.697 \text{ m/s}$$

and
$$V_2 = \frac{Q}{A_2} = \frac{.03}{.007854} = 3.82 \text{ m/s}$$

Applying Bernoulli's equation before and after contraction,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c \quad \ldots(i)$$

But $Z_1 = Z_2$

and h_c, the head loss due to contraction is given by equation (11.6) as

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = \frac{3.82^2}{2 \times 9.81}\left[\frac{1}{0.6} - 1\right]^2 = 0.33$$

Substituting these values in equation (*i*), we get

$$\frac{p_1}{\rho g} + \frac{1.697^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{3.82^2}{2 \times 9.81} + 0.33$$

or
$$\frac{p_1}{\rho g} + 0.1467 = \frac{p_2}{\rho g} + .7438 + .33$$

$$\therefore \quad \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = .7438 + .33 - .1467 = 0.9271 \text{ m of water}$$

$$\therefore \quad (p_1 - p_2) = \rho g \times 0.9271 = 1000 \times 9.81 \times 0.9271 \text{ N/m}^2$$

$$= 0.909 \times 10^4 \text{ N/m}^2 = 0.909 \text{ N/cm}^2$$

$\therefore$ Pressure loss across contraction

$= p_1 - p_2 =$ **0.909 N/cm². Ans.**

Problem 11.14 *In Fig. 11.3 below, when a sudden contraction is introduced in a horizontal pipe line from 50 cm to 25 cm, the pressure changes from 10,500 kg/m² (103005 N/m²) to 6900 kg/m² (67689 N/m²). Calculate the rate of flow. Assume co-efficient of contraction of jet to be 0.65.*

Following this if there is a sudden enlargement from 25 cm to 50 cm and if the pressure at the 25 cm section is 6900 kg/m² (67689 N/m²) what is the pressure at the 50 cm enlarged section ?

Solution. Given :

Dia. of large pipe, $D_1 = 50 \text{ cm} = 0.5 \text{ m}$

Area, $A_1 = \frac{\pi}{4} (.5)^2 = 0.1963 \text{ m}^2$

Dia. of smaller pipe, $D_2 = 25 \text{ cm} = 0.25 \text{ m}$

∴ Area, $A_2 = \frac{\pi}{4} (.25)^2 = 0.04908 \text{ m}^2$

Pressure in large pipe, $p_1 = 10500 \text{ kg/m}^2$ or 103005 N/m^2

Pressure in smaller pipe, $p_2 = 6900 \text{ kg/m}^2$ or 67689 N/m^2

Co-efficient of contraction, $C_c = 0.65$

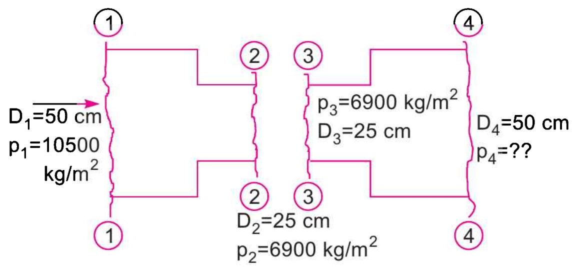


Fig. 11.3

Head lost due to contraction is given by equation (11.6),

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1.0\right)^2 = \frac{V_2^2}{2g}\left(\frac{1}{0.65} - 1\right)^2 = 0.2899\,\frac{V_2^2}{2g} \qquad ...(i)$$

From continuity equation, we have

$$A_1V_1 = A_2V_2 \text{ or } V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2}$$

$$= \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.50}{0.25}\right)^2 \times V_2 = \frac{V_2}{4} \qquad ...(ii)$$

Applying Bernoulli's equation at sections 1-1 and 2-2,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c$$

But $Z_1 = Z_2$ (as pipe is horizontal)

$$\therefore \qquad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

Substituting the values of p_1, p_2, h_c and V_1, we get

$$\frac{103005}{1000 \times 9.81} + \frac{(V_2/4)^2}{2g} = \frac{67689}{1000 \times 9.81} + \frac{V_2^2}{2g} + .2899\ \frac{V_2^2}{2g}$$

or $$10.5 + \frac{V_2^2}{16 \times 2g} = 6.9 + 1.2899\ \frac{V_2^2}{2g}$$

or $$10.5 - 6.9 = 1.2899\ \frac{V_2^2}{2g} - \frac{1}{16} \times \frac{V_2^2}{2g} = 1.2274\ \frac{V_2^2}{2g}$$

or $$3.6 = 1.2274 \times \frac{V_2^2}{2g}$$

$$\therefore \qquad V_2 = \sqrt{\frac{3.6 \times 2 \times 9.81}{1.2274}} = 7.586 \text{ m/s}$$

(*i*) Rate of flow of water, $Q = A_2 V_2 = 0.04908 \times 7.586$

$= \mathbf{0.3723\ m^3/s\ or\ 372.3\ lit/s.}$ **Ans.**

(*ii*) Applying Bernoulli's equation to sections 3-3 and 4-4,

$$\frac{p_3}{\rho g} + \frac{V_3^2}{2g} + Z_3 = \frac{p_4}{\rho g} + \frac{V_4^2}{2g} + Z_4 + \text{head loss due to sudden enlargement } (h_e)$$

But $p_3 = 6900$ kg/m^2, or 67689 N/m^2

$V_3 = V_2 = 7.586$ m/s

$$V_4 = V_1 = \frac{V_2}{4} = \frac{7.586}{4} = 1.8965$$

$Z_3 = Z_4$

And head loss due to sudden enlargement is given by equation (11.5) as

$$h_e = \frac{(V_3 - V_4)^2}{2g} = \frac{(7.586 - 1.8965)^2}{2 \times 9.81} = 1.65 \text{ m}$$

Substituting these values in Bernoulli's equation, we get

$$\frac{67689}{1000 \times 9.81} + \frac{7.586^2}{2 \times 9.81} = \frac{p_4}{1000 \times 9.81} + \frac{1.8965^2}{2 \times 9.81} + 1.65$$

or $$6.9 + 2.933 = \frac{p_4}{1000 \times 9.81} + 0.183 + 1.65$$

$$\therefore \qquad \frac{p_4}{1000 \times 9.81} = 6.9 + 2.933 - 0.183 - 1.65 = 9.833 - 1.833 = 8.00$$

$$\therefore \qquad p_4 = 8 \times 1000 \times 9.81 = \mathbf{78480\ N/m^2.\ Ans.}$$

11.4.3 Loss of Head at the Entrance of a Pipe. This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir. This loss is similar to the loss of head due to sudden contraction. This loss depends on the form of entrance. For a sharp edge entrance, this loss is slightly more than a rounded or bell mouthed entrance. In practice the value of loss of head at the entrance (or inlet) of a pipe with sharp cornered entrance is taken = $0.5 \frac{V^2}{2g}$, where V = velocity of liquid in pipe.

This loss is denoted by h_i

$$\therefore \qquad h_i = 0.5 \frac{V^2}{2g} \qquad \text{...(11.8)}$$

11.4.4 Loss of Head at the Exit of Pipe. This is the loss of head (or energy) due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet (if outlet of the pipe is free) or it is lost in the tank or reservoir (if the outlet of the pipe is connected to the tank or reservoir). This loss is equal to $\frac{V^2}{2g}$, where V is the velocity of liquid at the outlet of pipe. This loss is denoted h_o.

$$\therefore \qquad h_o = \frac{V^2}{2g} \qquad \text{...(11.9)}$$

where V = velocity at outlet of pipe.

11.4.5 Loss of Head Due to an Obstruction in a Pipe. Whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the cross-section of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place as shown in Fig. 11.3 (*a*)

Consider a pipe of area of cross-section A having an obstruction as shown in Fig. 11.3 (*a*).

Let a = Maximum area of obstruction

A = Area of pipe

V = Velocity of liquid in pipe

Then $(A - a)$ = Area of flow of liquid at section 1-1.

As the liquid flows and passes through section 1-1, a vena-contracta is formed beyond section 1-1, after which the stream of liquid widens again and velocity of flow at section 2-2 becomes uniform and equal to velocity, V in the pipe. This situation is similar to the flow of liquid through sudden enlargement.

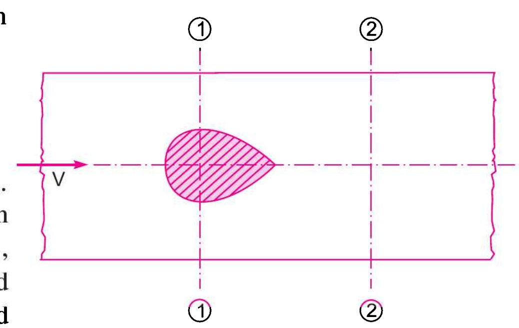


Fig. 11.3 (*a*) *An obstruction in a pipe.*

Let V_c = Velocity of liquid at vena-contracta.

Then loss of head due to obstruction = loss of head due to enlargement from vena-contracta to section 2-2.

$$= \frac{(V_c - V)^2}{2g} \qquad ...(i)$$

From continuity, we have $a_c \times V_c = A \times V$ ...(*ii*)

where a_c = area of cross-section at vena-contracta

If C_c = co-efficient of contraction

Then $$C_c = \frac{\text{area at vena - contracta}}{(A - a)} = \frac{a_c}{(A - a)}$$

$\therefore$ $a_c = C_c \times (A - a)$

Substituting this value in (*ii*), we get

$$C_c \times (A - a) \times V_c = A \times V \quad \therefore \quad V_c = \frac{A \times V}{C_c\,(A - a)}$$

Substituting this value of V_c in equation (*i*), we get

$$\text{Head loss due to obstruction} = \frac{(V_c - V)^2}{2g} = \frac{\left(\frac{A \times V}{C_c\,(A - a)} - V\right)^2}{2g} = \frac{V^2}{2g}\left(\frac{A}{C_c\,(A - a)} - 1\right)^2 \qquad ...(11.10)$$

11.4.6 Loss of Head due to Bend in Pipe. When there is any bend in a pipe, the velocity of flow changes, due to which the separation of the flow from the boundary and also formation of eddies takes place. Thus the energy is lost. Loss of head in pipe due to bend is expressed as

$$h_b = \frac{kV^2}{2g}$$

where h_b = loss of head due to bend, V = velocity of flow, k = co-efficient of bend

The value of k depends on

(*i*) Angle of bend, (*ii*) Radius of curvature of bend, (*iii*) Diameter of pipe.

11.4.7 Loss of Head in Various Pipe Fittings. The loss of head in the various pipe fittings such as valves, couplings etc., is expressed as

$$= \frac{kV^2}{2g} \qquad ...(11.11)$$

where V = velocity of flow, k = co-efficient of pipe fitting.

Problem 11.15 *Water is flowing through a horizontal pipe of diameter 200 mm at a velocity of 3 m/s. A circular solid plate of diameter 150 mm is placed in the pipe to obstruct the flow. Find the loss of head due to obstruction in the pipe if* $C_c = 0.62$.

Solution. Given :

Dia. of pipe, $D = 200 \text{ mm} = 0.20 \text{ m}$

Velocity, $V = 3.0 \text{ m/s}$

Area of pipe, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.2)^2 = 0.03141 \text{ m}^2$

Dia. of obstruction, $d = 150 \text{ mm} = 0.15 \text{ m}$

$\therefore$ Area of obstruction, $a = \frac{\pi}{4} (.15)^2 = 0.01767 \text{ m}^2$

$$C_c = 0.62$$

The head lost due to obstruction is given by equation (11.10) as

$$= \frac{V^2}{2g}\left(\frac{A}{C_c\,(A-a)} - 1.0\right)^2$$

$$= \frac{3\times 3}{2\times 9.81}\left[\frac{.03141}{0.62\,[.03141-.01767]} - 1.0\right]^2$$

$$= \frac{9}{2\times 9.81}\ [3.687 - 1.0]^2 = \mathbf{3.311\ m.\ Ans.}$$

Problem 11.16 *Determine the rate of flow of water through a pipe of diameter 20 cm and length 50 m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the height of water in the tank is 4 m above the centre of the pipe. Consider all minor losses and take f = .009 in the formula* $h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g}$.

Solution. Dia. of pipe, $d = 20$ cm $= 0.20$ m

Length of pipe, $L = 50$ m

Height of water, $H = 4$ m

Co-efficient of friction, $f = .009$

Let the velocity of water in pipe $= V$ m/s.

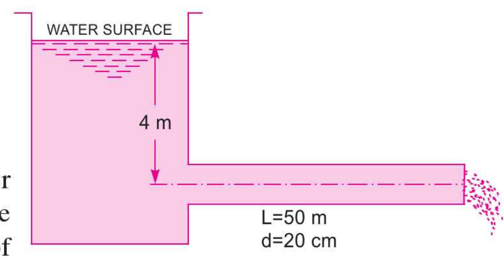


Fig. 11.4

Applying Bernoulli's equation at the top of the water surface in the tank and at the outlet of pipe, we have [Taking point 1 on the top and point 2 at the outlet of pipe].

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{all losses}$$

Considering datum line passing through the centre of pipe

$$0 + 0 + 4.0 = 0 + \frac{V_2^2}{2g} + 0 + (h_i + h_f)$$

or

$$4.0 = \frac{V_2^2}{2g} + h_i + h_f$$

But the velocity in pipe $= V$, $\therefore V = V_2$

$$\therefore \quad 4.0 = \frac{V^2}{2g} + h_i + h_f \qquad ...(i)$$

From equation (11.8), $h_i = 0.5\,\frac{V^2}{2g}$ and h_f from equation (11.1) is given as

$$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$$

Substituting these values, we have

$$4.0 = \frac{V^2}{2g} + \frac{0.5\,V^2}{2g} + \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$$

$$= \frac{V^2}{2g}\left[1.0 + 0.5 + \frac{4 \times .009 \times 50}{0.2}\right] = \frac{V^2}{2g}\ [1.0 + 0.5 + 9.0]$$

$$= 10.5 \times \frac{V^2}{2g}$$

$\therefore$ $$V = \sqrt{\frac{4 \times 2 \times 9.81}{10.5}} = 2.734 \text{ m/sec}$$

$\therefore$ Rate of flow, $$Q = A \times V = \frac{\pi}{4} \times (0.2)^2 \times 2.734 = 0.08589 \text{ m}^3\text{/s}$$

$$= \textbf{85.89 litres/s. Ans.}$$

Problem 11.17 *A horizontal pipe line 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the centre of the pipe. Considering all losses of head which occur, determine the rate of flow. Take f = .01 for both sections of the pipe.*

Solution. Given :

Total length of pipe, $L = 40$ m

Length of 1st pipe, $L_1 = 25$ m

Dia. of 1st pipe, $d_1 = 150$ mm $= 0.15$ m

Length of 2nd pipe, $L_2 = 40 - 25 = 15$ m

Dia. of 2nd pipe, $d_2 = 300$ mm $= 0.3$ m

Height of water, $H = 8$ m

Co-efficient of friction, $f = 0.01$

Applying Bernoulli's theorem to the free surface of water in the tank and outlet of pipe as shown in Fig. 11.5 and taking reference line passing through the centre of pipe.

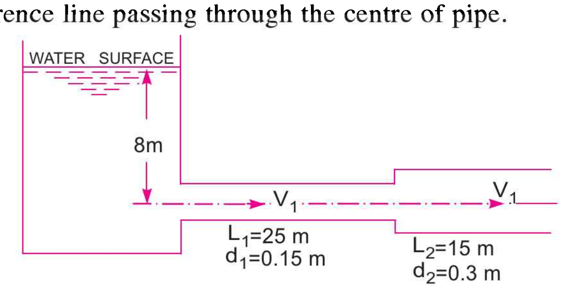


Fig. 11.5

$$0 + 0 + 8 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + \text{all losses}$$

or
$$8.0 = 0 + \frac{V_2^2}{2g} + h_i + h_{f_1} + h_e + h_{f_2} \quad ...(i)$$

where h_i = loss of head at entrance = $0.5 \frac{V_1^2}{2g}$

h_{f_1} = head lost due to friction in pipe 1 = $\frac{4 \times f \times L_1 \times V_1^2}{d_1 \times 2g}$

h_e = loss head due to sudden enlargement = $\frac{(V_1 - V_2)^2}{2g}$

h_{f_2} = Head lost due to friction in pipe 2 = $\frac{4 \times f \times L_2 \times V_2^2}{d_2 \times 2g}$

But from continuity equation, we have
$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4} d_2^2 \times V_2}{\frac{\pi}{4} d_1^2} = \left(\frac{d_2}{d_1}\right)^2 \times V_2 = \left(\frac{0.3}{.15}\right)^2 \times V_2 = 4V_2$$

Substituting the value of V_1 in different head losses, we have
$$h_i = \frac{0.5\, V_1^2}{2g} = \frac{0.5 \times (4V_2)^2}{2g} = \frac{8V_2^2}{2g}$$

$$h_{f_1} = \frac{4 \times 0.01 \times 25 \times (4V_2)^2}{0.15 \times 2 \times g}$$

$$= \frac{4 \times .01 \times 25 \times 16}{0.15} \times \frac{V_2^2}{2g} = 106.67 \frac{V_2^2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

$$h_{f_2} = \frac{4 \times .01 \times 15 \times V_2^2}{0.3 \times 2g} = \frac{4 \times .01 \times 15}{0.3} \times \frac{V_2^2}{2g} = 2.0 \times \frac{V_2^2}{2g}$$

Substituting the values of these losses in equation (*i*), we get
$$8.0 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67 \frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2 \times \frac{V_2^2}{2g}$$

$$= \frac{V_2^2}{2g} [1 + 8 + 106.67 + 9 + 2] = 126.67 \frac{V_2^2}{2g}$$

$$\therefore \quad V_2 = \sqrt{\frac{8.0 \times 2 \times g}{126.67}} = \sqrt{\frac{8.0 \times 2 \times 9.81}{126.67}} = \sqrt{1.2391} = 1.113 \text{ m/s}$$

$\therefore$ Rate of flow, $Q = A_2 \times V_2 = \frac{\pi}{4} (0.3)^2 \times 1.113 = 0.07867 \text{ m}^3\text{/s}$ = **78.67 litres/s. Ans.**

Problem 11.18 *Determine the difference in the elevations between the water surfaces in the two tanks which are connected by a horizontal pipe of diameter 300 mm and length 400 m. The rate of flow of water through the pipe is 300 litres/s. Consider all losses and take the value of f = .008.*

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Length, $L = 400 \text{ m}$

Discharge, $Q = 300 \text{ lit/s} = 0.3 \text{ m}^3\text{/s}$

Co-efficient of friction, $f = 0.008$

Velocity, $$V = \frac{Q}{\text{Area}} = \frac{0.3}{\frac{\pi}{4} \times (.3)^2} = 4.244 \text{ m/s}$$

Let the two tanks are connected by a pipe as shown in Fig. 11.6.

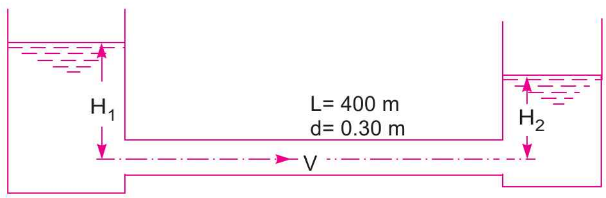


Fig. 11.6

Let H_1 = height of water in 1st tank above the centre of pipe

H_2 = height of water in 2nd tank above the centre of pipe

Then difference in elevations between water surfaces = $H_1 - H_2$

Applying Bernoulli's equation to the free surface of water in the two tanks, we have

$$H_1 = H_2 + \text{losses}$$

$$= H_2 + h_i + H_{f_1} + h_o \qquad \text{...}(i)$$

where h_i = Loss of head at entrance $= 0.5 \dfrac{V^2}{2g} = \dfrac{0.5 \times 4.244^2}{2 \times 9.81} = 0.459 \text{ m}$

h_{f_1} = Loss of head due to friction $= \dfrac{4 \times f \times L \times V^2}{d \times 2g} = \dfrac{4 \times .008 \times 400 \times 4.244^2}{0.3 \times 2 \times 9.81} = 39.16 \text{ m}$

h_o = Loss of head at outlet $= \dfrac{V^2}{2g} = \dfrac{4.244^2}{2 \times 9.81} = 0.918 \text{ m}$

Substituting these values in (*i*), we get

$$H_1 = H_2 + 0.459 + 39.16 + 0.918 = H_2 + 40.537$$

$\therefore$ $H_1 - H_2$ = Difference in elevations

= **40.537 m. Ans.**

Problem 11.19 *The friction factor for turbulent flow through rough pipes can be determined by Karman-Prandtl equation,* $\dfrac{1}{\sqrt{f}} = 2 \log_{10} (R_0/k) + 1.74$

where f = friction factor, R_0 = pipe radius, k = average roughness.

Two reservoirs with a surface level difference of 20 metres are to be connected by 1 metre diameter pipe 6 km long. What will be the discharge when a cast iron pipe of roughness k = 0.3 mm is used ? What will be the percentage increase in the discharge if the cast iron pipe is replaced by a steel pipe of roughness k = 0.1 mm ? Neglect all local losses.

Solution. Given :

Difference in levels, $h = 20$ m

Dia. of pipe, $d = 1.0$ m $\quad\therefore$ Radius, $R_0 = 0.5$ m = 500 mm

Length of pipe, $L = 6$ km $= 6 \times 1000 = 6000$ m

Roughness of cast iron pipe, $k = 0.3$ mm

Roughness of steel pipe, $k = 0.1$ mm

1st Case. ***Cast Iron Pipe.*** First find the value of friction factor using

$$\frac{1}{\sqrt{f}} = 2\log_{10}(R_0/k) + 1.74 \quad ...(i)$$

$$= 2\log_{10}(500/0.3) + 1.74 = 8.1837$$

$$\therefore \quad f = \left(\frac{1}{8.1837}\right)^2 = 0.0149$$

Local losses are to be neglected. This means only head loss due to friction is to be considered. Head loss due to friction is

$$20 = \frac{f \times L \times V^2}{d \times 2g}$$

[Here f is the friction factor and not co-efficient of friction
$\because$ Friction factor = 4 × co-efficient of friction]

$$20 = \frac{.0149 \times 6000 \times V^2}{1.0 \times 2 \times 9.81} = 4.556\ V^2$$

$$\therefore \quad V = \sqrt{\frac{20}{4.556}} = 2.095 \text{ m/s}$$

$\therefore$ Discharge, $Q_1 = V \times \text{Area} = 2.095 \times \frac{\pi}{4} \times d^2 = 2.095 \times \frac{\pi}{4} \times 1^2 = 1.645 \text{ m}^3/\text{s}$

2nd Case. ***Steel Pipe.*** $k = 0.1$ mm, $R_0 = 500$ mm

Substituting these values in equation (*i*), we get

$$\frac{1}{\sqrt{f}} = 2\log_{10}(500/0.1) + 1.74 = 9.1379$$

$$\therefore \quad f = \left(\frac{1}{9.1379}\right)^2 = 0.0119$$

Head loss due to friction, $20 = \frac{f \times L \times V^2}{d \times 2g}$ or $20 = \frac{.0119 \times 6000 \times V^2}{1.0 \times 2 \times 9.81} = 3.639\ V^2$

$\therefore$ $$V = \sqrt{\frac{20}{3.639}} = 2.344 \text{ m/s}$$

$\therefore$ Discharge, $$Q_2 = V \times \text{Area} = 2.344 \times \frac{\pi}{4} \times 1^2 = 1.841 \text{ m}^3\text{/s}$$

percentage increase in the discharge $= \dfrac{Q_2 - Q_1}{Q_1} \times 100 = \dfrac{(1.841 - 1.645)}{1.645} \times 100 =$ **11.91%. Ans.**

Problem 11.20 *Design the diameter of a steel pipe to carry water having kinematic viscosity $\nu = 10^{-6}$ m²/s with a mean velocity of 1 m/s. The head loss is to be limited to 5 m per 100 m length of pipe. Consider the equivalent sand roughness height of pipe $k_s = 45 \times 10^{-4}$ cm. Assume that the Darcy Weisbach friction factor over the whole range of turbulent flow can be expressed as*

$$f = 0.0055\left[1 + \left(20 \times 10^3 \frac{k_s}{D} + \frac{10^6}{R_e}\right)^{1/3}\right]$$

where D = Diameter of pipe and R_e = Reynolds number.

Solution. Given :

Kinematic viscosity, $\nu = 10^{-6}$ m²/s

Mean velocity, $V = 1$ m/s

Head loss, $h_f = 5$ m in a length $L = 100$ m

Value of $k_s = 45 \times 10^{-4}$ cm $= 45 \times 10^{-6}$ m

Value of $$f = 0.0055\left[1 + \left(20 \times 10^3 \frac{k_s}{D} + \frac{10^6}{R_e}\right)^{1/3}\right] \quad ...(i)$$

Using Darcy Weisbach equation, $h_f = \dfrac{4 \times f \times L \times V^2}{D \times 2g}$

or $$f = \frac{h_f \times D \times 2g}{4 \times L \times V^2} = \frac{5 \times D \times 2 \times 9.81}{4 \times 100 \times 1^2} = 0.2452\, D$$

Now the Reynolds number is given by,

$$R_e = \frac{\rho VD}{\mu} = \frac{V \times D}{\nu} \qquad \left(\because \nu = \frac{\mu}{\rho}\right)$$

$$= \frac{1 \times D}{10^{-6}} = 10^6 D$$

Substituting the values of f, R_e and k_s in equation (i), we get

$$0.2452\, D = 0.0055\left[1 + \left(20 \times 10^3 \times \frac{45 \times 10^{-6}}{D} + \frac{10^6}{10^6 D}\right)^{1/3}\right]$$

or $$\frac{0.2452}{0.0055} D = \left[1 + \left(\frac{0.9}{D} + \frac{1}{D}\right)^{1/3}\right]$$

or $$44.58\ D = \left[1 + \left(\frac{1.9}{D}\right)^{1/3}\right] \text{ or } 44.58\ D - 1 = \left(\frac{1.9}{D}\right)^{1/3}$$

or $$(44.58\ D - 1)^3 = \frac{1.9}{D} \text{ or } D\ (44.58\ D - 1)^3 = 1.9 \qquad \ldots(ii)$$

Equation (*ii*) is solved by hit and trial method.

(*i*) Let $D = 0.1$ m, then L.H.S. of equation (*ii*) becomes as

L.H.S. $= 0.1\ (44.58 \times 0.1 - 1)^3 = 0.1 \times 3.458^3 = 4.135$

This is more than the R.H.S.

(*ii*) Let $D = 0.08$ m, then L.H.S. of equation (*ii*) becomes as

L.H.S. $= 0.08\ (44.58 \times 0.08 - 1)^3 = 0.08\ (2.5664)^3 = 1.352$

This is less than the R.H.S.

Hence value of D lies between 0.1 and 0.08

(*iii*) Let $D = 0.085$, then L.H.S. of equation (*ii*) becomes as

L.H.S. $= 0.085\ (44.58 \times 0.085 - 1)^3 = 1.844$

This value is slightly less than R.H.S. Hence increase the value of D slightly.

(*iv*) Let $D = 0.0854$ m, then L.H.S. of equation (*ii*) becomes as

L.H.S. $= 0.0854\ (44.58 \times 0.0854 - 1)^3 = 1.889$

This value is nearly equal to R.H.S.

$\therefore$ Correct value of $D =$ **0.0854 m. Ans.**

Problem 11.21 *A pipe line AB of diameter 300 mm and of length 400 m carries water at the rate of 50 litres/s. The flow takes place from A to B where point B is 30 metres above A. Find the pressure at A if the pressure at B is 19.62 N/cm². Take f = .008.*

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Length of pipe, $L = 400 \text{ m}$

Discharge, $Q = 50 \text{ litres/s} = 0.05 \text{ m}^3/\text{s}$

$\therefore$ Velocity, $$V = \frac{Q}{\text{Area}} = \frac{0.05}{\frac{\pi}{4}d^2} = \frac{0.05}{\frac{\pi}{4} \times (.3)^2} = 0.7074 \text{ m/s}$$

Pressure at B, $p_B = 19.62 \text{ N/cm}^2 = 19.62 \times 10^4 \text{ N/m}^2$

$f = .008$

Applying Bernoulli's equations at points A and B and taking datum line passing through A, we have

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + h_f$$

But $V_A = V_B$ [$\because$ Dia. is same]

$z_A = 0,\ z_B = 30$

and $$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$$

$$\therefore \quad \frac{p_A}{\rho g} + 0 = \frac{19.62 \times 10^4}{1000 \times 9.81} + 30 + \frac{4 \times .008 \times 400 \times .7074^2}{0.3 \times 2 \times 9.81}$$

$$= 20 + 30 + 1.088 = 51.088$$

$$\therefore \quad p_A = 51.088 \times 1000 \times 9.81 \text{ N/m}^2$$

$$= \frac{51.088 \times 1000 \times 9.81}{10^4}$$

$$= \mathbf{50.12 \text{ N/cm}^2. \text{ Ans.}}$$

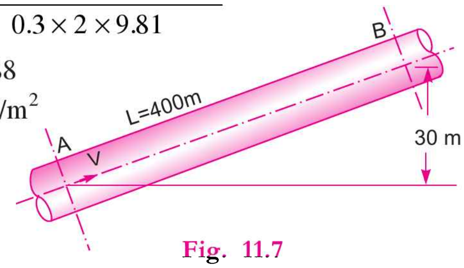


Fig. 11.7

▶ 11.5 HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

The concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes. They are defined as :

11.5.1 Hydraulic Gradient Line. It is defined as the line which gives the sum of pressure head $\left(\frac{p}{w}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L. (Hydraulic Gradient Line).

11.5.2 Total Energy Line. It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L. (Total Energy Line).

Problem 11.22 *For the problem 11.16, draw the Hydraulic Gradient Line (H.G.L.) and Total Energy Line (T.E.L.).*

Solution. Given :

$$L = 50 \text{ m}, \; d = 200 \text{ mm} = 0.2 \text{ m}$$

$$H = 4 \text{ m}, \; f = .009$$

Velocity, V through pipe is calculated in problem 11.16 and its value is $V = 2.734$ m/s

Now, h_i = Head lost at entrance of pipe

$$= 0.5 \frac{V^2}{2g} + \frac{0.5 \times 2.734^2}{2 \times 9.81} = 0.19 \text{ m}$$

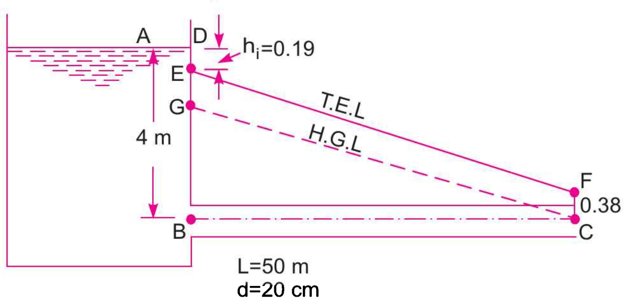


Fig. 11.8

and h_f = Head loss due to friction

$$= \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times 0.009 \times 50 \times (2.734)^2}{0.2 \times 2 \times 9.81} = 3.428 \text{ m.}$$

(*a*) **Total Energy Line (T.E.L.).** Consider three points, *A*, *B* and *C* on the free surface of water in the tank, at the inlet of the pipe and at the outlet of the pipe respectively as shown in Fig. 11.8. Let us find total energy at these points, taking the centre of pipe as reference line.

1. Total energy at $A = \frac{p}{\rho g} + \frac{V^2}{2g} + z = 0 + 0 + 4.0 = 4$ m
2. Total energy at B = Total energy at $A - h_i = 4.0 - 0.19 = 3.81$ m
3. Total energy at $C = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c = 0 + \frac{V^2}{2g} + 0 = \frac{2.734^2}{2 \times 9.81} = 0.38$ m.

Hence total energy line will coincide with free surface of water in the tank. At the inlet of the pipe, it will decrease by h_i (= 0.19 m) from free surface and at outlet of pipe total energy is 0.38 m. Hence in Fig. 11.8,

(*i*) Point *D* represents total energy at *A*
(*ii*) Point *E*, where $DE = h_i$, represents total energy at inlet of the pipe
(*iii*) Point *F*, where $CF = 0.38$ represents total energy at outlet of pipe.
Join *D* to *E* and *E* to *F*. Then *DEF* represents the total energy line.

(*b*) **Hydraulic Gradient Line (H.G.L.).** H.G.L. gives the sum of $(p/w + z)$ with reference to the datum-line. Hence hydraulic gradient line is obtained by subtracting $\frac{V^2}{2g}$ from total energy line. At outlet of the pipe, total energy $= \frac{V^2}{2g}$. By subtracting $\frac{V^2}{2g}$ from total energy at this point, we shall get point *C*, which lies on the centre line of pipe. From *C*, draw a line *CG* parallel to *EF*. Then *CG* represents the hydraulic gradient line.

Problem 11.23 *For the problem 11.17, draw the hydraulic gradient and total energy line.*

Solution. Refer to problem 11.17.

Given: $L_1 = 25$ m, $d_1 = 0.15$ m
$L_2 = 15$ m, $d_2 = 0.3$ m, $f = .01$, $H = 8$ m

The velocity V_2 as calculated in problem 11.17 is

$$V_2 = 1.113 \text{ m/s}$$
$$V_1 = 4V_2 = 4 \times 1.113 = 4.452 \text{ m/s}$$

The various head losses are $h_i = 0.5 \times \frac{V_1^2}{2g} = \frac{0.5 \times 4.452^2}{2 \times 9.81} = 0.50$ m

$$h_{f_1} = \frac{4f \times L_1 \times V_1^2}{d_1 \times 2g} = \frac{4 \times .01 \times 25 \times (4.452)^2}{0.15 \times 2 \times 9.81} = 6.73 \text{ m}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4.452 - 1.11)^2}{2 \times 9.81} = 0.568 \text{ m}$$

$$h_{f_2} = \frac{4 \times f \times L_2 \times V_2^2}{d_2 \times 2g} = \frac{4 \times .01 \times 15 \times (1.113)^2}{0.3 \times 2 \times 9.81} = 0.126 \text{ m}$$

$$h_o = \frac{V_2^2}{2g} = \frac{1.113^2}{2 \times 9.81} = 0.063 \text{ m}$$

Also $$V_1^2/2g = \frac{4.452^2}{2 \times 9.81} = 1.0 \text{ m}.$$

Total Energy Line

(*i*) Point *A* lies on free surface of water.

(*ii*) Take $AB = h_i = 0.5$ m.

(*iii*) From *B*, draw a horizontal line. Take *BL* equal to the length of pipe, *i.e.*, L_1. From *L* draw a vertical line downward.

(*iv*) Cut the line $LC = h_{f_1} = 6.73$ m.

(*v*) Join the point *B* to *C*. From *C*, take a line *CD* vertically downward equal to $h_e = 0.568$ m.

(*vi*) From *D*, draw *DM* horizontal and from point *F* which is lying on the centre of the pipe, draw a vertical line in the upward direction, meeting at *M*. From *M*, take a distance $ME = h_{f_2} = 0.126$. Join *DE*.

Then line *ABCDE* represents the total energy line.

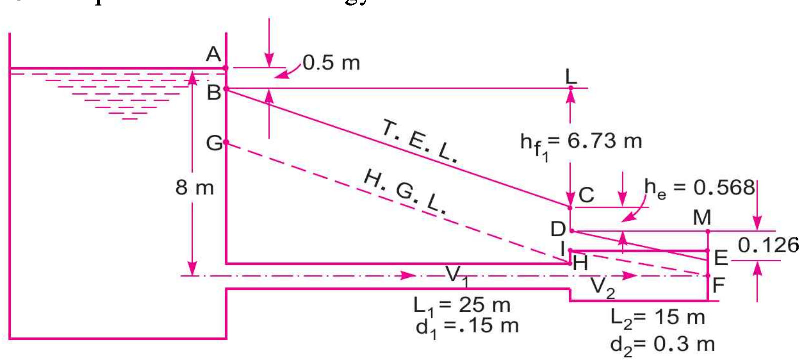


Fig. 11.9

Hydraulic Gradient Line (H.G.L.)

(*i*) From *B*, take $BG = \frac{V_1^2}{2g} = 1.0$ m.

(*ii*) Draw the line *GH* parallel to the line *BC*.

(*iii*) From *F*, draw a line *FI* parallel to the line *ED*.

(*iv*) Join the point *H* and *I*.

Then the line *GHIF* represents the hydraulic gradient line (H.G.L.).

Problem 11.24 *For Problem 11.18, draw the hydraulic gradient and total energy line.*

Solution. Refer to Problem 11.18,

Given : $d = 300$ mm $= 0.3$ m

$L = 400$ m, $Q = 300$ litres/s $= 0.3$ m^3/s

$f = .008$

Let $H_1 = 50$ m. But $H_1 - H_2 = 40.537$ m (Calculated in Problem 11.18)

$\therefore$ $H_2 = 50 - 40.537 = 9.463$ m.

The calculated losses are :

(*i*) $h_i = 0.459$ m (*ii*) $h_{f_1} = 39.16$ m

(*iii*) $h_o = 0.918$ m

(*a*) **T.E.L.**

(*i*) Point *A* is on the free surface of water in 1st tank. From *A*, take $AB = h_i = 0.459$ m.

(*ii*) Draw a horizontal line *BF*. Take *BF* equal to the length of pipe. From *F*, draw a vertical line in the downward direction. Cut $FC = h_{f_1} = 39.16$ m.

(*iii*) Join *BC*. From *C* take $CD = h_0 = 0.918$ m. The point *D* should coincide with free surface of water in 2nd tank. Then line *ABCD* is the total energy line.

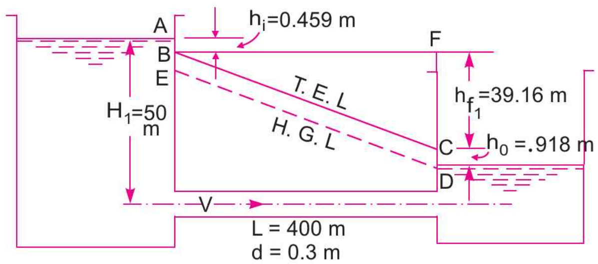


Fig. 11.10

(*b*) **H.G.L.** From *D*, draw a line *DE* parallel to line *BC*. Then *DE* is the H.G.L.

Or

From *B*, take $BE = \dfrac{V^2}{2g} = 0.918$ m and from *E* draw a line *ED* parallel to *BC*. The point *D* should coincide with free surface of water in the 2nd tank. Then line *ED* represents the H.G.L.

Problem 11.25 *The rate of flow of water pumped into a pipe ABC, which is 200 m long, is 20 litres/s. The pipe is laid on an upward slope of 1 in 40. The length of the portion AB is 100 m and its diameter is 100 mm, while the length of the portion BC is also 100 m but its diameter is 200 mm. The change of diameter at B is sudden. The flow is taking place from A to C, where the pressure at A is 19.62 N/cm² and end C is connected to a tank. Find the pressure at C and draw the hydraulic gradient and total energy line. Take f = .008.*

Solution. Given :

Length of pipe, $ABC = 200$ m

Discharge, $Q = 20$ litres/s $= 0.02\ m^3/s$

Slope of pipe, $i = 1 \text{ in } 40 = \dfrac{1}{40}$

Length of pipe, $AB = 100$ m, Dia. of pipe $AB = 100$ mm $= 0.1$ m

Length of pipe, $BC = 100$ m, Dia. of pipe $BC = 200$ mm $= 0.2$ m

Pressure at *A*, $p_A = 19.62\ N/cm^2 = 19.62 \times 10^4\ N/m^2$

Co-efficient of friction, $f = .008$

Velocity of water in pipe *AB*, $V_1 = \dfrac{\text{Discharge}}{\text{Area of } AB} = \dfrac{0.02}{\frac{\pi}{4}(.1)^2} = 2.54$ m/s

Velocity of water in pipe BC, $V_2 = \dfrac{Q}{\text{Area of } BC} = \dfrac{0.02}{\frac{\pi}{4}(.2)^2} = 0.63$ m/s

Applying Bernoulli's equation to points A and C,

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c + \text{total loss from } A \text{ to } C \qquad \ldots(i)$$

Total loss from A to C = Loss due to friction in pipe AB + loss of head due to enlargement at B + loss of head due to friction in pipe BC. ...(*ii*)

Now loss of head due to friction in pipe AB,

$$h_{f_1} = \frac{4fLV^2}{d \times 2g} = \frac{4 \times .008 \times 100 \times (2.54)^2}{0.1 \times 2 \times 9.81} = 10.52 \text{ m}$$

Loss of head due to friction in pipe BC,

$$h_{f_2} = \frac{4 \times .008 \times 100 \times (0.63)^2}{0.2 \times 2 \times 9.81} = 0.323 \text{ m}$$

Loss of head due to enlargement at B,

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(2.54 - .63)^2}{2 \times 9.81} = 0.186 \text{ m}$$

$\therefore$ Total loss from A to C $= h_{f_1} + h_e + h_{f_2} = 10.52 + .186 + .323 = 11.029 \simeq 11.03$ m

Substituting this value in (*i*), we get

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c + 11.03 \qquad \ldots(iii)$$

Taking datum line passing through A, we have

$$z_A = 0$$

$$z_c = \frac{1}{40} \times \text{total length of pipe} = \frac{1}{40} \times 200 = 5 \text{ m}$$

Also $p_A = 19.62 \times 10^4$ N/m^2

$V_A = V_1 = 2.54$ m/s, $V_c = V_2 = 0.63$ m/s

Substituting these values in (*iii*), we get

$$\frac{19.62 \times (10)^4}{1000 \times 9.81} + \frac{(2.54)^2}{2 \times 9.81} + 0 = \frac{p_c}{\rho g} + \frac{(0.63)^2}{2 \times 9.81} + 5.0 + 11.03$$

or $$20 + 0.328 = \frac{p_c}{\rho g} + 0.02 + 5.0 + 11.03$$

$\therefore$ $$20.328 = \frac{p_c}{\rho g} + 16.05$$

$\therefore \quad \dfrac{p_c}{\rho g} = 20.328 - 16.05 = 4.278$ m

or $\quad p_c = 4.278 \times 1000 \times 9.81$ N/m^2

$= \dfrac{4.278 \times 1000 \times 9.81}{10^4}$ N/cm^2 = **4.196 N/cm^2. Ans.**

Hydraulic Gradient and Total Energy Line

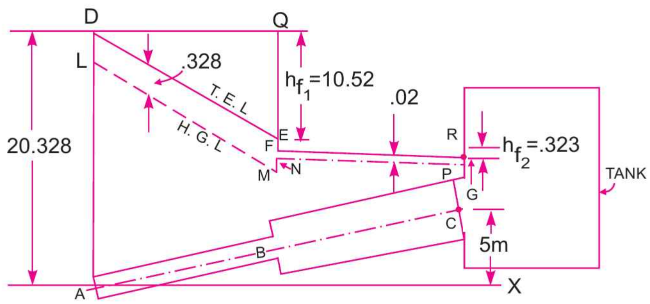


Fig. 11.11

Pipe AB. Assuming the datum line passing through *A*, then total energy at *A*

$$= \frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{(2.54)^2}{2 \times 9.81} + 0 = 20 + 0.328$$

$= 20.328$ m

Total energy at *B* $\quad$ = Total energy at $A - h_{f_1} = 20.328 - 10.52 = 9.808$ m

Also $\quad V_c^2/2g = \dfrac{(0.63)^2}{2 \times 9.81} = 0.02.$

Total Energy Line. Draw a horizontal line *AX* as shown in Fig. 11.11. The centre-line of the pipe is drawn in such a way that slope of pipe is 1 in 40. Thus the point *C* will be at a height of $\dfrac{1}{40} \times 200 = 5$ m from the line *AX*. Now draw a vertical line *AD* equal to total energy at *A*, *i.e.*, *AD* = 20.328 m. From point *D*, draw a horizontal line and from point *B*, a vertical line, meeting at *Q*. From *Q*, take vertical distance $QE = h_{f_1} = 10.52$ m. Join *DE*. From *E*, take $EF = h_e = 0.186$ m. From *F*, draw a horizontal line and from *C*, a vertical line meeting at *R*. From *R* take $RG = h_{f_2} = 0.323$ m. Join *F* to *G*. Then *DEFG* represents the total energy line.

Hydraulic Gradient Line. Draw the line *LM* parallel to the line *DE* at a distance in the downward direction equal to 0.328 m. Also draw the line *PN* parallel to the line *GF* at a distance of $\dfrac{V_c^2}{2g} = 0.02$. Join point *M* to *N*. Then line *LMNP* represents the hydraulic gradient line.

Problem 11.26 *A pipe line, 300 mm in diameter and 3200 m long is used to pump up 50 kg per second of an oil whose density is 950 kg/m^3 and whose kinematic viscosity is 2.1 stokes. The centre of the pipe line at the upper end is 40 m above than that at the lower end. The discharge at the upper end is atmospheric. Find the pressure at the lower end and draw the hydraulic gradient and the total energy line.*

Solution. Given :

Dia. of pipe, $d = 300$ mm $= 0.3$ m

Length of pipe, $L = 3200$ m

Mass, $M = 50$ kg/s $= \rho \cdot Q$

$\therefore$ Discharge, $Q = \dfrac{50}{\rho} = \dfrac{50}{950} = 0.0526$ m^3/s

$\because$ Density, $\rho = 950$ kg/m^3

Kinematic viscosity, $\nu = 2.1$ stokes $= 2.1$ cm^2/s

$= 2.1 \times 10^{-4}$ m^2/s

Height of upper end $= 40$ m

Pressure at upper end $=$ atmospheric $= 0$

Reynolds number, $R_e = \dfrac{V \times d}{\nu}$, where $V = \dfrac{\text{Discharge}}{\text{Area}} = \dfrac{0.0526}{\frac{\pi}{4}(0.3)^2} = 0.744$ m/s

$$\therefore \quad R_e = \frac{0.744 \times 0.30}{2.1 \times 10^{-4}} = 1062.8$$

$\therefore$ Co-efficient of friction, $f = \dfrac{16}{R_e} = \dfrac{16}{1062.8} = 0.015$

Head lost due to friction, $h_f = \dfrac{4 \times f \times L \times V^2}{d \times 2g}$

$$= \frac{4 \times 0.015 \times 3200 \times (0.744)^2}{0.3 \times 2 \times 9.81} = 18.05 \text{ m of oil}$$

Applying the Bernoulli's equation at the lower and upper end of the pipe and taking datum line passing through the lower end, we have

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

But $z_1 = 0$, $z_2 = 40$ m, $V_1 = V_2$ as diameter is same

$p_2 = 0$, $h_f = 18.05$ m

$\therefore$ Substituting these values, we have

$$\frac{p_1}{\rho g} = 40 + 18.05 = 58.05 \text{ m of oil}$$

$\therefore$ $p_1 = 58.05 \times \rho g = 58.05 \times 950 \times 9.81$ [$\because$ ρ for oil $= 950$]

$= 540997$ N/m^2 $= \dfrac{540997}{10^{-4}}$ N/cm^2 $=$ **54.099 N/cm^2. Ans.**

H.G.L. and T.E.L.

$$\frac{V^2}{2g} = \frac{(.744)^2}{2 \times 9.81} = 0.0282 \text{ m}$$

$$\frac{p_1}{\rho g} = 58.05 \text{ m of oil}$$

$$\frac{p_2}{\rho g} = 0$$

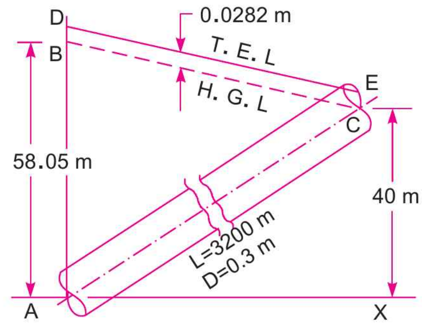


Fig. 11.12

Draw a horizontal line AX as shown in Fig. 11.12. From A, draw the centre line of the pipe in such a way that point C is a distance of 40 m above the horizontal line. Draw a vertical line AB through A such that AB = 58.05 m. Join B with C. Then BC is the hydraulic gradient line.

Draw a line DE parallel to BC at a height of 0.0282 m above the hydraulic gradient line. Then DE is the total energy line.

▶ 11.6 FLOW THROUGH SYPHON

Syphon is a long bent pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at a lower level when the two reservoirs are separated by a hill or high level ground as shown in Fig. 11.13.

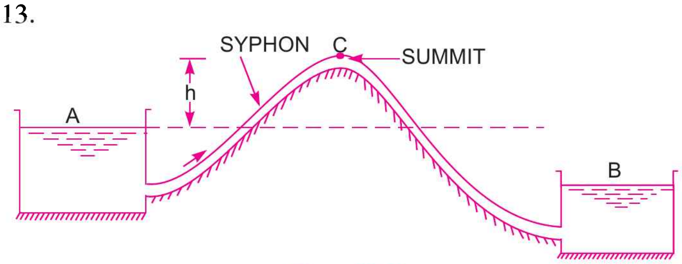


Fig. 11.13

The point C which is at the highest of the syphon is called the summit. As the point C is above the free surface of the water in the tank A, the pressure at C will be less than atmospheric pressure. Theoretically, the pressure at C may be reduced to – 10.3 m of water but in actual practice this pressure is only – 7.6 m of water or 10.3 – 7.6 = 2.7 m of water absolute. If the pressure at C becomes less than 2.7 m of water absolute, the dissolved air and other gases would come out from water and collect at the summit. The flow of water will be obstructed. Syphon is used in the following cases :

1. To carry water from one reservoir to another reservoir separated by a hill or ridge.
2. To take out the liquid from a tank which is not having any outlet.
3. To empty a channel not provided with any outlet sluice.

Problem 11.27 *A syphon of diameter 200 mm connects two reservoirs having a difference in elevation of 20 m. The length of the syphon is 500 m and the summit is 3.0 m above the water level in the upper reservoir. The length of the pipe from upper reservoir to the summit is 100 m. Determine the discharge through the syphon and also pressure at the summit. Neglect minor losses. The co-efficient of friction, f = .005.*

Solution. Given :

Dia. of syphon, d = 200 mm = 0.20 m

Difference in level of two reservoirs, H = 20 m

Length of syphon, L = 500 m

Height of summit from upper reservoir, $h = 3.0$ m
Length of syphon upto summit, $L_1 = 100$ m
Co-efficient of friction, $f = .005$

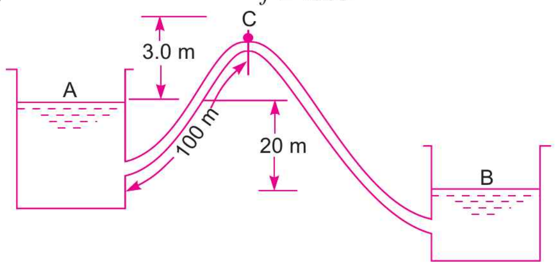


Fig. 11.14

If minor losses are neglected then the loss of head takes place only due to friction.
Applying Bernoulli's equation to points A and B,

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + \text{Loss of head due to friction from } A \text{ to } B$$

or $$0 + 0 + z_A = 0 + 0 + z_B + h_f \; [\because p_A = p_B = \text{atmospheric pressure}, V_A = V_B = 0]$$

$\therefore$ $$z_A - z_B = h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$$

But $$z_A - z_B = 20 \text{ m}$$

$\therefore$ $$20 = \frac{4 \times .005 \times 500 \times V^2}{0.20 \times 2 \times 9.81} = 2.548\ V^2$$

$\therefore$ $$V = \sqrt{\frac{20}{2.548}} = 2.80 \text{ m/s}$$

$\therefore$ Discharge, $$Q = \text{Velocity} \times \text{Area}$$

$$= 2.80 \times \frac{\pi}{4}(.2)^2 = 0.0879 \text{ m}^3\text{/s} = \textbf{87.9 litres/s. Ans.}$$

Pressure at Summit. Applying Bernoulli's equation to points A and C,

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c + \text{Loss of head due to friction between } A \text{ and } C$$

or $$0 + 0 + 0 = \frac{p_c}{\rho g} + \frac{V^2}{2g} + 3.0 + h_{f_1} \qquad [\text{Taking datum line passing through } A]$$

$\therefore$ $$0 = \frac{p_c}{\rho g} + \frac{2.8^2}{2 \times 9.81} + 3.0 + \frac{4 \times .005 \times 100 \times (2.8)^2}{0.2 \times 2 \times 9.81} \quad [V_c = V = 2.80]$$

$$= \frac{p_c}{\rho g} + 0.399 + 3.0 + 4.00 = \frac{p_c}{\rho g} + 7.399$$

$\therefore$ $$\frac{p_c}{\rho g} = \textbf{– 7.399 m of water. Ans.}$$

Problem 11.28 *A syphon of diameter 200 mm connects two reservoirs having a difference in elevation of 15 m. The total length of the syphon is 600 m and the summit is 4 m above the water level in the upper reservoir. If the separation takes place at 2.8 m of water absolute, find the maximum length of syphon from upper reservoir to the summit. Take f = .004 and atmospheric pressure = 10.3 m of water.*

Solution. Given :

Dia. of syphon, $d = 200 \text{ mm} = 0.2 \text{ m}$

Difference of level in two reservoirs $= 15 \text{ m}$

Total length of pipe $= 600 \text{ m}$

Height of summit from upper reservoir $= 4 \text{ m}$

Pressure head at summit, $\frac{p_c}{\rho g} = 2.8$ m of water absolute

Atmospheric pressure head, $\frac{p_c}{\rho g} = 10.3$ m of water absolute

Co-efficient of friction, $f = .004$

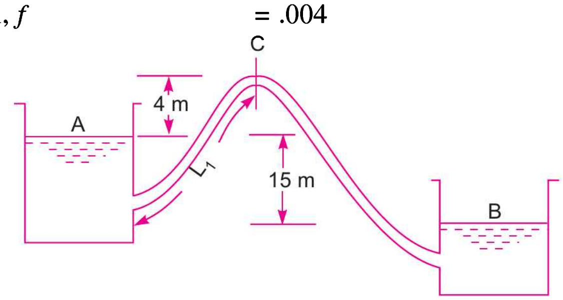


Fig. 11.15 (*a*)

Applying Bernoulli's equation to points A and C and taking the datum line passing through, A,

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c + \text{Loss of head due to friction between } A \text{ and } C$$

Substituting the values of pressures in terms of absolute, we have

$$10.3 + 0 + 0 = 2.8 + \frac{V^2}{2g} + 4.0 + h_{f_1} \qquad [\because V_c = \text{velocity in pipe} = V]$$

$$\therefore \qquad h_{f_1} = 10.3 - 2.8 - 4.0 - \frac{V^2}{2g} = 3.5 - \frac{V^2}{2g} \qquad \ldots(i)$$

Applying Bernoulli's equation to points A and B and taking datum line passing through B,

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + \text{Loss of head due to friction from } A \text{ to } B$$

But $\frac{p_A}{\rho g} = \frac{p_B}{\rho g} = \text{atmospheric pressure}$

$V_A = 0,\ V_B = 0,\ z_A = 15,\ z_B = 0$

$$\therefore \qquad 0 + 0 + 15 = 0 + 0 + 0 + h_f$$

$$\therefore \qquad h_f = 15 \text{ or } \frac{4 \times f \times L \times V^2}{d \times 2g} = 15$$

or $$\frac{4 \times .004 \times 600 \times V^2}{0.2 \times 2 \times 9.81} = 15 \text{ or } V = \sqrt{\frac{15 \times 0.2 \times 2 \times 9.81}{4 \times .004 \times 600}} = 2.47 \text{ m/s}$$

Substituting this value of V in equation (i), we get

$$h_{f_1} = 3.5 - \frac{2.47^2}{2 \times 9.81} = 3.5 - 0.311 = 3.189 \text{ m} \qquad ...(ii)$$

But $$h_{f_2} = \frac{4 \times f \times L_1 \times V^2}{d \times 2g} \qquad ...(iii)$$

where L_1 = inlet leg of syphon or length of syphon from upper reservoir to the summit.

$$h_{f_1} = \frac{4 \times .004 \times L_1 \times (2.47)^2}{0.2 \times 2 \times 9.81} = 0.0248 \times L_1$$

Substituting this value in equation (ii),

$$0.0248\, L_1 = 3.189$$

$$\therefore \qquad L_1 = \frac{3.189}{.0248} = \mathbf{128.58 \text{ m. Ans.}}$$

Problem 11.29 *A syphon of diameter 200 mm connects two reservoirs whose water surface level differ by 40 m. The total length of the pipe is 8000 m. The pipe crosses a ridge. The summit of ridge is 8 m above the level of water in the upper reservoir. Determine the minimum depth of the pipe below the summit of the ridge, if the absolute pressure head at the summit of syphon is not to fall below 3.0 m of water. Take f = 0.006 and atmospheric pressure head = 10.3 m of water. The length of syphon from the upper reservoir to the summit is 500 m. Find the discharge also.*

Solution. Given :

Dia. of syphon, d = 200 mm = 0.20 m

Difference in levels of two reservoirs, H = 40 m

Total length of pipe, L = 8000 m

Height of ridge summit from water level in upper reservoir = 8 m

Let the depth of the pipe below the summit of ridge = x m

∴ Height of syphon from water surface in the upper reservoir = $(8 - x)$ m

Pressure head at C, $\frac{p_c}{\rho g}$ = 3.0 m of water absolute

Atmospheric pressure head, $\frac{p_a}{\rho g}$ = 10.3 m of water

Co-efficient of friction f = .006

Length of syphon from upper reservoir to the summit, L_1 = 500 m

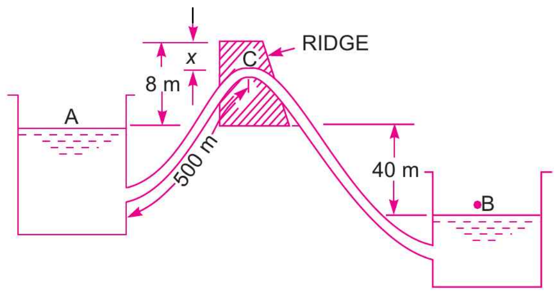


Fig. 11.15 (*b*)

Applying Bernoulli's equation to points A and B and taking datum line passing through B, we have

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + \text{head loss due to friction } A \text{ to } B$$

or
$$0 + 0 + 40 = 0 + 0 + 0 + \frac{4 \times f \times L \times V^2}{d \times 2g}$$

$\therefore$
$$40 = \frac{4 \times 0.006 \times 8000 \times V^2}{0.2 \times 2 \times 9.81}$$

$\therefore$
$$V = \sqrt{\frac{40 \times 0.2 \times 2 \times 9.81}{4 \times .006 \times 8000}} = 0.904 \text{ m/s}$$

Now applying Bernoulli's equation to points A and C and assuming datum line passing through A, we have

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c + \text{head loss due to friction from } A \text{ to } C$$

Substituting $\frac{p_A}{\rho g}$ and $\frac{p_c}{\rho g}$ in terms of absolute pressure

$$10.3 + 0 + 0 = 3.0 + \frac{V^2}{2g} + (8 - x) + \frac{4 \times f \times L_1 \times V^2}{d \times 2g}$$

or
$$10.3 = 3.0 + \frac{(0.904)^2}{2 \times 9.81} + (8 - x) + \frac{4 \times .006 \times 500 \times (0.904)^2}{0.2 \times 2 \times 9.81}$$

$$= 3.0 + 0.041 + (8 - x) + 2.499 = 13.54 - x$$

$\therefore$
$$x = 13.54 - 10.3 = \mathbf{3.24\ m.\ Ans.}$$

Discharge,
$$Q = \text{Area} \times \text{Velocity} = \frac{\pi}{4} \times (.2)^2 \times 0.904 = \mathbf{0.0283\ m^3/s.\ Ans.}$$

▶ 11.7 FLOW THROUGH PIPES IN SERIES OR FLOW THROUGH COMPOUND PIPES

Pipes in series or compound pipes are defined as the pipes of different lengths and different diameters connected end to end (in series) to form a pipe line as shown in Fig. 11.16.

Let, L_1, L_2, L_3 = length of pipes 1, 2 and 3 respectively

d_1, d_2, d_3 = diameter of pipes 1, 2, 3 respectively

V_1, V_2, V_3 = velocity of flow through pipes 1, 2, 3

f_1, f_2, f_3 = co-efficient of frictions for pipes 1, 2, 3

H = difference of water level in the two tanks.

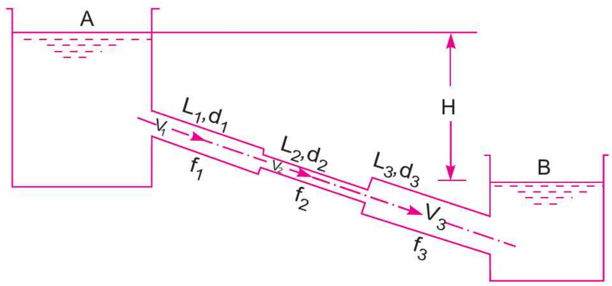


Fig. 11.16

The discharge passing through each pipe is same.

$$\therefore \quad Q = A_1V_1 = A_2V_2 = A_3V_3$$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$\therefore \quad H = \frac{0.5\,V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5\,V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g} \quad \text{...(11.12)}$$

If minor losses are neglected, then above equation becomes as

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} \quad \text{...(11.13)}$$

If the co-efficient of friction is same for all pipes

i.e., $f_1 = f_2 = f_3 = f$, then equation (11.13) becomes as

$$H = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} + \frac{4fL_3V_3^2}{d_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right] \quad \text{...(11.14)}$$

Problem 11.30 *The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 300 m, 170 m and 210 m and of diameters 300 mm, 200 mm and 400 mm respectively, is 12 m. Determine the rate of flow of water if co-efficient of friction are .005, .0052 and .0048 respectively, considering : (i) minor losses also (ii) neglecting minor losses.*

Solution. Given :

Difference of water level, $H = 12$ m

Length of pipe 1, $L_1 = 300$ m and dia., $d_1 = 300$ mm $= 0.3$ m

Length of pipe 2, $L_2 = 170$ m and dia., $d_2 = 200$ mm $= 0.2$ m

Length of pipe 3, $L_3 = 210$ m and dia., $d_3 = 400$ mm $= 0.4$ m

Also, $f_1 = .005, f_2 = .0052$ and $f_3 = .0048$

(*i*) **Considering Minor Losses.** Let V_1, V_2 and V_3 are the velocities in the 1st, 2nd and 3rd pipe respectively.

From continuity, we have $A_1V_1 = A_2V_2 = A_3V_3$

$$\therefore \qquad V_2 = \frac{A_1V_1}{A_2} = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2}V_1 = \frac{d_1^2}{d_2^2}V_1 = \left(\frac{0.3}{.2}\right)^2 \times V_1 = 2.25\ V_1$$

and
$$V_3 = \frac{A_1V_1}{A_3} = \frac{d_1^2}{d_3^2}V_1 = \left(\frac{0.3}{0.4}\right)^2 V_1 = 0.5625\ V_1$$

Now using equation (11.12), we have

$$H = \frac{0.5\,V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5\,V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g}$$

Substituting V_2 and V_3,
$$12.0 = \frac{0.5\,V_1^2}{2g} + \frac{4 \times .005 \times 300 \times V_1^2}{0.3 \times 2g} + \frac{0.5 \times (2.25 V_1^2)^2}{2g}$$

$$+ 4 \times 0.0052 \times 170 \times \frac{(2.25\,V_1)^2}{0.2 \times 2g} + \frac{(2.25\,V_1 - .562\,V_1)^2}{2g} + \frac{4 \times .0048 \times 210 \times (.5625\,V_1)^2}{0.4 \times 2g} + \frac{(.5625\,V_1)^2}{2g}$$

or
$$12.0 = \frac{V_1^2}{2g}\,[0.5 + 20.0 + 2.53 + 89.505 + 2.847 + 3.189 + 0.316]$$

$$= \frac{V_1^2}{2g}\,[118.887]$$

$$\therefore \qquad V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{118.887}} = 1.407 \text{ m/s}$$

$\therefore$ Rate of flow, Q = Area × Velocity = $A_1 \times V_1$

$$= \frac{\pi}{4}(d_1)^2 \times V_1 = \frac{\pi}{4}(.3)^2 \times 1.407 = 0.09945 \text{ m}^3/\text{s}$$

= 99.45 litres/s. Ans.

(*ii*) **Neglecting Minor Losses.** Using equation (11.13), we have

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

or
$$12.0 = \frac{V_1^2}{2g}\left[\frac{4 \times .005 \times 300}{0.3} + \frac{4 \times .0052 \times 170 \times (2.25)^2}{0.2} + \frac{4 \times .0048 \times 210 \times (.5625)^2}{0.4}\right]$$

$$= \frac{V_1^2}{2g} [20.0 + 89.505 + 3.189] = \frac{V_1^2}{2g} \times 112.694$$

$\therefore$ $$V_1 = \sqrt{\frac{2 \times 9.81 \times 12.0}{112.694}} = 1.445 \text{ m/s}$$

$\therefore$ Discharge, $Q = V_1 \times A_1 = 1.445 \times \frac{\pi}{4} (.3)^2 = 0.1021 \text{ m}^3\text{/s} =$ **102.1 litres/s. Ans.**

Problem 11.30 (A). *Three pipes of 400 mm, 200 mm and 300 mm diameters have lengths of 400 m, 200 m, and 300 m respectively. They are connected in series to make a compound pipe. The ends of this compound pipe are connected with two tanks whose difference of water levels is 16 m. If co-efficient of friction for these pipes is same and equal to 0.005, determine the discharge through the compound pipe neglecting first the minor losses and then including them.*

Solution. Given :

Difference of water levels, $H = 16$ m

Length and dia. of pipe 1, $L_1 = 400$ m and $d_1 = 400$ mm $= 0.4$ m

Length and dia. of pipe 2, $L_2 = 200$ m and $d_2 = 200$ mm $= 0.2$ m

Length and dia. of pipe 3, $L_3 = 300$ m and $d_3 = 300$ mm $= 0.3$ m

Also $f_1 = f_2 = f_3 = 0.005$

(*i*) **Discharge through the compound pipe first neglecting minor losses.**

Let V_1, V_2 and V_3 are the velocities in the 1st, 2nd and 3rd pipe respectively.

From continuity, we have $A_1V_1 = A_2V_2 = A_3V_3$

$\therefore$ $$V_2 = \frac{A_1V_1}{A_2} = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_2^2} \times V_1 = \frac{d_1^2}{d_2^2} V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 4V_1$$

and $$V_3 = \frac{A_1V_1}{A_3} = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_3^2} \times V_1 = \frac{d_1^2}{d_3^2} V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 1.77V_1$$

Now using equation (11.13), we have

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

or $$16 = \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 200 \times (4V_1)^2}{0.2 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 300}{0.3 \times 2 \times 9.81} \times (1.77\ V_1)^2$$

$$= \frac{V_1^2}{2 \times 9.81} \left(\frac{4 \times 0.005 \times 400}{0.4} + \frac{4 \times 0.005 \times 200 \times 16}{0.2} + \frac{4 \times 0.005 \times 300 \times 3.157}{0.3}\right)$$

$$16 = \frac{V_1^2}{2 \times 9.81} (20 + 320 + 63.14) = \frac{V_1^2}{2 \times 9.81} \times 403.14$$

$\therefore$ $$V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{403.14}} = 0.882 \text{ m/s}$$

$\therefore$ Discharge, $Q = A_1 \times V_1 = \frac{\pi}{4}(0.4)^2 \times 0.882 =$ **0.1108 m³/s. Ans.**

(*ii*) **Discharge through the compound pipe considering minor losses also.**

Minor losses are :

(*a*) At inlet, $h_i = \frac{0.5\,V_1^2}{2g}$

(*b*) Between 1st pipe and 2nd pipe, due to contraction,

$$h_c = \frac{0.5\,V_2^2}{2g} = \frac{0.5\left(4V_1^2\right)}{2g} \qquad (\because V_2 = 4V_1)$$

$$= \frac{0.5 \times 16 \times V_1^2}{2g} = 8 \times \frac{V_1^2}{2g}$$

(*c*) Between 2nd pipe and 3rd pipe, due to sudden enlargement,

$$h_e = \frac{\left(V_2 - V_3\right)^2}{2g} = \frac{\left(4V_1 - 1.77V_1\right)^2}{2g} \qquad (\because V_3 = 1.77\,V_1)$$

$$= (2.23)^2 \times \frac{V_1^2}{2g} = 4.973\,\frac{V_1^2}{2g}$$

(*d*) At the outlet of 3rd pipe, $h_o = \frac{V_3^2}{2g} = \frac{\left(1.77V_1\right)^2}{2g} = 1.77^2 \times \frac{V_1^2}{2g} = 3.1329\,\frac{V_1^2}{2g}$

The major losses are $= \frac{4f_1 \times L_1 \times V_1^2}{d_1 \times 2g} + \frac{4f_2 \times L_2 \times V_2^2}{d_2 \times 2g} + \frac{4f_3 \times L_3 \times V_3^2}{d_3 \times 2g}$

$$= \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 200 \times \left(4V_1\right)^2}{0.2 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 300 \times \left(1.77V_1\right)^2}{0.3 \times 2 \times 9.81}$$

$$= 403.14 \times \frac{V_1^2}{2 \times 9.81}$$

$\therefore$ Sum of minor losses and major losses

$$= \left[\frac{0.5\,V_1^2}{2g} + 8 \times \frac{V_1^2}{2g} + 4.973\,\frac{V_1^2}{2g} + 3.1329\,\frac{V_1^2}{2g}\right] + 403.14\,\frac{V_1^2}{2g}$$

$$= 419.746\,\frac{V_1^2}{2g}$$

But total loss must be equal to H (or 16 m)

$\therefore \qquad 419.746 \times \frac{V_1^2}{2g} = 16 \qquad \therefore \quad V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{419.746}} = 0.864$ m/s

$\therefore$ Discharge, $Q = A_1 V_1 = \frac{\pi}{4}(0.4)^2 \times 0.864 =$ **0.1085 m³/s. Ans.**

▶ 11.8 EQUIVALENT PIPE

This is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is called equivalent size of the pipe. The length of equivalent pipe is equal to sum of lengths of the compound pipe consisting of different pipes.

Let L_1 = length of pipe 1 and d_1 = diameter of pipe 1

L_2 = length of pipe 2 and d_2 = diameter of pipe 2

L_3 = length of pipe 3 and d_3 = diameter of pipe 3

H = total head loss

L = length of equivalent pipe

d = diameter of the equivalent pipe

Then $L = L_1 + L_2 + L_3$

Total head loss in the compound pipe, neglecting minor losses

$$H = \frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{4 f_3 L_3 V_3^2}{d_3 \times 2g} \qquad \text{...(11.14}A\text{)}$$

Assuming $f_1 = f_2 = f_3 = f$

Discharge, $Q = A_1 V_1 = A_2 V_2 = A_3 V_3 = \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2 = \frac{\pi}{4} d_3^2 V_3$

$\therefore$ $V_1 = \frac{4Q}{\pi d_1^2}$, $V_2 = \frac{4Q}{\pi d_2^2}$ and $V_3 = \frac{4Q}{\pi d_3^2}$

Substituting these values in equation (11.14*A*), we have

$$H = \frac{4 f L_1 \times \left(\frac{4Q}{\pi d_1^2}\right)^2}{d_1 \times 2g} + \frac{4 f L_2 \left(\frac{4Q}{\pi d_2^2}\right)^2}{d_2 \times 2g} + \frac{4 f L_3 \left(\frac{4Q}{\pi d_3^2}\right)^2}{d_3 \times 2g}$$

$$= \frac{4 \times 16 f Q^2}{\pi^2 \times 2g} \left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] \qquad \text{...(11.15)}$$

Head loss in the equivalent pipe, $H = \frac{4 f \,.\, L \,.\, V^2}{d \times 2g}$ [Taking same value of f as in compound pipe]

where $V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4} d^2} = \frac{4Q}{\pi d^2}$

$$\therefore \qquad H = \frac{4 f \,.\, L \,.\left(\frac{4Q}{\pi d^2}\right)^2}{d \times 2g} = \frac{4 \times 16 Q^2 f}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right] \qquad \text{...(11.16)}$$

Head loss in compound pipe and in equivalent pipe is same hence equating equations (11.15) and (11.16), we have

$$\frac{4 \times 16 f Q^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5}+\frac{L_2}{d_2^5}+\frac{L_3}{d_3^5}\right] = \frac{4 \times 16 Q^2 f}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right]$$

or
$$\frac{L_1}{d_1^5}+\frac{L_2}{d_2^5}+\frac{L_3}{d_3^5} = \frac{L}{d^5} \quad \text{or} \quad \frac{L}{d^5} = \frac{L_1}{d_1^5}+\frac{L_2}{d_2^5}+\frac{L_3}{d_3^5} \qquad ...(11.17)$$

Equation (11.17) is known as Dupuit's equation. In this equation $L = L_1 + L_2 + L_3$ and d_1, d_2 and d_3 are known. Hence the equivalent size of the pipe, *i.e.*, value of d can be obtained.

Problem 11.31 *Three pipes of lengths 800 m, 500 m and 400 m and of diameters 500 mm, 400 mm and 300 mm respectively are connected in series. These pipes are to be replaced by a single pipe of length 1700 m. Find the diameter of the single pipe.*

Solution. Given :

Length of pipe 1, $L_1 = 800$ m and dia., $d_1 = 500$ mm $= 0.5$ m

Length of pipe 2, $L_2 = 500$ m and dia., $d_2 = 400$ mm $= 0.4$ m

Length of pipe 3, $L_3 = 400$ m and dia., $d_3 = 300$ mm $= 0.3$ m

Length of single pipe, $L = 1700$ m

Let the diameter of equivalent single pipe $= d$

Applying equation (11.17), $\frac{L}{d^5} = \frac{L_1}{d_1^5}+\frac{L_2}{d_2^5}+\frac{L_3}{d_3^5}$

or
$$\frac{1700}{d^5} = \frac{800}{.5^5}+\frac{500}{.4^5}+\frac{400}{0.3^5} = 25600 + 48828.125 + 164609 = 239037$$

$$\therefore \quad d^5 = \frac{1700}{239037} = .007118$$

$$\therefore \quad d = (.007188)^{0.2} = 0.3718 = \textbf{371.8 mm. Ans.}$$

▶ 11.9 FLOW THROUGH PARALLEL PIPES

Consider a main pipe which divides into two or more branches as shown in Fig. 11.17 and again join together downstream to form a single pipe, then the branch pipes are said to be connected in parallel. The discharge through the main is increased by connecting pipes in parallel.

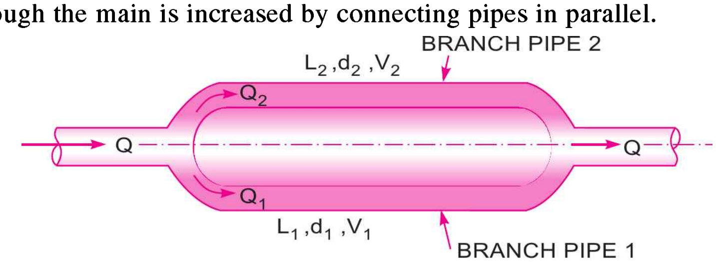


Fig. 11.17

The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes. Hence from Fig. 11.17, we have

$$Q = Q_1 + Q_2 \qquad ...(11.18)$$

In this, arrangement, the loss of head for each branch pipe is same.

$\therefore$ Loss of head for branch pipe 1 = Loss of head for branch pipe 2 or $$\frac{4f_1L_1V_1^2}{d_1\times 2g}=\frac{4f_2L_2V_2^2}{d_2\times 2g} \qquad ...(11.19)$$

If $$f_1=f_2,\text{ then } \frac{L_1V_1^2}{d_1\times 2g}=\frac{L_2V_2^2}{d_2\times 2g} \qquad ...(11.20)$$

Problem 11.32 *A main pipe divides into two parallel pipes which again forms one pipe as shown in Fig. 11.17. The length and diameter for the first parallel pipe are 2000 m and 1.0 m respectively, while the length and diameter of 2nd parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is 3.0 m³/s. The co-efficient of friction for each parallel pipe is same and equal to .005.*

Solution. Given :

Length of pipe 1, $L_1 = 2000$ m

Dia. of pipe 1, $d_1 = 1.0$ m

Length of pipe 2, $L_2 = 2000$ m

Dia. of pipe 2, $d_2 = 0.8$ m

Total flow, $Q = 3.0\ m^3/s$

$f_1 = f_2 = f = .005$

Let Q_1 = discharge in pipe 1

Q_2 = discharge in pipe 2

From equation (11.18), $Q = Q_1 + Q_2 = 3.0$ ...(*i*)

Using equation (11.19), we have

$$\frac{4f_1L_1V_1^2}{d_1\times 2g}=\frac{4f_2L_2V_2^2}{d_2\times 2g}$$

$$\frac{4\times .005\times 2000\times V_1}{1.0\times 2\times 9.81}=\frac{4\times .005\times 2000\times V_2^2}{0.8\times 2\times 9.81}$$

or $$\frac{V_1^2}{1.0}=\frac{V_2^2}{0.8} \text{ or } V_1{}^2=\frac{V_2^2}{0.8}$$

∴ $$V_1=\frac{V_2}{\sqrt{0.8}}=\frac{V_2}{.894} \qquad ...(ii)$$

Now $$Q_1=\frac{\pi}{4}d_1{}^2\times V_1=\frac{\pi}{4}(1)^2\times\frac{V_2}{.894} \qquad \left[\because V_1=\frac{V_2}{.894}\right]$$

and $$Q_2=\frac{\pi}{4}d_2{}^2\times V_2=\frac{\pi}{4}(.8)^2\times V_2=\frac{\pi}{4}\times .64\times V_2$$

Substituting the value of Q_1 and Q_2 in equation (*i*), we get

$$\frac{\pi}{4}\times\frac{V_2}{0.894}+\frac{\pi}{4}\times .64\ V_2=3.0 \text{ or } 0.8785\ V_2+0.5026\ V_2=3.0$$

or $$V_2[.8785+.5026]=3.0 \text{ or } V=\frac{3.0}{1.3811}=2.17\text{ m/s.}$$

Substituting this value in equation (*ii*),

$$V_1 = \frac{V_2}{.894} = \frac{2.17}{0.894} = 2.427 \text{ m/s}$$

Hence $$Q_1 = \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} \times 1^2 \times 2.427 = \mathbf{1.906 \ m^3/s. \ Ans.}$$

∴ $$Q_2 = Q - Q_1 = 3.0 - 1.906 = \mathbf{1.094 \ m^3/s. \ Ans.}$$

Problem 11.33 *A pipe line of 0.6 m diameter is 1.5 km long. To increase the discharge, another line of the same diameter is introduced parallel to the first in the second half of the length. Neglecting minor losses, find the increase in discharge if 4f = 0.04. The head at inlet is 300 mm.*

Solution. Given :

Dia. of pipe line, $D = 0.6$ m

Length of pipe line, $L = 1.5$ km $= 1.5 \times 1000 = 1500$ m

$4f = 0.04$ or $f = .01$

Head at inlet, $h = 300$ mm $= 0.3$ m

Head at outlet, = atmospheric head = 0

∴ Head loss, $h_f = 0.3$ m

Length of another parallel pipe, $L_1 = \frac{1500}{2} = 750$ m

Dia. of another parallel pipe, $d_1 = 0.6$ m

Fig. 11.18 shows the arrangement of pipe system.

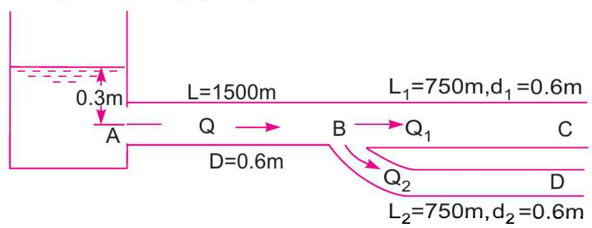


Fig. 11.18

1st Case. Discharge for a single pipe of length 1500 m and dia. = 0.6 m.

This head lost due to friction in single pipe is $h_f = \frac{4fLV^{*2}}{d \times 2g}$

where V^* = velocity of flow for single pipe

or $$0.3 = \frac{4 \times .01 \times 1500 \times V^{*2}}{0.6 \times 2g}$$

∴ $$V^* = \sqrt{\frac{0.3 \times 0.6 \times 2 \times 9.81}{4 \times .01 \times 1500}} = 0.2426 \text{ m/s}$$

∴ $$\text{Discharge, } Q^* = V^* \times \text{Area} = 0.2426 \times \frac{\pi}{4} (.6)^2 = 0.0685 \text{ m}^3\text{/s} \quad ...(i)$$

2nd Case. When an additional pipe of length 750 m and diameter 0.6 m is connected in parallel with the last half length of the pipe.

Let Q_1 = discharge in 1st parallel pipe

Q_2 = discharge in 2nd parallel pipe

$\therefore \quad Q = Q_1 + Q_2$

where Q = discharge in main pipe when pipes are parallel.

But as the length and diameters of each parallel pipe is same

$$\therefore \quad Q_1 = Q_2 = Q/2$$

Consider the flow through pipe ABC or ABD

Head loss through ABC = Head lost through AB + head lost through BC ...(*ii*)

But head lost due to friction through ABC = 0.3 m given

Head loss due to friction through $AB = \dfrac{4 \times f \times 750 \times V^2}{0.6 \times 2 \times 9.81}$, where V = velocity of flow through AB

$$= \frac{Q}{\text{Area}} = \frac{Q}{\frac{\pi}{4}(0.6)^2} = \frac{40}{\pi \times .36}$$

$\therefore$ Head loss due to friction through AB

$$= \frac{4 \times .01 \times 750}{0.6 \times 2 \times 9.81} \times \left(\frac{4Q}{\pi \times .36}\right)^2 = 31.87\ Q^2$$

Head loss due to friction through BC

$$= \frac{4 \times f \times L_1 \times V_1^2}{d_1 \times 2g}$$

$$= \frac{4 \times .01 \times 750}{0.6 \times 2 \times 9.81} \times \left[\frac{Q}{2 \times \frac{\pi}{4}(.6)^2}\right] \quad \left[\because V_1 = \frac{\text{Distance}}{\frac{\pi}{4}(.6)^2} = \frac{Q}{2 \times \frac{\pi}{4} \times (.6)^2}\right]$$

$$= \frac{4 \times .01 \times 750}{0.6 \times 2 \times 9.81} \times \frac{16}{4 \times \pi^2 \times .36^2}\ Q^2 = 7.969\ Q^2$$

Substituting these values in equation (*ii*), we get

$$0.3 = 31.87\ Q^2 + 7.969\ Q^2 = 39.839\ Q^2$$

$$\therefore \quad Q = \sqrt{\frac{0.3}{39.839}} = 0.0867\ \text{m}^3/\text{s}$$

$\therefore$ Increase in discharge = $Q - Q^*$ = 0.0867 – 0.0685 = **0.0182 m^3/s. Ans.**

Problem 11.34 *A pumping plant forces water through a 600 mm diameter main, the friction head being 27 m. In order to reduce the power consumption, it is proposed to lay another main of appropriate diameter along the side of the existing one, so that two pipes may work in parallel for the entire length and reduce the friction head to 9.6 m only. Find the diameter of the new main if, with the exception of diameter, it is similar to the existing one in every respect.*

Solution. Given :

Dia. of single main pipe, d = 600 mm = 0.6 m

Friction head, h_f = 27 m

Friction head for two parallel pipes = 9.6 m

1st Case.

For a single main [Fig. 11.19 (*a*)]

$$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g} \quad \text{or} \quad 27.0 = \frac{4 \times f \times L \times V^2}{0.6 \times 2 \times 9.81}$$

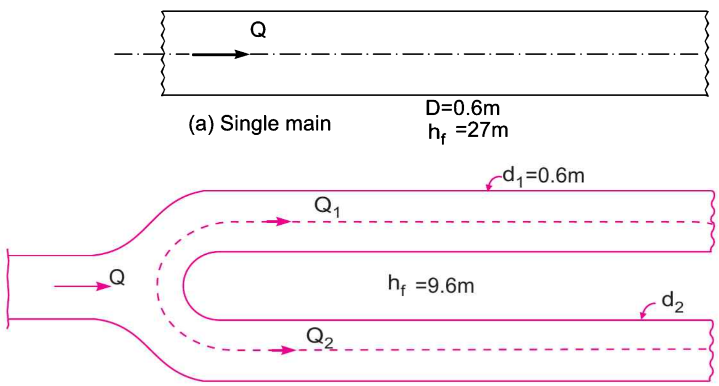
$\therefore$
$$fLV^2 = \frac{27.0 \times 0.6 \times 2 \times 9.81}{4} = \frac{317.844}{5} = 79.461, \text{ where } V = \frac{Q}{A}$$

$\therefore$
$$f.L.\frac{Q^2}{A^2} = 79.461 \qquad \text{...}(i)$$

2nd Case. Two pipes are in parallel [Fig. 11.19 (*b*)]

Loss of head in any one pipe = 9.6 m

$\therefore$ For 1st pipe,
$$h_{f_1} = \frac{4 \,.\, f \,.\, L \,.\, V_1^2}{d_1 \times 2g} = 9.6$$



(b) Two parallel pipes

Fig. 11.19

But
$$L_1 = L,\ V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{A} \qquad \left[\because A_1 = A = \frac{\pi}{4}(.6)^2\right]$$
$$d_1 = d = 0.6$$

$\therefore$
$$\frac{4 \,.\, f \,.\, L}{0.6 \times 2 \times 9.81}\frac{Q_1^2}{A^2} = 0.6$$

or
$$f \times L \times \frac{Q_1^2}{A^2} = \frac{9.6 \times 0.6 \times 2 \times 9.81}{4} = 28.2528 \qquad \text{...}(ii)$$

For the 2nd pipe,
$$h_{f_2} = \frac{4f \times L_2 \times V_2^2}{d_2 \times 2g} = 9.6, \qquad \text{where } L_2 = L,\ V_2 = \frac{Q_2}{A_2}$$

$\therefore$
$$\frac{4f \times L \times Q_2^2}{d_2 \times 2g \times A_2^2} = 9.6$$

or
$$\frac{f \times L \times Q_2^2}{d_2 \times A_2^2} = \frac{9.6 \times 2 \times 9.81}{4} = 47.088 \qquad \text{...}(iii)$$

Dividing equation (*i*) by equation (*ii*), we get

$$\frac{Q^2}{Q_1^2} = \frac{79.461}{28.2528} = 2.8125$$

$\therefore$ $$\frac{Q}{Q_1} = \sqrt{2.8125} = 1.667$$

$\therefore$ $$Q_1 = \frac{Q}{1.667} = .596\ Q$$

But $$Q_1 + Q_2 = Q$$

$\therefore$ $$Q_2 = Q - Q_1 = Q - .596\ Q = 0.404\ Q$$

Dividing equation (*ii*) by equation (*iii*),

$$\frac{Q_1^2 \times d_2 \times A_2^2}{A^2 \times Q_2^2} = \frac{28.2528}{47.088} = 0.6$$

But $$A_2 = \frac{\pi}{4}\ d_2^{\ 2} \text{ and } A = \frac{\pi}{4}\ d^2 = \frac{\pi}{4}\ (.6)^2 = \frac{\pi}{4} \times .36$$

$\therefore$ $$\frac{Q_1^2}{Q_2^2} \times \frac{d_2 \times \left(\frac{\pi}{4}\right)^2 \times d_2^4}{\left(\frac{\pi}{4}\right)^2 \times (.36)^2} = 0.6 \quad \text{or} \quad \left(\frac{.596\ Q}{.404\ Q}\right)^2 \times \frac{d_2^5}{.36^2} = 0.6$$

or $$d_2^{\ 5} = 0.6 \times .36^2 \times \left(\frac{.404}{.596}\right)^2 = 0.03537$$

$\therefore$ $$d_2 = (.03537)^{1/5} = 0.5125 \text{ m} = \mathbf{512.5\ mm.\ Ans.}$$

Problem 11.35 *A pipe of diameter 20 cm and length 2000 m connects two reservoirs, having difference of water levels as 20 m. Determine the discharge through the pipe.*

If an additional pipe of diameter 20 cm and length 1200 m is attached to the last 1200 m length of the existing pipe, find the increase in the discharge. Take f = .015 and neglect minor losses.

Solution. Given :

Dia. of pipe, $d = 20 \text{ cm} = 0.20 \text{ m}$

Length of pipe, $L = 2000 \text{ m}$

Difference of water levels, $H = 20 \text{ m}$

Co-efficient of friction, $f = 0.015$

1st Case. When a single pipe connects the two reservoirs

$$H = \frac{4 . f . L . V^2}{d \times 2g} = \frac{4f . L}{d \times 2g}\left(\frac{Q}{\frac{\pi}{4} d^2}\right)^2 \qquad \left[\because V = \frac{Q}{\frac{\pi}{4} d^2}\right]$$

$$= \frac{32 f . L . Q^2}{\pi^2 \times g \times d^5}$$

or
$$20 = \frac{32 \times .015 \times 2000 \times Q^2}{\pi^2 \times 9.81 \times (0.2)^5} = 30985.07\ Q^2$$

∴
$$Q = \sqrt{\frac{20}{30985.07}} = \mathbf{0.0254\ m^3/s.\ Ans.}$$

2nd Case.

Let Q_1 = discharge through pipe *CD*,

Q_2 = discharge through pipe *DE*,

Q_3 = discharge through pipe *DF*.

Length of pipe *CD*, $L_1 = 800$ m and its dia., $d_1 = 0.20$ m

Length of pipe *DE*, $L_2 = 1200$ m and its dia., $d_2 = 0.20$ m

Length of pipe *DF*, $L_3 = 1200$ m and its dia., $d_3 = 0.20$ m.

Since the diameters and lengths of the pipes *DE* and *DF* are equal. Hence Q_2 will be equal to Q_3. Also for parallel pipes, we have

$$Q_1 = Q_2 + Q_3 = Q_2 + Q_2 = 2Q_2 \qquad [\because Q_2 = Q_3]$$

∴
$$Q_2 = \frac{Q_1}{2}$$

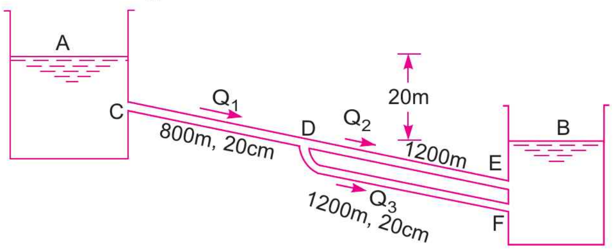


Fig. 11.20

Applying Bernoulli's equation to points *A* and *B* and taking the flow through *CDE*, we have

$$20 = \frac{4f.L_1.V_1^2}{d_1 \times 2g} + \frac{4f.L_2.V_2^2}{d_2 \times 2g}$$

where
$$V_1 = \frac{Q_1}{\frac{\pi}{4}(.2)^2} = \frac{4Q_1}{\pi \times .04}, V_2 = \frac{Q_2}{\frac{\pi}{4}(.2)^2} = \frac{4Q_2}{\pi \times .04} = \frac{4 \times \frac{Q_1}{2}}{\pi \times .04} = \frac{2Q_1}{\pi \times .04}$$

$$= \frac{4 \times .015 \times 800}{0.2 \times 2 \times 9.81} \times \left(\frac{4Q_1}{\pi \times .04}\right)^2 + \frac{4 \times .015 \times 1200}{0.2 \times 2 \times 9.81} \times \left(\frac{2Q_1}{\pi \times .04}\right)^2$$

$$= 12394\ Q_1^2 + 4647\ Q_1^2 = 17041\ Q_1^2$$

∴
$$Q_1 = \sqrt{\frac{20}{17041}} = 0.0342\ \text{m}^3/\text{s}$$

Increase in discharge = $Q_1 - Q$ = 0.0342 – 0.0254 = **.0088 m³/s. Ans.**

Problem 11.36 *Two pipes have a length L each. One of them has a diameter D, and the other a diameter d. If the pipes are arranged in parallel, the loss of head, when a total quantity of water Q flows through them is h, but, if the pipes are arranged in series and the same quantity Q flows through them, the loss of head is H. If* $d = \frac{D}{2}$*, find the ratio of H to h, neglecting secondary losses and assuming the pipe co-efficient f has a constant value.*

Solution. Given :

Length of pipe 1, $L_1 = L$ and its dia. $d_1 = D$

Length of pipe 2, $L_2 = L$ and its dia., $d_2 = d$

Total discharge $= Q$

Head loss when pipes are arranged in parallel $= h$

Head loss when pipes are arranged in series $= H$

$$d = \frac{D}{2} \text{ and } f \text{ is constant}$$

1st Case. When pipes are connected to parallel

$$Q = Q_1 + Q_2 \quad \text{...(i)}$$

Loss of head in each pipe $= h$

For pipe AB, $\frac{4fL_1V_1^2}{d_1 \times 2g} = h$, where $V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{\frac{\pi}{4}D^2} = \frac{4Q_1}{\pi D^2}$

$$d_1 = D$$

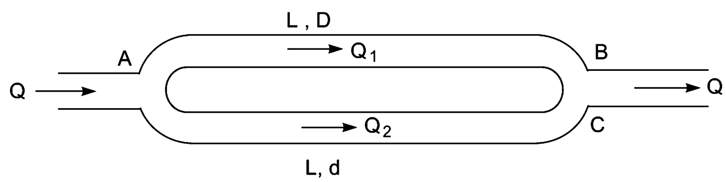


Fig. 11.21

$$\therefore \quad \frac{4fL \times \left(\frac{4Q_1}{\pi D^2}\right)^2}{D \times 2g} = h \text{ or } \frac{32fLQ_1^2}{\pi^2 D^5 \times g} = h \quad \text{...(ii)}$$

For pipe AC, $\frac{32fLQ_2^2}{\pi^2 d^5 \times g} = h$ ...(iii)

$$\therefore \quad \frac{32fLQ_1^2}{\pi^2 D^5 g} = \frac{32fLQ_2^2}{\pi^2 d^5 g} \text{ or } \frac{Q_1^2}{D^5} = \frac{Q_2^2}{d^5}$$

or $$\left(\frac{Q_1}{Q_2}\right)^2 = \frac{D^5}{d^5} = \frac{(2d)^5}{d^5} \quad [\because D = 2d]$$

$$= 2^5 = 32$$

$$\therefore \quad \frac{Q_1}{Q_2} = \sqrt{32} = 5.657 \text{ or } Q_1 = 5.657\, Q_2$$

Substituting the values of Q_1 in equation (*i*), we get

$$Q = 5.657\ Q_2 + Q_2 = 6.657\ Q_2$$

$$\therefore \quad Q_2 = \frac{Q}{6.657} = 0.15\ Q \quad ...(iv)$$

$$\text{From } (i) \therefore \quad Q_1 = Q - Q_2 = Q - 0.15\ Q = 0.85\ Q \quad ...(v)$$

2nd Case. When the pipes are connected in series.

Total loss = Sum of head losses in the two pipes

$$\therefore \quad H = \frac{4f.L.V_1^2}{d_1 \times 2g} + \frac{4f.L.V_2^2}{d_2 \times 2g}$$

where $V_1 = \frac{Q}{\frac{\pi}{4}D^2} = \frac{4Q}{\pi D^2}$, $V_2 = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$

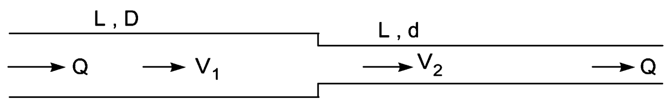


Fig. 11.22

$$\therefore \quad H = \frac{4f.L \times \left(\frac{4Q}{\pi D^2}\right)^2}{D \times 2g} + \frac{4fL\left(\frac{4Q}{\pi d^2}\right)^2}{d \times 2g}$$

$$\text{or} \quad H = \frac{32fLQ^2}{D^5\pi^2 \times g} + \frac{32fLQ^2}{d^5\pi^2 \times g} \quad ...(vi)$$

From equation (*ii*), $\frac{32fL}{\pi^2 D^5 \times g} = \frac{h}{Q_1^2}$

and from equation (*iii*), $\frac{32fL}{\pi^2 d^5 \times g} = \frac{h}{Q_2^2}$

Substituting these values in equation (*vi*), we have

$$H = Q^2 \times \frac{h}{Q_1^2} + Q^2 \times \frac{h}{Q_2^2} = \frac{Q^2}{Q_1^2}h + \frac{Q^2}{Q_2^2}h = h\left[\frac{Q^2}{Q_1^2} + \frac{Q^2}{Q_2^2}\right]$$

$$\therefore \quad \frac{H}{h} = \frac{Q^2}{Q_1^2} + \frac{Q^2}{Q_2^2}$$

But from equations (*iv*) and (*v*), $Q_1 = .85\ Q$ and $Q_2 = 0.15\ Q$

$$\therefore \quad \frac{H}{h} = \frac{Q^2}{.85^2 Q^2} + \frac{Q^2}{.15^2 Q^2} = \frac{1}{.85^2} + \frac{1}{.15^2} = 1.384 + 44.444 = \mathbf{45.828.\ Ans.}$$

Problem 11.36 (A). *Three pipes of the same length L, diameter D, and friction factor f are connected in parallel. Determine the diameter of the pipe of length L and friction factor f which will carry the same discharge for the same head loss. Use the formula* $h_f = f \times L \times V^2/2g\ D$.

Solution. Given :

Length of each pipe $= L$

Diameter of each pipe $= D$

Friction factor of each pipe $= f$

Head loss, $h_f = f \times L \times V^2/2gD$

When the three pipes are connected in parallel, then head loss in each pipe will be same. And total head loss will be equal to the head loss in each pipe.

Let h_f = Total head loss,

h_{f_1} = Head loss in 1st pipe,

h_{f_2} = Head loss in 2nd pipe, and h_{f_3} = Head loss in 3rd pipe.

Then
$$h_f = h_{f_1} = h_{f_2} = h_{f_3} \text{ or } h_f = \frac{f \times L \times V^2}{2gD} \qquad ...(i)$$

Let Q_1 = Discharge through 1st pipe, Q_2 = Discharge through 2nd pipe,

Q_3 = Discharge through 3rd pipe, and Q = Total discharge.

When the three pipes are connected in parallel, then

$$Q = Q_1 + Q_2 + Q_3 = 3 \times Q_1 \qquad (\because Q_1 = Q_2 = Q_3)$$
$$= 3 \times A_1 \times V_1$$
$$= 3 \times \frac{\pi}{4} D^2 \times V \left(\text{where } A_1 = \frac{\pi}{4} D^2 \text{ and } V_1 = V\right) \qquad ...(ii)$$

For a single pipe (or length L ; friction factor f) which will carry same discharge as the three pipes in parallel

Let d = dia. of the single pipe

v = velocity through single pipe

Then discharge, Q = Area × Velocity = $\left(\frac{\pi}{4} d^2\right) \times v$ ...(*iii*)

Equating the two values of discharge, given by equations (*ii*) and (*iii*), we get

$$3 \times \frac{\pi}{4} D^2 \times V = \frac{\pi}{4} d^2 \times v \text{ or } 3 \times \frac{D^2}{d^2} = \frac{v}{V} \qquad ...(iv)$$

The head loss for the single pipe is also equal to the total head loss for three pipes when they are in parallel.

But head loss for the single pipe of length L, dia. d, friction factor f and velocity v is given by

$$h_f = \frac{f \times L \times v^2}{d \times 2g} \qquad ...(v)$$

Equating the two values of h_f given by equations (*i*) and (*v*), we get

$$\frac{f \times L \times V^2}{D \times 2g} = \frac{f \times L \times v^2}{d \times 2g} \text{ or } \frac{V^2}{D} = \frac{v^2}{d}$$

or
$$\frac{d}{D} = \frac{v^2}{V^2} \quad \text{or} \quad \left(\frac{d}{D}\right)^{1/2} = \frac{v}{V}$$

Substituting the value of v/V in equation (*iv*), we get

$$3 \times \frac{D^2}{d^2} = \left(\frac{d}{D}\right)^{1/2} \text{ or } 3 = \left(\frac{d}{D}\right)^{1/2} \times \left(\frac{d}{D}\right)^2 = \left(\frac{d}{D}\right)^{5/2}$$

or $$\frac{d}{D} = 3^{2/5} = 3^{0.4} = 1.55$$

∴ $d =$ **1.55 D. Ans.**

Hence dia. of single pipe should be 1.55 times the dia. of the three pipes connected in parallel.

Problem 11.37 *For a town water supply, a main pipe line of diameter 0.4 m is required. As pipes more than 0.35 m diameter are not readily available, two parallel pipes of the same diameter were used for water supply. If the total discharge in the parallel pipes is same as in the single main pipe, find the diameter of the parallel pipe. Assume the co-efficient of friction same for all pipes.*

Solution. Given :

Dia. of single main pipe line, $d = 0.4$ m

Let the length of single pipe line $= L$

Co-efficient of friction $= f$

Loss of head due to friction in single pipe $= \dfrac{4fLV^2}{d \times 2g} = \dfrac{4fLV^2}{0.4 \times 2 \times g}$ ...(*i*)

where V = Velocity of flow in the single pipe.

In case of parallel pipe, as the diameters and lengths of the two pipes are same. Hence discharge in each pipe will be half the discharge of single main pipe. As discharge in each parallel pipe is same, hence velocity will also be same.

Let V_* = Velocity in each parallel pipe

d_* = Dia. of each parallel pipe

Then loss of head due to friction in parallel pipes $= \dfrac{4f \times L \times V_*^2}{d_* \times 2g}$ ...(*ii*)

Equating the two losses given by equations (*i*) and (*ii*), we have

$$\frac{4f \,.\, L \,.\, V^2}{0.4 \times 2g} = \frac{4f \times L \times V_*^2}{d_* \times 2g}$$

Cancelling $\dfrac{4fL}{2g}$, $$\frac{V^2}{0.4} = \frac{V_*^2}{d_*^2} \text{ or } \frac{V^2}{V_*^2} = \frac{0.4}{d_*} \quad ...(iii)$$

From continuity

Total flow in single main = sum of flow in two parallel pipes

or Velocity of main × Area = 2 × Velocity in each parallel pipe × Area

$$V \times \frac{\pi}{4}(0.4)^2 = 2 \times V_* \times \frac{\pi}{4} d^2{}_* \text{ or } \frac{V}{V_*} = \frac{2 \times \frac{\pi}{4} d_*^2}{\frac{\pi}{4}(0.4)^2} = \frac{2d_*^2}{0.16}$$

Squaring both sides, $$\frac{V^2}{V_*^2} = \frac{4d_*^4}{0.0256} \quad ...(iv)$$

Comparing equations (*iii*) and (*iv*), we get

$$\frac{0.4}{d_*} = \frac{4d_*^4}{.0256} \quad \text{or} \quad d_*^5 = \frac{0.4 \times .0256}{4} = .00256$$

$\therefore$ $d_* = (.00256)^{1/5} = 0.303$ m = **30.3 cm. Ans.**

$\therefore$ Use two pipes of 30.3 cm diameter.

Problem 11.38 *An old water supply distribution pipe of 250 mm diameter of a city is to be replaced by two parallel pipes of smaller equal diameter having equal lengths and identical friction factor values. Find out the new diameter required.*

Solution. Given :

Dia. of old pipe, D = 250 mm = 0.25 m

Let d = Dia. of each of parallel pipes

Q = Discharge in old pipe

Q_1 = Discharge in first parallel pipe

Q_2 = Discharge in second parallel pipe

f = Friction factor.

When a single pipe is replaced by two parallel pipes, the head loss will be same in the single pipe and in each of the parallel pipes. Also the discharge in single pipe will be equal to the total discharge in two parallel pipes *i.e.,*

$$h_f = h_{f_1} = h_2 \quad \text{...(i)}$$

and

$$Q = Q_1 + Q_2 \quad \text{...(ii)}$$

As the dia. of each parallel pipe is same and also length of each parallel pipe is equal, hence

$$Q_1 = Q_2 \text{ or } Q_1 = Q_2 = Q/2$$

Now h_f = Head loss in single pipe

$$= \frac{f \times L \times V^2}{D \times 2g}, \quad \text{where } f = \text{Friction factor}$$

$$= \frac{f \times L \times \left(\dfrac{Q}{\dfrac{\pi}{4} \times 0.25^2}\right)^2}{0.25 \times 2 \times 9.81} \quad \left(\because V = \frac{Q}{\text{Area}} = \frac{Q}{\dfrac{\pi}{4} D^2}\right)$$

$$= \frac{f \times L \times (4Q)^2}{0.25 \times 2 \times 9.81 \times \left(\pi \times 0.25^2\right)^2} \quad \text{...(iii)}$$

h_{f_1} = Head loss in 1st parallel pipe

$$= \frac{f \times L \times (V_1)^2}{d \times 2g}$$ ($\because$ Dia. of parallel pipe = d and V_1 is the velocity in 1st parallel pipe)

$$= \frac{f \times L \times \left(\dfrac{Q}{2 \times \dfrac{\pi}{4} d^2}\right)^2}{d \times 2g} \quad \left(\because V_1 = \frac{Q_1}{A_1} = \frac{\dfrac{Q}{2}}{\dfrac{\pi}{4} d^2} \text{ as } Q_1 = \frac{Q}{2}\right)$$

$$= \frac{f \times L \times (4Q)^2}{d \times 2 \times 9.81 \times \left(2 \times \pi \times d^2\right)^2} \qquad ...(iv)$$

But $\qquad h_f = h_{f_1}$

or $$\frac{f \times L \times (4Q)^2}{0.25 \times 2 \times 9.81 \times (\pi \times 0.25^2)^2} = \frac{f \times L \times (4Q)^2}{d \times 2 \times 9.81 \times \left(2 \times \pi \times d^2\right)^2}$$

or $$d \times (2\pi d^2)^2 = 0.25 \times (\pi \times 0.25^2)^2$$

or $$d \times 4 \times d^4 = 0.25 \times 0.25^4$$

or $$d^5 = \frac{0.25^5}{4} \text{ or } d = \frac{0.25}{(4)^{1/5}} = \frac{0.25}{1.3195} = \mathbf{0.1894\ m \simeq 0.19\ m.\ Ans.}$$

Problem 11.39 *A pipe of diameter 0.4 m and of length 2000 m is connected to a reservoir at one end. The other end of the pipe is connected to a junction from which two pipes of lengths 1000 m and diameter 300 mm run in parallel. These parallel pipes are connected to another reservoir, which is having level of water 10 m below the water level of the above reservoir. Determine the total discharge if f = 0.015. Neglect minor losses.*

Solution. Given :

Dia. of pipe, $\qquad d = 0.4$ m

Length of pipe, $\qquad L = 2000$ m

Dia. of parallel pipes, $\qquad d_1 = d_2 = 300$ mm $= 0.30$ m

Length of parallel pipes, $\qquad L_1 = L_2 = 1000$ m

Difference of water level in two reservoir, $H = 10$ m, $f = .015$

Applying Bernoulli's equation to points E and F. Taking flow through ABC.

$$10 = \frac{4fLV^2}{d \times 2g} + \frac{4f \times L_1 \times V_1^2}{d_1 \times 2g}$$

$$= \frac{4 \times .015 \times 2000 \times V^2}{0.4 \times 2 \times 9.81} + \frac{4 \times .015 \times 1000 \times V_1^2}{0.3 \times 2 \times 9.81}$$

$$= 15.29\ V^2 + 10.19\ V_1^2 \qquad ...(i)$$

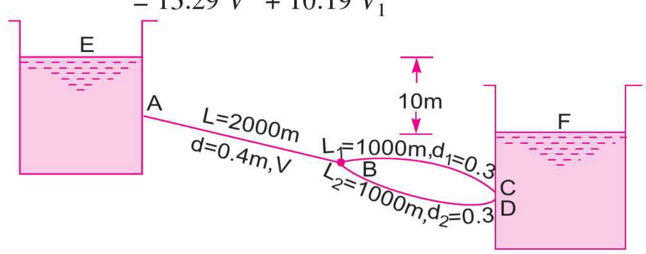


Fig. 11.23

From continuity equation

Discharge through AB = discharge through BC + discharge through BD

or $$\frac{\pi}{4} d^2 \times V = \frac{\pi}{4} d_1^2 \times V_1 + \frac{\pi}{4} d_1^2 V_2$$

But $d_1 = d_2$ and also the lengths of pipes BC and BD are equal and hence discharge through BC and BD will be same. This means $V_1 = V_2$ also

$$\therefore \quad \frac{\pi}{4} d^2 V = \frac{\pi}{4} d_1^2 \times V_1 + \frac{\pi}{4} d_1^2 \times V_1 \qquad [\because d_1 = d_2, V_1 = V_2]$$

$$= 2 \times \frac{\pi}{4} d_1^2 \times V_1 \text{ or } d^2 V = 2 d_1^2 V_1$$

or $$(0.4)^2 \times V = 2 \times (0.3)^2 V_1 \text{ or } .16V = 0.18\, V_1$$

$$\therefore \quad V_1 = \frac{0.16}{0.18} V = 0.888\, V$$

Substituting this value of V_1 in equation (i), we get

$$10 = 15.29\, V^2 + (10.19)(.888)^2 V^2 = 15.29\, V^2 + 8.035\, V^2 = 23.325\, V^2$$

$$\therefore \quad V = \sqrt{\frac{10}{23.325}} = 0.654 \text{ m/s}$$

$\therefore$ Discharge $= V \times \text{Area}$

$$= 0.654 \times \frac{\pi}{4} d^2 = 0.654 \times \frac{\pi}{4} (0.4)^2 = \mathbf{.0822 \text{ m}^3/\text{s. Ans.}}$$

Problem 11.40 *Two sharp ended pipes of diameters 50 mm and 100 mm respectively, each of length 100 m are connected in parallel between two reservoirs which have a difference of level of 10 m. If the co-efficient of friction for each pipe is (4f) 0.32, calculate the rate of flow for each pipe and also the diameter of a single pipe 100 m long which would give the same discharge, if it were substituted for the original two pipes.*

Solution. Given :

Dia. of 1st pipe, $d_1 = 50 \text{ mm} = 0.05 \text{ m}$

Length of 1st pipe, $L_1 = 100 \text{ m}$

Dia. of 2nd pipe, $d_2 = 100 \text{ mm} = 0.10 \text{ m}$

Length of 2nd pipe, $L_2 = 100 \text{ m}$

Difference in level in reservoirs, $H = 10 \text{ m}$

Co-efficient of friction $4f = 0.32$

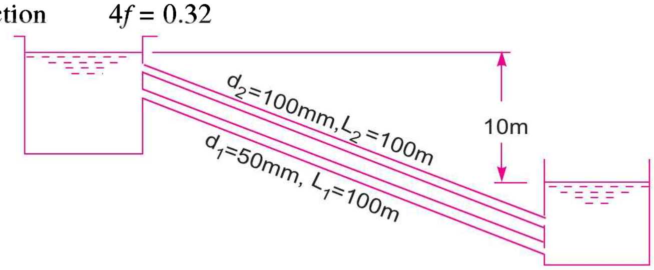


Fig. 11.24

Let V_1 = velocity of flow in pipe 1, and

V_2 = velocity of flow in pipe 2.

When the pipes are connected in parallel, the loss of head will be same in both the pipes.

For the first pipe, loss of head is given as

$$H = \frac{4f \times L_1 \times V_1^2}{d_1 \times 2g} = \frac{0.32 \times 100 \times V_1^2}{0.05 \times 2 \times 9.81} \qquad (\because 4f = .32)$$

or $$10 = 32.619\ V_1^2$$

$\therefore$ $$V_1 = \sqrt{\frac{10}{32.619}} = 0.5535 \text{ m/s}$$

$\therefore$ Rate of flow in 1st pipe, $Q_1 = V_1 \times A_1 = 0.5536 \times \frac{\pi}{4} (d_1)^2$

$$= .5536 \times \frac{\pi}{4} (0.05)^2 = .001087 \text{ m}^3\text{/s} = \textbf{1.087 litres/s. Ans.}$$

For the 2nd pipe, loss of head is given by,

$$10 = H = \frac{4f \times L_2 \times V_2^2}{d_2 \times 2g} = \frac{0.32 \times 100 \times V_2^2}{0.10 \times 2 \times 9.81}$$

$\therefore$ $$V_2 = \sqrt{\frac{10 \times .10 \times 2 \times 9.81}{.32 \times 100}} = 0.783 \text{ m/s}$$

$\therefore$ Rate of flow in 2nd pipe, $Q_2 = A_2 \times V_2 = \frac{\pi}{4} d_2^2 \times V_2$

$$= \frac{\pi}{4} (.1)^2 \times .783 = 0.00615 \text{ m}^3\text{/s} = \textbf{6.15 litres/s. Ans.}$$

Let D = diameter of a single pipe which is substituted for the two original pipes

L = length of single pipe = 100 m

V = velocity through pipe

The discharge through single pipe,

$$Q = Q_1 + Q_2 = 1.087 + 6.15 = 7.237 \text{ litres/s} = .007237 \text{ m}^3\text{/s}$$

$\therefore$ $$V = \frac{Q}{\text{Area}} = \frac{.007237}{\frac{\pi}{4} D^2} = \frac{4 \times .007237}{\pi D^2} = \frac{.009214}{D^2} \text{ m/s}$$

Loss of head through single pipe is

$$H = \frac{4f \times L \times V^2}{D \times 2g} = \frac{0.32 \times 100 \times \left(\frac{.009214}{D^2}\right)^2}{D \times 2 \times 9.81}$$

or $$10.0 = \frac{.32 \times 100 \times .009214^2}{2 \times 9.81 \times D^5} = \frac{.0001384}{D^5}$$

or $$D^5 = \frac{.0001384}{10} = .00001384$$

$\therefore$ $$D = (.00001384)^{1/5} = 0.1067 \text{ m} = \textbf{106.7 mm. Ans.}$$

Problem 11.41 *Two reservoir are connected by a pipe line of diameter 600 mm and length 4000 m. The difference of water level in the reservoirs is 20 m. At a distance of 1000 m from the upper reservoir, a small pipe is connected to the pipe line. The water can be taken from the small pipe. Find the discharge to the lower reservoir, if*

(i) No water is taken from the small pipe, and

(ii) 100 litres/s of water is taken from small pipe.

Take f = .005 and neglect minor losses.

Solution. Given :

Dia. of pipe, $d = 600$ mm $= 0.60$ m

Length of pipe, $L = 400$ m

Difference of water level, $H = 20$ m, $f = .005$

(*i*) **No water is taken from small pipe**

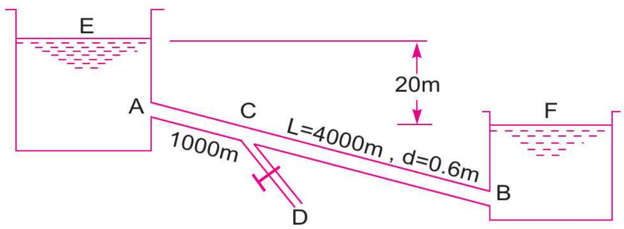


Fig. 11.25

The head loss due to friction in pipe $AB = \dfrac{4f \times L \times V^2}{d \times 2g}$ or $20 = \dfrac{4 \times .005 \times 4000 \times V^2}{0.6 \times 2 \times 9.81}$

$$\therefore \quad V = \sqrt{\frac{20 \times 0.6 \times 2 \times 9.81}{4 \times .005 \times 4000}} = \sqrt{2.943} = 1.715 \text{ m/s}$$

$\therefore$ Discharge, $Q = \text{Area} \times V = \dfrac{\pi}{4}(0.6)^2 \times 1.715 =$ **0.485 m³/s. Ans.**

(*ii*) **100 litres of water is taken from small pipe**

Let Q_1 = discharge through pipe AC

Q_2 = discharge through pipe CB

Then for parallel pipes $Q_1 = Q_2 + 100$ litres/s $= Q_2 + 0.1$ m³/s

$\therefore$ $Q_2 = (Q_1 - 0.1)$ m³/s ...(*i*)

Length of pipe AC, $L_1 = 1000$ m

Length of pipe CB, $L_2 = 4000 - 1000 = 3000$ m

Applying Bernoulli's equation to points E and F and taking flow through ABC, we have

$$20 = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} \quad \text{...(ii)}$$

where V_1 = velocity through pipe $AC = \dfrac{Q_1}{\frac{\pi}{4}(0.6)^2} = \dfrac{4Q_1}{\pi \times .36}$

d_1 = dia. of pipe $AC = 0.6$

V_2 = velocity through pipe $CB = \dfrac{Q_2}{\frac{\pi}{4}(0.6)^2} = \dfrac{4Q_2}{\pi \times .36}$

d_2 = dia. of pipe $CB = 0.6$

Substituting these values in equation (*ii*), we get

$$20 = \frac{4 \times .005 \times 1000}{0.6 \times 2 \times 9.81} \times \left(\frac{4Q_1}{\pi \times .36}\right)^2 + \frac{4 \times .005 \times 3000}{0.6 \times 2 \times 9.81} \times \left(\frac{4Q_2}{\pi \times .36}\right)^2$$

$$20 = 21.25\, Q_1^2 + 63.75\, Q_2^2 \qquad \text{...(iii)}$$

But from (*i*), $Q_2 = Q_1 - 0.1$ or $Q_1 = Q_2 + 0.1$

Substituting the value of Q_1 in equation (*iii*), we get

$$20 = 21.25\,(Q_2 + 0.1)^2 + 63.75\, Q_2^2$$
$$= 21.55\,[Q_2^2 + .01 + 0.2\, Q_2] + 63.75\, Q_2^2$$
$$= 21.25\, Q_2^2 + 0.2125 + 4.250\, Q_2 + 63.75\, Q_2^2$$
$$= 85\, Q_2^2 + 4.25\, Q_2 + .2125$$

or $85\, Q_2^2 + 4.25\, Q_2 - 19.7875 = 0$

This is a quadratic equation in Q_2

$$\therefore \qquad Q_2 = \frac{-4.25 \pm \sqrt{4.25^2 + 4 \times 85 \times 19.7875}}{2 \times 85}$$

$$= \frac{-4.25 \pm \sqrt{18.0625 + 6727.75}}{170} = \frac{-44.25 \pm 82.13}{170} = \frac{82.13 - 4.25}{170}$$

$= 0.458\ \text{m}^3/\text{s}$ (Neglecting negative root)

$\therefore$ Discharge to lower reservoir = Q_2 = **0.458 m^3/s. Ans.**

▶ 11.10 FLOW THROUGH BRANCHED PIPES

When three or more reservoirs are connected by means of pipes, having one or more junctions, the system is called a branching pipe system. Fig. 11.26 shows three reservoirs at different levels connected to a single junction, by means of pipes which are called branched pipes. The lengths, diameters and co-efficient of friction of each pipes is given. It is required to find the discharge and direction of flow in each pipe. The basic equations used for solving such problems are :

1. **Continuity equation** which means the inflow of fluid at the junction should be equal to the outflow of fluid.
2. **Bernoulli's equation, and**
3. **Darcy-Weisbach equation**

Also it is assumed that reservoirs are very large and the water surface levels in the reservoirs are constant so that steady conditions exist in the pipes. Also minor losses are assumed very small. The flow from reservoir *A* takes place to junction *D*. The flow from junction *D* is towards reservoirs *C*. Now the flow from junction *D* towards reservoir *B* will take place only when piezometric head at *D* $\left(\text{which is equal to } \dfrac{p_D}{\rho g} + Z_D\right)$ is more than the piezometric head at *B* (*i.e.*, Z_B). Let us consider that flow is from *D* to reservoir *B*.

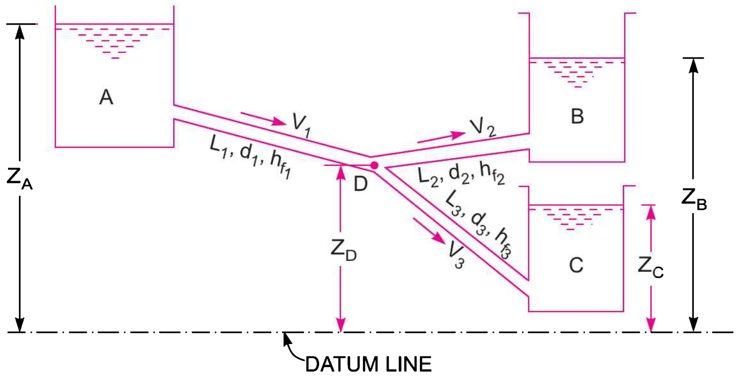


Fig. 11.26

For flow from A to D from Bernoulli's equation

$$Z_A = Z_D + \frac{p_D}{\rho g} + h_{f_1} \qquad ...(i)$$

For flow from D to B from Bernoulli's equation

$$Z_D + \frac{p_D}{\rho g} = Z_B + h_{f_2} \qquad ...(ii)$$

For flow from D to C from Bernoulli's equation

$$Z_D + \frac{p_D}{\rho g} = Z_C + h_{f_3} \qquad ...(iii)$$

From continuity equation,

Discharge through AD = Discharge through DB + Discharge through DC

$$\therefore \qquad \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 \times V_2 + \frac{\pi}{4} d_3^2 V_3$$

or

$$d_1^2 V_1 = d_2^2 V_2 + d_3^2 V_3 \qquad ...(iv)$$

There are four unknowns *i.e.*, V_1, V_2, V_3 and $\frac{p_D}{\rho g}$ and there are four equations (*i*), (*ii*), (*iii*) and (*iv*). Hence unknown can be calculated.

Problem 11.42 *Three reservoirs A, B and C are connected by a pipe system shown in Fig. 11.27. Find the discharge into or from the reservoirs B and C if the rate of flow from reservoirs A is 60 litres/s. Find the height of water level in the reservoir C. Take f = .006 for all pipes.*

Solution. Given :

Length of pipe AD, L_1 = 1200 m

Dia. of pipe AD, d_1 = 30 cm = 0.30 m

Discharge through AD, Q_1 = 60 litres/s = 0.06 m^3/s

Height of water level in A from reference line, Z_A = 40 m

For pipe DB, length L_2 = 600 m, dia., d_2 = 20 cm = 0.20 m, Z_B = 38.0

For pipe DC, length L_3 = 800 m, dia., d_3 = 30 cm = 0.30 m

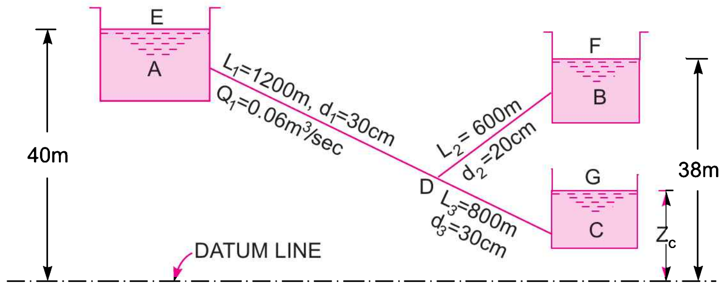


Fig. 11.27

Applying Bernoulli's equations to points E and D, $Z_A = Z_D + \frac{p_D}{\rho g} + h_{f_1}$

where $h_{f_1} = \frac{4 \cdot f \cdot L_1 \cdot V_1^2}{d_1 \times 2g}$, where $V_1 = \frac{Q_1}{\text{Area}} = \frac{0.06}{\frac{\pi}{4}(.3)^2} = 0.848$ m/sec

$$h_{f_1} = \frac{4 \times .006 \times 1200 \times .848^2}{0.3 \times 2 \times 9.81} = 3.518 \text{ m}$$

$$\therefore \quad Z_A = Z_D + \frac{p_D}{\rho g} + 3.518 \text{ or } 40.0 = Z_D + \frac{p_D}{\rho g} + 3.518$$

$$\therefore \quad \left(Z_D + \frac{p_1}{\rho g}\right) = 40.0 - 3.518 = 36.482 \text{ m}$$

Hence piezometric head at D = 36.482. But Z_B = 38 m. Hence water flows from B to D. Applying Bernoulli's equation to points B and D

$$Z_B = \left(Z_D + \frac{p_D}{\rho g}\right) + h_{f_2} \text{ or } 38 = 36.482 + h_{f_2}$$

$$\therefore \quad h_{f_2} = 38 - 36.482 = 1.518 \text{ m}$$

$$\text{But} \quad h_{f_2} = \frac{4 \cdot f \cdot L_2 \cdot V_2^2}{d_2 \times 2g} = \frac{4 \times .006 \times 600 \times V_2^2}{0.2 \times 2 \times 9.81}$$

$$\therefore \quad 1.518 = \frac{4 \times .006 \times 600 \times V_2^2}{0.2 \times 2 \times 9.81}$$

$$\therefore \quad V_2 = \sqrt{\frac{1.518 \times 0.2 \times 2 \times 9.81}{4 \times .006 \times 600}} = 0.643 \text{ m/s.}$$

$$\therefore \text{ Discharge,} \quad Q_2 = V_2 \times \frac{\pi}{4}(d_2)^2 = 0.643 \times \frac{\pi}{2} \times (.2)^2$$

$= 0.0202$ m³/s = **20.2 litres/s. Ans.**

Applying Bernoulli's equation to points D and C

$$Z_D + \frac{p_D}{\rho g} = Z_C + h_{f_3}$$

or
$$36.482 = Z_C + \frac{4f \cdot L_3 \cdot V_3^2}{d_3 \times 2g}, \text{ where } V_3 = \frac{Q_3}{\frac{\pi}{4} d_3^2}$$

But from continuity $Q_1 + Q_2 = Q_3$

$\therefore$
$$Q_3 = Q_1 + Q_2 = 0.06 + 0.0202 = 0.0802 \text{ m}^3/\text{s}$$

$\therefore$
$$V_3 = \frac{Q_3}{\frac{\pi}{4}(.3)^2} = \frac{0.0802}{\frac{\pi}{4}(.09)} = 1.134 \text{ m/s}$$

$\therefore$
$$36.482 = Z_C + \frac{4 \times .006 \times 800 \times 1.134^2}{0.3 \times 2 \times 9.81} = Z_C + 4.194$$

$\therefore$
$$Z_C = 36.482 - 4.194 = \mathbf{32.288 \text{ m. Ans.}}$$

Problem 11.43 *Three reservoirs, A, B and C are connected by a pipe system shown in Fig. 11.28. The lengths and diameters of pipes 1, 2 and 3 are 800 m, 1000 m, 800 m, and 300 mm, 200 mm and 150 mm respectively. Determine the piezometric head at junction D. Take f = .005.*

Solution. Given :

The length of pipe 1, $L_1 = 800$ m and its dia., $d_1 = 300$ mm $= 0.3$ m

The length of pipe 2, $L_2 = 1000$ m and its dia., $d_2 = 200$ mm $= 0.2$ m

The length of pipe 3, $L_3 = 800$ m and its dia., $d_3 = 150$ mm $= 0.15$ m

Height of reservoir, A from datum line, $Z_A = 60$ m

Similarly, $Z_B = 40$ m and $Z_C = 30$ m.

The direction of flow in pipes are shown (given) in Fig. 11.28. Applying Bernoulli's equation to points A and D

$$Z_A = \left(Z_D + \frac{p_D}{\rho g}\right) + h_{f_1}$$

or
$$\left[Z_A - \left(Z_D + \frac{p_D}{\rho g}\right)\right] = h_{f_1} = \frac{4 \times f \times L_1 \times V_1^2}{d_1 \times 2g} = \frac{4 \times .005 \times 800 \times V_1^2}{0.3 \times 2 \times 9.81}$$

or
$$60 - \left(Z_D + \frac{p_D}{\rho g}\right) = 2.718\ V_1^2 \qquad \ldots(i)$$

Applying Bernoulli's equation to points D and B

$$\left(Z_D + \frac{p_D}{\rho g}\right) = Z_B + h_{f_2} = 40 + \frac{4f \times L_2 \times V_2^2}{d_2 \times 2g}$$

$$= 40 + \frac{4 \times .005 \times 1000 \times V_2^2}{0.2 \times 2 \times 9.81} = 40.0 + 5.09\ V_2^2$$

or
$$\left(Z_D + \frac{p_D}{\rho g}\right) - 40.0 = 5.09\ V_2^2 \qquad \ldots(ii)$$

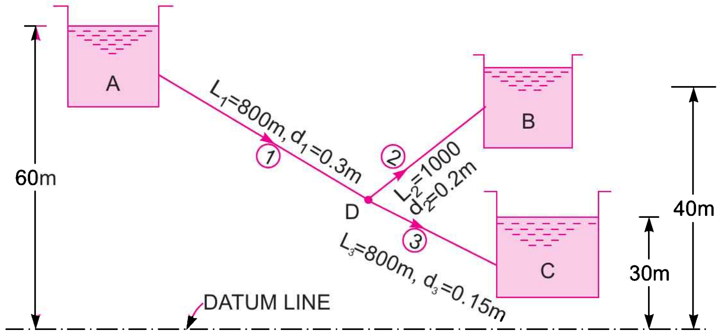


Fig. 11.28

Applying Bernoulli's equation to points D and C

$$\left(Z_D + \frac{p_D}{\rho g}\right) = Z_C + h_{f_3} = 30 + \frac{4f \times L_3 \times V_3^2}{d_3 \times 2g} = 30 + \frac{4 \times .005 \times 800 \times V_3^2}{0.15 \times 2 \times 9.81}$$

or $$\left(Z_D + \frac{p_D}{\rho g}\right) = 30.0 + 5.436\ V_3^2 \quad ...(iii)$$

Adding (i) and (ii), we have $60 - 40 = 2.718\ V_1^2 + 5.09\ V_2^2$

or $$20 = 2.718\ V_1^2 + 5.09\ V_2^2 \quad ...(iv)$$

Adding (i) and (iii), we have $60 = 2.718\ V_1^2 + 30.0 + 5.436\ V_3^2$

or $$60 - 30 = 30 = 2.718\ V_1^2 + 5.436\ V_3^2 \quad ...(v)$$

Also from continuity equation, we have

$$Q_1 = Q_2 + Q_3$$

or $$\frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} d_2^2 V_2 + \frac{\pi}{4} d_3^2 V_3 \text{ or } d_1^2 V_1 = d_2^2 V_2 + d_3^2 V_3$$

or $$0.3^2 V_1 = 0.2^2 V_2 + 0.15^2 \times V_3 \text{ or } .09 V_1 = .04\ V_2 + .0225\ V_3 \quad ...(vi)$$

Now from (iv), $$V_2 = \sqrt{\frac{20 - 2.718\ V_1^2}{5.09}} \quad ...(vii)$$

And from (v), $$V_3 = \sqrt{\frac{30 - 2.718\ V_1^2}{5.436}} \quad ...(viii)$$

Substituting the value of V_2 and V_3 in (vi), we get

$$0.09\ V_1 = .04 \sqrt{\frac{20 - 2.718\ V_1^2}{5.09}} + .0225 \sqrt{\frac{30 - 2.718\ V_1^2}{5.436}}$$

Squaring both sides, we get

$$(0.09\ V_1)^2 = (.04)^2 \times \left(\frac{20 - 2.718\ V_1^2}{5.09}\right) + (0.0225)^2 \times \frac{30 - 2.718\ V_1^2}{5.436} + 2 \times .04$$

$$\times .0225 \times \sqrt{\frac{20 - 2.718\ V_1^2}{5.09}} \times \sqrt{\frac{30 - 2.718\ V_1^2}{5.436}}$$

or $\quad .0081\ V_1^2 = .00628 - .000854\ V_1^2 + .00279 - .000253\ V_1^2 + .0018$

or $\quad .0081\ V_1^2 + .000854\ V_1^2 + .000253\ V_1^2 = .00628 + .00279 + .0018 = .01087$

or $\quad .009207\ V_1^2 = .01087$

$$\therefore \quad V_1 = \sqrt{\frac{.01087}{.009207}} = 1.086 \text{ m/s}$$

Substituting this value of V_1 in (*vii*) and (*viii*)

$$V_2 = \sqrt{\frac{20 - 2.718 \times V_1^2}{5.09}} = \sqrt{\frac{20 - 2.718 \times 1.086^2}{5.09}} = 1.816 \text{ m/s}$$

$$\therefore \quad V_3 = \sqrt{\frac{30 - 2.718 \times 1.086^2}{5.436}} = 2.22 \text{ m/s}$$

Piezometric head at $\quad D = Z_D + \dfrac{p_D}{\rho g} = 30.0 + 5.436 \times V_3^2$

$$= 30.0 + 5.436 \times (2.22)^2 = \mathbf{56.79 \text{ m. Ans.}}$$

Problem 11.44 *A pipe line 60 cm diameter bifurcates at a Y-junction into two branches 40 cm and 30 cm in diameter. If the rate of flow in the main pipe is 1.5 m^3/s and mean velocity of flow in 30 cm diameter pipe is 7.5 m/s, determine the rate of flow in the 40 cm diameter pipe.*

Solution. Given :

Dia. of main pipe, $\quad D = 60 \text{ cm} = 0.6 \text{ m}$

Dia. of branch pipe 1, $\quad D_1 = 40 \text{ cm} = 0.4 \text{ m}$

Dia. of branch pipe 2, $\quad D_2 = 30 \text{ cm} = 0.3 \text{ m}$

Velocity in branch pipe 2, $\quad V_2 = 7.5 \text{ m/s}$

Rate of flow in main pipe, $\quad Q = 1.5 \text{ m}^3\text{/s}$

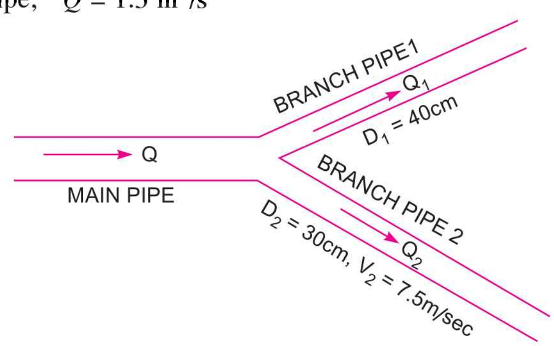


Fig. 11.29

Let $\quad Q_1$ = Rate of flow in branch pipe 1,

$\quad Q_2$ = Rate of flow in branch pipe 2,

$\quad Q$ = Rate of flow in main pipe,

Now rate of flow in main pipe is equal to the sum of rate of flow in branch pipes.

$$\therefore \quad Q = Q_1 + Q_2 \quad \ldots(i)$$

But Q_2 = Area of branch pipe 2 × Velocity in branch pipe 2

$$= A_2 \times V_2 = \frac{\pi}{4} D_2^2 \times V_2 = \frac{\pi}{4} (0.3)^2 \times 7.5 = 0.53 \text{ m}^3/\text{s}$$

Substituting the values of Q and Q_2 in equation (*i*), we get

$$1.5 = Q_1 + 0.53$$

$$\therefore \quad Q_1 = 1.5 - 0.53 = \mathbf{0.97 \text{ m}^3/\text{s. Ans.}}$$

▶ 11.11 POWER TRANSMISSION THROUGH PIPES

Power is transmitted through pipes by flowing water or other liquids flowing through them. The power transmitted depends upon (*i*) the weight of liquid flowing through the pipe and (*ii*) the total head available at the end of the pipe. Consider a pipe AB connected to a tank as shown in Fig. 11.30. The power available at the end B of the pipe and the condition for maximum transmission of power will be obtained as mentioned below :

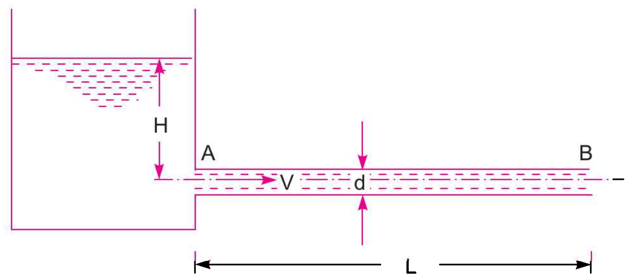


Fig. 11.30 *Power transmission through pipe.*

Let L = length of the pipe,
d = diameter of the pipe,
H = total head available at the inlet of pipe,
V = velocity of flow in pipe,
h_f = loss of head due to friction, and f = co-efficient of friction.

The head available at the outlet of the pipe, if minor losses are neglected

= Total head at inlet – loss of head due to friction

$$= H - h_f = H - \frac{4f \times L \times V^2}{d \times 2g} \qquad \left\{\because h_f = \frac{4f \times L \times V^2}{d \times 2g}\right\}$$

Weight of water flowing through pipe per sec,

$$W = \rho g \times \text{volume of water per sec} = \rho g \times \text{Area} \times \text{Velocity}$$

$$= \rho g \times \frac{\pi}{4} d^2 \times V$$

∴ The power transmitted at the outlet of the pipe

= weight of water per sec × head at outlet

$$= \left(\rho g \times \frac{\pi}{4} d^2 \times V\right) \times \left(H - \frac{4f \times L \times V^2}{d \times 2g}\right) \text{ Watts}$$

∴ Power transmitted at outlet of the pipe,

$$P = \frac{\rho g}{1000} \times \frac{\pi}{4} d^2 \times V \left(H - \frac{4fLV^2}{d \times 2g}\right) \text{ kW} \qquad \text{...(11.21)}$$

Efficiency of power transmission,

$$\eta = \frac{\text{Power available at outlet of the pipe}}{\text{Power supplied at the inlet of the pipe}}$$

$$= \frac{\text{Weight of water per sec} \times \text{Head available at outlet}}{\text{Weight of water per sec} \times \text{Head at inlet}}$$

$$= \frac{W \times (H - h_f)}{W \times H} = \frac{H - h_f}{H}. \qquad ...(11.22)$$

11.11.1 Condition for Maximum Transmission of Power. The condition for maximum transmission of power is obtained by differentiating equation (11.21) with respect to V and equating the same to zero.

Thus $$\frac{d}{dV}(P) = 0$$

or $$\frac{d}{dV}\left[\frac{\rho g}{1000} \times \frac{\pi}{4} d^2 \left(HV - \frac{4fLV^3}{d \times 2g}\right)\right] = 0$$

or $$\frac{\rho g}{1000} \times \frac{\pi}{4} d^2 \left(H - \frac{4 \times 3 \times f \times L \times V^2}{d \times 2g}\right) = 0$$

or $$H - 3 \times \frac{4fLV^2}{d \times 2g} = 0 \text{ or } H - 3 \times h_f = 0 \qquad \left(\because \frac{4fLV^2}{d \times 2g} = h_f\right)$$

$$\therefore \quad H = 3h_f \text{ or } h_f = \frac{H}{3} \qquad ...(11.23)$$

Equating (11.23) is the condition for maximum transmission of power. It states that power transmitted through a pipe is maximum when the loss of head due to friction is one-third of the total head at inlet.

11.11.2 Maximum Efficiency of Transmission of Power. Efficiency of power transmission through pipe is given by equation (11.22) as

$$\eta = \frac{H - h_f}{H}$$

For maximum power transmission through pipe the condition is given by equation (11.23) as

$$h_f = \frac{H}{3}$$

Substituting the value of h_f in efficiency, we get maximum η,

$$\eta_{max} = \frac{H - H/3}{H} = 1 - \frac{1}{3} = \frac{2}{3} \text{ or } 66.7\%. \qquad ...(11.24)$$

Problem 11.45 *A pipe of diameter 300 mm and length 3500 m is used for the transmission of power by water. The total head at the inlet of the pipe is 500 m. Find the maximum power available at the outlet of the pipe, if the value of f = .006.*

Solution. Given :

Diameter of the pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Length of the pipe, $L = 3500$ m
Total head at inlet, $H = 500$ m
Co-efficient of friction, $f = .006$
For maximum power transmission, using equation (11.23)

$$h_f = \frac{H}{3} = \frac{500}{3} = 166.7 \text{ m}$$

Now
$$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times .006 \times 3500 \times V^2}{0.3 \times 2 \times 9.81} = 14.27\, V^2$$

Equating the two values of h_f, we get

$$166.7 = 14.27\, V^2 \text{ or } V = \sqrt{\frac{166.7}{14.27}} = 3.417 \text{ m/s}$$

∴ Discharge,
$$Q = V \times \text{Area}$$
$$= 3.417 \times \frac{\pi}{4} (d)^2 = 3.417 \times \frac{\pi}{4} (.3)^2 = 0.2415 \text{ m}^3/\text{s}$$

Head available at the end of the pipe

$$= H - h_f = H - \frac{H}{3} = \frac{2H}{3} = \frac{2 \times 500}{3} = 333.33 \text{ m}$$

∴ Maximum power available
$$= \frac{\rho g \times Q \times \text{head at the end of pipe}}{1000} \text{ kW}$$
$$= \frac{1000 \times 9.81 \times .2415 \times 333.33}{1000} \text{ kW} = \mathbf{689.7\ kW.\ Ans.}$$

Problem 11.46 *A pipe line of length 2000 m is used for power transmission. If 110.3625 kW power is to be transmitted through the pipe in which water having a pressure of 490.5 N/cm^2 at inlet is flowing. Find the diameter of the pipe and efficiency of transmission if the pressure drop over the length of pipe is 98.1 N/cm^2. Take f = .0065.*

Solution. Given :
Length of pipe, $L = 2000$ m
Power transmitted $= 110.3625$ kW
Pressure at inlet, $p = 490.5 \text{ N/cm}^2 = 490.5 \times 10^4 \text{ N/m}^2$

∴ Pressure head at inlet,
$$H = \frac{p}{\rho g} = \frac{490.5 \times 10^4}{1000 \times 9.81} = 500 \text{ m} \qquad [\because \rho = 1000]$$

Pressure drop $= 98.1 \text{ N/cm}^2 = 98.1 \times 10^4 \text{ N/m}^2$

∴ Loss of head,
$$h_f = \frac{98.1 \times 10^4}{\rho g} = \frac{98.1 \times 10^4}{1000 \times 9.81} = 100 \text{ m}$$

Co-efficient of friction, $f = .0065$
Head available at the end of the pipe $= H - h_f = 500 - 100 = 400$ m
Let the diameter of the pipe $= d$

Now power transmitted is given by, $P = \dfrac{\rho g \times Q \times (H - h_f)}{1000}$ kW

or $$110.3625 = \frac{1000 \times 9.81 \times Q \times 400}{1000}$$

$\therefore$ $$Q = \frac{110.3625 \times 1000}{1000 \times 9.81 \times 400} = 0.02812$$

But discharge, Q = Area × Velocity = $\dfrac{\pi}{4} d^2 \times V$

$\therefore$ $$\frac{\pi}{4} d^2 \times V = .02812$$

$\therefore$ $$V = \frac{.2812 \times 4}{\pi d^2} = \frac{0.0358}{d^2} \quad \text{...}(i)$$

The head lost due to friction, $h_f = \dfrac{4f \times L \times V^2}{d \times 2g}$

But $h_f = 100$ m

$\therefore$ $$100 = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times .0065 \times 2000 \times V^2}{d \times 2 \times 9.81}$$

$$= \frac{2.65 \times V^2}{d} = \frac{2.65}{d} \times \left(\frac{.0358}{d^2}\right)^2 = \frac{.003396}{d^5}$$

$\because$ From equation (i), $V = \dfrac{.0358}{d^2}$

$\therefore$ $$100 = \frac{.003396}{d^5}$$

or $$d = \left(\frac{.003396}{100}\right)^{1/5} = 0.1277 \text{ m} = \mathbf{127.7\ mm.\ Ans.}$$

Efficiency of power transmission is given by equation (11.22),

$$\eta = \frac{H - h_f}{H} = \frac{500 - 100}{500} = \mathbf{0.80 = 80\%.\ Ans.}$$

Problem 11.47 *For Problem 11.46, find : (i) the diameter of the pipe corresponding to maximum efficiency of transmission, (ii) diameter of the pipe corresponding to 90% efficiency of transmission.*

Solution. (i) Diameter of pipe corresponding to maximum efficiency.

Let the dia. of pipe for $\eta_{max} = d$

But from equation (11.24), $\eta_{max} = 66.67\% = \dfrac{2}{3}$

or $$\frac{H - h_f}{H} = \frac{2}{3} \quad \text{or} \quad \frac{500 - h_f}{500} = \frac{2}{3}$$

or $$h_f = 500 - 500 \times \frac{2}{3} = \frac{1500 - 1000}{3} = \frac{500}{3} = 166.7 \text{ m}$$

The other data given from Problem 11.46,

Power transmitted $= 110.3625$

Length of pipe, $L = 2000$ m

Co-efficient of friction, $f = .0065$

Power transmitted is given by the relation,

$$P = \frac{\rho g \times Q \times (H - h_f)}{1000}$$

or $$110.3625 = \frac{1000 \times 9.81 \times Q \times (500 - 166.7)}{1000}$$

or $$Q = \frac{110.3625 \times 1000}{1000 \times 9.81 \times (500 - 166.7)} = 0.03375 \text{ m}^3/\text{s}$$

But Q = area of pipe × velocity of flow

$$= \frac{\pi}{4} d^2 \times V \text{ \{where } V = \text{velocity of flow\}}$$

$\therefore$ $$0.03375 = \frac{\pi}{4} d^2 \times V$$

$\therefore$ $$V = \frac{0.03375 \times 4}{\pi \times d^2} = \frac{0.04297}{d^2} \quad \text{...}(i)$$

Now the head lost due to friction, $h_f = \dfrac{4fLV^2}{d \times 2g}$

But $h_f = 166.7$ m

$\therefore$ $$166.7 = \frac{4 \times .0065 \times 2000 \times V^2}{d \times 2 \times 9.81}$$

$$= \frac{2.65\, V^2}{d} = \frac{2.65}{d} \times \left(\frac{.04297}{d^2}\right)^2 = \frac{.00489}{d^5} \qquad \left(\because\ V = \frac{.04297}{d^2}\right)$$

$\therefore$ $$d^5 = \frac{.00489}{166.7} = .00002933$$

$\therefore$ $d = (.00002933)^{1/5} = 0.1240$ m = **124 mm. Ans.**

(*ii*) Let the diameter of pipe, when efficiency of transmission is 90% = d

$$\eta = 90\% = 0.9$$

But η is given by equation (11.22) as, $\eta = \dfrac{H - h_f}{H} = 0.9$

But $H = 500$ m

$\therefore \qquad \frac{500 - h_f}{500} = 0.9$ or $500 - 500 \times 0.9 = h_f$ or $500 - 450 = h_f$

$\therefore \qquad h_f = 500 - 450 = 50$ m

The other given data is, $\quad P = 110.3625,\ L = 2000,\ f = .0065$

Using relation for power transmission, $P = \frac{\rho g \times Q \times (H - h_f)}{1000}$

or $$110.3625 = \frac{1000 \times 9.81 \times Q \times (500 - 50)}{1000}$$

$$Q = \frac{110.3625 \times 1000}{1000 \times 9.81 \times (500 - 50)} = .025 \text{ m}^3/\text{s}$$

But $$Q = \frac{\pi}{4} d^2 \times V$$

$\therefore$ $$\frac{\pi}{4} d^2 \times V = .025 \text{ or } V = \frac{.025 \times 4}{\pi d^2} = \frac{0.03183}{d^2} \qquad \ldots(i)$$

Now the head lost due to friction, $h_f = \frac{4fLV^2}{d \times 2g}$

or $$50 = \frac{4 \times .0065 \times 2000 \times}{d \times 2g} \times \left(\frac{.03183}{d^2}\right)^2 = \frac{.002685}{d^5}$$

$\therefore$ $$d^5 = \frac{.002685}{50} = .0000537$$

$$d = (.0000537)^{1/5} = .1399 \text{ m} \simeq \textbf{140 mm. Ans.}$$

▶ 11.12 FLOW THROUGH NOZZLES

Fig. 11.31 shows a nozzle fitted at the end of a long pipe. The total energy at the end of the pipe consists of pressure energy and kinetic energy. By fitting the nozzle at the end of the pipe, the total energy is converted into kinetic energy. Thus nozzles are used, where higher velocities of flow are required. The examples are :

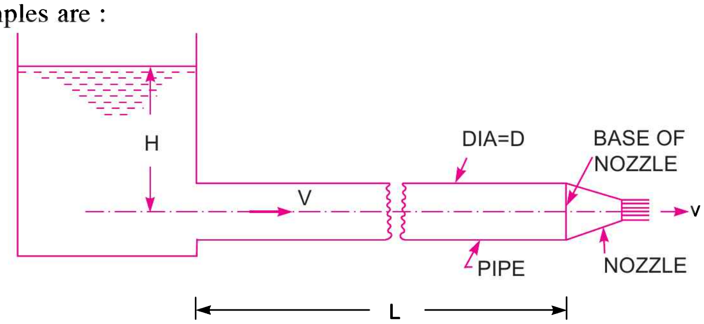


Fig. 11.31 *Nozzle fitted to a pipe.*

1. In case of Pelton turbine, the nozzle is fitted at the end of the pipe (called penstock) to increase velocity.
2. In case of the extinguishing fire, a nozzle is fitted at the end of the hose pipe to increase velocity.

Let D = diameter of the pipe, L = length of the pipe,

A = area of the pipe = $\frac{\pi}{4} D^2$,

V = velocity of flow in pipe,

H = total head at the inlet of the pipe,

d = diameter of nozzle at outlet,

v = velocity of flow at outlet of nozzle,

a = area of the nozzle at outlet = $\frac{\pi}{4} d^2$,

f = co-efficient of friction for pipe.

Loss of head due to friction in pipe, $h_f = \frac{4fLV^2}{2g \times D}$

∴ Head available at the end of the pipe or at the base of nozzle

= Head at inlet of pipe – head lost due to friction

$$= H - h_f = \left(H - \frac{4fLV^2}{2g \times D}\right)$$

Neglecting minor losses and also assuming losses in the nozzle negligible, we have

Total head at inlet of pipe = total head (energy) at the outlet of nozzle + losses

But total head at outlet of nozzle = kinetic head = $\frac{v^2}{2g}$

$$\therefore \quad H = \frac{v^2}{2g} + h_f = \frac{v^2}{2g} + \frac{4fLV^2}{2gD} \qquad \left(\because h_f = \frac{4fLV^2}{2gD}\right) \quad ...(i)$$

From continuity equation in the pipe and outlet of nozzle,

$$AV = av$$

$$\therefore \quad V = \frac{av}{A}$$

Substituting this value in equation (i), we get

$$H = \frac{v^2}{2g} + \frac{4fL}{2gD} \times \left(\frac{av}{A}\right)^2 = \frac{v^2}{2g} + \frac{4fLa^2v^2}{2g \times D \times A^2} = \frac{v^2}{2g}\left(1 + \frac{4fLa^2}{DA^2}\right)$$

$$\therefore \quad v = \sqrt{\frac{2gH}{\left(1 + \frac{4fL}{D} \times \frac{a^2}{A^2}\right)}} \quad ...(11.25)$$

∴ Discharge through nozzle = $a \times v$.

11.12.1 Power Transmitted Through Nozzle. The kinetic energy of the jet at the outlet of nozzle $= \frac{1}{2} mv^2$

Now mass of liquid at the outlet of nozzle per second $= \rho av$

$\therefore$ Kinetic energy of the jet at the outlet per sec. $= \frac{1}{2} \rho av \times v^2 = \frac{1}{2} \rho av^3$

$\therefore$ Power in kW at the outlet of nozzle $= (\text{K.E./sec}) \times \frac{1}{1000} = \dfrac{\frac{1}{2}\rho av^3}{1000}$

$\therefore$ Efficiency of power transmission through nozzle,

$$\eta = \frac{\text{Power at outlet of nozzele}}{\text{Power at the inlet of pipe}} = \frac{\dfrac{\frac{1}{2}\rho av^3}{1000}}{\dfrac{\rho g . Q . H}{1000}}$$

$$= \frac{\frac{1}{2}\rho av . v^2}{\rho g . Q . H} = \frac{\frac{1}{2}\rho av . v^2}{\rho g . av . H} \qquad \{\because Q = av\}$$

$$= \frac{v^2}{2gH} = \left[\frac{1}{1 + \frac{4fL}{D} \times \frac{a^2}{A^2}}\right] \qquad \text{...(11.26)}$$

$$\left(\because \text{ From equation (11.25), } \frac{v^2}{2gH} = \left[\frac{1}{1 + \frac{4fL}{D}\frac{a^2}{A^2}}\right]\right)$$

11.12.2 Condition for Maximum Power Transmitted Through Nozzle. We know that, the total head at inlet of pipe = total head at the outlet of the nozzle + losses

i.e., $$H = \frac{v^2}{2g} + h_f \qquad \left[\because \text{ total head at outlet of nozzle} = \frac{v^2}{2g} \text{ and } h_f = \frac{4 . f . L . V^2}{D \times 2g} = \text{loss of liquid in pipe}\right]$$

$$= \frac{v^2}{2g} + \frac{4 . f . L . V^2}{D \times 2g}$$

$$\therefore \qquad \frac{v^2}{2g} = \left(H - \frac{4 . f . L . V^2}{D \times 2g}\right)$$

But power transmitted through nozzle $= \dfrac{\frac{1}{2}\rho av^3}{1000} = \dfrac{\frac{1}{2}\rho av}{1000} \times v^2 = \dfrac{\frac{1}{2}\rho av}{1000}\left[2g\left(H - \dfrac{4 . f . L . V^2}{D \times 2g}\right)\right]$

$$= \frac{\rho gav}{1000}\left[H - \frac{4fLV^2}{D \times 2g}\right] \quad \text{...(11.27)}$$

Now from continuity equation, $AV = av$

$$\therefore \qquad V = \frac{av}{A}$$

Substituting the value of V in equation (11.27), we get

$$\text{Power transmitted through nozzle} = \frac{\rho gav}{1000}\left[H - \frac{4fLa^2}{D \times 2g}\frac{v^2}{A^2}\right]$$

The power (P) will be maximum, when $\frac{d(P)}{dv} = 0$

or $$\frac{d}{dv}\left[\frac{\rho gav}{1000}\left(H - \frac{4fL}{D \times 2g}\frac{a^2v^2}{A^2}\right)\right] = 0$$

or $$\frac{d}{dv}\left[\frac{\rho ga}{1000}\left(Hv - \frac{4fL}{D \times 2g}\frac{a^2v^3}{A^2}\right)\right] = 0$$

or $$\left[\frac{\rho ga}{1000}\left(H - 3\frac{4fL}{D \times 2g}\frac{a^2v^2}{A^2}\right)\right] = 0 \text{ or } H - 3 \times \frac{4fL}{D \times 2g} \times V^2 = 0 \left(\because\ V = \frac{av}{A}\right)$$

or $$H - 3h_f = 0 \qquad \left(\because\ \frac{4fLV^2}{D \times 2g} = h_f = \text{head loss in pipe}\right)$$

or $$h_f = \frac{H}{3} \quad \text{...(11.28)}$$

Equation (11.28) gives the condition for maximum power transmitted through nozzle. It states that power transmitted through nozzle is maximum when the head lost due to friction in pipe is one-third the total head supplied at the inlet of pipe.

11.12.3 Diameter of Nozzle for Maximum Transmission of Power Through Nozzle.

For maximum transmission of power, the condition is given by equation (11.28) as, $h_f = \frac{H}{3}$

But $$h_f = \frac{4f \,.\, L \,.\, V^2}{D \times 2g}$$

$$\therefore \qquad \frac{4fLV^2}{D \times 2g} = \frac{H}{3} \text{ or } H = 3 \times \frac{4fLV^2}{D \times 2g}$$

But H is also = total head at outlet of nozzle + losses

$$= \frac{v^2}{2g} + h_f = \frac{v^2}{2g} + \frac{4fLV^2}{D \times 2g}$$

Equating the two values of H, we get

$$3 \times \frac{4fLV^2}{D \times 2g} = \frac{v^2}{2g} + \frac{4fLV^2}{D \times 2g} \quad \text{or} \quad \frac{12fLV^2}{D \times 2g} - \frac{4fLV^2}{D \times 2g} = \frac{v^2}{2g}$$

or
$$\frac{8fLV^2}{D \times 2g} = \frac{v^2}{2g} \qquad ...(i)$$

But from continuity, $AV = av$ or $V = \frac{av}{A}$.

Substituting this value of V in equation (i), we get

$$\frac{8fL}{D \times 2g} \times \frac{a^2v^2}{A^2} = \frac{v^2}{2g} \quad \text{or} \quad \frac{8fL}{D} \times \frac{a^2}{A^2} = 1 \qquad \left(\text{Divide by } \frac{v^2}{2g}\right) ...(ii)$$

or
$$\frac{8fL}{D} \times \frac{\left(\frac{\pi}{4}d^2\right)^2}{\left(\frac{\pi}{4}D^2\right)^2} = 1 \text{ or } \frac{8fL}{D} \times \frac{d^4}{D^4} = 1 \text{ or } d^4 = \frac{D^5}{8fL}$$

$\therefore$
$$d = \left(\frac{D^5}{8fL}\right)^{1/4} \qquad ...(11.29)$$

From equation (ii),
$$\frac{8fL}{D} = \frac{A^2}{a^2}$$

$\therefore$
$$\frac{A}{a} = \sqrt{\frac{8fL}{D}} \qquad ...(11.30)$$

Equation (11.30) gives the ratio of the area of the supply pipe to the area of the nozzle and hence from this equation, the diameter of the nozzle can be obtained.

Problem 11.48 *A nozzle is fitted at the end of a pipe of length 300 m and of diameter 100 mm. For the maximum transmission of power through the nozzle, find the diameter of nozzle. Take f = .009.*

Solution. Given :

Length of pipe, $L = 300$ m

Diameter of pipe, $D = 100$ mm $= 0.1$ m

Co-efficient of friction, $f = .009$

Let the diameter of nozzle $= d$

For maximum transmission of power, the diameter of nozzle is given by relation (11.29) as

$$d = \left(\frac{D^5}{8fL}\right)^{1/4} = \left(\frac{0.1^5}{8 \times .009 \times 300}\right)^{1/4} = 0.02608 \text{ m} = \mathbf{26.08\ mm.\ Ans.}$$

Problem 11.49 *The head of water at the inlet of a pipe 2000 m long and 500 mm diameter is 60 m. A nozzle of diameter 100 mm at its outlet is fitted to the pipe. Find the velocity of water at the outlet of the nozzle if f = .01 for the pipe.*

Solution. Given :

Head of water at inlet of pipe, $H = 60$ m

Length of pipe, $L = 2000$ m
Dia. of pipe, $D = 500$ mm $= 0.50$ m
Dia. of nozzle at outlet, $d = 100$ mm $= 0.1$ m
Co-efficient of friction, $f = .01$

The velocity at outlet of nozzle is given by equation (11.25) as

$$v = \sqrt{\frac{2gH}{\left(1 + \frac{4fL}{D} \times \frac{a^2}{A^2}\right)}} = \sqrt{\frac{2 \times 9.81 \times 60}{1 + \frac{4 \times .01 \times 2000}{0.5}\left(\frac{\frac{\pi}{4}d^2}{\frac{\pi}{4}D^2}\right)^2}}$$

$$= \sqrt{\frac{2 \times 9.81 \times 60}{1 + \frac{4 \times .01 \times 2000}{0.5} \times \left(\frac{0.1 \times .1}{0.5 \times .5}\right)^2}} = \mathbf{30.61 \text{ m/s. Ans.}}$$

Problem 11.50 *Find the maximum power transmitted by a jet of water discharging freely out of nozzle fitted to a pipe = 300 m long and 100 mm diameter with co-efficient of friction as 0.01. The available head at the nozzle is 90 m.*

Solution. Given :

Length of pipe, $L = 300$ m
Dia. of pipe, $D = 100$ mm $= 0.1$ m
Co-efficient of friction, $f = .01$
Head available at nozzle, $= 90$ m

For maximum power transmission through the nozzle, the diameter at the outlet of nozzle is given by equation (11.29) as

$$d = \left(\frac{D^5}{8fL}\right)^{1/4} = \left[\frac{(0.1)^5}{8 \times .01 \times 300}\right]^{1/4} = .0254 \text{ m}$$

$\therefore$ Area at the nozzle, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.0254)^2 = .0005067 \text{ m}^2$.

The nozzle at the outlet, discharges water into atmosphere and hence the total head available at the nozzle is converted into kinetic head.

$\therefore$ Head available at outlet $= v^2/2g$ or $90 = v^2/2g$

$$\therefore \quad v = \sqrt{2 \times 9.81 \times 90} = 42.02 \text{ m/s}$$

Discharge through nozzle, $Q = a \times v = .0005067 \times 42.02 = 0.02129 \text{ m}^3/\text{s}$

$$\therefore \text{ Maximum power transmitted} = \frac{\rho g \times Q \times \text{Head at outlet of nozzle}}{1000}$$

$$= \frac{1000 \times 9.81 \times 0.02129 \times 90}{1000} = \mathbf{18.796 \text{ kW. Ans.}}$$

Problem 11.51 *The rate of flow of water through a pipe of length 2000 m and diameter 1 m is 2 m^3/s. At the end of the pipe a nozzle of outside diameter 300 mm is fitted. Find the power transmitted* *through the nozzle if the head of water at inlet of the pipe is 200 m and co-efficient of friction for pipe is 0.01.*

Solution. Given :

Length of pipe, $L = 2000$ m

Dia. of pipe, $D = 1$ m

Discharge, $Q = 2$ m^3/s

Dia. of nozzle, $d = 300$ mm $= 0.3$ m

Head at inlet of pipe, $H = 200$ m

Co-efficient of friction, $f = .01$

Now area of pipe, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 1^2 = 0.7854$ m^2

Velocity of water through pipe, $V = \frac{Q}{A} = \frac{2.0}{0.7854} = 2.546$ m/s

Power transmitted through nozzle is given by equation (11.27) as

$$P = \frac{\rho g \,.\, a \,.\, v}{1000}\left[H - \frac{4fLV^2}{D \times 2g}\right]$$

$$= \frac{1000 \times 9.81 \times 2.0}{1000}\left[200 - \frac{4 \times .01 \times 2000 \times (2.546)^2}{1 \times 2 \times 9.81}\right] \quad (\because av = Q)$$

$$= \mathbf{3405.43\ kW.\ Ans.}$$

▶ 11.13 WATER HAMMER IN PIPES

Consider a long pipe AB as shown in Fig. 11.32 connected at one end to a tank containing water at a height of H from the centre of the pipe. At the other end of the pipe, a valve to regulate the flow of water is provided. When the valve is completely open, the water is flowing with a velocity, V in the pipe. If now the valve is suddenly closed, the momentum of the flowing water will be destroyed and consequently a wave of high pressure will be set up. This wave of high pressure will be transmitted along the pipe with a velocity equal to the velocity of sound wave and may create noise called knocking. Also this wave of high pressure has the effect of hammering action on the walls of the pipe and hence it is also known as water hammer.

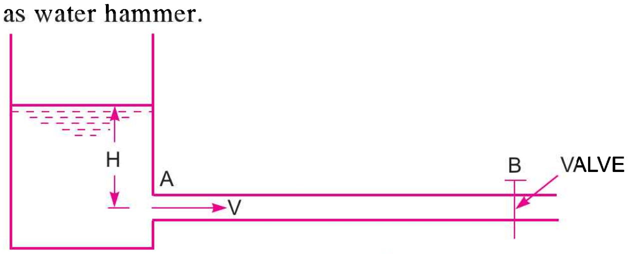


Fig. 11.32 *Water hammer.*

The pressure rise due to water hammer depends upon : (*i*) the velocity of flow of water in pipe, (*ii*) the length of pipe, (*iii*) time taken to close the valve, (*iv*) elastic properties of the material of the pipe. The following cases of water hammer in pipes will be considered :

1. Gradual closure of valve,
2. Sudden closure of valve and considering pipe rigid, and

3. Sudden closure of valve and considering pipe elastic.

11.13.1 Gradual Closure of Valve. Let the water is flowing through the pipe AB shown in Fig. 11.32, and the valve provided at the end of the pipe is closed gradually.

Let A = area of cross-section of the pipe AB,
L = length of pipe,
V = velocity of flow of water through pipe,
T = time in second required to close the valve, and
p = intensity of pressure wave produced.

Mass of water in pipe $AB = \rho \times$ volume of water $= \rho \times A \times L$

The valve is closed gradually in time 'T' seconds and hence the water is brought from initial velocity V to zero velocity in time seconds.

$$\therefore \text{ Retardation of water } = \frac{\text{Change of velocity}}{\text{Time}} = \frac{V-0}{T} = \frac{V}{T}$$

$$\therefore \text{ Retarding force } = \text{Mass} \times \text{Retardation} = \rho AL \times \frac{V}{T} \quad ...(i)$$

If p is the intensity of pressure wave produced due to closure of the valve, the force due to pressure wave,

$$= p \times \text{area of pipe} = p \times A \quad ...(ii)$$

Equating the two forces, given by equations (*i*) and (*ii*),

$$\rho AL \times \frac{V}{T} = p \times A$$

$$\therefore \quad p = \frac{\rho LV}{T} \quad ...(11.31)$$

$$\text{Head of pressure, } \quad H = \frac{p}{\rho g} = \frac{\rho LV}{\rho g \times T} = \frac{\rho LV}{\rho \times g \times T} \text{ or } H = \frac{LV}{gT} \quad ...(11.32)$$

(*i*) The valve closure is said to be gradual if $T > \frac{2L}{C}$ ...(11.33)

where t = time in sec, C = velocity of pressure wave

(*ii*) The valve closure is said to be sudden if $T < \frac{2L}{C}$ ...(11.34)

where C = velocity of pressure wave.

11.13.2 Sudden Closure of Valve and Pipe is Rigid. Equation (11.31) gives the relation between increase of pressure due to water hammer in pipe and the time required to close the valve. If $t = 0$, the increase in pressure will be infinite. But from experiments, it is observed that the increase in pressure due to water hammer is finite, even for a very rapid closure of valve. Thus equation (11.31) is valid only for (*i*) incompressible fluids and (*ii*) when pipe is rigid. But when a wave of high pressure is created, the liquids get compressed to some extent and also pipe material gets stretched. For a sudden closure of valve [the valve of t is small and hence a wave of high pressure is created] the following two cases will be considered :

(*i*) Sudden closure of valve and pipe is rigid, and
(*ii*) Sudden closure of valve and pipe is elastic.

Consider a pipe AB in which water is flowing as shown in Fig. 11.32. Let the pipe is rigid and valve fitted at the end B is closed suddenly.

Let A = Area of cross-section of pipe AB,
L = Length of pipe,
V = Velocity of flow of water through pipe,
p = Intensity of pressure wave produced,
K = Bulk modulus of water.

When the valve is closed suddenly, the kinetic energy of the flowing water is converted into strain energy of water if the effect of friction is neglected and pipe wall is assumed perfectly rigid.

$$\therefore \quad \text{Loss of kinetic energy} = \frac{1}{2} \times \text{mass of water in pipe} \times V^2$$

$$= \frac{1}{2} \times \rho AL \times V^2 \qquad (\because \text{ mass} = \rho \times \text{volume} = \rho \times A \times L)$$

$$\text{Gain of strain energy} = \frac{1}{2}\left(\frac{p^2}{K}\right) \times \text{volume} = \frac{1}{2}\frac{p^2}{K} \times AL$$

Equating loss of kinetic energy to gain of strain energy

$$\therefore \quad \frac{1}{2}\rho AL \times V^2 = \frac{1}{2}\frac{p^2}{K} \times AL$$

or

$$p^2 = \frac{1}{2}\rho AL \times V^2 \times \frac{2K}{AL} = \rho K V^2$$

$$\therefore \quad p = \sqrt{\rho K V^2} = V\sqrt{K\rho} = V\sqrt{\frac{K\rho^2}{\rho}} \qquad \text{...(11.35)}$$

$$= \rho V \times C \qquad \left(\because \sqrt{K/\rho} = C\right) \text{...(11.36)}$$

where C = velocity* of pressure wave.

11.13.3 Sudden Closure of Valve and Pipe is Elastic. Consider the pipe AB in which water is flowing as shown in Fig. 11.32. Let the thickness 't' of the pipe wall is small compared to the diameter D of the pipe and also let the pipe is elastic.

Let E = Modulus of Elasticity of the pipe material,
$\frac{1}{m}$ = Poisson's ratio for pipe material,
p = Increase of pressure due to water hammer,
t = Thickness of the pipe wall,
D = Diameter of the pipe.

When the valve is closed suddenly, a wave of high pressure of intensity p will be produced in the water. Due to this high pressure p, circumferential and longitudinal stresses in the pipe wall will be produced.

Let f_l = Longitudinal stress in pipe
f_c = Circumferential stress in pipe,

The magnitude of these stresses are given as $f_l = \frac{pD}{4t}$ and $f_c = \frac{pD}{2t}$

Now from the knowledge of strength of material we know, strain energy stored in pipe material per unit volume

* For derivation of velocity of pressure wave, please refer to chapter 15.

$$= \frac{1}{2E}\left[f_l^2 + f_c^2 - \frac{2f_l \times f_c}{m}\right]$$

$$= \frac{1}{2E}\left[\left(\frac{pD}{4t}\right)^2 + \left(\frac{pD}{2t}\right)^2 - \frac{2 \times \frac{pD}{4t} \times \frac{pD}{2t}}{m}\right]$$

$$= \frac{1}{2E}\left[\frac{p^2D^2}{16t^2} + \frac{p^2D^2}{4t^2} - \frac{p^2D^2}{4mt^2}\right]$$

Taking $\frac{1}{m} = \frac{1}{4}$ $\left(i.e., \text{Poisson ratio} = \frac{1}{4}\right)$

∴ Strain energy stored in pipe material per unit volume

$$= \frac{1}{2E}\left[\frac{p^2D^2}{16t^2} + \frac{p^2D^2}{4t^2} - \frac{p^2D^2}{4t^2 \times 4}\right] = \frac{1}{2E} \times \frac{p^2D^2}{4t^2} = \frac{p^2D^2}{8Et^2}$$

Total volume of pipe material = $\pi D \times t \times L$.

∴ Total strain energy stored in pipe material

= Strain energy per unit volume × total volume

$$= \frac{p^2D^2}{8Et^2} \times \pi D \times t \times L = \frac{p^2\pi D^3 L}{8Et}$$

$$= \frac{p^2 \times \pi D^2 \times DL}{8Et} = \frac{p^2 A \times DL}{2Et} \qquad \left(\because \frac{\pi D^2}{4} = \text{Area of pipe} = A\right)$$

Now loss of kinetic energy of water $= \frac{1}{2} mV^2 = \frac{1}{2} \rho AL \times V^2$ $(\because m = \rho AL)$

Gain of strain energy in water $= \frac{1}{2}\left(\frac{p^2}{K}\right) \times \text{volume} = \frac{1}{2}\frac{p^2}{K} \times AL$

Then, loss of kinetic energy of water = Gain of strain energy in water + Strain energy stored in pipe material.

$$\therefore \qquad \frac{1}{2}\rho AL \times V^2 = \frac{1}{2}\left(\frac{p^2}{K}\right) \times AL + \frac{p^2 A \times DL}{2Et}$$

Divide by AL,

$$\frac{\rho V^2}{2} = \frac{1}{2}\frac{p^2}{K} + \frac{p^2 D}{2Et} = \frac{p^2}{2}\left[\frac{1}{K} + \frac{D}{Et}\right] \text{ or } \rho V^2 = p^2\left[\frac{1}{K} + \frac{D}{Et}\right]$$

$$\therefore \qquad p^2 = \frac{\rho V^2}{\frac{1}{K} + \frac{D}{Et}} \text{ or } p = \sqrt{\frac{\rho V^2}{\frac{1}{K} + \frac{D}{Et}}} = V \times \sqrt{\frac{\rho}{\frac{1}{K} + \frac{D}{Et}}} \qquad \text{...(11.37)}$$

11.13.4 Time Taken by Pressure Wave to Travel from the Valve to the Tank and from Tank to the Valve

Let T = The required time taken by pressure wave

L = Length of the pipe

C = Velocity of pressure wave

Then total distance = $L + L = 2L$

$$\therefore \text{ Time, } \quad T = \frac{\text{Distance}}{\text{Velocity of pressure wave}} = \frac{2L}{C}. \quad \text{...(11.38)}$$

Problem 11.52 *The water is flowing with a velocity of 1.5 m/s in a pipe of length 2500 m and of diameter 500 mm. At the end of the pipe, a valve is provided. Find the rise in pressure if the valve is closed in 25 seconds. Take the value of C = 1460 m/s.*

Solution. Given :

Velocity of water, $V = 1.5$ m/s

Length of pipe, $L = 2500$ m

Diameter of pipe, $D = 500$ mm $= 0.5$ m

Time to close the valve, $T = 25$ seconds

Value of, $C = 1460$ m/s

Let the rise in pressure $= p$

The ratio, $\frac{2L}{C} = \frac{2 \times 2500}{1460} = 3.42$

From equation (11.33), we have if $T > \frac{2L}{C}$, the closure of valve is said to be gradual.

Here $T = 25$ sec and $\frac{2L}{C} = 3.42$

$\therefore$ $T > \frac{2L}{C}$ and hence valve is closed gradually.

For gradually closure of valve, the rise in pressure is given by equation (11.31) as

$$p = \frac{\rho V L}{T} = 1000 \times 2500 \times \frac{1.5}{25} = 150000 \text{ N/m}^2$$

$$= \frac{150000}{10^4} \frac{\text{N}}{\text{cm}^2} = \mathbf{15.0 \frac{N}{cm^2}}. \textbf{ Ans.}$$

Problem 11.53 *If in Problem 11.52, the valve is closed in 2 sec, find the rise in pressure behind the valve. Assume the pipe to be rigid one and take Bulk modulus of water.*
i.e., $K = 19.62 \times 10^4$ N/cm^2.

Solution. Given :

$V = 1.5$ m/s, $L = 2500$ m

$D = 500$ mm $= 0.5$ m

Time to close the valve, $T = 2$ sec

Bulk modulus of water, $K = 19.62 \times 10^4$ N/cm^2

$= 19.62 \times 10^4 \times 10^4$ N/m^2 $= 19.62 \times 10^8$ N/m^2

Velocity of pressure wave is given by,

$$C = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{19.62 \times 10^8}{1000}} = 1400 \text{ m/s} \qquad (\because \rho = 1000)$$

The ratio, $$\frac{2L}{C} = \frac{2 \times 2500}{1400} = 3.57 \qquad \therefore \quad T < \frac{2L}{C}.$$

$\therefore$ From equation (11.34), if $T < \frac{2L}{C}$, valve is closed suddenly. For sudden closure of valve, when pipe is rigid, the rise in pressure is given by equation (11.35) or (11.36) as

$$p = V\sqrt{K\rho} = 1.5\sqrt{19.62 \times 10^8 \times 1000} \qquad (\because \rho = 1000)$$
$$= 210.1 \times 10^4 \text{ N/m}^2 = \mathbf{210.1 \text{ N/cm}^2. \text{ Ans.}}$$

Problem 11.54 *If in Problem 11.52, the thickness of the pipe is 10 mm and the valve is suddenly closed at the end of the pipe, find the rise in pressure if the pipe is considered to be elastic. Take E = 19.62 × 10^10 N/m^2 for pipe material and K = 19.62 × 10^4 N/cm^2 for water. Calculate the circumferential stress and longitudinal stress developed in the pipe wall.*

Solution. Given :

$V = 1.5$ m/s, $L = 2500$ m, $D = 0.5$ m

Thickness of pipe, $t = 10 \text{ mm} = .01 \text{ m}$

Modulus of elasticity, $E = 19.62 \times 10^{10} \text{ N/m}^2$

Bulk modulus, $K = 19.62 \times 10^4 \text{ N/cm}^2 = 19.62 \times 10^8 \text{ N/m}^2$

For sudden closure of the valve for an elastic pipe, the rise in pressure is given by equation (11.37) as

$$p = V \times \sqrt{\frac{\rho}{\left(\frac{1}{K} + \frac{D}{Et}\right)}} = 1.5 \times \sqrt{\frac{1000}{\left(\frac{1}{19.62 \times 10^8} + \frac{0.5}{19.62 \times 10^{10} \times .01}\right)}}$$

$$= 1.5 \times \sqrt{\frac{1000}{\left(5.09 \times 10^{-10} + 2.54 \times 10^{-10}\right)}}$$

$$= 1715510 \text{ N/m}^2 = \mathbf{171.55 \text{ N/cm}^2. \text{ Ans.}}$$

Circumferential stress (f_c) is given by

$$= \frac{p \times D}{2t} = \frac{171.55 \times 0.5}{2 \times .01} = 4286.9 \text{ N/m}^2$$

Longitudinal stress is given by, $f_l = \frac{p \times D}{4t} = \frac{171.55 \times 0.5}{4 \times .01} = \mathbf{2143.45 \text{ N/m}^2. \text{ Ans.}}$

Problem 11.55 *A valve is provided at the end of a cast iron pipe of diameter 150 mm and of thickness 10 mm. The water is flowing through the pipe, which is suddenly stopped by closing the valve. Find the maximum velocity of water, when the rise of pressure due to sudden closure of valve is 196.2 N/cm^2. Take K for water as 19.62 × 10^4 N/cm^2 and E for cast iron pipe as 11.772 × 10^6 N/cm^2.*

Solution. Given :

Diameter of pipe, $D = 150 \text{ mm} = 0.15 \text{ m}$

Thickness of pipe, $t = 10 \text{ mm} = .01 \text{ m}$

Rise of pressure, $p = 196.2 \text{ N/cm}^2 = 196.2 \times 10^4 \text{ N/m}^2$

Bulk modulus, $K = 19.62 \times 10^4 \text{ N/cm}^2 = 19.62 \times 10^8 \text{ N/m}^2$

Modulus of elasticity, $E = 11.772 \times 10^6 \text{ N/cm}^2 = 11.772 \times 10^{10} \text{ N/m}^2$

For sudden closure of valve and when pipe is elastic, the pressure rise is given by equation (11.37) as

$$p = V \times \sqrt{\frac{\rho}{\left(\frac{1}{K} + \frac{D}{Et}\right)}} = V \times \sqrt{\frac{1000}{\left(\frac{1}{19.62 \times 10^8} + \frac{0.15}{11.772 \times 10^{10} \times .01}\right)}}$$

or

$$196.2 \times 10^4 = V \times \sqrt{\frac{1000}{5.09 \times 10^{-10} + 1.274 \times 10^{-10}}}$$

$$= V \times \sqrt{\frac{1000}{6.364 \times 10^{-10}}} = V \times 125.27 \times 10^4$$

$\therefore$

$$V = \frac{196.2 \times 10^4}{125.27 \times 10^4} = 1.566 \text{ m/s}$$

$\therefore$ Maximum velocity **= 1.566 m/s. Ans.**

▶ 11.14 PIPE NETWORK

A pipe network is an interconnected system of pipes forming several loops or circuits. The pipe network is shown in Fig. 11.33. The examples of such networks of pipes are the municipal water distribution systems in cities and laboratory supply system. In such system, it is required to determine the distribution of flow through the various pipes of the network. The following are the necessary conditions for any network of pipes :

(*i*) The flow into each junction must be equal to the flow out of the junction. This is due to continuity equation.

(*ii*) The algebraic sum of head losses round each loop must be zero. This means that in each loop, the loss of head due to flow in clockwise direction must be equal to the loss of head due to flow in anticlockwise direction.

(*iii*) The head loss in each pipe is expressed as $h_f = rQ^n$. The value of r depends upon the length of pipe, diameter of pipe and co-efficient of friction of pipe. The value of n for turbulent flow is 2. We know that,

$$h_f = \frac{4 \times f \times L \times V^2}{D \times 2g} = \frac{4fL \times \left(\frac{Q}{A}\right)^2}{D \times 2g} \qquad \left(\because V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2}\right)$$

$$= \frac{4fL \times Q^2}{D \times 2g \times \left(\frac{\pi}{4}D^2\right)^2} = \frac{4fL \times Q^2}{D \times 2g \times \left(\frac{\pi}{4}\right)^2 \times D^4}$$

$$= \frac{4f \times L \times Q^2}{2g \times \left(\frac{\pi}{4}\right)^2 \times D^5}$$

$$= rQ^2 \quad ...(11.39) \quad \left(\text{where } \frac{4f \times L}{2g \times \left(\frac{\pi}{4}\right)^2 \times D^5} = r\right)$$

This head loss will be positive, when the pipe is a part of loop and the flow in the pipe is clockwise.

Generally, the pipe network problems are difficult to solve analytically. Hence the methods of successive approximations are used. '**Hardy Cross Method**' is one such method which is commonly used.

11.14.1 Hardy Cross Method. The procedure for Hardy Cross Method is as follows :

1. In this method a trial distribution of discharges is made arbitrary but in such a way that continuity equation is satisfied at each junction (or node).
2. With the assumed values of Q, the head loss in each pipe is calculated according to equation (11.39).
3. Now consider any loop (or circuits). The algebraic sum of head losses round each loop must be zero. This means that in each loop, the loss of head due to flow in clockwise direction must be equal to the loss of head due to flow in anticlockwise direction.
4. Now calculate the net head loss around each loop considering the head loss to be positive in clockwise flow and to be negative in anticlockwise flow.

If the net head loss due to assumed values of Q round the loop is zero, then the assumed values of Q in that loop is correct. But if the net head loss due to assumed values of Q is not zero, then the assumed values of Q are corrected by introducing a correction ΔQ for the flows, till the circuit is balanced.

The correction factor ΔQ* is obtained by

$$\Delta Q = \frac{-\sum r\, Q_0^n}{\sum r\, n\, Q_0^{n-1}} \quad ...(11.40)$$

For turbulent flow, the value of $n = 2$ and hence above correction factor becomes as

$$\Delta Q = \frac{-\sum r\, Q_0^2}{\sum 2r\, Q_0} \quad ...(11.41)$$

5. If the value of ΔQ comes out to be positive, then it should be added to the flows in the clockwise direction ($\because$ the flows in clockwise direction in a loops are considered positive) and subtracted from the flows in the anticlockwise direction.
6. Some pipes may be common to two circuits (or two loops), then the two corrections are applied to these pipes.

---

* Let for any pipe Q_0 = assumed discharge and Q = correct discharge, then

$$Q = Q_0 + \Delta Q$$

$\therefore$ Head loss for the pipe, $\quad h_f = rQ^2 = r(Q_0 + \Delta Q)^2$.

For complete circuit, the net head loss, $\Sigma h_f = \Sigma\,(rQ^2) = \Sigma r\,(Q_0 + \Delta Q)^2 = \Sigma r(Q_0^2 + 2Q_0\,\Delta Q + \Delta Q^2)$

$= \Sigma r\,({Q_0}^2 + 2Q_0\,\Delta Q)$ As ΔQ is small compared with Q_0 and hence ΔQ^2 can be neglected.

$$\therefore \quad \Sigma\, rQ^2 = \Sigma\, r{Q_0}^2 + \Sigma r \times 2Q_0\Delta Q$$

For the correct distribution, the net head loss for a circuit should be zero (*i.e.*, $\Sigma h_f = \Sigma\,(rQ^2) = 0$)

$$\therefore \quad \Sigma rQ_0^2 + \Sigma r \times 2Q_0\Delta Q = 0$$

or $\Sigma rQ_0^2 + \Delta Q\ \Sigma r \times 2Q_0 = 0$ [As ΔQ is same for one circuit, hence it can be taken out of the summation]

$$\therefore \quad \Delta Q = \frac{-\Sigma\, r\, Q_0^2}{\Sigma\, 2r\, Q_0}.$$

7. After the corrections have been applied to each pipe in a loop and to all loops, a second trial calculation is made for all loops. The procedure is repeated till ΔQ becomes negligible.

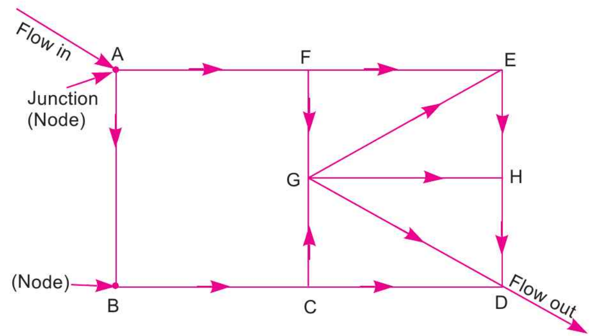


Fig. 11.33 *Pipe network.*
[Loops are : *ABCGFA*, *FEGF*, *GEHG*, *GHDG and GCDG*]

Problem 11.56 *Calculate the discharge in each pipe of the network shown in Fig. 11.34. The pipe network consists of 5 pipes. The head loss h_f in a pipe is given by $h_f = rQ^2$. The values of r for various pipes and also the inflow or outflows at nodes are shown in the figure.*

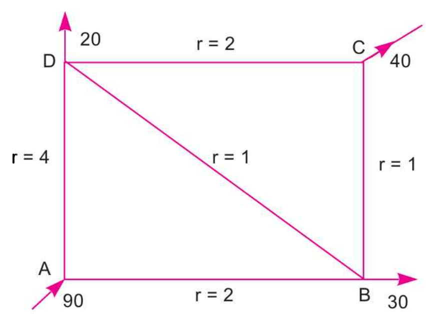


Fig. 11.34

Solution. Given :

Inflow at node $A = 90$, outflow at $B = 30$, at $C = 40$ and at $D = 20$.

Values of r for $AB = 2$, for $BC = 1$, for $CD = 2$, for $AD = 4$ and for $BD = 1$.

For the first trial, the discharges are assumed as shown in Fig. 11.34 (*a*) so that continuity is satisfied at each node (*i.e.*, flow into a node = flow out of the node). For this distribution of discharge, the corrections ΔQ for the loops *ABD* and *BCD* are calculated.

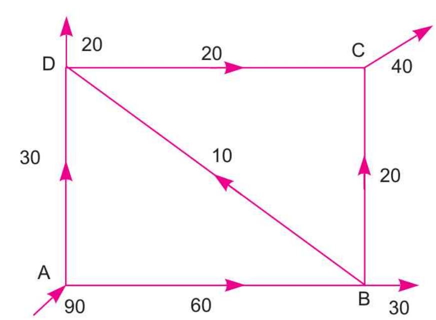


Fig. 11.34(*a*)

First Trial

Loop ADB

Pipe	r	Q_0	$h_f = rQ_0^2$	$2rQ_0$
AD	4	30	$4 \times 30^2 = 3600$	$2 \times 4 \times 30 = 240$
DB	1	10	$-1 \times 10^2 = -100$	$2 \times 1 \times 10 = 20$
AB	2	60	$-2 \times 60^2 = -7200$	$2 \times 2 \times 60 = 240$
			$\Sigma rQ_0^2 = -3700,$	$\Sigma 2rQ_0 = 500,$

$$\therefore \quad \Delta Q = \frac{-\Sigma r Q_0^2}{\Sigma 2r Q_0} = \frac{-(-3700)}{500} = 7.4$$

In the loop *ADB*, the head loss h_f is negative in pipes *DB* and *AB* as the direction of discharges in these pipes is anticlockwise.

As ΔQ is positive for loop *ADB,* hence it should be added to the flow in the clockwise direction and subtractedfrom the flow in the anticlockwise direction. Hence the corrected flow for second trial for loop *ADB* will be as follows :

Pipe *AD* = 30 + 7.4 = 37.4 (flow is clockwise)
Pipe *AB* = 60 – 7.4 = 52.6 (flow is anticlockwise)
Pipe *BD* = 10 – 7.4 = 2. 6 (flow is anticlockwise)

Loop DCB

Pipe	r	Q_0	$h_f = rQ_0^2$	$2rQ_0$
DC	2	20	$2 \times 20^2 = 800$	$2 \times 2 \times 20 = 80$
CB	1	20	$-1 \times 20^2 = -400$	$2 \times 1 \times 20 = 40$
BD	1	10	$1 \times 10^2 = 100$	$2 \times 1 \times 10 = 20.$
			$\Sigma 2\ rQ_0^2 = 500$	$\Sigma 2rQ_0 = 140$

$$\therefore \quad \Delta Q = \frac{-\Sigma r Q_0^2}{\Sigma 2r Q_0} \frac{-500}{140} = -3.57 \simeq -3.6.$$

The head loss in pipe *BC* for loop *DCB* is negative as the direction of discharge in pipe *BC* in anticlockwise.

As ΔQ is negative for loop *DCB*, hence it should be subtracted from the flow in the clockwise direction and added to the flow in the anticlockwise direction. Hence corrected flow for second trial for loop *DCB* will be as follows :

Pipe *DC* = 20 – 3.6 = 16.4
Pipe *BC* = 20 + 3.6 = 23.6
Pipe *BD** = 2.6 – 3.6 = – 1

Note. The pipe *BD* is common to two loops (*i.e.*, loop *ADB* and loop *DCB*). Hence this pipe will get two corrections. After the two corrections, the resultant flow in pipe *BD* is negative in loop *DCB*. Hence the direction of flow will be anticlockwise in pipe *BD* for loop *DCB*.

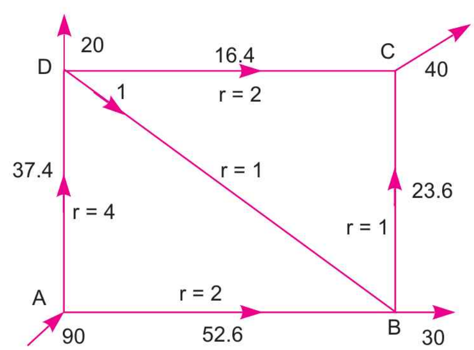


Fig. 11.34 (*b*)

The distribution of discharges in various pipes for second trial is shown in Fig. 11.34 (*b*). For second trial the correction ΔQ for loops *ADB* and *DCB* are calculated as follows :

Loop ADB

Pipe	r	Q_0	$h_f = rQ_0^2$	$2rQ_0$
AD	4	37.4	$4 \times 37.4^2 = 5595$	$2 \times 4 \times 37.4 = 299.2$
DB	1	1	$1 \times 1^2 = 1$	$2 \times 1 \times 1 = 2$
AB	2	52.6	$-2 \times 52.6^2 = -5533.5$	$2 \times 2 \times 52.6 = 210.4$
			$\Sigma rQ_0^2 = 62.54$,	$\Sigma 2rQ_0 = 511.6$

$$\therefore \Delta Q = \frac{-\sum r Q_0^2}{\sum 2r Q_0} = \frac{62.54}{-511.6}$$

$$= -0.122 \simeq -0.1$$

As ΔQ is negative, hence it should be subtracted from the flow in the clockwise direction and added to the flow in the anticlockwise direction

As the correction (ΔQ) is small (*i.e.*, $\Delta Q = -0.1$), this correction is applied and further trials are discontinued.

Hence corrected flow for loop *ADB* will be as follows :

For pipe *AD*, $Q_0 = 37.4 - 0.1 = 37.3$ (as flow is clockwise)

For pipe *DB*, $Q_0 = 1 - 0.1 = 0.9$ (as flow is clockwise)

For pipe *AB*, $Q_0 = 52.6 + 0.1 = 52.7$ (as flow is anti-clockwise)

Loop DCB

Pipe	r	Q_0	$h_f = rQ_0^2$	$2rQ_0$
DC	2	16.4	$2 \times 16.4^2 = 537.9$	$2 \times 2 \times 16.4 = 65.6$
CB	1	23.6	$-1 \times 23.6^2 - 556.9$	$2 \times 1 \times 23.6 = 47.2$
BD	1	1	$-1 \times 1^2 = -1$	$2 \times 1 \times 1 = 2$
			$\Sigma rQ_0^2 = -20$,	$\Sigma 2rQ_0 = 114.8$

$$\therefore \Delta Q = \frac{-\sum r Q_0^2}{\sum 2r Q_0} = \frac{-(-20)}{114.8}$$

$$= \frac{20}{114.8} = 0.174$$

$$\simeq 0.2$$

As ΔQ is positive, hence it should be added to the flow in the clockwise direction and subtracted from the flow in the anticlockwise direction.

As the correction (ΔQ) is small (*i.e.*, $\Delta Q = 0.2$), this correction is applied and further trials are discontinued.

Hence corrected flow for loop *DCB* will be as follows :

For pipe *DC*, $Q_0 = 16.4 + 0.2 = 16.6$ (clockwise flow)

For pipe *CB*, $Q_0 = 23.6 - 0.2 = 23.4$ (anticlockwise flow)

For pipe *BD*, $Q_0 = 0.9 - 0.2 = 0.7$ (anticlockwise flow)

The final distribution of discharges in each pipe is as follows :

Discharge in pipe *AD* = 37.3 from *A* to *D*

AB = 52.7 from *A* to *B*

DB = 0.7 from *D* to *B*

DC = 16.6. from *D* to *C*

BC = 23.4 from *B* to *C*

The final discharge in each pipe is shown in Fig. 11.34 (*c*)

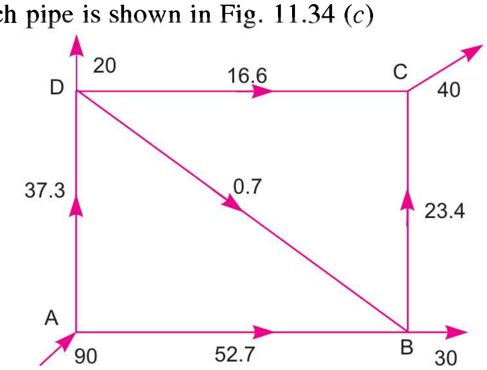


Fig. 11.34 (*c*)

Note. The pipe *DB* is common to two loop (*i.e.*, loops *ADB* and loop *DBC*). Hence this pipe will get two corrections. For loop *ADB*, the correction $\Delta Q = -0.1$ and hence the corrected flow in pipe *DB* is $1 - 0.1 = 0.9$. Now again, the correction is applied to pipe *DB* when we consider loop *DBC*. For loop *DBC*, the correction $\Delta Q = 0.2$ but flow is anticlockwise and hence the final correct flow in pipe *DB* will be $0.9 - 0.2 = 0.7$.

HIGHLIGHTS

1. The energy loss in pipe is classified as major energy loss and minor energy losses. Major energy loss is due to friction while minor energy losses are due to sudden expansion of pipe, sudden contraction of pipe, bend in pipe and an obstruction in pipe.

2. Energy loss due to friction is given by Darcy Formula, $h_f = \dfrac{4fLV^2}{d \times 2g}$.

3. The head loss due to friction in pipe can also be calculated by Chezy's formula.

$$V = C\sqrt{mi} \quad \text{Chezy's formula}$$

where C = Chezy's Constant

m = Hydrualic mean depth = $\dfrac{d}{4}$ (for pipe running full)

V = Velocity of flow

i = Loss of head per unit length = $\dfrac{h_f}{L}$

$\therefore$ $h_f = L \times i$, where i is obtained from Chezy's formula.

4. Loss of head due to sudden expansion of pipe, $h_c = \dfrac{(V_1 - V_2)^2}{2g}$

where V_1 = Velocity in small pipe, V_2 = Velocity in large pipe.

5. Loss of head due to sudden contraction of pipe, $h_c = \left(\dfrac{1}{C_c} - 1\right)\dfrac{V_2^2}{2g}$

where C_c = co-efficient of contraction $= 0.375\,\dfrac{V_2^2}{2g}$ ...(For $C_c = 0.62$)

$= 0.5\,\dfrac{V_2^2}{2g}$ ...(if value of C_c is not given)

6. Loss of head at the entrance of a pipe, $h_i = 0.5\,\dfrac{V^2}{2g}$.

7. Loss of head at the exit of pipe, $h_o = \dfrac{V^2}{2g}$.

8. The line representing the sum of pressure head and datum head with respect to some reference line is called hydraulic gradient line (H.G.L.) while the line representing the sum of pressure head, datum head and velocity head with respect to some reference line is known as total energy line (T.E.L.).

9. Syphon is a long bent pipe used to transfer liquids from a reservoir at a higher level to another reservoir at a lower level, when the two reservoirs are separated by a high level ground.

10. The maximum vacuum created at the summit of syphon is only 7.4 m of water.

11. When pipes of different lengths and different diameters are connected end to end, pipes are called in series or compound pipes. The rate of flow through each pipe connected in series is same.

12. A single pipe of uniform diameter, having same discharge and same loss of head as compound pipe consisting of several pipes of different lengths and diameters, is known as equivalent pipe. The diameter of equivalent pipe is called equivalent size of the pipe.

13. The equivalent size of the pipe is obtained from

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

where L = equivalent length of pipe = $L_1 + L_2 + L_3$

d_1, d_2, d_3 = diameters of pipes connected in series

d = equivalent size of the pipes.

14. When the pipes are connected in parallel, the loss of head in each pipe is same. The rate of flow in main pipe is equal to sum of the rate of flow in each pipe, connected in parallel.

15. For solving problems for branched pipes, the three basic, equations *i.e.*, continuity, Bernoulli's and Darcy's equations are used.

16. Power transmitted in kW through pipe is given by $P = \frac{\rho g \times Q \times (H - h_f)}{1000}$

where Q = discharge through pipe = area × velocity = $\frac{\pi}{4} d^2 \times V$

H = total head at inlet of pipe

h_f = head lost due to friction

$= \frac{4 fLV^2}{d \times 2g}$, where L = Length of pipe

In S.I. units, power transmitted is given by, Power = $\frac{\rho g \times Q \times (H - h_f)}{1000}$ kW.

17. Efficiency of power transmission through pipes, $\eta = \frac{H - h_f}{H}$.

18. Condition for maximum transmission of power through pipe, $h_f = \frac{H}{3}$ and maximum efficiency = 66.7%.

19. The velocity of water at the outlet of the nozzle is $v = \sqrt{\frac{2gH}{\left(1 + \frac{4 fL}{D} \times \frac{a^2}{A^2}\right)}}$

where H = head at the inlet of the pipe, L = length of the pipe,

D = diameter of the pipe, a = area of the nozzle at outlet,

A = area of the pipe.

20. The power transmitted through nozzle, $P = \frac{\rho g \times Q}{1000} \left[H - \frac{4 fLV^2}{D \times 2g} \right]$

and the efficiency of power transmission through nozzle, $\eta = \frac{1}{\left[1 + \frac{4 fL}{D} \times \frac{a^2}{A^2}\right]}$

21. Condition for maximum power transmission through nozzle, $h_f = \frac{H}{3}$.

22. Diameter of nozzle for maximum power transmission through nozzle is, $d = \left(\frac{D^5}{8fL}\right)^{1/4}$

where d = diameter of the nozzle at outlet, D = diameter of the pipe,
L = length of the pipe, f = co-efficient of friction for pipe.

23. When a liquid is flowing through a long pipe fitted with a valve at the end of the pipe and the valve is closed suddenly, a pressure wave of high intensity is produced behind the valve. This pressure wave of high intensity is having the effect of hammering action on the walls of the pipe. This phenomenon is known as water hammer.

24. The intensity of pressure rise due to water hammer is given by

$$p = \frac{\rho LV}{T} \quad \text{... when valve is closed gradually.}$$

$$= V\sqrt{K\rho} \quad \text{... when valve is closed suddenly and pipe is assumed rigid}$$

$$= V \times \sqrt{\frac{\rho}{\frac{1}{K} + \frac{D}{Et}}} \quad \text{... when valve is closed suddenly and pipe is elastic.}$$

where L = Length of pipe, V = Velocity of flow,
T = Time required to close the valve, K = Bulk modulus of water,
D = Diameter of the pipe, E = Modulus of elasticity for pipe material.
t = Thickness of the pipe wall,

25. If the time required to close the valve :

$$T > \frac{2L}{C} \quad \text{... the valve closure is said to be gradual,}$$

$$T < \frac{2L}{C} \quad \text{... the valve closure is said to be sudden}$$

where L = length of pipe,

C = velocity of pressure wave produced due to water hammer = $\sqrt{\frac{K}{\rho}}$.

EXERCISE

(A) THEORETICAL PROBLEMS

1. How will you determine the loss of head due to friction in pipes by using (*i*) Darcy Formula and (*ii*) Chezy's formula ?
2. (*a*) What do you understand by the terms : Major energy loss and minor energy losses in pipes ?
   (*b*) What do you understand by total energy line, hydraulic gradient line, pipes in series, pipes in parallel and equivalent pipe ?
3. (*a*) Derive an expression for the loss of head due to : (*i*) Sudden enlargement and (*ii*) Sudden contraction of a pipe.
   (*b*) Obtain expression for head loss in a sudden expansion in the pipe. List all the assumptions made in the derivation.
4. Define and explain the terms : (*i*) Hydraulic gradient line and (*ii*) Total energy line.

5. Show that the loss of head due to sudden expansion in pipe line is a function of velocity head.
6. What is a syphon ? On what principle it works ?
7. What is a compound pipe ? What will be loss of head when pipes are connected in series ?
8. Explain the terms : (*i*) Pipes in parallel (*ii*) Equivalent pipe and (*iii*) Equivalent size of the pipe.
9. Find an expression for the power transmission through pipes. What is the condition for maximum transmission of power and corresponding efficiency of transmission ?
10. Prove that the head loss due to friction is equal to one-third of the total head at inlet for maximum power transmission through pipes or nozzles.
11. Prove that the velocity through nozzle is given by $v = \sqrt{\dfrac{2gH}{1 + \dfrac{4fL}{D} \times \dfrac{a^2}{A^2}}}$

    where a = Area of nozzle at outlet, A = Area of the pipe.
12. Show that the diameter of the nozzle for maximum transmission of power is given by $d = \left(\dfrac{D^5}{8fL}\right)^{1/4}$

    where D = Diameter of pipe, L = Length of pipe.
13. Find an expression for the ratio of the outlet area of the nozzle to the area of the pipe for maximum transmission of power.
14. Explain the phenomenon of Water Hammer. Obtain an expression for the rise of pressure when the flowing water in a pipe is brought to rest by closing the valve gradually.
15. Show that the pressure rise due to sudden closure of a valve at the end of a pipe, through which water is flowing is given by $p = V\sqrt{\dfrac{d}{\dfrac{1}{K} + \dfrac{D}{Et}}}$

    where V = Velocity of flow, D = Diameter of pipe, E = Young's Modulus, K = Bulk Modulus and t = Thickness of pipe.
16. Three pipes of different diameters and different lengths are connected in series to make a compound pipe. The ends of this compound pipe are connected with two tanks whose difference of water level is H. If co-efficient of friction for these pipes is same, then derive the formula for the total head loss, neglecting first the minor losses and then including them.
17. For the two cases of flow in a sudden contraction in a pipeline and flow in a sudden expansion in a pipe line, draw the flow pattern, piezometric grade line and total energy line.
18. What do you mean by "equivalent pipe" and "flow through parallel pipes" ?
19. (*a*) Define and explain the terms : (*i*) Hydraulic gradient line and (*ii*) total energy line.
    (*b*) What do you mean by equivalent pipe ? Obtain an expression for equivalent pipe.
    (Delhi University, December 2002)

(B) NUMERICAL PROBLEMS

1. Find the head loss due to friction in a pipe of diameter 250 mm and length 60 m, through which water is flowing at a velocity of 3.0 m/s using (*i*) Darcy formula and (*ii*) Chezy's Formula for which $C = 55$. Take ν for water = .01 stoke. [**Ans.** (*i*) 1.182, (*ii*) 2.856]
2. Find the diameter of a pipe of length 2500 m when the rate of flow of water through the pipe is 0.25 m^3/s and head loss due to friction is 5 m. Take $C = 50$ in Chezy's formula. [**Ans.** 605 mm]

3. An oil of Kinematic Viscosity 0.5 stoke is flowing through a pipe of diameter 300 mm at the rate of 320 litres per sec. Find the head lost due to friction for a length of 60 m of the pipe. [**Ans.** 5.14 m]

4. Calculate the rate of flow of water through a pipe of diameter 300 mm, when the difference of pressure head between the two ends of a pipe 400 m apart is 5 m of water. Take the value of f = .009 in the formula

$$h_f = \frac{4fLV^2}{d \times 2g}$$ [**Ans.** 0.101 m^3/s]

5. The discharge through a pipe is 200 litres/s. Find the loss of head when the pipe is suddenly enlarged from 150 mm to 300 mm diameter. [**Ans.** 3.672 m]

6. The rate of flow of water through a horizontal pipe is 0.3 m^3/s. The diameter of the pipe is suddenly enlarged from 250 mm to 500 mm. The pressure intensity in the smaller pipe is 13.734 N/cm^2. Determine : (*i*) loss of head due to sudden enlargement, (*ii*) pressure intensity in the large pipe and (*iii*) power lost due to enlargement. [**Ans.** (*i*) 1.07 m, (*ii*) 14.43 N/cm^2, (*iii*) 3.15 kW]

7. A horizontal pipe of diameter 400 mm is suddenly contracted to a diameter of 200 mm. The pressure intensities in the large and smaller pipe is given as 14.715 N/cm^2 and 12.753 N/cm^2 respectively. If C_c = 0.62, find the loss of head due to contraction. Also determine the rate of flow of water. [**Ans.** (*i*) 0.571 m, (*ii*) 171. 7 litres/s]

8. Water is flowing through a horizontal pipe of diameter 300 mm at a velocity of 4 m/s. A circular solid plate of diameter 200 mm is placed in the pipe to obstruct the flow. If C_c = 0.62, find the loss of head due to obstruction in the pipe. [**Ans.** 2.953 m]

9. Determine the rate of flow of water through a pipe of diameter 10 cm and length 60 cm when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The height of water in the tank from the centre of the pipe is 5 cm. Pipe is given as horizontal and value of f = .01. Consider minor losses. [**Ans.** 15.4 litres/s]

10. A horizontal pipe-line 50 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 30 m of its length from the tank, the pipe is 200 mm diameter and its diameter is suddenly enlarged to 400 mm. The height of water level in the tank is 10 m above the centre of the pipe. Considering all minor losses, determine the rate of flow. Take f = .01 for both sections of the pipe. [**Ans.** 164.13 litres/s]

11. Determine the difference in the elevations between the water surfaces in the two tanks which are connected by a horizontal pipe of diameter 400 mm and length 500 m. The rate of flow of water through the pipe is 200 litres/s. Consider all losses and take the value of f = .009. [**Ans.** 11.79 m]

12. For the problems 9, 10 and 11 draw the hydraulic gradient lines (H.G.L.) and total energy lines (T.E.L.)

13. A syphon of diameter 150 mm connects two reservoirs having a difference in elevation of 15 m. The length of the syphon is 400 m and summit is 4.0 m above the water level in the upper reservoir. The length of the pipe from upper reservoir to the summit is 80 m. Determine the discharge through the syphon and also pressure at the summit. Neglect minor losses. The co-efficient of friction, f = .005. [**Ans.** 41.52 litres/s, – 7.281 m of water]

14. A syphon of diameter 200 mm connects two reservoirs having a difference in elevation as 20 m. The total length of the syphon is 800 m and the summit is 5 m above the water level in the upper reservoir. If the separation takes place at 2.8 m of water absolute find the maximum length of syphon from upper reservoir to the summit. Take f = .004 and atmospheric pressure = 10.3 m of water. [**Ans.** 87.52 m]

15. Three pipes of lengths 800 m, 600 m and 300 m and of diameters 400 mm, 300 mm and 200 mm respectively are connected in series. The ends of the compound pipe is connected to two tanks, whose water surface levels are maintained at a difference of 15 m. Determine the rate of flow of water through the pipes if f = .005. What will be diameter of a single pipe of length 1700 m and f = .005, which replaces the three pipes ? [**Ans.** 0.0848 m^3/s, 266.5 mm]

16. Two pipes of lengths 2500 m each and diameters 80 cm and 60 cm respectively, are connected in parallel. The co-efficient of friction for each pipe is 0.006. The total flow is equal to 250 litres/s. Find the rate of flow in each pipe. [**Ans.** 0.1683 m^3/s, 0.0817 m^3/s]

17. A pipe of diameter 300 mm and length 1000 m connects two reservoirs, having difference of water levels as 15 m. Determine the discharge through the pipe. If an additional pipe of diameter 300 mm and length 600 m is attached to the last 600 m length of the existing pipe, find the increase in the discharge. Take f = .02 and neglect minor losses. [**Ans.** 0.0742 m^3/s, 0.0258 m^3/s]

18. Two sharp ended pipes of diameters 60 mm and 100 mm respectively, each of length 150 m are connected in parallel between two reservoirs which have a difference of level of 15 m. If co-efficient of friction for each pipe is 0.08, calculate the rate of flow for each pipe and also the diameter of a single pipe 150 m long which would give the same discharge if it were substituted for the original two pipes. [**Ans.** 0.0017, .00615, 110 mm]

19. Three reservoirs A, B and C are connected by a pipe system having length 700 m, 1200 m and 500 m and diameters 400 mm, 300 mm and 200 mm respectively. The water levels in reservoir A and B from a datum line are 50 m and 45 m respectively. The level of water in reservoir C is below the level of water in reservoir B. Find the discharge into or from the reservoirs B and C if the rate of flow from reservoir A is 150 litres per sec. Find the height of water level in the reservoir C. Take f = .005 for all pipes. [**Ans.** .005 m^3/s, .095 m^3/s, 24.16 m]

20. A pipe of diameter 300 mm and length 3000 m is used for the transmission of power by water. The total head at the inlet of the pipe is 400 m. Find the maximum power available at the outlet of the pipe. Take f = .005. [**Ans.** 667.07 kW]

21. A pipe line of length 2100 m is used for transmitting 103 kW. The pressure at the inlet of the pipe is 392.4 N/cm^2. If the efficiency of transmission is 80%, find the diameter of the pipe. Take f = .005. [**Ans.** 136 mm]

22. A nozzle is fitted at the end of a pipe of length 400 m and of diameter 150 mm. For the maximum transmission of power through the nozzle, find the diameter of the nozzle. Take f = .008. [**Ans.** 41.5 mm]

23. The head of water at the inlet of a pipe of length 1500 m and of diameter 400 mm is 50 m. A nozzle of diameter 80 mm at the outlet, is fitted to the pipe. Find the velocity of water at the outlet of the nozzle if f = .01 for the pipe. [**Ans.** 28.12 m/s]

24. The rate of flow of water through a pipe of length 1500 m and diameter 800 mm is 2 m^3/s. At the end of the pipe a nozzle of outside diameter 200 mm is fitted. Find the power transmitted through the nozzle if the head of water at the inlet of the pipe is 180 m and f = .01 for pipe. [**Ans.** 2344.7 kW]

25. The water is flowing with a velocity of 2 m/s in a pipe of length 2000 m and of diameter 600 mm. At the end of the pipe, a valve is provided. Find the rise in pressure if the valve is closed in 20 seconds. Take the value of C = 1420 m/s. [**Ans.** 20 N/cm^2]

26. If the valve in the problem 25 is closed in 1.5 sec, find the rise in pressure. Take bulk modulus of water = 19.62×10^4 N/cm^2 and consider pipe as rigid one. [**Ans.** 186.75 N/cm^2]

27. If in the problem 25, the thickness of the pipe is 10 mm and the valve is closed suddenly. Find the rise in pressure if the pipe is considered to be elastic. Take value of $E = 19.62 \times 10^6$ N/cm^2 for pipe material and $K = 19.62 \times 10^4$ N/cm^2 for water. Calculate the circumferential stress and longitudinal stress developed in the pipe wall. [**Ans.** p = 221.47 N/cm^2, f_c = 6644.1 N/cm^2, f_l = 3322 N/cm^2]

28. The difference in water surface levels in two tanks, which are connected by two pipes in series of lengths 600 m and 400 m and of diameters 30 cm and 20 cm respectively, is 15 m. Determine the rate of flow of water if the co-efficient of friction is 0.005 for both the pipes. Neglect minor losses.

29. Water is flowing vertically downwards through a 10 cm diameter pipe at the rate of 50 l.p.s. At a particular location the pipe suddenly enlarges to 20 cm diameter. A point P is located 50 cm above the section of enlargement and another point Q is located 50 cm below it in the enlarged portion. A pressure gauge connected at P gives a reading of 19.62 N/cm^2. Calculate the pressure at location Q neglecting friction loss between P and Q but considering the loss due to sudden enlargement. What will be the pressure at Q if the same discharge flows upwards assuming that the pressure P remains the same ? Consider the loss due to contraction with $C_c = 0.60$ but neglect friction loss between P and Q. [**Ans.** 21.36 N/cm², 23.4 N/cm²]

30. Two tanks are connected with the help of two pipes in series. The lengths of the pipes are 1000 m and 800 m whereas the diameters are 400 mm and 200 mm respectively. The co-efficient of friction for both the pipes is 0.008. The difference of water level in the two tanks is 15 m. Find the rate of flow of water through the pipes, considering all losses. Also draw the total energy line and hydraulic gradient lines for the system. [**Ans.** 0.0464 m³/s]

[**Hint .** $L_1 = 1000$ m ; $L_2 = 800$ m ; $d_1 = 400$ mm $= 0.4$ m ; $d_2 = 200$ mm $= 0.2$ m, $f = 0.008$; $H = 15$ m.

Now $$H = h_i + hf_1 + h_c + hf_2 + h_o,$$

$$H = \frac{0.5\,V_1^2}{2g} + \frac{4f \times L_1 \times V_1^2}{d \times 2g} + \frac{0.5\,V_2^2}{2g} + \frac{4f \times L_2 \times V_2^2}{d_2 \times 2g} + \frac{V_2^2}{2g}$$

or $$15 = \frac{0.5\,V_1^2}{2 \times 9.81} + \frac{4 \times 0.008 \times 1000 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{0.5\,V_2^2}{2 \times 9.81} + \frac{4 \times 0.008 \times 800 \times V_2^2}{0.2 \times 2 \times 9.81} + \frac{V_2^2}{2g}$$

Also $$A_1V_1 = A_2V_2 \text{ or } V_2 = 4V_1$$

$\therefore$ $$15 = 0.02548V_1^2 + 4.0775V_1^2 + 0.02548V_2^2 + 6.524V_2^2 + 0.05097V_2^2$$
$$= 4.103V_1^2 + 6.6V_2^2 = 4.103V_1^2 + 6.6 \times (4V_1)^2$$
$$= 4.103V_1^2 + 105.607V_1^2 = 109.71V_1^2$$

$\therefore$ $$V_1 = \sqrt{\frac{15}{109.71}} = 0.3697 \text{ m/s} \therefore Q = A_1V_1 = \frac{\pi}{4}(.4)^2 \times 0.3697 = 0.0464 \text{ m}^3\text{/s}]$$

31. A pipe of diameter 25 cm and length 2000 m connects two reservoirs, having difference of water level 25 m. Determine the discharge through the pipe. If an additional pipe of diameter 25 cm and length 1000 m is attached to the last 1000 m length of the existing pipe, find the increase in discharge. Take $f = 0.015$. Neglect minor losses. (*Delhi University, December 2002*) [**Ans.** (*i*) 49.62 l/s, (*ii*) 13.14 l/s]