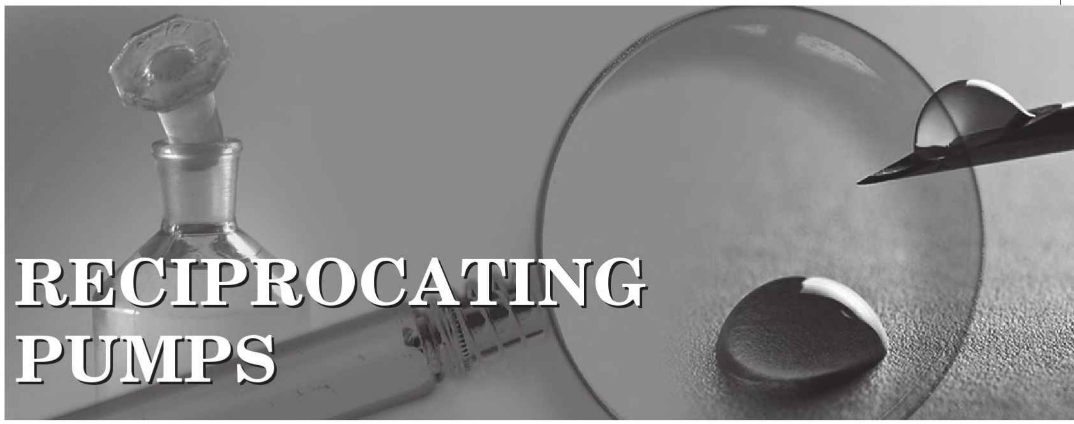
20 CHAPTER

RECIPROCATING PUMPS



▶ 20.1 INTRODUCTION

In the last chapter, we have defined the pumps as the hydraulic machines which convert the mechanical energy into hydraulic energy which is mainly in the form of pressure energy. If the mechanical energy is converted into hydraulic energy, by means of centrifugal force acting on the liquid, the pump is known as centrifugal pump. But if the mechanical energy is converted into hydraulic energy (or pressure energy) by sucking the liquid into a cylinder in which a piston is reciprocating (moving backwards and forwards), which exerts the thrust on the liquid and increases its hydraulic energy (pressure energy), the pump is known as reciprocating pump.

▶ 20.2 MAIN PARTS OF A RECIPROCATING PUMP

The following are the main parts of a reciprocating pump as shown in Fig. 20.1 :

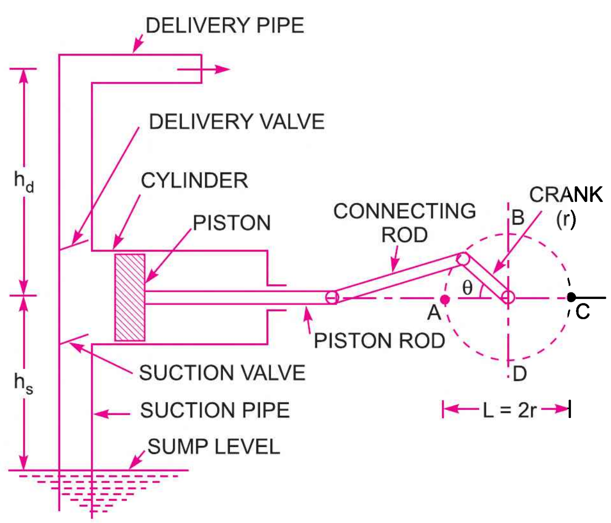


Fig. 20.1 *Main parts of a reciprocating pump.*

1. A cylinder with a piston, piston rod, connecting rod and a crank,
2. Suction pipe,
3. Delivery pipe,
4. Suction valve, and
5. Delivery valve.

▶ 20.3 WORKING OF A RECIPROCATING PUMP

Fig. 20.1 shows a single acting reciprocating pump, which consists of a piston which moves forwards and backwards in a close fitting cylinder. The movement of the piston is obtained by connecting the piston rod to crank by means of a connecting rod. The crank is rotated by means of an electric motor. Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder. The suction and delivery valves are one way valves or non-return valves, which allow the water to flow in one direction only. Suction valve allows water from suction pipe to the cylinder which delivery valve allows water from cylinder to delivery pipe only.

When crank starts rotating, the piston moves to and fro in the cylinder. When crank is at A., the piston is at the extreme left position in the cylinder. As the crank is rotating from A to C, (*i.e.*, from $\theta = 0^o$ to $\theta = 180°$), the piston is moving towards right in the cylinder. The movement of the piston towards right creates a partial vacuum in the cylinder. But on the surface of the liquid in the sump atmospheric pressure is acting, which is more than the pressure inside the cylinder. Thus, the liquid is forced in the suction pipe from the sump. This liquid opens the suction valve and enters the cylinder.

When crank is rotating from C to A (*i.e.*, from $\theta = 180°$ to $\theta = 360°$), the piston from its extreme right position starts moving towards left in the cylinder. The movement of the piston towards left increases the pressure of the liquid inside the cylinder more than atmospheric pressure. Hence suction valve closes and delivery valve opens. The liquid is forced into the delivery pipe and is raised to a required height.

20.3.1 Discharge Through a Reciprocating Pump. Consider a single* acting reciprocating pump as shown in Fig. 20.1.

Let D = Diameter of the cylinder

A = Cross-sectional area of the piston or cylinder

$$= \frac{\pi}{4} D^2$$

r = Radius of crank

N = r.p.m. of the crank

L = Length of the stroke = $2 \times r$

h_s = Height of the axis of the cylinder from water surface in sump.

h_d = Height of delivery outlet above the cylinder axis (also called delivery head)

Volume of water delivered in one revolution or discharge of water in one revolution

= Area × Length of stroke = $A \times L$

Number of revolution per second, $= \frac{N}{60}$

∴ Discharge of the pump per second,

Q = Discharge in one revolution × No. of revolution per second

$$= A \times L \times \frac{N}{60} = \frac{ALN}{60} \qquad \text{...(20.1)}$$

* Single acting means the water is acting on one side of the piston only.

Weight of water delivered per second,

$$W = \rho \times g \times Q = \frac{\rho g A L N}{60}. \qquad \text{...(20.2)}$$

20.3.2 Work done by Reciprocating Pump. Work done by the reciprocating pump per second is given by the reaction as

Work done per second = Weight of water lifted per second × Total height through which water is lifted

$$= W \times (h_s + h_d) \qquad \text{...}(i)$$

where $(h_s + h_d)$ = Total height through which water is lifted.

From equation (20.2), Weight, W, is given by

$$W = \frac{\rho g \times ALN}{60}.$$

Substituting the value of W in equation (i), we get

$$\text{Work done per second} = \frac{\rho g \times ALN}{60} \times (h_s + h_d) \qquad \text{...(20.3)}$$

∴ Power required to drive the pump, in kW

$$P = \frac{\text{Work done per second}}{1000} = \frac{\rho g \times ALN \times (h_s + h_d)}{60 \times 1000}$$

$$= \frac{\rho g \times ALN \times (h_s + h_d)}{60{,}000} \text{ kW} \qquad \text{...(20.4)}$$

20.3.3 Discharge, Work done and Power Required to Drive a Double-acting Pump. In case of double-acting pump, the water is acting on both sides of the piston as shown in Fig. 20.2. Thus, we require two suction pipes and two delivery pipes for double-acting pump. When there is a suction stroke on one side of the piston, there is at the same time a delivery stroke on the other side of the piston. Thus for one complete revolution of the crank there are two delivery strokes and water is delivered to the pipes by the pump during these two delivery strokes.

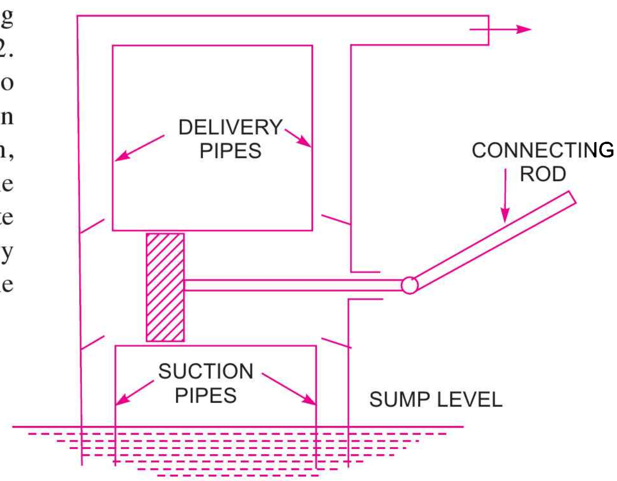


Fig. 20.2

Let D = Diameter of the piston,

d = Diameter of the piston rod

∴ Area on one side of the piston,

$$A = \frac{\pi}{4} D^2$$

Area on the other side of the piston, where piston rod is connected to the piston,

$$A_1 = \frac{\pi}{4} D^2 - \frac{\pi}{4} d^2 = \frac{\pi}{4} (D^2 - d^2).$$

∴ Volume of water delivered in one revolution of crank

$$= A \times \text{Length of stroke} + A_1 \times \text{Length of stroke}$$

$$= AL + A_1L = (A + A_1)L = \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}\left(D^2 - d^2\right)\right] \times L$$

∴ Discharge of pump per second

= Volume of water delivered in one revolution × No. of revolution per second

$$= \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}\left(D^2 - d^2\right)\right] \times L \times \frac{N}{60}$$

If 'd' the diameter of the piston rod is very small as compared to the diameter of the piston, then it can be neglected and discharge of pump per second,

$$Q = \left(\frac{\pi}{4}D^2 + \frac{\pi}{4}D^2\right) \times \frac{L \times N}{60} = 2 \times \frac{\pi}{4}D^2 \times \frac{L \times N}{60} = \frac{2ALN}{60} \quad \text{...(20.5)}$$

Equation (20.5) gives the discharge of a double-acting reciprocating pump. This discharge is two times the discharge of a single-acting pump.

Work done by double-acting reciprocating pump

Work done per second = Weight of water delivered × Total height

$$= \rho g \times \text{Discharge per second} \times \text{Total height}$$

$$= \rho g \times \frac{2ALN}{60} \times (h_s + h_d) = 2\rho g \times \frac{ALN}{60} \times (h_s + h_d) \quad \text{...(20.6)}$$

∴ Power required to drive the double-acting pump in kW,

$$P = \frac{\text{Work done per second}}{1000} = 2\rho g \times \frac{ALN}{60} \times \frac{(h_s + h_d)}{1000}$$

$$= \frac{2\rho g \times ALN \times (h_s + h_d)}{60{,}000} \quad \text{...(20.7)}$$

▶ 20.4 SLIP OF RECIPROCATING PUMP

Slip of a pump is defined as the difference between the theoretical discharge and actual discharge of the pump. The discharge of a single-acting pump given by equation (20.1) and of a double-acting pump given by equation (20.5) are theoretical discharge. The actual discharge of a pump is less than the theoretical discharge due to leakage. The difference of the theoretical discharge and actual discharge is known as slip of the pump. Hence, mathematically.

$$\text{Slip} = Q_{th} - Q_{act} \quad \text{...(20.8)}$$

But slip is mostly expressed as percentage slip which is given by,

$$\text{Percentage slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

$$= (1 - C_d) \times 100 \qquad \left(\because \frac{Q_{act}}{Q_{th}} = C_d\right) \quad \text{...(20.9)}$$

where C_d = Co-efficient of discharge.

20.4.1 Negative Slip of the Reciprocating Pump. Slip is equal to the difference of theoretical discharge and actual discharge. If actual discharge is more than the theoretical discharge, the slip of the pump will become –ve. In that case, the slip of the pump is known as negative slip.

Negative slip occurs when delivery pipe is short, suction pipe is long and pump is running at high speed.

▶ 20.5 CLASSIFICATION OF RECIPROCATING PUMPS

The reciprocating pumps may be classified as :

1. According to the water being in contact with one side or both sides of the piston, and
2. According to the number of cylinders provided.

If the water is in contact with one side of the piston, the pump is known as single-acting. On the other hand, if the water is in contact with both sides of the piston, the pump is called double-acting. Hence, classification according to the contact of water is :

(*i*) Single-acting pump, and (*ii*) Double-acting pump.

According to the number of cylinder provided, the pumps are classified as :

(*i*) Single cylinder pump, (*ii*) Double cylinder pump, and

(*iii*) Triple cylinder pump.

Problem 20.1 *A single-acting reciprocating pump, running at 50 r.p.m., delivers 0.01 m^3/s of water. The diameter of the piston is 200 mm and stroke length 400 mm. Determine :*

(i) The theoretical discharge of the pump, (ii) Co-efficient of discharge, and (iii) Slip and the percentage slip of the pump.

Solution. Given :

Speed of the pump, $N = 50$ r.p.m.

Actual discharge, $Q_{act} = .01\ m^3/s$

Dia. of piston, $D = 200\ mm = .20\ m$

∴ Area, $A = \frac{\pi}{4}(.2)^2 = .031416\ m^2$

Stroke, $L = 400\ mm = 0.40\ m.$

(*i*) Theoretical discharge for single-acting reciprocating pump is given by equation (20.1) as

$$Q_{th} = \frac{A \times L \times N}{60} = \frac{.031416 \times .40 \times 50}{60} = \mathbf{0.01047\ m^3/s.\ Ans.}$$

(*ii*) Co-efficient of discharge is given by

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{.01047} = \mathbf{0.955.\ Ans.}$$

(*iii*) Using equation (20.8), we get

$$\text{Slip} = Q_{th} - Q_{act} = .01047 - .01 = \mathbf{0.00047\ m^3/s.\ Ans.}$$

And percentage slip
$$= \frac{(Q_{th} - Q_{act})}{Q_{th}} \times 100 = \frac{(.01047 - .01)}{.01047} \times 100$$

$$= \frac{.00047}{.01047} \times 100 = \mathbf{4.489\%.\ Ans.}$$

Problem 20.2 *A double-acting reciprocating pump, running at 40 r.p.m., is discharging 1.0 m*^{3} *of water per minute. The pump has a stroke of 400 mm. The diameter of the piston is 200 mm. The delivery and suction head are 20 m and 5 m respectively. Find the slip of the pump and power required to drive the pump.*

Solution. Given:

Speed of pump, $N = 40$ r.p.m.

Actual discharge, $Q_{act} = 1.0 \text{ m}^3/\text{min} = \frac{1.0}{60} \text{ m}^3/\text{s} = 0.01666 \text{ m}^3/\text{s}$

Stroke, $L = 400 \text{ mm} = 0.40 \text{ m}$

Diameter of piston, $D = 200 \text{ mm} = 0.20 \text{ m}$

$\therefore$ Area, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (.2)^2 = 0.031416 \text{ m}^2$

Suction head, $h_s = 5$ m

Delivery head, $h_d = 20$ m.

Theoretical discharge for double-acting pump is given by equation (20.5) as,

$$Q_{th} = \frac{2ALN}{60} = \frac{2 \times .031416 \times 0.4 \times 40}{60} = .01675 \text{ m}^3/\text{s}.$$

Using equation (20.8), Slip $= Q_{th} - Q_{act} = .01675 - .01666 =$ **.00009 m^3/s. Ans.**

Power required to drive the double-acting pump is given by equation (20.7) as,

$$P = \frac{2 \times \rho g \times ALN \times (h_s + h_d)}{60{,}000} = \frac{2 \times 1000 \times 9.81 \times .031416 \times .4 \times 40 \times (5 + 20)}{60{,}000}$$

$= \mathbf{4.109 \text{ kW}}$. **Ans.**

▶ 20.6 VARIATION OF VELOCITY AND ACCELERATION IN THE SUCTION AND DELIVERY PIPES DUE TO ACCELERATION OF THE PISTON

It is mentioned in Art. 20.3 that when crank starts rotating, the piston moves forwards and backwards in the cylinder. At the extreme left position and right position of the piston in the cylinder, the velocity of the piston is zero. The velocity of the piston is maximum at the centre of the cylinder. This means that at the start of a stroke (may be suction or delivery stroke), the velocity of the piston is zero and this velocity becomes maximum at the centre of each stroke and again becomes zero at the end of each stroke. Thus at the beginning of each stroke, the piston will be having an acceleration and at the end of each stroke, the piston will be having a retardation. The water in the cylinder is in contact with the piston and hence the water, flowing from the suction pipe or to the delivery pipe will have an acceleration at the beginning of each stroke and a retardation at the end of each stroke. This means the velocity of flow of water in the suction and delivery pipe will not be uniform. Hence, an accelerative or retarding head will be acting on the water flowing through the suction or delivery pipe. This accelerative or retarding head will change the pressure inside the cylinder.

If the ratio of length of connecting rod to the radius of crank (*i.e.*, L/r) is very large, then the motion of the piston can be assumed as simple harmonic in nature. Fig. 20.3 shows the cylinder of a reciprocating single-acting pump, fitted with a piston which is connected to the crank. Let the crank is rotating at a constant angular speed.

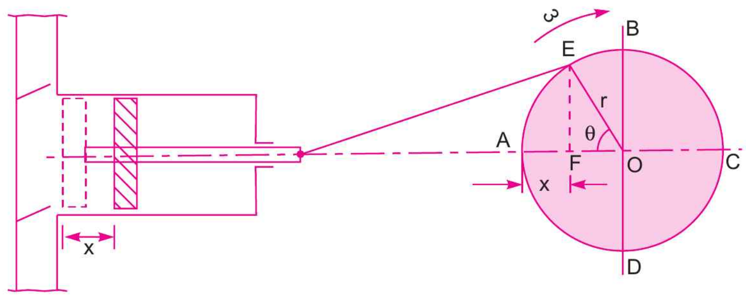


Fig. 20.3 *Velocity and acceleration of piston.*

Let ω = Angular speed of the crank in rad./s,

A = Area of the cylinder,

a = Area of the pipe (suction or delivery),

l = Length of the pipe (suction or delivery), and

r = Radius of the crank.

In the beginning, the crank is at A (which is called inner dead centre) and the piston in the cylinder is at a position shown by dotted lines. The crank is rotating with an angular velocity ω and let in time 't' seconds, the crank turns through an angle θ (in radians) from A (*i.e.*, inner dead centre). The displacement of the piston in time 't' is 'x' as shown in Fig. 20.3.

Now θ = Angle turned by crank in radians in time 't'

$= \omega t$ ...(*i*)

The distance x travelled by the piston is given as

x = Distance $AF = AO - FO$

$= r - r\cos\theta$ ($\because AO = r, FO = r\cos\theta$)

$= r - r\cos(\omega t)$ ($\because$ From (*i*), $\theta = \omega t$) ...(*ii*)

The velocity of the piston is obtained by differentiating equation (*ii*) with respect to 't' .

$\therefore$ Velocity of piston, $V = \dfrac{dx}{dt} = \dfrac{d}{dt}[r - r\cos(\omega t)]$

$= 0 - r[-\sin\omega t] \times \omega$ ($\because$ r is constant)

$= \omega r \sin \omega t.$ ...(20.10)

Now from continuity equation, the volume of water flowing into cylinder per second is equal to the volume of water flowing from the pipe per second.

$\therefore$ Velocity of water in cylinder $\times$ Area of cylinder

= Velocity of water in pipe $\times$ Area of pipe

or $V \times A = v \times a$ ($\because$ Velocity of water in cylinder = Velocity of piston = V)

where v = Velocity of water in pipe

$$\therefore \quad v = \frac{V \times A}{a} = \frac{A}{a} \times V$$

$$= \frac{A}{a}\,\omega r \sin \omega t \quad [\because \text{From (20.10)}, V = \omega r \sin \omega t] \qquad ...(20.11)$$

The acceleration of water in pipe is obtained by differentiating equation (20.11) with respect to 't'.

∴ Acceleration of water in pipe

$$= \frac{dv}{dt} = \frac{d}{dt}\left(\frac{A}{a}\omega r \sin \omega t\right) = \frac{A}{a}\omega^2 r \cos \omega t \quad \text{....(20.12)}$$

Mass of water in pipe = ρ × Volume of water in pipe

$$= \rho \times [\text{Area of pipe} \times \text{Length of pipe}] = \rho \times [a \times l] = \rho al$$

∴ Force required to accelerate the water in the pipe

= Mass of water in pipe × Acceleration of water in pipe

$$= \rho al \times \frac{A}{a}\omega^2 r \cos \omega t$$

∴ Intensity of pressure due to acceleration

$$= \frac{\text{Force required to accelerate the water}}{\text{Area of pipe}}$$

$$= \frac{\rho al \times \frac{A}{a}\omega^2 r \cos \omega t}{a} = \rho l \times \frac{A}{a}\omega^2 r \cos \omega t$$

$$= \rho l \times \frac{A}{a}\omega^2 r \cos \theta \quad (\because \omega t = \theta)$$

∴ Pressure head (h_a) due to acceleration

$$h_a = \frac{\text{Intensity of pressure due to acceleration}}{\text{Weight density of liquid}}$$

$$= \frac{\rho l \times \frac{A}{a}\omega^2 r \cos \theta}{\rho g} = \frac{l}{g} \times \frac{A}{a}\omega^2 r \cos \theta. \quad \text{...(20.13)}$$

The pressure head due to acceleration in the suction and delivery pipes is obtained from equation (20.13) by using subscripts 's' and 'd' as

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s}\omega^2 r \cos \theta \quad \text{...(20.14)}$$

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d}\omega^2 r \cos \theta. \quad \text{...(20.15)}$$

The pressure head (h_a) due to acceleration, given by equation (20.13) varies with θ. The values of 'h_a' for different values of θ are :

1. When θ = 0°, $h_a = \frac{l}{g} \times \frac{A}{a}\omega^2 r$ as cos 0° = 1

2. When θ = 90°, $h_a = 0$ as cos 90° = 0

3. When θ = 180°, $h_a = -\frac{l}{g} \times \frac{A}{a}\omega^2 r$ as cos 180° = – 1

∴ Maximum pressure head due to acceleration

$$(h_a)_{max} = \frac{l}{g} \times \frac{A}{a} \omega^2 r \qquad ...(20.16)$$

▶ 20.7 EFFECT OF VARIATION OF VELOCITY ON FRICTION IN THE SUCTION AND DELIVERY PIPES

The velocity of water in suction or delivery pipe is given by equation (20.11) as

$$v = \frac{A}{a} \omega r \sin \omega t = \frac{A}{a} \omega r \sin \theta \qquad ...(i)$$

Loss of head due to friction in pipes is given by

$$h_f = \frac{4flv^2}{d \times 2g} \qquad ...(ii)$$

where f = Co-efficient of friction, l = Length of pipe,

d = Diameter of pipe, and v = Velocity of water in pipe.

Substituting equation (*i*) into equation (*ii*), we get

$$h_f = \frac{4fl}{d \times 2g} \times \left[\frac{A}{a} \omega r \sin \theta\right]^2 \qquad ...(20.17)$$

The variation of h_f with θ is parabolic. The loss of head due to friction in suction and delivery pipes is obtained from equation (20.17) by using subscripts '*s*' for suction pipe and '*d*' for delivery pipe as

$$h_{fs} = \frac{4fl_s}{d_s \times 2g} \times \left[\frac{A}{a_s} \omega r \sin \theta\right]^2 \qquad ...(20.18)$$

$$h_{fd} = \frac{4fl_d}{d_d \times 2g} \times \left[\frac{A}{a_d} \omega r \sin \theta\right]^2 \qquad ...(20.19)$$

The loss of head due to friction in pipes given by equation (20.17) varies with θ as :

1. When θ = 0°, $\sin \theta = 0$ ∴ $h_f = \frac{4fl}{d \times 2g} \times 0 = 0$

2. When θ = 90°, $\sin 90° = 1$ ∴ $h_f = \frac{4fl}{d \times 2g} \times \left[\frac{A}{a} \omega r\right]^2$

3. When θ = 180°, $\sin 180° = 0$ ∴ $h_f = 0$

∴ Maximum value of loss of head due to friction ;

$$(h_f)_{max} = \frac{4fl}{d \times 2g} \times \left[\frac{A}{a} \omega r\right]^2 \qquad ...(20.20)$$

Problem 20.3 *The cylinder bore diameter of a single-acting reciprocating pump is 150 mm and its stroke is 300 mm. The pump runs at 50 r.p.m. and lifts water through a height of 25 m. The delivery pipe is 22 m long and 100 mm in diameter. Find the theoretical discharge and the theoretical power required to run the pump. If the actual discharge is 4.2 litres/s, find the percentage slip. Also determine the acceleration head at the beginning and middle of the delivery stroke.*

Solution. Given :

Dia. of cylinder, $D = 150 \text{ mm} = 0.15 \text{ m}$

$\therefore$ Area, $A = \left(\frac{\pi}{4}\right) \times 0.15^2 = 0.01767 \text{ m}^2$

Stroke, $L = 300 \text{ mm} = 0.3 \text{ m}$

Speed of pump, $N = 50$ r.p.m.

Total height through which water is lifted,

$H = 25 \text{ m}$

Length of delivery pipe, $l_d = 22 \text{ m}$

Diameter of delivery pipe, $d_d = 100 \text{ mm} = 0.1 \text{ m}$

Actual discharge, $Q_{act} = 4.2 \text{ litres/s} = \frac{4.2}{1000} \text{ m}^3\text{/s} = 0.0042 \text{ m}^3\text{/s}.$

(*i*) *Theoretical discharge* (Q_{th})

Theoretical discharge for a single-acting reciprocating pump is given by equation (20.1), as

$$Q_{th} = \frac{A \times L \times N}{60} = \frac{0.01767 \times 0.3 \times 50}{60} = 0.0044175 \text{ m}^3\text{/s}$$

$= 0.0044175 \times 1000$ litres/s = **4.4175 litres/s. Ans.**

(*ii*) *Theoretical power* (P_t)

Theoretical power is given by, $P_t = \frac{(\text{Theoretical weight of water lifted/s}) \times \text{Total height}}{1000}$

$$= \frac{\rho \times g \times Q_{th} \times H}{1000}$$

$$= \frac{1000 \times 9.81 \times 0.0044175 \times 25}{1000} \qquad (\because Q_{th} = 0.0044175 \text{ m}^3\text{/s})$$

= **1.0833 kW. Ans.**

(*iii*) *The percentage slip*

The percentage slip is given by,

$$\% \text{ slip} = \left(\frac{Q_{th} - Q_{act}}{Q_{th}}\right) \times 100 = \left(\frac{4.4175 - 4.2}{4.4175}\right) \times 100 = \mathbf{4.92\%}. \textbf{ Ans.}$$

(*iv*) *Acceleration head at the beginning of delivery stroke.*

The acceleration head in the delivery pipe is given by equation (20.15) as :

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \times \cos\theta$$

where a_d = Area of delivery pipe = $\frac{\pi}{4} \times (0.1)^2 = 0.007854$

$$\omega = \text{Angular speed} = \frac{2\pi N}{60} = \frac{2\pi \times 50}{60} = 5.236$$

$$r = \text{Crank radius} = \frac{L}{2} = \frac{0.3}{2} = 0.15 \text{ m}$$

$$\therefore \qquad h_{ad} = \frac{22}{9.81} \times \frac{0.01767}{0.007854} \times 5.236^2 \times 0.15 \times \cos\theta = 20.75 \times \cos\theta$$

At the beginning of delivery stroke, $\theta = 0°$ and hence $\cos\theta = 1$

$\therefore \qquad h_{ad}$ = **20.75 m. Ans.**

(v) *Acceleration head at the middle of delivery stroke.*

At the middle of delivery stroke, $\theta = 90°$ and hence $\cos\theta = 0$.

$\therefore \qquad h_{ad} = 20.75 \times 0 =$ **0. Ans.**

▶ 20.8 INDICATOR DIAGRAM

The indicator diagram for a reciprocating pump is defined as the graph between the pressure head in the cylinder and the distance travelled by piston from inner dead centre for one complete revolution of the crank. As the maximum distance travelled by the piston is equal to the stroke length and hence the indicator diagram is a graph between pressure head and stroke length of the piston for one complete revolution. The pressure head is taken as ordinate and stroke length as abscissa.

20.8.1 Ideal Indicator Diagram. The graph between pressure head in the cylinder and stroke length of the piston for one complete revolution of the crank under ideal conditions is known as ideal indicator diagram. Fig. 20.4 shows the ideal indicator diagram, in which line *EF* represents the atmospheric pressure head equal to 10.3 m of water.

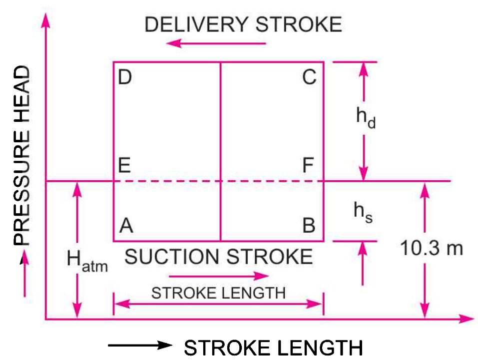


Fig. 20.4 *Ideal indicator diagram.*

Let H_{atm} = Atmospheric pressure head = 10.3 m of water,

L = Length of the stroke,

h_s = Suction head, and

h_d = Delivery head.

During suction stroke, the pressure head in the cylinder is constant and equal to suction head (h_s), which is below the atmospheric pressure head (H_{atm}) by a height of h_s. The pressure head during suction stroke is represented by a horizontal line *AB* which is below the line *EF* by a height of 'h_s'.

During delivery stroke, the pressure head in the cylinder is constant and equal to delivery head (h_d), which is above the atmospheric head by a height of (h_d). Thus, the pressure head during delivery stroke is represented by a horizontal line *CD* which is above the line *EF* by a height of h_d. Thus, for one complete revolution of the crank, the pressure head in the cylinder is represented by the diagram *A-B-C-D-A*. This diagram is known as ideal indicator diagram.

Now from equation (20.3), we know that the work done by the pump per second

$$= \frac{\rho \times g \times ALN}{60} \times (h_s + h_d)$$

$$= K \times L(h_s + h_d) \qquad \left(\text{where } K = \frac{\rho g AN}{60} = \text{Constant}\right)$$

$$\propto L \times (h_s + h_d) \qquad \ldots(i)$$

But from Fig. 20.4, area of indicator diagram

$$= AB \times BC = AB \times (BF + FC) = L \times (h_s + h_d).$$

Substituting this value in equation (*i*), we get

Work done by pump $\propto$ Area of indicator diagram. ...(20.21)

20.8.2 Effect of Acceleration in Suction and Delivery Pipes on Indicator Diagram.

The pressure head due to acceleration in the suction pipe is given by equation (20.14) as

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos \theta$$

When $\theta = 0°$, $\cos \theta = 1$, and $h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$

When $\theta = 90°$, $\cos \theta = 0$, and $h_{as} = 0$

When $\theta = 180°$, $\cos \theta = -1$, and $h_{as} = -\frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$.

Thus, the pressure head inside the cylinder during suction stroke will not be equal to 'h_s', as was the case for ideal indicator diagram, but it will be equal to the sum of 'h_s' and 'h_{as}'. At the beginning of suction stroke $\theta = 0°$, 'h_{as}' is +ve and hence the pressure head in the cylinder will be $(h_s + h_{as})$ below the atmospheric pressure head. At the middle of suction stroke $\theta = 90°$ and $h_{as} = 0$ and hence pressure head in the cylinder will be h_s below the atmospheric pressure head. At the end of suction stroke, $\theta = 180°$ and h_{as} is –ve and hence the pressure head in the cylinder will be $(h_s - h_{as})$ below the atmospheric pressure head. For suction stroke, the indicator diagram will be shown by $A'GB'$. Also the area of $A'AG$ = Area of BGB'.

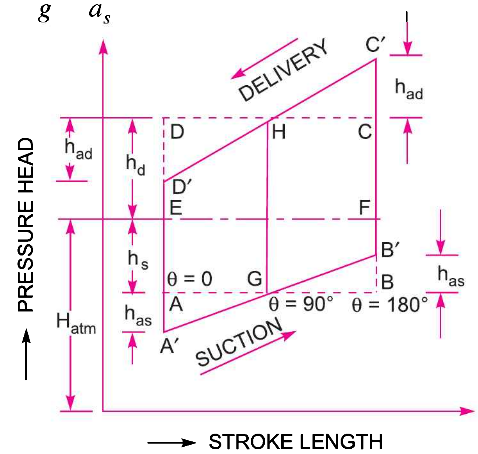


Fig. 20.5 *Effect of acceleration on indicator diagram.*

Similarly, the indicator diagram for the delivery stroke can be drawn. At the beginning of delivery stroke, h_{ad} is +ve and hence the pressure head in the cylinder will be $(h_d + h_{ad})$ above the atmospheric pressure head. At the middle of the delivery stroke, $h_{ad} = 0$ and hence pressure head in the cylinder is equal to h_d above the atmospheric pressure head. At the end of the delivery stroke, h_{ad} is –ve and hence pressure in the cylinder will be $(h_d - h_{ad})$ above the atmospheric pressure head. And thus the indicator diagram for delivery stroke is represented by the line $C'HD'$. Also, the area of $CC'H$ = Area of $DD'H$.

From Fig. 20.5, it is now clear that due to acceleration in suction and delivery pipe, the indicator diagram has changed from $ABCD$ to $A'B'C'D'$. But the area of indicator diagram $ABCD$ = Area $A'B'C'D'$. Now from equation (20.21), work done by pump is proportional to the area of indicator diagram. Hence the work done by the pump on the water remains same.

Problem 20.4 *The length and diameter of a suction pipe of a single-acting reciprocating pump are 5 m and 10 cm respectively. The pump has a plunger of diameter 15 cm and a stroke length of 35 cm. The centre of the pump is 3 m above the water surface in the pump. The atmospheric pressure head is 10.3 m of water and pump is running at 35 r.p.m. Determine :*

(*i*) *Pressure head due to acceleration at the beginning of the suction stroke,*
(*ii*) *Maximum pressure head due to acceleration, and*
(*iii*) *Pressure head in the cylinder at the beginning and at the end of the stroke.*

Solution. Given :

Length of suction pipe, $l_s = 5$ m

Diameter of suction pipe, $d_s = 10$ cm $= 0.1$ m

∴ Area, $a_s = \frac{\pi}{4} d_s^2 = \frac{\pi}{4} \times 0.1^2 = .007854 \text{ m}^2$

Diameter of plunger, $D = 15$ cm $= 0.15$ m

∴ Area of plunger, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times .15^2 = .01767 \text{ m}^2$

Stroke length, $L = 35$ cm $= 0.35$ m

∴ Crank radius, $r = \frac{L}{2} = \frac{0.35}{2} = 0.175$ m

Suction head, $h_s = 3$ m

Atmospheric pressure head, $H_{atm} = 10.3$ m of water

Speed of pump, $N = 35$ r.p.m.

Angular speed of the crank is given by,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 35}{60} = 3.665 \text{ rad/s.}$$

(*i*) The pressure head due to acceleration in the suction pipe is given by equation (20.14) as

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \cos\theta$$

At the beginning of stroke, $\theta = 0^o$ and hence $\cos\theta = 1$

∴ $$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r = \frac{5}{9.81} \times \frac{.01767}{.007854} \times 3.665^2 \times .175 = \mathbf{2.695 \text{ m. Ans.}}$$

(*ii*) Maximum pressure head due to acceleration in suction pipe is given by equation (20.16), as

$$(h_{as})_{\max} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r = \mathbf{2.695 \text{ m. Ans.}}$$

(*iii*) Pressure head in the cylinder at the beginning of the suction stroke (Refer to Fig. 20.5)

$= h_s + h_{as} = 3.0 + 2.695 = 5.695.$

This pressure head in the cylinder is below the atmospheric pressure head.

∴ Absolute pressure head in the cylinder at the beginning of suction stroke

= Atmospheric pressure head – 5.695

= 10.3 – 5.695 = **4.605 m of water (abs.) Ans.**

Similarly, pressure head in the cylinder at the end of suction stroke

$= h_s - h_{as} = 3.0 - 2.695 = 0.305$ m below atmospheric pressure head

= 10.3 – 0.305 = **9.995 m of water (abs.) Ans.**

Problem 20.5 *If in Problem 20.4, the length and diameter of the delivery pipe are 30 m and 10 cm respectively and water is delivered by the pump to a tank which is 20 m above the centre of the pump, determine :*

(*i*) *Pressure head due to acceleration at the beginning of delivery stroke,*
(*ii*) *Pressure head in the cylinder at the beginning of the delivery stroke, and*
(*iii*) *Pressure head in the cylinder at the end of the delivery stroke.*

Solution. Given :

Length of delivery pipe, $l_d = 30$ m

Diameter of delivery pipe, $d_d = 10$ cm $= 0.1$ m

∴ Area of delivery pipe, $a_d = \frac{\pi}{4} d_d^2 = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$

Diameter of plunger, $D = 15$ cm $= 0.15$ m

∴ Area of plunger, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(.15)^2 = .01767 \text{ m}^2$

Stroke length, $L = 35$ cm $= 0.35$ m

Crank radius, $r = \frac{L}{2} = \frac{0.35}{2} = 0.175$ m

Delivery head, $h_d = 20$ m

Speed of pump, $N = 35$ r.p.m.

Angular speed, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 35}{60} = 3.665$ rad/s.

(*i*) Using equation (20.15), we get the pressure head due to acceleration in delivery pipe as

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos\theta$$

At the beginning of delivery stroke, $\theta = 0^\circ$ and hence $\cos\theta = 1$.

∴ Pressure head due to acceleration at the beginning of delivery stroke becomes as

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r$$

$$= \frac{30}{9.81} \times \frac{.01767}{.007854} \times (3.665)^2 \times 0.175 = \textbf{16.17 m. Ans.}$$

(*ii*) From Fig. 20.5, the pressure head in the cylinder at the beginning of the delivery stroke

$= FC' = FC + CC' = (h_d + h_{ad})$ m of water above atmospheric head

$= 20 + 16.17 = 36.17$ m of water above atms.

$= 36.17$ + Atmospheric pressure head

$= 36.17 + 10.3 =$ **46.47 m (abs.) Ans.**

(*iii*) The pressure head in the cylinder at the end of delivery stroke

$= ED'$ above atmospheric pressure head

$= (ED - DD') = (h_d - h_{ad})$

$= 20 - 16.17 = 3.83$ m of water above atms.

$= 3.83 + 10.3 =$ **14.13 m (abs.) Ans.**

Problem 20.6 *A single-acting reciprocating pump has piston diameter 12.5 cm and stroke length 30 cm. The centre of the pump is 4 m above the water level in the sump. The diameter and length of suction pipe are 7.5 cm and 7 m respectively. The separation occurs if the absolute pressure head in the cylinder during suction stroke falls below 2.5 m of water. Calculate the maximum speed at which the pump can run without separation. Take atmospheric pressure head = 10.3 m of water.*

Solution. Given :

Diameter of piston, $D = 12.5 \text{ cm} = 0.125 \text{ m}$

∴ Area, $A = \frac{\pi}{4}(.125)^2 = .01227 \text{ m}^2$

Stroke length, $L = 30 \text{ cm} = 0.30 \text{ m}$

∴ Crank radius, $r = \frac{L}{2} = \frac{0.30}{2} = 0.15 \text{ m}$

Suction head, $h_s = 4.0 \text{ m}$

Diameter of suction pipe, $d_s = 7.5 \text{ cm} = 0.075 \text{ m}$

∴ Area of suction pipe, $a_s = \frac{\pi}{4}(.075)^2 = .004418 \text{ m}^2$

Length of suction pipe, $l_s = 7.0 \text{ m}$

Separation pressure head, $h_{sep} = 2.5 \text{ m (absolute)}$

Atmospheric pressure head, $H_{atm} = 10.3 \text{ m}$

From the indicator diagram, drawn in Fig. 20.5, it is clear that the absolute pressure head during suction stroke is minimum at the beginning of the stroke. Thus, the separation can take place at the beginning of the stroke only. In that case the pressure head in the cylinder at the beginning of stroke becomes = h_{sep}.

But pressure head in the cylinder at the beginning of suction stroke

$= (h_s + h_{as})$ m below atmospheric pressure head

$=$ Atmospheric pressure head $- (h_s + h_{as})$ m absolute

$= H_{atm} - (h_s + h_{as})$ m (abs.)

$= 10.3 - (4.0 + h_{as})$

∴ $h_{sep} = 10.3 - (4.0 + h_{as})$

$2.5 = 10.3 - 4.0 - h_{as}$

or $h_{as} = 10.3 - 4.0 - 2.5 = 3.80 \text{ m.}$ ...(*i*)

But from equation (20.14), h_{as} at the beginning of suction stroke is given by the relation

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s}\omega^2 r \qquad (\because \theta = 0^\circ, \because \cos\theta = 1) \quad ...(ii)$$

Equating equations (*i*) and (*ii*), we get

$$3.80 = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r = \frac{7.0}{9.81} \times \frac{.01227}{.004418} \times \omega^2 \times .15$$

∴ $$\omega^2 = \frac{3.80 \times 9.81 \times .004418}{7.0 \times .01227 \times .15} = 12.783$$

or $$\omega = \sqrt{12.783} = 3.575 \text{ radian/s.}$$

But $$\omega = \frac{2\pi N}{60}$$

$\therefore$ $$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 3.575}{2\pi} = \textbf{34.14 r.p.m. Ans.}$$

Thus, the maximum speed at which the pump can run without separation is 34.14 r.p.m.

Problem 20.7 *The diameter and stroke length of a single-acting reciprocating pump are 100 mm and 300 mm respectively. The water is lifted to a height of 20 m above the centre of the pump. Find the maximum speed at which the pump may be run so that no separation occurs during the delivery stroke if the diameter and length of delivery pipe are 50 mm and 25 m respectively. Separation occurs if the absolute pressure head in the cylinder during delivery stroke falls below 2.50 m of water.*

Take atmospheric pressure head = 10.3 m of water.

Solution. Given :

Diameter of pump, $D = 100$ mm $= 0.1$ m

Stroke length , $L = 300$ mm $= 0.30$ m

$\therefore$ Crank radius, $r = \frac{L}{2} = \frac{0.30}{2} = 0.15$ m

Delivery head, $h_d = 20$ m

Diameter of delivery pipe, $d_d = 50$ mm $= 0.05$ m

Length of delivery pipe, $l_d = 25$ m

Separation pressure head, $h_{sep} = 2.5$ m (abs.)

Atmospheric pressure head, $H_{atm} = 10.3$ m of water.

Fig. 20.6 show the indicator diagram for delivery stroke only. The absolute pressure head during delivery stroke is minimum at the end of the stroke only. It means, if separation is to take place, it will occur only at the end of the delivery stroke where pressure head will be equal to separation pressure head (h_{sep}). The absolute pressure head at the end of delivery stroke from Fig. 20.6 is equal to $D'M$.

where $D'M = DM - DD'$

$$= (DE + EM) - DD' \quad (\because DM = DE + EM)$$

$$= (h_d + H_{atm}) - h_{ad}$$

$\therefore$ $$h_{sep} = (h_d + H_{atm}) - h_{ad}$$

or $$2.5 = (20 + 10.3) - h_{ad}$$

$\therefore$ $$h_{ad} = (20 + 10.3) - 2.5 = 27.8 \text{ m}$$

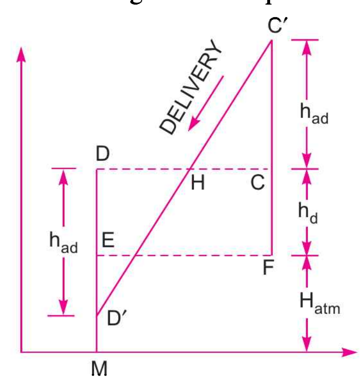


Fig. 20.6

But the acceleration head (h_{ad}) at the end of delivery stroke is given by

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r$$

$$27.8 = \frac{25}{9.81} \times \frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} d_d^2} \times \omega^2 \times 0.15 = \frac{25}{9.81} \times \frac{D^2}{d_d^2} \times \omega^2 \times .15$$

$$= \frac{25}{9.81} \times \left(\frac{0.1}{.05}\right)^2 \times \omega^2 \times .15 = 1.529\omega^2$$

$\therefore$ $$\omega = \sqrt{\frac{27.8}{1.529}} = 4.264 \text{ radians/s.}$$

But we know, $\omega = \frac{2\pi N}{60}$

$\therefore$ $$H = \frac{60 \times \omega}{2\pi} = \frac{60 \times 4.264}{2\pi} = \textbf{40.72 r.p.m. Ans.}$$

Problem 20.8 *A single-acting reciprocating pump raises water to a height of 20 m through a delivery pipe 35 m long and 140 mm in diameter. The bore and stroke of piston are 250 mm and 400 mm respectively. Cavitation occurs at 2.5 m of water absolute. Find the speed at which the pump can run without separation on delivery side if the pipe rises first vertically and then runs horizontally. Will there be any change in the maximum speed if the pipe first runs horizontally and then rises vertically.*

Solution. Given :

Delivery head, $h_d = 20$ m

Length of delivery pipe, $l_d = 35$ m

Dia. of delivery pipe, $d_d = 140$ mm $= 0.14$ m

Dia. of piston, $D = 250$ mm $= 0.25$ m

Stroke length, $L = 400$ mm $= 0.40$ m

$\therefore$ Crank radius, $r = \frac{L}{2} = \frac{0.40}{2} = 0.20$ m

Separation pressure head, $h_{sep} = 2.5$ m (abs.)

Atmospheric pressure head, $H_{atm} = 10.3$ m

The separation on delivery side can occur only at the end of delivery stroke as the pressure head during delivery stroke is minimum at the end of delivery stroke only. The acceleration head (h_{ad}) at the end of delivery stroke is given by,

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r = \frac{35}{9.81} \times \frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} d_d^2} \times \omega^2 \times 0.20$$

$$= \frac{35}{9.81} \times \frac{0.25^2}{0.14^2} \times \omega^2 \times 0.20.$$

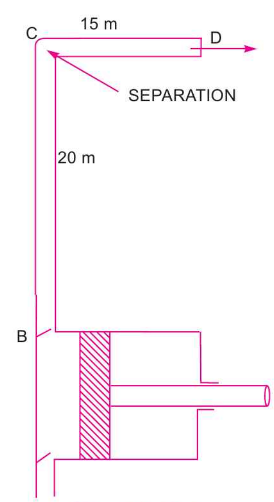


Fig. 20.6(*a*)

1st Case. The pipe rises first vertically and then horizontally as shown in Fig. 20.6 (*a*). In this case, the possibility of separation is at the point C at the end of the delivery stroke.

The pressure head at the end of delivery stroke at B will be equal to atmospheric pressure head plus delivery head minus acceleration head.

$\therefore$ Pressure head at $B = H_{atm} + h_d - h_{ad}$

The pressure head at C = Pressure head at $B - h_d$

$$= (H_{atm} + h_d - h_{ad}) - h_d = H_{atm} - h_{ad}$$

Now if separation is to take place at C, then the pressure head at C is 2.5 m.

Equating the two pressure heads at C, we get

$$2.5 = H_{atm} - h_{ad}$$

or $$h_{ad} = H_{atm} - 2.5 = 10.3 - 2.5 = 7.8 \text{ m}$$

or $$\frac{35}{9.81} \times \frac{0.25^2}{0.14^2} \times \omega^2 \times 0.20 = 7.8$$

or $$\omega = \sqrt{\frac{9.81 \times 0.14^2 \times 7.8}{35 \times 0.25^2 \times 0.20}} = 1.85 \text{ rad/s}$$

∴ $$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 1.85}{2\pi} = \textbf{17.68 r.p.m. Ans.}$$

2nd Case. The pipe first runs horizontally and then rises vertically as shown in Fig. 20.6 (*b*).

In this case, the possibility of separation is at the point C at the end of delivery stroke. But the pressure head at C is same as pressure head at B and C are in the horizontal plane. Hence, at the end of delivery stroke, the pressure head at B is equal to $H_{atm} + h_d - h_{ad}$.

For this condition

$$h_{sep} = H_{atm} + h_d - h_{ad}$$

or $$2.5 = 10.3 + 20 - h_{ad}$$

or $$h_{ad} = 10.3 + 20 - 2.5 = 27.8$$

or $$\frac{35}{9.81} \times \frac{0.25^2}{0.14^2} \times \omega^2 \times 0.20 = 27.8$$

or $$\omega = \sqrt{\frac{9.81 \times 0.14^2 \times 27.8}{35 \times 0.25^2 \times 0.20}} = 3.495$$

∴ $$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 3.495}{2\pi}$$

$$= \textbf{33.37 r.p.m. Ans.}$$

∴ Change in maximum speed

$$= 33.36 - 17.68 = \textbf{15.69 r.p.m. Ans.}$$

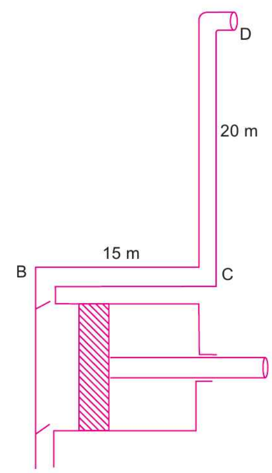


Fig. 20.6(*b*)

Problem 20.9 *A single-acting reciprocating pump has a plunger of 10 cm diameter and a stroke of length 200 mm. The centre of the pump is 4 m above the water level in the sump and 14 m below the level of water in a tank to which water is delivered by the pump. The diameter and length of suction pipe are 40 mm and 6 m while of the delivery pipe are 30 mm and 18 m respectively. Determine the maximum speed at which the pump may be run without separation, if separation occurs at 7.848 N/cm² below the atmospheric pressure. Take atmospheric pressure head = 10.3 m of water.*

Solution. Given :

Diameter of plunger, $D = 100 \text{ mm} = 0.10 \text{ m}$

Stroke length, $L = 200 \text{ mm} = 0.20 \text{ m}$

∴ Crank radius, $r = \frac{L}{2} = \frac{0.20}{2} = 0.10 \text{ m}$

Suction head, $h_s = 4 \text{ m}$

Delivery head, $h_d = 14 \text{ m}$

Dia. of suction pipe, $d_s = 40 \text{ mm} = 0.04 \text{ m}$

Length of suction pipe, $l_s = 6 \text{ m}$

Dia. of delivery pipe, $d_d = 30 \text{ mm} = .03 \text{ m}$

Length of delivery pipe, $l_d = 18 \text{ m}$

Separation pressure, $p_{sep} = \frac{7.848\ N}{\text{cm}^2} = \frac{7.848 \times 10^4}{\text{m}^2}$

$\therefore$ Separation pressure head, $h_{sep} = \frac{p_{sep}}{\rho g} = \frac{7.848 \times 10^4}{1000 \times 9.81} = 8.0 \text{ m below atmosphere}$

$= (H_{atm} - 8.0) \text{ absolute} = (10.3 - 8.0) = 2.3 \text{ m (abs.)}$

where H_{atm} = Atmospheric pressure head = 10.3 m.

(*i*) **Speed of pump without separation during suction stroke.** During suction stroke, possibility of separation is only at the beginning of the stroke. The pressure head in the cylinder at the beginning of suction stroke

$= (h_s + h_{as})$ m below atmospheric pressure head

$= 10.3 - (h_s + h_{as})$ m (abs.)

$\therefore \quad h_{sep} = 10.3 - (h_s + h_{as})$

or $\quad 2.3 = 10.3 - (h_s + h_{as}) = 10.3 - 4 - h_{as}$

$\therefore \quad h_{as} = 10.3 - 4 - 2.3 = 4 \text{ m.}$ ...(*i*)

But 'h_{as}' at the beginning of suction stroke is given by,

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r = \frac{6}{9.81} \times \frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} d_s^2} \times \omega^2 \times .10$$

$$= \frac{6}{9.81} \times \left(\frac{0.1}{.04}\right)^2 \times \omega^2 \times .1 = 0.3822\ \omega^2 \quad \text{...(ii)}$$

Equating the values of h_{as} given by equations (*i*) and (*ii*),

$$4 = 0.3822\ \omega^2$$

$$\therefore \quad \omega = \sqrt{\frac{4}{.382}} = 3.235 \text{ rad/s}$$

But ω is also $= \frac{2\pi N}{60}$

$$\therefore \quad N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 3.235}{2\pi} = 30.89 \text{ r.p.m.}$$

$\therefore$ Maximum speed of the pump without separation during suction stroke only is 30.89 r.p.m.

(*ii*) **Speed of pump without separation during delivery stroke.** During delivery stroke, the possibility of separation is only at the end of the delivery stroke. The pressure head in the cylinder at the end of the delivery stroke from Fig. 20.6

$$= (H_{atm} + h_d) - h_{ad} \text{ m (abs.)} = (10.3 + 14) - h_{ad}.$$

If separation is to be avoided this pressure should be equal to separation pressure head.

$\therefore$ $$h_{sep} = (10.3 + 14) - h_{ad} \text{ or } 2.30 = (10.3 + 14) - h_{ad}$$

$\therefore$ $$h_{ad} = 10.3 + 14.0 - 2.30 = 22.0 \text{ m}$$

But 'h_{ad}' is given by the relation

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 \times r$$

$\therefore$ $$22.0 = \frac{18.0}{9.81} \times \frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} d_d^2} \times \omega^2 \times r = \frac{18.0}{9.81} \times \frac{D^2}{d_d^2} \times \omega^2 \times r$$

$$= \frac{18.0}{9.81} \times \left(\frac{.1}{.03}\right)^2 \times \omega^2 \times 0.10 = 2.04\, \omega^2$$

$\therefore$ $$\omega = \sqrt{\frac{22.0}{2.04}} = 3.284 \text{ rad/s.}$$

But $$\omega = \frac{2\pi N}{60}$$

$\therefore$ $$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 3.284}{2 \times \pi} = 31.36 \text{ r.p.m.}$$

$\therefore$ Maximum speed of the pump without separation during delivery stroke is 31.36 r.p.m.

Thus the maximum speed of the pump without separation during suction and delivery stroke is the minimum of these two speeds, *i.e.*, minimum of 30.89 and 31.36 r.p.m.

$\therefore$ Maximum speed = **30.89 r.p.m. Ans.**

20.8.3 Effect of Friction in Suction and Delivery Pipes on Indicator Diagram. The loss of head due to friction in suction and delivery pipes is given by equations (20.18) and (20.19) as

$$h_{fs} = \frac{4fl_s}{d_s \times 2g} \times \left(\frac{A}{a_s} \omega r \sin\theta\right)^2 \text{ and } h_{fd} = \frac{4fl_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \omega r \sin\theta\right)^2$$

It is clear from the above equations that the variation of h_{fs} or h_{fd} is parabolic with θ.

During the suction or delivery stroke, the pressure head inside the cylinder will change as given below :

(*i*) At the beginning of the suction or delivery stroke, $\theta = 0°$ and hence $\sin\theta = 0$. This means h_{fs} and $h_{fd} = 0$.

(*ii*) At the middle of the suction or delivery stroke, $\theta = 90°$ and hence $\sin\theta = 1$. This means

$$h_{fs} = \frac{4fl_s}{d_s \times 2g} \times \left(\frac{A}{a_s} \omega r\right)^2 \text{ and } h_{fd} = \frac{4fl_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \omega r\right)^2.$$

(*iii*) At the end of the suction or delivery stroke, $\theta = 180°$ and hence $\sin\theta = 0°$. This means h_{fs} and $h_{fd} = 0$.

As the variation of h_{fs} or h_{fd} with θ is parabolic in nature, the indicator diagram during suction and delivery stroke with friction in suction pipe and delivery pipe will be as shown in Fig. 20.7.

The area of the parabolas *AGB* and *CHD* represents the work done against friction in suction and delivery pipes.

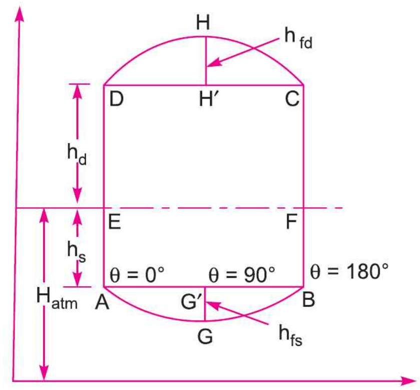


Fig. 20.7 *Effect of friction on indicator diagram.*

Now $$\text{area } AGB = AB \times \frac{2}{3} GG'$$

$$= AB \times \frac{2}{3} h_{fs} = L \times \frac{2}{3} h_{fs}$$

where $$h_{fs} = \frac{4 \times f \times l_s}{d_s \times 2g} \times \left(\frac{A}{a_s}\omega r\right)^2$$

Similarly, $$\text{area } CHD = CD \times \frac{2}{3} \times HH' = CD \times \frac{2}{3} h_{fd}$$

$$= L \times \frac{2}{3} h_{fd} \qquad (\because \; CD = L = \text{Length of stroke})$$

where $$h_{fd} = \frac{4 f l_d}{d_d \times 2g} \times \left(\frac{A}{a_d}\omega r\right)^2.$$

20.8.4 Effect of Acceleration and Friction in Suction and Delivery Pipes on Indicator Diagram. Fig. 20.8 shows the combined effect of acceleration and friction in suction and delivery pipes. The pressure head in the cylinder during suction and delivery strokes will change as given below :

(*i*) At the beginning of the suction stroke, $\theta = 0°$ and hence h_{as} from equation (20.14) is equal to $\frac{l_s}{g} \times \frac{A}{a_s}\omega^2 r$. But $h_{fs} = 0$. Thus, the pressure head in the cylinder will be $(h_s + h_{as})$ below the atmospheric pressure head.

(*ii*) At the middle of the suction stroke, $h_{as} = 0$ but $h_{fs} = \frac{4 \times f \times l_s}{d_s \times 2g} \times \left(\frac{A}{a_s}\omega r\right)^2$. Thus, the pressure head in the cylinder will be $(h_s + h_{fs})$ below the atmospheric pressure head.

(*iii*) At the end of the suction stroke, $h_{as} = -\frac{l_s}{g} \times \frac{A}{a_s}\omega^2 r$ but $h_{fs} = 0$. Thus, the pressure head in the cylinder will be $(h_s - h_{as})$ below the atmospheric pressure head.

(*iv*) At the beginning of the delivery stroke, $h_{ad} = -\frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r$ but $h_{fd} = 0$. Thus, the pressure head in the cylinder will be $(h_d + h_{ad})$ above the atmospheric pressure head.

(*v*) In the middle of the delivery stroke, $h_{ad} = 0$ and $h_{fd} = \frac{4fl_d}{d_d \times 2g} \times \left(\frac{A}{a_d}\omega r\right)^2$. Thus the pressure head in the cylinder will be $(h_d + h_{fd})$ above the atmospheric pressure head.

(*vi*) At the end of the delivery stroke, $h_{ad} = -\frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r$ and $h_{fd} = 0$. Thus, the pressure head in the cylinder will be $(h_d - h_{ad})$ above the atmospheric pressure head.

Thus, the indicator diagram with acceleration and friction in suction and delivery pipes will become as shown in Fig. 20.8.

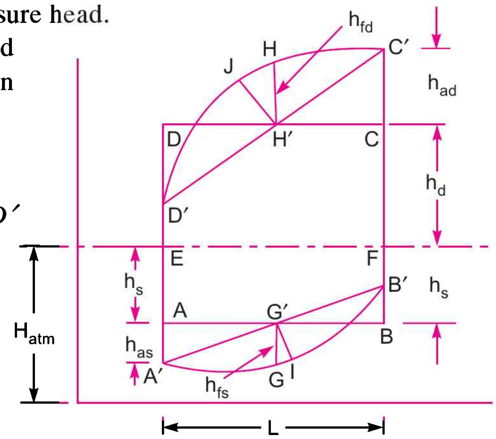


Fig. 20.8 *Effect of acceleration and friction on indicator diagram.*

Area of the indicator diagram $A'GB'C'HD'$

$$= \text{Area of } A'G'B'C'H'D' + \text{Area of parabola } A'GB' + \text{Area of parabola } C'HD'$$

But area of $A'G'B'C'H'D'$

$= \text{Area of } ABCD$

$= (h_s + h_d) \times L$

Area of parabola $A'GB'$

$$= A'B' \times \frac{2}{3} \times G'I = \frac{2}{3} \times (A'B' \times G'I)$$

$$= \frac{2}{3} \times (AB \times GG') = \frac{2}{3} \times L' h_{fs}$$

Similarly, area of parabola $C'HD'$

$$= C'D' \times \frac{2}{3} H'J = \frac{2}{3}(C'D' \times H'J)$$

$$= \frac{2}{3} \times (CD \times H'H) = \frac{2}{3}(L \times h_{fd}) = \frac{2}{3} L \times h_{fd}$$

∴ Area of indicator diagram

$$= (h_s + h_d) \times L + \frac{2}{3} \times L \times h_{fs} + \frac{2}{3} \times L \times h_{fd}$$

$$= \left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right) \times L$$

But from equation (20.21), we know that work done by pump is proportional to the area of the indicator diagram.

∴ Work done by pump per second $\propto \left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right) \times L$

$$= KL\left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right)$$

where K = a constant of proportionality

$$= \frac{\rho g A N}{60} \qquad \text{...for a single-acting}$$

$$= \frac{2\rho gAN}{60} \qquad \text{...for a double-acting}$$

∴ Work done by pump per second for a single-acting

$$= \frac{\rho gALN}{60}\left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right) \qquad \text{...(20.22)}$$

and for a double-acting

$$= \frac{2\rho gALN}{60}\left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right) \qquad \text{...(20.23)}$$

Problem 20.10 *A single-acting reciprocating pump has a stroke length of 15 cm. The suction pipe is 7 metre long and the ratio of the suction diameter to the plunger diameter is 3/4. The water level in the sump is 2.5 metres below the axis of the pump cylinder, and the pipe connecting the sump and pump cylinder is 7.5 cm diameter. If the crank is running at 75 r.p.m., determine the pressure head on the piston :*

(*i*) *in the beginning of the suction stroke,* (*ii*) *in the end of the suction stroke, and*

(*iii*) *in the middle of the suction stroke.*

Take co-efficient of friction as 0.01.

Solution. Given :

Stroke length, $L = 15 \text{ cm} = 0.15 \text{ m}$

∴ Crank radius, $r = \frac{L}{2} = \frac{0.15}{2} = 0.075 \text{ m}$

Length of suction pipe, $l_s = 7.0 \text{ m}$

$$\frac{\text{Suction pipe diameter}}{\text{Plunger diameter}} = \frac{d_s}{D} = \frac{3}{4}$$

$$\therefore \quad \frac{\text{Area of suction pipe}}{\text{Area of plunger}} = \frac{a_s}{A} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Suction head, $h_s = 2.5$

Diameter of suction pipe, $d_s = 7.5 \text{ cm} = 0.075 \text{ m}$

Crank speed, $N = 75 \text{ r.p.m.}$

∴ Angular speed, $\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 75}{60} = 2.5\,\pi \text{ rad/s.}$

Friction co-efficient, $f = 0.01$

The pressure head due to acceleration in suction pipe is given by equation (20.14), as

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 \times r\cos\theta$$

$$= \frac{7.0}{9.81} \times \frac{16}{9} \times (2.5\,\pi)^2 \times 0.075\cos\theta \qquad \left(\because \frac{A}{a_s} = \frac{16}{9}\right)$$

$$= 5.87\cos\theta$$

The loss of head due to friction in suction pipe is given by equation (20.18)as

$$h_{fs} = \frac{4fl_s}{d_s \times 2g} \times \left(\frac{A}{a_s}\omega r\sin\theta\right)^2$$

$$= \frac{4 \times 0.01 \times 7.0}{0.075 \times 2 \times 9.81} \times \left(\frac{16}{9} \times 2.5\pi \times 0.075 \times \sin\theta\right)^2 = 0.208 \sin^2\theta.$$

(*i*) *Pressure head on the piston in the beginning of suction stroke :*

(Refer to Fig. 20.8). At the beginning of suction stroke, $\theta = 0^o$ and pressure head

$= (h_s + h_{as})$ m below atmospheric pressure head

$= 2.5 + 5.87 =$ **8.37 m vacuum. Ans.**

(*ii*) *Pressure head on the piston at the end of suction stroke :*

At the end of the suction stroke, $\theta = 180^o$ and hence $\cos\theta$

$= -1$ and $\sin\theta = 0$

The pressure head $= (h_s - h_{as})$ m below atmospheric pressure head

$= H_{atm} - (h_s - h_{as})$ m abs. $= H_{atm} - (2.5 - 5.87)$ m abs.

$= H_{atm} - (-3.37)$ m abs. $= H_{atm} + 3.37$ m abs. $=$ **3.37 m (gauge). Ans.**

(*iii*) *Pressure head on the piston in the middle of suction stroke :*

In the middle of suction stroke, $\theta = 90°$ and hence $\cos\theta = 0$ and $\sin\theta = 1$. The pressure head

$= (h_s + h_{fs})$ m below atmospheric pressure head

$= (2.5 + 0.208)$ m vacuum $=$ **2.708 m vacuum. Ans.**

Problem 20.11 *The diameter and stroke length of a single-acting reciprocating pump are 12 cm and 20 cm respectively. The lengths of suction and delivery pipes are 8 m and 25 m respectively and their diameters are 7.5 cm. If the pump is running at 40 r.p.m. and suction and delivery heads are 4 m and 14 m respectively, find the pressure head in the cylinder :*

(*i*) *at the beginning of the suction and delivery stroke,*

(*ii*) *in the middle of suction and delivery stroke, and*

(*iii*) *at the end of the suction and delivery stroke.*

Take atmospheric pressure head = 10.30 metres of water and f = .009 for both pipes.

Solution. Given :

Diameter of cylinder, $D = 12$ cm $= 0.12$ m

Stroke length, $L = 20$ cm $= 0.20$ m

∴ Crank radius, $r = \frac{L}{2} = \frac{.20}{2} = 0.10$ m

Length of suction pipe, $l_s = 8$ m

Length of delivery pipe, $l_d = 25$ m

Dia. of suction pipe, $d_s = 7.5$ cm $= 0.075$ m

Dia. of delivery pipe, $d_d = 7.5$ cm $= 0.075$ m

Speed of pump, $N = 40$ r.p.m.

Suction head, $h_s = 4$ m

Delivery head, $h_d = 14$ m

Atmospheric pressure head, $H_{atm} = 10.3$ m of water

Co-efficient of friction, $f = .009$.

We know that angular velocity, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60} = 4.188$ rad/s.

Using equation (20.14), the pressure head due to acceleration in suction pipe is obtained as

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \cos\theta = \frac{8}{9.81} \times \frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} d_s^2} \times 4.188^2 \times 0.1 \times \cos\theta$$

$$= \frac{8}{9.81} \times \left(\frac{D}{d_s}\right)^2 \times 4.188^2 \times 0.1 \times \cos\theta$$

$$= \frac{8}{9.81} \times \left(\frac{.120}{.075}\right)^2 \times 4.188^2 \times 0.1 \times \cos\theta = 3.66 \times \cos\theta \text{ m.}$$

Similarly, the pressure head due to acceleration in delivery pipe is obtained from equation (20.15) as

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r \cos\theta$$

$$= \frac{25}{9.81} \times \frac{D^2}{d_d^2} \times 4.188^2 \times 0.1 \times \cos\theta$$

$$= \frac{25}{9.81} \times \left(\frac{.120}{.075}\right)^2 \times 4.188^2 \times 0.1 \times \cos\theta = 11.44 \times \cos\theta \text{ m.}$$

Using equation (20.18) for the loss of head due to friction in suction pipe,

$$h_{fs} = \frac{4 f l_s}{d_s \times 2g} \times \left(\frac{A}{a_s} \omega r \sin\theta\right)^2$$

$$= \frac{4 \times .009 \times 8}{.075 \times 2 \times 9.81} \times \left(\frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} d_s^2} \times 4.188 \times .1 \times \sin\theta\right)^2$$

$$= \frac{4 \times .009 \times 8}{.075 \times 2 \times 9.81} \times \left(\frac{.12^2}{.075^2} \times 4.188 \times .1\right)^2 \times \sin^2\theta = 0.225 \sin^2\theta.$$

Similarly, loss of head due to friction in delivery pipe is obtained from equation (20.19) as

$$h_{fd} = \frac{4 \times f \times l_d}{d_d \times 2g} \times \left[\frac{A}{a_d} \omega r \sin\theta\right]^2$$

$$= \frac{4 \times .009 \times 25}{0.075 \times 2 \times 9.81} \left[\frac{D^2}{d_d^2} \times 4.188 \times 0.1 \times \sin\theta\right]^2$$

$$= 0.6116 \left[\frac{.12^2}{.075^2} \times 4.188 \times 0.1\right]^2 \times \sin^2\theta = 0.703 \sin^2\theta.$$

(*i*) **Pressure head in the cylinder at the beginning of suction and delivery strokes (Fig. 20.8).** At the beginning of suction and delivery strokes, $\theta = 0°$.

$\therefore \quad h_{as} = 3.66 \cos 0° = 3.66$ m

and $\quad h_{ad} = 11.44 \cos 0° = 11.44$ m

Pressure head in the cylinder at the beginning of suction stroke

$= (h_s + h_{as})$ below atmospheric pressure head

$= H_{atm} - (h_s + h_{as})$ m (abs.)

$= 10.3 - (1 + 3.66) = 10.3 - 7.66 =$ **2.64 m (abs.) Ans.**

Pressure head in the cylinder at the beginning of delivery stroke

$= (h_d + h_{ad})$ above atmospheric pressure head

$= H_{atm} + (h_d + h_{ad})$ m (abs.)

$= 10.3 + (14 + 11.44) =$ **35.74 m (abs.) Ans.**

(*ii*) **Pressure head in the cylinder in the middle of suction and delivery strokes.** In the middle of suction and delivery strokes, $\theta = 90°$.

$\therefore \quad h_{as} = 0, h_{ad} = 0, h_{fs} = 0.225 \sin^2 90 = 0.225$ m and

$h_{fd} = 0.703 \sin^2 90 = 0.703$ m.

Pressure head in the cylinder at the beginning of suction stroke

$= (h_s + h_{fs})$ below the atmospheric pressure head

$= H_{atm} - (h_s + h_{fs})$ m (abs.)

$= 10.3 - (4 + 0.225) = 10.3 - 4.225 =$ **6.075 m (abs.) Ans.**

Pressure head in the cylinder at the beginning of delivery stroke

$= (h_d + h_{fd})$ above atmospheric pressure head

$= H_{atm} + (h_d + h_{fd})$ m (abs.)

$= 10.3 + (14 + 0.703) =$ **125.003 m (abs.) Ans.**

(*iii*) **Pressure head in the cylinder at the end of suction and delivery strokes.** At the end of suction and delivery strokes, $\theta = 180°$.

$\therefore \quad \cos\theta = -1$ and $\sin\theta = 0$

$\therefore \quad h_{as} = 3.66 \times (-1) = -3.66$ m

and $\quad h_{ad} = 11.44 \times (-1) = -11.44$ m.

Pressure head in the cylinder at the end of suction stroke

$= (h_s - 3.66)$ below the atmospheric pressure head

$= H_{atm} - (h_s - 3.66)$ m (abs.) $= 10.3 - (4 - 3.66) =$ **9.96 m (abs.) Ans.**

Pressure head in the cylinder at the end of delivery stroke

$= (h_d - 11.44)$ above the atmospheric pressure head

$= H_{atm} + (h_d - 11.44)$ m (abs.)

$= 10.3 + (14 - 11.44) =$ **12.86 m (abs.) Ans.**

Problem 20.12 *For the single-acting reciprocating pump, given in Problem 20.11, find the power required to drive the pump, if water is flowing through the pump.*

Solution. Using equation (20.22) for the work done by the pump per second for a single-acting, we get

$$\text{Work done per sec} = \frac{\rho gALN}{60}\left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right)$$

where $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times .12^2 = 0.01131 \text{ m}^2$

L = Stroke length = 0.20 m, N = Speed = 40 r.p.m.

ρ = Density of water = 1000 kg/m^3, h_s = 4 m, h_d = 14 m

h_{fs} = Maximum loss of head due to friction in suction pipe = 0.225 m

h_{fd} = Maximum loss of head due to friction in delivery pipe = 0.703 m

$$\therefore \quad \text{Work done per second} = \frac{1000 \times 9.81 \times 0.01131 \times 0.20 \times 40}{60}\left(4 + 14 + \frac{2}{3} \times .225 + \frac{2}{3} \times .703\right)$$

$$= 14.793 \times (4 + 14 + 0.15 + 0.468) = 275.42 \text{ Nm/s}$$

$\therefore$ Power required to drive the pump in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{275.42}{1000} = 0.2754 \text{ kW.}$$

20.8.5 Maximum Speed of a Reciprocating Pump. Maximum speed of a reciprocating pump is determined from the fact that the pressure in the cylinder during suction and delivery stroke, should not fall below the vapour pressure of the liquid, flowing through suction and delivery pipe. If the pressure in the cylinder is below the vapour pressure, the dissolved gases will be liberated from the liquid and cavitation * will take place. Also the continuous flow of liquid will not exist which means separation of liquid will take place. The pressure at which separation takes place is known as separation pressure and the head corresponding to separation pressure is called separation pressure head. It is denoted by h_{sep}. For water, the limiting value of separation pressure head (h_{sep}) is 7.8 below atmospheric pressure head or 10.3 – 7.8 = 2.5 m abs. The separation may take place during the suction stroke or during delivery stroke. The maximum speed of the reciprocating pump during suction and delivery strokes is calculated as :

(a) **Maximum Speed during Suction Stroke.** From the indicator diagram, drawn in Fig. 20.8, it is clear that the absolute pressure head during suction stroke is minimum at the beginning of the stroke. Thus, the separation can take place at the beginning of the stroke only. In that case, the abs. pressure head in the cylinder at the beginning of the stroke will be equal to separation pressure head (h_{sep}).

$$\therefore \quad h_{sep} = H_{atm} - (h_s + h_{as}) \text{ (abs.)}$$

$$\text{or} \quad h_{as} = H_{atm} - h_s - h_{sep} \qquad ...(i)$$

Generally, the values of h_{sep} and h_s (suction head) are given and hence 'h_{as}', the pressure head due to acceleration at the beginning of suction stroke can be obtained. The value of 'h_{as}' is also given by equation (20.14) as

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \qquad ...(ii)$$

Equating the two values of h_{as} given by equations (*i*) and (*ii*)

$$H_{atm} - h_s - h_{sep} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \qquad ...(iii)$$

* Please refer to Art. 19.11 on page 980 for definition.

From equation (*iii*), the value of the ω (and hence speed N) can be obtained. This speed is the maximum speed of the reciprocating pump without separation during suction stroke.

(b) **Maximum Speed during Delivery Stroke.** During delivery stroke, the probability of separation is only at the end of the delivery stroke. The pressure head in the cylinder at the end of the delivery stroke from Fig. 20.8.

$$= (H_{atm} + h_d) - h_{ad} \text{ m (abs.)}$$

If separation is to be avoided, the above pressure head should be more than the separation pressure head (h_{sep}). In the limiting case

$$h_{sep} = (H_{atm} + h_d) - h_{ad} \text{ or } h_{ad} = (H_{atm} + h_d) - h_{sep}$$

But 'h_{ad}', the pressure head due to acceleration at the end of the delivery stroke is also given by

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 \times r$$

Equating the two values of h_{ad}, we get

$$(H_{atm} + h_d) - h_{sep} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 \times r \quad ...(iv)$$

From the above equation (*iv*), the value of ω and hence speed N, can be calculated. This is the maximum speed of the reciprocating pump without separation during delivery stroke only.

The minimum of the two speeds given by above two cases (*a*) and (*b*) is the maximum speed of the reciprocating pump without separation during suction and delivery strokes.

Problem 20.13 *Find the maximum speed of a single-acting reciprocating pump to avoid separation, which occurs at 3.0 m of water (abs.) The pump has a cylinder of diameter 10 cm and a stroke length of 20 cm.The pump draws water from a sump and delivers to a tank. The water level in the sump is 3.5 m below the pump axis and in the tank the water level is 13 m above the pump axis. The diameter and length of the suction pipe are 4 cm and 5 m while of delivery pipe the diameter and length are 3 cm, 20 m respectively. Take atmospheric pressure head = 10.3 m of water.*

Solution. Given :

Separation pressure head, $h_{sep} = 3.0$ m of water (abs.)

Dia. of cylinder, $D = 10$ cm $= 0.10$ m

Stroke length, $L = 20$ cm $= 0.20$ m

$\therefore$ Crank radius, $r = \frac{L}{2} = \frac{0.20}{3} = 0.10$ m

Suction head, $h_s = 3.5$ m

Delivery head, $h_d = 13$ m

Dia. of suction pipe, $d_s = 4$ cm $= .04$ m

Length of suction pipe, $l_s = 5$ m

Dia. of delivery pipe, $d_d = 3$ cm $= .03$ m

Length of delivery pipe, $l_d = 20$ m

Atmos. pressure head, $H_{atm} = 10.3$ m.

(*a*) Maximum speed during suction stroke without separation is obtained from the relation, given by equation (*iii*),

$$H_{atm} - h_s - h_{sep} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r$$

or $$10.3 - 3.5 - 3.0 = \frac{5}{9.81} \times \frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} d_s^2} \times \omega^2 \times 0.10 \qquad \left(\because \; A = \frac{\pi}{4} D^2, a_s = \frac{\pi}{4} d_s^2\right)$$

or $$3.8 = \frac{5}{9.81} \times \frac{.1 \times .1}{.04 \times .04} \times \omega^2 \times 0.1 = .3185\ \omega^2$$

$\therefore$ $$\omega = \sqrt{\frac{3.8}{.3185}} = 3.454 \text{ rad/s.}$$

But $$\omega = \frac{2\pi N}{60} \quad \therefore \quad \frac{2\pi N}{60} = 3.454$$

$\therefore$ $$N = \frac{60 \times 3.454}{2\pi} = 32.98 \text{ r.p.m.} \qquad ...(i)$$

(*b*) Maximum speed during delivery stroke without separation is obtained from equation (*iv*),

$$(H_{atm} + h_d) - h_{sep} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 \times r$$

$$(10.3 + 13.0) - 3.0 = \frac{20}{9.81} \times \frac{D^2}{d_d^2} \times \omega^2 \times .1 = \frac{20}{9.81} \times \frac{0.1 \times .1}{.03 \times .03} \times \omega^2 \times .1$$

or $$20.3 = 2.265\ \omega^2$$

$\therefore$ $$\omega = \sqrt{\frac{20.3}{2.265}} = 2.994 \text{ rad/s.}$$

But $$\omega = \frac{2\pi N}{60} = 2.994$$

$\therefore$ $$N = \frac{60 \times 2.994}{2\pi} = 28.59 \text{ r.p.m.} \qquad ...(ii)$$

The minimum of the two speeds given by equations (*i*) and (*ii*) is the maximum speed of the pump, without separation.

$\therefore$ Maximum speed without separation = **28.59 r.p.m. Ans.**

▶ 20.9 AIR VESSELS

An air vessel is a closed chamber containing compressed air in the top portion and liquid (or water) at the bottom of the chamber. At the base of the chamber there is an opening through which the liquid (or water) may flow into the vessel or out from the vessel. When the liquid enters the air vessel, the air gets compressed further and when the liquid flows out the vessel, the air will expand in the chamber.

An air vessel is fitted to the suction pipe and to the delivery pipe at a point close to the cylinder of a single-acting reciprocating pump :

(*i*) to obtain a continuous supply of liquid at a uniform rate,

(*ii*) to save a considerable amount of work in overcoming the frictional resistance in the suction and delivery pipes, and (*iii*) to run the pump at a high speed without separation.

Fig. 20.9 shows the single-acting reciprocating pump to which air vessels are fitted to the suction and delivery pipes. The air vessels act like an intermediate reservoir. During the first half of the suction stroke, the piston moves with acceleration, which means the velocity of water in the suction pipe is more than the mean velocity and hence the discharge of water entering the cylinder will be more than the mean discharge. This excess quantity of water will be supplied from the air vessel to the cylinder in such a way that the velocity in the suction pipe below the air vessel is equal to mean velocity of flow. During the second half of the suction stroke, the piston moves with retardation and hence velocity of flow in the suction pipe is less than the mean velocity of flow. Thus, the discharge entering the cylinder will be less than the mean discharge. The velocity of water in the suction pipe due to air vessel is equal to mean velocity of flow and discharge required in cylinder is less than the mean discharge. Thus, the excess water flowing in suction pipe will be stored into air vessel, which will be supplied during the first half of the next suction stroke.

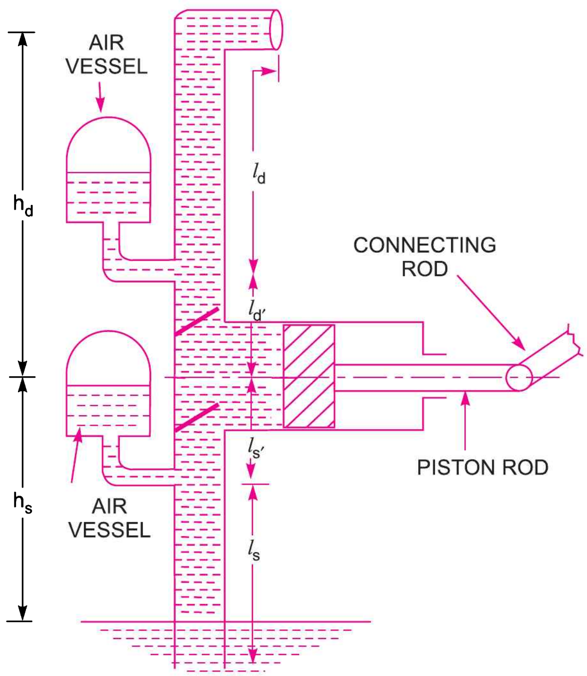


Fig. 20.9 *Air vessels fitted to reciprocating pump.*

When the air vessel is fitted to the delivery pipe, during the first half of delivery stroke, the piston moves with acceleration and forces the water into the delivery pipe with a velocity more than the mean velocity. The quantity of water in excess of the mean discharge will flow into the air vessel. This will compress the air inside the vessel. During the second half of the delivery stroke, the piston moves with retardation and the velocity of water in the delivery pipe will be less than the mean velocity. The water already stored into the air vessel will start flowing into the delivery pipe and the velocity of flow in the delivery pipe beyond the point to which air vessel is fitted will become equal to the mean velocity. Hence, the rate of flow of water in the delivery pipe will be uniform.

Let A = Cross-sectional area of the cylinder,

a = Cross-sectional area of suction or delivery pipe,

l_d = Length of delivery pipe beyond the air vessel,
l_d' = Length of delivery pipe between cylinder and air vessel,
l_s' = Length of suction pipe between cylinder and air vessel,
l_s = Length of suction pipe below air vessel,
h_{ad} = Pressure head due to acceleration in delivery pipe,
h_{as} = Pressure head due to acceleration in suction pipe,
h_{fd} = Loss of head due to friction in delivery pipe beyond the air vessel,
h_{fd}' = Loss of head due to friction in delivery pipe between cylinder and air vessel,
h_{fs} = Loss of head due to friction in suction pipe below the air vessel, and
h_{fs}' = Loss of head due to friction in suction pipe between cylinder and air vessel.

The effect of acceleration will be observed only in the lengths l_d' and l_s' which may be made very small by fitting air vessels very close to the cylinder. The velocity of flow of water in the length l_d and l_s will be equal to mean velocity of flow.

For a single-acting, discharge per second is given by equation (20.1), as

$$Q = \frac{ALN}{60}, \qquad \text{where } L = \text{Length of stroke}$$

$\therefore$ Mean velocity,

$$\bar{V} = \frac{\text{Discharge}}{\text{Area of pipe}} = \frac{Q}{a} = \frac{ALN}{60a}$$

$$= \frac{AL}{60a} \times \frac{60\omega}{2\pi} \qquad \left(\because \ \omega = \frac{2\pi N}{60} \text{ or } N = \frac{60\omega}{2\pi}\right)$$

$$= \frac{A}{a} \times L \times \frac{\omega}{2\pi} = \frac{A}{a} \times 2r \times \frac{\omega}{2\pi} \qquad (\because \ L = 2r)$$

$$= \frac{A}{a} \times \frac{\omega r}{\pi} \qquad \text{...(20.24)}$$

The velocity of water in the suction or delivery pipes for the lengths l_s' and l_d' due to acceleration and retardation of the piston is given by equation (20.11) as

$$v = \frac{A}{a} \omega r \sin \omega t = \frac{A}{a} \omega r \sin \theta \qquad (\because \ \theta = \omega t)$$

(a) **Pressure head in the cylinder during delivery stroke.** The pressure head due to acceleration in the delivery pipe of length l_d' (between air vessel and cylinder) is given by equation (20.15) as

$$h_{ad} = \frac{l_d'}{g} \times \frac{A}{a_d} \omega^2 r \cos \theta \qquad \text{...}(i)$$

Loss of head due to friction in the delivery pipe for lengths l_d' is given as

$$h_{fd}' = \frac{4f \times l_d' \times v^2}{d \times 2g}$$

where for delivery pipe,

$$v = \frac{A}{a_d} \ \omega r \sin \theta$$

$$d = \text{dia. of delivery pipe} = d_d$$

$$\therefore \qquad h_{fd}' = \frac{4f \times l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d} \omega r \sin\theta\right)^2 \qquad ...(ii)$$

Loss of head due to friction in the delivery pipe for the length beyond the air vessel (*i.e.,* length l_d),

$$h_{fd} = \frac{4f \times l_d \times (\overline{V}_d)^2}{d_d \times 2g}$$

where $\overline{V}_d$ = Mean velocity in delivery pipe = $\frac{A}{a_d} \times \frac{\omega r}{\pi}$ [from equation (20.24)]

$$\therefore \qquad h_{fd} = \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 \qquad ...(iii)$$

The pressure head in the cylinder during delivery stroke is given as :

(*i*) At the beginning of the delivery stroke, $\theta = 0°$, $\sin\theta = 0$ and $\cos\theta = 1$ and hence total pressure head

$$= (h_d + h_{ad} + h_{fd}' + h_{fd}) + \text{velocity head at the outlet of delivery}$$

$$= h_d + h_{ad} + h_{fd}' + h_{fd} + \frac{\overline{V}_d^2}{2g} \qquad (\because \text{ Velocity at outlet is equal to mean velocity})$$

$$= h_d + \frac{l_d'}{g} \times \frac{A}{a_d} \omega^2 r + 0 + \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2}{2g} \qquad \left(\because \overline{V}_d = \frac{A}{a_d} \times \frac{\omega r}{\pi}\right)$$

$$= h_d + \frac{l_d'}{g} \times \frac{A}{a_d} \omega^2 r + \frac{4fl_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 \qquad ...(20.25)$$

(*ii*) In the middle of the stroke, $\theta = 90°$, $\sin\theta = 1$ and $\cos\theta = 0$ and hence total pressure head

$$= h_d + h_{ad} + h_{fd}' + h_{fd} + \frac{\overline{V}_d^2}{2g} \text{ above atmospheric pressure head}$$

$$= h_d + 0 + \frac{4f \times l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d} \omega r\right)^2 + \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2$$

$$= h_d + \frac{4f \times l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d} \omega r\right)^2 + \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 \qquad ...(20.26)$$

(*iii*) At the end of the delivery stroke, $\theta = 180°$, $\sin\theta = 0$ and $\cos\theta = -1$ and hence total pressure head

$$= h_d - \frac{l_d'}{g} \times \frac{A}{a_d} \omega^2 r + \frac{4f \times l_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 \qquad ...(20.27)$$

In equations (20.25), (20.26) and (20.27), the quantities

$$\left(\frac{l_d'}{g} \times \frac{A}{a_d} \omega^2 r\right) \text{ and } \left[\frac{4f \times l_d'}{d_a \times 2g} \times \left(\frac{A}{a_d} \omega r\right)^2\right]$$

are very small and can be neglected.

(*b*) **Pressure head in the cylinder during suction stroke.** The pressure head due to acceleration in the suction pipe of length l_s' (between air vessel and cylinder) is given as

$$h_{as}' = \frac{l_s'}{g} \times \frac{A}{a_s} \omega^2 r \cos\theta$$

Loss of friction head in the suction pipe of length l_s' given as

$$h_{fs}' = \frac{4f \times l_s' \times v_s^2}{d_s \times 2g}$$

where for suction pipe, the velocity (v_s) is given by equation (20.11) as $v_s = \dfrac{A}{a_s} \omega r \sin\theta$

Substituting the value of v_s in h_{fs}', we get

$$h_{fs}' = \frac{4 \times f \times l_s'}{d_s \times 2g} \times \left(\frac{A}{a_s} \omega r \sin\theta\right)^2$$

Loss of head due to friction in the suction pipe for the length below the air vessel (*i.e.,* lengths l_s),

$$h_{fs} = \frac{4f \times l_s}{d_s \times 2g} \times \left(\overline{V_s}\right)^2$$

where $\overline{V_s}$ is the mean velocity of flow and is given by equation (20.24) as $\overline{V_s} = \dfrac{A}{a_s} \times \dfrac{\omega r}{\pi}$

$$\therefore \qquad h_{fs} = \frac{4fl_s}{d_s \times 2g} \times \left(\frac{A}{a_s} \times \frac{\omega r}{\pi}\right)^2$$

Hence the pressure head in the cylinder during suction stroke is given as

(*i*) At the beginning of suction stroke $\theta = 0°$, $\sin\theta = 0$ and $\cos\theta = 1$ and hence pressure head

$$= \left(h_s + h_{as}' + h_{fs}' + h_{fs}\right)_{\theta = 0°} + \frac{\overline{V_s}^2}{2g} \text{ below atmospheric pressure head}$$

$$= h_s + \frac{l_s'}{g} \times \frac{A}{a_s} \omega^2 r + 0 + \frac{4fl_s}{d_s \times 2g} \times \left(\frac{A}{a_s} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_s} \times \frac{\omega r}{\pi}\right)^2$$

$$= h_s + \frac{l_s'}{g} \times \frac{A}{a_s} \omega^2 r + \frac{4fl_s}{d_s \times 2g}\left(\frac{A}{a_s} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_s} \times \frac{\omega r}{\pi}\right)^2 \qquad \text{...(20.28)}$$

below atmospheric pressure head.

(*ii*) In the middle of suction stroke, $\theta = 90°$, $\sin\theta = 1$ and $\cos\theta = 0$ and hence pressure head

$$= \left(h_s + h_{as}' + h_{fs}' + h_{fs}\right)_{\theta = 90°} + \frac{\overline{V_s}^2}{2g} \text{ below atmospheric pressure head}$$

$$= h_s + 0 + \frac{4f \times l_s'}{d_s \times 2g} \times \left(\frac{A}{a_s} \omega r\right)^2 + \frac{4fl_s}{d_s \times 2g} \times \left(\frac{A}{a_s} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_s} \times \frac{\omega r}{\pi}\right)^2$$

$$= h_s + \frac{4fl_s'}{d_s \times 2g} \times \left(\frac{A}{a_s}\omega r\right)^2 + \frac{4fl_s}{d_s \times 2g} \times \left(\frac{A}{a_s} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_s} \times \frac{\omega r}{\pi}\right)^2 \quad ...(20.29)$$

(*iii*) At the end of the suction stroke, $\theta = 180°$, $\sin\theta = 0$ and $\cos\theta = -1$.

Hence pressure head $= \left(h_s + h'_{as} + h'_{fs} + h_{fs}\right)_{\theta = 180°} + \frac{\overline{V_s}^2}{2g}$ below atmospheric pressure head

$$= h_s - \frac{l_s'}{g} \times \frac{A}{a_s}\omega^2 r + 0 + \frac{4fl_s}{d_s \times 2g} \times \left(\frac{A}{a_s} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2}\left(\frac{A}{a_s} \times \frac{\omega r}{\pi}\right)^2 \quad ...(20.30)$$

In equations (20.28), (20.29) and (20.30), the quantities $\left(\frac{l_s'}{g} \times \frac{A}{a_s}\omega^2 r\right)$ and $\left[\frac{4f \times l_s'}{d_s \times 2g} \times \left(\frac{A}{a_s}\omega r\right)^2\right]$ are very small and may be neglected.

(*c*) **Work done by reciprocating pump with air vessels.** Work done by reciprocating pump fitted with air vessels to the suction and delivery pipe

$$= \text{Weight of water discharged per second}$$

$$\times \left[h_s + h_d + h_{fs} + h_{fd} + \frac{\overline{V_s}^2}{2g} + \frac{\overline{V_d}^2}{2g} + \frac{2}{3}h_{fs}' + \frac{2}{3}h_{fd}'\right] \text{ N-m/s.}$$

where weight of water discharged per second for a single-acting pump, $W = \frac{wALN}{60} = \frac{\rho gALN}{60}$.

Also the values of h_{fs}', h_{fd}', $\frac{\overline{V_s}^2}{2g}$ and $\frac{\overline{V_d}^2}{2g}$ are very small and hence they can be neglected.

$\therefore$ Work done per sec
$$= \frac{\rho gALN}{60}\left[h_s + h_d + h_{fs} + h_{fd}\right] \quad ...(20.31)$$

(*d*) **Work saved by fitting air vessel.** By fitting air vessel the head loss due to friction in suction and delivery pipe is reduced. This reduction in the head loss saves a certain amount of energy, which can be calculated by finding the work done against friction without air vessel and with air vessel. The difference of the two gives the saving in work done.

(*i*) **Work done against friction without air vessels.** Consider a single-acting reciprocating pump without any air vessels on the pipes. The velocity of flow through the pipes is given by equation (20.11) as

$$v = \frac{A}{a}\,\omega \times r \sin\theta$$

and loss of head due to friction is given by equation (20.17) as $h_f = \frac{4fl}{d \times 2g} \times \left[\frac{A}{a}\omega r \sin\theta\right]^2$.

The variation of h_f with θ is parabolic in nature and hence indicator diagram for the loss of head due to friction in pipes will be a parabola. The work done by pump against friction per stroke is equal to the area of the indicator diagram due to friction.

∴ Work done by pump per stroke against friction,

$$W_1 = \text{Area of the parabola} = \frac{2}{3} \times \text{Base} \times \text{Height}$$

$$= \frac{2}{3} \times L \times \left[\frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2\right] \qquad (\because \text{ Height} = h_f \text{ at } \theta = 90°)$$

$$= \frac{2}{3} \times L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2. \qquad ...(20.32)$$

(*ii*) **Work done against friction with air vessels.** By fitting an air vessel to the pump, the velocity of flow through the pipes (except for lengths l_s' and l_d' which may be considered negligible) is uniform and equal to mean velocity of flow, which is given by equation (20.24) as

$$\overline{V} = \frac{A}{a} \times \frac{\omega r}{\pi}$$

∴ Loss of head due to friction with air vessel is given as

$$= \frac{4fl \times \overline{V}^2}{d \times 2g} = \frac{4fl}{d \times 2g} \times \left(\frac{A}{a} \times \frac{\omega r}{\pi}\right)^2$$

The head loss due to friction with air vessel is independent of θ and hence indicator diagram will be a rectangle.

∴ Work done by pump per stroke against friction,

$$W_2 = \text{Area of the rectangle}$$

$$= \text{Base} \times \text{Height} = L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a} \times \frac{\omega r}{\pi}\right)^2$$

$$= \frac{1}{\pi^2} \times L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2 \qquad ...(20.33)$$

The work given by equation (20.33) is less than the work given by equation (20.32). Hence, by fitting an air vessel work is saved.

(*iii*) **Work saved in a single-acting reciprocating pump.** Hence, saving in work done per stroke is obtained by subtracting equation (20.33) from equation (20.32).

∴ Work saved per stroke $= W_1 - W_2$

$$= \frac{2}{3} L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2 - \frac{1}{\pi^2} L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2$$

$$= L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2 \left[\frac{2}{3} - \frac{1}{\pi^2}\right] \qquad ...(20.34)$$

The percentage of the work saved per stroke

$$= \left(\frac{W_1 - W_2}{W_1}\right) \times 100 = \frac{L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2 \left[\frac{2}{3} - \frac{1}{\pi^2}\right]}{\frac{2}{3} L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2} \times 100$$

$$= \frac{\left(\frac{2}{3}\right) - \left(\frac{1}{\pi^2}\right)}{\left(\frac{2}{3}\right)} \times 100 = 84.8\%.$$

(*iv*) **Work saved in a double-acting reciprocating pump.** The work lost in friction per stroke in case of double-acting reciprocating pump without air vessel is the same as given in case of single-acting reciprocating pump. Hence, it is given by equation (20.32) as

$$W_1 = \frac{2}{3} \times L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2$$

When the air vessel is fitted to the pipe near the cylinder, the mean velocity of flow, $\overline{V}$ for double-acting is given by,

$$\overline{V} = \frac{\text{Discharge}}{\text{Area of pipe}} = \frac{Q}{a} = \frac{2ALN}{60a}$$

$$= \frac{2A \times 2r}{60a} \times \frac{60\omega}{2\pi} \qquad \left(\because \; L = 2r \text{ and } N = \frac{60\omega}{2\pi}\right)$$

$$= \frac{2A}{a} \times \frac{\omega r}{\pi}$$

∴ Loss of head due to friction for double-acting

$$= \frac{4fl \times \overline{V^2}}{d \times 2g} = \frac{4fl}{d \times 2g} \times \left(\frac{2A}{a} \times \frac{\omega r}{\pi}\right)^2$$

∴ Work lost due to friction per stroke

$$W_2 = \text{Area of the rectangle}$$

$$= L \times \frac{4fl}{d \times 2g} \times \left(\frac{2A}{a} \times \frac{\omega r}{\pi}\right)^2$$

$$= \frac{4}{\pi^2} \times L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a} \times \omega r\right)^2 \qquad \text{....(20.34}A\text{)}$$

∴ Saving in work done per stroke $= \dfrac{W_1 - W_2}{W_1}$

$$= \frac{\frac{2}{3} \times L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a} \times \omega r\right)^2 - \frac{4}{\pi^2} \times L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2}{\frac{2}{3} \times L \times \frac{4fl}{d \times 2g} \times \left(\frac{A}{a} \times \omega r\right)^2} = \frac{\left(\frac{2}{3}\right) - \left(\frac{4}{\pi^2}\right)}{\left(\frac{2}{3}\right)} = 0.392 = 39.2\%.$$

(*e*) **Discharge of liquid into and from the air vessel. 1.** Let the air vessel is fitted to both suction and delivery pipes of a *single-acting* reciprocating pump. Due to air vessel, the liquid in suction and delivery pipes beyond air vessel will be moving with a constant mean velocity. This mean velocity in the pipes is given by equation (20.24) as

$$\overline{V} = \frac{A}{a} \times \frac{\omega r}{\pi}$$

The mean discharge ($\overline{Q}$) in the pipes (suction or delivery) will be equal to,

$$\overline{Q} = \overline{V} \times \text{area of pipe}$$

$$= \left(\frac{A}{a} \times \frac{\omega r}{\pi}\right) \times a \qquad (\because \quad a = \text{area of pipe})$$

$$= \frac{A \times \omega \times r}{\pi} \qquad \text{...(20.35)}$$

The velocity of piston in the cylinder at any instant is given by equation (20.10) as

$$V = \omega r \times \sin(\omega t) = \omega r \times \sin\theta \qquad (\because \quad \omega t = \theta)$$

Hence instantaneous discharge to or from the cylinder of the pump will be as

$$Q_i = \text{Velocity of piston} \times \text{Area of piston}$$

$$= (\omega r \sin\theta) \times A \qquad (\because \quad A = \text{Area of piston})$$

$$= A\omega r \sin\theta \qquad \text{...(20.36)}$$

The difference of the two discharges given by equations (20.36) and (20.35) will be equal to the rate of flow of liquid into or from the air vessel.

∴ Rate of flow of liquid into the air vessel

$$= \left(A\omega r \sin\theta - \frac{A\omega r}{\pi}\right) = A\omega r\left(\sin\theta - \frac{1}{\pi}\right) \qquad \text{...(20.37)}$$

(*i*) *For air vessel fitted to delivery pipe.* The liquid will be flowing into the air vessel if equation (20.37) is positive. But if equation (20.37) is negative, then liquid will flow from the air vessel. And if equation (20.37) is zero, then no flow is taking place from or to the air vessel.

(*ii*) *For air vessel fitted to suction pipe.* If equation (20.37) is positive, then liquid is flowing from the air vessel. If equation (20.37) is negative, then liquid is flowing into the air vessel.

For no flow of liquid into or from the air vessel, the equation (20.37) should be zero.

$$\therefore \quad A\omega r\left(\sin\theta - \frac{1}{\pi}\right) = 0 \text{ or } \sin\theta - \frac{1}{\pi} = 0$$

or

$$\sin\theta = \frac{1}{\pi} = 0.3183$$

$$\therefore \quad \theta = 18^\circ\, 34' \text{ or } 161^\circ\, 26'$$

2. For double-acting pump. The discharge for double-acting is given by equation (20.5) as,

$$Q = \frac{2ALN}{60} \qquad \left(\omega = \frac{2\pi N}{60} \text{ or } N = \frac{60\,\omega}{2\pi}\right)$$

$$= \frac{2AL}{60} \times \frac{60\omega}{2\pi} = \frac{AL\omega}{\pi} = \frac{A \times 2r \times \omega}{\pi} \qquad (\because \quad L = 2r)$$

$$= \frac{2A\omega r}{\pi}$$

The above discharge is the mean discharge. Hence for double-acting mean discharge (Q) is given by

$$Q = \frac{2A\omega r}{\pi}$$

But the instantaneous discharge (Q_i) to or from the cylinder of the pump will be

$$Q_i = \text{Velocity of piston} \times \text{Area of piston}$$

$$= (\omega r \sin \theta) \times A = A\omega r \sin \theta$$

Hence, rate of flow of liquid into air vessel

$$= Q_i - Q = A\omega r \sin \theta - \frac{2A\omega r}{\pi} = A\omega r \left(\sin \theta - \frac{2}{\pi}\right) \quad \text{...(20.38)}$$

(i) For air vessel fitted to delivery pipe. If equation (20.38) is positive, the liquid is flowing into the air vessel fitted to delivery pipe. If equation (20.38) is negative, then liquid is flowing from the air vessel. And if equation (20.38) is zero then no flow is taking place into or from the air vessel.

(ii) For air vessel fitted to suction pipe. If equation (20.38) is positive, the liquid is flowing from the air vessel fitted to suction pipe. If equation (20.38) is negative, the liquid is flowing into the air vessel.

For no flow of liquid into or from the air vessel, equation (20.38) should be zero.

$$\therefore \quad A\omega r \left(\sin \theta - \frac{2}{\pi}\right) = 0 \text{ or } \sin \theta = \frac{2}{\pi} = 0.6366$$

$$\therefore \quad \theta = 39° \, 32' \text{ or } 140° \, 28'.$$

Problem 20.14 *The cylinder of a single-acting reciprocating pump is 15 cm in diameter and 30 cm in stroke. The pump is running at 30 r.p.m. and discharge water to a height of 12 m. The diameter and length of the delivery pipe are 10 cm and 30 m respectively. If a large air vessel is fitted in the delivery pipe at a distance of 2 m from the centre of the pump, find the pressure head in the cylinder.*

(*i*) *At the beginning of the delivery stroke, and*

(*ii*) *In the middle of the delivery stroke. Take $f = .01$*

Solution. Given :

Diameter of cylinder, $D = 15 \text{ cm} = 0.15 \text{ m}$

$\therefore$ Area, $A = \frac{\pi}{4} \times .15^2 = 0.01767 \text{ m}^2$

Stroke length, $L = 30 \text{ cm} = 0.30 \text{ m}$

$\therefore$ Crank radius, $r = \frac{L}{2} = \frac{0.30}{2} = 0.15 \text{ m}$

Speed of pump, $N = 30$ r.p.m.

$\therefore$ Angular speed $\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 30}{60} = \pi$ rad/s.

Delivery head, $h_d = 12 \text{ m}$

Diameter of delivery pipe, $d_d = 10 \text{ cm} = 0.10 \text{ m}$

$\therefore$ Area, $a_d = (\pi/4)\,(.1)^2 = .007854$

Length of delivery pipe, $l = 30 \text{ m}$

Length of air vessel from the centre of the cylinder,

$$l_d' = 2 \text{ m}$$

$\therefore$ Length of delivery pipe above the air vessel,

$$l_d = l - l_d' = 30 - 2 = 28 \text{ m}$$

Co-efficient of friction, $f = 0.01.$

(*i*) The pressure head in the cylinder at the beginning of the delivery stroke is given by equation (20.25) as

$$= h_d + \frac{l_d'}{g} \times \frac{A}{a_d}\omega^2 r + \frac{4fl_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2$$

$$= 12 + \frac{2}{9.81} \times \frac{.01767}{.007854} \times \pi^2 \times .15 + \frac{4 \times .01 \times 28}{0.1 \times 2 \times 9.81}\left(\frac{.01767}{.007854} \times \frac{\pi \times .15}{\pi}\right)^2$$

$$+ \frac{1}{2 \times 9.81}\left(\frac{.01767}{.007854} \times \frac{\pi \times .15}{\pi}\right)^2$$

$= 12 + .6709 + 0.065 + .0058 =$ **12.75 m. Ans.**

(*ii*) The pressure head in the cylinder in the middle of the delivery stroke is given by equation (20.26) as

$$= h_d + \frac{4f \times l_d'}{d_d \times 2g} \times \left(\frac{A}{a_d}\omega r\right)^2 + \frac{4fl_d}{d_d \times 2g} \times \left(\frac{A}{a_d} \times \frac{\omega r}{\pi}\right)^2 + \frac{1}{2g}\left(\frac{A}{a_d}\frac{\omega r}{\pi}\right)^2$$

$$= 12 + \frac{4 \times .01 \times 2}{0.1 \times 2 \times 9.81} \times \left(\frac{.01767}{.007854} \times \pi \times .15\right)^2 + .065 + .0058$$

$= 12 + .0458 + .065 + .0058 =$ **12.116 m. Ans.**

Problem 20.15 *A single-acting reciprocating pump is to raise a liquid of density 1200 kg per cubic metre through a vertical height of 11.5 metres, from 2.5 metres below pump axis to 9 metres above it. The plunger, which moves with S.H.M., has diameter 125 mm and stroke 225 mm. The suction and delivery pipes are 75 mm diameter and 3.5 metres and 13.5 metres long respectively. There is a large air vessel placed on the delivery pipe near the pump axis. But there is no air vessel on the suction pipe. If separation takes place at 8.829 N/cm² below atmospheric pressure, find :*

(*i*) *maximum speed, with which the pump can run without separation taking place, and*

(*ii*) *power required to drive the pump, if f = 0.02.*

Neglect slip for the pump.

Solution. Given :

Liquid density, $\rho = 1200$ kg/m^3

Total vertical height $= 11.5$ m

Suction head, $h_s = 2.5$ m

Delivery head, $h_d = 9$ m

Dia. of plunger, $D = 125$ mm $= 0.125$ m

Area of plunger, $A = \frac{\pi}{4} \times .125^2 = 0.0123$ m^2

Stroke length, $L = 225$ mm $= 0.225$ m

∴ Crank radius, $r = \frac{L}{2} = \frac{.225}{2} = .1125$

Dia. of suction and delivery pipe, $d = 75$ mm $= 0.075$ m

Area of suction and delivery pipe, $a = \frac{\pi}{4}(0.075)^2 = 0.00442$ m^2

Length of suction pipe, $l_s = 3.5$ m
Length of delivery pipe, $l_d = 13.5$ m.

Air vessel is placed on the delivery side only. Hence, the velocity in the delivery pipe will be uniform. And there will be no accelerating head on delivery side.

Separation pressure $= 8.829 \dfrac{\text{N}}{\text{cm}^2}$ below atmospheric pressure

$= 8.829 \times 10^4 \dfrac{\text{N}}{\text{m}^2}$ below atmospheric pressure

$\therefore$ Separation pressure head, $h_{sep} = \dfrac{\text{Separation pressure}}{\rho \times g}$

$= \dfrac{8.829 \times 10^4}{1200 \times 9.81}$ m below atmosphere

$= 7.5$ m below atmosphere ...(*i*)

(*i*) *Maximum speed, with which pump can run without separation taking place.*

Let N = Max. speed with which pump can run without separation taking place.

The separation can take place only at the beginning of suction stroke. As air vessel is not fitted on the suction pipe, there will be accelerating head acting on suction side.

Pressure head at the beginning of suction stroke

$= h_s + h_{as}$ below atmosphere

This pressure should be equal to h_{sep} in the limiting case

$\therefore \quad 7.5 = h_s + h_{as} = 2.5 + h_{as}$

$\therefore \quad h_{as} = 7.5 - 2.5 = 5.0$ m

But 'h_{as}' at the beginning of suction stroke

$$= \frac{l_s}{g} \times \frac{A}{a} \omega^2 r$$

$$\therefore \quad 5.0 = \frac{3.5}{9.81} \times \frac{0.0123}{.00442} \times \omega^2 \times .1125$$

$$\therefore \quad \omega = \sqrt{\frac{5.0 \times 9.81 \times .00442}{3.5 \times .0123 \times .1125}} = 6.69 \text{ radians/s.}$$

But $\omega = \dfrac{2\pi N}{60}$

$$\therefore \quad N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 6.69}{2\pi} = \textbf{63.88 r.p.m. Ans.}$$

$\therefore$ Maximum speed with which the pump can run without separation taking place is 63.88 r.p.m.

(*ii*) *Power required to drive the pump.*

New discharge (Q) of the single-acting pump is given by equation (20.1) as

$$Q = \frac{ALN}{60} = \frac{0.0123 \times 0.225 \times 63.88}{60} = 0.00294 \text{ m}^3\text{/s.}$$

Velocity of liquid in delivery pipe will be uniform.

$\therefore$ Q = Area of delivery pipe × Velocity = $a \times v$

$$\therefore \quad v = \frac{Q}{a} = \frac{0.00294}{0.00442} = 0.665 \text{ m/s.}$$

$\therefore$ Head loss due to friction in delivery pipe,

$$h_{fd} = \frac{4f \times l_d \times v^2}{d \times 2g} = \frac{4 \times .02 \times 13.5 \times (.665)^2}{.075 \times 2 \times 9.81} = 0.324 \text{ m.}$$

During suction stroke, the value of maximum h_{fs} is given by

$$h_{fs} = \frac{4 \times f \times l_s}{d \times 2g} \times \left(\frac{A}{a}\omega r\right)^2 = \frac{4 \times .02 \times 3.5}{.075 \times 2 \times 9.81}\left(\frac{.0123}{.00442} \times 6.69 \times .1125\right)^2$$

$$= 0.834 \text{ m}$$

Now power required to drive the pump in kW

$$= \frac{\text{Work done/s}}{1000} = \frac{\rho \times g \times Q}{1000} \times \left[h_s + h_d + \frac{2}{3}h_{fs} + h_{fd}\right]$$

$$= \frac{1200 \times 9.81 \times .00294}{1000} \times \left[2.5 + 9.0 + \frac{2}{3} \times .834 + .324\right]$$

= **0.428 kW. Ans.**

Problem 20.16 *A double-acting reciprocating piston pump is pumping water (diameter of the piston 250 mm, diameter of piston rod, which is on one side of the piston 50 mm, piston stroke 380 mm). The suction and discharge heads are 4.5 m and 18.6 m respectively. Find the work done by the piston during outward stroke. Would the work done change for the inward stroke?*

Solution. Given :

Dia. of piston, $D = 250 \text{ mm} = 0.25 \text{ m}$

$\therefore$ Area of piston, $A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 0.25^2 \text{ m}^2 = 0.0491 \text{ m}^2$

Dia. of piston rod, $d = 50 \text{ mm} = 0.05 \text{ m}$

$\therefore$ Area of piston rod, $a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0.05^2 = 0.001963 \text{ m}^2$

Stroke length, $L = 380 \text{ mm} = 0.380 \text{ m}$

Suction head, $h_s = 4.5 \text{ m}$

Discharge or delivery head, $h_d = 18.6 \text{ m}$

Find work done during outward stroke

In a double-acting pump for outward stroke, suction side will be towards the piston and delivery side will be towards the piston rod.

Hence, total work done during outward stroke

= Weight of water lifted × height through which water is lifted
+ Weight of water delivered × height through which water is delivered

$= \rho \times g \times Q_1 \times h_s + \rho \times g \times Q_2 \times h_d$

where during outward stroke, $Q_1 = A \times L = 0.0491 \times 0.380 = 0.01865 \text{ m}^3$

$Q_2 = (A - a) \times L$

$= (0.0491 - 0.001963) \times 0.380 = 0.01791 \text{ m}^3$

and $\rho \times g = 1000 \times 9.81 \text{ N/m}^3$

∴ Total work done during outward stroke

$= (1000 \times 9.81 \times 0.01865 \times 4.5 + 1000 \times 9.81 \times 0.01791 \times 18.6) \text{ Nm}$

$= 9.81 \times 0.01865 \times 4.5 + 9.81 \times 0.01791 \times 18.6 \text{ (kJ)}$

$= (0.8233 + 3.268) \text{ kJ}$ (∵ J = Nm)

$= \mathbf{4.0913 \text{ kJ}}$. **Ans.**

For inward stroke, suction side will be towards the piston rod whereas the delivery side will be towards the piston.

∴ Total work done during inward stroke

$= \rho \times g \times Q_2 \times h_s + \rho \times g \times Q_1 \times h_d = \rho \times g\, (Q_2 \times h_s + Q_1 \times h_d)$

$= 1000 \times 9.81\, (0.01791 \times 4.5 + 0.01865 \times 18.6) \text{ Nm}$

$= 1000 \times 9.81\, (0.0806 + 0.3468) \text{ J}$

$= 9.81\, (0.0806 + 0.3468) \text{ kJ} = 4.192 \text{ kJ}$

Hence, work done during inward stroke will be different. **Ans.**

Problem 20.17 *A single-acting reciprocating pump has a plunger diameter of 250 mm and stroke of 450 mm and it is driven with S.H.M. at 60 r.p.m. The length and diameter of delivery pipe are 60 m and 100 mm respectively. Determine the power saved in overcoming friction in the delivery pipe by fitting an air vessel on the delivery side of the pump. Assume friction factor = 0.01.*

Solution. Given :

Dia. of plunger, $D = 250 \text{ mm} = 0.25 \text{ m}$

∴ Area, $A = \dfrac{\pi}{4} \times 0.25^2$

Stroke length, $L = 450 \text{ mm} = 0.45 \text{ m}$

∴ Crank radius, $r = \dfrac{L}{2} = \dfrac{0.45}{2} = 0.225 \text{ m}$

Speed, $N = 60 \text{ r.p.m.}$

∴ Angular speed, $\omega = 2\pi N/60 = 2\pi \times 60/60 = 2\pi \text{ rad/s.}$

Length of delivery pipe, $L = 60 \text{ m}$

Dia. of delivery pipe, $d = 100 \text{ mm} = 0.1 \text{ m}$

∴ Area of pipe, $a = \dfrac{\pi}{4} \times 0.1^2$

Friction factor, $f = 0.01$

Power saved is given by,

Power saved $= \rho \times g \times Q \times \left[\dfrac{2}{3}(h_f)_{\text{without air vessel}} - (h_f)_{\text{with air vessel}}\right]$

where $\rho \times g = 1000 \times 9.81 \ \dfrac{\text{N}}{\text{m}^3}$

$$Q = \frac{ALN}{60} = \frac{\pi}{4} \times \frac{0.25^2 \times 0.45 \times 60}{60} = 0.02209 \text{ m}^3/\text{s}$$

Without air vessel,

$$h_f = \frac{f^* \times L \times v^2}{d \times 2g} = \frac{f \times L}{d \times 2g} \times \left(\frac{A}{a}\omega \times r\right)^2 \qquad \left[\because \ v = \frac{A}{a} \times \omega \times r\right]$$

$$= \frac{0.01 \times 60}{0.1 \times 2 \times 9.81} \times \frac{\left(\frac{\pi}{4} \times 0.25^2 \times 2\pi \times 0.225\right)^2}{\frac{\pi}{4} \times 0.1^2} = 23.87 \text{ m}$$

With air vessel,

$$h_f = \frac{f^* \times L \times \overline{V}^2}{d \times 2g}$$

$$= \frac{f^* \times L}{d \times 2g} \times \left(\frac{A}{a} \times \frac{\omega r}{\pi}\right)^2, \text{ where } \overline{V}^2 = \frac{A}{a} \times \frac{\omega \times r}{\pi}$$

$$= \frac{0.01 \times 60}{0.1 \times 2 \times 9.81} \times \left(\frac{\frac{\pi}{4} \times 0.25^2}{\frac{\pi}{4} \times 0.1^2} \times \frac{2\pi \times 0.225}{\pi}\right)^2 = 2.419 \text{ m.}$$

$\therefore$ Power saved

$$= \rho \times g \times Q \times \left[\frac{2}{3}(h_f)_{\text{without air vessel}} - (h_f)_{\text{with air vessel}}\right]$$

$$= 1000 \times 9.81 \times 0.02209 \left[\frac{2}{3} \times 23.87 - 2.419\right] \text{ W}$$

$$= 9.81 \times 0.02209 \left[\frac{2}{3} \times 23.87 - 2.419\right] \text{ kW} = \mathbf{2.924 \ kW. \ Ans.}$$

Problem 20.18 *A double-acting reciprocating pump runs at 120 r.p.m. When its suction pipe of 100 mm diameter is fitted with an air vessel on its suction side. The diameter of cylinder and stroke are 150 mm and 450 mm respectively. If piston is to be driven with S.H.M., find the rate of flow from or into the air vessel when the crank makes angles of 30°, 90° and 120° with the inner dead centre. Find also the crank angles at which there is no flow into or from the air vessel.*

Solution. Given :

Speed, $N = 120$ r.p.m.

$\therefore$ Angular speed, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 120}{60} = 4\pi$ rad/s

* Here friction factor is given and hence the formula is $h_f = \dfrac{f \times L \times V^2}{d \times 2g}$ and not $\dfrac{4f \times L \times V^2}{d \times 2g}$.

Dia. of suction pipe = 100 mm = 0.1 m

$\therefore$ Area of suction pipe, $a = \frac{\pi}{4}(0.1)^2 = 0.007854\ \text{m}^2$

Dia. of cylinder = 150 mm = 0.15 m

$\therefore$ Area of cylinder, $A = \frac{\pi}{4}(0.15)^2 = 0.01767\ \text{m}^2$

Stroke length, $L = 450$ mm = 0.45 m

$\therefore$ Crank radius, $r = \frac{L}{2} = \frac{0.45}{2} = 0.225$ m

Find rate of flow from or into air vessel when θ = 30°, 90° and 120°

The rate of flow of liquid for double-acting pump into the air vessel is given by equation (20.38).

$\therefore$ Rate of flow of liquid into air vessel

$$= A\omega r\left(\sin\theta - \frac{2}{\pi}\right) = 0.01767 \times 4\pi \times 0.225\left(\sin\theta - \frac{2}{\pi}\right) = 0.04996\left(\sin\theta - \frac{2}{\pi}\right)$$

In this problem, air vessel is fitted to the suction pipe. Hence if the above rate of flow is positive, the liquid will be flowing from the air vessel. And if the above rate of flow is negative, the liquid will be flowing into the air vessel.

(*i*) *For θ = 30°.*

The above rate of flow $= 0.04996\left(\sin 30° - \frac{2}{\pi}\right) = 0.04996(0.5 - 0.6366) =$ **– 0.00682 m³/s. Ans.**

Since the rate of flow is negative, hence the flow is taking place into the air vessel.

(*ii*) *For θ = 90°.*

The rate of flow becomes $= 0.04996\left(\sin 90° - \frac{2}{\pi}\right) = 0.04996(1 - 0.6366) =$ **0.0181 m³/s. Ans.**

As it is positive, hence rate of flow is taking place from the air vessel.

(*iii*) *For θ = 120°.*

The rate of flow becomes $= 0.04996\left(\sin 120° - \frac{2}{\pi}\right) = 0.04996(0.866 - 0.6366) =$ **0.01146 m³/s. Ans.**

As it is positive, hence rate of flow is taking place from the air vessel.

(*iv*) *Crank angle at which there is no flow into or from air vessel*

Let θ = angle at which there is no flow. But rate of flow

$$= 0.04996\left(\sin\theta - \frac{2}{\pi}\right)$$

For no flow from or into air vessel

$$0.04996\left(\sin\theta - \frac{2}{\pi}\right) = 0 \text{ or } \sin\theta = \frac{2}{\pi} = 0.6366$$

$\therefore$ $\theta = \sin^{-1} 0.6366 =$ **39° 32′** and **140° 28′. Ans.**

▶ 20.10 COMPARISON BETWEEN CENTRIFUGAL PUMPS AND RECIPROCATING PUMPS

Centrifugal pumps	*Reciprocating pumps*
1. The discharge is continuous and smooth.	1. The discharge is fluctuating and pulsating.
2. It can handle large quantity of liquid.	2. It handles small quantity of liquid only.
3. It can be used for lifting highly viscous liquids.	3. It is used only for lifting pure water or less viscous liquids.
4. It is used for large discharge through smaller heads.	4. It is meant for small discharge and high heads.
5. Cost of centrifugal pump is less as compared to reciprocating pump.	5. Cost of reciprocating pump is approximately four times the cost of centrifugal pump.
6. Centrifugal pump runs at high speed. They can be coupled to electric motor.	6. Reciprocating pump runs at low speed. Speed is limited due to consideration of separation and cavitation.
7. The operation of centrifugal pump is smooth and without much noise. The maintenance cost is low.	7. The operation of reciprocating pump is complicated and with much noise. The maintenance cost is high.
8. Centrifugal pump needs smaller floor area and installation cost is low.	8. Reciprocating pump requires large floor area and installation cost is high.
9. Efficiency is high.	9. Efficiency is low.

HIGHLIGHTS

1. A reciprocating pump consists of a cylinder with a piston, a suction pipe, a delivery pipe, a suction valve and a delivery valve.

2. Discharge through a pump per second is given as

$$Q = \frac{ALN}{60} \qquad \text{...For a single-acting}$$

$$= \frac{2ALN}{60} \qquad \text{...For a double-acting.}$$

3. Work done by reciprocating pump per second

$$= \frac{\rho g ALN}{60}(h_s + h_d) \qquad \text{...For a single-acting}$$

$$= \frac{2\rho g ALN}{60}(h_s + h_d) \qquad \text{...For a double-acting.}$$

4. Slip of a pump is defined as the difference between the theoretical discharge and actual discharge of the pump.

5. The pressure head (h_a) due to acceleration in the suction and delivery pipes is given as

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s}\omega^2 r \cos\theta \qquad \text{...For suction pipe}$$

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d}\omega^2 r \cos\theta \qquad \text{... For delivery pipe.}$$

6. The loss of head due to friction in suction and delivery pipes is obtained from

$$h_f = \frac{4fl}{d \times 2g} \times \left(\frac{A}{a}\omega r \sin\theta\right)^2.$$

7. Indicator diagram is a graph between the pressure head in the cylinder and the distance travelled by the piston from inner dead centre for one complete revolution of the crank.

8. Work done by the pump is proportional to the area of the indicator diagram. Area of ideal indicator diagram is the same as the area of indicator diagram due to acceleration in suction and delivery pipes.

9. Work done by the pump per second due to acceleration and friction in suction and delivery pipes

$$= \frac{\rho g A L N}{60}\left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right) \quad \text{...For a single-acting}$$

$$= \frac{2\rho g A L N}{60}\left(h_s + h_d + \frac{2}{3}h_{fs} + \frac{2}{3}h_{fd}\right) \quad \text{...For a double-acting.}$$

10. Air vessel is used to obtain a continuous supply of water at uniform rate, to save a considerable amount of work and to run the pump at a high speed without separation.

11. Mean velocity ($\overline{V}$) for a single-acting pump is given as

$$\overline{V} = \frac{A}{a}\frac{\omega r}{\pi}.$$

12. Work done by reciprocating with air vessels fitted to suction and delivery pipes

$$\simeq \frac{\rho g A L N}{60}\left[h_s + h_d + h_{fs} + h_{fd}\right].$$

13. Work saved by fitting air vessels in a single-acting reciprocating pump is 84.8% while in a double-acting reciprocating pump, the work saved is 39.2%.

EXERCISE

(A) THEORETICAL PROBLEMS

1. What is a reciprocating pump ? Describe the principle and working of a reciprocating pump with a neat sketch. Why is a reciprocating pump not coupled directly to the motor ? Discuss the reason in detail.
2. Differentiate : (*i*) Between a single-acting and double-acting reciprocating pump, (*ii*) Between a single cylinder and double cylinder reciprocating pump.
3. Define slip, percentage slip and negative slip of a reciprocating pump.
4. How will you classify the reciprocating pumps ?
5. What is the effect of acceleration of the piston on the velocity and acceleration of the water in suction and delivery pipes? Obtain an expression for the pressure head due to acceleration in the suction and delivery pipes.
6. Find an expression for the head lost due to friction in suction and delivery pipes.
7. Define indicator diagram. How will you prove that area of indicator diagram is proportional to the work done by the reciprocating pump?
8. What is the effect of acceleration in suction and delivery pipes on indicator diagram ? Does the area of the indicator diagram change as compared to the area of ideal indicator diagram ?
9. Draw an indicator diagram, considering the effect of acceleration and friction in suction and delivery pipes. Find an expression for the work done per second in case of single-acting reciprocating pump.
10. What is an air vessel ? Describe the function of the air vessel for reciprocating pumps.

11. Show from first principle that the work saved, against friction in the delivery pipe of a single-acting reciprocating pump, by fitting an air vessel is 84.8% while for a double-acting reciprocating pump the work saved is only 39.20%.

12. What is negative slip in a reciprocating pump ? Explain with neat sketches the function of air vessels in a reciprocating pump.

13. Differentiate, with examples between :

(*i*) Turbines and pumps, (*ii*) Impulse and reaction turbines,
(*iii*) Radial and axial flow turbines, flow turbines, (*iv*) Inward and outward radial, and
(*v*) Kaplan and propeller turbines.

14. Explain in brief how and when separation of flow takes place in a reciprocating pump. Discuss the preventive measures usually adopted for effective reduction of separation in such a pump.

15. Why is it that the speed of a reciprocating pump without air vessels is not high ? Explain with sketches.

16. Derive an expression for the head lost due to friction in the delivery pipe of a reciprocating pump with and without an air vessel.

(B) NUMERICAL PROBLEMS

1. A single-acting reciprocating pump running at 30 r.p.m., delivers 0.012 m^3/s of water. The diameter of the piston is 25 cm and stroke length is 50 cm. Determine : (*i*) The theoretical discharge of the pump, (*ii*) Co-efficient of discharge, and (*iii*) Slip and percentage slip of the pump.
[**Ans.** (*i*) 0.01227 m^3/s, (*ii*) 0.978, (*iii*) .00027 m^2/s and 2.20%]

2. A double-acting reciprocating pump, running at 50 r.p.m. is discharging 900 litres of water per minute. The pump has stroke of 400 mm. The diameter of piston is 250 mm. The delivery and suction heads are 25 m and 4 m respectively. Find the slip of the pump and power required to drive the pump.
[**Ans.** .0027 m^3/s, 9.3 kW]

3. A single-acting reciprocating pump has a cylinder of a diameter 150 mm and of stroke length 300 mm. The centre of the pump is 4 m above the water surface in the sump. The atmospheric pressure head is 10.3 m of water and pump is running at 40 r.p.m. If the length and diameter of the suction pipe are 5 m and 10 cm respectively, determine the pressure head due to acceleration in the cylinder : (*i*) At the beginning of the suction stroke, and (*ii*) In the middle of suction stroke. [**Ans.** (*i*) 3.018 m, (*ii*) 0]

4. If in Problem 3, the length and diameter of delivery pipe are 35 m and 100 mm respectively and water is delivered by the pump to a tank which is 25 m above the centre of the pump, determine the pressure head in the cylinder : (*i*) At the beginning of the delivery stroke, (*ii*) In the middle of the stroke, and (*iii*) At the end of the delivery stroke. [**Ans.** (*i*) 5.426 m (abs.), (*ii*) 35.3 m (abs.), (*iii*) 14.174 m (abs.)]

5. A single-acting reciprocating pump has piston diameter 15 cm and stroke length 30 cm. The centre of the pump is 5 m above the water level in the sump. The diameter and length of the suction pipe are 10 cm and 8 m respectively. The separation occurs if the absolute pressure head in the cylinder during suction stroke falls below 2.5 m of water. Calculate the maximum speed at which the pump can run without separation. Take atmospheric pressure head = 10.3 m of water. [**Ans.** 30.45 r.p.m.]

6. A single-acting reciprocating pump has a plunger of 100 mm diameter and a stroke length 200 mm. The centre of the pump is 3 m above the water level in the sump and 20 m below the water level in a tank to which water is delivered by the pump. The diameter and length of suction pipe are 50 mm and 5 m while of the delivery pipe are 40 mm and 30 m respectively. Determine the maximum speed at which the pump may be run without separation, if separation occurs at 7.3575 N/cm^2 below the atmospheric pressure. Take atmospheric pressure head = 10.3 m of water. [**Ans.** 36.22 r.p.m.]

7. The diameter and stroke length of a single-acting reciprocating pump are 100 mm and 200 mm respectively. The lengths of suction and delivery pipes are 10 m and 30 m respectively and their diameters are 50 mm. If the pump is running at 30 r.p.m. and suction and delivery heads are 3.5 m and 20 m respectively, find the pressure head in the cylinder : (*i*) at the beginning of the suction and delivery stroke, (*ii*) in the middle of suction and delivery stroke, and (*iii*) at the end of the suction and delivery stroke. Take atmospheric pressure head = 10.3 m of water and co-efficient of friction = .009 for both pipes.

[**Ans.** (*i*) 2.776 m (abs.), 42.373 m (abs.), (*ii*) 6.22 m, 43.34 m, (*iii*) Not possible, 18.225 m]

8. For Problem 7, find the power required to drive the pump, if the liquid flowing through the pump is water.

[**Ans.** 0.25 kW]

9. The cylinder of a single-acting reciprocating pump is 125 mm in diameter and 250 mm in stroke. The pump is running at 40 r.p.m. and discharge water to a height of 15 m. The diameter and length of the delivery pipe are 100 mm and 30 m respectively. If a large air vessel is fitted in the delivery pipe at a distance of 1.5 m from the centre of the pump, find the pressure head in the cylinder : (*i*) At the beginning of the delivery stroke, and (*ii*) In the middle of the delivery stroke. Take the efficiency of friction = .01.

[**Ans.** (*i*) 15.566 m, (*ii*) 15.07 m]