19 CHAPTER

CENTRIFUGAL PUMPS

▶ 19.1 INTRODUCTION

The hydraulic machines which convert the mechanical energy into hydraulic energy are called pumps. The hydraulic energy is in the form of pressure energy. If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump.

The centrifugal pump acts as a reverse of an inward radial flow reaction turbine. This means that the flow in centrifugal pumps is in the radial outward directions. The centrifugal pump works on the principle of forced vortex flow which means that when a certain mass of liquid is rotated by an external torque, the rise in pressure head of the rotating liquid takes place. The rise in pressure head at any point of the rotating liquid is proportional to the square of tangential velocity of the liquid at that point $\left(i.e., \text{ rise in pressure head } = \frac{V^2}{2g} \text{ or } \frac{\omega^2 r^2}{2g}\right)$. Thus at the outlet of the impeller, where radius is more, the rise in pressure head will be more and the liquid will be discharged at the outlet with a high pressure head. Due to this high pressure head, the liquid can be lifted to a high level.

▶ 19.2 MAIN PARTS OF A CENTRIFUGAL PUMP

The following are the main parts of a centrifugal pump :

1. Impeller.
2. Casing.
3. Suction pipe with a foot valve and a strainer.
4. Delivery pipe.

All the main parts of the centrifugal pump are shown in Fig. 19.1.

1. Impeller. The rotating part of a centrifugal pump is called 'impeller'. It consists of a series of backward curved vanes. The impeller is mounted on a shaft which is connected to the shaft of an electric motor.

2. Casing. The casing of a centrifugal pump is similar to the casing of a reaction turbine. It is an air-tight passage surrounding the impeller and is designed in such a way that the kinetic energy of the water discharged at the outlet of the impeller is converted into pressure energy before the water leaves the casing and enters the delivery pipe. The following three types of the casings are commonly adopted :

(*a*) Volute casing as shown in Fig. 19.1.
(*b*) Vortex casing as shown in Fig. 19.2 (*a*).
(*c*) Casing with guide blades as shown in Fig. 19.2 (*b*).

(*a*) **Volute Casing.** Fig 19.1 shows the volute casing, which surrounds the impeller. It is of spiral type in which area of flow increases gradually. The increase in area of flow decreases the velocity of flow. The decrease in velocity increases the pressure of the water flowing through the casing. It has been observed that in case of volute casing, the efficiency of the pump increases slightly as a large amount of energy is lost due to the formation of eddies in this type of casing.

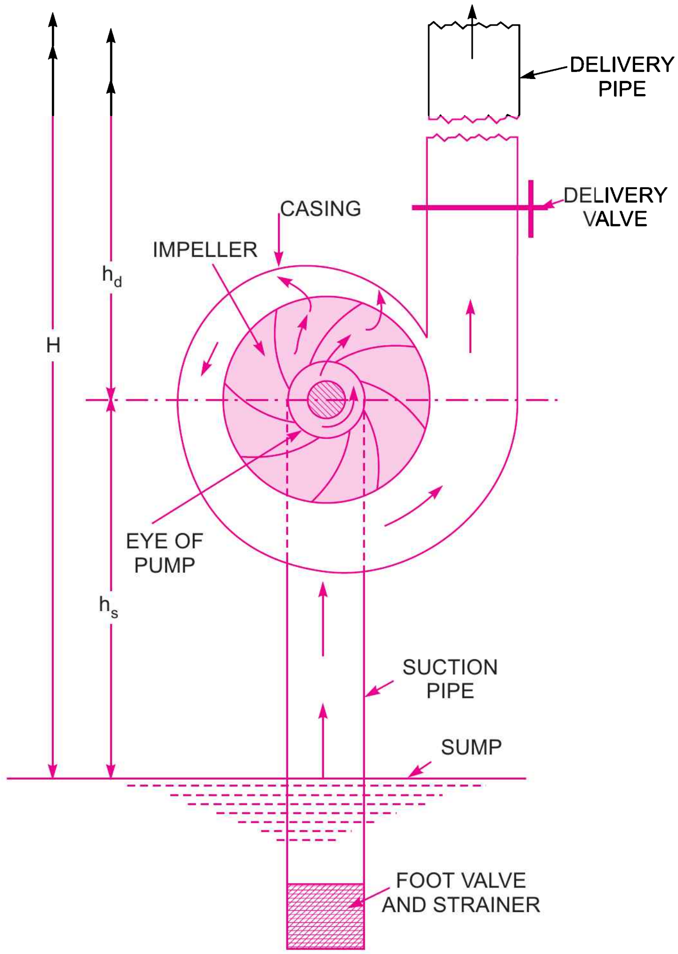

Fig. 19.1 *Main parts of a centrifugal pump.*

(*b*) **Vortex Casing.** If a circular chamber is introduced between the casing and the impeller as shown in Fig. 19.2 (*a*), the casing is known as Vortex Casing. By introducing the circular chamber, the loss of energy due to the formation of eddies is reduced to a considerable extent. Thus the efficiency of the pump is more than the efficiency when only volute casing is provided.

(*c*) **Casing with Guide Blades.** This casing is shown in Fig. 19.2 (*b*) in which the impeller is surrounded by a series of guide blades mounted on a ring which is known as diffuser. The guide vanes are designed in such a way that the water from the impeller enters the guide vanes without stock.

Also the area of the guide vanes increases, thus reducing the velocity of flow through guide vanes and consequently increasing the pressure of water. The water from the guide vanes then passes through the surrounding casing which is in most of the cases concentric with the impeller as shown in Fig. 19.2 (*b*).

3. Suction Pipe with a Foot valve and a Strainer. A pipe whose one end is connected to the inlet of the pump and other end dips into water in a sump is known as suction pipe. A foot valve which is a non-return valve or one-way type of valve is fitted at the lower end of the suction pipe. The foot valve opens only in the upward direction. A strainer is also fitted at the lower end of the suction pipe.

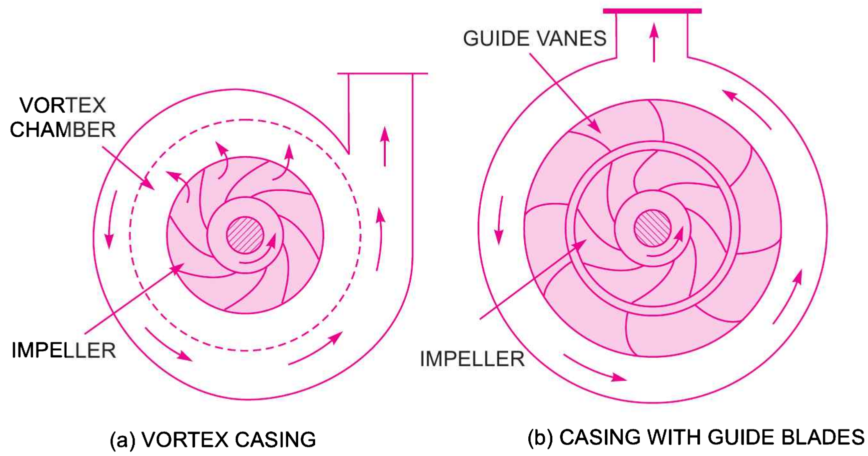

Fig. 19.2 *Different types of casing.*

4. Delivery Pipe. A pipe whose one end is connected to the outlet of the pump and other end delivers the water at a required height is known as delivery pipe.

▶ 19.3 WORK DONE BY THE CENTRIFUGAL PUMP (OR BY IMPFLLER) ON WATER

In case of the centrifugal pump, work is done by the impeller on the water. The expression for the work done by the impeller on the water is obtained by drawing velocity triangles at inlet and outlet of the impeller in the same way as for a turbine. The water enters the impeller radially at inlet for best efficiency of the pump, which means the absolute velocity of water at inlet makes an angle of 90° with the direction of motion of the impeller at inlet. Hence angle $\alpha = 90°$ and $V_{w_1} = 0$. For drawing the velocity triangles, the same notations are used as that for turbines. Fig. 19.3 shows the velocity triangles at the inlet and outlet tips of the vanes fixed to an impeller.

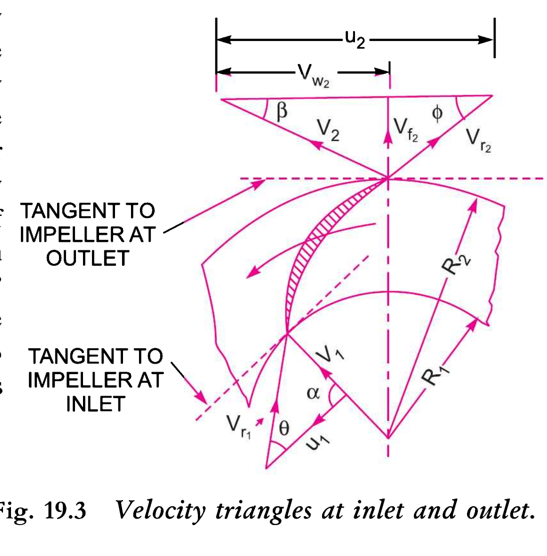

Fig. 19.3 *Velocity triangles at inlet and outlet.*

Let N = Speed of the impeller in r.p.m.,

D_1 = Diameter of impeller at inlet,

u_1 = Tangential velocity of impeller at inlet,

$= \frac{\pi D_1 N}{60}$

D_2 = Diameter of impeller at outlet,

u_2 = Tangential velocity of impeller at outlet

$= \frac{\pi D_2 N}{60}$

V_1 = Absolute velocity of water at inlet,

V_{r_1} = Relative velocity of water at inlet,

α = Angle made by absolute velocity (V_1) at inlet with the direction of motion of vane,

θ = Angle made by relative velocity (V_{r_1}) at inlet with the direction of motion of vane, and V_2, V_{r_2}, β and ϕ are the corresponding values at outlet.

As the water enters the impeller radially which means the absolute velocity of water at inlet is in the radial direction and hence angle $\alpha = 90°$ and $V_{w_1} = 0$.

A centrifugal pump is the reverse of a radially inward flow reaction turbine. But in case of a radially inward flow reaction turbine, the work done by the water on the runner per second per unit weight of the water striking per second is given by equation (18.19) as

$$= \frac{1}{g}[V_{w_1}u_1 - V_{w_2}u_2]$$

$\therefore$ Work done by the impeller on the water per second per unit weight of water striking per second

$$= -\text{[Work done in case of turbine]}$$

$$= -\left[\frac{1}{g}\left(V_{w_1}u_1 - V_{w_2}u_2\right)\right] = \frac{1}{g}\left[V_{w_2}u_2 - V_{w_1}u_1\right]$$

$$= \frac{1}{g}V_{w_2}u_2 \quad (\because \quad V_{w_1} = 0 \text{ here}) \quad ...(19.1)$$

Work done by impeller on water per second

$$= \frac{W}{g}.V_{w_2}u_2 \quad ...(19.2)$$

where W = Weight of water = $\rho \times g \times Q$

where Q = Volume of water

and Q = Area $\times$ Velocity of flow = $\pi D_1 B_1 \times V_{f_1}$

$= \pi D_2 B_2 \times V_{f_2}$ ...(19.2A)

where B_1 and B_2 are width of impeller at inlet and outlet and V_{f_1} and V_{f_2} are velocities of flow at inlet and outlet.

Equation (19.1) gives the head imparted to the water by the impeller or energy given by impeller to water per unit weight per second.

▶ 19.4 DEFINITIONS OF HEADS AND EFFICIENCIES OF A CENTRIFUGAL PUMP

1. Suction Head (h_s). It is the vertical height of the centre line of the centrifugal pump above the water surface in the tank or pump from which water is to be lifted as shown in Fig. 19.1. This height is also called suction lift and is denoted by 'h_s'.

2. Delivery Head (h_d). The vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered is known as delivery head. This is denoted by 'h_d'.

3. Static Head (H_s). The sum of suction head and delivery head is known as static head. This is represented by 'H_s' and is written as

$$H_s = h_s + h_d. \quad ...(19.3)$$

4. Manometric Head (H_m). The manometric head is defined as the head against which a centrifugal pump has to work. It is denoted by 'H_m'. It is given by the following expressions :

(*a*) H_m = Head imparted by the impeller to the water – Loss of head in the pump

$$= \frac{V_{w_2} u_2}{g} - \text{Loss of head in impeller and casing} \quad ...(19.4)$$

$$= \frac{V_{w_2} u_2}{g} \text{ ...if loss of pump is zero} \quad ...(19.5)$$

(*b*) H_m = Total head at outlet of the pump – Total head at the inlet of the pump

$$= \left(\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right) \quad ...(19.6)$$

where $\frac{p_o}{\rho g}$ = Pressure head at outlet of the pump = h_d

$\frac{V_o^2}{2g}$ = Velocity head at outlet of the pump

= Velocity head in delivery pipe = $\frac{V_d^2}{2g}$

Z_o = Vertical height of the outlet of the pump from datum line, and

$\frac{p_i}{\rho g}, \frac{V_i^2}{2g}, Z_i$ = Corresponding values of pressure head, velocity head and datum head at the inlet of the pump,

i.e., h_s, $\frac{V_s^2}{2g}$ and Z_s respectively.

(*c*) $$H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g} \quad ...(19.7)$$

where h_s = Suction head, h_d = Delivery head,

h_{f_s} = Frictional head loss in suction pipe, h_{f_d} = Frictional head loss in delivery pipe, and

V_d = Velocity of water in delivery pipe.

5. Efficiencies of a Centrifugal Pump. In case of a centrifugal pump, the power is transmitted from the shaft of the electric motor to the shaft of the pump and then to the impeller. From the impeller, the power is given to the water. Thus power is decreasing from the shaft of the pump to the impeller and then to the water. The following are the important efficiencies of a centrifugal pump :

(*a*) Manometric efficiency, η_{man} (*b*) Mechanical efficiency, η_m and

(*c*) Overall efficiency, η_o.

(*a*) **Manometric Efficiency (η_{man}).** The ratio of the manometric head to the head imparted by the impeller to the water is known as manometric efficiency. Mathematically, it is written as

$$\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$$

$$= \frac{H_m}{\left(\frac{V_{w_2} u_2}{g}\right)} = \frac{gH_m}{V_{w_2} u_2} \qquad ...(19.8)$$

The power at the impeller of the pump is more than the power given to the water at outlet of the pump. The ratio of the power given to water at outlet of the pump to the power available at the impeller, is known as manometric efficiency.

The power given to water at outlet of the pump $= \frac{WH_m}{1000}$ kW

The power at the impeller $= \frac{\text{Work done by impeller per second}}{1000}$ kW

$$= \frac{W}{g} \times \frac{V_{w_2} \times u_2}{1000} \text{ kW}$$

$$\eta_{man} = \frac{\frac{W \times H_m}{1000}}{\frac{W}{g} \times \frac{V_{w_2} \times u_2}{1000}} = \frac{g \times H_m}{V_{w_2} \times u_2}.$$

(*b*) **Mechanical Efficiency (η_m).** The power at the shaft of the centrifugal pump is more than the power available at the impeller of the pump. The ratio of the power available at the impeller to the power at the shaft of the centrifugal pump is known as mechanical efficiency. It is written as

$$\eta_m = \frac{\text{Power at the impeller}}{\text{Power at the shaft}}$$

The power at the impeller in kW $= \frac{\text{Work done by impeller per second}}{1000}$

$$= \frac{W}{g} \times \frac{V_{w_2} u_2}{1000} \qquad \text{[Using equation (19.2)]}$$

$$\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} u_2}{1000}\right)}{\text{S.P.}} \qquad ...(19.9)$$

where S.P. = Shaft power.

(*c*) **Overall Efficiency (η_o).** It is defined as ratio of power output of the pump to the power input to the pump. The power output of the pump in kW

$$= \frac{\text{Weight of water lifted} \times H_m}{1000} = \frac{WH_m}{1000}$$

Power input to the pump = Power supplied by the electric motor

= S.P. of the pump.

$$\therefore \qquad \eta_o = \frac{\left(\frac{WH_m}{1000}\right)}{\text{S.P.}} \qquad ...(19.10)$$

Also $\qquad \eta_o = \eta_{man} \times \eta_m. \qquad ...(19.11)$

Problem 19.1 *The internal and external diameters of the impeller of a centrifugal pump are 200 mm and 400 mm respectively. The pump is running at 1200 r.p.m. The vane angles of the impeller at inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.*

Solution. Given :

Internal diameter of impeller, $D_1 = 200 \text{ mm} = 0.20 \text{ m}$

External diameter of impeller, $D_2 = 400 \text{ mm} = 0.40 \text{ m}$

Speed, $N = 1200$ r.p.m.

Vane angle at inlet, $\theta = 20°$

Vane angle at outlet, $\phi = 30°$

Water enters radially* means, $\alpha = 90°$ and $V_{w_1} = 0$

Velocity of flow, $V_{f_1} = V_{f_2}$

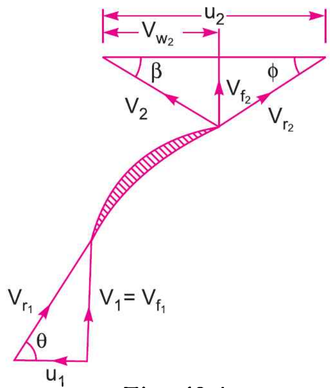

Fig. 19.4

Tangential velocity of impeller at inlet and outlet are,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.20 \times 1200}{60} = 12.56 \text{ m/s}$$

and $$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13 \text{ m/s.}$$

From inlet velocity triangle, $\tan\theta = \dfrac{V_{f_1}}{u_1} = \dfrac{V_{f_1}}{12.56}$

$\therefore$ $V_{f_1} = 12.56 \tan\theta = 12.56 \times \tan 20° = 4.57$ m/s

$\therefore$ $V_{f_2} = V_{f_1} = 4.57$ m/s.

From outlet velocity triangle, $\tan\phi = \dfrac{V_{f_2}}{u_2 - V_{w_2}} = \dfrac{4.57}{25.13 - V_{w_2}}$

or $$25.13 - V_{w_2} = \frac{4.57}{\tan\phi} = \frac{4.57}{\tan 30°} = 7.915$$

$\therefore$ $V_{w_2} = 25.13 - 7.915 = 17.215$ m/s.

The work done by impeller per kg of water per second is given by equation (19.1) as

$$= \frac{1}{g} V_{w_2} u_2 = \frac{17.215 \times 25.13}{9.81} = \textbf{44.1 Nm/N. Ans.}$$

Problem 19.2 *A centrifugal pump is to discharge 0.118 m³/s at a speed of 1450 r.p.m. against a head of 25 m. The impeller diameter is 250 mm, its width at outlet is 50 mm and manometric efficiency is 75%. Determine the vane angle at the outer periphery of the impeller.*

Solution. Given :

Discharge, $Q = 0.118 \text{ m}^3/\text{s}$

Speed, $N = 1450$ r.p.m.

Head, $H_m = 25$ m

Diameter at outlet, $D_2 = 250 \text{ mm} = 0.25 \text{ m}$

Width at outlet, $B_2 = 50 \text{ mm} = 0.05 \text{ m}$

Manometric efficiency, $\eta_{man} = 75\% = 0.75.$

Let vane angle at outlet $= \phi$

Tangential velocity of impeller at outlet,

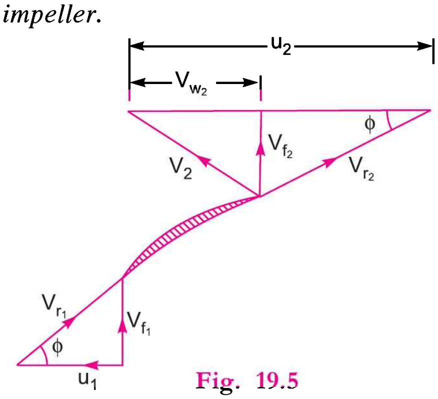

Fig. 19.5

* If in the problem, this condition is not given even then the water is assumed to be entering radially unless stated otherwise

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1450}{60} = 18.98 \text{ m/s}$$

Discharge is given by $Q = \pi D_2 B_2 \times V_{f_2}$

$$\therefore \quad V_{f_2} = \frac{Q}{\pi D_2 B_2} = \frac{0.118}{\pi \times 0.25 \times .05} = 3.0 \text{ m/s.}$$

Using equation (19.8), $\eta_{man} = \dfrac{gH_m}{V_{w_2} u_2} = \dfrac{9.81 \times 25}{V_{w_2} \times 18.98}$

$$\therefore \quad V_{w_2} = \frac{9.81 \times 25}{\eta_{man} \times 18.98} = \frac{9.81 \times 25}{0.75 \times 18.98} = 17.23.$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{\left(u_2 - V_{w_2}\right)} = \frac{3.0}{(18.98 - 17.23)} = 1.7143$$

$$\therefore \quad \phi = \tan^{-1} 1.7143 = 59.74° \text{ or } \mathbf{59° \; 44'.} \textbf{ Ans.}$$

Problem 19.3 *A centrifugal pump delivers water against a net head of 14.5 metres and a design speed of 1000 r.p.m. The vanes are curved back to an angle of 30° with the periphery. The impeller diameter is 300 mm and outlet width is 50 mm. Determine the discharge of the pump if manometric efficiency is 95%.*

Solution. Given :

Net head, $H_m = 14.5$ m

Speed, $N = 1000$ r.p.m.

Vane angle at outlet, $\phi = 30°$

Impeller diameter means the diameter of the impeller at outlet

∴ Diameter, $D_2 = 300 \text{ mm} = 0.30$ m

Outlet width, $B_2 = 50 \text{ mm} = 0.05$ m

Manometric efficiency, $\eta_{man} = 95\% = 0.95$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1000}{60} = 15.70 \text{ m/s.}$$

Now using equation (19.8), $\eta_{man} = \dfrac{gH_m}{V_{w_2} \times u_2}$

$$\therefore \quad 0.95 = \frac{9.81 \times 14.5}{V_{w_2} \times 15.70}$$

$$\therefore \quad V_{w_2} = \frac{0.95 \times 14.5}{0.95 \times 15.70} = 9.54 \text{ m/s.}$$

Refer to Fig. 19.5. From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} \text{ or } \tan 30° = \frac{V_{f_2}}{(15.70 - 9.54)} = \frac{V_{f_2}}{6.16}$$

$$\therefore \quad V_{f_2} = 6.16 \times \tan 30° = 3.556 \text{ m/s.}$$

∴ Discharge, $Q = \pi D_2 B_2 \times V_{f_2}$

$$= \pi \times 0.30 \times 0.05 \times 3.556 \text{ m}^3\text{/s} = \mathbf{0.1675 \text{ m}^3\text{/s.}} \textbf{ Ans.}$$

Problem 19.4 *A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 r.p.m. works against a total head of 40 m. The velocity of flow through the impeller is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50 mm, determine :*

(*i*) *Vane angle at inlet,* (*ii*) *Work done by impeller on water per second, and*

(*iii*) *Manometric efficiency.*

Solution. Given :

Speed, $N = 1000$ r.p.m.

Head, $H_m = 40$ m

Velocity of flow, $V_{f_1} = V_{f_2} = 2.5$ m/s

Vane angle at outlet, $\phi = 40°$

Outer dia. of impeller, $D_2 = 500$ mm $= 0.50$ m

Inner dia. of impeller, $D_1 = \dfrac{D_2}{2} = \dfrac{0.50}{2} = 0.25$ m

Width at outlet, $B_2 = 50$ mm $= 0.05$ m

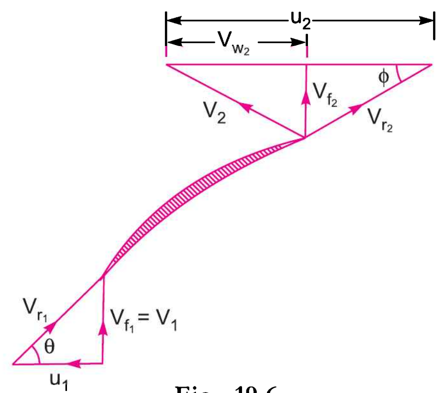

Fig. 19.6

Tangential velocity of impeller at inlet and outlet are

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/s}$$

and $$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 1000}{60} = 26.18 \text{ m/s.}$$

Discharge is given by, $Q = \pi D_2 B_2 \times V_{f_2} = \pi \times 0.50 \times .05 \times 2.5 = 0.1963 \text{ m}^3\text{/s.}$

(*i*) **Vane angle at inlet (θ).**

From inlet velocity triangle $\tan\theta = \dfrac{V_{f_1}}{u_1} = \dfrac{2.5}{13.09} = 0.191$

$$\therefore \quad \theta = \tan^{-1} .191 = 10.81° \text{ or } \mathbf{10° \, 48'}. \quad \textbf{Ans.}$$

(*ii*) **Work done by impeller on water per second** is given by equation (19.2) as

$$= \frac{W}{g} \times V_{w_2} u_2 = \frac{\rho \times g \times Q}{g} \times V_{w_2} \times u_2$$

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times V_{w_2} \times 26.18 \quad \ldots(i)$$

But from outlet velocity triangle, we have

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.5}{(26.18 - V_{w_2})}$$

$$\therefore \quad 26.18 - V_{w_2} = \frac{2.5}{\tan\phi} = \frac{2.5}{\tan 40°} = 2.979$$

$$\therefore \quad V_{w_2} = 26.18 - 2.979 = 23.2 \text{ m/s.}$$

Substituting this value of V_{w_2} in equation (*i*), we get the work done by impeller as

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times 23.2 \times 26.18$$

$$= \mathbf{119227.9 \text{ Nm/s.}} \quad \textbf{Ans.}$$

(*iii*) **Manometric efficiency (η_{man}).** Using equation (19.8), we have

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 0.646 = \mathbf{64.4\%. \ Ans.}$$

Problem 19.5 *A centrifugal pump discharges 0.15 m³/s of water against a head of 12.5 m, the speed of the impeller being 600 r.p.m. The outer and inner diameters of impeller are 500 mm and 250 mm respectively and the vanes are bent back at 35° to the tangent at exit. If the area of flow remains 0.07 m² from inlet to outlet, calculate :*

(i) Manometric efficiency of pump, (ii) Vane angle at inlet, and

(iii) Loss of head at inlet to impeller when the discharge is reduced by 40% without changing the speed.

Solution. Given :

Discharge, $Q = 0.15$ m³/s

Head, $H_m = 12.5$ m

Speed, $N = 600$ r.p.m.

Outer dia., $D_2 = 500$ mm $= 0.50$ m

Inner dia., $D_1 = 250$ mm $= 0.25$ m

Vane angle at outlet, $\phi = 35°$

Area of flow, $= 0.07$ m²

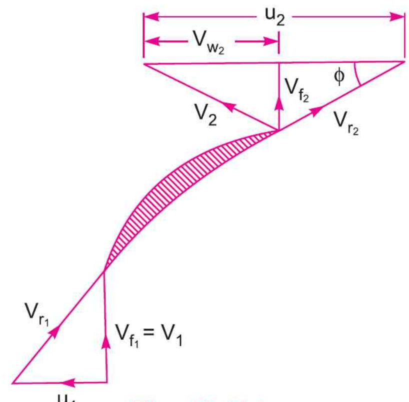

Fig. 19.6 (*a*)

As area of flow is constant from inlet to outlet, then velocity of flow will be constant from inlet to outlet.

Discharge = Area of flow × Velocity of flow

or 0.15 = 0.07 × Velocity of flow

∴ Velocity of flow $= \frac{0.15}{0.07} = 2.14$ m/s.

∴ $V_{f_1} = V_{f_2} = 2.14$ m/s.

Tangential velocity of impeller at inlet and outlet are

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 600}{60} = 7.85 \text{ m/s}$$

and

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 600}{60} = 15.70 \text{ m/s}$$

From outlet velocity triangle, $V_{w_2} = u_2 - \frac{V_{f_2}}{\tan \phi} = 15.70 - \frac{2.14}{\tan 35°} = 12.64$ m/s

(*i*) *Manometric efficiency of the pump*

Using equation (19.8), we have $\eta_{man} = \frac{g \times H_m}{V_{w_2} \times u_2} = \frac{9.81 \times 12.5}{12.64 \times 15.7} = 0.618$ or **61.8%. Ans.**

(*ii*) *Vane angle at inlet* (θ)

From inlet velocity triangle, $\tan \theta = \frac{V_{f_1}}{u_1} = \frac{2.14}{7.85} = 0.272$

∴ $\theta = \tan^{-1} 0.272 = \mathbf{15° \ 12'. \ Ans.}$

(*iii*) *Loss of head at inlet to impeller when discharge is reduced by 40% without changing the speed.*

When there is an increase or decrease in the discharge from the normal discharge, a loss of head occurs at entry due to shock. In this case, discharge is reduced by 40%. Hence the new discharge is given by,

$$Q^* = 0.6 \times Q$$

where $Q = 0.15 \text{ m}^3/\text{s}$

As area of flow is constant, hence new velocity of flow ($V_{f_1}{}^*$) will be given by,

$$V_{f_1}{}^* = \frac{Q^*}{\text{Area of flow}}$$

$$= \frac{0.6 \times Q}{0.07} = \frac{0.6 \times 0.15}{0.07} = 1.284 \text{ m/s.}$$

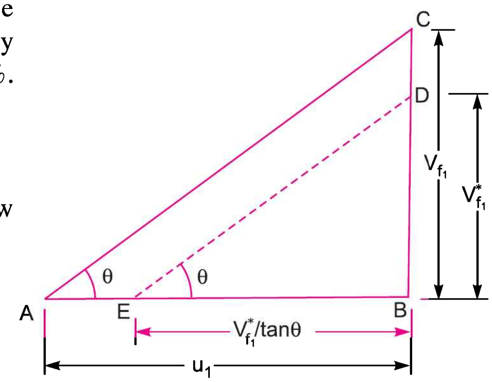

Fig. 19.6 (*b*)

Fig. 19.6 (*b*) shows the velocity triangle at inlet corresponding to normal discharge and reduced discharge. *ABC* is the velocity triangle due to normal discharge. Triangle *BDE* is corresponding to reduced discharge *BD* = 1.284 m/s and *DE* is parallel to *AC*.

The blade angle θ at inlet cannot change and hence *DE* will be parallel to *AC*.

There will be a sudden change in the tangential velocity from *AB* to *BE*. Hence due to this shock, there will be a loss of head at inlet.

$$\therefore \quad \text{Head lost at inlet} = \frac{(\text{change in tangential velocity at inlet})^2}{2g}$$

$$= \frac{(AB - BE)^2}{2g} = \frac{\left(u_1 - \dfrac{V_{f_1}^*}{\tan\theta}\right)^2}{2g} = \frac{\left(7.85 - \dfrac{1.284}{\tan 15.2°}\right)^2}{2 \times 9.81} = \textbf{0.5 m. Ans.}$$

Problem 19.6 *The outer diameter of an impeller of a centrifugal pump is 400 mm and outlet width is 50 mm. The pump is running at 800 r.p.m. and is working against a total head of 15 m. The vanes angle at outlet is 40° and manometric efficiency is 75%. Determine :*

(i) velocity of flow at outlet, *(ii) velocity of water leaving the vane,*

(iii) angle made by the absolute velocity at outlet with the direction of motion at outlet, and

(iv) discharge.

Solution. Given :

Outer diameter, $D_2 = 400 \text{ mm} = 0.4 \text{ m}$

Width at outlet, $B_2 = 50 \text{ mm} = 0.05 \text{ m}$

Speed, $N = 800$ r.p.m.

Head, $H_m = 15$ m

Vane angle at outlet, $\phi = 40°$

Manometric efficiency, $\eta_{man} = 75\% = 0.75$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 800}{60} = 16.75 \text{ m/s.}$$

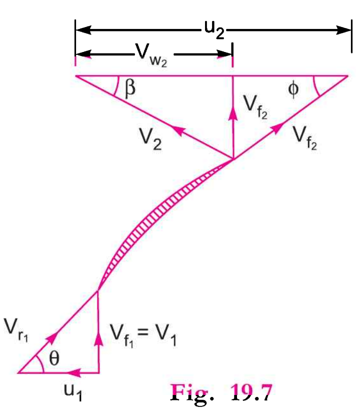

Fig. 19.7

Using equation (19.8), $\eta_{man} = \dfrac{gH_m}{V_{w_2} u_2}$

$$0.75 = \frac{9.81 \times 15}{V_{w_2} \times 16.75}$$

$$\therefore \quad V_{w_2} = \frac{9.81 \times 15}{0.75 \times 16.75} = 11.71 \text{ m/s.}$$

From the outlet velocity triangle, we have

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{V_{f_2}}{(16.75 - 11.71)} = \frac{V_{f_2}}{5.04}$$

(*i*) $\therefore \quad V_{f_2} = 5.04 \tan\phi = 5.04 \times \tan 40° =$ **4.23 m/s. Ans.**

(*ii*) **Velocity of water leaving the vane** (V_2).

$$V_2 = \sqrt{V_{f_2}^2 + V_{w_2}^2} = \sqrt{4.23^2 + 11.71^2}$$

$$= \sqrt{17.89 + 137.12} = \mathbf{12.45 \text{ m/s.}} \quad \textbf{Ans.}$$

(*iii*) **Angle made by absolute velocity at outlet** (β),

$$\tan\beta = \frac{V_{f_2}}{V_{w_2}} = \frac{4.23}{11.71} = 0.36$$

$$\therefore \quad \beta = \tan^{-1} 0.36 = 19.80° \text{ or } \mathbf{19° \, 48'}. \quad \textbf{Ans.}$$

(*iv*) **Discharge** through pump is given by,

$$Q = \pi D_2 B_2 \times V_{f_2} = \pi \times 0.4 \times 0.05 \times 4.23 = \mathbf{0.265 \text{ m}^3\text{/s.}} \quad \textbf{Ans.}$$

Problem 19.7 *A centrifugal pump is running at 1000 r.p.m. The outlet vane angle of the impeller is 45° and velocity of flow at outlet is 2.5 m/s. The discharge through the pump is 200 litres/s when the pump is working against a total head of 20 m. If the manometric efficiency of the pump is 80%, determine :*

(*i*) *the diameter* of the impeller, and* (*ii*) *the width of the impeller at outlet.*

Solution. Given :

Speed, $N = 1000$ r.p.m.

Outlet vane angle, $\phi = 45°$

Velocity of flow at outlet, $V_{f_2} = 2.5$ m/s

Discharge, $Q = 200$ litres/s $= 0.2$ m^2/s

Head, $H_m = 20$ m

Manometric efficiency, $\eta_{man} = 80\% = 0.80$

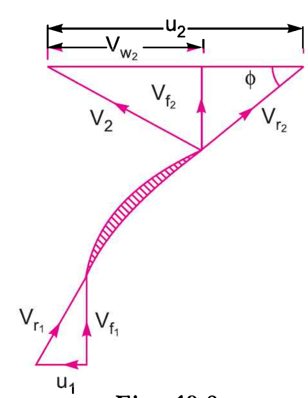

Fig. 19.8

From outlet velocity triangle, we have

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

or $$u_2 - V_{w_2} = \frac{V_{f_2}}{\tan\phi} = \frac{2.5}{\tan 45} = 2.5$$

$$\therefore \quad V_{w_2} = (u_2 - 2.5) \quad \text{...}(i)$$

* Diameter of impeller means the outside diameter.

Using equation (19.8), $$\eta_{man} = \frac{gH_m}{V_{w_2} u_2}$$

$$0.80 = \frac{9.81 \times 20}{V_{w_2} u_2}$$

$$\therefore \quad V_{w_2} u_2 = \frac{9.81 \times 20}{0.80} = 245.25 \quad \text{...}(ii)$$

Substituting the value of V_{w_2} from equation (*i*) in (*ii*), we get

$$(u_2 - 2.5)u_2 = 245.25$$

$$u_2^2 - 2.5u_2 - 245.25 = 0$$

which is a quadratic equation in u_2 and its solution is

$$u_2 = \frac{2.5 \pm \sqrt{(2.5)^2 + 4 \times 245.25}}{2} = \frac{2.5 + \sqrt{6.25 + 981}}{2}$$

$$= \frac{2.5 \pm 31.42}{2} = 16.96 \text{ or } - 14.46$$

$$\therefore \quad u_2 = 16.96 \quad (\because \text{ –ve value is not possible})$$

(*i*) *Diameter of impeller* (D_2).

Using, $$u_2 = \frac{\pi D_2 N}{60}$$

$$\therefore \quad 16.96 = \frac{\pi D_2 N}{60} = \frac{\pi \times D_2 \times 1000}{60}$$

$$\therefore \quad D_2 = \frac{16.96 \times 60}{\pi \times 1000} = 0.324 \text{ m} = \mathbf{324 \text{ mm. Ans.}}$$

(*ii*) *Width of impeller at outlet* (B_2).

Discharge, $$Q = \pi D_2 B_2 V_{f_2}$$

$$0.2 = \pi \times .324 \times B_2 \times 2.5$$

$$\therefore \quad B_2 = \frac{0.2}{\pi \times .324 \times 2.5} = 0.0786 \text{ m} = \mathbf{78.6 \text{ mm. Ans.}}$$

Problem 19.7 (A) *A centrifugal pump has the following dimensions : inlet radius = 80 mm ; outlet radius = 160 mm ; width of impeller at the inlet = 50 mm ; β_1 = 0.45 radians ; β_2 = 0.25 radians ; width of impeller at outlet = 50 mm.*

Assuming shockless entry determine the discharge and the head developed by the pump when the impeller rotates at 90 radians/second.

Solution. Given :

Inlet radius, R_1 = 80 mm = 0.08 m

Outlet radius R_2 = 160 mm = 0.16 m

Width at inlet, B_1 = 50 mm = 0.05 m

Width at outlet, B_2 =50 mm = 0.05 m

Angles, β_1 = 0.45 radians and β_2 = 0.25 radians.

Here β_1 is the vane angle at inlet and β_2 is the vane angle at outlet.

$\therefore$ Vane angle at inlet, $\theta = \beta_1 = 0.45$ radians

Vane angle at outlet, $\phi = \beta_2 = 0.25$ radians.

Angular velocity, $\omega = 90$ rad/s

Find :

(*i*) discharge, and (*ii*) head developed.

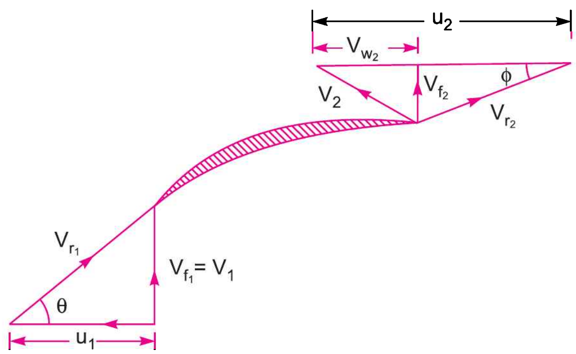

Fig. 19.8 (*a*)

Now tangential velocity of impeller at inlet and outlet are,

$$u_1 = \frac{\pi D_1 \times N}{60} = \frac{2\pi N}{60} \times \frac{D_1}{2} = \omega \times R_1 = 90 \times 0.08 = 7.2 \text{ m/s}$$

and $$u_2 = \omega \times R_2 = 90 \times 0.16 = 14.4 \text{ m/s}$$

From inlet velocity triangle, $\tan\theta = \dfrac{V_{f_1}}{u_1}$

$\therefore$ $$V_{f_1} = u_1 \times \tan\theta = 7.2 \times \tan(0.45 \text{ rad}) = 7.2 \times 0.483 = 3.478 \text{ m/s}$$

(*i*) *Discharge* (*Q*)

Discharge is given by, $$Q = \pi D_1 \times B_1 \times V_{f_1} = \pi \times (2R_1) \times B_1 \times V_{f_1}$$
$$= \pi \times 2 \times 0.08 \times 0.05 \times 3.478 \text{ m}^3\text{/s} = \mathbf{0.0874\ m^3/s.\ Ans.}$$

(*ii*) *Head developed* (H_m)

For the shockless entry, the losses of the pump will be zero. Hence, the head developed (H_m) will be given by equation (19.5).

$\therefore$ $$H_m = \frac{V_{w_2} \times u_2}{g} \quad ...(i)$$

where from outlet velocity triangle, $V_{w_2} = u_2 - V_{f_2} \times \cot\phi$

The value of V_{f_2} is obtained from $Q = \pi D_2 \times B_2 \times V_{f_2}$

or $$0.0874 = \pi \times (2R_2) \times B_2 \times V_{f_2}$$
$$= \pi \times (2 \times 0.16) \times 0.05 \times V_{f_2}$$

$\therefore$ $$V_{f_2} = \frac{0.0874}{\pi \times 2 \times 0.16 \times 0.05} = 1.7387 \text{ m/s}$$

$\therefore$ $$V_{w_2} = u_2 - V_{f_2} \times \cot\phi$$

$= 14.4 - 1.7387 \times \cot (0.25 \text{ radians})$

$= 14.4 - 1.7387 \times 3.9163 = 14.4 - 6.809 = 7.591$ m/s

Substituting this value in equation (*i*) above, we get

$$H_m = \frac{V_{w_2} \times u_2}{g} = \frac{7.591 \times 14.4}{9.81} = \mathbf{11.142 \text{ m. Ans.}}$$

Problem 19.8 *The internal and external diameter of an impeller of a centrifugal pump which is running at 1000 r.p.m., are 200 mm and 400 mm respectively. The discharge through pump is 0.04 m^3/s and velocity of flow is constant and equal to 2.0 m/s. The diameters of the suction and delivery pipes are 150 mm and 100 mm respectively and suction and delivery heads are 6 m (abs.) and 30 m (abs.) of water respectively. If the outlet vane angle is 45° and power required to drive the pump is 16.186 kW, determine :*

(i) Vane angle of the impeller at inlet, (ii) The overall efficiency of the pump, and

(iii) Manometric efficiency of the pump.

Solution. Given :

Speed, $N = 1000$ r.p.m.
Internal dia., $D_1 = 200$ mm $= 0.2$ m
External dia., $D_2 = 400$ mm $= 0.4$ m
Discharge, $Q = 0.04$ m^3/s
Velocity of flow, $V_{f_1} = V_{f_2} = 2.0$ m/s
Dia. of suction pipe, $D_s = 150$ mm $= 0.15$ m
Dia. of delivery pipe, $D_d = 100$ mm $= 0.10$ m
Suction head, $h_s = 6$ m (abs.)
Delivery head, $h_d = 30$ m (abs.)
Outlet vane angle, $\phi = 45°$
Power required to drive the pump, $P = 16.186/$ kW

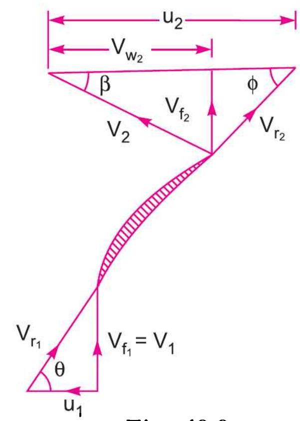

Fig. 19.9

(*i*) **Vane angle of the impeller at inlet (θ).**

From inlet velocity, we have $\tan \theta = \frac{V_{f_1}}{u_1} = \frac{2.0}{u_1}$, where $u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 1000}{60} = 10.47$ m/s

$$\therefore \quad \tan \theta = \frac{2.0}{10.47} = 0.191 \text{ or } \theta = \tan^{-1} .191 = \mathbf{10° \ 48'. \text{ Ans.}}$$

(*ii*) **Overall efficiency of the pump (η_o).**

Using equation (19.10), we have $\eta_o = \frac{\left(\frac{WH_m}{1000}\right)}{\text{S.P.}}$

where S.P. = Power required to drive the pump and equal to P here.

$$\eta_o = \frac{\left(\frac{\rho \times g \times Q \times H_m}{1000}\right)}{P} = \frac{\rho g \times Q \times H_m}{1000 \times P}$$

$$= \frac{1000 \times 9.81 \times .04 \times H_m}{1000 \times 16.186} = 0.02424 \ H_m \qquad ...(i)$$

Now H_m is given by equation (19.6) as

$$H_m = \left(\frac{p_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right) \qquad ...(ii)$$

where $\frac{p_o}{\rho g}$ = Pressure head at outlet of pump = h_d =30 m

$\frac{V_o^2}{2g}$ = Velocity head at outlet of pump = $\frac{V_d^2}{2g}$

$\frac{p_i}{\rho g}$ = Pressure head at inlet of pump = h_s = 6 m

$\frac{V_i^2}{2g}$ = Velocity head at inlet of pump = $\frac{V_s^2}{2g}$

Z_o and Z_i = Vertical height at outlet and inlet of the pump from datum line.

If $Z_o = Z_i$ then equation(*ii*) becomes as

$$H_m = \left(30 + \frac{V_d^2}{2g}\right) - \left(6 + \frac{V_s^2}{2g}\right) \qquad ...(iii)$$

Now $$V_d = \frac{\text{Discharge}}{\text{Area of delivery pipe}} = \frac{0.04}{\frac{\pi}{4}(D_d)^2} = \frac{.04}{\frac{\pi}{4} \times .1^2} = 5.09 \text{ m/s}$$

And $$V_s = \frac{.04}{\text{Area of suction pipe}} = \frac{.04}{\frac{\pi}{4} D_s^2} = \frac{.04}{\frac{\pi}{4} \times .15^2} = 2.26 \text{ m/s.}$$

Substituting these values in equation (*iii*), we get

$$H_m = \left(30 + \frac{5.09^2}{2 \times 9.81}\right) - \left(6 + \frac{2.26^2}{2 \times 9.81}\right)$$

$$= (30 + 1.32) - (6 + .26) = 31.32 - 6.26 = 25.06 \text{ m.}$$

Substituting the value of 'H_m' in equation (*i*), we get

$$\eta_o = .02424 \times 25.06 = 0.6074 = \textbf{60.74\%. Ans.}$$

(*iii*) **Manometric efficiency of the pump (η_{man}).**

Tangential velocity at outlet is given by

$$u_2 = \frac{\pi D_2 \times N}{60} = \frac{\pi \times 0.4 \times 1000}{60} = 20.94 \text{ m/s.}$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.0}{20.94 - V_{w_2}}$$

$$\therefore \quad 20.94 - V_{w_2} = \frac{2.0}{\tan \phi} = \frac{2.0}{\tan 45} = 2.0$$

$$\therefore \quad V_{w_2} = 20.94 - 2.0 = 18.94.$$

Using equation (19.8), $$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 25.06}{18.94 \times 20.94} = 0.6198 = \textbf{61.98\%. Ans.}$$

Problem 19.9 *Find the power required to drive a centrifugal pump which delivers 0.04 m³/s of water to a height of 20 m through a 15 cm diameter pipe and 100 m long. The overall efficiency of the pump is 70% and co-efficient of friction 'f' = 0.15 in the formula* $h_f = \dfrac{4fLV^2}{d \times 2g}$.

Solution. Given :

Discharge, $Q = 0.04 \text{ m}^3/\text{s}$

Height, $H_s = h_s + h_d = 20$ m

Dia. of pipe, $D_s = D_d = 15$ cm $= 0.15$ m

Length, $L_s + L_d = L = 100$ m

Overall efficiency, $\eta_o = 70\% = 0.70$

Co-efficient of friction, $f = .015$

Velocity of water in pipe, $V_s = V_d = V = \dfrac{\text{Discharge}}{\text{Area of pipe}} = \dfrac{0.04}{\frac{\pi}{4}(.15)^2} = 2.26$ m/s.

Frictional head loss in pipe,

$$(h_{f_s} + h_{f_d}) = \frac{4fLV^2}{d \times 2g} = \frac{4 \times .015 \times 100 \times 2.26^2}{.15 \times 2 \times 9.81} = 10.41 \text{ m.}$$

Using equation (19.7), we get manometric head as

$$H_m = (h_s + h_d) + (h_{f_s} + h_{f_d}) + \frac{V_d^2}{2g}$$

$$= 20 + 10.41 + \frac{2.26^2}{2 \times 9.81} \qquad (\because \quad h_s + h_d = H_s = 20 \text{ m})$$

$$= 30.41 + 0.26 = 30.67 \text{ m.}$$

Overall efficiency is given by equation (19.10) as

$$\eta_o = \frac{\left(\dfrac{WH_m}{1000}\right)}{\text{S.P.}} = \frac{\rho g \times Q \times H_m}{1000 \times \text{S.P.}}$$

$$\therefore \qquad \text{S.P.} = \frac{\rho g \times Q \times H_m}{1000 \times \eta_o} = \frac{1000 \times 9.81 \times .04 \times 30.67}{1000 \times 0.70} = \mathbf{17.19 \text{ kW. Ans.}}$$

S.P. is the power required to drive the centrifugal pump.

Problem 19.10 *Show that the pressure rise in the impeller of a centrifugal pump when frictional and other losses in the impeller are neglected is given by*

$$\frac{1}{2g}\left[V_{f_1}^2 + u_2^2 - V_{f_2}^2 \operatorname{cosec}^2 \phi\right]$$

where V_{f_1} *and* V_{f_2} *are velocity of flow at inlet and outlet,*

u_2 = *tangential velocity of impeller at outlet, and* ϕ = *vane angle at outlet.*

Solution. Let suffix 1 represents the values at the inlet and suffix 2 represents the values at the outlet of the impeller.

Applying Bernoulli's equation at the inlet and outlet of the impeller and neglecting losses from inlet to outlet, Total energy at inlet = Total energy at outlet – Work done by impeller on water

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\left(\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+Z_2\right)-\text{Work done by impeller on water per kg of water}$$

$$=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+Z_2-\frac{V_{w_2}u_2}{g} \qquad \text{(taking flow radial at inlet)}$$

If inlet and outlet of the impeller are at the same height,

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}-\frac{V_{w_2}u_2}{g} \qquad (\because \; Z_1=Z_2)$$

$$\therefore \qquad \left(\frac{p_2}{\rho g}-\frac{p_1}{\rho g}\right)=\frac{V_1^2}{2g}-\frac{V_2^2}{2g}+\frac{V_{w_2}u_2}{g}$$

But $\frac{p_2}{\rho g}-\frac{p_1}{\rho g}$ = Pressure rise in impeller

$$\therefore \quad \text{Pressure rise in impeller}=\frac{V_1^2}{2g}-\frac{V_2^2}{2g}+\frac{V_{w_2}u_2}{g} \qquad ...(i)$$

From Fig. 19.9, we have

From inlet velocity triangle, $V_1 = V_{f_1}$ ...(*ii*)

From outlet velocity triangle, $\tan\phi=\frac{V_{f_2}}{(u_2-V_{w_2})}$ or $u_2-V_{w_2}=\frac{V_{f_2}}{\tan\phi}$

$$\therefore \qquad V_{w_2}=u_2-\frac{V_{f_2}}{\tan\phi}=u_2-V_{f_2}\cot\phi \qquad ...(iii)$$

Also

$$V_2^2=V_{f_2}^2+V_{w_2}^2=V_{f_2}^2+(u_2-V_{f_2}\cot\phi)^2$$
$$=V_{f_2}^2+(u_2^2+V_{f_2}^2\cot^2\phi-2u_2V_{f_2}\cot\phi)$$
$$=V_{f_2}^2+V_{f_2}^2\cot^2\phi+u_2^2-2u_2V_{f_2}\cot\phi$$
$$=V_{f_2}^2(1+\cot^2\phi)+u_2^2-2u_2V_{f_2}\cot\phi$$
$$=V_{f_2}^2\operatorname{cosec}^2\phi+u_2^2-2u_2V_{f_2}\cot\phi \qquad (\because\; 1+\cot^2\phi=\operatorname{cosec}^2\phi) \quad ...(iv)$$

Substituting the values of V_1, V_{w_2} and V_2^2 given by equations (*ii*), (*iii*) and (*iv*) in equation (*i*), we get

$$\text{Pressure rise}=\frac{V_{f_1}^2}{2g}-\frac{1}{2g}(V_{f_2}^2\operatorname{cosec}^2\phi+u_2^2-2u_2V_{f_2}\cot\phi)+\frac{(u_2-V_{f_2}\cot\phi)\times u_2}{g}$$

$$=\frac{1}{2g}[V_{f_1}^2-V_{f_2}^2\operatorname{cosec}^2\phi-u_2^2+2u_2V_{f_2}\cot\phi+2u_2^2-2u_2V_{f_2}\times\cot\phi]$$

$$=\frac{1}{2g}[V_{f_1}^2-V_{f_2}^2\operatorname{cosec}^2\phi+u_2^2]$$

$$=\frac{1}{2g}[V_{f_1}^2+u_2^2-V_{f_2}^2\operatorname{cosec}^2\phi]. \qquad ...(19.12)$$

Problem 19.11 *Find the rise in pressure in the impeller of a centrifugal pump through which water is flowing at the rate of 0.01 m^3/s. The internal and external diameters of the impeller are 15 cm and 30 cm respectively. The widths of the impeller at inlet and outlet are 1.2 cm and 0.6 cm. The pump is running at 1500 r.p.m. The water enters the impeller radially at inlet and impeller vane angle at outlet is 45°. Neglect losses through the impeller.*

Solution. Given :

Discharge, $Q = .10 \text{ m}^3/\text{s}$

Internal dia., $D_1 = 15 \text{ cm} = 0.15 \text{ m}$

External dia., $D_2 = 30 \text{ cm} = 0.30 \text{ m}$

Width at inlet, $B_1 = 1.2 \text{ cm} = 0.012 \text{ m}$

Width at outlet, $B_2 = 0.6 \text{ cm} = 0.006 \text{ m}$

Speed, $N = 1500$ r.p.m.

Vane angle at inlet, $\phi = 45°$

Velocity of flow at outlet, $V_{f_1} = \dfrac{Q}{\text{Area of flow at inlet}} = \dfrac{.01}{\pi D_1 B_1} = \dfrac{.01}{\pi \times .15 \times .012} = 1.768 \text{ m/s}$

Velocity of flow at outlet, $V_{f_2} = \dfrac{Q}{\pi D_2 B_2} = \dfrac{.01}{\pi \times .30 \times .006} = 1.768 \text{ m/s}.$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1500}{60} = 23.56 \text{ m/s}.$$

Using equation (19.12),

$$\text{Pressure rise} = \frac{1}{2g}\left[V_{f_1}^2 + u_2^2 - V_{f_2}^2 \operatorname{cosec}^2 \phi\right]$$

$$= \frac{1}{2g}\left[1.768^2 + 23.56^2 - 1.768^2 \operatorname{cosec}^2 45°\right]$$

But $\operatorname{cosec}^2 45° = 1 + \cot^2 45° = 1 + \dfrac{1}{\tan^2 45°} = 1 + 1 = 2$

$$\therefore \quad \text{Pressure rise} = \frac{1}{2 \times 9.81}\left[1.768^2 + 23.56^2 - 1.768^2 \times 2.0\right]$$

$$= \frac{1}{2 \times 9.81}\left[3.1258 + 555.07 - 6.25\right] = \mathbf{28.13 \text{ m. Ans.}}$$

Problem 19.12 *Prove that the manometric head of a centrifugal pump running at speed N and giving a discharge Q may be written as :*

$$H_{mano} = AN^2 + BNQ + CQ^2$$

where A, B and C are constants.

Solution. From equation (19.4), we know that the manometric head is equal to the head imparted by the impeller to the water minus the losses of head in the impeller and casing.

$$\therefore \quad \text{Manometric head} = \frac{V_{w_2} u_2}{g} - \text{Losses of head in impeller and casing}$$

or

$$H_{mano} = \frac{V_{w_2} u_2}{g} - \frac{K V_2^2}{2g} \quad \ldots(i)$$

where V_2 = Absolute velocity of water at outlet of impeller and

$K\dfrac{V_2^2}{2g}$ is the part of head not converted into pressure head and is actually lost in eddies.

Now
$$u_2 = \text{Velocity of impeller at outlet} = \frac{\pi D_2 N}{60} = K_1 N$$

where $K_1 = \dfrac{\pi D_2}{60}$ and is a constant.

From equation (19.2 *A*), we know that

$$Q = \pi D_2 B_2 \times V_{f_2}$$

$$\therefore \quad V_{f_2} = \frac{Q}{\pi D_2 B_2} = K_2 Q$$

where $K_2 = \dfrac{1}{\pi D_2 B_2}$ and is a constant for a given pump.

From Fig. 19.10, it is clear that

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\therefore \quad u_2 - V_{w_2} = \frac{V_{f_2}}{\tan \phi} = V_{f_2} \cot \phi$$

or
$$V_{w_2} = u_2 - V_{f_2} \cot \phi$$
$$= u_2 - K_2 Q \cot \phi \qquad (\because V_{f_2} = K_2 Q)$$
$$= u_2 - K_3 Q \qquad \text{where } K_3 = K_2 \cot \phi$$
$$= K_1 N - K_3 Q \qquad (\because u_2 = K_1 N)$$

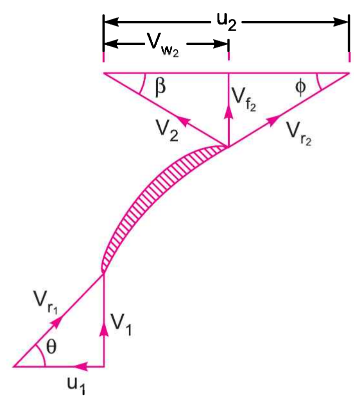

Fig. 19.10

Now from outlet velocity triangle, we know that

$$V_2^2 = V_{f_2}^2 + V_{w_2}^2$$
$$= (K_2 Q)^2 + (K_1 N - K_3 Q)^2 \qquad (\because V_{f_2} = K_2 Q \text{ and } V_{w_2} = K_1 N - K_3 Q)$$

Substituting the values of V_{w_2}, u_2 and V_2 in equation (*i*), we get

$$H_{mano} = \frac{(K_1 N - K_3 Q)(K_1 N)}{g} - \frac{K\left[K_2^2\, Q^2 + K_1^2 N^2 + K_3^2 Q^2 - 2K_1 N \times K_3 Q\right]}{2g}$$

$$= \frac{1}{2g}\left[2(K_1^2 N^2 - K_1 K_3 N Q) - KK_2^2 Q^2 - KK_1^2 N^2 - KK_3^2\, Q^2 + 2KK_1 K_3 N Q\right]$$

$$= \frac{1}{2g}\left[N^2(2K_1^2 - KK_1^2) + NQ(2KK_1 K_3 - 2K_1 K_3) + Q^2(-KK_2^2 - KK_3^2)\right]$$

$$= AN^2 + BNQ + CQ^2$$

where $A = \dfrac{2K_1^2 - KK_1^2}{2g}$, $B = \dfrac{2KK_1 K_3 - 2K_1 K_3}{2g}$ and $C = \dfrac{-KK_2^2 - KK_3^2}{2g}$ and they are constant.

▶ 19.5 MINIMUM SPEED FOR STARTING A CENTRIFUGAL PUMP

If the pressure rise in the impeller is more than or equal to manometric head (H_m), the centrifugal pump will start delivering water. Otherwise, the pump will not discharge any water, though the impeller is rotating. When impeller is rotating, the water in contact with the impeller is also rotating. This is the case of forced vortex. In case of forced vortex, the centrifugal head or head due to pressure rise in the impeller

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g} \quad \text{...}(i)$$

where ωr_2 = Tangential velocity of impeller at outlet = u_2, and
ωr_1 = Tangential velocity of impeller at inlet = u_1.

$\therefore$ Head due to pressure rise in impeller $= \dfrac{u_2^2}{2g} - \dfrac{u_1^2}{2g}$

The flow of water will commence only if

Head due to pressure rise in impeller $\geq H_m$ or $\dfrac{u_2^2}{2g} - \dfrac{u_1^2}{2g} \geq H_m$.

For minimum speed, we must have

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m \quad \text{...(19.13)}$$

But from equation (19.8), we have

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2}$$

$$\therefore \quad H_m = \eta_{man} \times \frac{V_{w_2} u_2}{g}.$$

Substituting this value of H_m in equation (19.13),

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{man} \times \frac{V_{w_2} u_2}{g} \quad \text{...(19.14)}$$

Now $u_2 = \dfrac{\pi D_2 N}{60}$ and $u_1 = \dfrac{\pi D_1 N}{60}$.

Substituting the values of u_2 and u_1 in equation (19.14),

$$\frac{1}{2g}\left(\frac{\pi D_2 N}{60}\right)^2 - \frac{1}{2g}\left(\frac{\pi D_1 N}{60}\right)^2 = \eta_{man} \times \frac{V_{w_2} \times \pi D_2 N}{g \times 60}$$

Dividing by $\dfrac{\pi N}{g \times 60}$, we get

$$\frac{\pi N D_2^2}{120} - \frac{\pi N D_1^2}{120} = \eta_{man} \times V_{w_2} \times D_2$$

or

$$\frac{\pi N}{120}[D_2^2 - D_1^2] = \eta_{man} \times V_{w_2} \times D_2$$

$$\therefore \quad N = \frac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi\left[D_2^2 - D_1^2\right]} \quad \text{...(19.15)}$$

Equation (19.15) gives the minimum starting speed of the centrifugal pump.

Problem 19.13 *The diameters of an impeller of a centrifugal pump at inlet and outlet are 30 cm and 60 cm respectively. Determine the minimum starting speed of the pump if it works against a head of 30 m.*

Solution. Given :

Dia. of impeller at inlet, $D_1 = 30 \text{ cm} = 0.30 \text{ m}$

Dia. of impeller at outlet, $D_2 = 60 \text{ cm} = 0.60 \text{ m}$

Head, $H_m = 30 \text{ cm}$

Let the minimum starting speed $= N$

Using equation (19.13) for minimum speed,

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$$

where $u_2 = \dfrac{\pi \times D_2 \times N}{60} = \dfrac{\pi \times 0.6 \times N}{60} = 0.03141\, N$

$$u_1 = \frac{\pi \times D_1 \times N}{60} = \frac{\pi \times .3 \times N}{60} = 0.0157\, N$$

$$\therefore \quad \frac{1}{2g}(0.3141\, N)^2 - \frac{1}{2g}(.0157\, N)^2 = 30$$

or $(.03141\, N)^2 - (.0157\, N)^2 = 30 \times 2 \times g = 30 \times 2 \times 9.81$

or $$N^2 = \frac{30 \times 2 \times 9.81}{(.03141^2 - .0157^2)} = \frac{588.6}{.0009866 - .0002465} = 795297.9$$

$\therefore \quad N = \sqrt{795297.9} =$ **891.8 r.p.m. Ans.**

Problem 19.14 *The diameters of an impeller of a centrifugal pump at inlet and outlet are 30 cm and 60 cm respectively. The velocity of flow at outlet is 2.0 m/s and the vanes are set back at an angle of 45° at the outlet. Determine the minimum starting speed of the pump if the manometric efficiency is 70%.*

Solution. Given :

Diameter at inlet, $D_1 = 30 \text{ cm} = 0.30 \text{ m}$

Diameter at outlet, $D_2 = 60 \text{ cm} = 0.60 \text{ m}$

Velocity of flow at outlet, $V_{f_2} = 2.0 \text{ m/s}$

Vane angle at outlet, $\phi = 45°$

Manometric efficiency, $\eta_{man} = 70\% = 0.70$.

Let the minimum starting speed $= N$.

From Fig. 19.9, for velocity triangle at outlet, we have

$$\tan \phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} \quad \text{or} \quad u_2 - V_{w_2} = \frac{V_{f_2}}{\tan \phi} = \frac{2.0}{\tan 45°} = 2.0$$

$$\therefore \quad V_{w_2} = u_2 - 2.0$$

But $$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.60 \times N}{60} = 0.03141 \text{ N}$$

$\therefore \quad V_{w_2} = (0.03141N - 2.0).$

Using equation (19.15) for minimum starting speed,

$$N = \frac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi\left[D_2^2 - D_1^2\right]} = \frac{120 \times 0.70 \times (.03141\,N - 2.0) \times 0.6}{\pi\left[.6^2 - .3^2\right]}$$

$$= \frac{50.4(.03141\,N - 2.0)}{\pi[.36 - .09]} = 59.417\ [.03141\ \text{N} - 2.0]$$

$$= 1.866\ \text{N} - 118.834$$

or $\quad 1.866\,N - N = 118.834$ or $.886\,N = 118.834$

$$\therefore \quad N = \frac{118.834}{0.866} = \mathbf{137.22\ r.p.m.\ Ans.}$$

Problem 19.15 *A centrifugal pump with 1.2 m diameter runs at 200 r.p.m. and pumps 1880 litres/s, the average lift being 6 m. The angle which the vanes make at exit with the tangent to the impeller is 26° and the radial velocity of flow is 2.5 m/s. Determine the manometric efficiency and the least speed to start pumping against a head of 6 m, the inner diameter of the impeller being 0.6 m.*

Solution. Given :

Dia. at outlet, $D_2 = 1.2$ m

Speed, $N = 200$ r.p.m.

Discharge, $Q = 1880$ litres/s $= 1.88\ m^3/s$

Manometric head, $H_m = .6$ m

Angle of vane at outlet, $\phi = 26°$

Velocity of flow at outlet, $V_{f_2} = 2.5$ m/s

Dia. at inlet, $D_1 = 0.6$ m

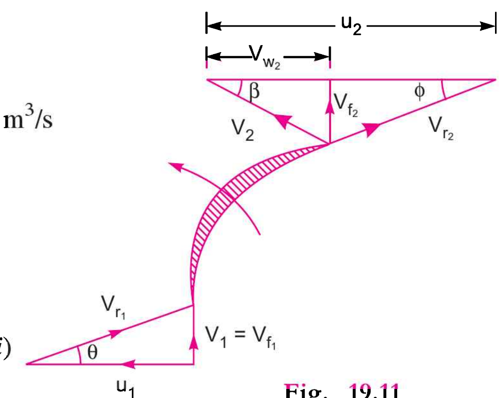

Fig. 19.11

(*i*) *Manometric efficiency* (η_{man})

Using equation (19.8), $\quad \eta_{man} = \dfrac{gH_m}{V_{w_2} \times u_2} \quad$ ...(*i*)

But $\quad u_2 = \dfrac{\pi D_2 N}{60} = \dfrac{\pi \times 1.2 \times 200}{60} = 12.56$ m/s

and $\quad \tan\phi = \dfrac{V_{f_2}}{u_2 - V_{w_2}}$ or $u_2 - V_{w_2} = \dfrac{V_{f_2}}{\tan\phi} = \dfrac{2.5}{\tan 26°} = 5.13$

$\therefore \quad V_{w_2} = u_2 - 5.13 = 12.56 - 5.13 = 7.43$ m/s.

Substituting these values in equation (*i*), we get

$$\eta_{man} = \frac{9.81 \times 6.0}{7.43 \times 12.56} = 0.63 = \mathbf{63\%.\ Ans.}$$

(*ii*) *Least speed to start the pump* :

Least speed to start the pump is given by equation(19.13),

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m \qquad \text{...}(ii)$$

where u_2 and u_1 are the tangential velocities of the vane at outlet and inlet respectively, corresponding to least speed of the pump.

But $u_2 = \omega \times r_2$ and $u_1 = \omega \times r_1$.

Substituting these values in equation (*ii*), we get

$$\frac{(\omega \times r_2)^2}{2g} - \frac{(\omega \times r_1)^2}{2g} = H_m = 6.0 \text{ or } \frac{\omega^2}{2g}\left[r_2^2 - r_1^2\right] = 6.0$$

or $$\frac{\omega^2}{2 \times 9.81}\left[0.6^2 - 0.3^2\right] = 6.0 \quad \left(\because \quad r_2 = \frac{D_2}{2} = \frac{1.2}{2} = 0.6 \text{ m and } r_1 = \frac{D_1}{2} = \frac{0.6}{2} = 0.3 \text{ m}\right)$$

$$\therefore \quad \omega^2 = \frac{6.0 \times 2.0 \times 9.81}{0.36 - .09} = 436 \quad \therefore \quad \omega = \sqrt{436} = 20.88 = \frac{2\pi N}{60}$$

$$\therefore \quad N = \frac{60 \times 20.88}{2 \times \pi} = \mathbf{200 \text{ r.p.m.}} \text{ Ans.}$$

▶ 19.6 MULTISTAGE CENTRIFUGAL PUMPS

If a centrifugal pump consists of two or more impellers, the pump is called a multistage centrifugal pump. The impellers may be mounted on the same shaft or on different shafts. A multistage pump is having the following two important functions :

1. To produce a high head, and
2. To discharge a large quantity of liquid.

If a high head is to be developed, the impellers are connected in series (or on the same shaft) while for discharging large quantity of liquid, the impellers (or pumps) are connected in parallel.

19.6.1 Multistage Centrifugal Pumps for High Heads. For developing a high head, a number of impellers are mounted in series or on the same shaft as shown in Fig. 19.12.

The water from suction pipe enters the 1st impeller at inlet and is discharged at outlet with increased pressure. The water with increased pressure from the outlet of the 1st impeller is taken to the inlet of the 2nd impeller with the help of a connecting pipe as shown in Fig. 19.12. At the outlet of the 2nd impeller, the pressure of water will be more than the pressure of water at the outlet of the 1st impeller. Thus if more impellers are mounted on the same shaft, the pressure at the outlet will be increased further.

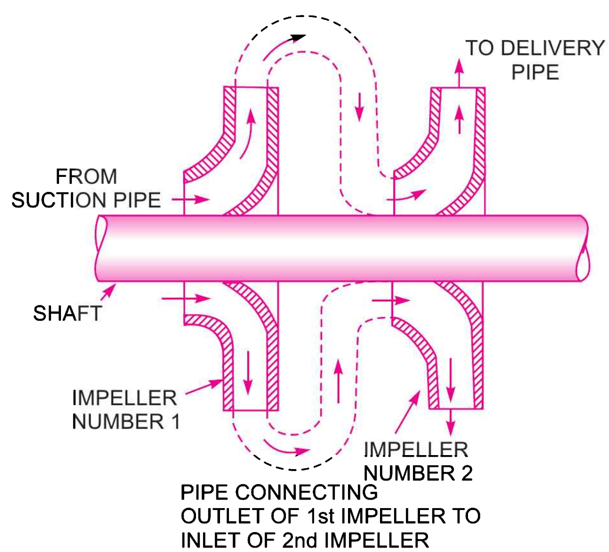

Fig. 19.12 *Two-stage pumps with impellers in series.*

Let n = Number of identical impellers mounted on the same shaft,

H_m = Head developed by each impeller.

Then total head developed

$$= n \times H_m \quad \text{...(19.16)}$$

The discharge passing through each impeller is same

19.6.2 Multistage Centrifugal Pumps for High Discharge.

For obtaining high discharge, the pumps should be connected in parallel as shown in Fig. 19.13. Each of the pumps lifts the water from a common pump and discharges water to a common pipe to which the delivery pipes of each pump is connected. Each of the pump is working against the same head.

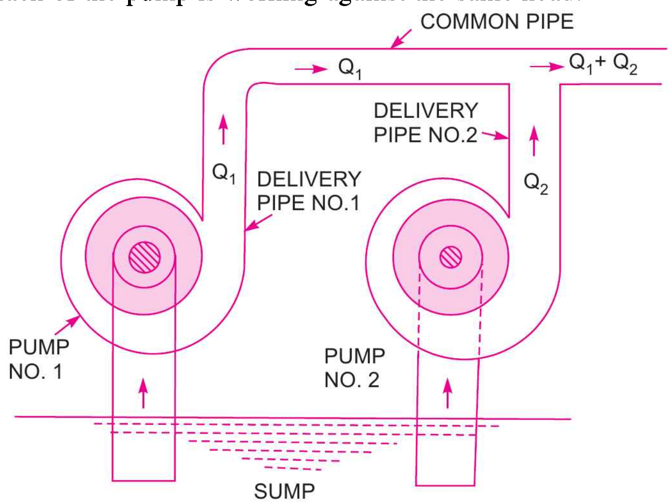

Fig. 19.13 *Pumps in parallel.*

Let n = Number of identical pumps arranged in parallel.

Q = Discharge from one pump.

$\therefore$ Total discharge $= n \times Q$ ...(19.17)

Problem 19.16 *A three stage centrifugal pump has impellers 40 cm in diameter and 2 cm wide at outlet. The vanes are curved back at the outlet at 45° and reduce the circumferential area by 10%. The manometric efficiency is 90% and the overall efficiency is 80%. Determine the head generated by the pump when running at 1000 r.p.m. delivering 50 litres per second. What should be the shaft horse power ?*

Solution. Given :

Number of stages, $n = 3$

Dia. of impeller at outlet, $D_2 = 40$ cm $= 0.40$ m

Width at outlet, $B_2 = 2$ cm $= 0.02$ m

Vane angle at outlet, $\phi = 45°$

Reduction in area at outlet $= 10\% = 0.1$

$\therefore$ Area of flow at outlet $= 0.9 \times \pi D_2 \times B_2 = 0.9 \times \pi \times .4 \times .02 = 0.02262 \text{ m}^2$

Manometric efficiency, $\eta_{man} = 90\% = 0.90$

Overall efficiency, $\eta_o = 80\% = 0.80$

Speed, $N = 1000$ r.p.m.

Discharge, $Q = 50$ litres/s $= 0.05$ m^3/s

Determine : (*i*) Head generated by the pump and

(*ii*) Shaft power.

Velocity of flow at outlet, $V_{f_2} = \dfrac{\text{Discharge}}{\text{Area of flow}} = \dfrac{0.05}{.02262} = 2.21$ m/s

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1000}{60} = 20.94 \text{ m/s}$$

Refer to Fig. 19.9. From velocity triangle at outlet,

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\therefore \quad u_2 - V_{w_2} = \frac{V_{f_2}}{\tan \phi} = \frac{2.21}{\tan 45^\circ} = 2.21 \text{ m/s}$$

$$\therefore \quad V_{w_2} = u_2 - 2.21 = 20.94 - 2.21 = 18.73 \text{ m/s}$$

Using equation (19.8), $\eta_{man} = \dfrac{gH_m}{V_{w_2} u_2}$, $0.90 = \dfrac{9.81 \times H_m}{18.73 \times 20.94}$

$$\therefore \quad H_m = \frac{0.90 \times 18.73 \times 20.94}{9.81} = 35.98 \text{ m.}$$

Using equation (19.16) for total head generated by pump,

$= n \times H_m = 3 \times 35.98 =$ **107.94 m. Ans.**

$\therefore$ Power output of the pump $= \dfrac{\text{Weight of water lifted} \times \text{Total head}}{1000}$

$$= \frac{\rho g \times Q \times 107.94}{1000} = \frac{1000 \times 9.81 \times 0.05 \times 107.94}{1000} = 52.94 \text{ kW.}$$

Using equation (19.10), we have $\eta_o = \dfrac{\text{Power output of pump}}{\text{Power input to the pump}} = \dfrac{52.94}{\text{S.P.}}$

$\therefore$ Shaft power $= \dfrac{52.94}{\eta_o} = \dfrac{52.94}{0.80} =$ **66.175 kW. Ans.**

Problem 19.17 *A four-stage centrifugal pump has four identical impellers, keyed to the same shaft. The shaft is running at 400 r.p.m. and the total manometric head developed by the multistage pump is 40 m. The discharge through the pump is 0.2 m^3/s. The vanes of each impeller are having outlet angle as 45°. If the width and diameter of each impeller at outlet is 5 cm and 60 cm respectively, find the manometric efficiency.*

Solution. Given :

Number of stage, $n = 4$

Speed, $N = 400$ r.p.m.

Total manometric head $= 40$ m

$\therefore$ Manometric head for each stage, $H_m = \dfrac{40}{4} = 10.0$ m

Discharge, $Q = 0.2\ m^3/s$

Outlet vane angle, $\phi = 45°$

Width at outlet, $B_2 = 5$ cm $= 0.05$ m

Dia. at outlet, $D_2 = 60$ cm $= 0.6$ m

Tangential velocity of impeller at outlet, $u_2 = \dfrac{\pi D_2 N}{60} = \dfrac{\pi \times 0.6 \times 400}{60} = 12.56$ m/s

Velocity of flow at outlet, $V_{f_2} = \dfrac{\text{Discharge}}{\text{Area of flow}} = \dfrac{0.20}{\pi D_2 B_2} = \dfrac{0.20}{\pi \times 0.6 \times 0.5} = 2.122$ m/s

Refer to Fig. 19.9. From velocity triangle at outlet,

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$u_2 - V_{w_2} = \frac{V_{f_2}}{\tan \phi} = \frac{2.122}{\tan 45°} = 2.122 \text{ m/s}$$

$\therefore$ $V_{w_2} = u_2 - 2.122 = 12.56 - 2.122 = 10.438$

Using equation (19.8), $\eta_{man} = \dfrac{gH_m}{V_{w_2} u_2} = \dfrac{9.81 \times 10.0}{10.438 \times 12.56} =$ **0.7482** or **74.82%. Ans.**

▶ 19.7 SPECIFIC SPEED OF A CENTRIFUGAL PUMP (N_s)

The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver *one cubic metre* of liquid per second against a head of *one metre*. It is denoted by 'N_s'.

19.7.1 Expression for Specific Speed for a Pump.

The discharge, Q, for a centrifugal pump is given by the relation

$$Q = \text{Area} \times \text{Velocity of flow}$$

$$= \pi D \times B \times V_f \text{ or } Q \propto D \times B \times V_f \quad \text{...(i)}$$

where D = Diameter of the impeller of the pump and

B = Width of the impeller.

We know that $B \propto D$

$\therefore$ From equation *(i)*, we have $Q \propto D^2 \times V_f$ ...*(ii)*

We also know that tangential velocity is given by

$$u = \frac{\pi DN}{60} \propto DN \quad \text{...(iii)}$$

Now the tangential velocity (u) and velocity of flow (V_f) are related to the manometric head (H_m) as

$$u \propto V_f \propto \sqrt{H_m} \quad \text{...(iv)}$$

Substituting the value of u in equation (*iii*), we get

$$\sqrt{H_m} \propto DN \text{ or } D \propto \frac{\sqrt{H_m}}{N}$$

Substituting the values of D in equation (*ii*),

$$Q \propto \frac{H_m}{N^2} \times V_f$$

$$\propto \frac{H_m}{N^2} \times \sqrt{H_m} \qquad [\because \text{ From equation } (iv),\ V_f \propto \sqrt{H_m}\,]$$

$$\propto \frac{H_m^{3/2}}{N^2}$$

$$\therefore \qquad Q = K\,\frac{H_m^{3/2}}{N^2} \qquad \text{...}(v)$$

where K is a constant of proportionality.

If $H_m = 1$ m and $Q = 1$ m^3/s, N becomes $= N_s$.

Substituting these values in equation (v), we get

$$1 = K\,\frac{1^{3/2}}{N_s^2} = \frac{K}{N_s^2}$$

$$\therefore \qquad K = N_s^{\,2}$$

Substituting the value of K in equation (v), we get

$$Q = N_s^{\,2}\frac{H_m^{3/2}}{N^2} \quad \text{or} \quad N_s^{\,2} = \frac{N^2 Q}{H_m^{3/2}}$$

$$\therefore \qquad N_s = \frac{N\sqrt{Q}}{H_m^{3/4}} \qquad \text{...(19.18)}$$

▶ 19.8 MODEL TESTING OF CENTRIFUGAL PUMPS

Before manufacturing the large sized pumps, their models which are in complete similarity with the actual pumps (also called prototypes) are made. Tests are conducted on the models and performance of the prototypes are predicted. The complete similarity between the model and actual pump (prototype) will exist if the following conditions are satisfied :

1. Specific speed of model = Specific speed of prototype

$$(N_s)_m = (N_s)_p \quad \text{or} \quad \left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_m = \left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_p \qquad \text{...(19.19)}$$

2. Tangential velocity (u) is given by $u = \dfrac{\pi DN}{60}$ also $u \propto \sqrt{H_m}$

$$\therefore \qquad \sqrt{H_m} \propto DN$$

$$\therefore \qquad \frac{\sqrt{H_m}}{DN} = \text{Constant} \qquad \text{...(19.19 } A)$$

or $$\left(\frac{\sqrt{H_m}}{DN}\right)_m = \left(\frac{\sqrt{H_m}}{DN}\right)_p \quad ...(19.20)$$

3. From equation (*ii*) of Art. 19.7.1, we have

$$Q \propto D^2 \times V_f \quad \text{where } V_f \propto u \propto DN$$
$$\propto D^2 \times D \times N$$
$$\propto D^3 \times N$$

$$\therefore \quad \frac{Q}{D^3 N} = \text{Constant or } \left(\frac{Q}{D^3 N}\right)_m = \left(\frac{Q}{D^3 N}\right)_p \quad ...(19.21)$$

4. Power of the pump, $$P = \frac{\rho \times g \times Q \times H_m}{75}$$

$$\therefore \quad P \propto Q \times H_m$$
$$\propto D^3 \times N \times H_m \quad (\because \; Q \propto D^3 N)$$
$$\propto D^3 N \times D^2 N^2 \quad (\because \; \sqrt{H_m} \propto DN)$$
$$\propto D^5 N^3$$

$$\therefore \quad \frac{P}{D^5 N^3} = \text{Constant} \quad \text{or} \quad \left(\frac{P}{D^5 N^3}\right)_m = \left(\frac{P}{D^5 N^3}\right)_p \quad ...(19.22)$$

Problem 19.18 *A single-stage centrifugal pump with impeller diameter of 30 cm rotates at 2000 r.p.m. and lifts 3 m³ of water per second to a height of 30 m with an efficiency of 75%. Find the number of stages and diameter of each impeller of a similar multistage pump to lift 5 m³ of water per second to a height of 200 metres when rotating at 1500 r.p.m.*

Solution. Given :

Single-stage pump :

Diameter of impeller, $D_1 = 30 \text{ cm} = 0.30 \text{ m}$

Speed, $N_1 = 2000$ r.p.m.

Discharge, $Q_1 = 3 \text{ m}^3/\text{s}$

Height, $H_{m_1} = 30$ m

Efficiency, $\eta_{man} = 75\% = 0.75$.

Multistage similar pump :

Discharge, $Q_2 = 5 \text{ m}^3/\text{s}$

Total height $= 200$ m

Let the height per stage $= H_{m_2}$

Speed, $N_2 = 1500$ r.p.m.

Diameter of each impeller $= D_2$

Specific speed should be same. Hence, applying equation (19.19) as

$$\left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_1 = \left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_2$$

$$\therefore \quad \frac{N_1\sqrt{Q_1}}{H_{m_1}^{3/4}} = \frac{N_2\sqrt{Q_2}}{H_{m_2}^{3/4}} \quad \text{or} \quad \frac{2000\times\sqrt{3}}{30^{3/4}} = \frac{1500\times\sqrt{5}}{H_{m_2}^{3/4}}$$

$$\therefore \quad H_{m_2}^{3/4} = \frac{1500\times\sqrt{5}\times 30^{3/4}}{2000\times\sqrt{3}} = \frac{1500}{2000}\times\sqrt{\frac{5}{3}}\times 12.818 = 12.411$$

$$\therefore \quad H_{m_2} = (12.411)^{4/3} = 28.71 \text{ m}$$

$$\therefore \quad \text{Number of stages} = \frac{\text{Total head}}{\text{Head per stage}} = \frac{200}{28.71} = 6.96 \simeq \mathbf{7.\ Ans.}$$

Using equation (19.20), we have

$$\frac{\sqrt{H_{m_1}}}{D_1 N_1} = \frac{\sqrt{H_{m_2}}}{D_2 N_2} \quad \text{or} \quad \frac{\sqrt{30}}{0.30\times 2000} = \frac{\sqrt{28.71}}{D_2\times 1500}$$

$$\therefore \quad D_2 = \frac{0.30\times 2000\times\sqrt{28.71}}{1500\times\sqrt{30}} = 0.3913 \text{ m} = \mathbf{391.3\ mm.\ \ Ans.}$$

Problem 19.19 *Find the number of pumps required to take water from a deep well under a total head of 89 m. All the pumps are identical and are running at 800 r.p.m. The specific speed of each pump is given as 25 while the rated capacity of each pump is 0.16 m^3/s.*

Solution. Given :

Total head = 89 m

Speed, $N = 800$ r.p.m.

Specific speed, $N_s = 25$

Rate capacity, $Q = 0.16 \text{ m}^3\text{/s}$

Let H_m = Head developed by each pump.

Using equation (19.18), $N_s = \dfrac{N\sqrt{Q}}{H_m^{3/4}}$

$$25 = \frac{800\times\sqrt{0.16}}{H_m^{3/4}}$$

$$\therefore \quad H_m^{3/4} = \frac{800\times\sqrt{0.16}}{25} = 12.8$$

$$\therefore \quad H_m = (12.8)^{4/3} = 29.94 \text{ m}$$

$$\therefore \quad \text{Number of pumps required} = \frac{\text{Total head}}{\text{Head developed by one pump}} = \frac{89}{29.94} \simeq \mathbf{3.\ Ans.}$$

As the total head is more than the head developed by one pump, the pumps should be connected in series.

Problem 19.20 *Two geometrically similar pumps are running at the same speed of 1000 r.p.m. One pump has an impeller diameter of 0.30 metre and lifts water at the rate of 20 litres per second against a head of 15 metres. Determine the head and impeller diameter of the other pump to deliver half the discharge.*

Solution. Given :

For pump No. 1,

Speed, $N_1 = 1000$ r.p.m.

Diameter, $D_1 = 0.30$ m

Discharge, $Q_2 = 20$ litres/s $= 0.02\ m^3/s$

Head, $H_{m_1} = 15$ m

For pump No.2,

Speed, $N_2 = 1000$ r.p.m.

Discharge, $Q_2 = \frac{Q_1}{2} = \frac{20}{2} = 10$ litres/s $= 0.01\ m^3/s$.

Let D_2 = Diameter of impeller

H_{m_2} = Head developed.

Using equation (19.19), $\frac{N_1\sqrt{Q_1}}{H_{m_1}^{3/4}} = \frac{N_2\sqrt{Q_2}}{H_{m_2}^{3/4}}$

$$\therefore \qquad \frac{1000 \times \sqrt{.02}}{15^{3/4}} = \frac{1000 \times \sqrt{.01}}{H_{m_2}^{3/4}}$$

or
$$H_{m_2}^{3/4} = \frac{1000 \times \sqrt{.01} \times 15^{3/4}}{1000 \times \sqrt{.02}} = \sqrt{\frac{.01}{.02}} \times 7.622 = 5.389$$

$$\therefore \qquad H_{m_2} = (5.389)^{4/3} = \mathbf{9.44\ m.\ Ans.}$$

Using equation (19.20), $\left(\frac{\sqrt{H_m}}{DN}\right)_1 = \left(\frac{\sqrt{H_m}}{DN}\right)_2$ or $\frac{\sqrt{H_{m_1}}}{D_1N_1} = \frac{\sqrt{H_{m_2}}}{D_2N_2}$

$$\frac{\sqrt{15}}{0.3 \times 1000} = \frac{\sqrt{9.44}}{D_2 \times 1000}$$

$$\therefore \qquad D_2 = \frac{\sqrt{9.44} \times 0.3}{\sqrt{15}} = 0.238 \text{ m} = \mathbf{238.0\ mm.\ Ans.}$$

Problem 19.21 *The diameter of a centrifugal pump, which is discharging 0.03 m^3/s of water against a total head of 20 m is 0.40 m. The pump is running at 1500 r.p.m. Find the head, discharge and ratio of powers of a geometrically similar pump of diameter 0.25 m when it is running at 3000 r.p.m.*

Solution. Given :

Centrifugal pump,

Discharge, $Q_1 = .03\ m^3/s$

Head, $H_{m_1} = 20$ m

Diameter, $D_1 = 0.40$ m

Speed, $N_1 = 1500$ r.p.m.

Geometrically similar pump,

Diameter, $D_2 = 0.25$ m

Speed, $N_2 = 3000$ r.p.m.

Let Head on similar group $= H_{m_2}$

Discharge on similar pump $= Q_2$

Using equation (19.21), $\left(\frac{Q}{D^3 N}\right)_1 = \left(\frac{Q}{D^3 N}\right)_2$

$$\therefore \quad \frac{Q_1}{D_1^3 N_1} = \frac{Q_2}{D_2^3 N_2}$$

$$\frac{.03}{.40^3 \times 1500} = \frac{Q_2}{0.25^3 \times 3000}$$

$$\therefore \quad Q_2 = \frac{.03 \times .25^3 \times 3000}{.40^3 \times 1500} = .03 \times \left(\frac{.25}{.40}\right)^3 \times 2.0 = \mathbf{0.01465\ m^3/s.\ Ans.}$$

Using equation (19.20), we have

$$\left(\frac{\sqrt{H_m}}{DN}\right)_1 = \left(\frac{\sqrt{H_m}}{DN}\right)_2$$

or $$\frac{\sqrt{H_{m_1}}}{D_1 N_1} = \frac{\sqrt{H_{m_2}}}{D_2 N_2} \quad \therefore \quad \frac{\sqrt{20}}{0.40 \times 1500} = \frac{\sqrt{H_{m_2}}}{0.25 \times 3000}$$

or $$\sqrt{H_{m_2}} = \frac{\sqrt{20} \times 0.25 \times 3000}{0.40 \times 1500} = 5.59$$

$$\therefore \quad H_{m_2} = (5.59)^2 = \mathbf{31.25\ m.\ Ans.}$$

Using equation (19.22), we have

$$\left(\frac{P}{D^5 N^3}\right)_1 = \left(\frac{P}{D^5 N^3}\right)_2$$

$$\therefore \quad \frac{P_1}{D_1^5 N_1^3} = \frac{P_2}{D_2^5 N_2^3} \quad \text{or} \quad \frac{P_1}{P_2} = \frac{D_1^5 N_1^3}{D_2^5 N_2^3} = \left(\frac{D_1}{D_2}\right)^5 \times \left(\frac{N_1}{N_2}\right)^3$$

$$= \left(\frac{0.40}{0.25}\right)^5 \times \left(\frac{1500}{3000}\right)^3 = 10.485 \times .125 = \mathbf{1.31.\ Ans.}$$

Problem 19.22 *A one-fifth scale model of a pump was tested in a laboratory at 1000 r.p.m. The head developed and the power input at the best efficiency point were found to be 8 m and 30 kW respectively. If the prototype pump has to work against a head of 25 m, determine its working speed, the power required to drive it and the ratio of the flow rates handled by the two pumps.*

Solution. Given :

One-fifth scale model means that the ratio of linear dimensions of a model and its prototype is equal to 1/5.

Speed of model, $N_m = 1000$ r.p.m.

Head of model, $H_m = 8$ m

Power of model, $P_m = 30$ kW

Head of prototype, $H_p = 25$ m

Let N_p = Speed of prototype

P_p = Power of prototype

Q_p = Flow rate of prototype

Q_m = Flow rate of model

(*i*) *Speed of prototype*

Using equation (19.20), we get

$$\left(\frac{\sqrt{H}}{DN}\right)_m = \left(\frac{\sqrt{H}}{DN}\right)_p \quad \text{or} \quad \frac{\sqrt{H_m}}{D_m N_m} = \frac{\sqrt{H_p}}{D_p N_p}$$

or
$$N_p = \frac{\sqrt{H_p}}{\sqrt{H_m}} \times \frac{D_m}{D_p} \times N_m$$

$$= \frac{\sqrt{25}}{\sqrt{8}} \times \frac{1}{5} \times 1000 \qquad \left(\because \frac{D_m}{D_p} = \frac{1}{5}\right)$$

$= \mathbf{353.5}$ **r.p.m. Ans.**

(*ii*) *Power developed by prototype*

Using equation (19.22), we get

$$\left(\frac{P}{D^5 N^3}\right)_m = \left(\frac{P}{D^5 N^3}\right)_p \quad \text{or} \quad \frac{P_m}{D_m^5 N_m^3} = \frac{P_p}{D_p^5 N_p^3}$$

or
$$P_p = P_m \times \left(\frac{D_p}{D_m}\right)^5 \times \left(\frac{N_p}{N_m}\right)^3 = 30 \times 5^5 \times \left(\frac{353.5}{1000}\right)^3 \qquad \left(\because \frac{D_p}{D_m} = \frac{5}{1}\right)$$

$= 30 \times 3125 \times 0.04419 = \mathbf{4143}$ **kW. Ans.**

(*iii*) *Ratio of the flow rates of two pumps (i.e., model and prototype)*

$$\left(\frac{Q}{D^3 N}\right)_m = \left(\frac{Q}{D^3 N}\right)_p \quad \text{or} \quad \frac{Q_m}{D_m^3 N_m} = \frac{Q_p}{D_p^3 N_p}$$

or
$$\frac{Q_p}{Q_m} = \frac{D_p^3 N_p}{D_m^3 N_m} = \left(\frac{D_p}{D_m}\right)^3 \times \frac{N_p}{N_m} = 5^3 \times \frac{353.5}{1000} \qquad \left(\because \frac{D_p}{D_m} = \frac{5}{1}\right)$$

$= \mathbf{44.1875}$. **Ans.**

▶ 19.9 PRIMING OF A CENTRIFUGAL PUMP

Priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the delivery pipe upto the delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting the pump. Thus the air from these parts of the pump is removed and these parts are filled with the liquid to be pumped.

The work done by the impeller per unit weight of liquid per sec is known as the head generated by the pump. Equation (19.1) gives the head generated by the pump as $= \frac{1}{g} V_{w_2} u_2$ metre. This equation is independent of the density of the liquid. This means that when pump is running in air, the head generated is in terms of metre of air. If the pump is primed with water, the head generated is same metre of water. But as the density of air is very low, the generated head of air in terms of equivalent metre of water head is negligible and hence the water may not be sucked from the pump. To avoid this difficulty, priming is necessary.

▶ 19.10 CHARACTERISTIC CURVES OF CENTRIFUGAL PUMPS

Characteristic curves of centrifugal pumps are defined as those curves which are plotted from the results of a number of tests on the centrifugal pump. These curves are necessary to predict the behaviour and performance of the pump when the pump is working under different flow rate, head and speed. The following are the important characteristic curves for pumps :

1. Main characteristic curves,
2. Operating characteristic curves, and
3. Constant efficiency or Muschel curves.

19.10.1 Main Characteristic Curves. The main characteristic curves of a centrifugal pump consists of variation of head (manometric head, H_m), power and discharge with respect to speed. For plotting curves of manometric head *versus* speed, discharge is kept constant. For plotting curves of discharge *versus* speed, manometric head (H_m) is kept constant. And for plotting curves of power *versus* speed, the manometric head and discharge are kept constant. Fig. 19.14 shows main characteristic curves of a pump.

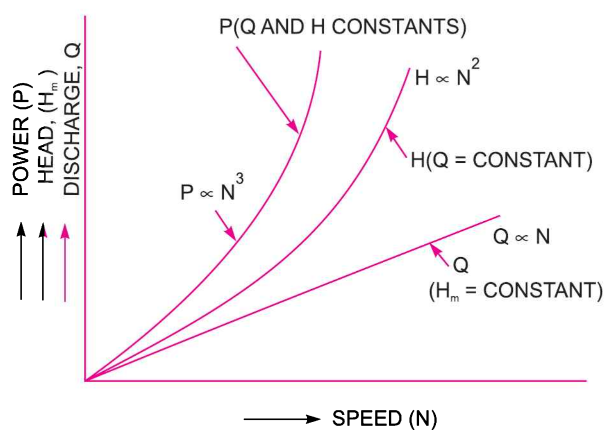

Fig. 19.14 *Main characteristic curves of a pump.*

For plotting the graph of H_m versus speed (N), the discharge is kept constant. From equation (19.19 A), it is clear that $\sqrt{H_m}/DN$ is a constant or $H_m \propto N^2$. This means that head developed by a pump is proportional to N^2. Hence the curve of H_m v/s N is a parabolic curves as shown in Fig. 19.14.

From equation (19.22), it is clear that P/D^5N^3 is a constant. Hence $P \propto N^3$. This means that the curve P v/s N is a cubic curve as shown in Fig. 19.14.

Equation (19.21), shows that $\frac{Q}{D^3N}$ = constant. This means $Q \propto N$ for a given pump. Hence the curve Q v/s N is a straight line as shown in Fig. 19.14.

19.10.2 Operating Characteristic Curves. If the speed is kept constant, the variation of manometric head, power and efficiency with respect to discharge gives the operating characteristics of the pump. Fig. 19.15 shows the operating characteristic curves of a pump.

The input power curve for pumps shall not pass through the origin. It will be slightly away from the origin on the y-axis, as even at zero discharge some power is needed to overcome mechanical losses.

The head curve will have maximum value of head when discharge is zero.

The output power curve will start from origin as at $Q = 0$, output power (ρQgH) will be zero.

The efficiency curve will start from origin as at $Q = 0$, $\eta = 0$ $\left(\because \quad \eta = \frac{\text{Output}}{\text{Input}}\right)$

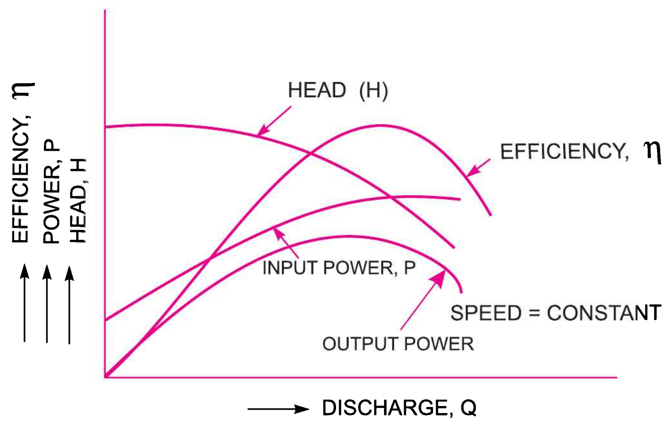

Fig. 19.15 *Operating characteristic curves of a pump.*

19.10.3 Constant Efficiency Curves. For obtaining constant efficiency curves for a pump, the head *versus* discharge curves and efficiency *versus* discharge curves for different speed are used. Fig. 19.16 (a) shows the head *versus* discharge curves for different speeds. The efficiency *versus* discharge curves for the different speeds are as shown in Fig. 19.16 (b). By combining these curves ($H \sim Q$ curves and $\eta \sim Q$ curves), constant efficiency curves are obtained as shown in Fig. 19.16 *(a)*.

For plotting the constant efficiency curves (also known as iso-efficiency curves), horizontal lines representing constant efficiencies are drawn on the $\eta \sim Q$ curves. The points, at which these lines cut the efficiency curves at various speeds, are transferred to the corresponding $H \sim Q$ curves. The points having the same efficiency are then joined by smooth curves. These smooth curves represents the iso-efficiency curves.

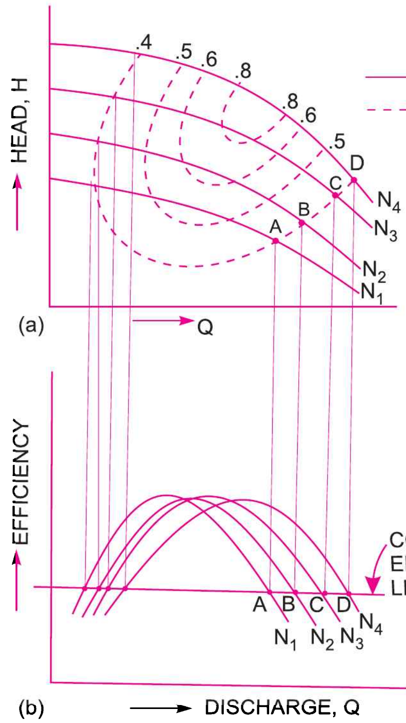

Fig. 19.16 *Constant efficiency curves of a pump.*

▶ 19.11 CAVITATION

Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure. When the vapour bubbles collapse, a very high pressure is created. The metallic surfaces, above which these vapour bubbles collapse, is subjected to these high pressures, which cause pitting action on the surface. Thus cavities are formed on the metallic surface and also considerable noise and vibrations are produced.

Cavitation includes formation of vapour bubbles of the flowing liquid and collapsing of the vapour bubbles. Formation of vapour bubbles of the flowing liquid take place only whenever the pressure in any region falls below vapour pressure. When the pressure of the flowing liquid is less than its vapour pressure, the liquid starts boiling and vapour bubbles are formed. These vapour bubbles are carried along with the flowing liquid to higher pressure zones where these vapours condense and bubbles collapse. Due to sudden collapsing of the bubbles on the metallic surface, high pressure is produced and metallic surfaces are subjected to high local stresses. Thus the surfaces are damaged.

19.11.1 Precaution Against Cavitation. The following precautions should be taken against cavitation :

(*i*) The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure. If the flowing liquid is water, then the absolute pressure head should not be below 2.5 m of water.

(*ii*) The special materials or coatings such as aluminium-bronze and stainless steel, which are cavitation resistant materials, should be used.

19.11.2 Effects of Cavitation. The following are the effects of cavitation :

(*i*) The metallic surfaces are damaged and cavities are formed on the surfaces.

(*ii*) Due to sudden collapse of vapour bubble, considerable noise and vibrations are produced.

(*iii*) The efficiency of a turbine decreases due to cavitation. Due to pitting action, the surface of the turbine blades becomes rough and the force exerted by water on the turbine blades decreases. Hence, the work done by water or output horse power becomes less and thus efficiency decreases.

19.11.3 Hydraulic Machines Subjected to Cavitation. The hydraulic machines subjected to cavitation are reaction turbines and centrifugal pumps.

19.11.4 Cavitation in Turbines. In turbines, only reaction turbines are subjected to cavitation. In reaction turbines the cavitation may occur at the outlet of the runner or at the inlet of the draft-tube where the pressure is considerably reduced (*i.e.*, which may be below the vapour pressure of the liquid flowing through the turbine). Due to cavitation, the metal of the runner vanes and draft-tube is gradually eaten away, which results in lowering the efficiency of the turbine. Hence, the cavitation in a reaction turbine can be noted by a sudden drop in efficiency. In order to determine whether cavitation will occur in any portion of a reaction turbine, the critical value of Thoma's cavitation factor (σ, sigma) is calculated.

Thoma's Cavitation Factor for Reaction Turbines. Prof. D. Thoma suggested a dimensionless number, called after his name Thoma's cavitation factor σ (sigma), which can be used for determining the region where cavitation takes place in reaction turbines. The mathematical expression for the Thoma's cavitation factor is given by

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H} \qquad ...(19.23)$$

where H_b = Barometric pressure head in m of water,

H_{atm} = Atmospheric pressure head in m of water,

H_v = Vapour pressure head in m of water,

H_s = Suction pressure at the outlet of reaction turbine in m of water or height of turbine runner above the tail water surface,

H = Net head on the turbine in m.

19.11.5 Cavitation in Centrifugal Pumps. In centrifugal pumps the cavitation may occur at the inlet of the impeller of the pump, or at the suction side of the pumps, where the pressure is considerably reduced. Hence if the pressure at the suction side of the pump drops below the vapour pressure of the liquid then the cavitation may occur. The cavitation in a pump can be noted by a sudden drop in efficiency and head. In order to determine whether cavitation will occur in any portion of the suction side of the pump, the critical value of Thoma's cavitation factor (σ) is calculated.

Thoma's Cavitation Factor for Centrifugal Pumps. The mathematical expression for Thoma's cavitation factor for centrifugal pump is given by

$$\sigma = \frac{(H_b) - H_S - h_{LS}}{H} = \frac{(H_{atm} - H_V) - H_S - h_{LS}}{H} \qquad ...(19.24)$$

where H_{atm} = Atmospheric pressure head in m of water or absolute pressure head at the liquid surface in pump,

H_V = Vapour pressure head in m of water,

H_S = Suction pressure head in m of water,

h_{LS} = Head lost due to friction in suction pipe, and

H = Head developed by the pump.

The value of Thoma's cavitation factor (σ) for a particular type of turbine or pump is calculated from equations (19.23) or (19.24). This value of Thoma's cavitation factor (σ) is compared with critical cavitation factor (σ_c) for that type of turbine pump. If the value of σ is greater than σ_c, the cavitation will not occur in that turbine or pump. The critical cavitation factor (σ_c) may be obtained from tables or empirical relationships.

The following empirical relationships are used for obtaining the value of σ_c for different turbines :

For Francis turbines, $$\sigma_c = 0.625 \left(\frac{N_s}{380.78}\right)^2 \quad ...(19.25)$$

$$\simeq 431 \times 10^{-8}\, Ns^2 \quad ...(19.26)$$

For Propeller turbines, $$\sigma_c = 0.28 + \left[\frac{1}{7.5}\left(\frac{N_s}{380.78}\right)^3\right] \quad ...(19.27)$$

In the above expressions N_s is in (r.p.m., kW, m) units. If N_s is in (r.p.m., h.p., m) units, the empirical relationships would be as follows :

For Francis turbines, $$\sigma_c = 0.625 \left(\frac{N_s}{444}\right)^2 \quad ...(19.28)$$

$$\simeq 317 \times 10^{-8} \times N_s^2 \quad ...(19.29)$$

For Propeller turbines, $$\sigma_c = 0.28 + \left[\frac{1}{7.5}\left(\frac{N_s}{444}\right)^3\right] \quad ...(19.30)$$

Problem 19.23 *A Francis turbine has been manufactured to develop 15000 horse power at the head of 81 m and speed 375 r.p.m. The mean atmospheric pressure at the site is 1.03 kgf/cm² and vapour pressure 0.03 kgf/cm². Calculate the maximum permissible height of the runner above the tail water level to ensure cavitation free operation. The critical cavitation factor for Francis turbine is given by*

$$\sigma_c = 317 \times 10^{-8} \times N_s^2$$

where N_s is the specific speed of the turbine in M.K.S. units.

Solution. Given :

Horse power developed, $P = 15000$

Head, $H = 81$ m

Speed, $N = 375$ r.p.m.

Atmospheric pressure, $p_a = 1.03 \text{ kgf/cm}^2 = 1.03 \times 9.81 \text{ N/cm}^2$

$= 1.03 \times 9.81 \times 10^4 \text{ N/m}^2$

$\therefore$ Atmospheric pressure head in meter of water,

$$H_{atm} = \frac{p_a}{\rho g} = \frac{1.03 \times 9.81 \times 10^4}{1000 \times 9.81} = 10.3 \text{ m}$$

Vapour pressure, $p_v = 0.03$ kgf/cm^2 = 0.03×9.81 N/cm^2 = $0.03 \times 9.81 \times 10^4$ N/m^2

∴ Vapour pressure head in meter of water,

$$H_v = \frac{p_v}{\rho g} = \frac{0.03 \times 9.81 \times 10^4}{1000 \times 9.81} = 0.3 \text{ m}$$

Critical cavitation factor, $\sigma_c = 317 \times 10^{-8}\ N_s^2$ ...(*i*)

where N_s is the specific speed of the turbine in M.K.S. units *i.e.*, (r.p.m., h.p., m) units.

Now specific speed of turbine is given by

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{375 \times \sqrt{15000}\ *}{81^{5/4}} = 189 \text{ r.p.m.}$$

Substituting this value in equation (*i*), we get

$$\sigma_c = 317 \times 10^{-8} \times 189^2 = 0.1132$$

Now let H_s = Suction pressure head in meter of water at the outlet of Francis turbine or height of the turbine runner above the tail water surface.

Now using equation (19.23), we get

$$\sigma_c = \frac{H_{atm} - H_v - H_s}{H} \quad \text{or} \quad 0.1132 = \frac{10.3 - 0.3 - H_s}{81}$$

or $0.1132 \times 81 = 10 - H_s$ or $H_s = 10 - 0.1132 \times 81 =$ **0.8308 m. Ans.**

Hence, maximum permissible height is 0.83 m above the tail water level.

▶ 19.12 MAXIMUM SUCTION LIFT (or SUCTION HEIGHT)

Fig. 19.17 shows a centrifugal pump that lifts a liquid from a sump. The free surface of liquid is at a depth of h_s below the pump axis. The liquid is flowing with a velocity v_s in the suction pipe.

Let h_s = Suction height (or lift)

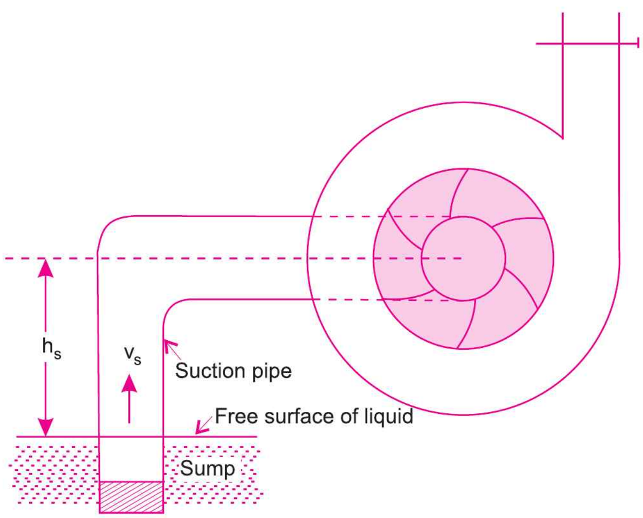

Fig. 19.17

* Here power *P* should be taken in horse power and not in kW.

Applying Bernoulli's equation at the free surface of liquid in the sump and section 1 in the suction pipe just at the inlet of the pump and taking the free surface of liquid as datum line, we get

$$\frac{p_a}{\rho g} + \frac{V_a^2}{2g} + Z_a = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + h_L \quad \text{...}(i)$$

where p_a = Atmospheric pressure on the free surface of liquid,
V_a = Velocity of liquid at the free surface of liquid $\simeq 0$,
Z_a = Height of free surface from datum line = 0,
p_1 = Absolute pressure at the inlet of pump,
V_1 = Velocity of liquid through suction pipe = v_s,
Z_1 = Height of inlet of pump from datum line = h_s,
h_L = Loss of head in the foot valve, strainer and suction pipe = h_{f_s}.

Hence equation (*i*), after substituting the above values becomes as

$$\frac{p_a}{\rho g} + 0 + 0 = \frac{p_1}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

or

$$\frac{p_a}{\rho g} = \frac{p_1}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

or

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right) \quad \text{...}(ii)$$

For finding the maximum suction lift, the pressure at the inlet of the pump should not be less than the vapour pressure of the liquid. Hence for the limiting case, taking the pressure at the inlet of pump equal to vapour pressure of the liquid, we get

$p_1 = p_v$, where p_v = vapour pressure of the liquid in absolute units.

Now the equation (*ii*) becomes as

$$\frac{p_v}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

or

$$\frac{p_a}{\rho g} = \frac{p_v}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s} \quad (\because \; p_1 = p_v) \text{ ...}(iii)$$

Now

$$\frac{p_a}{\rho g} = \text{Atmospheric pressure head } = H_a \text{ (meter of liquid)}$$

$$\frac{p_v}{\rho g} = \text{Vapour pressure head} = H_v \text{ (meter of liquid)}$$

Now, equation (*iii*) becomes as

$$H_a = H_v + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

or

$$h_s = H_a - H_v - \frac{v_s^2}{2g} - h_{f_s} \quad \text{...(19.31)}$$

Equation (19.31) gives the value of maximum suction lift (or maximum suction height) for a centrifugal pump. Hence, the suction height of any pump should not be more than that given by equation (19.31). If the suction height of the pump is more, then vaporization of liquid at inlet of pump will take place and there will be a possibility of cavitation.

▶ 19.13 NET POSITIVE SUCTION HEAD (NPSH)

The term NPSH (Net Positive Suction Head) is very commonly used in the pump industry. Actually the minimum suction conditions are more frequently specified in terms of NPSH.

The net positive suction head (NPSH) is defined as the *absolute* pressure head at the inlet to the pump, minus the vapour pressure head (in absolute units) plus the velocity head.

∴ NPSH = Absolute pressure head at inlet of the pump – vapour pressure head (absolute units) + velocity head

$$= \frac{p_1}{\rho g} - \frac{p_v}{\rho g} + \frac{v_s^2}{2g} \quad (\because \text{ Absolute pressure at inlet of pump} = p_1) \quad \ldots (19.32)$$

But from equation (*ii*) of Art. 19.12, the absolute pressure head at inlet of the pump is given by as

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

Substituting this value in equation (19.32) , we get

$$\text{NPSH} = \left[\frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)\right] - \frac{p_v}{\rho g} + \frac{v_s^2}{2g}$$

$$= \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - h_s - h_{f_s}$$

$$= H_a - H_v - h_s - h_{f_s}$$

$$\left(\because \frac{p_a}{\rho g} = H_a = \text{Atmospheric pressure head}, \frac{p_v}{\rho g} = H_v = \text{Vapour pressure head}\right)$$

$$= \left[\left(H_a - h_s - h_{f_s}\right) - H_v\right] \quad \ldots (19.33)$$

The right hand side of equation (19.33) is the total suction head. Hence NPSH is equal to total suction head. Thus NPSH may also be defined as the total head required to make the liquid flow through the suction pipe to the pump impeller.

For any pump installation, a distinction is made between the required NPSH and the available NPSH. The value of required NPSH is given by the pump manufacturer. This value can also be determined experimentally. For determining its value, the pump is tested and minimum value of h_s is obtained at which the pump gives maximum efficiency without any objectional noise (*i.e.,* cavitation free). The required NPSH varies with the pump design, speed of the pump and capacity of the pump.

When the pump is installed, the available NPSH is calculated from equation (19.33). In order to have cavitation free operation of centrifugal pump, the available NPSH should be greater than the required NPSH.

▶ 19.14 CAVITATION IN CENTRIFUGAL PUMP

Thoma's cavitation factor is used to indicate whether cavitation will occur in pumps. Equation (19.24) gives the value of Thoma's cavitation factor for pumps as

$$\sigma = \frac{\left(H_{atm} - H_v\right) - H_s - h_{f_s}}{H}$$

$$= \frac{H_a - H_v - h_s - h_{f_s}}{H_m} \quad (\because H_s = h_s \text{ and } h_{L_s} = h_{f_s}) \; (H = H_m \text{ for pumps})$$

But from equation (19.33), we have

$$H_a - H_v - h_s - h_{f_s} = \text{NPSH} \qquad \text{(Net position suction head)}$$

$$\therefore \qquad \sigma = \frac{\text{NPSH}}{H_m} \qquad ...(19.34)$$

If the value of σ (calculated from equation 19.34) is less than the critical value, σ_c then cavitation will occur in the pumps. The value of σ_c depends upon the specific speed of the pump $\left(N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}\right)$.

The following empirical relation is used to determine the value of σ_c.

$$\sigma_c = 0.103 \left(\frac{N_s}{1000}\right)^{4/3}$$

$$= 0.103 \frac{N_s^{4/3}}{(10^3)^{4/3}} = \frac{0.103\, N_s^{4/3}}{10^4}$$

$$= 1.03 \times 10^{-3} N_s^{4/3} \qquad ...(19.35)$$

Problem 19.24 *A centrifugal pump rotating at 1000 r.p.m. delivers 160 litres/s of water against a head of 30 m. The pump is installed at a place where atmospheric pressure is 1×10^5 P_a (abs.) and vapour pressure of water is 3 kPa (abs.). The head loss in suction pipe is equivalent to 0.2 m of water. Calculate :*

(i) Minimum NPSH, and

(ii) Maximum allowable height of the pump from free surface of water in the sump.

Solution. Given :

$N = 1000$ r.p.m.; $Q = 160$ litres/s $= 0.16$ m^3/s.; $H_m = 30$ m

$p_a = 1 \times 10^5$ Pa (abs.) $= 1 \times 10^5$ N/m^2 (abs.) ; $p_v = 3$ kPa (abs.) $= 3 \times 10^3$ N/m^2 (abs.)

$h_{f_s} = 0.2$ m.

(*i*) *Minimum NPSH*

Using equation (19.34), we get

$$\sigma = \frac{\text{NPSH}}{H_m}$$

From the above equation, it is clear that NPSH is directly proportional to Thoma's cavitation factor (σ). NPSH will be minimum when σ is minimum. But the minimum value of σ for no cavitation is σ_c. Hence when $\sigma = \sigma_c$ then NPSH will be minimum.

$$\therefore \qquad \sigma_c = \frac{(\text{NPSH})_{\min}}{H_m}$$

or

$$(\text{NPSH})_{\min} = H_m \times \sigma_c \qquad ...(i)$$

Now the critical value of σ *i.e.*, σ_c is given by equation (19.35) as

$$\sigma_c = 1.03 \times 10^{-3} \times N_s^{4/3} \qquad ...(ii)$$

where N_s = Specific speed of pump $= \frac{N\sqrt{Q}}{H_m^{3/4}}$

$$= 1000 \times \frac{\sqrt{0.16}}{30^{3/4}} \quad (\because N = 1000 \text{ r.p.m.}, Q = 0.16 \text{ m}^3/\text{s and } H_m = 30 \text{ m})$$

Substituting the value of N_s in equation (*ii*), we get

$$\sigma_c = 1.03 \times 10^{-3} \times \left[\frac{1000 \times \sqrt{0.16}}{30^{3/4}}\right]^{4/3}$$

$$= 1.03 \times 10^{-3} \times \frac{1000^{4/3} \times 0.16^{2/3}}{30} = \frac{1.03 \times 10^{-3} \times 10^4 \times 0.2947}{30}$$

$$= 0.1012$$

Substituting the value of σ_c in equation (*i*), we get

$$(\text{NPSH})_{min.} = H_m \times 0.1012$$

$$= 30 \times 0.1012 = \mathbf{3.036\ m.\ Ans.} \qquad (\because H_m = 30 \text{ m})$$

(*ii*) *Maximum allowable height of the pump from free surface of water in the sump* (*i.e.*, h_s)

Let $(h_s)_{max}$ = Max. allowable height of pump from free surface of water.

Using equation (19.33)

$$\text{NPSH} = H_a - H_v - h_s - h_{f_s} \qquad \text{...}(i)$$

From the above equation, it is clear that for a given value of atmospheric pressure head $\left(H_a = \frac{p_a}{\rho g}\right)$, given vapour pressure head $\left(H_v = \frac{p_v}{\rho g}\right)$ and given loss of head due to friction (h_{f_s}), the value of suction head (h_s) will be maximum if NPSH is minimum.

$$\therefore \quad (\text{NPSH})_{min} = H_a - H_v - (h_s)_{max} - h_{f_s} \qquad \text{...}(ii)$$

$$\therefore \quad (h_s)_{max} = H_a - H_v - h_{f_s} - (\text{NPSH})_{min} \qquad \text{...}(iii)$$

Now

$$H_a = \frac{p_a}{\rho g} = \frac{1 \times 10^5}{1000 \times 9.81} = 10.193 \text{ m of water}$$

$$H_v = \frac{p_v}{\rho g} = \frac{3 \times 10^3}{1000 \times 9.81} = 0.305 \text{ m of water}$$

$$h_{f_s} = 0.2 \text{ m and } (\text{NPSH})_{min} = 3.036 \text{ m}$$

Substituting the values of H_a, H_v, h_{f_s} and $(\text{NPSH})_{min.}$ in equation (*iii*), we get

$$(h_s)_{max} = 10.193 - 0.305 - 0.2 - 3.036$$

$$= \mathbf{6.652\ m.\ Ans.}$$

HIGHLIGHTS

1. The hydraulic machine which converts the mechanical energy into pressure energy by means of centrifugal force is called centrifugal pump.
2. The centrifugal pump acts as a reverse of an inward radial flow reaction turbine. The work done by the impeller (rotating part of the pump) on the water per second per unit weight of water per second flowing through the pump is given as

$$= \frac{1}{g} V_{w_2} \times u_2$$

where V_{w_2} = Velocity of whirl at outlet, and

u_2 = Tangential velocity of wheel at outlet.

3. The vertical height of the centre-line of the centrifugal pump from the water surface in the pump is called the suction head (h_s).

4. Delivery head (h_d) is the vertical distance between the centre-line of the pump and the water surface in the tank to which water is lifted.

5. Manometric head (H_m) is the head against which a centrifugal pump has to work. It is given as

(*a*) $$H_m = \frac{V_{w_2} \times u_2}{g} - \text{ Loss of head in impeller and casing}$$

$$= \frac{V_{w_2} \times u_2}{g} \quad \text{...if losses in pump is zero}$$

(*b*) H_m = Total head at outlet – Total head at inlet of pump

$$= \left(\frac{p_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right)$$

(*c*) $$H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g}.$$

6. The efficiencies of a pump are : (*i*) Manometric efficiency (h_{man}), (*ii*) Mechanical efficiency (η_m), and (*iii*) Overall efficiency (η_o). Mathematically they are given as

$$\eta_{man} = \frac{gH_m}{V_{w_2} \times u_2}$$

$$\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} \times u}{75}\right)}{\text{S.P.}}, \text{ where } W = w \times Q$$

$$\eta_o = \frac{W \times H_m}{1000 \times \text{S.P.}}$$

7. The minimum speed for starting a centrifugal pump is given by $N = \dfrac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi\left[D_2^2 - D_1^2\right]}$.

8. If a centrifugal pump consists of two or more impellers, the pump is called a multistage pump. To produce a high head, the impellers are connected in series while to discharge a large quantity of liquid, the impellers are connected in parallel.

9. The specific speed of a centrifugal pump is defined as the speed at which a pump runs when the head developed is one metre and discharge is one cubic metre. Mathematically, it is given as

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}, \text{ where } H_m = \text{Manometric hand.}$$

10. For complete similarity between the model and actual centrifugal pump (prototype) the following conditions should be satisfied :

(a) $\left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_{\text{model}} = \left(\frac{N\sqrt{Q}}{H_m^{3/4}}\right)_{\text{prototype}}$ (b) $\left(\frac{\sqrt{H_m}}{DN}\right)_{\text{model}} = \left(\frac{\sqrt{H_m}}{DN}\right)_{\text{prototype}}$

(c) $\left(\frac{Q}{D^3N}\right)_{\text{model}} = \left(\frac{Q}{D^3N}\right)_{\text{prototype}}$ (d) $\left(\frac{P}{D^5N^3}\right)_{\text{model}} = \left(\frac{P}{D^5N^3}\right)_{\text{prototype}}$

11. Characteristic curves are used for predicting the behaviour and performance of a pump when it is working under different flow rate, head and speed.

12. Cavitation is defined as the phenomenon of formation of vapour bubbles and sudden collapsing of the vapour bubbles.

EXERCISE

(A) THEORETICAL PROBLEMS

1. Define a centrifugal pump. Explain the working of a single-stage centrifugal pump with sketches.

2. Differentiate between the volute casing and vortex casing for the centrifugal pump.

3. Obtain an expression for the work done by impeller of a centrifugal pump on water per second per unit weight of water.

4. Define the terms : suction head, delivery head, static head and manometric head.

5. What do you mean by manometric efficiency, mechanical efficiency and overall efficiency of a centrifugal pump?

6. How will you obtain an expression for the minimum speed for starting a centrifugal pump?

7. What is the difference between single-stage and multistage pumps? Describe multistage pump with (*a*) impellers in parallel, and (*b*) impellers in series.

8. Define specific speed of a centrifugal pump. Derive an expression for the same.

9. How does the specific speed of a centrifugal pump differ from that of a turbine ?

10. What is priming ? Why is it necessary ?

11. How the model testing of the centrifugal pumps are made?

12. What do you understand by characteristic curves of a pump? What is the significance of the characteristic curves?

13. Define cavitation. What are the effects of cavitation ? Give the necessary precautions against cavitation.

14. How will you determine the possibility of the cavitation to occur in the installation of a turbine or a pump?

15. Why are centrifugal pumps used sometimes in series and sometimes in parallel ? Draw the following characteristic curves for a centrifugal pump :

Head, power and efficiency *versus* discharge with constant speed.

16. Draw and discuss the operating characteristics of a centrifugal pump.

17. (*a*) What is cavitation and what are its causes ? How will you prevent the cavitation in hydraulic machines ?

(*b*) What is cavitation? State its effects on the performance of water turbines and also state how to prevent cavitation in water turbines.

18. Briefly state the significance of similarity parameters in hydraulic pumps.

19. The frictional torque T of a disc of diameter D rotating at a speed of N in a fluid of viscosity μ and density ρ in a turbulent flow is given by :

$$T = D^5 N^2 \rho\, \phi\left[\frac{\mu}{D^2 N \rho}\right]$$

Prove this by method of dimensions.

20. With a neat sketch, explain the principle and working of a centrifugal pump.

21. Explain the following terms as they are applied to a centrifugal pump :
(*i*) Static suction lift ; (*ii*) static suction head ; (*iii*) static discharge head ; and (*iv*) total static head.

22. (*a*) How does a volute casing differ from a vortex casing for the centrifugal pump ?
(*b*) What is priming ? Why is it necessary ?
(*c*) What do you mean by pump characteristics ? Briefly explain the uses of such characteristics.
(*Jawaharlal Nehru Technical University, S 2002*)

(B) NUMERICAL PROBLEMS

1. The internal and external diameters of the impeller of a centrifugal pump are 300 mm and 600 mm respectively. The pump is running at 1000 r.p.m. The vane angles at inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water. [**Ans.** 68.89 Nm/N]

2. A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1200 r.p.m. works against a total head of 75 m. The velocity of flow through the impeller is constant and equal to 3 m/s. The vanes are set back at an angle of 30° at outlet. If the outer diameter of the impeller is 600 mm and width at outlet is 50 mm, determine : (*a*) vane angle at inlet, (*b*) work done per second by impeller, (*c*) manometric efficiency. [**Ans.** (*a*) 9° 2′, (*b*) 346.37 kW, (*c*) 60%]

3. A centrifugal pump is running at 1000 r.p.m. The outlet vane angle of the impeller is 30° and velocity of flow at outlet is 3 m/s. The pump is working against a total head of 30 m and the discharge through the pump is 0.3 m^3/s. If the manometric efficiency of the pump is 75%, determine: (*i*) the diameter of the impeller, and (*ii*) the width of the impeller at outlet. [**Ans.** (*i*) 43.1 cm, (*ii*) 7.4 cm]

4. Find the power required to drive a centrifugal pump which delivers 0.02 m^3/s of water to a height of 30 m through a 10 cm diameter pipe and 90 m long. The overall efficiency of the pump is 70% and f = .009 in the formula

$$h_f = \frac{4 f l V^2}{d \times 2g}.$$ [**Ans.** 11.5 kW]

5. Find the rise in pressure in the impeller of a centrifugal pump through which water is flowing at the rate of 15 litre/s. The internal and external diameters of the impeller are 20 cm and 40 cm respectively. The widths of impeller at inlet and outlet are 1.6 cm and 0.8 cm. The pump is running at 1200 r.p.m. The water enters the impeller radially at inlet and impeller vane angle at outlet is 30°. Neglect losses through the impeller. [**Ans.** 31.85 m]

6. The diameters of an impeller of a centrifugal pump at inlet and outlet are 20 cm and 40 cm respectively. Determine the minimum speed for starting the pump if it works against a head of 25 m. [**Ans.** 1221.2 r.p.m.]

7. The diameter of an impeller of a centrifugal pump at inlet and outlet are 300 mm and 600 mm respectively. The velocity of flow at outlet is 2.5 m/s and vanes are set back at an angle of 45° at outlet. Determine the minimum starting speed of the pump if the manometric efficiency is 75%. [**Ans.** 159.31 r.p.m.]

8. A three-stage centrifugal pump has impeller 40 cm in diameter and 2.5 cm wide at outlet. The vanes are curved back at the outlet at 30° and reduce the circumferential area by 15%. The manometric efficiency is 85% and overall efficiency is 75%. Determine the head generated by the pump when running at 12000 r.p.m. and discharge is 0.06 m^3/s. Find the shaft power also. [**Ans.** 138.75 m, 108.89 kW]

9. Find the number of pumps required to take water from a deep well under a total head of 156 m. Also the pumps are identical and are running at 1000 r.p.m. The specific speed of each pump is given as 20 while the rated capacity of each pump is 150 litre/s. [**Ans.** 3]

10. The diameter of a centrifugal pump, which is discharging 0.035 m^3/s of water against a total head of 25 m is 0.05 m. The pump is running at 1200 r.p.m. Find the head, discharge and ratio of powers of a geometrically similar pump of diameter 0.3 m when it is running at 2000 r.p.m. [**Ans.** 25 m, .0126 m^3/s, 2.777]

11. A centrifugal pump is to discharge 0.12 m^3 at a speed of 1400 r.p.m. against a head of 30 m. The diameter and width of the impeller at outlet are 25 cm and 5 cm respectively. If the manometric efficiency is 75%, determine the vane angle at outlet.