4 Pipes

4.1 Introduction

Pipes are a common feature of water supply systems and have many advantages over open channels. They are completely enclosed, usually circular in section and always flow full of water. This is in contrast to channels which are open to the atmosphere and can have many different shapes and sizes – but more about channels in Chapter 5. One big advantage of pipes is that water can flow uphill as well as downhill so land topography is not such a constraint when taking water from one location to another.

There are occasions when pipes do not flow full – one example is gravity flow sewers. They take sewage away from homes and factories and often only flow partially full under the force of gravity in order to avoid pumping. They look like pipes and are indeed pipes but hydraulically they behave like open channels. The reason pipes are used for this purpose is that sewers are usually buried below ground to avoid public health problems and it would be difficult to bury an open channel!

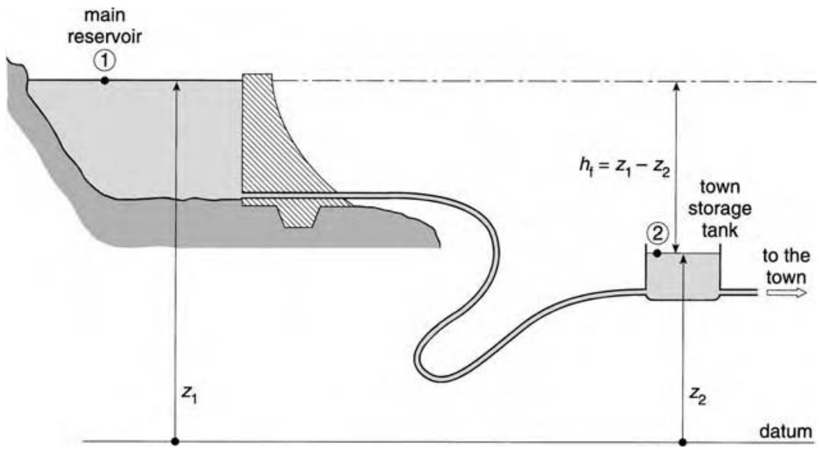
4.2 A typical pipe flow problem

Pipe flow problems usually involve calculating the right size of pipe to use for a given discharge. A typical example is a water supply to a village (Figure 4.1). A pipeline connects a main storage reservoir to a small service (storage) tank just outside the village which then supplies water to individual houses. The required discharge (Q m^3/s) for the village is determined by the water demand of each user and the number of users being supplied. We now need to determine the right size of pipe to use to ensure that this discharge is supplied from the main storage reservoir to the service tank.

A formula to calculate pipe size would be ideal. However, to get there we first need to look at the energy available to 'push' water through the system, so the place to start is the energy equation. But this is a real fluid problem and so energy losses due to friction must be taken into account. So writing the energy equation for two points in this system – point 1 is at the main reservoir and point 2 at the service tank – and allowing for the energy loss as water flows between the two:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

Points 1 and 2 are carefully chosen in order to simplify the equation and also the solution. Point 1 is at the surface of the main reservoir where the pressure p_1 is atmospheric pressure and



4.1 A typical pipe flow problem.

so equal to zero (remember we are working in gauge pressures). Point 2 is also at the water surface in the service tank so p_2 is zero as well. The water velocities v_1 and v_2 in the reservoir and the tank are very small and so the kinetic energy terms are also very small and can be assumed to be zero. This leaves just the potential energy terms z_1 and z_2 and the energy loss term h_f. So the energy equation simplifies down to:

$$h_f = z_1 - z_2$$

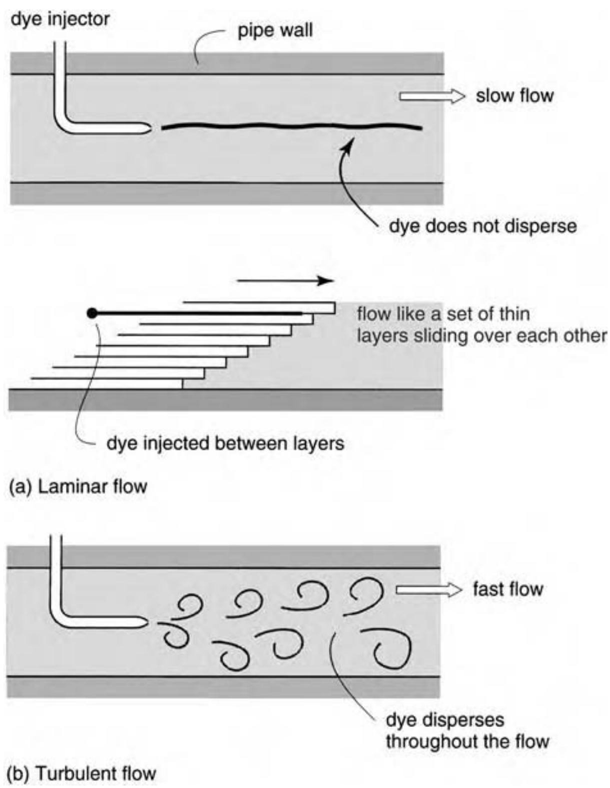
$z_1 - z_2$ is the difference in water levels between the reservoir and the storage tank and this represents the energy available to 'push' water through the system. h_f is the energy loss due to friction in the pipe. The energy available is usually known and so this means we also know the amount of energy that can be lost through friction. The question now is – is there a formula that links this energy loss h_f with the pipe diameter? The short answer is yes – but it has taken some 150 years of research to sort this out. So let us first step through this bit of history and see what it tells us about pipe flow.

4.3 A formula to link energy loss and pipe size

Some of the early research work on pipe friction was done by Osborne Reynolds (1842–1912), a mathematician and engineer working at the University in Manchester in UK. He measured the pressure loss in pipes of different lengths and diameters at different discharges with some interesting results. At low flows he found that the energy loss varied directly with the velocity. So when the velocity was doubled the energy loss also doubled. But at high flows the energy loss varied as the square of the velocity. So when the velocity was doubled the energy loss increased four-fold. Clearly, Reynolds was observing two quite different types of flow. This thinking led to Reynolds classic experiment that established the difference between what is now referred to as *laminar* and *turbulent* flow and formulae which would enable the energy loss to be calculated for each flow type from a knowledge of the pipes themselves.

4.3.1 Laminar and turbulent flow

Reynolds experiment involved setting up a glass tube through which he could pass water at different velocities (Figure 4.2). A thin jet of coloured dye was injected into the flow so that the flow patterns were visible.



4.2 Laminar and turbulent flow.

When the water moved slowly the dye remained in a thin line as it followed the flow path of the water down the pipe. This was described as *laminar flow*. It was as though the water was moving as a series of very thin layers – like a pack of cards – each one sliding over the other, and the dye had been injected between two of the layers. This type of flow rarely exists in nature and so is not of great practical concern in hydraulics. However, you can see it occasionally under very special conditions. Examples include smoke rising in a thin column from a chimney on a very still day or a slow flow of water from a tap that looks so much like a glass rod that you feel you could get hold of it. Blood flow in our bodies is usually laminar.

The second and more common type of flow he identified was *turbulent flow*. This occurred when water was moving faster. The dye was broken up as the water whirled around in a random manner and was dissipated throughout the flow. Turbulence was a word introduced by Lord Kelvin (1824–1907) to describe this kind of flow behaviour.

There are very clear visual differences between laminar and turbulent flow but what was not clear was how to predict which one would occur in any given set of circumstances. Velocity was obviously important. As velocity increased so the flow would change from laminar to turbulent flow. But it was obvious that from the experiments that velocity was not the only factor. It was Reynolds who first suggested that the type of flow depended not just on velocity (v) but also on mass density (ρ), viscosity (μ) and pipe diameter (d). He put these factors together in a way which is now called the *Reynolds Number* in recognition of his work.

$$\text{Reynolds No.} = \frac{\rho v d}{\mu}$$

Note that Reynolds Number has no dimensions. All the dimensions cancel out. Reynolds found that he could use this number to reliably predict when laminar and turbulent flow would occur.

$R < 2000$ flow would always be laminar
$R > 4000$ flow would always be turbulent

Between $R = 2000$ and 4000 he observed a very unstable zone as the flow seemed to jump from laminar to turbulent and back again as if the flow could not decide which of the two conditions it preferred. This is a zone to avoid as both the pressure and flow fluctuate widely in an uncontrolled manner.

Reynolds Number also shows just how important is viscosity in pipe flow. Low Reynolds Number ($R < 2000$) means that viscosity (μ) is large compared with the term ρvd. So viscosity is important in laminar flow and cannot be ignored. High Reynolds Number ($R > 4000$) means viscosity is small compared with the ρvd term and so it follows that viscosity is less important in turbulent flow. This is the reason why engineers ignore the viscosity of water when designing pipes and channels as it has no material effect on the solution. Ignoring viscosity also greatly simplifies pipeline design.

It has since been found that Reynolds Number is very useful in other ways besides telling us the difference between laminar and turbulent flow. It is used extensively in hydraulic modelling (physical models – not mathematical models) for solving complex hydraulic problems. When a problem cannot be solved using some formula, another approach is to construct a small-scale model in a laboratory and test it to see how it performs. The guideline for modelling pipe systems (or indeed any fully enclosed system) is to ensure that the Reynolds Number in the model is similar to the Reynolds Number in the real situation. This ensures that the forces and velocities are similar so that the model, as near as possible, produces similar results to those in the real pipe systems.

Although it is useful to know that laminar flow exists it is not important in practical hydraulics for designing pipes and channels and so only turbulent flow is considered in this text. Turbulent flow is very important to us in our daily lives. Indeed it would be difficult for us to live if it was not for the mixing that takes place in turbulent flow which dilutes fluids. When we breathe out, the carbon dioxide from our lungs is dissipated into the surrounding air through turbulent mixing. If it did not disperse in this way we would have to move our heads to avoid breathing in the same gases as we had just breathed out. Car exhaust fumes are dispersed in a similar way, otherwise we could be quickly poisoned by the intake of concentrated carbon monoxide.

4.3.2 A formula for turbulent flow

Several formulae link energy loss with pipe size for turbulent flow but one of the most commonly used today is that devised by Julius Weisbach (1806–1871) and Henry Darcy (1803–1858). It is often referred to as the *Darcy–Weisbach equation*:

$$h_f = \frac{\lambda l v^2}{2gd}$$

where λ is a friction factor; l is pipe length (m); v is velocity (m/s); g is gravity constant (9.81 m/s^2) and d is pipe diameter (m).

This formula shows that energy loss depends on pipe length, velocity and diameter but also on the friction between the pipe and the flow as represented by λ:

- *Length* has a direct influence on energy loss. The longer the pipeline the greater the energy loss.
- *Velocity* has a great influence on energy loss because it is the square of the velocity that counts. When the velocity is doubled (say by increasing the discharge), the energy loss increases four-fold. It is usual practice in water supply systems to keep the velocity below 1.6 m/s. This is done primarily to avoid excessive energy losses but it also helps to reduce water hammer problems (see Chapter 5, Section 5.9).
- *Pipe diameter* has the most dramatic effect on energy loss. As the pipe diameter is reduced so the energy losses increase, not only because of the direct effect of d in the formula but also because of its effect on the velocity v (remember the discharge equation $Q = va$). The overall effect of reducing the diameter by half (say from 300 to 150 mm) is to increase h_f by 32-times. See box below for an illustration of this.
- *Pipe friction* λ Unfortunately this is not just a simple measure of pipe roughness; it depends on several other factors which are discussed more fully in the next section.

Take care when using the Darcy–Weisbach formula as some text books, particularly American, use f as the friction factor and not and they are not the same. The link between them is $\lambda = 4f$.

EXAMPLE: HOW PIPE DIAMETER AFFECTS ENERGY LOSS

A pipeline 1000 m long carries a flow of 100 l/s. Calculate the energy loss when the pipe diameter is 0.3 m, 0.25 m, 0.2 m and 0.15 m and $\lambda = 0.04$.

The first step is to calculate the velocities for each pipe diameter using the discharge equation:

$$Q = va$$

And so:

$$v = \frac{Q}{a}$$

Use this equation to calculate velocity v for each diameter and then use the Darcy–Weisbach equation to calculate h_f. The results are shown in Table 4.1. Notice the very large rise in head loss as the pipe diameter is reduced. Clearly the choice of pipe diameter is a critical issue in any pipeline system.

Table 4.1 Results of Darcy–Weisbach equation to calculate h_f.

Diameter (m)	*Pipe area (m^2)*	*Velocity (m/s)*	*Head loss h_f (m)*
0.30	0.07	1.43	13.6
0.25	0.049	2.04	33.3
0.20	0.031	3.22	103.7
0.15	0.018	5.55	418.6

4.4 The λ story

It would be convenient if λ was just a constant number for a given pipe that depended only on its roughness and hence its resistance to the flow. But few things are so simple and λ is no exception. Some of the earliest work on pipe friction was done by Paul Blazius in 1913. He carried out a wide range of experiments on different pipes and different flows and came to the conclusion that λ depended only on the Reynolds Number and surprisingly, the roughness of the pipe seemed to have no effect at all on friction.

From this he developed a formula for λ (note R is Reynolds Number):

$$\lambda = \frac{0.316}{R^{0.25}}$$

Another investigator was Johann Nikuradse who may well have been puzzled by the Blazius results. He set up a series of laboratory experiments in the 1930s with different pipe sizes and flows and he roughened the inside of the pipes with sand grains of a known size in order to create different but known roughness. His data showed that values of λ were independent of Reynolds Number and depended only on the roughness of the pipe. Clearly, either someone was wrong or they were both right and each was looking at something different.

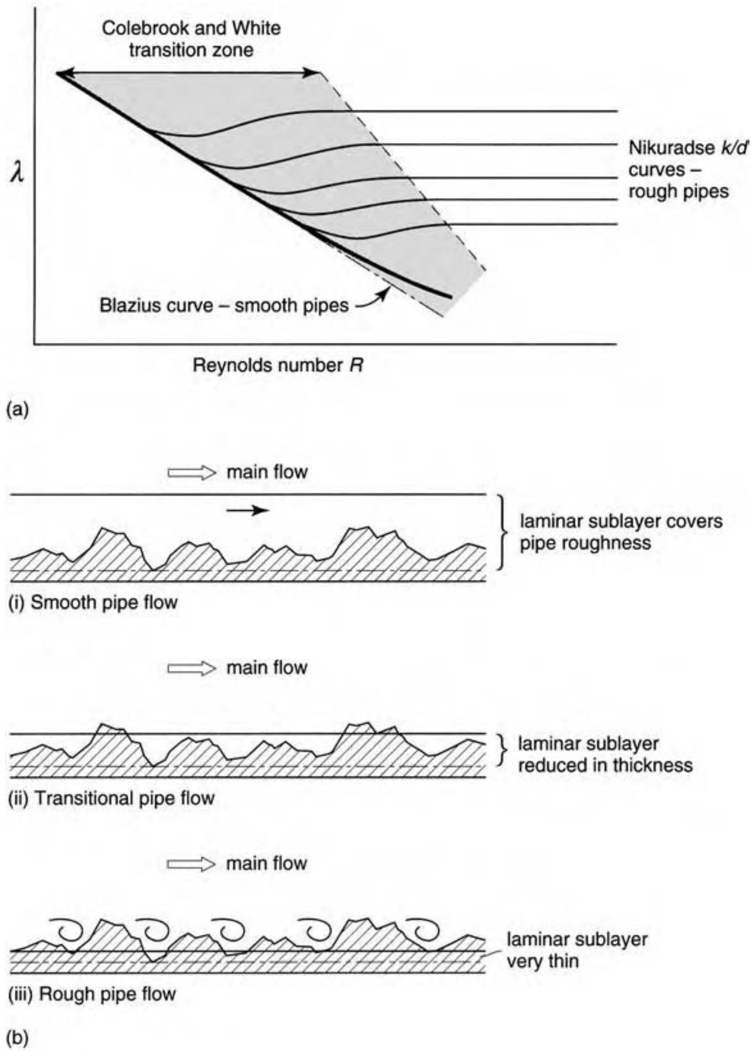
4.4.1 Smooth and rough pipes

We now know that both investigators were right but they were looking at different aspects of the same problem. Blazius was looking at flows with relatively low Reynolds Numbers (4000 to 100 000) and his results refer to what are now called *smooth pipes*. Nikuradse's experiments dealt with high Reynolds Number flows (greater than 100 000) and his results refer to what are now called *rough pipes*. Both Blazius and Nikuradse results are shown graphically in Figure 4.3a. This is a graph with a special logarithmic scale for Reynolds Number so that a wide range of values can be shown on the same graph. It shows how λ varies with both Reynolds Number and pipe roughness which is expressed as the height of the sand grains (k) divided by the pipe diameter (d). The Blazius formula produces a single line on this graph and is almost a straight line.

The terms rough and smooth refer as much to the flow conditions in pipes as to the pipes themselves and so, paradoxically, it is possible for the same pipe to be described as both rough and smooth. Roughness and smoothness are also relative terms. How the inside of a pipe feels to touch is not a good guide to its smoothness in hydraulic terms. Pipes which are smooth to the touch can still be quite rough hydraulically. However, a pipe that feels rough to touch will be very rough hydraulically and very high energy losses can be expected.

As there are two distinct types of flow it implies that there must be some point or zone where the flow changes from one to the other. This is indeed the case. It is not a specific point but a zone known as the *transition zone* when λ depends on both Reynolds Number and pipe roughness (Figure 4.3a). This zone was successfully investigated by C.F. Colebrook and C.M. White working at Imperial College in London in the 1930s and they developed a formula to cover this flow range. This is not quoted here as it is quite a complex formula and in practice there is no need to use it because it has now been simplified to design charts. These can be used to select pipe sizes for a wide range of hydraulic conditions. The use of typical pipe charts is described later in this chapter in Section 4.8.

The transition zone between smooth and rough pipe flow should *not* be confused with the transition zone from laminar to turbulent flow, as is often done. The flow is fully turbulent for all smooth and rough pipes and the transition is from smooth to rough pipe flow.



4.3 The λ story.

To summarise the different flows in pipes:

laminar flow

↓

transition from laminar to turbulent flow
(this zone is very unstable and should be avoided)

↓

turbulent flow
smooth pipe flow

↓

transition from smooth to rough pipe flow

↓

rough pipe flow

4.4.2 A physical explanation

Since those early experiments, modern scientific techniques have enabled investigators to look more closely at what happens close to a pipe wall. This has resulted in a physical explanation for smooth and rough pipe flow (Figure 4.3b). Investigators have found that even when the flow is turbulent there exists a very thin layer of fluid – less than 1 mm thick – close to the boundary that is laminar. This is called the laminar sub-layer. At low Reynolds Numbers the laminar sub-layer is at its thickest and completely covers the roughness of the pipe. The main flow is unaffected by the boundary roughness and is influenced mainly by viscosity in the laminar sub-layer. It seems that the layer covers the roughness like a blanket and protects the flow from the pipe wall. This is the smooth pipe flow that Blazius investigated. As Reynolds Number increases, the laminar sub-layer becomes thinner and roughness elements start to protrude into the main flow. The flow is now influenced both by viscosity and pipe roughness. This is the transition zone. As Reynolds Number is further increased the sub-layer all but disappears and the roughness of the pipe wall takes over and dominates the friction. This is rough pipe flow which Nikuradse investigated.

Commercially manufactured pipes are not artificially roughened with sand like experimental pipes, they are manufactured as smooth as possible to reduce energy losses. For this reason they tend to come within the transition zone where λ varies with both Reynolds Number and pipe roughness.

4.5 Hydraulic gradient

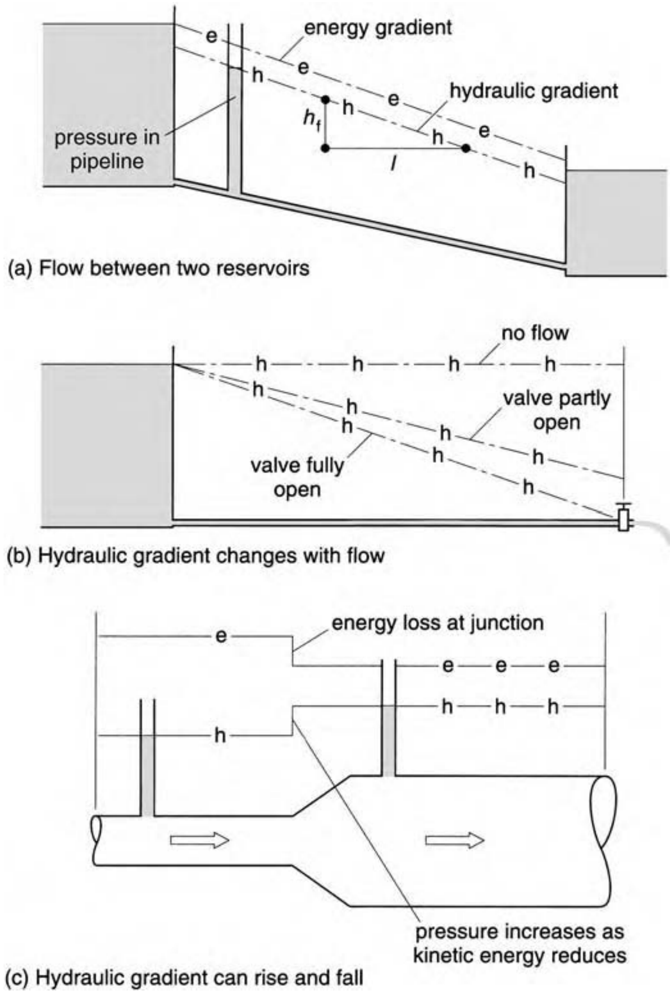
One way of showing energy losses in a pipeline is to use a diagram (Figure 4.4a). The total energy is shown as a line drawn along the pipe length and marked *e—e—e*. This line always slopes downwards in the direction of the flow and demonstrates that energy is continually being lost through friction. It connects the water surfaces in the two tanks. There is a small step at the downstream tank to represent the energy loss at the outlet from the pipeline into the tank. Note that the energy line is not necessarily parallel to the pipeline. The pipeline usually just follows the natural ground surface profile.

Although total energy is of interest, pressure is more important because this determines how strong the pipes must be to avoid bursts. For this reason a second line is drawn below the energy line, but parallel to it, to represent the pressure (pressure energy) and is marked *h—h—h*. This shows the pressure change along the pipeline. Imagine standpipes are attached to the pipe. Water would rise up to this line to represent the pressure head (Figure 4.4a). The difference between the two lines is the kinetic energy. Notice how both the energy line and the hydraulic gradient are straight lines. This shows that the rate of energy loss and the pressure loss are uniform (at the same rate). The slope of the pressure line is called the *hydraulic gradient*. It is calculated as follows:

$$\text{hydraulic gradient} = \frac{h_f}{l}$$

where h_f is change in pressure (m); l is the pipe length over which the pressure change takes place (m).

The hydraulic gradient has no dimensions as it comes from dividing a length in metres by a head difference in metres. However, it is often expressed in terms of metres head per metre length of pipeline. As an example a hydraulic gradient of 0.02 means for every one metre of pipeline there will be a pressure loss of 0.02 m. This may also be written as 0.02 m/m or as 2 m/100 m of pipeline. This reduces the number of decimal places that must be dealt with and means that for



4.4 Hydraulic gradient.

every 100 m of pipeline 2 m of head is lost through friction. So if a pipeline is 500 m long (there are five 100 m lengths) the pressure loss over 500 m will be $5 \times 2 = 10$ m head.

The hydraulic gradient is not a fixed line for a pipe; it depends on the flow (Figure 4.4b). When there is no flow the gradient is horizontal but when there is full flow the gradient is at its steepest. Adjusting the outlet valve will produce a range of gradients between these two extremes.

The energy gradient can only slope downwards in the direction of flow to show how energy is lost, but the hydraulic gradient can slope upwards as well as downwards. An example of this is a pipe junction when water flows from a smaller pipe into a larger one (Figure 4.4c). As water enters the larger pipe the velocity reduces and so does the kinetic energy. Although there is some energy loss when the flow expands (this causes the energy line to drop suddenly) most of the loss of kinetic energy is recovered as pressure energy and so the pressure rises slightly.

Two more points of detail about the energy and hydraulic gradients (Figure 4.4a). At the first reservoir, the energy gradient starts at the water surface but the hydraulic gradient starts just below it. This is because the kinetic energy increases as water enters the pipe so there is a corresponding drop in the pressure energy. As the flow enters the second reservoir the energy line is just above the water surface. This is because there is a small loss in energy as the flow expands from the pipe into the reservoir. The hydraulic gradient is located just below the water level because there is still some kinetic energy in the flow. When it enters the reservoir this changes back to pressure energy. The downstream water level represents the final energy condition in the system. These changes close to the reservoirs are really very small in comparison to the friction losses along the pipe and so they play little or no part in the design of the pipeline.

Normally pipelines are located well below the hydraulic gradient. This means that the pressure in the pipe is always positive – see the standpipes in Figure 4.4a. Even though it may rise and fall as it follows the natural ground profile, water will flow as long as it is always below the hydraulic gradient and provided the outlet is below the inlet. There are limits to how far below the hydraulic gradient a pipeline can be located. The further below the higher will be the pressure in the pipe and the risk of a burst if the pressure exceeds the limits set by the pipe manufacturer.

4.6 Energy loss at pipe fittings

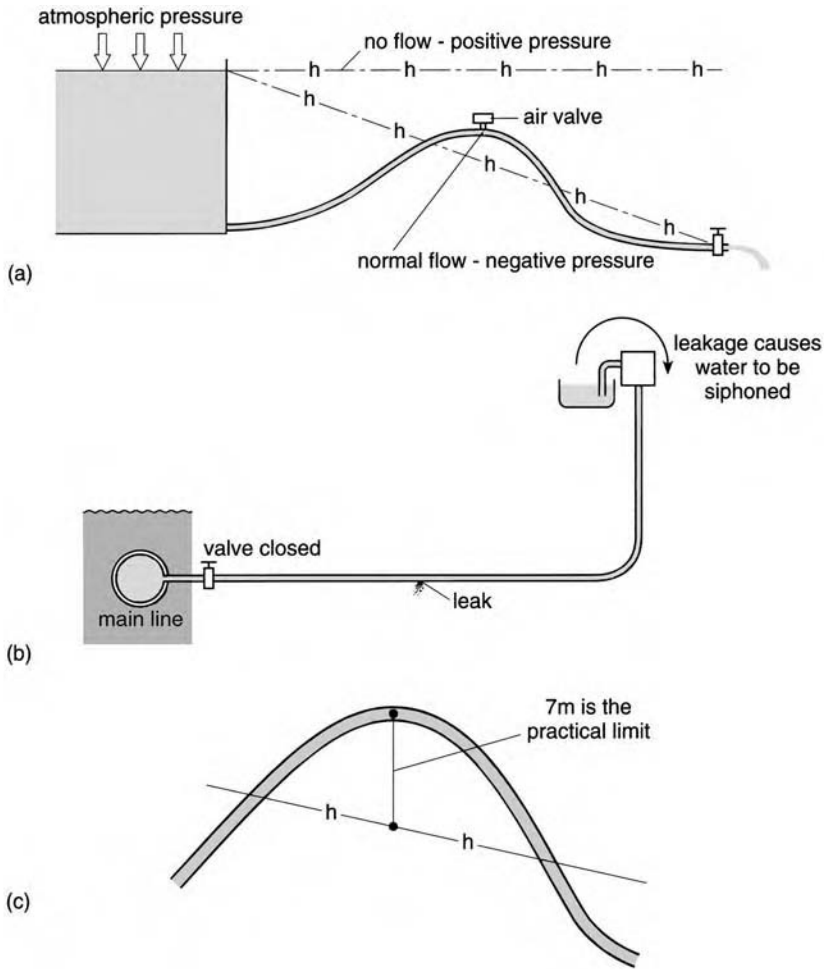
Although there is an energy loss at the pipe connection with the reservoir in Figure 4.4a this is usually very small in comparison with the loss in the main pipeline and so it is often ignored. Similar losses occur at pipe bends, reducers, pipe junctions and valves and although each one is small, together they can add up. They can all be calculated individually but normal design practice is to simply increase the energy loss in the main pipeline by 10% to allow for all these minor losses.

4.7 Siphons

Siphon is the name given to sections of pipe that rise above the hydraulic gradient. Normally pipes are located well below the hydraulic gradient and this ensures that the pressure is always positive and so well above atmospheric pressure. Under these conditions water flows freely under gravity provided the outlet is lower than the inlet (Figure 4.4a). But when part of a pipeline is located above the hydraulic gradient, even though the outlet is located below the inlet, water will not flow without some help (Figure 4.5a). This is because the pressure in the section of pipe above the hydraulic gradient is negative.

4.7.1 How they work

Before water will flow, all the air must be taken out of the pipe to create a vacuum. When this happens atmospheric pressure on the open water surface pushes water into the pipe to fill the vacuum and once it is full of water it will begin to flow. Under these conditions the pipe is working as a siphon. Taking the air out of a pipeline is known as *priming*. Sometimes a pump is needed to extract the air but if the pipeline can be temporarily brought below the hydraulic gradient the resulting positive pressure will push the air out and it will prime itself. This can be done by closing the main valve at the end of the pipeline so that the hydraulic gradient rises to a horizontal line at the same level as the reservoir surface. An air valve on top of the siphon then releases the air. Once the pipe is full of water, the main valve can then be opened and the pipeline flows normally.



4.5 Siphons.

Even pipelines that normally operate under positive pressures have air valves. These release air which accumulates at high spots along the line. So it is good practice to include an air valve at such locations. They can be automatic valves or just simple gate valves that are opened manually occasionally to release air.

It can sometimes be difficult to spot an air valve that is above the hydraulic gradient and this can lead to problems. An engineer visiting a remote farm saw what he thought was a simple gated air valve on a high spot on a pipeline supplying the farm with water. Air does tend to accumulate over time and can restrict the flow. So he thought he would do the farmer a favour and open the valve to bleed off any air that had accumulated. After a while he realised that the hissing sound was not air escaping from the pipe but air rushing in. The pipe was in fact above the hydraulic gradient and was working as a siphon at that point and the valve was only there to let air out during the priming process. The pressure inside the pipe was in fact negative and so when he opened the valve air was sucked and this de-primed the siphon. Realising his mistake he quickly closed the valve and went on down to the farmhouse. The farmer was most upset. What a coincidence – just as a water engineer had arrived, his water supply had suddenly stopped and an engineer was on hand to fix it for him!

If your car ever runs out of petrol a siphon can be a useful means of taking some fuel from a neighbour's tank. Insert a flexible small diameter tube into the tank and suck out all the air (making sure not to get a mouthful of petrol). When the petrol begins to flow catch it in a container and then transfer it to your car. Make sure that the outlet is lower than the liquid level in the tank otherwise the siphon will not work.

Another very practical use for siphons is to detect leakage in domestic water mains (sometimes called rising mains) from the supply outside in the street to a house (Figure 4.5b). This can be important for those on a water meter who pay high prices for their water. A leaky pipe in this situation would be very costly. The main valve to the house must first be closed. Then seal the cold water tap inside the house by immersing the outlet in a pan of water and opening the tap. If there is any leakage in the main pipe then water will be siphoned back out of the pan into the main. The rate of flow will indicate the extent of the leakage.

Siphons can be very useful in situations where the land topography is undulating between a reservoir and the water users. It is always preferable to locate a pipe below the hydraulic gradient by putting it in a deep trench but this may not always be practicable. In situations where siphoning is unavoidable the pipeline must not be more than 7 m above the hydraulic grade line. Remember atmospheric pressure drives a siphon and the absolute limit is 10 m head of water. So 7 m is a safe practical limit. When pipelines are located in mountainous regions the limit needs to be lower than this due to the reduced atmospheric pressure.

The pressure inside a working siphon is less than atmospheric pressure and so it is negative when referred to as a gauge pressure (measured above or below atmospheric pressure as the datum), for example, a -7 m head. Sometimes siphon pressures are quoted as absolute pressures (measured above vacuum pressure as the datum). So -7 m gauge pressure is the same as $+3$ m absolute pressure. This is calculated as follows:

$$\begin{aligned} \text{gauge pressure} &= -7 \text{ m head} \\ \text{absolute pressure} &= \text{atmospheric pressure} + \text{gauge pressure} \\ &= 10 - 7 = 3 \text{ m head absolute} \end{aligned}$$

4.8 Selecting pipe sizes in practice

The development of λ as a pipe roughness coefficient is an interesting story and this nicely leads into the use of the Darcy–Weisbach formula for linking energy loss with the various pipe parameters. There are several examples using this formula in the boxes and they demonstrate well the effects of pipe length, diameter and velocity on energy loss. So it is a useful learning tool.

Engineers in different industries and in different countries have also used other formulae often developed empirically to fit their particular circumstances. But these are gradually being abandoned and replaced by the Colebrook-White formula which accurately deals with most commercially available pipes. The task of using the formula, which is a rather complicated one, is made simple by the fact that it is now available as a set of design charts (Figure 4.6). The charts are also easier to use because discharge can be related directly to pipe diameter whereas Darcy–Weisbach formula only links to velocity and so requires an extra step (continuity equation) to get to discharge.

The boxes provide examples of the use of Darcy–Weisbach formula and design charts based on Colebrook-White formula.

EXAMPLE: CALCULATING PIPE DIAMETER USING DARCY–WEISBACH FORMULA

A 2.5 km long pipeline connects a reservoir to a smaller storage tank outside a town which then supplies water to individual houses. Determine the pipe diameter when the discharge required between the reservoir and the tank is 0.35 m^3/s and the difference in their water levels is 30 m. Assume the value of λ is 0.03.

This problem can be solved using the energy equation. The first step is to write down the equation for two points in the system. Point 1 is at the water surface of the main reservoir and point 2 is at the surface of the tank. Friction losses are important in this example and so these must also be included:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

This equation can be greatly simplified. p_1 and p_2 are both at atmospheric pressure and are zero. The water velocities v_1 and v_2 in the two tanks are very small and so the kinetic energy terms are also very small and can be assumed to be zero. This leaves just the potential energy terms z_1 and z_2 and the energy loss term h_f so the equation simplifies to:

$$h_f = z_1 - z_2$$

Using the Darcy–Weisbach formula for h_f:

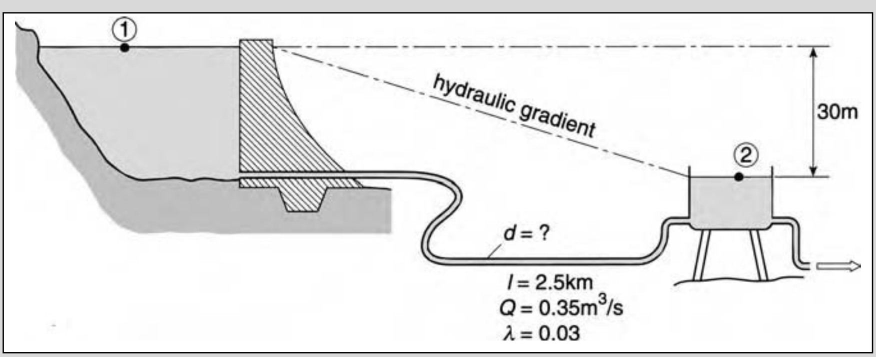
$$h_f = \frac{\lambda l v^2}{2gd}$$

And so:

$$\frac{\lambda l v^2}{2gd} = z_1 - z_2$$

Diameter d is unknown but so is the velocity in the pipe. So first calculate velocity v using the continuity equation:

$$Q = va$$
$$v = \frac{Q}{a}$$



4.6 Calculating the pipe diameter.

Calculate area *a*:

$$a = \frac{\pi d^2}{4}$$

And use this value to calculate *v*:

$$v = \frac{4Q}{\pi d^2} = \frac{4 \times 0.35}{3.14 \times d^2} = \frac{0.446}{d^2}$$

Note that as *d* is not known it is not yet possible to calculate a value for *v* and so this must remain as an algebraic expression for the moment.

Put all the known values into the Darcy–Weisbach equation:

$$\frac{0.03 \times 2500 \times 0.198}{2 \times 9.81 \times d \times d^4} = 0.35$$

Rearrange this to calculate *d*:

$$d^5 = \frac{0.03 \times 2500 \times 0.198}{2 \times 9.81 \times 30} = 0.025$$

Calculate the fifth root of 0.025 to find *d*:

$$d = 0.47 \text{ m} = 470 \text{ mm}$$

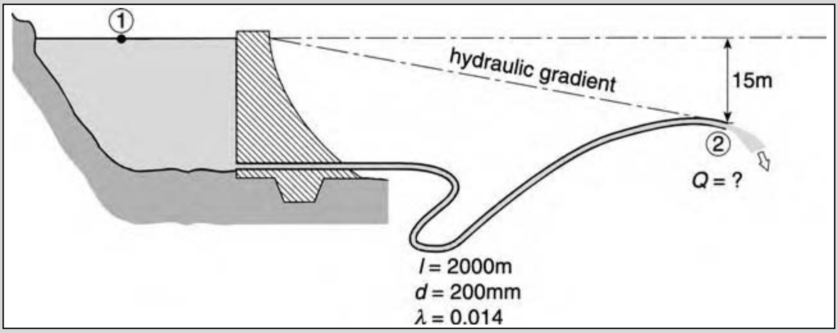
The nearest pipe size to this would be 500 mm. So this is the size of pipe needed to carry this flow between the reservoir and the tank.

This may seem rather involved mathematically but another approach, and perhaps a simpler one, is to guess the size of pipe and then put this into the equation and see if it gives the right value of discharge. This 'trial and error' approach is the way most engineers approach the problem. The outcome will show if the chosen size is too small or too large. A second or third guess will usually produce the right answer. If you are designing pipes on a regular basis you soon learn to 'guess' the right size for a particular installation. The design then becomes one of checking that your guess was the right one.

Try this design example again using the design chart in Figure 4.6 above to see if you get the same answer.

EXAMPLE: CALCULATING DISCHARGE FROM A PIPELINE

A 200 mm diameter pipeline 2000 m long is connected to a reservoir and its outlet is 15 m below the reservoir water level and discharges freely into the atmosphere. Calculate the discharge from the pipe when the friction factor is 0.014.



4.7 Measuring the discharge.

To solve this problem use the energy equation between point 1 at the surface of the reservoir and point 2 just inside the water jet emerging from the pipe outlet:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

This equation can be greatly simplified because p_1 is at atmospheric pressure and so is zero. Also p_2 is very near atmospheric pressure because the position of 2 is in the jet as it emerges from the pipe into the atmosphere. If it was above atmospheric pressure then the jet would flow laterally under the pressure. It does not do this and so the pressure can be assumed to be close to atmospheric pressure. Therefore p_2 is zero. The water velocity v_1 is zero in the reservoir and v_2 at the outlet is very small in comparison with the potential energy of 30 m and so this can be assumed to be zero also. This leaves just the potential energy terms z_1 and z_2 and the energy loss term h_f so the equation simplifies to:

$$z_1 - z_2 = h_f = \frac{\lambda l v^2}{2gd}$$

Put in the known values and calculate velocity v:

$$15 = \frac{0.014 \times 2000 \times v^2}{2 \times 9.81 \times 0.2}$$

$$v^2 = \frac{15 \times 2 \times 9.81 \times 0.2}{0.014 \times 2000} = 2.1$$

$$v = 1.45 \text{ m/s}$$

Use the continuity equation calculate the discharge:

$$Q = va$$

Calculate area a:

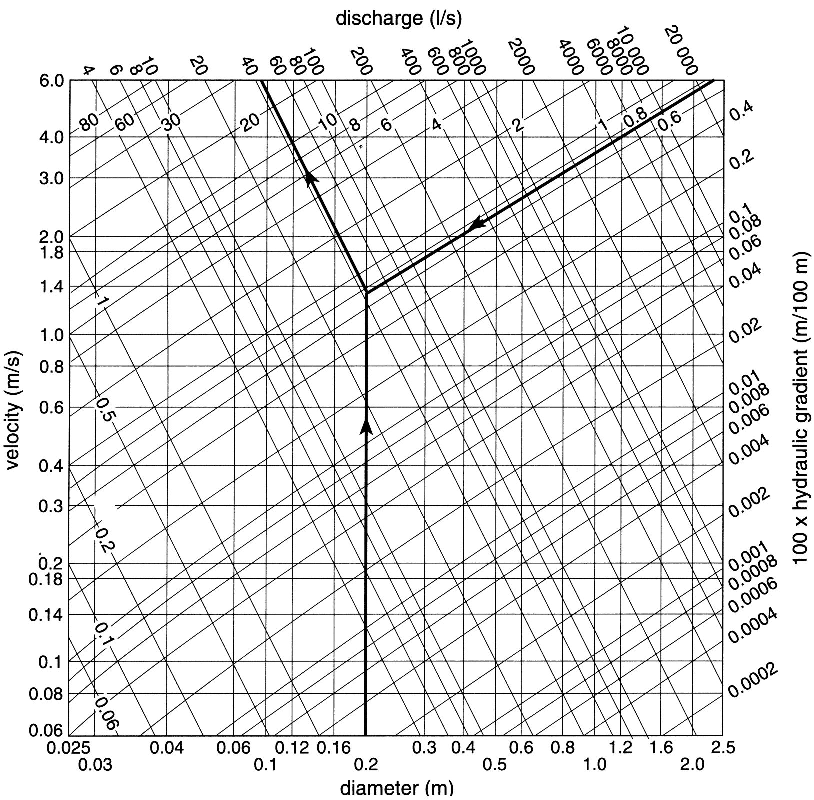
$$a = \frac{\pi d^2}{4} = \frac{\pi 0.2^2}{4} = 0.031 \text{ m}^2$$

$$Q = 1.45 \times 0.031$$

$$Q = 0.045 \text{ m}^3\text{/s or 45 l/s}$$

4.8.1 Using hydraulic design charts

Pipe charts are now an increasingly common way of designing pipes. An excellent and widely used source is *Hydraulic Design of Channels and Pipes* by Peter Ackers (see references for details). This is a book of design charts based on the Colebrook-White equation. The equation best describes the transitional flow between smooth and rough pipe flow referred to in Section 4.4.1 which covers all commercially available pipes. An example of one of these charts is shown in Figure 4.8. It does not use λ values but expresses friction as the height of the roughness on the inside of a pipe. So for this chart, surface roughness is $k = 0.03$ mm and this is representative of asbestos cement and PVC pipes in reasonably good condition. The chart's range of flows is considerable; from less than 0.1 l/s to 20 000 l/s (or 20 m^3/s) with pipe diameters from 0.025 m to 2.5 m. This should satisfy most pipe designers. In the box is an example showing what a design chart looks like.



4.8 Typical pipe design chart.

EXAMPLE: CALCULATING DISCHARGE FROM A PIPELINE USING A DESIGN CHART

Using the same example as for the Darcy–Weisbach equation. A 200 mm diameter pipeline 2000 m long is connected to a reservoir and the outlet is 15 m below the reservoir water level. Calculate the discharge from the pipe when the pipe roughness value k is 0.03 mm.

The design chart uses the hydraulic gradient to show the rate of head loss in a pipeline. So the first step is to calculate the hydraulic gradient from the information given above. The pipe is 2000 m long and the head loss from the reservoir to the pipe outlet is 15 m so:

$$\text{hydraulic gradient} = \frac{h}{l} = \frac{15}{2000} = 0.0075$$

$$\text{or} \quad = 0.75 \text{ m}/100 \text{ m}$$

Using the chart locate the intersection of the lines for a hydraulic gradient of 0.75 m/100 m and a diameter of 200 mm. This locates the discharge line and the value of the discharge:

$$Q = 0.045 \text{ m}^3/\text{s} \quad \text{or} \quad 45 \text{ l/s}$$

Note that the chart can also be used in reverse to determine the diameter of a pipe and head loss for a given discharge.

There are four important practical points to note from the examples.

The first point refers to the first worked example which showed how mathematically cumbersome it can be to determine the diameter by calculation. The easier way is to do what most engineers do; they guess the diameter and then check by calculation that their chosen pipe is the right one. This might seem a strange way of approaching a problem but it is quite common in engineering. It always helps to know approximately the answer to a problem before beginning to solve it. An experienced engineer usually knows what answer to expect; the calculation then becomes just a way of confirming this. (This is one of the basic unwritten laws of engineering – that you need to know the answer to the problem before you begin so that you know that you have the right answer when you get there.) This foresight is important because when you calculate a pipe diameter how will you know that you have arrived at the right answer? There will not be any answers available to the real problems in the field as there are in this text book and there may not be anyone around to ask if it is the right answer. You need to know that you have the right answer and this comes largely from experience of similar design problems. New designers are unlikely to have this experience, but they have to start somewhere and one way is to rely initially on the experience of others and to learn from them. This is the apprenticeship that all engineers go through to gain experience and become competent designers.

The second point to note is that there is no one unique pipe diameter that must be used in any given situation. If, for example, calculations show that a 100 mm pipe is sufficient then any pipe larger than this will also carry the flow. The question of which one is the most acceptable is usually determined by several design criteria. One is that the velocity should not exceed 1.6 m/s. Another might be a limit on the head that can be lost through friction. A third could be a limit on the pipe sizes available. There are only certain standard sizes which are manufactured and not all these may be readily available in some countries. A final deciding factor is cost – which pipe is the cheapest to buy and to operate?

The third point to consider is the value for pipe roughness. It is easy to choose the value for a new pipe but how long will it be before the roughness increases? What will the value be in, say, 10 years from now when the pipe is still being used? The roughness will undoubtedly increase through general wear and tear. If there has been lime scale deposits or algae slime build up on the inside of the pipe or the pipe has been misused and damaged then the roughness will be significantly greater. So when choosing the most appropriate value for design it is important to think ahead to what the roughness might be later in the life of the pipe. This is where engineering becomes an art and all the engineer's experience is brought to bear in selecting the right roughness value for design purposes. If the selected roughness is too low then the pipe may not give good, long service. If it is too high then this will result in the unnecessary expense of having pipes which are too big for the job in hand.

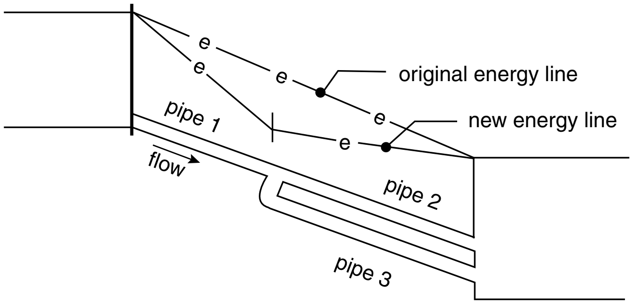
The final point, and often the most important to consider, is cost. Small diameter pipes are usually cheaper than larger diameter ones but they require more energy to deliver the discharge because of the greater friction. This is particularly important when water is pumped and energy costs are high. The trade-off between the two must take account of both capital and operating costs if a realistic comparison is to be made between alternatives. This aspect of pipeline design is covered in more detail in Chapter 8 Pumps.

4.8.2 Sizing pipes for future demand

Pipe sizes are often selected using a discharge based on present water demands and little thought is given to how this might change in the future. Also there is always a temptation to select small pipes to satisfy current demand simply because they are cheaper than larger ones. These two factors can lead to trouble in the future. If demand increases and higher discharges are required from the same pipe, the energy losses can rise sharply and so a lot more energy is needed to run the system.

As an example, a 200 mm diameter pumped pipeline 500 m long supplies a small town with a discharge of 50 l/s. Several years later the demand doubles to 100 l/s. This increases the velocity in the pipe from 1.67 to 3.34 m/s (i.e. it doubles) which pushes up the energy loss from 5 m to 20 m (i.e. a four-fold increase). This increase in head loss plus the extra flow means that eight times more energy is needed to operate the system and extra pumps may be required to provide the extra power input. A little extra thought at the planning stage and a little more investment at the beginning could save a lot of extra pumping cost later.

Increasing the energy available is one way of increasing the discharge in a pipeline to meet future demand. But another way is to increase the effective diameter of the pipe. The practical way of doing this is to lay a second pipe parallel to the first one. It may not be necessary to lay the second pipe along the entire length. Pipes are expensive and so from a cost point of view only the minimum length of parallel pipe should be laid to meet the demand (Figure 4.9). The discharge in pipe 1 will equal the discharge in pipe 2. This is the original pipeline carrying the (inadequate) discharge between the tanks – note the energy line which represents the uniform energy loss along the pipeline. Pipe 3 is the new pipe laid parallel to the original pipe and so the combined effect of pipes 2 and 3 is to increase the cross-sectional area carrying the discharge and decrease the energy loss along the parallel section of the pipeline (the velocity is lower because of the increased area). The effect of reducing the energy loss in the parallel section is to make more energy available to move water through pipeline 1 and so the overall discharge is increased. Note how the energy line for pipe 1 in the new system is steeper showing that it is carrying a higher discharge. The energy line for the parallel pipes has a more gentle gradient due to the overall reduction in velocity in this section. The length of parallel pipe depends on the required increase in discharge. Should the discharge demand increase further in the future the length of parallel pipeline can be extended to suit. The job for the designer is to decide on the diameter and length of pipe 3.



4.9 Parallel pipes can increase discharge with the same energy.

EXAMPLE: CALCULATING LENGTH OF A PARALLEL PIPE

A 1000 m long pipeline 150 mm diameter supplies water from a reservoir to an offtake point. Calculate the discharge at the offtake when the head available is 10 m. Since the pipeline was installed the water demand has doubled and so a parallel 250 mm dia pipeline is to be installed alongside the original pipeline (Figure 4.9). Calculate the length of new pipe required to double the discharge. Assume the friction factor for the pipelines is $k = 0.03$ mm. Use the pipe design chart in Figure 4.8 above.

First calculate the original discharge. Calculate the hydraulic gradient (100 h/l) and together with the pipe diameter determine the discharge from the pipe design chart. The data and the results are tabulated as follows:

Pipe	*Pipe dia (mm)*	*Length (m)*	*Friction k (mm)*	*Hyd grad (m/100 m)*	*Discharge (l/s)*
Original pipe	150	1000	0.03	1.0	23

The demand has now doubled to 46 l/s. So the additional 23 l/s is supplied by introducing a pipe of 250 mm diameter – pipe 3 – alongside the original pipeline but as yet of unknown length. This length cannot be calculated directly and requires some iteration. In other words, some intelligent guess work. It is convenient at this stage to divide the original pipeline into two parts – pipe 2 which has the same length as pipe 3, and pipe 1 which is from the reservoir to the point where the two parallel pipes join.

First determine the hydraulic gradient in the two parallel pipes – pipes 2 and 3 – so that the two pipelines carry a combined discharge of 46 l/s. The gradient will be the same for each pipeline as they have the same pressure at the points of connection and discharge. Mark on the pipe chart vertical lines representing the two pipe diameters. Look for a hydraulic gradient that intersects the pipe ‘lines’ so that the sum of the two discharges is 46 l/s. This occurs at a hydraulic gradient of 0.18 m/100 m which results in discharges of 10 and 35 l/s totalling 45 l/s. This is close enough to 46 l/s.

Next determine the hydraulic gradient for pipe 1 for a discharge of 46 l/s. From the chart this is 3.2 m/100 m.

Using this information it is now possible to set up an equation to calculate the length of pipe 3.

The sum of the head loss in pipes 1 and 3 (remember the loss in pipe 3 will be the same as pipe 2) is 10 m. So:

$$h_1 + h_3 = 10 \text{ m}$$

Now:

$$\frac{100h_1}{L_1} = \text{hydraulic gradient} = 3.2$$

$$h_1 = \frac{L_1 \times 3.2}{100}$$

Similarly:

$$h_3 = \frac{L_3 \times 0.18}{100}$$

So:

$$\frac{L_1 \times 3.2}{100} + \frac{L_3 \times 0.18}{100} = 10$$

Both L_1 and L_3 are unknown. So to 'eliminate' one of the unknowns substitute for L_1 in terms of L_3:

$$L_1 = 1000 - L_3$$

Substitute this into the above equation:

$$\frac{(1000 - L_3) \times 3.2}{100} + \frac{L_3 \times 0.18}{100} = 10$$

$$(1000 - L_3)\ 3.2 + 0.18L_3 = 1000$$

$$3200 - 3.2L_3 + 0.18L_3 = 1000$$

$$3.38L_3 = 2200$$

$$L_3 = \frac{2200}{3.38} = 650 \text{ m}$$

So the length of pipe 3 is 650 m. This is also the length of pipe 2. Pipe 1 will be 350 m. The following table summarises the results:

Pipe	*Pipe dia (mm)*	*Friction k (mm)*	*Hyd grad (m/100 m)*	*Discharge (l/s)*	*Length (m)*
Pipe 1	150	0.03	3.20	46	350
Pipe 2	150	0.03	0.18	10	650
Pipe 3	250	0.03	0.18	35	650

This is one example where using a formula makes the problem easier to solve as it avoids the iterative approach. Try to solve the problem using the Darcy–Weisbach formula and the continuity equation with a value of $\lambda = 0.04$ for the pipes.

4.9 Pipe networks

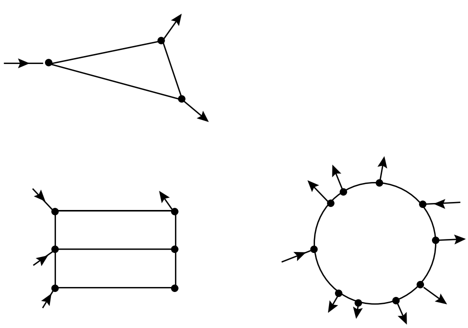
Most water supply systems are not just single pipes but involve several branching pipes or a pipe network. These supply water to several reservoirs or dwellings in a village or town (Figure 4.10). Some networks are simple and involve just a few pipes but some are quite complicated involving many different pipes and connections. Sometimes the pipes form a ring or loop and this ensures that if one section of pipe fails for some reason then flow can be maintained from another direction. It also has hydraulic advantages. Each offtake point is supplied from two directions and so the pipe sizes in the ring can be smaller than if the point was fed from a single pipe.

The simplest example of a ring or loop network is a triangular pipe layout (Figure 4.11). Water flows into the loop at point A and flows out at points B and C. So the water flows away from point A towards B and C. But there will also be a flow in pipe BC and this could be in the direction BC or CB depending on the pressure difference between B and C. If the pressure at B is higher than C then there will be a flow from B to C. Conversely if the pressure at C is higher than the pressure at B then the flow will be from C to B.

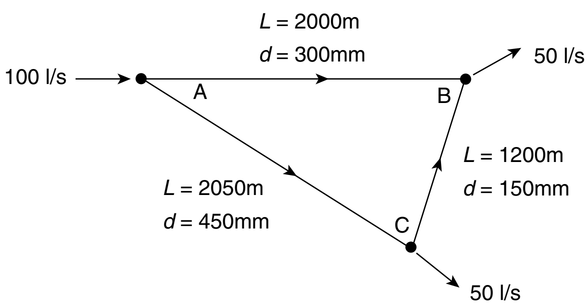
There are three rules to solving the problem of pressures and discharges in a network:

1 The sum of all the discharges at a junction is zero;
2 The flow in one leg of a network will be in the direction of the pressure drop;
3 The sum of the head losses in a closed loop will be zero for a start and finish at any junction in the loop.

These rules apply to all networks, not just the simple ones. However, the calculations can get rather involved. An example of how the rules are applied to a simple network is shown in the box.



4.10 Pipe networks.



4.11 Triangular pipe network.

EXAMPLE: CALCULATING DISCHARGES AND PRESSURES IN A PIPE NETWORK

A simple network of three pipes forms a triangle ABC. The following data are available

Pipe	*Diameter (mm)*	*Length (m)*	*Friction factor k (mm)*
AB	300	2000	0.03
BC	150	1200	0.03
CA	450	2050	0.03

The discharge entering the system at point A is 100 l/s and this meets the demand at B of 50 l/s and at C of 50 l/s. Calculate the discharges in each pipe and the pressures at B and C if A is supplied at a pressure of 100 m head of water. Use the pipe design chart in Figure 4.8.

Start by looking at the data, apply some common sense, and rule (1) get an assessment of the likely discharges in each pipe. AB and AC are approximately the same in length and AC has a large diameter. So assume Q_{AC} is 60 l/s, Q_{AB} is 40 l/s and Q_{CB} is 10 l/s. Note the assumed flow directions indicated by the arrows.

Next calculate the head loss in each pipe using the pipe chart in Figure 4.8. Notice how the discharges in pipes BC and CA are listed as negative. The sign comes from considering the discharges positive in a clockwise direction around the loop.

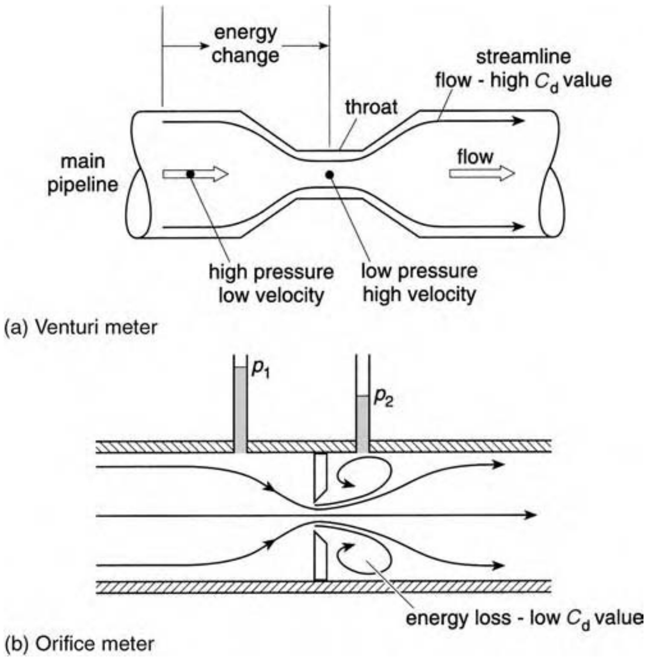
Pipe	*Diameter (mm)*	*Discharge (l/s)*	*Hydraulic gradient (m per 100 m)*	*Length (m)*	*Head loss (m)*
AB	300	40	0.14	2000	+2.80
BC	150	−10	0.2	1200	−2.40
CA	450	−60	0.025	2050	−0.51

Applying rule (3) from the start point A and moving in a clockwise direction add up all the head losses in the pipes, that is, 2.80 – 2.40 – 0.51 = 0.11 m. This sum should come to zero. To make the above sum come to zero slightly decrease the discharge in pipe AB, recalculate the head losses again and see if the sum of the head losses comes to zero. In this case the value is close to zero so this suggests that the discharge values chosen are close to the right ones. So there is no need for further iteration.

4.10 Measuring discharge in pipes

Discharges in pipelines can be measured using a *venturi meter* or an *orifice plate* (Figure 4.12). Both devices rely on changing the components of the total energy of flow from which discharge can be calculated (see Section 3.7.6). The venturi meter was developed by an American, Clemens Herschel (1842–1930) who was looking for a way to measure water abstraction from a river by industrialists. Although the principles of this measuring device were well established by Bernoulli it was Herschel who, being troubled by unlicensed and unmeasured removal of water by pipelines from a canal by paper mills, developed it into the device used today.

A venturi meter comprises a short, narrow section of pipe (throat) followed by a gradually expanding tube. This causes the flow velocity to increase (remember continuity) and so the kinetic energy increases also. As the total energy remains the same throughout the system it follows that there must be a corresponding reduction in pressure energy. By measuring this change in pressure using a pressure gauge or a manometer and using the continuity and energy



4.12 Measuring discharge in pipelines.

equations, the following formula for discharge in the pipe can be obtained:

$$Q = C_d a_1 \sqrt{\frac{2gH}{m^2 - 1}}$$

$$m = \frac{a_1}{a_2}$$

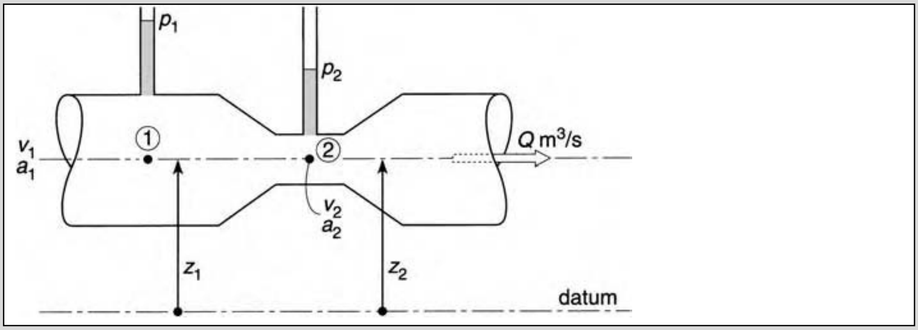
where a_1 is area of main pipe (m^2); a_2 is area of venturi throat (m^2); H is the head difference between pipe and throat (m); g is gravity constant (9.81 m/s^2); C_d is coefficient of discharge.

This is the theory, but in practice there are some minor energy losses in a venturi meter, so a coefficient discharge C_d is introduced to obtain the true discharge. Care is needed when using this formula. Some textbooks quote the formula in terms of a_2 rather than a_1 and this changes several of the terms. It is the same formula from the same fundamental base but it can be confusing. The safest way is to avoid the formula and work directly from the energy and continuity equations. A derivation of the formula and an example of calculating discharge working from energy and continuity are shown in the boxes.

DERIVATION: FORMULA FOR DISCHARGE IN A VENTURI METER

First write down the energy equation for the venturi meter. Point 1 is in the main pipe and point 2 is located in the throat of the venturi. It is assumed that there is no energy loss between the two points. This is a reasonable assumption as contracting flows suppress turbulence which is the main cause of energy loss.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$



4.13 Venturi meter for measuring discharge.

As the venturi is horizontal:

$$z_1 = z_2$$

And so:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

Now rearrange this equation so that all the pressure terms and all the velocity terms are brought together:

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

The left-hand side of this equation is the pressure difference between points 1 and 2 which can be measured using pressure gauges or a differential manometer. This is a manometer with one limb connected to the pipe and the other limb connected to the throat (see Section 2.9.4).

At this point it is not possible to calculate the velocities because both v_1 and v_2 are unknown. A second equation is needed to do this – the continuity equation.

Write the continuity equation for points 1 and 2 in the venturi:

$$a_1 v_1 = a_2 v_2$$

Rearrange this:

$$v_2 = \frac{a_1}{a_2} v_1$$

Now a_1 and a_2 are the cross-sectional areas of the pipe and venturi respectively and can be calculated from the pipe and venturi throat diameters respectively. Substituting for v_2 in the energy equation

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \left(\frac{a_1^2}{a_2^2}\right) \frac{v_1^2}{2g} - \frac{v_1^2}{2g}$$

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{v_1^2}{2g}\left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

Put:

$$H = \frac{p_1}{\rho g} - \frac{p_2}{\rho g}$$

where H is the difference in head between the pipe (point 1) and the venturi throat point 2)

Rearrange the equation for v_1

$$v_1 = \sqrt{2gH}\left(\frac{a_2}{\sqrt{a_1^2 - a_2^2}}\right)$$

Use the continuity equation to calculate discharge:

$$Q = a_1 v_1$$

And so:

$$Q = \sqrt{2gH}\left(\frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\right)$$

Put:

$$m = \frac{a_1}{a_2}$$

And so:

$$Q = a_1\sqrt{\frac{2gH}{m^2 - 1}}$$

Introduce a coefficient of discharge C_d:

$$Q = C_d a_1\sqrt{\frac{2gH}{m^2 - 1}}$$

In the case of the venturi meter $C_d = 0.97$ which means that the energy losses are small and the energy theory works very well ($C_d = 1.0$ would mean the theory was perfect). For orifice plates the same theory and formula can be used but the value of C_d is quite different at $C_d = 0.6$. The theory is not so good for this case because there is a lot of energy loss (Figure 4.12b). The water is not channelled smoothly from one section to another as in the venturi but is forced to make abrupt changes as it passes through the orifice and expands downstream. Such abruptness causes a lot of turbulence which results in energy loss. (The C_d value is similar to that for orifice flow from a tank – see Section 3.6.3.)

EXAMPLE: CALCULATING DISCHARGE USING A VENTURI METER

A 120 mm diameter venturi meter is installed in a 250 mm diameter pipeline to measure discharge. Calculate the discharge when the pressure difference between the pipe and the venturi throat is 2.5 m of head of water and C_d is 0.97.

Although there is a formula for discharge it can be helpful to work from first principles. Not only does this reinforce the principle but it also avoids possible errors in using a rather involved formula which can easily be misquoted. So this example is worked from the energy and continuity equations.

First step write down the energy equation:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As the venturi is horizontal:

$$z_1 = z_2$$

And so:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

Now rearrange this equation so that all the pressure terms and all the velocity terms are brought together:

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But:

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 2.5 \text{ m}$$

Remember that the pressure terms are in m head of water and it is the difference that is important and not the individual pressures:
And so:

$$\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = 2.5 \text{ m}$$

It is not possible to solve this equation directly as both v_1 and v_2 are unknown. So use continuity to obtain another equation for v_1 and v_2:

$$a_1 v_1 = a_2 v_2$$

Rearrange this:

$$v_2 = \frac{a_1}{a_2} v_1$$

Next step calculate the areas a_1 and a_2.

Area of pipe:

$$a_1 = \frac{\pi d_1^2}{4} = \frac{\pi 0.25^2}{4} = 0.05 \text{ m}^2$$

Area of venturi:

$$a_2 = \frac{\pi d_2^2}{4} = \frac{\pi 0.12^2}{4} = 0.011 \text{ m}^2$$

$$v_2 = \frac{0.05}{0.011} \quad v_1 = 4.55\ v_1$$

Substitute this value for v_2 in the energy equation:

$$\frac{4.55^2 v_1^2}{2g} - \frac{v_1^2}{2g} = 2.5 \text{ m}$$

$$20.7\ v_1^2 - v_1^2 = 2.5 \times 2 \times 9.81 = 49.05$$

$$v_1 = \sqrt{\frac{49.05}{19.7}} = 1.57 \text{ m/s}$$

Calculate Q:

$$Q = C_d\ v_1\ a_1$$
$$= 0.97 \times 1.57 \times 0.05$$
$$Q = 0.076 \text{ m}^3\text{/s}$$

4.11 Momentum in pipes

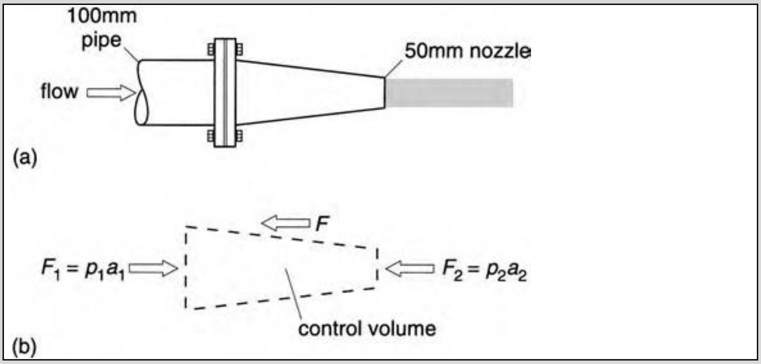
The momentum equation is used in pipe flow to calculate forces on pipe fittings such as nozzles, pipe bends and valves. In more advanced applications it is used in the design of pumps and turbines where water flow creates forces on pump and turbine impellers.

To solve force and momentum problems a concept known as the *control volume* is used. This is a way of isolating part of a system being investigated so that the momentum equation can be applied to it. To see how this works an example is given in the box below showing how the force on a pipe reducer (or nozzle) can be calculated.

EXAMPLE: CALCULATING THE FORCE ON A NOZZLE

A 100 mm diameter fire hose discharges 15 l/s from a 50 mm diameter nozzle. Calculate the force on the nozzle.

To solve this problem all three hydraulic equations are needed; energy, continuity and momentum. The energy and continuity equations are needed to calculate the pressure in the 100 mm pipe and momentum is then used to calculate the force on the nozzle.



4.14 Calculating the force on a nozzle.

The first step is to calculate the pressure p_1 in the 100 mm pipe. Use the energy equation:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

The pressure in the jet as it emerges from the nozzle into the atmosphere is p_2. The jet is at the same pressure as the atmosphere and so the pressure p_2 is zero. The potential energies z_1 and z_2 are equal to each other because the nozzle and the pipe are horizontal and so they cancel out.

So the energy equation becomes:

$$\frac{p_1}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

The value of p_1 is unknown and so are v_1 and v_2. So the next step is to calculate the velocities from the discharge equation:

$$v_1 = \frac{Q}{a_1} \quad \text{and} \quad v_2 = \frac{Q}{a_2}$$

Area of pipe:

$$a_1 = \frac{\pi d_1^2}{4} = \frac{\pi 0.1^2}{4} = 0.0078 \text{ m}^2$$

Area of jet:

$$a_2 = \frac{\pi d_2^2}{4} = \frac{\pi 0.05^2}{4} = 0.0019 \text{ m}^2$$

Next calculate the velocities:

$$v_1 = \frac{Q}{a_1} = \frac{0.015}{0.0078} = 1.92 \text{ m/s}$$

And:

$$v_2 = \frac{Q}{a_2} = \frac{0.015}{0.0019} = 7.9 \text{ m/s}$$

Put all the known values into the energy equation:

$$\frac{p_1}{\rho g} = \frac{7.9^2}{2 \times 9.81} - \frac{1.92^2}{2 \times 9.81} = 2.99$$

$$p_1 = 2.99 \times 1000 \times 9.81 = 29\,332 \text{ N/m}^2$$

The final step is to calculate the force F on the nozzle using the momentum equation. To do this the concept of the *control volume* isolates that part of the system being investigated. All the forces which help to maintain the control volume are then identified. F_1 and F_2 are forces due to the water pressure in the pipe and the jet and F is the force exerted on the water by the nozzle. Although the force F_2 is shown acting against the flow remember that there is an equal and opposite force acting in the direction of the flow (Newton's third law).

Use the momentum equation:

$$F_1 - F_2 - F = \rho Q\,(v_2 - v_1)$$

F is the force we wish to calculate but at this stage both F_1 and F_2 are also unknown. So the next step is to calculate F_1 and F_2 using known values of pressure and pipe area:

$$\begin{aligned} F_1 &= p_1 \times a_1 \\ &= 29\,332 \times 0.0078 = 228.8 \text{ N} \end{aligned}$$

And:

$$F_2 = p_2 \times a_2$$

But p_2 is zero and so:

$$F_2 = 0$$

Finally all the information for the momentum equation is available to calculate F:

$$\begin{aligned} 228.8 - F - 0 &= 1000 \times 0.015\,(7.9 - 1.92) \\ F &= 228.8 - 89.7 \\ F &= 139 \text{ N} \end{aligned}$$

As this is a fire hose, a firm grip would be required to hold it in place. If a fireman let go of the nozzle, the unbalanced force of 139 N would cause it to rapidly shoot backwards and this could do serious injury if it hit someone. A larger nozzle and discharge would probably need two firemen to hold and control it.

Forces, often large ones, also occur at pipe bends and this is not always appreciated. For example, a 90° bend on a 0.5 m diameter pipe operating at 30 m head carrying a discharge of 0.3 m^3/s would produce a force of 86 kN. This is a large thrust (over 8 tons) – which means that the bend must be held firmly in place if it is not to move and burst the pipe (Figure 4.15). A good way to deal with this is to bury the pipe below ground and encase the bend in concrete to stop it moving. The side of the trench must also be very firm for the concrete to push against it.

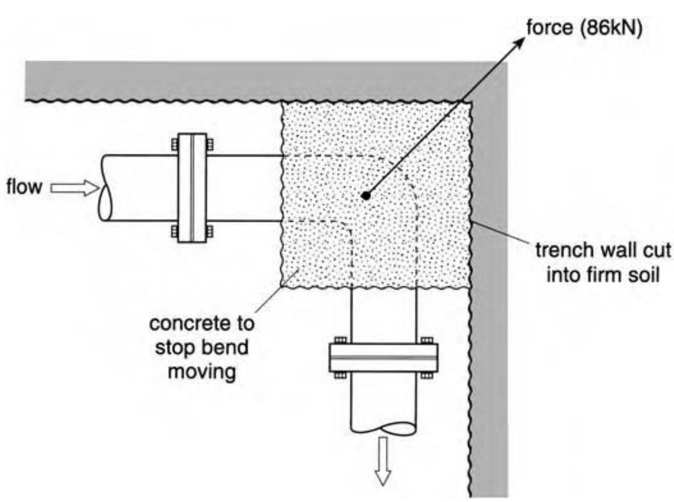
4.12 Pipe materials

Pipes are a major cost in any water supply or irrigation scheme. They must carry the design flow, resist all external and internal forces, be durable and have a long useful life. So it is important to know something about what pipes are available, how they are installed and used, what valves and fittings are needed and how to test them once they are in place.

4.12.1 Specifying pipes

Pipe manufacture is normally controlled by specifications laid down by national and international standards organisations. In the UK the British Standards Institute (BSI) has developed its own standards for pipes although a notable exception to this is the aluminium pipe used in many irrigation systems which has an international standard adopted by UK. The standards were developed for the water supply industry which demand high quality manufacture and rigorous testing of all pipes and fittings. The International Standards Organisation (ISO) does a similar job on an international scale. Many standards are the same and some BSI publications for example also have an ISO number.

Although the internal bore of a pipe determines its flow capacity it is not generally used to specify pipe size. The reason for this is that pipes are also classified according to pressure which means they have different wall thicknesses. If the outside diameter of a pipe is fixed, which is often the case from a pipe joining point of view, the internal bore will be different for different wall thicknesses. To add to this confusion different pipe materials (e.g. PVC, steel) will have different wall thicknesses. To overcome this problem manufacturers quote a *nominal pipe size*, which in many cases is neither the inside nor the outside diameter! – it is just an indication of



4.15 Force on a pipe bend.

the pipe diameter for selection purposes. It is also normal to specify the safe working pressure at which the pipes can be used.

4.12.2 Materials

Pipes are made from a wide variety of materials but the most common in water systems are steel, asbestos cement and various plastics. Steel pipes tend to be used only where very high pressures are encountered or in conjunction with other pipe materials where extra strength is needed, such as under roads or across ditches. Larger pipes are made from steel plate bent or rolled to shape and butt welded. But small sizes up to 450 mm are made from hot steel ingots which are pierced and rolled into a cylinder of the right dimensions. Corrosion is a problem with steel and so pipes are wrapped with bituminous materials or galvanising. Buried pipes often use cathodic protection. Corrosion is an electrolytic process set up between the pipe and the surrounding soil. It is a bit like a battery with the steel pipe acting as an anode which gradually corrodes. The process is reversed by making the pipe a cathode either by connecting it to an expendable anodic material such as magnesium or by passing a small electric current through the pipe material.

Asbestos cement pipes are still used in many countries for underground mains and are made from cement and asbestos fibre mixed in a slurry and deposited layer upon layer on a rolling mandrel. The pipe is then dipped in cold bitumen for protection and the end turned down to a specific diameter for jointing purposes. Unlike steel, asbestos cement does not corrode easily but it is easily damaged by shock loads and must be handled with care. Sometimes damage is not always apparent as in the case of hairline fractures. These only show up when the pipe is being site tested and leaks occur. Joints are made using an asbestos cement collar and rubber sealing rings similar to those used for PVC pipes. Bends and fittings cannot be made from asbestos cement and so ductile iron is used. An important criterion is that the outside diameter of the ductile steel must be the same as the asbestos cement so they can be effectively joined together.

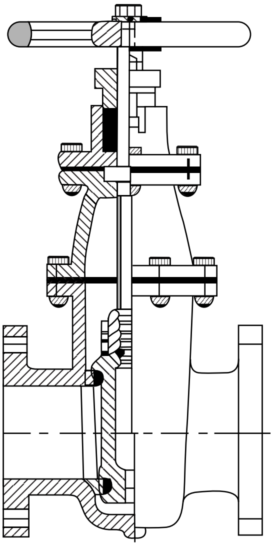
Several plastic materials are in common use for making pipes. Unplasticised polyvinyl chloride (UPVC) is a rigid material and an alternative to asbestos cement, which is falling out of favour because of the health risks associated with asbestos fibres. Pipes come in various pressure classes and are colour coded for easy recognition. They are virtually corrosion free, light in weight, flexible and easily jointed by a spigot and socket system using either a chemical solvent or rubber ring to create a seal. But laying PVC pipes needs care. They are easily damaged by sharp stones in the soil and distorted by poor compaction of material on the bottom and sides of trenches. Hydraulically PVC has a very low friction characteristic, but it can be easily damaged internally by sand and silt particles in the flowing water.

In contrast polythene pipe is very flexible and comes in high density and low density forms. Low density pipe is cheaper than high density and is used extensively for trickle and some sprinkler irrigation systems where pipes are laid out on the ground. The pipe wall is thin and so it is easily damaged by sharp tools or animals biting through it. However, it will stand up to quite high water pressures. High density pipes are much stronger, but more expensive and are used extensively for domestic water supply systems.

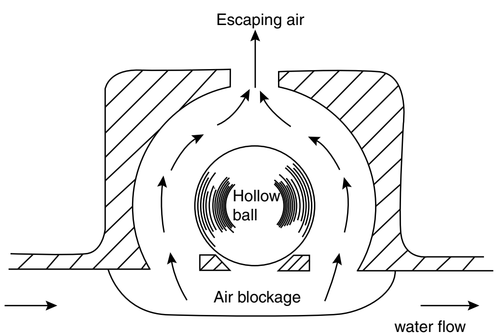
4.13 Pipe fittings

Pipelines require a wide range of valves and special fittings to make sure the discharge is always under control. Some of the more important are sluice valves, air valves, non-return valves and control valves (Figure 4.16).

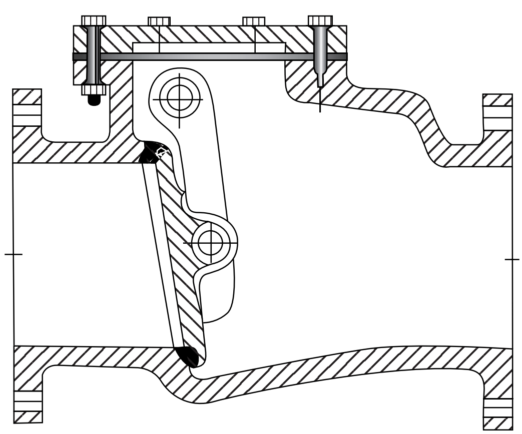
Sluice valves are essentially on-off valves. They can be used to control pressure and discharge but they are rather crude and need constant attention. It is only the last 10% of the gate opening



(a) Sluice valve.

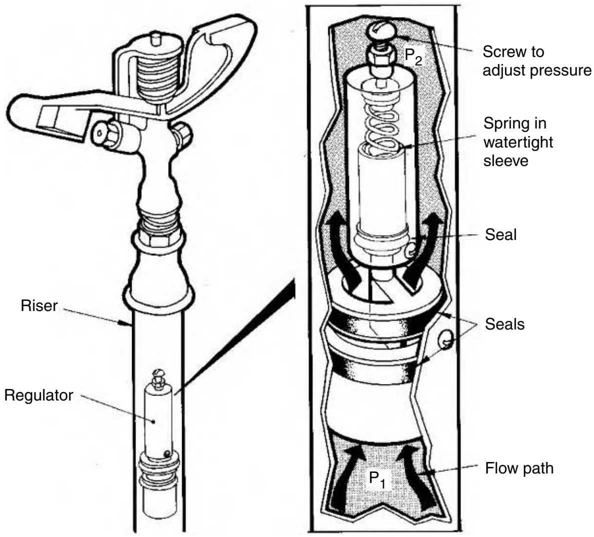


(b) Air valve.



(c) Non-return valve.

4.16 Pipe fittings.



(d) Pressure control valve.

4.16 Continued.

that has any real controlling influence. Moving the gate over the remaining 90% does not really influence the discharge, it just changes the energy from pressure to kinetic energy so the flow goes through the narrow opening faster. The valve body contains a gate which can be lowered using a screw device to close off the flow. The gate slides in a groove in the valve body and relies on the surfaces of the gate and the groove being forced together to make a seal. It was developed over 100 years ago for the water industry and has not changed materially since then. The gate can be tapered so that it does not jam in the gate guide but the taper gate must be opened fully otherwise it may start to vibrate once it is partially open. Sluice valves do require some effort to open them as water pressure builds up on one side producing an unbalanced head. The term 'cracking open' a valve is sometimes used to describe the initial effort needed to get the valve moving. A 100 mm valve fitted with a hand wheel is difficult to open when there is an unbalanced pressure of over 8 bar. Larger valves often used gearing mechanisms to move them and so opening and closing valves can be a slow business. This is an advantage as it reduces the problems of water hammer (see Section 4.14).

Butterfly valves do much the same job as sluice valves – they consist of a disc which rotates about a spindle across the diameter of the pipe. They are easier to operate than sluice valves requiring less force to open them. In spite of this they have not always been favoured by designers because the disc in an obstacle to the flow where debris can accumulate. It is also possible to close a small butterfly valve quite rapidly and this can cause water hammer problems.

Air valves are a means of letting unwanted air out of pipelines. Water can contain large volumes of dissolved air which can come out of solution when the pressure drops, particularly at high spots on pipelines (see Section 4.7). If air is allowed to accumulate it can block the flow or at least considerably reduce it. Air valves allow air to escape. There are two types depending on whether the pipeline is above or below the hydraulic gradient. For pipes below the hydraulic gradient where the water pressure is positive, the most common situation, a ball valve is used. When air collects in the pipe the ball falls onto its seating allowing air to escape. As the air escapes water rises into the valve which pushes the ball up to close off the valve. When the pipeline is above the hydraulic gradient the pressure inside the pipe is negative (below atmospheric pressure). In this case the ball valve would not work as air would be sucked in and this would allow the pipe to fill with air and break the siphon. A manual valve is needed to remove the air but a bellows fitting can also be used to suck out unwanted air. This process is very much like the priming described both for siphons (see Section 4.7) and pumps (see Section 8.4.1).

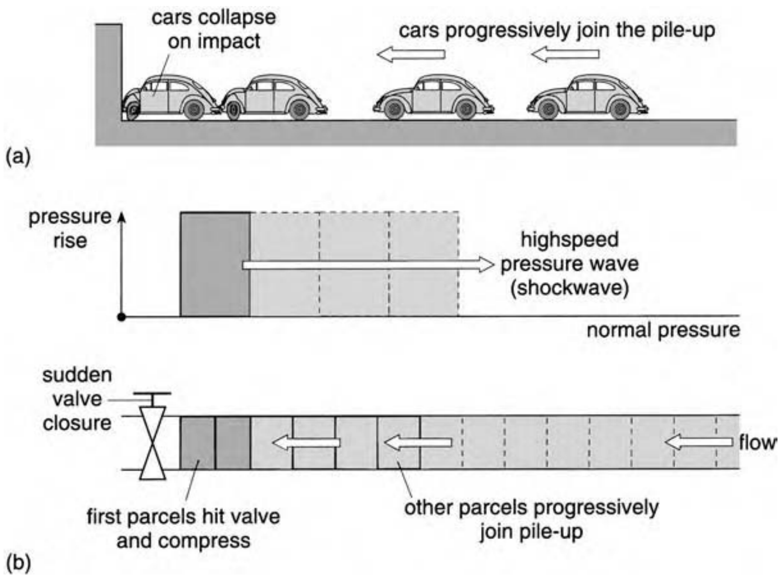
The non-return valve – sometimes called a reflux valve – does what it says. It allows water to flow one way only and prevents return flow once the main flow stops. These are essential fittings on pump delivery pipes to prevent damage from water hammer, and on water supply pipelines to prevent contamination from being 'sucked' into the pipe when it is being shut down. An example is an outside tap for watering the garden with a hose pipe. If for some reason another tap on the same domestic supply system is opened at the same time, the flow in the hose pipe can stop and even be reversed. This can suck soil particles and harmful bacteria into the pipeline and contaminate the flow. To avoid this problem all outside taps should be fitted with a non-return valve – in the UK it is a legal requirement.

Control valves are available for a wide range of control issues such as discharge and pressure control, pressure reducing, pressure sustaining and surge control. Some valves now are very sophisticated and expensive but each works on a simple principle. One simple example is a valve to regulate pressure under agricultural sprinklers. Sprinklers work best when the operating pressure is constant and at the value recommended by the manufacturer. A small regulator placed under each sprinkler ensures that each sprinkler operates at the desired pressure. Normally pressures p_1 and p_2 are equal and close to the sprinkler operating pressure (Figure 4.16d). But if the pressure p_1 rises then p_2 also starts to rise. This pushes down the sleeve closing up the waterway and reducing the flow to the sprinkler. The effect of this is to reduce the pressure p_2. If p_1 starts to fall then p_2 also falls, the sleeve opens and allows more flow through to maintain the pressure p_2. The regulated pressure p_2 is controlled by a spring in compression inside the sleeve. This can be adjusted using a small screw. For example, if a higher pressure p_2 is required then the screw is turned clockwise increasing the spring compression. This stops the movement of the sleeve until the higher pressure p_2 is reached. Although this is a simple mechanism most pressure regulating valves work on this principle. Note that pressure control valves will only reduce the pressure, they cannot increase it beyond the working pressure in the pipeline.

4.14 Water hammer

Most people will already have experienced this but may not have realised it nor appreciated the seriousness of it. When a domestic water tap is turned off quickly, there is sometimes a loud banging noise in the pipes and sometimes the pipes start to vibrate. The noise is the result of a high pressure wave which moves rapidly through the pipes as a result of the rapid closure of the tap. This is known as *water hammer* and although it may not be too serious in domestic plumbing it can have disastrous consequences in larger pipelines and may result in pipe bursts.

Water hammer occurs when flowing water is suddenly stopped. It behaves in a similar way to traffic flowing along a road when suddenly one car stops for no clear reason (Figure 4.17). The car travelling close behind then crashes into it and the impact causes the cars to crumple. The next one crashes into the other two and so on until there is quite a pile up. Notice that all the cars do not crash at the same time. A few seconds pass between each impact and so it takes several seconds before they all join the pile up. If you are watching this from a distance it would appear as if there was a wave moving up the line of cars as each joins the pile up. The speed of the wave is equal to the speed at which successive impacts occur. It is worth pointing out that cars are designed to collapse on impact so as to absorb the kinetic energy. If they were built more rigidly then all the energy on impact would be transferred to the driver and the passengers and not even seat belts would hold you in such circumstances.



4.17 Water hammer.

Now imagine water flowing along a pipeline at the end of which is a valve that is closed suddenly (Figure 4.17b). If water was not compressible then it would behave like a long solid rod and would crash into the valve with such enormous force (momentum change) that it would probably destroy the valve. Fortunately, water is compressible and it behaves in a similar manner to the vehicles it squashes on impact. Think of the flow being made up of small 'parcels' of water. The first parcel hits the valve and compresses (like the first car); the second crashes into the first and compresses and so on until all the water is stopped. This does not happen instantly but takes several seconds before all the water feels the impact and stops. The result is a sudden, large pressure rise at the valve and a pressure wave which travels rapidly along the pipe. This is referred to as a *shock wave* because of its suddenness.

The pressure wave is not just one way. Once it reaches the end of the pipeline it reflects back towards the valve again. It is like a coiled spring that moves back and forth and gradually stops. This oscillating motion can go on for several minutes in a pipe until friction slowly reduces the pressure back to the normal operating level.

The extent of the pressure rise depends on how fast the water was travelling (velocity) and how quickly the valve was closed. It does not depend on the initial pipeline pressure as is often thought. It can be calculated using a formula developed by Nicholai Joukowsky (1847–1921) who carried out the first successful analysis of this problem:

$$\Delta h = \frac{cv}{g}$$

where Δh is rise in pressure (m); c is velocity of the shock wave (m/s); v is water velocity (m/s); g is gravity constant (9.81 m/s^2).

The shock wave travels at very high velocity between 1200 and 1400 m/s. It depends on the diameter of the pipe and the material from which the pipe is made as some materials absorb the energy of compression of the water better than others. An example in the box shows just how high the pressure can rise.

EXAMPLE: CALCULATING PRESSURE RISE IN A PIPELINE DUE TO WATER HAMMER

Using the Joukowsky equation determine the pressure rise in a pipeline when it is suddenly closed. The normal pipeline velocity is 1.0 m/s and the shock wave velocity is 1200 m/s.

If the pipeline is 10 km long determine how long it takes for the pressure wave to travel the length of the pipeline.

Using the Joukowsky equation:

$$\Delta h = \frac{cv}{g}$$

$$= \frac{1200 \times 1.0}{9.81}$$

pressure rise = 122 m

So the pressure rise would be 122 m head of water or 12.2 bar. This is on top of the normal operating pressure of the pipe and could well cause the pipe to burst.

Calculate the time it takes the wave to travel the length of the pipe:

$$\text{time} = \frac{\text{distance}}{\text{velocity}} = \frac{10\ 000}{1200} = 8 \text{ s}$$

It takes only 8 s for the shock wave to travel the 10 km length of the pipe and 16 s for this to return to the valve.

The example of course, is an extreme one as it is difficult to close a valve instantaneously. It is also wrong to assume that the pipe is rigid. All materials stretch when they are under pressure and so the pipe itself will absorb some of the pressure energy by expanding. All these factors help to reduce the pressure rise but they do not stop it. Even if the pressure rise is only half the above value (say 6 bar) it is still a high pressure to suddenly cope with and this too is enough to burst the pipe. When pipes burst they usually open up along their length rather than around their circumference. They burst open rather like unzipping a coat.

Reducing water hammer problems is similar to reducing car crash problems. When cars are moving slowly then the force of impact is not as great. Also when the first car slows down gradually then the others are unlikely to crash into it. Similarly, when water is moving more slowly the pressure rises when a valve closes is reduced (see Joukowsky equation). This is one of the reasons why most pipeline designers restrict velocities to below 1.6 m/s so as to reduce water hammer problems. Also, when valves are closed slowly, water slows down gradually and there is little or no pressure rise along the pipe.

In summary to reduce the effects of water hammer:

- Make sure water velocities are low (below 1.6 m/s).
- Close control valves slowly.

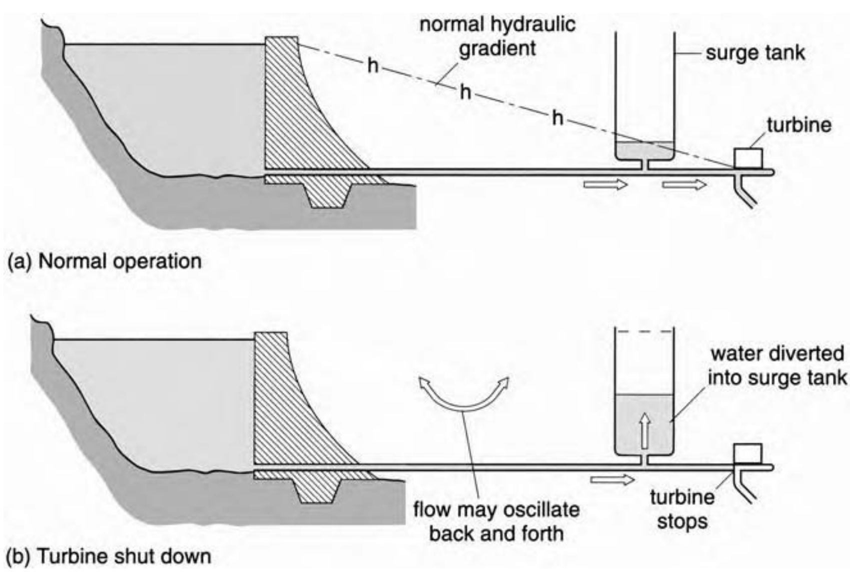
In some cases it is not always possible to avoid the sudden closure of a pipeline. For example, if a heavy vehicle drives over pipes laid out on the ground, such as might occur with fire hoses, it will squash them and stop the flow instantly. This will immediately cause the pressure to rise rapidly and this could burst the pipes. A similar situation can occur on farms where mobile irrigation machines use flexible pipes. It is easy for a tractor to accidentally drive over a pipe without realising that the resulting pressure rise can split open the pipeline and cause a lot of damage. In such situations where there are pipes on the ground and vehicles about, it is wise to use pipe bridges.

Some incidents though are not always easy to foresee. On a sprinkler irrigation scheme in east Africa pipe bridges were used to allow tractors to cross pipelines. But an elephant got into the farm and walked through the crop. It trod on an aluminium irrigation pipe and squashed it flat resulting in an instantaneous closure. This caused a massive pressure rise upstream and several pipes burst open!

4.15 Surge

Surge and water hammer are terms that are often confused because one is caused by the other. Surge is the large mass movement of water that sometimes takes place as a result of water hammer. It is much slower and can last for many minutes whereas water hammer may only last for a few seconds.

An example of the difference between the two can be most easily seen in a hydro-electric power station (Figure 4.18). Water flows down a pipeline from a large reservoir and is used to turn a turbine which is coupled to a generator that produces electricity. Turbines run at high speeds and require large quantities of water and so the velocities in the supply pipe can be very high. The demand for electricity can vary considerably over very short periods and problems occur when the demand falls and one or more of the turbines have to be shut down quickly. This is done by closing the valve on the supply pipe and this can cause water hammer. To protect a large part of the pipeline a *surge tank* is located as close to the power station as possible. This is a vertical chamber many times larger than the pipeline diameter. Water no longer required for the turbine is diverted into the tank and any water hammer shock waves coming up from



4.18 Surge in pipelines.

the valve closure are absorbed by the tank. Thus water hammer is confined to the pipeline between the turbine and the tank and so only this length of pipe needs to be constructed to withstand the high water hammer pressures. Gradually the tank fills with water and the flow from the reservoir slows down and eventually stops. Usually, the rushing water can cause the tank to overfill. In such cases water may flow back and forth between the tank and the reservoir for several hours. This slow but large movement of water is called *surge* and although it is the result of water hammer it is quite different in character.

Surge can also cause problems in pumping mains and these are discussed more fully in Section 8.14.

4.16 Some examples to test your understanding

1 A 150 mm pipeline is 360 m long and has a friction factor $\lambda = 0.02$. Calculate the head loss in the pipeline using the Darcy–Weisbach formula when the discharge is 0.05 m^3/s. Calculate the hydraulic gradient in m/100 m of pipeline (19.46 m; 5.4 m/100 m).

2 A pipeline 2.5 km long and 150 mm diameter supplies water from a reservoir to a small town storage tank. Calculate the discharge when the pipe outlet is freely discharging into the tank and the difference in level between the reservoir and the outlet is 15 m. Assume $\lambda = 0.04$ (0.012 m^3/s).

3 Water from a large reservoir flows through a pipeline, 1.8 km long and discharges into service tank. The first 600 m of pipe is 300 mm in diameter and the remainder is 150 mm in diameter. Calculate the discharge when the difference in water level between the two reservoirs is 25 m and $\lambda = 0.04$ for both pipes (0.02 m^3/s).

4 A venturi meter is fitted to a pipeline to measure discharge. The pipe diameter is 300 mm and the venturi throat diameter is 75 mm. Calculate the discharge in m^3/s when the difference in pressure between the pipe and the venturi throat is 400 mm of water. Assume the coefficient of discharge is 0.97 (0.07 m^3/s).

5 A pipeline is reduced in diameter from 500 mm to 300 mm using a concentric reducer pipe. Calculate the force on the reducer when the discharge is 0.35 m^3/s and the pressure in the 500 mm pipe is 300 kN/m^2 (37.4 kN).

6 A 500 mm diameter pipeline is fitted with a 90° bend. Calculate the resultant force on the bend if the normal operating pressure is 50 m head of water and the discharge is 0.3 m^3/s. Calculate the resultant force when there is no flow in the pipe but the system is still under pressure (138 kN; 137 kN).

7 Calculate the pressure rise in a 0.5 m diameter pipeline carrying a discharge of 0.3 m^3/s when a sluice valve is closed suddenly (187 m).