3 CHAPTER

HYDROSTATIC FORCES ON SURFACES

▶ 3.1 INTRODUCTION

This chapter deals with the fluids (*i.e.,* liquids and gases) at rest. This means that there will be no relative motion between adjacent or neighbouring fluid layers. The velocity gradient, which is equal to the change of velocity between two adjacent fluid layers divided by the distance between the layers, will be zero or $\frac{du}{dy} = 0$. The shear stress which is equal to $\mu \frac{\partial u}{\partial y}$ will also be zero. Then the forces acting on the fluid particles will be :

1. due to pressure of fluid normal to the surface,
2. due to gravity (or self-weight of fluid particles).

▶ 3.2 TOTAL PRESSURE AND CENTRE OF PRESSURE

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface. There are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined. The submerged surfaces may be :

1. Vertical plane surface,
2. Horizontal plane surface,
3. Inclined plane surface, and
4. Curved surface.

▶ 3.3 VERTICAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig. 3.1.

Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.

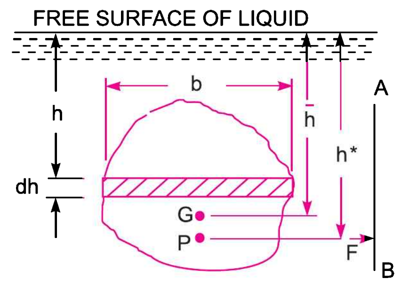


Fig. 3.1

(*a*) **Total Pressure (F)**. The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig. 3.1

Pressure intensity on the strip, $p = \rho gh$

(See equation 2.5)

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$

$= \rho gh \times b \times dh$

$\therefore$ Total pressure force on the whole surface,

$$F = \int dF = \int \rho gh \times b \times dh = \rho g \int b \times h \times dh$$

But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

$\therefore \qquad F = \rho g A \bar{h} \qquad$ ...(3.1)

For water the value of $\rho = 1000$ kg/m^3 and $g = 9.81$ m/s^2. The force will be in Newton.

(*b*) **Centre of Pressure (h*)**. Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from free surface of the liquid as shown in Fig. 3.1. Hence moment of the force F about free surface of the liquid $= F \times h^*$ ...(3.2)

Moment of force dF, acting on a strip about free surface of liquid

$= dF \times h \qquad \{\because \; dF = \rho gh \times b \times dh\}$

$= \rho gh \times b \times dh \times h$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho gh \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int bh^2 \, dh = \rho g \int h^2 \, dA \qquad (\because \; bdh = dA)$$

But $\int h^2 dA = \int bh^2 dh$

= Moment of Inertia of the surface about free surface of liquid

$= I_0$

$\therefore$ Sum of moments about free surface

$= \rho g I_0 \qquad$ ...(3.3)

Equating (3.2) and (3.3), we get

$$F \times h^* = \rho g I_0$$

But $$F = \rho g A \bar{h}$$

$\therefore$ $$\rho g A \bar{h} \times h^* = \rho g I_0$$

or $$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}} \qquad \text{...(3.4)}$$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \overline{h}^2$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_0 in equation (3.4), we get

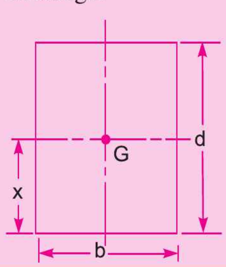
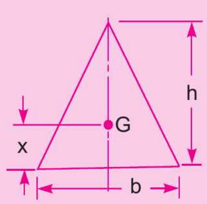
$$h^* = \frac{I_G + A\overline{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h} \qquad \text{...(3.5)}$$

In equation (3.5), $\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid. Hence from equation (3.5), it is clear that :

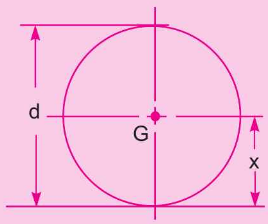
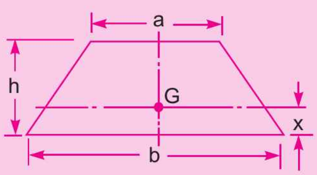
(*i*) Centre of pressure (*i.e.*, h^*) lies below the centre of gravity of the vertical surface.

(*ii*) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

Table 3.1 The moments of inertia and other geometric properties of some important plane surfaces

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Contd...

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base* (I_G)	*Moment of inertia about base* (I_0)
3. Circle 	$x = \dfrac{d}{2}$	$\dfrac{\pi d^2}{4}$	$\dfrac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\dfrac{2a+b}{a+b}\right)\dfrac{h}{3}$	$\dfrac{(a+b)}{2} \times h$	$\left(\dfrac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Problem 3.1 *A rectangular plane surface is 2 m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water surface, (b) 2.5 m below the free water surface.*

Solution. Given :

Width of plane surface, $b = 2$ m

Depth of plane surface, $d = 3$ m

(*a*) **Upper edge coincides with water surface (Fig. 3.2).** Total pressure is given by equation (3.1) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000$ kg/m^3, $g = 9.81$ m/s^2

$$A = 3 \times 2 = 6 \text{ m}^2,\ \bar{h} = \frac{1}{2}(3) = 1.5 \text{ m.}$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 1.5$$

$$= \mathbf{88290 \text{ N. Ans.}}$$

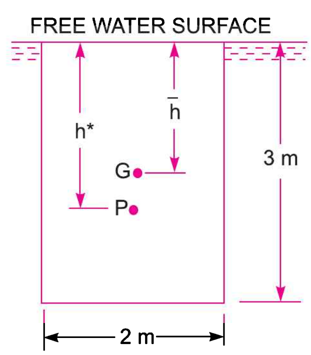


Fig. 3.2

Depth of centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$\therefore$ $$h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = \mathbf{2.0\ m.\ Ans.}$$

(*b*) **Upper edge is 2.5 m below water surface (Fig. 3.3).** Total pressure (*F*) is given by (3.1)

$$F = \rho g A \bar{h}$$

where $\bar{h}$ = Distance of C.G. from free surface of water

$$= 2.5 + \frac{3}{2} = 4.0 \text{ m}$$

$\therefore$ $$F = 1000 \times 9.81 \times 6 \times 4.0$$
$$= \mathbf{235440\ N.\ Ans.}$$

Centre of pressure is given by $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

where $I_G = 4.5$, $A = 6.0$, $\bar{h} = 4.0$

$\therefore$ $$h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

$$= 0.1875 + 4.0 = 4.1875 = \mathbf{4.1875\ m.\ Ans.}$$

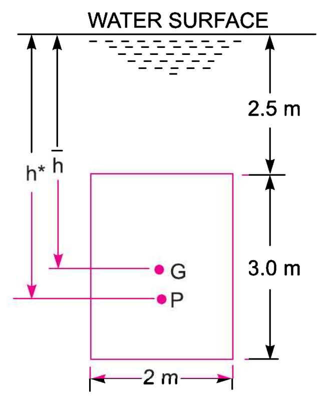


Fig. 3.3

Problem 3.2 *Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of the plate is 3 m below the free surface of water. Find the position of centre of pressure also.*

Solution. Given : Dia. of plate, $d = 1.5$ m

$\therefore$ Area, $$A = \frac{\pi}{4}(1.5)^2 = 1.767 \text{ m}^2$$

$$\bar{h} = 3.0 \text{ m}$$

Total pressure is given by equation (3.1),

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 1.767 \times 3.0 \text{ N}$$
$$= \mathbf{52002.81\ N.\ Ans.}$$

Position of centre of pressure (*h**) is given by equation (3.5),

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi d^4}{64} = \frac{\pi \times 1.5^4}{64} = 0.2485 \text{ m}^4$

$\therefore$ $$h^* = \frac{0.2485}{1.767 \times 3.0} + 3.0 = 0.0468 + 3.0$$
$$= \mathbf{3.0468\ m.\ Ans.}$$

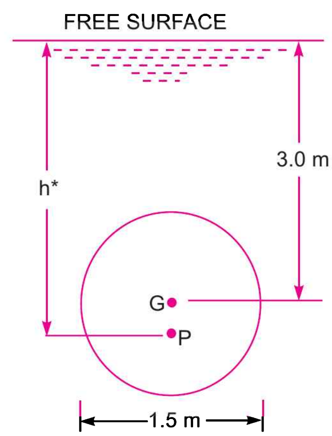


Fig. 3.4

Problem 3.3 *A rectangular sluice gate is situated on the vertical wall of a lock. The vertical side of the sluice is 'd' metres in length and depth of centroid of the area is 'p' m below the water surface. Prove that the depth of pressure is equal to* $\left(p + \frac{d^2}{12p}\right)$.

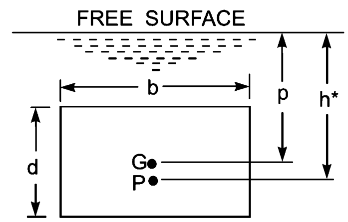


Fig. 3.5

Solution. Given :

Depth of vertical gate $= d$ m

Let the width of gate $= b$ m

$\therefore$ Area, $A = b \times d \text{ m}^2$

Depth of C.G. from free surface

$$\bar{h} = p \text{ m}.$$

Let h^* is the depth of centre of pressure from free surface, which is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}, \text{ where } I_G = \frac{bd^3}{12}$$

$$\therefore \quad h^* = \left(\frac{bd^3}{12} \Big/ b \times d \times p\right) + p = \mathbf{\frac{d^2}{12p} + p} \quad \text{or} \quad \mathbf{p + \frac{d^2}{12p}}. \textbf{ Ans.}$$

Problem 3.4 *A circular opening, 3 m diameter, in a vertical side of a tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter. Calculate :*

(i) the force on the disc, and

(ii) the torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 4 m.

Solution. Given :

Dia. of opening, $d = 3$ m

$\therefore$ Area, $A = \frac{\pi}{4} \times 3^2 = 7.0685 \text{ m}^2$

Depth of C.G., $\bar{h} = 4$ m

(*i*) Force on the disc is given by equation (3.1) as

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 7.0685 \times 4.0$$

$$= \mathbf{277368 \text{ N} = 277.368 \text{ kN}}. \textbf{ Ans.}$$

(*ii*) To find the torque required to maintain the disc in equilibrium, first calculate the point of application of force acting on the disc, *i.e.*, centre of pressure of the force F. The depth of centre of pressure (h^*) is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times 4.0} + 4.0 \qquad \left\{\because I_G = \frac{\pi}{64} d^4\right\}$$

$$= \frac{d^2}{16 \times 4.0} + 4.0 = \frac{3^2}{16 \times 4.0} + 4.0 = 0.14 + 4.0 = 4.14 \text{ m}$$

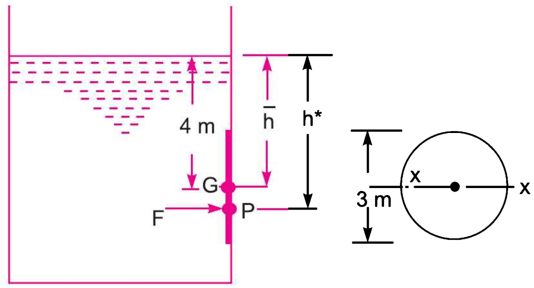


Fig. 3.6

The force F is acting at a distance of 4.14 m from free surface. Moment of this force about horizontal diameter X-X

$$= F \times \left(h^* - \bar{h}\right) = 277368\ (4.14 - 4.0) = \mathbf{38831\ Nm.\ Ans.}$$

Hence a torque of 38831 Nm must be applied on the disc in the clockwise direction.

Problem 3.5 *A pipe line which is 4 m in diameter contains a gate valve. The pressure at the centre of the pipe is 19.6 N/cm². If the pipe is filled with oil of sp. gr. 0.87, find the force exerted by the oil upon the gate and position of centre of pressure.*

Solution. Given :

Dia. of pipe, $d = 4$ m

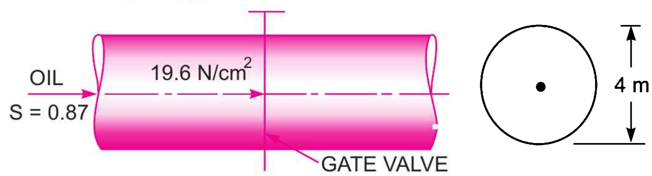


Fig. 3.7

∴ Area, $A = \frac{\pi}{4} \times 4^2 = 4\pi\ \text{m}^2$

Sp. gr. of oil, $S = 0.87$

∴ Density of oil, $\rho_0 = 0.87 \times 1000 = 870\ \text{kg/m}^3$

∴ Weight density of oil, $w_0 = \rho_0 \times g = 870 \times 9.81\ \text{N/m}^3$

Pressure at the centre of pipe, $p = 19.6\ \text{N/cm}^2 = 19.6 \times 10^4\ \text{N/m}^2$

∴ Pressure head at the centre $= \frac{p}{w_0} = \frac{19.6 \times 10^4}{870 \times 9.81} = 22.988$ m

∴ The height of equivalent free oil surface from the centre of pipe = 22.988 m.

The depth of C.G. of the gate valve from free oil surface $\bar{h} = 22.988$ m.

(*i*) Now the force exerted by the oil on the gate is given by

$$F = \rho g A \bar{h}$$

where ρ = density of oil = 870 kg/m³

$$F = 870 \times 9.81 \times 4\pi \times 22.988 = \mathbf{2465500\ N = 2.465\ MN.\ Ans.}$$

(*ii*) Position of centre of pressure (h^*) is given by (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times \bar{h}} + \bar{h} = \frac{d^2}{16\,\bar{h}} + \bar{h} = \frac{4^2}{16 \times 22.988} + 22.988$$

$$= 0.043 + 22.988 = \mathbf{23.031\ m.\ Ans.}$$

Or centre of pressure is below the centre of the pipe by a distance of 0.043 m. **Ans.**

Problem 3.6 *Determine the total pressure and centre of pressure on an isosceles triangular plate of base 4 m and altitude 4 m when it is immersed vertically in an oil of sp. gr. 0.9. The base of the plate coincides with the free surface of oil.*

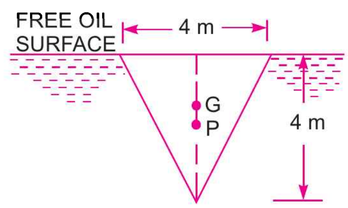


Fig. 3.8

Solution. Given :

Base of plate, $b = 4$ m

Height of plate, $h = 4$ m

$\therefore$ Area, $A = \frac{b \times h}{2} = \frac{4 \times 4}{2} = 8.0\ m^2$

Sp. gr. of oil, $S = 0.9$

$\therefore$ Density of oil, $\rho = 900\ kg/m^3$.

The distance of C.G. from free surface of oil,

$$\bar{h} = \frac{1}{3} \times h = \frac{1}{3} \times 4 = 1.33\ m.$$

Total pressure (F) is given by $F = \rho g A \bar{h}$

$$= 900 \times 9.81 \times 8.0 \times 1.33\ N = \mathbf{9597.6.\ N.\ Ans.}$$

Centre of pressure (h^*) from free surface of oil is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of triangular section about its C.G.

$$= \frac{bh^3}{36} = \frac{4 \times 4^3}{36} = 7.11\ m^4$$

$$\therefore \quad h^* = \frac{7.11}{8.0 \times 1.33} + 1.33 = 0.6667 + 1.33 = \mathbf{1.99\ m.\ Ans.}$$

Problem 3.7 *A vertical sluice gate is used to cover an opening in a dam. The opening is 2 m wide and 1.2 m high. On the upstream of the gate, the liquid of sp. gr. 1.45, lies upto a height of 1.5 m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume that the gate is hinged at the bottom.*

Solution. Given :

Width of gate, $b = 2$ m

Depth of gate, $d = 1.2$ m

$\therefore$ Area, $A = b \times d = 2 \times 1.2 = 2.4\ m^2$

Sp. gr. of liquid $= 1.45$

$\therefore$ Density of liquid, $\rho_1 = 1.45 \times 1000 = 1450$ kg/m^3

Let F_1 = Force exerted by the fluid of sp. gr. 1.45 on gate

F_2 = Force exerted by water on the gate.

The force F_1 is given by $F_1 = \rho_1 g \times A \times \bar{h}_1$

where $\rho_1 = 1.45 \times 1000 = 1450$ kg/m^2

$\bar{h}_1$ = Depth of C.G. of gate from free surface of liquid

$= 1.5 + \frac{1.2}{2} = 2.1$ m.

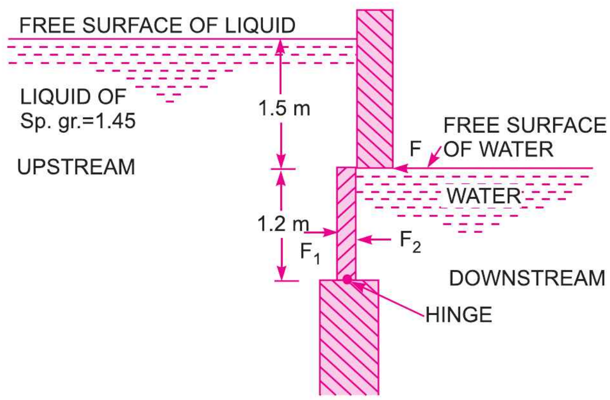


Fig. 3.9

$\therefore$ $F_1 = 1450 \times 9.81 \times 2.4 \times 2.1 = 71691$ N

Similarly, $F_2 = \rho_2 g.A\bar{h}_2$

where $\rho_2 = 1{,}000$ kg/m^3

$\bar{h}_2$ = Depth of C.G. of gate from free surface of water

$= \frac{1}{2} \times 1.2 = 0.6$ m

$\therefore$ $F_2 = 1000 \times 9.81 \times 2.4 \times 0.6 = 14126$ N

(*i*) **Resultant force on the gate** = $F_1 - F_2$ = 71691 − 14126 = **57565 N. Ans.**

(*ii*) **Position of centre of pressure of resultant force.** The force F_1 will be acting at a depth of h_1^* from free surface of liquid, given by the relation

$$h_1^* = \frac{I_G}{A\bar{h}_1} + \bar{h}_1$$

where $I_G = \frac{bd^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288$ m^4

$$\therefore \quad h_1^* = \frac{.288}{2.4 \times 2.1} + 2.1 = 0.0571 + 2.1 = 2.1571 \text{ m}$$

$\therefore$ Distance of F_1 from hinge

$= (1.5 + 1.2) - h_1^* = 2.7 - 2.1571 = 0.5429$ m

The force F_2 will be acting at a depth of h_2^* from free surface of water and is given by

$$h_2{}^* = \frac{I_G}{A\bar{h}_2} + \bar{h}_2$$

where $I_G = 0.288 \text{ m}^4$, $\bar{h}_2 = 0.6$ m, $A = 2.4 \text{ m}^2$,

$$h_2{}^* = \frac{0.288}{2.4 \times 0.6} + 0.6 = 0.2 + 0.6 = 0.8 \text{ m}$$

Distance of F_2 from hinge $= 1.2 - 0.8 = 0.4$ m

The resultant force 57565 N will be acting at a distance given by

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{57565}$$

$$= \frac{38921 - 5650.4}{57565} \text{ m above hinge}$$

= **0.578 m above the hinge. Ans.**

(*iii*) **Force at the top of gate which is capable of opening the gate.** Let F is the force required on the top of the gate to open it as shown in Fig. 3.9. Taking the moments of F, F_1 and F_2 about the hinge, we get

$$F \times 1.2 + F_2 \times 0.4 = F_1 \times .5429$$

or

$$F = \frac{F_1 \times .5429 - F_2 \times 0.4}{1.2}$$

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{1.2} = \frac{38921 - 5650.4}{1.2}$$

= **27725.5 N. Ans.**

Problem 3.8 *A caisson for closing the entrance to a dry dock is of trapezoidal form 16 m wide at the top and 10 m wide at the bottom and 6 m deep. Find the total pressure and centre of pressure on the caisson if the water on the outside is just level with the top and dock is empty.*

Solution. Given :

Width at top = 16 m

Width at bottom = 10 m

Depth, $d = 6$ m

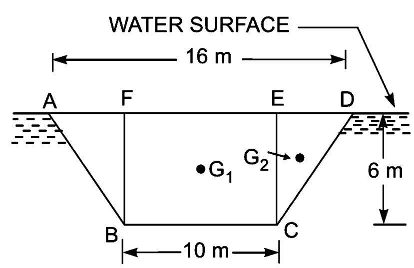


Fig. 3.10

Area of trapezoidal *ABCD*,

$$A = \frac{(BC + AD)}{2} \times d$$

$$= \frac{(10 + 16)}{2} \times 6 = 78 \text{ m}^2$$

Depth of C.G. of trapezoidal area *ABCD* from free surface of water,

$$\bar{h} = \frac{10 \times 6 \times 3 + \frac{(16 - 10)}{2} \times 6 \times \frac{1}{3} \times 6}{78}$$

$$= \frac{180 + 36}{78} = 2.769 \text{ m from water surface.}$$

(*i*) **Total Pressure (F).** Total pressure, F is given by

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 78 \times 2.769 \text{ N}$$
$$= 2118783 \text{ N} = \mathbf{2.118783 \text{ MN. Ans.}}$$

(*ii*) **Centre of Pressure (h*).** Centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of trapezoidal *ABCD* about its C.G.

Let I_{G_1} = M.O.I. of rectangle *FBCE* about its C.G.

I_{G_2} = M.O.I. of two Δs *ABF* and *ECD* about its C.G.

Then $$I_{G_1} = \frac{bd^3}{12} = \frac{10 \times 6^3}{12} = 180 \text{ m}^4$$

I_{G_1} is the M.O.I. of the rectangle about the axis passing through G_1.

∴ M.O.I. of the rectangle about the axis passing through the C.G. of the trapezoidal I_{G_1} + Area of rectangle × x_1^2

where x_1 is distance between the C.G. of rectangle and C.G. of trapezoidal

$$= (3.0 - 2.769) = 0.231 \text{ m}$$

∴ M.O.I. of *FBCE* passing through C.G. of trapezoidal

$$= 180 + 10 \times 6 \times (0.231)^2 = 180 + 3.20 = 183.20 \text{ m}^4$$

Now I_{G_2} = M.O.I. of Δ*ABD* in Fig. 3.11 about $G_2 = \dfrac{bd^3}{36}$

$$= \frac{(16 - 10) \times 6^3}{36} = 36 \text{ m}^4$$

The distance between the C.G. of triangle and C.G. of trapezoidal

$$= (2.769 - 2.0) = 0.769$$

∴ M.O.I. of the two Δs about an axis passing through C.G. of trapezoidal

$$= I_{G_2} + \text{Area of triangles} \times (.769)^2$$
$$= 36.0 + \frac{6 \times 6}{2} \times (.769)^2$$
$$= 36.0 + 10.64 = 46.64$$

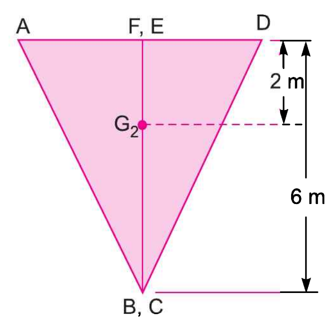


Fig. 3.11

∴ I_G = M.O.I. of trapezoidal about its C.G.
= M.O.I. of rectangle about the C.G. of trapezoidal
+ M.O.I. of triangles about the C.G. of the trapezoidal
= 183.20 + 46.64 = 229.84 m^4

∴ $$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $A = 78$, $\bar{h} = 2.769$

$$h^* = \frac{229.84}{78 \times 2.769} + 2.769 = 1.064 + 2.769 = \mathbf{3.833 \text{ m. Ans.}}$$

Alternate Method

The distance of the C.G. of the trapezoidal channel from surface *AD* is given by (refer to Table 3.1 on page 71)

$$x = \frac{(2a+b)}{(a+b)} \times \frac{h}{3}$$

$$= \frac{(2 \times 10 + 16)}{(10+16)} \times \frac{6}{3} \qquad (\because \; a = 10, b = 16 \text{ and } h = 6)$$

$$= \frac{36}{26} \times 2 = 2.769 \text{ m}$$

This is also equal to the distance of the C.G. of the trapezoidal from free surface of water.

$$\bar{h} = 2.769 \text{ m}$$

∴ Total pressure, $F = \rho g A \bar{h}$ $(\because \; A = 78)$

$= 1000 \times 9.81 \times 78 \times 2.769$ N = **2118783 N. Ans.**

Centre of Pressure, $(h^*) = \dfrac{I_G}{A\bar{h}} + \bar{h}$

Now I_G from Table 3.1 is given by,

$$I_G = \frac{(a^2 + 4ab + b^2)}{36(a+b)} \times h^3 = \frac{(10^2 + 4 \times 10 \times 16 + 16^2)}{36(10+16)} \times 6^3$$

$$= \frac{(100 + 640 + 256)}{36 \times 26} \times 216 = 229.846 \text{ m}^4$$

∴ $$h^* = \frac{229.846}{78 \times 2.769} + 2.769 \qquad (\because \; A = 78 \text{ m}^2)$$

= **3.833 m. Ans.**

Problem 3.9 *A trapezoidal channel 2 m wide at the bottom and 1 m deep has side slopes 1 : 1. Determine :*

(i) the total pressure, and

(ii) the centre of pressure on the vertical gate closing the channel when it is full of water.

Solution. Given :

Width at bottom = 2 m

Depth, $d = 1$ m

Side slopes = 1 : 1

∴ Top width, $AD = 2 + 1 + 1 = 4$ m

Area of rectangle *FBEC*, $A_1 = 2 \times 1 = 2 \text{ m}^2$

Area of two triangles *ABF* and *ECD*, $A_2 = \dfrac{(4-2)}{2} \times 1 = 1 \text{ m}^2$

∴ Area of trapezoidal *ABCD*, $A = A_1 + A_2 = 2 + 1 = 3 \text{ m}^2$

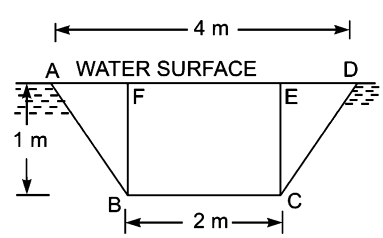


Fig. 3.12

Depth of C.G. of rectangle *FBEC* from water surface,

$$\bar{h}_1 = \frac{1}{2} = 0.5 \text{ m}$$

Depth of C.G. of two triangles *ABF* and *ECD* from water surface,

$$\bar{h}_2 = \frac{1}{3} \times 1 = \frac{1}{3} \text{ m}$$

∴ Depth of C.G. of trapezoidal *ABCD* from free surface of water

$$\bar{h} = \frac{A_1 \times \bar{h}_1 + A_2 \times \bar{h}_2}{(A_1 + A_2)} = \frac{2 \times 0.5 + 1 \times 0.33333}{(2+1)} = .44444$$

(*i*) **Total Pressure (F).** Total pressure *F* is given by

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 3.0 \times 0.44444 = \mathbf{13079.9\ N.\ Ans.}$$

(*ii*) **Centre of Pressure (h*).** M.O.I. of rectangle *FBCE* about its C.G.,

$$I_{G_1} = \frac{bd^3}{12} = \frac{2 \times 1^3}{12} = \frac{1}{6} \text{ m}^4$$

M.O.I. of *FBCE* about an axis passing through the C.G. of trapezoidal

or $$I_{G_1}^* = I_{G_1} + A_1 \times [\text{Distance between C.G. of rectangle and C.G. of trapezoidal}]^2$$

$$= \frac{1}{6} + 2 \times \left[\bar{h}_1 - \bar{h}\right]^2$$

$$= \frac{1}{6} + 2 \times [0.5 - .4444]^2 = .1666 + .006182 = 0.1727$$

M.O.I. of the two triangles *ABF* and *ECD* about their C.G.,

$$I_{G_2} = \frac{bd^3}{36} = \frac{(1+1) \times 1^3}{36} = \frac{2}{36} = \frac{1}{18} \text{ m}^4.$$

M.O.I. of the two triangles about the C.G. of trapezoidal,

$$I_{G_2}^* = I_{G_1} + A_2 \times [\text{Distance between C.G. of triangles and C.G. of trapezoidal}]^2$$

$$= \frac{1}{18} + 1 \times \left[\bar{h} - \bar{h}_2\right]^2 = \frac{1}{18} + 1 \times \left[.4444 - \frac{1}{3}\right]^2$$

$$= \frac{1}{18} + (.1111)^2 = 0.0555 + (.1111)^2$$

$$= .0555 + 0.01234 = 0.06789 \text{ m}^4$$

∴ M.O.I. of the trapezoidal about its C.G.

$$I_G = I_{G_1}^* + I_{G_2}^* = .1727 + .06789 = 0.24059 \text{ m}^4$$

Then centre of pressure (h^*) on the vertical trapezoidal,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{0.24059}{3 \times .4444} + .4444 = 0.18046 + .4444 = 0.6248$$

$$\simeq \mathbf{0.625\ m.\ Ans.}$$

Alternate Method

The distance of the C.G. of the trapezoidal channel from surface AD is given by (refer to Table 3.1 on page 71).

$$x = \frac{(2a + b)}{(a + b)} \times \frac{h}{3} = \frac{(2 \times 2 + 4)}{(2 + 4)} \times \frac{1}{3} \qquad (\because \quad a = 2, b = 4 \text{ and } h = 1)$$

$$= 0.444 \text{ m}$$

$\therefore$ $$\bar{h} = x = 0.444 \text{ m}$$

$\therefore$ Total pressure, $$F = \rho g A \bar{h} = 1000 \times 9.81 \times 3.0 \times .444 \qquad (\because \quad A = 3.0)$$

$$= \mathbf{13079 \text{ N. Ans.}}$$

Centre of pressure, $$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G from Table 3.1 is given by

$$I_G = \frac{(a^2 + 4ab + b^2)}{36(a + b)} \times h^3 = \frac{(2^2 + 4 \times 2 \times 4 + 4^2)}{36(2 + 4)} \times 1^3 = \frac{52}{36 \times 6} = 0.2407 \text{ m}^4$$

$\therefore$ $$h^* = \frac{0.2407}{3.0 \times .444} + .444 = \mathbf{0.625 \text{ m. Ans.}}$$

Problem 3.10 *A square aperture in the vertical side of a tank has one diagonal vertical and is completely covered by a plane plate hinged along one of the upper sides of the aperture. The diagonals of the aperture are 2 m long and the tank contains a liquid of specific gravity 1.15. The centre of aperture is 1.5 m below the free surface. Calculate the thrust exerted on the plate by the liquid and position of its centre of pressure.*

Solution. Given : Diagonals of aperture, $AC = BD = 2$ m

$\therefore$ Area of square aperture, A = Area of ΔACB + Area of ΔACD

$$= \frac{AC \times BO}{2} + \frac{AC \times OD}{2} = \frac{2 \times 1}{2} + \frac{2 \times 1}{2} = 1 + 1 = 2.0 \text{ m}^2$$

Sp. gr. of liquid = 1.15

$\therefore$ Density of liquid, $\rho = 1.15 \times 1000 = 1150$ kg/m^3

Depth of centre of aperture from free surface,

$$\bar{h} = 1.5 \text{ m.}$$

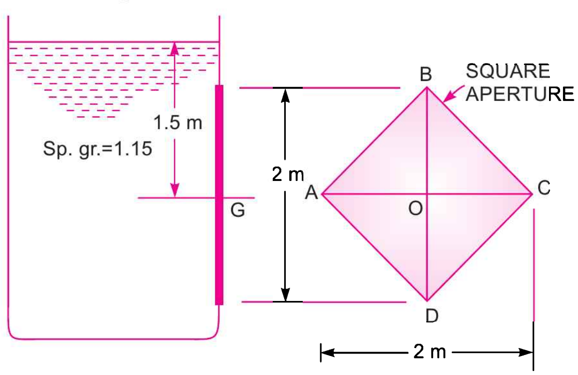


Fig. 3.13

(*i*) The thrust on the plate is given by

$$F = \rho g A\bar{h} = 1150 \times 9.81 \times 2 \times 1.5 = \mathbf{33844.5.\ Ans.}$$

(*ii*) Centre of pressure (h^*) is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of *ABCD* about diagonal *AC*

= M.O.I. of triangle *ABC* about *AC* + M.O.I. of triangle *ACD* about *AC*

$$= \frac{AC \times OB^3}{12} + \frac{AC \times OD^3}{12} \qquad \left(\because \text{ M.O.I. of a triangle about its base} = \frac{bh^3}{12}\right)$$

$$= \frac{2 \times 1^3}{12} + \frac{2 \times 1^3}{12} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \text{ m}^4$$

$$\therefore \qquad h^* = \frac{\frac{1}{3}}{2 \times 1.5} + 1.5 = \frac{1}{3 \times 2 \times 1.5} + 1.5 = \mathbf{1.611\ m.\ Ans.}$$

Problem 3.11 *A tank contains water upto a height of 0.5 m above the base. An immiscible liquid of sp. gr. 0.8 is filled on the top of water upto 1 m height. Calculate :*

(i) total pressure on one side of the tank,

(ii) the position of centre of pressure for one side of the tank, which is 2 m wide.

Solution. Given :

Depth of water = 0.5 m

Depth of liquid = 1 m

Sp. gr. of liquid = 0.8

Density of liquid, $\rho_1 = 0.8 \times 1000 = 800 \text{ kg/m}^3$

Density of water, $\rho_2 = 1000 \text{ kg/m}^3$

Width of tank = 2 m

(*i*) **Total pressure on one side** is calculated by drawing pressure diagram, which is shown in Fig. 3.14.

Intensity of pressure on top, $p_A = 0$

Intensity of pressure on *D* (or *DE*), $p_D = \rho_1 g.h_1$

$= 800 \times 9.81 \times 1.0 = 7848 \text{ N/m}^2$

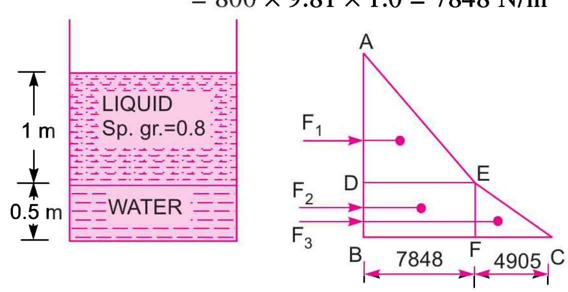


Fig. 3.14

Intensity of pressure on base (or *BC*), $p_B = \rho_1 g h_1 + \rho_2 g \times 0.5$

$= 7848 + 1000 \times 9.81 \times 0.5 = 7848 + 4905 = 12753 \text{ N/m}^2$

Now force F_1 = Area of ΔADE × Width of tank

$$= \frac{1}{2} \times AD \times DE \times 2.0 = \frac{1}{2} \times 1 \times 7848 \times 2.0 = 7848 \text{ N}$$

Force F_2 = Area of rectangle $DBFE$ × Width of tank

$= 0.5 \times 7848 \times 2 = 7848$ N

F_3 = Area of ΔEFC × Width of tank

$$= \frac{1}{2} \times EF \times FC \times 2.0 = \frac{1}{2} \times 0.5 \times 4905 \times 2.0 = 2452.5 \text{ N}$$

∴ Total pressure, $F = F_1 + F_2 + F_3$

$= 7848 + 7848 + 2452.5 =$ **18148.5 N. Ans.**

(*ii*) **Centre of Pressure (h*).** Taking the moments of all force about A, we get

$$F \times h^* = F_1 \times \frac{2}{3} AD + F_2 \left(AD + \frac{1}{2} BD\right) + F_3\left[AD + \frac{2}{3} BD\right]$$

$$18148.5 \times h^* = 7848 \times \frac{2}{3} \times 1 + 7848 \left(1.0 + \frac{0.5}{2}\right) + 2452.5 \left(1.0 + \frac{2}{3} \times .5\right)$$

$$= 5232 + 9810 + 3270 = 18312$$

∴ $$h^* = \frac{18312}{18148.5} = \textbf{1.009 m from top. Ans.}$$

Problem 3.12 *A cubical tank has sides of 1.5 m. It contains water for the lower 0.6 m depth. The upper remaining part is filled with oil of specific gravity 0.9. Calculate for one vertical side of the tank:*

(a) total pressure, and

(b) position of centre of pressure.

Solution. Given :

Cubical tank of sides 1.5 m means the dimensions of the tank are 1.5 m × 1.5 m × 1.5 m.

Depth of water = 0.6 m

Depth of liquid = 1.5 – 0.6 = 0.9 m

Sp. gr. of liquid = 0.9

Density of liquid, $\rho_1 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Density of water, $\rho_2 = 1000 \text{ kg/m}^3$

(*a*) **Total pressure** on one vertical side is calculated by drawing pressure diagram, which is shown in Fig. 3.15.

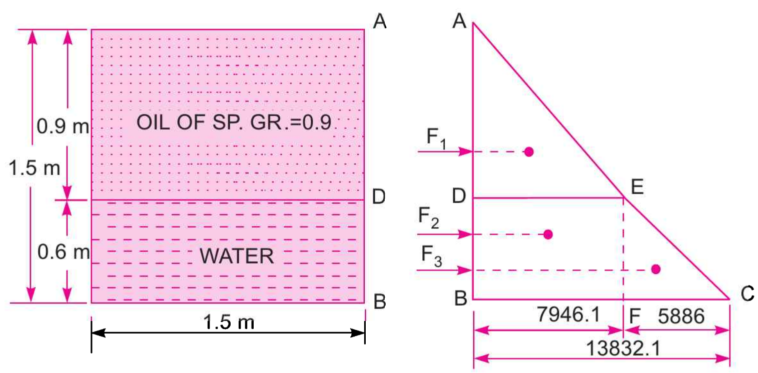


Fig. 3.15

Intensity of pressure at A, $p_A = 0$

Intensity of pressure at $D, p_D = \rho_1 g \times h = 900 \times 9.81 \times 0.9 = 7946.1$ N/m^2

Intensity of pressure at B, $p_B = \rho_1 g h_1 + \rho_2 g h_2 = 900 \times 9.81 \times 0.9 + 1000 \times 9.81 \times 0.6$

$= 7946.1 + 5886 = 13832.1$ N/m^2

Hence in pressure diagram :

$$DE = 7946.1 \text{ N/m}^2, \; BC = 13832.1 \text{ N/m}^2, \; FC = 5886 \text{ N/m}^2$$

The pressure diagram is split into triangle ADE, rectangle $BDEF$ and triangle EFC. The total pressure force consists of the following components :

(*i*) Force F_1 = Area of triangle ADE × Width of tank

$= \left(\frac{1}{2} \times AD \times DE\right) \times 1.5$ $\quad (\because$ Width = 1.5 m)

$= \left(\frac{1}{2} \times 0.9 \times 7946.1\right) \times 1.5$ N = 5363.6 N

This force will be acting at the C.G. of the triangle ADE, *i.e.*, at a distance of $\frac{2}{3} \times 0.9 = 0.6$ m below A

(*ii*) Force F_2 = Area of rectangle $BDEF$ × Width of tank

$= (BD \times DE) \times 1.5 = (0.6 \times 7946.1) \times 1.5 = 7151.5$

This force will be acting at the C.G. of the rectangle $BDEF$ *i.e.*, at a distance of $0.9 + \frac{0.6}{2} = 1.2$ m below A.

(*iii*) Force F_3 = Area of triangle EFC × Width of tank

$= \left(\frac{1}{2} \times EF \times FC\right) \times 1.5 = \left(\frac{1}{2} \times 0.6 \times 5886\right) \times 1.5 = 2648.7$ N

This force will be acting at the C.G. of the triangle EFC, *i.e.*, at a distance of $0.9 + \frac{2}{3} \times 0.6 = 1.30$ m below A.

∴ Total pressure force on one vertical face of the tank,

$$F = F_1 + F_2 + F_3$$

$= 5363.6 + 7151.5 + 2648.7 =$ **15163.8 N. Ans.**

(*b*) **Position of centre of pressure**

Let the total force F is acting at a depth of h^* from the free surface of liquid, *i.e.*, from A.

Taking the moments of all forces about A, we get

$$F \times h^* = F_1 \times 0.6 + F_2 \times 1.2 + F_3 \times 1.3$$

or

$$h^* = \frac{F_1 \times 0.6 + F_2 \times 1.2 + F_3 \times 1.3}{F}$$

$$= \frac{5363.6 \times 0.6 + 7151.5 \times 1.2 + 2648.7 \times 1.3}{15163.8}$$

= **1.005 m from A. Ans.**

▶ 3.4 HORIZONTAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface and equal to, $p = \rho g h$, where h is depth of surface.

Let A = Total area of surface

Then total force, F, on the surface

$$= p \times \text{Area} = \rho g \times h \times A = \rho g A \bar{h}$$

where $\bar{h}$ = Depth of C.G. from free surface of liquid = h

also h^* = Depth of centre of pressure from free surface = h.

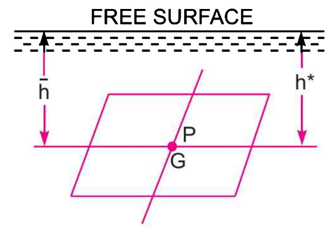


Fig. 3.16

Problem 3.13 *Fig. 3.17 shows a tank full of water. Find :*

(*i*) *Total pressure on the bottom of tank.*

(*ii*) *Weight of water in the tank.*

(*iii*) *Hydrostatic paradox between the results of (i) and (ii). Width of tank is 2 m.*

Solution. Given :

Depth of water on bottom of tank

$$h_1 = 3 + 0.6 = 3.6 \text{ m}$$

Width of tank = 2 m

Length of tank at bottom = 4 m

∴ Area at the bottom, $A = 4 \times 2 = 8 \text{ m}^2$

(*i*) Total pressure F, on the bottom is

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 8 \times 3.6$$

$$= \mathbf{282528 \text{ N. Ans.}}$$

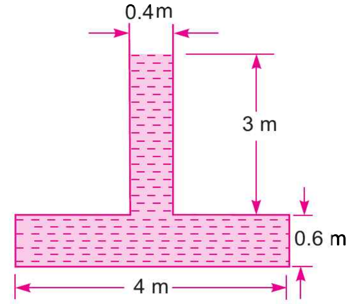


Fig. 3.17

(*ii*) Weight of water in tank = $\rho g \times$ Volume of tank

$$= 1000 \times 9.81 \times [3 \times 0.4 \times 2 + 4 \times .6 \times 2]$$

$$= 1000 \times 9.81 \ [2.4 + 4.8] = \mathbf{70632 \text{ N. Ans.}}$$

(*iii*) From the results of (*i*) and (*ii*), it is observed that the total weight of water in the tank is much less than the total pressure at the bottom of the tank. This is known as Hydrostatic paradox.

▶ 3.5 INCLINED PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid as shown in Fig. 3.18.

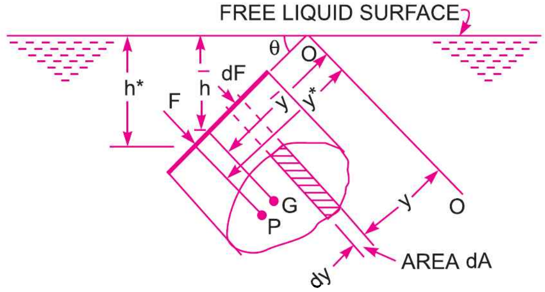


Fig. 3.18 *Inclined immersed surface.*

Let A = Total area of inclined surface

$\bar{h}$ = Depth of C.G. of inclined area from free surface

h^* = Distance of centre of pressure from free surface of liquid

θ = Angle made by the plane of the surface with free liquid surface.

Let the plane of the surface, if produced meet the free liquid surface at O. Then O-O is the axis perpendicular to the plane of the surface.

Let $\bar{y}$ = distance of the C.G. of the inclined surface from O-O

y^* = distance of the centre of pressure from O-O.

Consider a small strip of area dA at a depth 'h' from free surface and at a distance y from the axis O-O as shown in Fig. 3.18.

Pressure intensity on the strip, $p = \rho gh$

$\therefore$ Pressure force, dF, on the strip, $dF = p \times$ Area of strip $= \rho gh \times dA$

Total pressure force on the whole area, $F = \int dF = \int \rho ghdA$

But from Fig. 3.18,
$$\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin\theta$$

$$\therefore \quad h = y\sin\theta$$

$$\therefore \quad F = \int \rho g \times y \times \sin\theta \times dA = \rho g \sin\theta \int ydA$$

But
$$\int ydA = A\bar{y}$$

where $\bar{y}$ = Distance of C.G. from axis O-O

$$\therefore \quad F = \rho g \sin\theta\, \bar{y} \times A$$

$$= \rho g A\bar{h} \qquad (\because \bar{h} = \bar{y}\sin\theta) \qquad \text{...(3.6)}$$

Centre of Pressure (h*)

Pressure force on the strip, $dF = \rho ghdA$

$= \rho gy \sin\theta\, dA$ $\qquad [h = y\sin\theta]$

Moment of the force, dF, about axis O-O

$= dF \times y = \rho gy \sin\theta\, dA \times y = \rho g \sin\theta\, y^2 dA$

Sum of moments of all such forces about O-O

$$= \int \rho g \sin\theta\, y^2\, dA = \rho g \sin\theta \int y^2\, dA$$

But $\int y^2\, dA$ = M.O.I. of the surface about O-O = I_0

$\therefore$ Sum of moments of all forces about O-O = $\rho g \sin\theta\, I_0$ ...(3.7)

Moment of the total force, F, about O-O is also given by

$= F \times y^*$ ...(3.8)

where y^* = Distance of centre of pressure from O-O.

Equating the two values given by equations (3.7) and (3.8)

$$F \times y^* = \rho g \sin\theta\, I_0$$

or
$$y^* = \frac{\rho g \sin\theta\, I_0}{F} \qquad \text{...(3.9)}$$

Now
$$y^* = \frac{h^*}{\sin\theta}, \; F = \rho g A\bar{h}$$

and I_0 by the theorem of parallel axis $= I_G + A\bar{y}^2$.

Substituting these values in equation (3.9), we get

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A\bar{h}}\left[I_G + A\bar{y}^2\right]$$

$$\therefore \qquad h^* = \frac{\sin^2\theta}{A\bar{h}}\,[I_G + A\bar{y}^2]$$

But $$\frac{\bar{h}}{\bar{y}} = \sin\theta \quad \text{or} \quad \bar{y} = \frac{\bar{h}}{\sin\theta}$$

$$\therefore \qquad h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A\times\frac{\bar{h}^2}{\sin^2\theta}\right]$$

or $$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h} \qquad \text{...(3.10)}$$

If $\theta = 90°$, equation (3.10) becomes same as equation (3.5) which is applicable to vertically plane submerged surfaces.

In equation (3.10), I_G = M.O.I. of inclined surfaces about an axis passing through G and parallel to O-O.

Problem 3.14 (*a*) *A rectangular plane surface 2 m wide and 3 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge is 1.5 m below the free water surface.*

Solution. Given :

Width of plane surface, $b = 2$ m

Depth, $d = 3$ m

Angle, $\theta = 30°$

Distance of upper edge from free water surface = 1.5 m

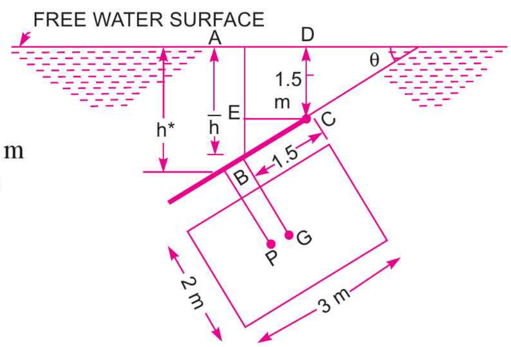


Fig. 3.19

(*i*) **Total pressure** force is given by equation (3.6) as

$$F = \rho g A\bar{h}$$

where $\rho = 1000$ kg/m^3

$A = b \times d = 3 \times 2 = 6$ m^2

$\therefore$ $\bar{h}$ = Depth of C.G. from free water surface

= 1.5 + 1.5 sin 30°

$\{\because \ \bar{h} = AE + EB = 1.5 + BC \sin 30° = 1.5 + 1.5 \sin 30°\}$

$= 1.5 + 1.5 \times \frac{1}{2} = 2.25$ m

$\therefore$ $F = 1000 \times 9.81 \times 6 \times 2.25 =$ **132435 N. Ans.**

(*ii*) **Centre of pressure (h*)**

Using equation (3.10), we have

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}, \quad \text{where } I_G = \frac{bd^3}{12} = \frac{2\times 3^3}{12} = 4.5 \text{ m}^4$$

$$\therefore \qquad h^* = \frac{4.5\times\sin^2 30°}{6\times 2.25} + 2.25 = \frac{4.5\times\frac{1}{4}}{6\times 2.25} + 2.25$$

$= 0.0833 + 2.25 =$ **2.3333 m. Ans.**

Problem 3.14 (*b*) *A rectangular plane surface 3 m wide and 4 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure force and position of centre of pressure, when the upper edge is 2 m below the free surface.*

Solution. Given :

$$b = 3 \text{ m}, \ d = 4 \text{ m}, \ \theta = 30°$$

Distance of upper edge from free surface of water = 2 m

(*i*) **Total pressure force** is given by equation (3.6) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000 \text{ kg/m}^3$,

$A = b \times d = 3 \times 4 = 12 \text{ m}^2$

and $\bar{h}$ = Depth of C.G. of plate from free water surface

$= 2 + \text{BE} = 2 + \text{BC} \sin\theta$

$= 2 + 2 \sin 30° = 2 + 2 \times \frac{1}{2} = 3$ m

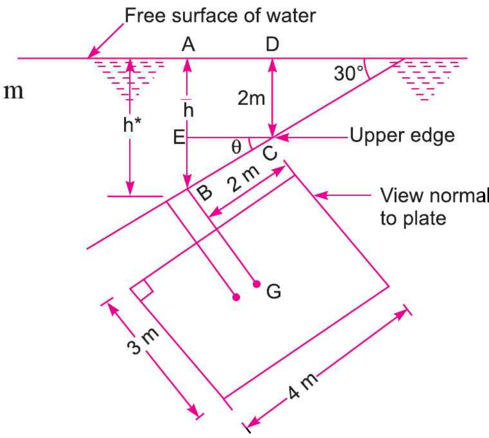


Fig. 3.19 (*a*)

∴ $F = 1000 \times 9.81 \times 12 \times 3 = 353167 \text{ N} = \mathbf{353.167 \text{ kN. Ans.}}$

(*ii*) **Centre of pressure (h*)**

Using equation (3.10), we have $h^* = \dfrac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

where $I_G = \dfrac{bd^3}{12} = \dfrac{3 \times 4^3}{12} = 16 \text{ m}^4$

∴ $$h^* = \frac{16 \times \sin^2 30°}{12 \times 3} + 3 = \frac{16 \times \frac{1}{4}}{36} + 3 = \mathbf{3.111 \text{ m. Ans.}}$$

Problem 3.15 (*a*) *A circular plate 3.0 m diameter is immersed in water in such a way that its greatest and least depth below the free surface are 4 m and 1.5 m respectively. Determine the total pressure on one face of the plate and position of the centre of pressure.*

Solution. Given :

Dia. of plate, $d = 3.0$ m

∴ Area, $A = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4} (3.0)^2 = 7.0685 \text{ m}^2$

Distance $DC = 1.5$ m, $BE = 4$ m

Distance of C.G. from free surface

$= \bar{h} = CD + GC \sin\theta = 1.5 + 1.5 \sin\theta$

But $\sin\theta = \dfrac{AB}{BC} = \dfrac{BE - AE}{BC} = \dfrac{4.0 - DC}{3.0} = \dfrac{4.0 - 1.5}{3.0}$

$= \dfrac{2.5}{3.0} = 0.8333$

∴ $\bar{h} = 1.5 + 1.5 \times .8333 = 1.5 + 1.249 = 2.749$ m

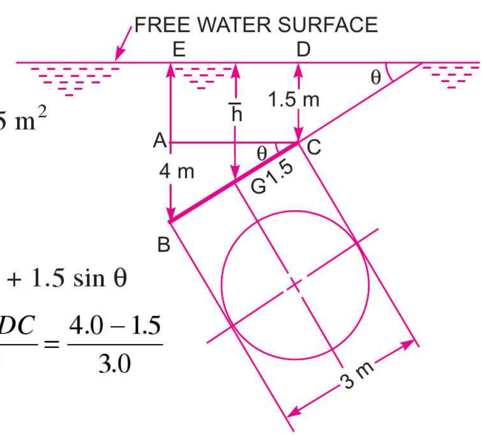


Fig. 3.20

(*i*) **Total pressure (F)**

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 7.0685 \times 2.749 = \textbf{190621 N. Ans.}$$

(*ii*) **Centre of pressure (h*)**

Using equation (3.10), we have $h^* = \dfrac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

where $I_G = \dfrac{\pi}{64} d^4 = \dfrac{\pi}{64}(3)^4 = 3.976 \text{ m}^4$

$$h^* = \frac{3.976 \times (.8333) \times .8333}{7.0685 \times 2.749} + 2.749 = 0.1420 + 2.749$$
$$= \textbf{2.891 m. Ans.}$$

Problem 3.15 (*b*) *If in the above problem, the given circular plate is having a concentric circular hole of diameter 1.5 m, then calculate the total pressure and position of the centre of pressure on one face of the plate.*

Solution. Given : [Refer to Fig. 3.20 (*a*)]

Dia. of plate, $d = 3.0$ m

∴ Area of solid plate $= \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4}(3)^2 = 7.0685 \text{ m}^2$

Dia. of hole in the plate, $d_0 = 1.5$ m

∴ Area of hole $= \dfrac{\pi}{4} {d_0}^2 = \dfrac{\pi}{4}(1.5)^2 = 1.7671 \text{ m}^2$

∴ Area of the given plate, A = Area of solid plate – Area of hole

$= 7.0685 - 1.7671 = 5.3014 \text{ m}^2$

Distance $CD = 1.5$, $BE = 4$ m

Distance of C.G. from the free surface,

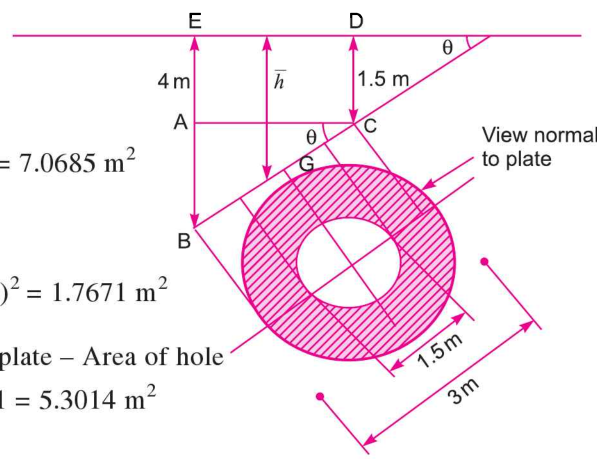


Fig. 3.20 (*a*)

$$\bar{h} = CD + GC \sin\theta$$
$$= 1.5 + 1.5 \sin\theta$$

But $\sin\theta = \dfrac{AB}{BC} = \dfrac{BE - AE}{BC} = \dfrac{4 - 1.5}{3} = \dfrac{2.5}{3}$

∴ $\bar{h} = 1.5 + 1.5 \times \dfrac{2.5}{3} = 1.5 + 1.25 = 2.75$ m

(*i*) **Total pressure force (F)**

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 5.3014 \times 2.75$$
$$= 143018 \text{ N} = \textbf{143.018 kN. Ans.}$$

(*ii*) **Position of centre of pressure (h*)**

Using equation (3.10), we have

$$h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi}{64}\,[d^4 - d_0^4] = \frac{\pi}{64}\,[3^4 - 1.5^4]\ \text{m}^4$

$$A = \frac{\pi}{4}\,[d^2 - d_0^2] = \frac{\pi}{4}\,[3^2 - 1.5^2]\ \text{m}^2$$

$$\sin\theta = \frac{2.5}{3} \text{ and } \bar{h} = 2.75$$

$$\therefore \qquad h^* = \frac{\frac{\pi}{64}\left[3^4 - 1.5^4\right]\times\left(\frac{2.5}{3}\right)^2}{\frac{\pi}{4}\left[3^2 - 1.5^2\right]\times 2.75} + 2.75$$

$$= \frac{\frac{1}{16}\left[3^2 + 1.5^2\right]\times\left(\frac{2.5}{3}\right)^2}{2.75} + 2.75 = \frac{1 \times 11.25 \times 6.25}{16 \times 2.75 \times 9} + 2.75$$

$$= 0.177 + 2.75 = \mathbf{2.927\ m.\ Ans.}$$

Problem 3.16 *A circular plate 3 metre diameter is submerged in water as shown in Fig. 3.21. Its greatest and least depths are below the surfaces being 2 metre and 1 metre respectively. Find : (i) the total pressure on front face of the plate, and (ii) the position of centre of pressure.*

Solution. Given :

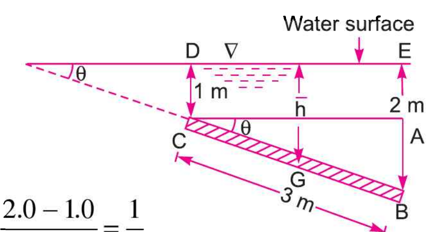


Fig. 3.21

Dia. of plate, $d = 3.0$ m

∴ Area, $A = \frac{\pi}{4}(3.0)^2 = 7.0685\ \text{m}^2$

Distance, $DC = 1$ m, $BE = 2$ m

In ΔABC, $\sin\theta = \frac{AB}{AC} = \frac{BE - AE}{BC} = \frac{BE - DC}{BC} = \frac{2.0 - 1.0}{3.0} = \frac{1}{3}$

The centre of gravity of the plate is at the middle of *BC*, *i.e.*, at a distance 1.5 m from *C*.

The distance of centre of gravity from the free surface of the water is given by

$$\bar{h} = CD + CG\sin\theta = 1.0 + 1.5 \times \frac{1}{3} \qquad \left(\because \sin\theta = \tfrac{1}{3}\right)$$

$$= 1.5\ \text{m}.$$

(*i*) Total pressure on the front face of the plate is given by

$$F = \rho g A\bar{h}$$

$$= 1000 \times 9.81 \times 7.0685 \times 1.5 = \mathbf{104013\ N.\ Ans.}$$

(*ii*) Let the distance of the centre of pressure from the free surface of the water be h^*. Then using equation (3.10), we have

$$h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64} (3)^4$, $A = \frac{\pi}{4} d^2$, $\bar{h} = 1.5$ m and $\sin\theta = \frac{1}{3}$

Substituting the values, we get

$$h^* = \frac{\frac{\pi}{64} d^4 \times \left(\frac{1}{3}\right)^2}{\frac{\pi}{4} d^2 \times 1.5} + 1.5 = \frac{d^2}{16} \times \frac{1}{9 \times 1.5} + 1.5$$

$$= \frac{3^2}{16 \times 9 \times 1.5} + 1.5 = .0416 + 1.5 = \mathbf{1.5416\ m.\ Ans.}$$

Problem 3.17 *A rectangular gate 5 m × 2 m is hinged at its base and inclined at 60° to the horizontal as shown in Fig. 3.22. To keep the gate in a stable position, a counter weight of 5000 kgf is attached at the upper end of the gate as shown in figure. Find the depth of water at which the gate begins to fall. Neglect the weight of the gate and friction at the hinge and pulley.*

Solution. Given :

Length of gate = 5 m

Width of gate = 2 m

$\theta = 60°$

Weight, W = 5000 kgf

= 5000 × 9.81 N

= 49050 N ($\because$ 1 kgf = 9.81 N)

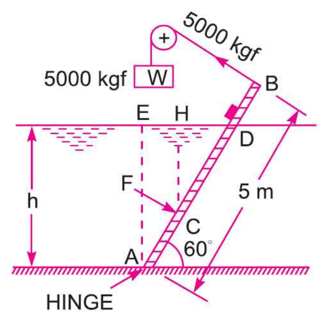


Fig. 3.22

As the pulley is frictionless, the force acting at B = 49050 N. First find the total force F acting on the gate AB for a given depth of water.

From figure, $$AD = \frac{AE}{\sin\theta} = \frac{h}{\sin 60^\circ} = \frac{5}{\sqrt{3}/2} = \frac{2h}{\sqrt{3}}$$

$\therefore$ Area of gate immersed in water, $A = AD \times \text{Width} \times \frac{2h}{\sqrt{3}} \times 2 = \frac{4h}{\sqrt{3}}\ \text{m}^2$

Also depth of the C.G. of the immersed area $= \bar{h} = \frac{h}{2} = 0.5\,h$

$\therefore$ Total force F is given by $F = \rho g A \bar{h} = 1000 \times 9.81 \times \frac{4h}{\sqrt{3}} \times \frac{h}{\sqrt{2}} = \frac{19620}{\sqrt{3}} h^2\text{N}$

The centre of pressure of the immersed surface, h^* is given by

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of the immersed area

$$= \frac{b \times (AD)^3}{12} = \frac{2}{12} \times \left(\frac{2h}{\sqrt{3}}\right)^3 \qquad \left\{\because\ AD = \frac{2h}{\sqrt{3}}\right\}$$

$$= \frac{16h^3}{12 \times 3 \times \sqrt{3}} = \frac{4h^3}{9 \times \sqrt{3}}\ \text{m}^4$$

$$\therefore \qquad h^* = \frac{4h^3}{9\times\sqrt{3}} \times \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{\frac{4h}{\sqrt{3}}\times\frac{h}{2}} + \frac{h}{2} = \frac{3h^3}{18h^2} + \frac{h}{2} = \frac{h}{6} + \frac{h}{2} = \frac{h+3h}{6} = \frac{2h}{3}$$

Now in the ΔCHD, $\quad CH = h^* = \frac{2h}{3}$, $\angle CDH = 60°$

$$\therefore \qquad \frac{CH}{CD} = \sin 60°$$

$$\therefore \qquad CD = \frac{CH}{\sin 60°} = \frac{h^*}{\sin 60°} = \frac{2h}{3\times\frac{\sqrt{3}}{2}} = \frac{4h}{3\times\sqrt{3}}$$

$$\therefore \qquad AC = AD - CD = \frac{2h}{\sqrt{3}} - \frac{4h}{3\sqrt{3}} = \frac{6h-4h}{3\sqrt{3}} = \frac{2h}{3\sqrt{3}} \text{ m}$$

Taking the moments about hinge, we get

$$49050 \times 5.0 = F \times AC = \frac{19620}{\sqrt{3}} h^2 \times \frac{2h}{3\sqrt{3}}$$

or $$.245250 = \frac{39240\, h^3}{3\times 3}$$

$$\therefore \qquad h^3 = \frac{9\times 245250}{39240} = 56.25$$

$$\therefore \qquad h = (56.25)^{1/3} = \mathbf{3.83 \text{ m. Ans.}}$$

Problem 3.18 *An inclined rectangular sluice gate AB, 1.2 m by 5 m size as shown in Fig. 3.23 is installed to control the discharge of water. The end A is hinged. Determine the force normal to the gate applied at B to open it.*

Solution. Given :

$$A = \text{Area of gate} = 1.2 \times 5.0 = 6.0 \text{ m}^2$$

Depth of C.G. of the gate from free surface of the water $= \bar{h}$

$$= DG = BC - BE$$

$$= 5.0 - BG \sin 45°$$

$$= 5.0 - 0.6 \times \frac{1}{\sqrt{2}} = 4.576 \text{ m}$$

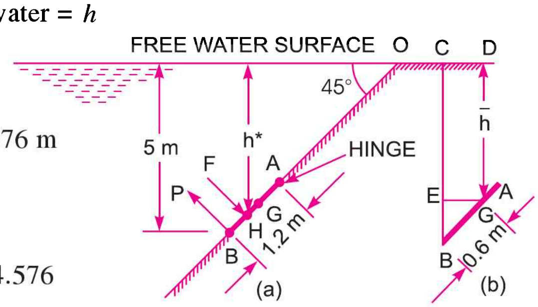


Fig. 3.23

The total pressure force (F) acting on the gate,

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 6.0 \times 4.576$$

$$= 269343 \text{ N}$$

This force is acting at H, where the depth of H from free surface is given by

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of gate = $\frac{bd^3}{12} = \frac{5.0 \times 1.2^3}{12}$ = 0.72 m

∴ Depth of centre of pressure $h^* = \frac{0.72 \times \sin^2 45°}{6 \times 4.576} + 4.576 = .013 + 4.576 = 4.589$ m

But from Fig. 3.23 (*a*), $\frac{h^*}{OH} = \sin 45°$

∴ Distance, $OH = \frac{h^*}{\sin 45°} = \frac{4.589}{\frac{1}{\sqrt{2}}} = 4.589 \times \sqrt{2} = 6.489$ m

Distance, $BO = \frac{5}{\sin 45°} = 5 \times \sqrt{2} = 7.071$ m

Distance, $BH = BO - OH = 7.071 - 6.489 = 0.582$ m

∴ Distance $AH = AB - BH = 1.2 - 0.582 = 0.618$ m

Taking the moments about the hinge *A*

$$P \times AB = F \times (AH)$$

where *P* is the force normal to the gate applied at *B*

∴ $P \times 1.2 = 269343 \times 0.618$

∴ $P = \frac{269343 \times 0.618}{1.2}$ = **138708 N. Ans.**

Problem 3.19 *A gate supporting water is shown in Fig. 3.24. Find the height h of the water so that the gate tips about the hinge. Take the width of the gate as unity.*

Solution. Given : $\theta = 60°$

Distance, $AC = \frac{h}{\sin 60°} = \frac{2h}{\sqrt{3}}$

where h = Depth of water.

The gate will start tipping about hinge *B* if the resultant pressure force acts at *B*. If the resultant pressure force passes through a point which is lying from *B* to *C* anywhere on the gate, the gate will tip over the hinge. Hence limiting case is when the resultant force passes through *B*. But the resultant force passes through the centre of pressure. Hence for the given position, point *B* becomes the centre of pressure. Hence depth of centre of pressure,

$$h^* = (h - 3) \text{ m}$$

But h^* is also given by $= \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$

Taking width of gate unity. Then

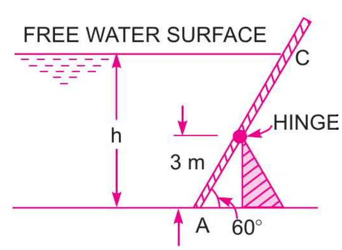


Fig. 3.24

Area, $A = AC \times 1 = \frac{2h}{\sqrt{3}} \times 1$; $\bar{h} = \frac{h}{2}$

$$I_G = \frac{bd^3}{12} = \frac{1 \times AC^3}{12} = \frac{1 \times \left(\frac{2h}{\sqrt{3}}\right)^3}{12} = \frac{8h^3}{12 \times 3 \times \sqrt{3}} = \frac{2h^3}{9 \times \sqrt{3}}$$

$$\therefore \qquad h^* = \frac{2h^3}{9\times\sqrt{3}} \times \frac{\sin^2 60^\circ}{\frac{2h}{\sqrt{3}}\times\frac{h}{2}} + \frac{h}{2} = \frac{2h^3\times\frac{3}{4}}{9h^2} + \frac{h}{2} = \frac{h}{6} + \frac{h}{2} = \frac{2h}{3}$$

Equating the two values of h^*,

$$h - 3 = \frac{2h}{3} \quad \text{or} \quad h - \frac{2h}{3} = 3 \quad \text{or} \quad \frac{h}{3} = 3$$

$$\therefore \qquad h = 3 \times 3 = 9 \text{ m}$$

$\therefore$ Height of water for tipping the gate = **9 m. Ans.**

Problem 3.20 *A rectangular sluice gate AB, 2 m wide and 3 m long is hinged at A as shown in Fig. 3.25. It is kept closed by a weight fixed to the gate. The total weight of the gate and weight fixed to the gate is 343350 N. Find the height of the water 'h' which will just cause the gate to open. The centre of gravity of the weight and gate is at G.*

Solution. Given :

Width of gate, $b = 2$ m ; Length of gate $L = 3$ m

$\therefore$ Area, $A = 2 \times 3 = 6 \text{ m}^2$

Weight of gate and $W = 343350$ N

Angle of inclination, $\theta = 45°$

Let h is the required height of water.

Depth of C.G. of the gate and weight = $\bar{h}$

From Fig. 3.25 (a),

$$\bar{h} = h - ED = h - (AD - AE)$$

$$= h - (AB \sin\theta - EG \tan\theta) \qquad \left\{\because \tan\theta = \frac{AE}{EG} \therefore AE = EG\tan\theta\right\}$$

$$= h - (3 \sin 45° - 0.6 \tan 45°)$$

$$= h - (2.121 - 0.6) = (h - 1.521) \text{ m}$$

The total pressure force, F is given by

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 6 \times (h - 1.521)$$

$$= 58860\,(h - 1.521) \text{ N.}$$

The total force F is acting at the centre of pressure as shown in Fig. 3.25 (b) at H. The depth of H from free surface is given by h^* which is equal to

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}, \text{ where } I_G = \frac{bd^3}{12} = \frac{2\times 3^3}{12} = \frac{54}{12} = 4.5 \text{ m}^4$$

$$\therefore \qquad h^* = \frac{4.5\times\sin^2 45^\circ}{6\times(h-1.521)} + (h - 1.521) = \frac{0.375}{(h-1.521)} + (h - 1.521) \text{ m}$$

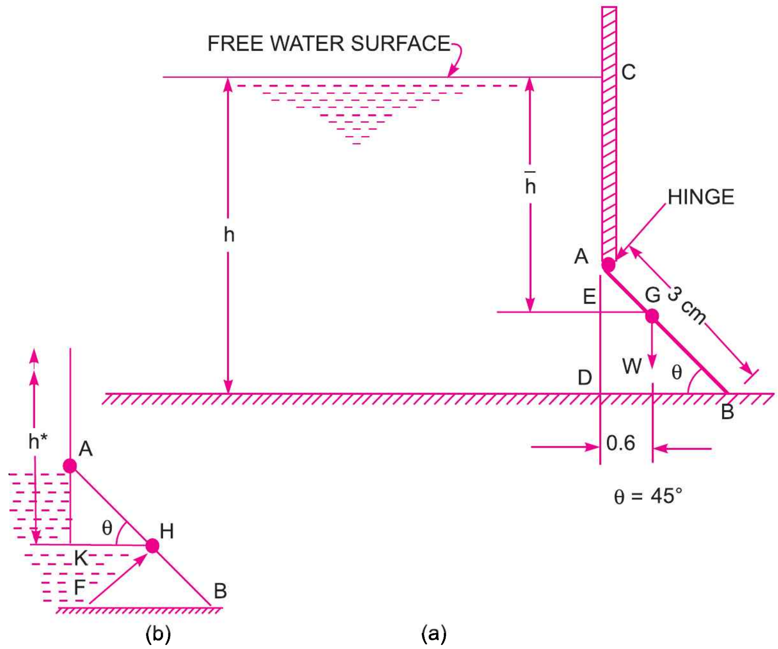


Fig. 3.25

Now taking moments about hinge A, we get

$$343350 \times EG = F \times AH$$

or $$343350 \times 0.6 = F \times \frac{AK}{\sin 45°}$$

$$\left[\text{From } \Delta AKH, \text{Fig. 3.25 } (b)\ AK = AH \sin \theta = AH \sin 45° \therefore AH = \frac{AK}{\sin 45°}\right]$$

$$= \frac{58860\,(h - 1.521) \times AK}{\sin 45°}$$

∴ $$AK = \frac{343350 \times 0.6 \times \sin 45°}{58860\,(h - 1.521)} = \frac{0.3535 \times 7}{(h - 1.521)} \quad \text{...}(i)$$

But $$AK = h^* - AC = \frac{.375}{(h - 1.521)} + (h - 1.521) - AC \quad \text{...}(ii)$$

But $$AC = CD - AD = h - AB \sin 45^{o} = h - 3 \times \sin 45^{o} = h - 2.121$$

∴ Substituting this value in (ii), we get

$$AK = \frac{.375}{h - 1.521} + (h - 1.521) - (h - 2.121)$$

$$= \frac{.375}{h - 1.521} + 2.121 - 1.521 = \frac{.375}{h - 1.521} + 0.6 \quad \text{...}(iii)$$

Equating the two values of AK from (i) and (iii)

$$\frac{0.3535 \times 7}{h - 1.521} = \frac{0.375}{h - 1.521} + 0.6$$

or $\quad 0.3535 \times 7 = 0.375 + 0.6\,(h - 1.521) = 0.375 + 0.6\,h - 0.6 \times 1.521$

or $\quad 0.6h = 2.4745 - .375 + 0.6 \times 1.521 = 2.0995 + 0.9126 = 3.0121$

$\therefore \quad h = \frac{3.0121}{0.6} = \mathbf{5.02\ m.\ Ans.}$

Problem 3.21 *Find the total pressure and position of centre of pressure on a triangular plate of base 2 m and height 3 m which is immersed in water in such a way that the plane of the plate makes an angle of 60° with the free surface of the water. The base of the plate is parallel to water surface and at a depth of 2.5 m from water surface.*

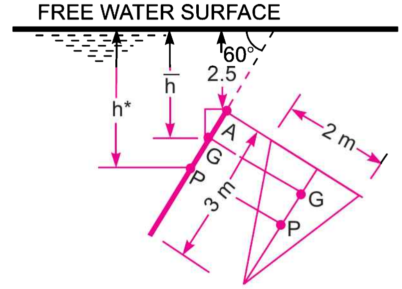


Fig. 3.26

Solution. Given :

Base of plate, $\quad b = 2$ m

Height of plate, $\quad h = 3$ m

$\therefore$ Area, $\quad A = \frac{b \times h}{2} = \frac{2 \times 3}{2} = 3\ m^2$

Inclination, $\quad \theta = 60°$

Depth of centre of gravity from free surface of water,

$$\bar{h} = 2.5 + AG \sin 60°$$

$$= 2.5 + \frac{1}{3} \times 3 \times \frac{\sqrt{3}}{2} \qquad \left\{\because\ AG = \frac{1}{3} \text{ of height of triangle}\right\}$$

$$= 2.5 + .866 \text{ m} = 3.366 \text{ m}$$

(*i*) **Total pressure force (F)**

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 3 \times 3.366 = \mathbf{99061.38\ N.\ Ans.}$$

(*ii*) **Centre of pressure (h*).** Depth of centre of pressure from free surface of water is given by

$$h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}$$

where $\quad I_G = \frac{bh^3}{36} = \frac{2 \times 3^3}{36} = \frac{3}{2} = 1.5\ m^4$

$\therefore \quad h^* = \frac{1.5 \times \sin^2 60°}{3 \times 3.366} + 3.366 = 0.111 + 3.366 = \mathbf{3.477\ m.\ Ans.}$

▶ 3.6 CURVED SURFACE SUB-MERGED IN LIQUID

Consider a curved surface *AB*, sub-merged in a static fluid as shown in Fig. 3.27. Let *dA* is the area of a small strip at a depth of *h* from water surface.

Then pressure intensity on the area *dA* is $= \rho g h$

and pressure force, $\quad dF = p \times \text{Area} = \rho g h \times dA \qquad$ ...(3.11)

This force *dF* acts normal to the surface.

Hence total pressure force on the curved surface should be

$$F = \int \rho g h dA \qquad ...(3.12)$$

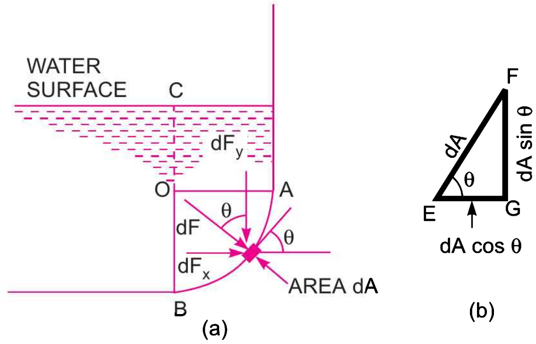


Fig. 3.27

But here as the direction of the forces on the small areas are not in the same direction, but varies from point to point. Hence integration of equation (3.11) for curved surface is impossible. The problem can, however, be solved by resolving the force dF in two components dF_x and dF_y in the x and y directions respectively. The total force in the x and y directions, *i.e.*, F_x and F_y are obtained by integrating dF_x and dF_y. Then total force on the curved surface is

$$F = \sqrt{F_x^2 + F_y^2} \quad \text{...(3.13)}$$

and inclination of resultant with horizontal is $\tan \phi = \dfrac{F_y}{F_x}$ ...(3.14)

Resolving the force dF given by equation (3.11) in x and y directions :

$$dF_x = dF \sin \theta = \rho g h dA \sin \theta \qquad \{\because dF = \rho g h dA\}$$

and $$dF_y = dF \cos \theta = \rho g h dA \cos \theta$$

Total forces in the x and y direction are :

$$F_x = \int dF_x = \int \rho g h dA \sin \theta = \rho g \int h dA \sin \theta \quad \text{...(3.15)}$$

and $$F_y = \int dF_y = \int \rho g h dA \cos \theta = \rho g \int h dA \cos \theta \quad \text{...(3.16)}$$

Fig. 3.27 (*b*) shows the enlarged area dA. From this figure, *i.e.*, ΔEFG,

$$EF = dA$$
$$FG = dA \sin \theta$$
$$EG = dA \cos \theta$$

Thus in equation (3.15), $dA \sin \theta = FG$ = Vertical projection of the area dA and hence the expression $\rho g \int h dA \sin \theta$ represents the total pressure force on the projected area of the curved surface on the vertical plane. Thus

F_x = Total pressure force on the projected area of the curved surface on vertical plane. ...(3.17)

Also $dA \cos \theta = EG$ = horizontal projection of dA and hence $h dA \cos \theta$ is the volume of the liquid contained in the elementary area dA upto free surface of the liquid. Thus $\int h dA \cos \theta$ is the total volume contained between the curved surface extended upto free surface.

Hence $\rho g \int h dA \cos \theta$ is the total weight supported by the curved surface. Thus

$$F_y = \rho g \int h dA \cos \theta$$

= weight of liquid supported by the curved surface upto free surface of liquid. ...(3.18)

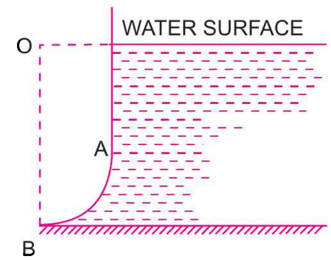


Fig. 3.28

In Fig. 3.28, the curved surface *AB* is not supporting any fluid. In such cases, F_y is equal to the weight of the imaginary liquid supported by *AB* upto free surface of liquid. The direction of F_y will be taken in upward direction.

Problem 3.22 *Compute the horizontal and vertical components of the total force acting on a curved surface AB, which is in the form of a quadrant of a circle of radius 2 m as shown in Fig. 3.29. Take the width of the gate as unity.*

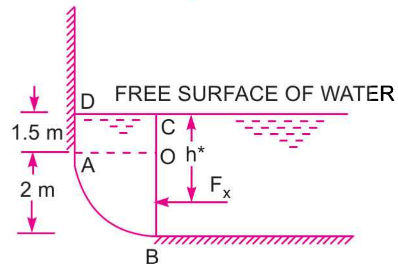


Fig. 3.29

Solution. Given :

Width of gate = 1.0 m

Radius of the gate = 2.0 m

∴ Distance $AO = OB = 2$ m

Horizontal force, F_x exerted by water on gate is given by equation (3.17) as

F_x = Total pressure force on the projected area of curved surface AB on vertical plane

= Total pressure force on *OB*

{projected area of curved surface on vertical plane = $OB \times 1$}

$$= \rho g A\bar{h}$$

$$= 1000 \times 9.81 \times 2 \times 1 \times \left(1.5 + \frac{2}{2}\right)$$

{∵ Area of $OB = A = BO \times 1 = 2 \times 1 = 2$,

$\bar{h}$ = Depth of C.G. of *OB* from free surface = $1.5 + \frac{2}{2}$ }

$F_x = 9.81 \times 2000 \times 2.5 =$ **49050 N. Ans.**

The point of application of F_x is given by $h^* = \dfrac{I_G}{A\bar{h}} + \bar{h}$

where I_G = M.O.I. of *OB* about its C.G. $= \dfrac{bd^3}{12} = \dfrac{1 \times 2^3}{12} = \dfrac{2}{3}$ m^4

$$\therefore \qquad h^* = \frac{\frac{2}{3}}{2 \times 2.5} + 2.5 = \frac{1}{7.5} + 2.5 \text{ m}$$

= 0.1333 + 2.5 = 2.633 m from free surface.

Vertical force, F_y, exerted by water is given by equation (3.18)

F_y = Weight of water supported by *AB* upto free surface

= Weight of portion *DABOC*

= Weight of *DAOC* + Weight of water *AOB*

= ρg [Volume of *DAOC* + Volume of *AOB*]

$$= 1000 \times 9.81 \left[AD \times AO \times 1 + \frac{\pi}{4}(AO)^2 \times 1\right]$$

$$= 1000 \times 9.81 \left[1.5 \times 2.0 \times 1 + \frac{\pi}{4} \times 2^2 \times 1\right]$$

$$= 1000 \times 9.81\ [3.0 + \pi]\text{N} = \mathbf{60249.1\ N.\ Ans.}$$

Problem 3.23 *Fig. 3.30 shows a gate having a quadrant shape of radius 2 m. Find the resultant force due to water per metre length of the gate. Find also the angle at which the total force will act.*

Solution. Given :

Radius of gate = 2 m

Width of gate = 1 m

Horizontal Force

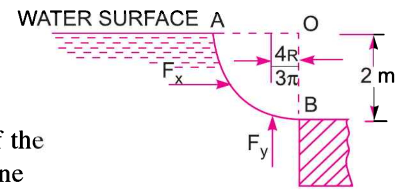


Fig. 3.30

F_x = Force on the projected area of the curved surface on vertical plane

= Force on $BO = \rho g A\bar{h}$

where A = Area of $BO = 2 \times 1 = 2\ \text{m}^2$, $\bar{h} = \frac{1}{2} \times 2 = 1$ m ;

$$F_x = 1000 \times 9.81 \times 2 \times 1 = 19620\ \text{N}$$

This will act at a depth of $\frac{2}{3} \times 2 = \frac{4}{3}$ m from free surface of liquid,

Vertical Force, F_y

F_y = Weight of water (imagined) supported by AB

$= \rho g \times$ Area of $AOB \times 1.0$

$$= 1000 \times 9.81 \times \frac{\pi}{4}\ (2)^4 \times 1.0 = 30819\ \text{N}$$

This will act at a distance of $\frac{4R}{3\pi} = \frac{4 \times 2.0}{3\pi} = 0.848$ m from OB.

∴ Resultant force, F is given by

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{19620 + 30819} = \sqrt{384944400 + 949810761}$$

$$= \mathbf{36534.4\ N.\ Ans.}$$

The angle made by the resultant with horizontal is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{30819}{19620} = 1.5708$$

∴ $\theta = \tan^{-1} 1.5708 = \mathbf{57° \ 31'.\ Ans.}$

Problem 3.24 *Find the magnitude and direction of the resultant force due to water acting on a roller gate of cylindrical form of 4.0 m diameter, when the gate is placed on the dam in such a way that water is just going to spill. Take the length of the gate as 8 m.*

Solution. Given :

Dia. of gate = 4 m

∴ Radius, R = 2 m

Length of gate, l = 8 m

Horizontal force, F_x acting on the gate is

$F_x = \rho g A \bar{h}$ = Force on projected area of curved surface ACB on vertical plane

= Force on vertical area AOB

where A = Area of $AOB = 4.0 \times 8.0 = 32.0 \text{ m}^2$

$\bar{h}$ = Depth of C.G. of $AOB = 4/2 = 2.0$ m

$\therefore \quad F_x = 1000 \times 9.81 \times 32.0 \times 2.0$

$= 627840$ N.

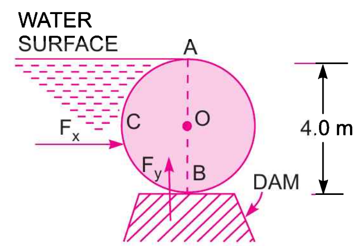


Fig. 3.31

Vertical force, F_y is given by

F_y = Weight of water enclosed or supported (actually or imaginary) by the curved surface ACB

$= \rho g \times$ Volume of portion ACB

$= \rho g \times$ Area of $ACB \times l$

$$= 1000 \times 9.81 \times \frac{\pi}{2}(R)^2 \times 8.0 = 9810 \times \frac{\pi}{2}(2)^2 \times 8.0 = 493104 \text{ N}$$

It will be acting in the upward direction.

$\therefore$ Resultant force, $F = \sqrt{F_x^2 + F_y^2} = \sqrt{627840 + 493104}$ = **798328 N. Ans.**

Direction of resultant force is given by $\tan\theta = \frac{F_y}{F_x} = \frac{493104}{627840} = 0.7853$

$\therefore \quad$ **θ = 31° 8′. Ans.**

Problem 3.25 *Find the horizontal and vertical component of water pressure acting on the face of a tainter gate of 90° sector of radius 4 m as shown in Fig. 3.32. Take width of gate unity.*

Solution. Given :

Radius of gate, $R = 4$ m

Horizontal component of force acting on the gate is

F_x = Force on area of gate projected on vertical plane

= Force on area ADB

$= \rho g A \bar{h}$

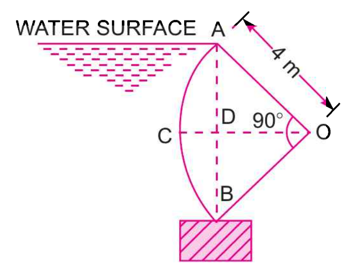


Fig. 3.32

where $A = AB \times$ Width of gate

$= 2 \times AD \times 1$ $\quad (\because AB = 2AD)$

$= 2 \times 4 \times \sin 45° = 8 \times .707 = 5.656 \text{ m}^2$ $\quad \{\because AD = 4 \sin 45°\}$

$$\bar{h} = \frac{AB}{2} = \frac{5.656}{2} = 2.828 \text{ m}$$

$\therefore \quad F_x = 1000 \times 9.81 \times 5.656 \times 2.828$ N = **156911 N. Ans.**

Vertical component

F_y = Weight of water supported or enclosed by the curved surface

= Weight of water in portion $ACBDA$

$= \rho g \times$ Area of $ACBDA \times$ Width of gate

$= 1000 \times 9.81 \times$ [Area of sector $ACBOA$ – Area of ΔABO] $\times 1$

$$= 9810 \times \left[\frac{\pi}{4}R^2 - \frac{AO \times BO}{2}\right] \quad [\because \Delta AOB \text{ is a right angled}]$$

$$= 9810 \times \left[\frac{\pi}{4}4^2 - \frac{4 \times 4}{2}\right] = \textbf{44796 N. Ans.}$$

Problem 3.26 *Calculate the horizontal and vertical components of the water pressure exerted on a tainter gate of radius 8 m as shown in Fig. 3.33. Take width of gate unity.*

Solution. The horizontal component of water pressure is given by

$F_x = \rho g A\bar{h}$ = Force on the area projected on vertical plane

= Force on the vertical area of BD

where $A = BD \times$ Width of gate $= 4.0 \times 1 = 4.0$ m

$$\bar{h} = \frac{1}{2} \times 4 = 2 \text{ m}$$

$$F_x = 1000 \times 9.81 \times 4.0 \times 2.0 = \textbf{78480 N. Ans.}$$

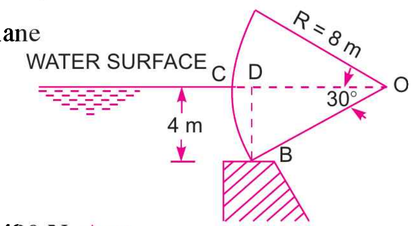


Fig. 3.33

Vertical component of the water pressure is given by

F_y = Weight of water supported or enclosed (imaginary) by curved surface CB

= Weight of water in the portion $CBDC$

$= \rho g \times$ [Area of portion $CBDC$] $\times$ Width of gate

$= \rho g \times$ [Area of sector CBO – Area of the triangle BOD] $\times 1$

$$= 1000 \times 9.81 \times \left[\frac{30}{360} \times \pi R^2 - \frac{BD \times DO}{2}\right]$$

$$= 9810 \times \left[\frac{1}{12}\pi \times 8^2 - \frac{4.0 \times 8.8 \cos 30°}{2}\right]$$

$$\{\because DO = BO \cos 30° = 8 \times \cos 30°\}$$

$$= 9810 \times [16.755 - 13.856] = \textbf{28439 N. Ans.}$$

Problem 3.27 *A cylindrical gate of 4 m diameter 2 m long has water on its both sides as shown in Fig. 3.34. Determine the magnitude, location and direction of the resultant force exerted by the water on the gate. Find also the least weight of the cylinder so that it may not be lifted away from the floor.*

Solution. Given :

Dia. of gate = 4 m

Radius = 2 m

(*i*) The forces acting on the left side of the cylinder are :

The horizontal component, F_{x_1}

where F_{x_1} = Force of water on area projected on vertical plane

= Force on area AOC

$= \rho g A\bar{h}$

$= 1000 \times 9.81 \times 8 \times 2$

$= 156960$ N

where $A = AC \times$ Width $= 4 \times 2 = 8 \text{ m}^2$

$= \bar{h} = \frac{1}{2} \times 4 = 2$ m

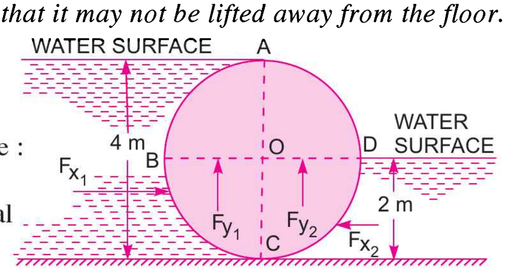


Fig. 3.34

$$F_{y_1} = \text{weight of water enclosed by } ABCOA$$

$$= 1000 \times 9.81 \times \left[\frac{\pi}{2} R^2\right] \times 2.0 = 9810 \times \frac{\pi}{2} \times 2^2 \times 2.0 = \mathbf{123276\ N.}$$

Right Side of the Cylinder

$$F_{x_2} = \rho g A_2 \bar{h}_2 = \text{Force on vertical area } CO$$

$$= 1000 \times 9.81 \times 2 \times 2 \times \frac{2}{2} \left\{ \quad A_2 = CO \times 1 = 2 \times 1 = 2 \text{ m}^2, \bar{h}_2 = \frac{2}{2} = 1.0 \right\}$$

$$= 39240 \text{ N}$$

$$F_{y_2} = \text{Weight of water enclosed by } DOCD$$

$$= \rho g \times \left[\frac{\pi}{4} R^2\right] \times \text{Width of gate}$$

$$= 1000 \times 9.81 \times \frac{\pi}{4} \times 2^2 \times 2 = 61638 \text{ N}$$

∴ Resultant force in the direction of x,

$$F_x = F_{x_1} - F_{x_2} = 156960 - 39240 = 117720 \text{ N}$$

Resultant force in the direction of y,

$$F_y = F_{y_1} + F_{y_2} = 123276 + 61638 = 184914 \text{ N}$$

(*i*) **Resultant force,** F is given as

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(117720)^2 + (184914)^2} = \mathbf{219206\ N.\ Ans.}$$

(*ii*) **Direction of resultant force** is given by

$$\tan \theta = \frac{F_y}{F_x} = \frac{184914}{117720} = 1.5707$$

∴ $\theta = \mathbf{57° \ 31'}$. **Ans.**

(*iii*) **Location of the resultant force**

Force, F_{x_1} acts at a distance of $\frac{2 \times 4}{3} = 2.67$ m from the top surface of water on left side, while F_{x_2} acts at a distance of $\frac{2}{3} \times 2 = 1.33$ m from free surface on the right side of the cylinder. The resultant force F_x in the direction of x will act at a distance of y from the bottom as

$$F_x \times y = F_{x_1} [4 - 2.67] - F_{x_2} [2 - 1.33]$$

or

$$117720 \times y = 156960 \times 1.33 - 39240 \times .67 = 208756.8 - 26290.8 = 182466$$

∴

$$y = \frac{182466}{117720} = 1.55 \text{ m from the bottom.}$$

Force F_{y_1} acts at a distance $\frac{4R}{3\pi}$ from AOC or at a distance $\frac{4 \times 2.0}{3\pi} = 0.8488$ m from AOC towards left of AOC.

Also F_{y_2} acts at a distance $\frac{4R}{3\pi} = 0.8488$ m from AOC towards the right of AOC. The resultant force F_y will act at a distance x from AOC which is given by

$$F_y \times x = F_{y_1} \times .8488 - F_{y_2} \times .8488$$

or $$184914 \times x = 123276 \times .8488 - 61638 \times .8488 = .8488\ [123276 - 61638] = 52318.4$$

$\therefore$ $$x = \frac{52318.4}{184914} = 0.2829 \text{ m from AOC.}$$

(*iv*) **Least weight of cylinder.** The resultant force in the upward direction is

$$F_y = 184914 \text{ N}$$

Thus the weight of cylinder should not be less than the upward force F_y. Hence least weight of cylinder should be at least.

$$= \mathbf{184914\ N.\ Ans.}$$

Problem 3.28 *Fig. 3.35 shows the cross-section of a tank full of water under pressure. The length of the tank is 2 m. An empty cylinder lies along the length of the tank on one of its corner as shown. Find the horizontal and vertical components of the force acting on the curved surface ABC of the cylinder.*

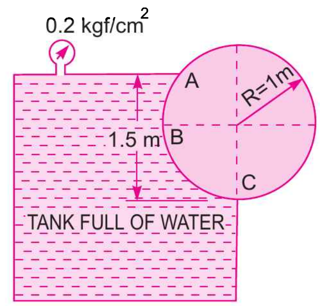


Fig. 3.35

Solution. Radius, $R = 1$ m

Length of tank, $l = 2$ m

Pressure, $p = 0.2 \text{ kgf/cm}^2 = 0.2 \times 9.81 \text{ N/cm}^2$

$= 1.962 \text{ N/cm}^2 = 1.962 \times 10^4 \text{ N/m}^2$

$\therefore$ Pressure head, $$h = \frac{p}{\rho g} = \frac{1.962 \times 10^4}{1000 \times 9.81} = 2 \text{ m}$$

$\therefore$ Free surface of water will be at a height of 2 m from the top of the tank.

$\therefore$ Fig. 3.36 shows the equivalent free surface of water.

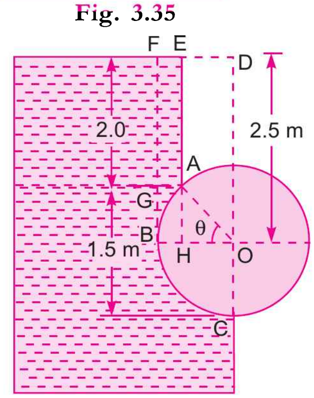


Fig. 3.36

(*i*) **Horizontal Component of Force**

$$F_x = \rho g A \bar{h}$$

where A = Area projected on vertical plane

$= 1.5 \times 2.0 = 3.0 \text{ m}^2$

$$\bar{h} = 2 + \frac{1.5}{2} = 2.75$$

$\therefore$ $$F_x = 1000 \times 9.81 \times 3.0 \times 2.75$$

$$= \mathbf{80932.5\ N.\ Ans.}$$

(*ii*) **Vertical Component of Force**

F_y = Weight of water enclosed or supported actually or imaginary by curved surface ABC

= Weight of water in the portion *CODE ABC*

= Weight of water in *CODFBC* – Weight of water in *AEFB*

But weight of water in *CODFBC*

= Weight of water in [*COB* + *ODFBO*]

$$= \rho g \left[\frac{\pi R^2}{4} + BO \times OD\right] \times 2 = 1000 \times 9.81 \left[\frac{\pi}{4} \times 1^2 + 1.0 \times 2.5\right] \times 2$$

$= 64458.5$ N

Weight of water in $AEFB = \rho g$ [Area of *AEFB*] $\times 2.0$

$$= 1000 \times 9.81 \text{ [Area of } (AEFG + AGBH - AHB)] \times 2.0$$

In ΔAHO, $\sin\theta = \dfrac{AH}{AO} = \dfrac{0.5}{1.0} = 0.5 \quad \therefore \quad \theta = 30°$

$$BH = BO - HO = 1.0 - AO\cos\theta = 1.0 - 1 \times \cos 30° = 0.134$$

Area, ABH = Area ABO – Area AHO

$$= \pi R^2 \times \frac{30}{360} - \frac{AH \times HO}{2.0} = \frac{\pi R^2}{12} - \frac{0.5 \times .866}{2} = 0.0453$$

$\therefore$ Weight of water in $AEFB$

$$= 9810 \times [AE \times AG + AG \times AH - 0.0453] \times 2.0$$
$$= 9810 \times [2.0 \times .134 + .134 \times .5 - .0453] \times 2.0$$
$$= 9810 \times [.268 + .067 - .0453] \times 2.0 = 5684 \text{ N}$$

$\therefore$ $F_y = 64458.5 - 5684 =$ **58774.5 N. Ans.**

Problem 3.29 *Find the magnitude and direction of the resultant water pressure acting on a curved face of a dam which is shaped according to the relation* $y = \dfrac{x^2}{9}$ *as shown in Fig. 3.37. The height of the water retained by the dam is 10 m. Consider the width of the dam as unity.*

Solution. Equation of curve AB is

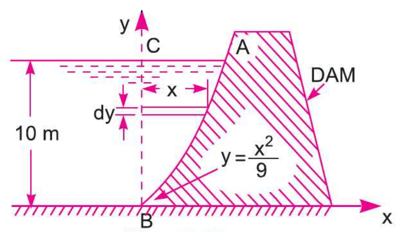


Fig. 3.37

$$y = \frac{x^2}{9} \quad \text{or} \quad x^2 = 9y$$

$\therefore$ $x = \sqrt{9y} = 3\sqrt{y}$

Height of water, $h = 10$ m

Width, $b = 1$ m

The horizontal component, F_x is given by

F_x = Pressure due to water on the curved area projected on vertical plane

= Pressure on area BC

$= \rho g A\bar{h}$

where $A = BC \times 1 = 10 \times 1 \text{ m}^2$, $\bar{h} = \frac{1}{2} \times 10 = 5$ m

$$F_x = 1000 \times 9.81 \times 10 \times 5 = 490500 \text{ N}$$

Vertical component, F_y is given by

F_y = Weight of water supported by the curve AB

= Weight of water in the portion ABC

= ρg[Area of ABC] $\times$ Width of dam

$$= \rho g \left[\int_0^{10} x \times dy\right] \times 1.0 \qquad \left\{\text{Area of strip} = xdy \quad \therefore \text{Area } ABC = \int_0^{10} xdy\right\}$$

$$= 1000 \times 9.81 \times \int_0^{10} 3\sqrt{y}\, dy \qquad \{\because x = 3\sqrt{y}\}$$

$$= 29430 \left[\frac{y^{3/2}}{3/2}\right]_0^{10} = 29430 \times \frac{2}{3}\left[y^{3/2}\right]_0^{10} = 19620\,[10^{3/2}]$$

$$= 19620 \times 31.622 = 620439 \text{ N}$$

∴ Resultant water pressure on dam

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(490500)^2 + (620439)^2}$$

$$= 790907 \text{ N} = \mathbf{790.907 \text{ kN. Ans.}}$$

Direction of the resultant is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{620439}{490500} = 1.265$$

∴ $\theta = \mathbf{51^\circ\ 40'. \text{ Ans.}}$

Problem 3.30 *A dam has a parabolic shape $y = y_0\left(\frac{x}{x_0}\right)^2$ as shown in Fig. 3.38 below having $x_0 = 6$ m and $y_0 = 9$ m. The fluid is water with density = 1000 kg/m^3. Compute the horizontal, vertical and the resultant thrust exerted by water per metre length of the dam.*

Solution. Given :

Equation of the curve *OA* is

$$y = y_0\left(\frac{x}{x_0}\right)^2 = 9\left(\frac{x}{6}\right)^2 = 9 \times \frac{x^2}{36} = \frac{x^2}{4}$$

or $x^2 = 4y$

∴ $x = \sqrt{4y} = 2y^{1/2}$

Width of dam, $b = 1$ m.

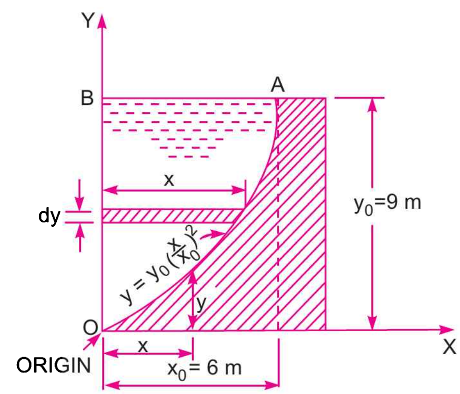


Fig. 3.38

(*i*) **Horizontal thrust exerted by water**

F_x = Force exerted by water on vertical surface *OB*, *i.e.*, the surface obtained by projecting the curved surface on vertical plane

$$= \rho g A\bar{h}$$

$$= 1000 \times 9.81 \times (9 \times 1) \times \frac{9}{2} = \mathbf{397305 \text{ N. Ans.}}$$

(*ii*) **Vertical thrust exerted by water**

F_y = Weight of water supported by curved surface *OA* upto free surface of water

= Weight of water in the portion *ABO*

= ρg × Area of *OAB* × Width of dam

$$= 1000 \times 9.81 \times \left[\int_0^9 x \times dy\right] \times 1.0$$

$$= 1000 \times 9.81 \times \left[\int_0^9 2y^{1/2} \times dy\right] \times 1.0 \qquad (\because x = 2y^{1/2})$$

$$= 19620 \times \left[\frac{y^{3/2}}{(3/2)}\right]_0^9 = 19620 \times \frac{2}{3}[9^{3/2}]$$

$$= 19620 \times \frac{2}{3} \times 27 = \mathbf{353160 \text{ N. Ans.}}$$

(*iii*) **Resultant thrust exerted by water**

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{397305 + 353160} = \mathbf{531574\ N.\ Ans.}$$

Direction of resultant is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{353160}{397305} = 0.888$$

$$\theta = \tan^{-1} 0.888 = \mathbf{41.63°.\ Ans.}$$

Problem 3.31 *A cylinder 3 m in diameter and 4 m long retains water on one side. The cylinder is supported as shown in Fig. 3.39. Determine the horizontal reaction at A and the vertical reaction at B. The cylinder weighs 196.2 kN. Ignore friction.*

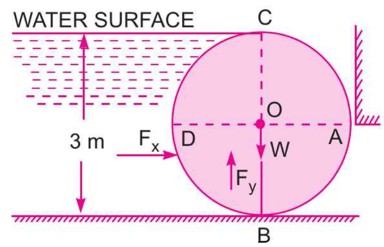


Fig. 3.39

Solution. Given :

Dia. of cylinder = 3 m

Length of cylinder = 4 m

Weight of cylinder, W = 196.2 kN = 196200 N

Horizontal force exerted by water

F_x = Force on vertical area BOC

$= \rho g A\bar{h}$

where $A = BOC \times l = 3 \times 4 = 12\ \text{m}^2$, $\bar{h} = \frac{1}{2} \times 3 = 1.5$ m

$F_x = 1000 \times 9.81 \times 12 \times 1.5 = 176580$ N

The vertical force exerted by water

F_y = Weight of water enclosed in $BDCOB$

$$= \rho g \times \left(\frac{\pi}{2}R^2\right) \times l = 1000 \times 9.81 \times \frac{\pi}{2} \times (1.5)^2 \times 4 = 138684\ \text{N}$$

Force F_y is acting in the upward direction.

For the equilibrium of cylinder

Horizontal reaction at $A = F_x = 176580$ N

Vertical reaction at B = Weight of cylinder – F_y

= 196200 – 138684 = **57516 N. Ans.**

▶ 3.7 TOTAL PRESSURE AND CENTRE OF PRESSURE ON LOCK GATES

Lock gates are the devices used for changing the water level in a canal or a river for navigation. Fig. 3.40 shows plan and elevation of a pair of lock gates. Let AB and BC be the two lock gates. Each gate is supported on two hinges fixed on their top and bottom at the ends A and C. In the closed position, the gates meet at B.

Let F = Resultant force due to water on the gate AB or BC acting are right angles to the gate

R = Reaction at the lower and upper hinge

P = Reaction at the common contact surface of the two gates and acting perpendicular to the contact surface.

Let the force P and F meet at O. Then the reaction R must pass through O as the gate AB is in the equilibrium under the action of three forces. Let θ is the inclination of the lock gate with the normal to the side of the lock.

In $\angle ABO$, $\angle OAB = \angle ABO = \theta$.

Resolving all forces along the gate AB and putting equal to zero, we get

$$R \cos\theta - P\cos\theta = 0 \text{ or } R = P \qquad ...(3.19)$$

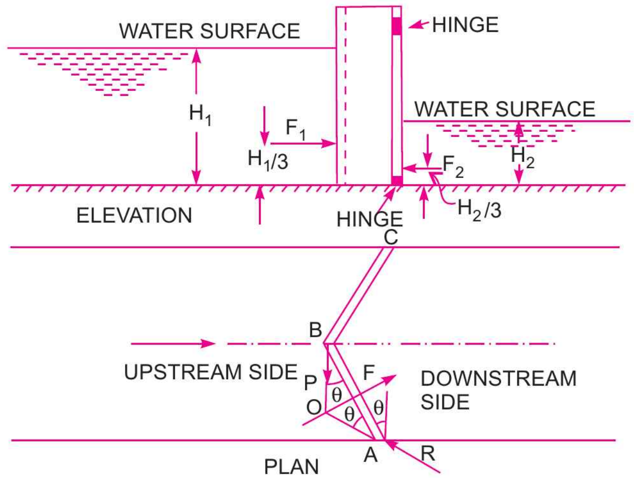


Fig. 3.40

Resolving forces normal to the gate AB

$$R \sin\theta + P\sin\theta - F = 0$$

or $$F = R\sin\theta + P\sin\theta = 2P\sin\theta \qquad \{\because R = P\}$$

$\therefore$ $$P = \frac{F}{2\sin\theta} \qquad ...(3.20)$$

To calculate P and R

In equation (3.20), P can be calculated if F and θ are known. The value of θ is calculated from the angle between the lock gates. The angle between the two lock gate is equal to $180^o - 2\theta$. Hence θ can be calculated. The value of F is calculated as :

Let H_1 = Height of water on the upstream side

H_2 = Height of water on the downstream side

F_1 = Water pressure on the gate on upstream side

F_2 = Water pressure on the gate on downstream side of the gate

l = Width of gate

Now $$F_1 = \rho g A_1 \bar{h}_1$$

$$= \rho g \times H_1 \times l \times \frac{H_1}{2}$$

$$= \rho g l \frac{H_1^2}{2} \qquad \left[\because A_1 = H_1 \times l, \bar{h}_1 = \frac{H_1}{2}\right]$$

Similarly, $$F_2 = \rho g A_2 \bar{h}_2 = \rho g \times (H_2 \times l) \times \frac{H_2}{2} = \frac{\rho g l H_2^2}{2}$$

$\therefore$ Resultant force $$F = F_1 - F_2 = \frac{\rho g l H_1^2}{2} - \frac{\rho g l H_2^2}{2}$$

Substituting the value of θ and F in equation (3.20), the value of P and R can be calculated.

Reactions at the top and bottom hinges

Let R_t = Reaction of the top hinge

R_b = Reaction of the bottom hinge

Then $R = R_t + R_b$

The resultant water pressure F acts normal to the gate. Half of the value of F is resisted by the hinges of one lock gates and other half will be resisted by the hinges of other lock gate. Also F_1 acts at a distance of $\frac{H_1}{3}$ from bottom while F_2 acts at a distance of $\frac{H_2}{3}$ from bottom.

Taking moments about the lower hinge

$$R_t \times \sin\theta \times H = \frac{F_1}{2} \times \frac{H_1}{3} - \frac{F_2}{2} \times \frac{H_2}{3} \quad ...(i)$$

where H = Distance between two hinges

Resolving forces horizontally

$$R_t \sin\theta + R_b \sin\theta = \frac{F_1}{2} - \frac{F_2}{2} \quad ...(ii)$$

From equations (*i*) and (*ii*), we can find R_t and R_b.

Problem 3.32 *Each gate of a lock is 6 m high and is supported by two hinges placed on the top and bottom of the gate. When the gates are closed, they make an angle of 120°. The width of lock is 5 m. If the water levels are 4 m and 2 m on the upstream and downstream sides respectively, determine the magnitude of the forces on the hinges due to water pressure.*

Solution. Given :

Height of lock = 6 m

Width of lock = 5 m

Width of each lock gate = AB

or $l = \frac{AD}{\cos 30°} = \frac{2.5}{\cos 30°}$

= 2.887 m

Angle between gates = 120°

$\therefore \quad \theta = \frac{180° - 120°}{2} = \frac{60°}{2} = 30°$

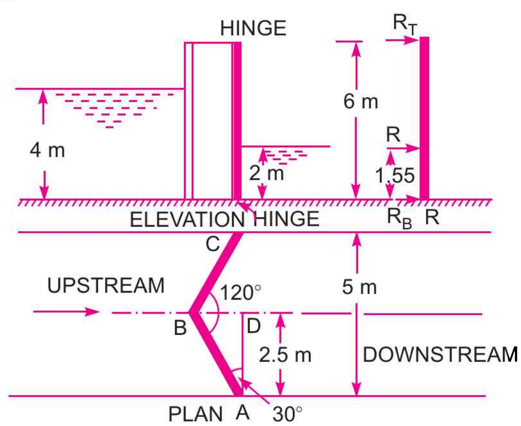


Fig. 3.41

Height of water on upstream side

$H_1 = 4$ m

and $H_2 = 2$ m

$\therefore$ Total water pressure on upstream side

$$F_1 = \rho g A_1 \bar{h}_1, \text{ where } A_1 = H_1 \times l = 4.0 \times 2.887 \text{ m}^2$$

$$= 1000 \times 9.81 \times 4 \times 2.887 \times 2.0 \qquad \left\{\bar{h}_1 = \frac{H_1}{2} = \frac{4}{2} = 2.0 \text{ m}\right\}$$

$$= 226571 \text{ N}$$

Force F_1 will be acting at a distance of $\frac{H_1}{3} = \frac{4}{3} = 1.33$ m from bottom.

Similarly, total water pressure on the downstream side

$$F_2 = \rho g A_2 \bar{h}_2, \text{ where } A_2 = H_2 \times l = 2 \times 2.887 \text{ m}^2$$

$$= 1000 \times 9.81 \times 2 \times 2.887 \times 1.0 \qquad \bar{h}_2 = \frac{H_2}{2} = \frac{2}{2} = 1.0 \text{ m}$$

$= 56643$ N

F_2 will act at a distance of $\frac{H_2}{3} = \frac{2}{3} = 0.67$ m from bottom,

Resultant water pressure on each gate

$$F = F_1 - F_2 = 226571 - 56643 = 169928 \text{ N.}$$

Let x is height of F from the bottom, then taking moments of F_1, F_2 and F about the bottom, we have

$$F \times x = F_1 \times 1.33 - F_2 \times 0.67$$

or $$169928 \times x = 226571 \times 1.33 - 56643 \times 0.67$$

$$\therefore \quad x = \frac{226571 \times 1.33 - 56643 \times 0.67}{169928} = \frac{301339 - 37950}{169928} = 1.55 \text{ m}$$

From equation (3.20), $P = \frac{F}{2\sin\theta} = \frac{169928}{2\sin 30} = 169928$ N.

From equation (3.19), $R = P = 169928$ N.

If R_T and R_B are the reactions at the top and bottom hinges, then $R_T + R_B = R = 169928$ N.

Taking movements of hinge reactions R_T, R_B and R about the bottom hinges, we have

$$R_T \times 6.0 + R_B \times 0 = R \times 1.55$$

$$\therefore \quad R_T = \frac{169928 \times 1.55}{6.0} = 43898 \text{ N}$$

$$\therefore \quad R_B = R - R_T = 169928 - 43898 = \mathbf{126030 \text{ N. Ans.}}$$

Problem 3.33 *The end gates ABC of a lock are 9 m high and when closed include an angle of 120°. The width of the lock is 10 m. Each gate is supported by two hinges located at 1 m and 6 m above the bottom of the lock. The depths of water on the two sides are 8 m and 4 m respectively. Find:*

(i) Resultant water force on each gate,

(ii) Reaction between the gates AB and BC, and

(iii) Force on each hinge, considering the reaction of the gate acting in the same horizontal plane as resultant water pressure.

Solution. Given :

Height of gate $= 9$ m

Inclination of gate $= 120°$

$$\therefore \quad \theta = \frac{180° - 120°}{2} = 30°$$

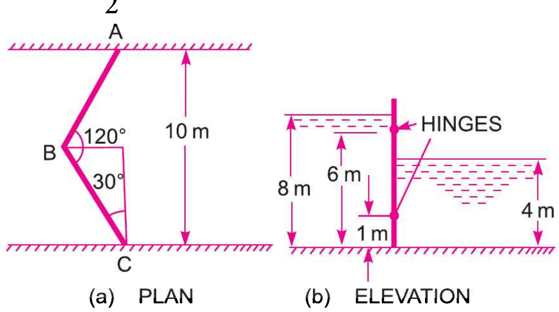


Fig. 3.42

Width of lock $= 10$ m

$\therefore$ Width of each lock $= \dfrac{5}{\cos 30°}$ or $l = 5.773$ m

Depth of water on upstream side, $H_1 = 8$ m

Depth of water on downstream side, $H_2 = 4$ m

(*i*) **Water pressure on upstream side**

$$F_1 = \rho g A_1 \bar{h}_1$$

where $A_1 = l \times H_1 = 5.773 \times 8 = 46.184$ m, $\bar{h}_1 = \dfrac{H_1}{2} = \dfrac{8}{2} = 4.0$ m

$$F_1 = 1000 \times 9.81 \times 46.184 \times 4.0 = 1812260 \text{ N} = 1812.26 \text{ kN}$$

Water pressure on downstream side,

$$F_2 = \rho g A_2 \bar{h}_2$$

where $A_2 = l \times H_2 = 5.773 \times 4 = 23.092$ m, $\bar{h}_2 = \dfrac{4}{2} = 2.0$

$$F_2 = 1000 \times 9.81 \times 23.092 \times 2.0 = 453065 \text{ N} = 453.065 \text{ kN}$$

$\therefore$ Resultant water pressure

$$= F_1 - F_2 = 1812.26 - 453.065 = 1359.195 \text{ kN}$$

(*ii*) **Reaction between the gates AB and BC.** The reaction (P) between the gates AB and BC is given by equation (3.20) as

$$F = \frac{F}{2 \sin \theta} = \frac{1359.195}{2 \times \sin 30°} = \textbf{1359.195 kN. Ans.}$$

(*iii*) **Force on each hinge.** If R_T and R_B are the reactions at the top and bottom hinges then

$$R_T + R_B = R$$

But from equation (3.19), $R = P = 1359.195$

$$\therefore \quad R_T + R_B = 1359.195$$

The force F_1 is acting at $\dfrac{H_1}{3} = \dfrac{8}{3} = 2.67$ m from bottom and F_2 at $\dfrac{H_2}{3} = \dfrac{4}{3} = 1.33$ m from bottom.

The resultant force F will act at a distance x from bottom is given by

$$F \times x = F_1 \times 2.67 - F_2 \times 1.33$$

or

$$x = \frac{F_1 \times 2.67 - F_2 \times 1.33}{F} = \frac{1812.26 \times 2.67 - 453.065 \times 1.33}{1359.195}$$

$$= \frac{4838.734 - 602.576}{1359.195} = 3.116 = 3.11 \text{ m}$$

Hence R is also acting at a distance 3.11 m from bottom.

Taking moments of R_T and R about the bottom hinge

$$R_T \times [6.0 - 1.0] = R \times (x - 1.0)$$

$$\therefore \quad R_T = \frac{R \times (x - 1.0)}{5.0} = \frac{1359.195 \times 2.11}{5.0} = 573.58 \text{ N}$$

$$\therefore \quad R_B = R - R_T = 1359.195 - 573.58$$

$$= \textbf{785.615 kN. Ans.}$$

▶ 3.8 PRESSURE DISTRIBUTION IN A LIQUID SUBJECTED TO CONSTANT HORIZONTAL/VERTICAL ACCELERATION

In chapters 2 and 3, the containers which contains liquids, are assumed to be at rest. Hence the liquids are also at rest. They are in static equilibrium with respect to containers. But if the container containing a liquid is made to move with a constant acceleration, the liquid particles initially will move relative to each other and after some time, there will not be any relative motion between the liquid particles and boundaries of the container. The liquid will take up a new position under the effect of acceleration imparted to its container. The liquid will come to rest in this new position relative to the container. The entire fluid mass moves as a single unit. Since the liquid after attaining a new position is in static condition relative to the container, the laws of hydrostatic can be applied to determine the liquid pressure. As there is no relative motion between the liquid particles, hence the shear stresses and shear forces between liquid particles will be zero. The pressure will be normal to the surface in contact with the liquid.

The following are the important cases under consideration :

(*i*) Liquid containers subject to constant horizontal acceleration.

(*ii*) Liquid containers subject to constant vertical acceleration.

3.8.1 Liquid Containers Subject to Constant Horizontal Acceleration. Fig. 3.43 (*a*) shows a tank containing a liquid upto a certain depth. The tank is stationary and free surface of liquid is horizontal. Let this tank is moving with a constant acceleration '*a*' in the horizontal direction towards right as shown in Fig. 3.43 (*b*). The initial free surface of liquid which was horizontal, now takes the shape as shown in Fig. 3.43 (*b*). Now *AB* represents the new free surface of the liquid. Thus the free surface of liquid due to horizontal acceleration will become a downward sloping inclined plane, with the liquid rising at the back end, the liquid falling at the front end. The equation for the free liquid surface can be derived by considering the equilibrium of a fluid element C lying on the free surface. The forces acting on the element *C* are :

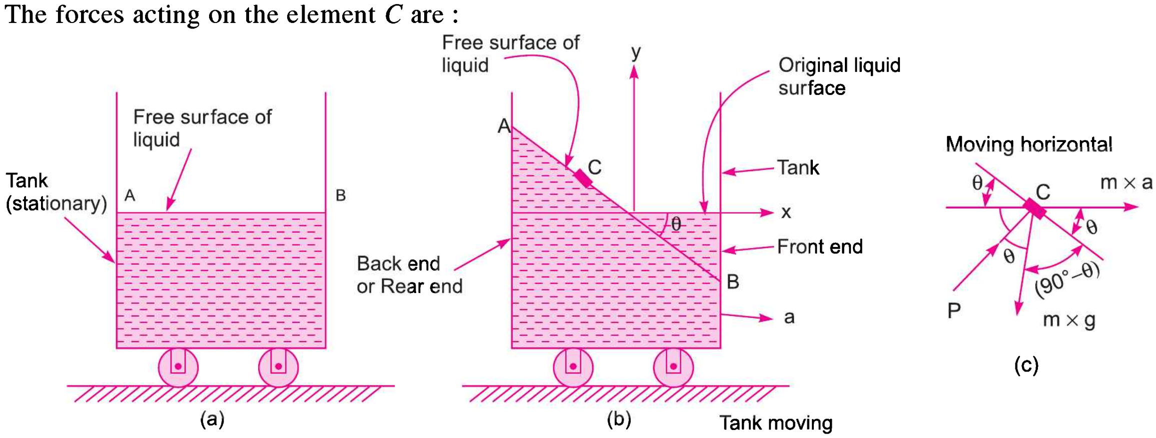


Fig. 3.43

(*i*) the pressure force *P* exerted by the surrounding fluid on the element *C*. This force is normal to the free surface.

(*ii*) the weight of the fluid element *i.e.*, $m \times g$ acting vertically downward.

(*iii*) accelerating force *i.e.*, $m \times a$ acting in horizontal direction.

Resolving the forces horizontally, we get

$$P \sin \theta + m \times a = 0$$

or $$P \sin \theta = - ma \quad \quad ...(i)$$

Resolving the forces vertically, we get

$$P \cos \theta - mg = 0$$

or $$P \cos \theta = m \times g \quad \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\tan \theta = - \frac{a}{g} \quad \left(\text{or } \frac{a}{g} \text{ Numerically} \right) \quad \quad ...(3.20A)$$

The above equation, gives the slope of the free surface of the liquid which is contained in a tank which is subjected to horizontal constant acceleration. The term (a/g) is a constant and hence $\tan \theta$ will be constant. The –ve sign shows that the free surface of liquid is sloping downwards. Hence the free surface is a straight plane inclined down at an angle θ along the direction of acceleration.

Now let us find the expression for the pressure at any point *D* in the liquid mass subjected to horizontal acceleration. Let the point *D* is at a depth of '*h*' from the free surface. Consider an elementary prism *DE* of height '*h*' and cross-sectional area *dA* as shown in Fig. 3.44.

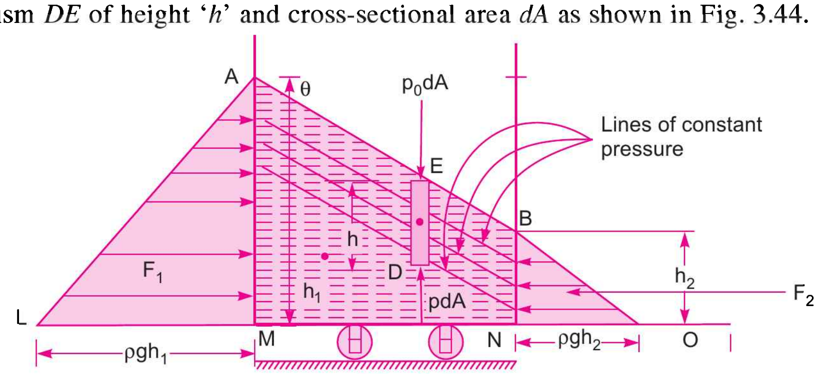


Fig. 3.44

Consider the equilibrium of the elementary prism *DE*.

The forces acting on this prism *DE* in the vertical direction are :

(*i*) the atmospheric pressure force $(p_0 \times dA)$ at the top end of the prism acting downwards,

(*ii*) the weight of the element $(\rho \times g \times h \times dA)$ at the C.G. of the element acting in the downward direction, and

(*iii*) the pressure force $(p \times dA)$ at the bottom end of the prism acting upwards.

Since there is no vertical acceleration given to the tank, hence net force acting vertically should be zero.

$$\therefore \quad p \times dA - p_0 \times dA - \rho gh\, dA = 0$$

or $$p - p_0 - \rho gh = 0 \quad \text{or} \quad p = p_0 + \rho gh$$

or $$p - p_0 = \rho gh$$

or gauge pressure at point *D* is given by

$$p = \rho gh$$

or pressure head at point *D*, $$\frac{p}{\rho g} = h.$$

From the above equation, it is clear that pressure head at any point in a liquid subjected to a constant horizontal acceleration is equal to the height of the liquid column above that point. Therefore the pressure distribution in a liquid subjected to a constant horizontal acceleration is same as hydrostatic pressure distribution. The planes of constant pressure are therefore, parallel to the inclined surface as shown in Fig. 3.44. This figure also shows the variation of pressure on the rear and front end of the tank.

If h_1 = Depth of liquid at the rear end of the tank

h_2 = Depth of liquid at the front end of the tank

F_1 = Total pressure force exerted by liquid on the rear side of the tank

F_2 = Total pressure force exerted by liquid on the front side of the tank,

then F_1 = (Area of triangle AML) × Width

$$= \left(\tfrac{1}{2} \times LM \times AM \times b\right) = \tfrac{1}{2} \times \rho g h_1 \times h_1 \times b = \frac{\rho g . b . h_1^2}{2}$$

and F_2 = (Area of triangle BNO) × Width

$$= \left(\tfrac{1}{2} \times BN \times NO\right) = \tfrac{1}{2} \times h_2 \times \rho g h_2 \times b = \frac{\rho g . b . h_2^2}{2}$$

where b = Width of tank perpendicular to the plane of the paper.

The values of F_1 and F_2 can also be obtained as [Refer to Fig. 3.44 (*a*)]

$$F_1 = \rho \times g \times A_1 \times \bar{h}_1, \text{ where } A_1 = h_1 \times b \text{ and } \bar{h}_1 = \frac{h_1}{2}$$

$$= \rho \times g \times (h_1 \times b) \times \frac{h_1}{2} = \frac{1}{2} \rho g . b . h_1^2$$

and

$$F_2 = \rho \times g \times A_2 \times \bar{h}_2, \text{ where } A_2 = h_2 \times b \text{ and } \bar{h}_2 = \frac{h_2}{2}$$

$$= \rho \times g \times (h_2 \times b) \times \frac{h_2}{2}$$

$$= \frac{1}{2} \rho g . b \times h_2^2.$$

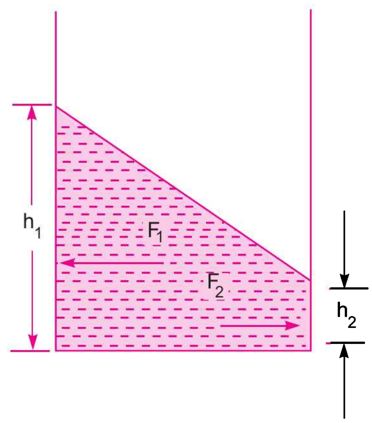


Fig. 3.44(*a*)

It can also be proved that the difference of these two forces (*i.e.*, $F_1 - F_2$) is equal to the force required to accelerate the mass of the liquid contained in the tank *i.e.*,

$$F_1 - F_2 = M \times a$$

where M = Total mass of the liquid contained in the tank

a = Horizontal constant acceleration.

Note : (*i*) If a tank completely filled with liquid and open at the top is subjected to a constant horizontal acceleration, then some of the liquid will spill out from the tank and new free surface with its slope given by equation $\tan \theta = -\frac{a}{g}$ will be developed.

(*ii*) If a tank partly filled with liquid and open at the top is subjected to a constant horizontal acceleration, spilling of the liquid may take place depending upon the magnitude of the acceleration.

(*iii*) If a tank completely filled with liquid and closed at the top is subjected to a constant horizontal acceleration, then the liquid would not spill out from the tank and also there will be no adjustment in the surface elevation of the liquid. But the equation $\tan \theta = -\frac{a}{g}$ is applicable for this case also.

(*iv*) The example for a tank with liquid subjected to a constant horizontal acceleration, is a fuel tank on an airplane during take off.

Problem 3.34 *A rectangular tank is moving horizontally in the direction of its length with a constant acceleration of 2.4 m/s^2. The length, width and depth of the tank are 6 m, 2.5 m and 2 m respectively. If the depth of water in the tank is 1 m and tank is open at the top then calculate :*

(*i*) *the angle of the water surface to the horizontal,*

(*ii*) *the maximum and minimum pressure intensities at the bottom,*

(*iii*) *the total force due to water acting on each end of the tank.*

Solution. Given :

Constant acceleration, $a = 2.4$ m/s^2.

Length = 6 m ; Width = 2.5 m and depth = 2 m.

Depth of water in tank, $h = 1$ m

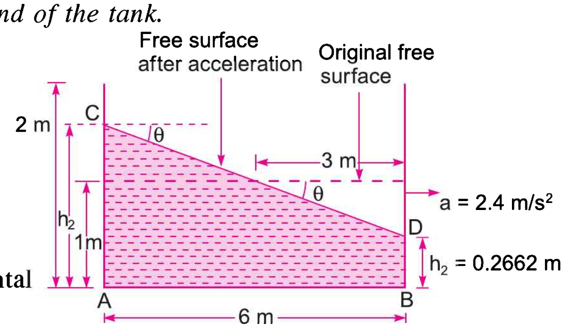


Fig. 3.45

(*i*) **The angle of the water surface to the horizontal**

Let θ = the angle of water surface to the horizontal

Using equation (3.20), we get

$$\tan\theta = -\frac{a}{g} = -\frac{2.4}{9.81} = -0.2446$$

(the –ve sign shows that the free surface of water is sloping downward as shown in Fig. 3.45)

∴ $\tan\theta = 0.2446$ (slope downward)

∴ $\theta = \tan^{-1} 0.2446 =$ **13.7446°** or **13° 44.6′**. **Ans.**

(*ii*) **The maximum and minimum pressure intensities at the bottom of the tank**

From the Fig. 3.45,

Depth of water at the front end,

$$h_1 = 1 - 3\tan\theta = 1 - 3 \times 0.2446 = 0.2662 \text{ m}$$

Depth of water at the rear end,

$$h_2 = 1 + 3\tan\theta = 1 + 3 \times 0.2446 = 1.7338 \text{ m}$$

The pressure intensity will be maximum at the bottom, where depth of water is maximum.

Now the maximum pressure intensity at the bottom will be at point *A* and it is given by,

$$p_{max} = \rho \times g \times h_2$$
$$= 1000 \times 9.81 \times 1.7338 \text{ N/m}^2 = \mathbf{17008.5\ N/m^2}. \ \textbf{Ans.}$$

The minimum pressure intensity at the bottom will be at point B and it is given by

$$p_{min} = \rho \times g \times h_1$$
$$= 1000 \times 9.81 \times 0.2662 = \mathbf{2611.4\ N/m^2}. \ \textbf{Ans.}$$

(*iii*) **The total force due to water acting on each end of the tank**

Let F_1 = total force acting on the front side (*i.e.*, on face *BD*)

F_2 = total force acting on the rear side (*i.e.*, on face *AC*)

Then $F_1 = \rho g A_1 \bar{h}_1$, where $A_1 = BD \times$ width of tank $= h_1 \times 2.5 = 0.2662 \times 2.5$

and $\bar{h}_1 = \frac{BD}{2} = \frac{h_1}{2} = \frac{0.2662}{2} = 0.1331$ m

$= 1000 \times 9.81 \times (0.2662 \times 2.5) \times 0.1331$

= 868.95 N. Ans.

and $F_2 = \rho.g.A_2.\bar{h}_2$, where $A_2 = AB \times$ width of tank $= h_2 \times 2.5 = 1.7338 \times 2.5$

$$\bar{h}_2 = \frac{AB}{2} = \frac{h_2}{2} = \frac{1.7338}{2} = 0.8669 \text{ m}$$

$$= 1000 \times 9.81 \times (1.7338 \times 2.5) \times 0.8669$$

$$= \mathbf{36861.8 \text{ N. Ans.}}$$

$\therefore$ Resultant force $= F_1 - F_2$

$$= 36861.8 \text{ N} - 868.95$$

$$= 35992.88 \text{ N}$$

Note. The difference of the forces acting on the two ends of the tank is equal to the force necessary to accelerate the liquid mass. This can be proved as shown below :

Consider the control volume of the liquid *i.e.*, control volume is *ACDBA* as shown in Fig. 3.46. The net force acting on the control volume in the horizontal direction must be equal to the product of mass of the liquid in control volume and acceleration of the liquid.

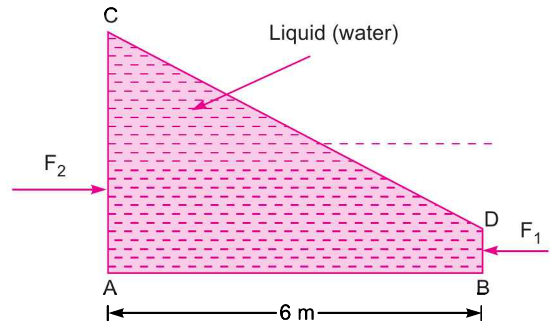


Fig. 3.46

$$\therefore \quad (F_1 - F_2) = M \times a$$

$$= (\rho \times \text{volume of control volume}) \times a$$

$$= (1000 \times \text{Area of } ABDCE \times \text{width}) \times 2.4$$

$$= \left[1000 \times \left(\frac{AC + BD}{2}\right) \times AB \times \text{width}\right] \times 2.4$$

$$\left[\because \text{ Area of trapezium} = \left(\frac{AC + BD}{2}\right) \times AB\right]$$

$$= 1000 \times \left(\frac{1.7338 + 0.2662}{2}\right) \times 6 \times 2.5 \times 2.4$$

$$= 36000 \text{ N}$$

($\because$ $AC = h_2 = 1.7338$ m, $BD = h_1 = 0.2662$ m, and $AB = 6$ m, width $= 2.5$ m)

The above force is nearly the same as the difference of the forces acting on the two ends of the tank. (*i.e.*, $35992.88 \simeq 36000$).

Problem 3.35 *The rectangular tank of the above problem contains water to a depth of 1.5 m. Find the horizontal acceleration which may be imparted to the tank in the direction of its length so that*

(*i*) *the spilling of water from the tank is just on the verge of taking place,*

(*ii*) *the front bottom corner of the tank is just exposed,*

(*iii*) *the bottom of the tank is exposed upto its mid-point.*

Also calculate the total forces exerted by the water on each end of the tank in each case. Also prove that the difference between these forces is equal to the force necessary to accelerate the mass of water tank.

Solution. Given :

Dimensions of the tank from previous problem,

$$L = 6 \text{ m, width } (b) = 2.5 \text{ m and depth} = 2 \text{ m}$$

Depth of water in tank, $h = 1.5$ m

Horizontal acceleration imparted to the tank

(*i*) (*a*) *When the spilling of water from the tank is just on the verge of taking place*

Let a = required horizontal acceleration

When the spilling of water from the tank is just on the verge of taking place, the water would rise upto the rear top corner of the tank as shown in Fig. 3.47 (*a*)

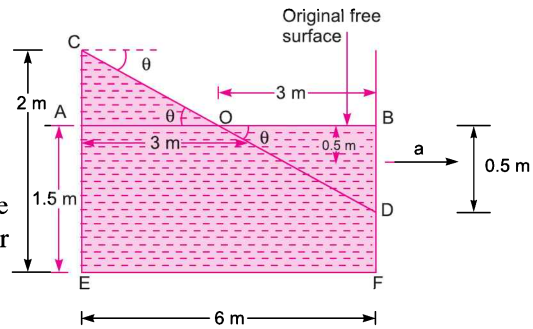


Fig. 3.47 (*a*) *Spilling of water is just on the verge of taking place.*

$$\therefore \qquad \tan\theta = \frac{AC}{AO} = \frac{(2-1.5)}{3} = \frac{0.5}{3} = 0.1667$$

But from equation (3.20) $\tan\theta = \frac{a}{g}$ (Numerically)

$$\therefore \qquad a = g \times \tan\theta = 9.81 \times 0.1667 = \mathbf{1.635\ m/s^2.\ Ans.}$$

(*b*) *Total forces exerted by water on each end of the tank*

The force exerted by water on the end *CE* of the tank is

$$F_1 = \rho g A_1 \bar{h}_1, \text{ where } A_1 = CE \times \text{width of the tank} = 2 \times 2.5$$

$$\bar{h}_1 = \frac{CE}{2} = \frac{2}{2} = 1 \text{ m}$$

$$= 1000 \times 9.81 \times (2 \times 2.5) \times 1$$

$$= \mathbf{49050\ N.\ Ans.}$$

The force exerted by water on the end *FD* of the tank is

$$F_2 = \rho g A_2 \times \bar{h}_2, \text{ where } A_2 = FD \times \text{width} = 1 \times 2.5$$

$$(\because AC = BD = 0.5 \text{ m}, \quad \therefore \quad FD = BF - BD = 1.5 - 0.5 = 1)$$

$$= 1000 \times 9.81 \times (1 \times 2.5) \times 0.5 \qquad \bar{h}_2 = \frac{FD}{2} = \frac{1}{2} = 0.5 \text{ m}$$

$$= \mathbf{12262.5\ N.\ Ans.}$$

(*c*) *Difference of the forces is equal to the force necessary to accelerate the mass of water in the tank*

Difference of the forces $= F_1 - F_2$

$$= 49050 - 12262.5 = 36787.5 \text{ N}$$

Volume of water in the tank before acceleration is imparted to it = $L \times b \times$ depth of water

$$= 6 \times 2.5 \times 1.5 = 22.5 \text{ m}^3.$$

The force necessary to accelerate the mass of water in the tank

= Mass of water in tank × Acceleration

= (ρ × volume of water) × 1.635 $\qquad (\because a = 1.635 \text{ m/s}^2)$

= 1000 × 22.5 × 1.635 [There is no spilling of water and volume of water = 22.5 m^3]

= 36787.5 N

Hence the difference between the forces on the two ends of the tank is equal to the force necessary to accelerate the mass of water in the tank.

Volume of water in the tank can also be calculated as volume = $\left(\frac{CE + FD}{2}\right) \times EF \times$ Width [Refer to Fig. 3.47 (*a*)]

$$= \left(\frac{2+1}{2}\right) \times 6 \times 2.5 = 22.5 \text{ m}^3.$$

(*ii*) (*a*) *Horizontal acceleration when the front bottom corner of the tank is just exposed*

Refer to Fig. 3.47 (*b*). In this case the free surface of water in the tank will be along *CD*.

Let a = required horizontal acceleration.

In this case, $\tan\theta = \frac{CE}{ED} = \frac{2}{6} = \frac{1}{3}$

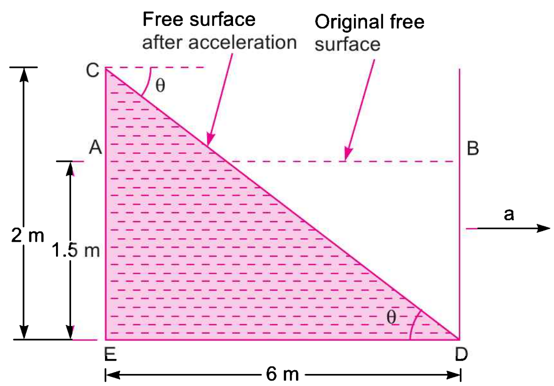


Fig. 3.47 (*b*)

But from equation (3.17),

$$\tan\theta = \frac{a}{g} \text{ (Numerically)}$$

$$\therefore \quad a = g \times \tan\theta = 9.81 \times \frac{1}{3} = \mathbf{3.27\ m/s^2.\ Ans.}$$

(*b*) *Total forces exerted by water on each end of the tank*

The force exerted by water on the end *CE* of the tank is

$$F_1 = \rho g \times A_1 \times \bar{h}_1$$

where $A_1 = CE \times \text{width} = 2 \times 2.5 = 5 \text{ m}^2$

$$\bar{h}_1 = \frac{CE}{2} = \frac{2}{2} = 1 \text{ m} \qquad = 1000 \times 9.81 \times 5 \times 1$$

$$= \mathbf{49050\ N.\ Ans.}$$

The force exerted by water on the end *BD* of the tank is zero as there is no water against the face *BD*

$\therefore \quad F_2 = 0$

$\therefore$ Difference of forces = 49050 – 0 = 49050 N

(*c*) *Difference of forces is equal to the force necessary to accelerate the mass of water in the tank.*

Volume of water in the tank = Area of *CED* × Width of tank

$$= \left(\frac{CE \times ED}{2}\right) \times 2.5 \qquad (\because \text{ Width of tank} = 2.5 \text{ m})$$

$$= \frac{2 \times 6}{2} \times 2.5 = 15 \text{ m}^3$$

$\therefore$ Force necessary to accelerate the mass of water in the tank

= Mass of water in tank × Acceleration

= (1000 × Volume of water) × 3.27

= 1000 × 15 × 3.27 = 49050 N

Difference of two forces is also = 49050 N

Hence difference between the forces on the two ends of the tank is equal to the force necessary to accelerate the mass of water in the tank.

(*iii*) (*a*) *Horizontal acceleration when the bottom of the tank is exposed upto its mid-point*

Refer to Fig. 3.47 (*c*). In this case the free surface of water in the tank will be along CD^*, where D^* is the mid-point of ED.

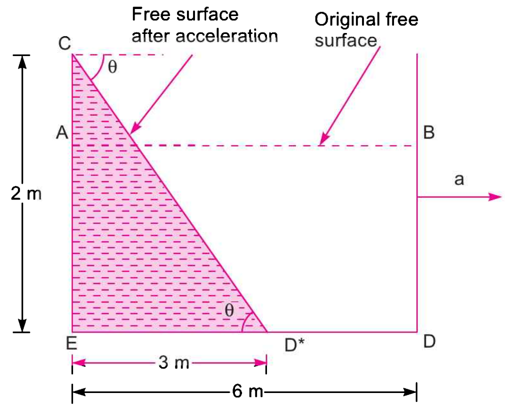


Fig. 3.47 (*c*)

Let a = required horizontal acceleration from Fig. 3.47 (*c*), it is clear that

$$\tan\theta = \frac{CE}{ED^*} = \frac{2}{3}$$

But from equation (3.20) numerically

$$\tan\theta = \frac{a}{g}$$

$$\therefore \quad a = g \times \tan\theta = 9.81 \times \frac{2}{3} = \mathbf{6.54\ m/s^2.\ Ans.}$$

(*b*) *Total forces exerted by water on each end of the tank*

The force exerted by water on the end CE of the tank is

$$F_1 = \rho \times g \times A_1 \times \bar{h}_1$$

where $A_1 = CE \times \text{Width} = 2 \times 2.5 = 5\ \text{m}^2$

$$\bar{h}_1 = \frac{CE}{2} = \frac{2}{2} = 1\ \text{m}$$

$$= 1000 \times 9.81 \times 5 \times 1$$

$$= \mathbf{49050\ N.\ Ans.}$$

The force exerted by water on the end BD is zero as there is no water against the face BD.

$$\therefore \quad F_2 = 0$$

$\therefore$ Difference of the forces $= F_1 - F_2 = 49050 - 0 = 49050$ N

(*c*) *Difference of the two forces is equal to the force necessary to accelerate the mass of water remaining in the tank*

Volume of water in the tank = Area CED^* × Width of tank

$$= \frac{CE \times ED^*}{2} \times 2.5 = \frac{2 \times 3}{2} \times 2.5 = 7.5\ \text{m}^3$$

Force necessary to accelerate the mass of water in the tank

= Mass of water × Acceleration

$= \rho \times$ Volume of water $\times 6.54$ $\quad (\because a = 6.54\ \text{m/s}^2)$

$= 1000 \times 7.5 \times 6.54$

$= 49050$ N

This is the same force as the difference of the two forces on the two ends of the tank.

Problem 3.36 *A rectangular tank of length 6 m, width 2.5 m and height 2 m is completely filled with water when at rest. The tank is open at the top. The tank is subjected to a horizontal constant linear acceleration of 2.4 m/s² in the direction of its length. Find the volume of water spilled from the tank.*

Solution. Given :

$$L = 6 \text{ m}, b = 2.5 \text{ m and height}, H = 2 \text{ m}$$

Horizontal acceleration, $a = 2.4 \text{ m/s}^2$.

The slope of the free surface of water after the tank is subjected to linear constant acceleration is given by equation (3.20) as

$$\tan\theta = \frac{a}{g} \text{ (Numerically)}$$

$$= \frac{2.4}{9.81} = 0.2446$$

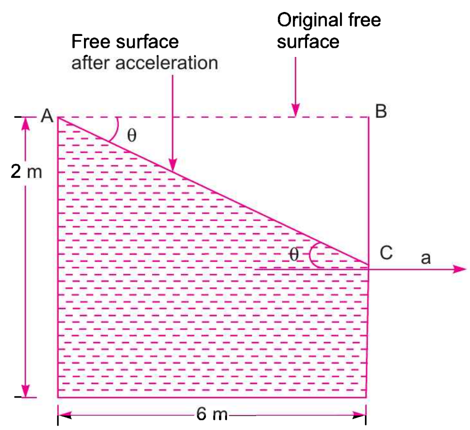


Fig. 3.48

From Fig. 3.48,

$$\tan\theta = \frac{BC}{AB}$$

∴ $BC = AB \times \tan\theta$

$= 6 \times 0.2446$

($\because$ AB = Length = 6 m ; $\tan\theta = 0.2446$)

$= 1.4676$ m

∴ Volume of water spilled = Area of ABC × Width of tank

$= \left(\frac{1}{2} \times AB \times BC\right) \times 2.5$ ($\because$ Width = 2.5 m)

$= \frac{1}{2} \times 6 \times 1.4676 \times 2.5$ ($\because$ $BC = 1.4676$ m)

= 11.007 m³. Ans.

3.8.2 Liquid Container Subjected to Constant Vertical Acceleration. Fig. 3.49 shows a tank containing a liquid and the tank is moving vertically upward with a constant acceleration. The liquid in the tank will be subjected to the same vertical acceleration. To obtain the expression for the pressure at any point in the liquid mass subjected to vertical upward acceleration, consider a vertical elementary prism of liquid *CDFE*.

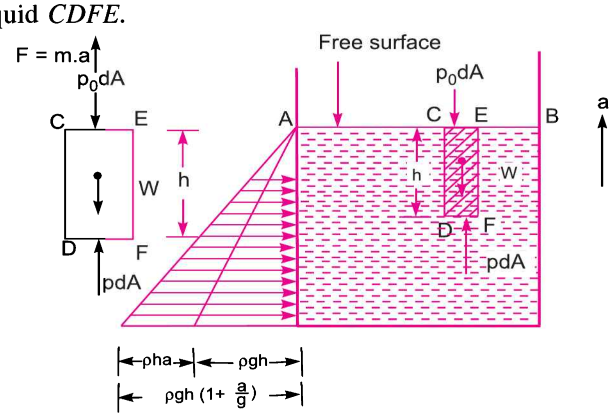


Fig. 3.49

Let dA = Cross-sectional area of prism

h = Height of prism

p_0 = Atmospheric pressure acting on the face CE

p = Pressure at a depth h acting on the face DF

The forces acting on the elementary prism are :

(*i*) Pressure force equal to $p_0 \times dA$ acting on the face *CE* vertically downward

(*ii*) Pressure force equal to $p \times dA$ acting on the face *DF* vertically upward

(*iii*) Weight of the prism equal to $\rho \times g \times dA \times h$ acting through C.G. of the element vertically downward.

According to Newton's second law of motion, the net force acting on the element must be equal to mass multiplied by acceleration in the same direction.

∴ Net force in vertically upward direction = Mass × acceleration

$$p \times dA - p_0 \times dA - \rho g dA \,.\, h = (\rho \times dA \times h) \times a \qquad (\because \text{Mass} = \rho \times dA \times h)$$

or
$$p - p_0 - \rho gh = \rho h \times a \qquad \text{(Cancelling } dA \text{ from both sides)}$$

or
$$p - p_0 = \rho gh + \rho ha$$

$$= \rho gh \left[1 + \frac{a}{g}\right] \qquad ...(3.21)$$

But $(p - p_0)$ is the gauge pressure. Hence gauge pressure at any point in the liquid mass subjected to a constant vertical upward acceleration, is given by

$$p_g = \rho gh \left[1 + \frac{a}{g}\right] \qquad ...(3.22)$$

$$= \rho gh + \rho ha \qquad ...(3.22A)$$

where $p_g = p - p_0$ = gauge pressure

In equation (3.22) ρ, *g* and *a* are constant. Hence variation of guage pressure is linear. Also when $h = 0$, $p_g = 0$. This means $p - p_0 = 0$ or $p = p_0$. Hence when $h = 0$, the pressure is equal to atmospheric pressure. Hence free surface of liquid subjected to constant vertical acceleration will be horizontal.

From equation (3.22*A*) it is also clear that the pressure at any point in the liquid mass is greater than the hydrostatic pressure (hydrostatic pressure is = ρgh) by an amount of $\rho \times h \times a$.

Fig. 3.49 shows the variation of pressure for the liquid mass subjected to a constant vertical upward acceleration.

If the tank containing liquid is moving vertically downward with a constant acceleration, then the gauge pressure at any point in the liquid at a depth of *h* from the free surface will be given by

$$(p - p_0) = \rho gh \left[1 - \frac{a}{g}\right] = \rho gh - \rho ha \qquad ...(3.23)$$

The above equation shows that the pressure at any point in the liquid mass is less than the hydrostatic pressure by an amount of ρha. Fig. 3.50 shows the variation of pressure for the liquid mass subjected to a constant vertical downward acceleration.

If the tank containing liquid is moving downward with a constant acceleration equal to *g* (*i.e.,* when $a = g$), then equation reduces to $p - p_0 = 0$ or $p = p_0$. This means the pressure at any point in the liquid is equal to surrounding atmospheric pressure. There will be no force on the walls or on the base of the tank.

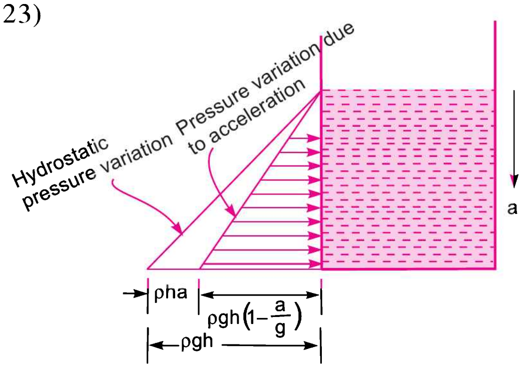


Fig. 3.50

Note. If a tank containing a liquid is subjected to a constant acceleration in the inclined direction, then the acceleration may be resolved along the horizontal direction and vertical direction. Then each of these cases may be separately analysed in accordance with the above procedure.

Problem 3.37 *A tank containing water upto a depth of 500 mm is moving vertically upward with a constant acceleration of 2.45 m/s². Find the force exerted by water on the side of the tank. Also calculate the force on the side of the tank when the width of tank is 2 m and*

(i) tank is moving vertically downward with a constant acceleration of 2.45 m/s², and

(ii) the tank is not moving at all.

Solution. Given :

Depth of water, $h = 500 \text{ mm} = 0.5 \text{ m}$

Vertical acceleration, $a = 2.45 \text{ m/s}^2$

Width of tank, $b = 2 \text{ m}$

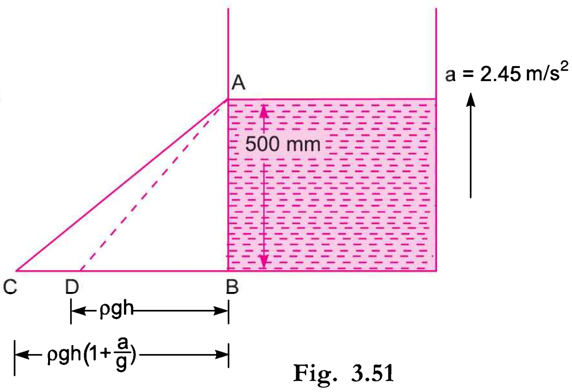


Fig. 3.51

To find the force exerted by water on the side of the tank when moving vertically upward, let us first find the pressure at the bottom of the tank.

The gauge pressure at the bottom (*i.e.,* at point *B*) for this case is given by equation as

$$p_B = \rho g h \left(1 + \frac{a}{g}\right)$$

$$= 1000 \times 9.81 \times 0.5 \left(1 + \frac{2.45}{9.81}\right) = 6131.25 \text{ N/m}^2$$

This pressure is represented by line *BC*.

Now the force on the side *AB* = Area of triangle *ABC* × Width of tank

$$= \left(\tfrac{1}{2} \times AB \times BC\right) \times b$$

$$= \left(\tfrac{1}{2} \times 0.5 \times 6131.25\right) \times 2 \qquad (\because BC = 6131.25 \text{ and } b = 2 \text{ m})$$

= 3065.6 N. Ans.

(*i*) **Force on the side of the tank, when tank is moving vertically downward.**

The pressure variation is shown in Fig. 3.52. For this case, the pressure at the bottom of the tank (*i.e.,* at point *B*) is given by equation (3.23) as

$$p_B = \rho g h \left(1 - \frac{a}{g}\right)$$

$$= 1000 \times 9.81 \times 0.5 \left(1 - \frac{2.45}{9.81}\right)$$

$$= 3678.75 \text{ N/m}^2$$

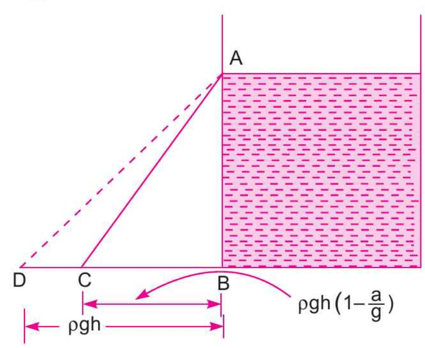


Fig. 3.52

This pressure is represented by line *BC*.

Now the force on the side *AB* = Area of triangle *ABC* × Width

$$= \left(\tfrac{1}{2} \times AB \times BC\right) \times b$$

$$= \left(\tfrac{1}{2} \times 0.5 \times 3678.75\right) \times 2 \qquad (\because BC = 3678.75,\ b = 2)$$

= 1839.37 N. Ans.

(*ii*) **Force on the side of the tank, when tank is stationary.**

The pressure at point *B* is given by,

$$p_B = \rho g h = 1000 \times 9.81 \times 0.5 = 4905 \text{ N/m}^2$$

This pressure is represented by line *BD* in Fig. 3.52

Force on the side *AB* = Area of triangle *ABD* × Width

$$= \left(\tfrac{1}{2} \times AB \times BD\right) \times b$$

$$= \left(\tfrac{1}{2} \times 0.5 \times 4905\right) \times 2 \qquad (\because BD = 4905)$$

= 2452.5 N. Ans.

For this case, the force on *AB* can also be obtained as

$$F_{AB} = \rho g A . \bar{h}$$

where $A = AB \times \text{Width} = 0.5 \times 2 = 1 \text{ m}^2$

$$\bar{h} = \frac{AB}{2} = \frac{0.5}{2} = 0.25 \text{ m} = 1000 \times 9.81 \times 1 \times 0.25$$

= 2452.5 N. Ans.

Problem 3.38 *A tank contains water upto a depth of 1.5 m. The length and width of the tank are 4 m and 2 m respectively. The tank is moving up an inclined plane with a constant acceleration of 4 m/s². The inclination of the plane with the horizontal is 30° as shown in Fig. 3.53. Find,*

(i) the angle made by the free surface of water with the horizontal.

(ii) the pressure at the bottom of the tank at the front and rear ends.

Solution. Given :

Depth of water, $h = 1.5$ m ; Length, $L = 4$ m and Width, $b = 2$ m

Constant acceleration along the inclined plane,

$$a = 4 \text{ m/s}^2$$

Inclination of plane, $\alpha = 30°$

Let θ = Angle made by the free surface of water after the acceleration is imparted to the tank

p_A = Pressure at the bottom of the tank at the front end and p_D = Pressure at the bottom of the tank at the rear end.

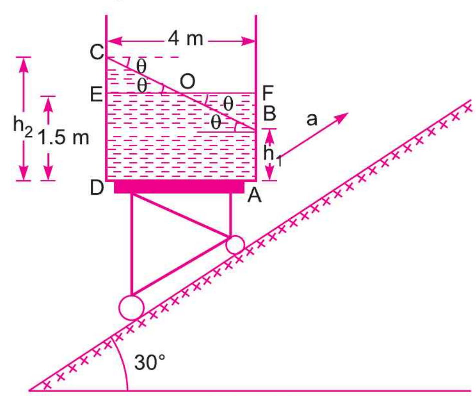


Fig. 3.53

This problem can be done by resolving the given acceleration along the horizontal direction and vertical direction. Then each of these cases may be separately analysed according to the set procedure.

Horizontal and vertical components of the acceleration are :

$$a_x = a \cos \alpha = 4 \cos 30° = 3.464 \text{ m/s}^2$$

$$a_y = a \sin \alpha = 4 \sin 30° = 2 \text{ m/s}^2$$

When the tank is stationary on the inclined plane, free surface of liquid will be along *EF* as shown in Fig. 3.53. But when the tank is moving upward along the inclined plane the free surface of liquid will be along *BC*. When the tank containing a liquid is moving up an inclined plane with a constant acceleration, the angle made by the free surface of the liquid with the horizontal is given by

$$\tan\theta = \frac{a_x}{a_y + g} = \frac{3.464}{2 + 9.81} = 0.2933$$

$\therefore$ $\theta = \tan^{-1} 0.2933 =$ **16.346° or 16° 20.8′. Ans.**

Now let us first find the depth of liquid at the front and rear end of the tank.

Depth of liquid at front end = $h_1 = AB$

Depth of liquid at rear end = $h_2 = CD$

From Fig. 3.53, in triangle COE, $\tan\theta = \dfrac{CE}{EO}$

or $CE = EO \tan\theta = 2 \times 0.2933$ $\quad$ ($\because$ $EO = 2$ m, $\tan\theta = 0.2933$)

$= 0.5866$ m

$\therefore$ $CD = h_2 = ED + CE = 1.5 + 0.5866 = 2.0866$ m

Similarly $h_1 = AB = AF - BF$

$= 1.5 - 0.5866$ $\quad$ ($\because$ $AF = 1.5$, $BF = CE = 0.5866$)

$= 0.9134$ m

The pressure at the bottom of tank at the rear end is given by,

$$p_D = \rho g h_2 \left(1 + \frac{a_y}{g}\right)$$

$$= 1000 \times 9.81 \times 2.0866 \left(1 + \frac{2}{9.81}\right) = \textbf{24642.7 N/m}^2\textbf{. Ans.}$$

The pressure at the bottom of tank at the front end is given by

$$p_A = \rho g h_1 \left(1 + \frac{a_y}{g}\right)$$

$$= 1000 \times 9.81 \times 0.9134 \left(1 + \frac{2}{9.81}\right) = \textbf{10787.2 N/m}^2\textbf{. Ans.}$$

HIGHLIGHTS

1. When the fluid is at rest, the shear stress is zero.
2. The force exerted by a static fluid on a vertical, horizontal or an inclined plane immersed surface,

$$F = \rho g A \bar{h}$$

where ρ = Density of the liquid,

A = Area of the immersed surface, and

$\bar{h}$ = Depth of the centre of gravity of the immersed surface from free surface of the liquid.

3. Centre of pressure is defined as the point of application of the resultant pressure.
4. The depth of centre of pressure of an immersed surface from free surface of the liquid,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} \quad \text{for vertically immersed surface.}$$

$$= \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h} \quad \text{for inclined immersed surface.}$$

5. The centre of pressure for a plane vertical surface lies at a depth of two-third the height of the immersed surface.

6. The total force on a curved surface is given by $F = \sqrt{F_x^2 + F_y^2}$

where F_x = Horizontal force on curved surface and is equal to total pressure force on the projected area of the curved surface on the vertical plane,

$= \rho g A \bar{h}$

and F_y = Vertical force on sub-merged curved surface and is equal to the weight of liquid actually or imaginary supported by the curved surface.

7. The inclination of the resultant force on curved surface with horizontal, $\tan \theta = \dfrac{F_y}{F_x}$.

8. The resultant force on a sluice gate, $F = F_1 - F_2$

where F_1 = Pressure force on the upstream side of the sluice gate and

F_2 = Pressure force on the downstream side of the sluice gate.

9. For a lock gate, the reaction between the two gates is equal to the reaction at the hinge, $R = P$.

Also the reaction between the two gates, $P = \dfrac{F}{2 \sin \theta}$

where F = Resultant water pressure on the lock gate $= F_1 - F_2$

and θ = Inclination of the gate with the normal to the side of the lock.

EXERCISE

(A) THEORETICAL PROBLEMS

1. What do you understand by 'Total Pressure' and 'Centre of Pressure' ?

2. Derive an expression for the force exerted on a sub-merged vertical plane surface by the static liquid and locate the position of centre of pressure.

3. Prove that the centre of pressure of a completely sub-merged plane surface is always below the centre of gravity of the sub-merged surface or at most coincide with the centre of gravity when the plane surface is horizontal.

4. Prove that the total pressure exerted by a static liquid on an inclined plane sub-merged surface is the same as the force exerted on a vertical plane surface as long as the depth of the centre of gravity of the surface is unaltered.

5. Derive an expression for the depth of centre of pressure from free surface of liquid of an inclined plane surface sub-merged in the liquid.

6. (*a*) How would you determine the horizontal and vertical components of the resultant pressure on a sub-merged curved surface ?

(*b*) Explain the procedure of finding hydrostatic forces on curved surfaces.

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7. Explain how you would find the resultant pressure on a curved surface immersed in a liquid.

8. Why the resultant pressure on a curved sub-merged surface is determined by first finding horizontal and vertical forces on the curved surface ? Why is the same method not adopted for a plane inclined surface sub-merged in a liquid ?

9. Describe briefly with sketches the various methods used for measuring pressure exerted by fluids.
10. Prove that the vertical component of the resultant pressure on a sub-merged curved surface is equal to the weight of the liquid supported by the curved surface.
11. What is the difference between sluice gate and lock gate ?
12. Prove that the reaction between the gates of a lock is equal to the reaction at the hinge.
13. Derive an expression for the reaction between the gates as $P = \dfrac{F}{2 \sin \theta}$

 where F = Resultant water pressure on lock gate, θ = inclination of the gate with normal to the side of the lock.
14. When will centre of pressure and centre of gravity of an immersed plane surface coincide ?
15. Find an expression for the force exerted and centre of pressure for a completely sub-merged inclined plane surface. Can the same method be applied for finding the resultant force on a curved surface immersed in the liquid ? If not, why ?
16. What do you understand by the hydrostatic equation ? With the help of this equation derive the expressions for the total thrust on a sub-merged plane area and the buoyant force acting on a sub-merged body.

(B) NUMERICAL PROBLEMS

1. Determine the total pressure and depth of centre of pressure on a plane rectangular surface of 1 m wide and 3 m deep when its upper edge is horizontal and (*a*) coincides with water surface (*b*) 2 m below the free water surface. [**Ans.** (*a*) 44145 N, 2.0 m, (*b*) 103005 N, 3.714 m]

2. Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that centre of plate is 2 m below the free surface of water. Find the position of centre of pressure also. [**Ans.** 34668.54 N, 2.07 m]

3. A rectangular sluice gate is situated on the vertical wall of a lock. The vertical side of the sluice is 6 m in length and depth of centroid of area is 8 m below the water surface. Prove that the depth of centre of pressure is given by 8.475 m.

4. A circular opening, 3 m diameter, in a vertical side of a tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter. Calculate : (*i*) the force on the disc, and (*ii*) the torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 6 m. [**Ans.** (*i*) 416.05 kN, (*ii*) 39005 Nm]

5. The pressure at the centre of a pipe of diameter 3 m is 29.43 N/cm^2. The pipe contains oil of sp. gr. 0.87 and is filled with a gate valve. Find the force exerted by the oil on the gate and position of centre of pressure. [**Ans.** 2.08 MN, .016 m below centre of pipe]

6. Determine the total pressure and centre of pressure on an isosceles triangular plate of base 5 m and altitude 5 m when the plate is immersed vertically in an oil of sp. gr. 0.8. The base of the plate is 1 m below the free surface of water. [**Ans.** 261927 N, 3.19 m]

7. The opening in a dam is 3 m wide and 2 m high. A vertical sluice gate is used to cover the opening. On the upstream of the gate, the liquid of sp. gr. 1.5, lies upto a height of 2.0 m above the top of the gate, whereas on the downstream side, the water is available upto a height of the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Assume that the gate is higher at the bottom. [**Ans.** 206010 N, 0.964 m above the hinge]

8. A caisson for closing the entrance to a dry dock is of trapezoidal form 16 m wide at the top and 12 m wide at the bottom and 8 m deep. Find the total pressure and centre of pressure on the caisson if the water on the outside is 1 m below the top level of the caisson and dock is empty.
[**Ans.** 3.164 MN, 4.56 m below water surface]

9. A sliding gate 2 m wide and 1.5 m high lies in a vertical plane and has a co-efficient of friction of 0.2 between itself and guides. If the gate weighs one tonne, find the vertical force required to raise the gate if its upper edge is at a depth of 4 m from free surface of water. [**Ans.** 37768.5 N]

10. A tank contains water upto a height of 1 m above the base. An immiscible liquid of sp. gr. 0.8 is filled on the top of water upto 1.5 m height. Calculate : (*i*) total pressure on one side of the tank, (*ii*) the position of centre of pressure for one side of the tank, which is 3 m wide. [**Ans.** 76518 N, 1.686 m from top]

11. A rectangular tank 4 m long, 1.5 m wide contains water upto a height of 2 m. Calculate the force due to water pressure on the base of the tank. Find also the depth of centre of pressure from free surface.
[**Ans.** 117720 N, 2 m from free surface]

12. A rectangular plane surface 1 m wide and 3 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge of the plate is 2 m below the free water surface. [**Ans.** 80932.5 N, 2.318 m]

13. A circular plate 3.0 m diameter is immersed in water in such a way that the plane of the plate makes an angle of 60° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge of the plate is 2 m below the free water surface.
[**Ans.** 228.69 kN, 3.427 m from free surface]

14. A rectangular gate 6 m × 2 m is hinged at its base and inclined at 60° to the horizontal as shown in Fig. 3.54. To keep the gate in a stable position, a counter weight of 29430 N is attached at the upper end of the gate. Find the depth of water at which the gate begins to fall. Neglect the weight of the gate and also friction at the hinge and pulley. [**Ans.** 3.43 m]

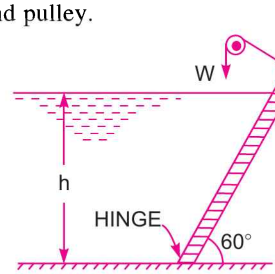

Fig. 3.54

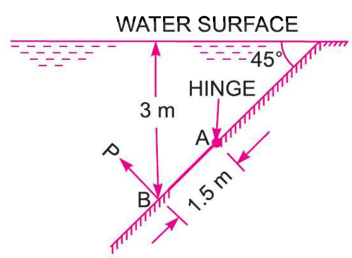

Fig. 3.55

15. An inclined rectangular gate of width 5 m and depth 1.5 m is installed to control the discharge of water as shown in Fig. 3.55. The end *A* is hinged. Determine the force normal to the gate applied at *B* to open it.
[**Ans.** 97435.8 N]

16. A gate supporting water is shown in Fig. 3.56. Find the height '*h*' of the water so that the gate begins to tip about the hinge. Take the width of the gate as unity. [**Ans.** $3 \times \sqrt{3}$ m]

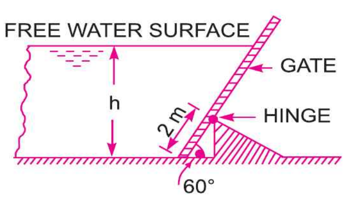

Fig. 3.56

17. Find the total pressure and depth of centre of pressure on a triangular plate of base 3 m and height 3 m which is immersed in water in such a way that plane of the plate makes an angle of 60° with the free surface. The base of the plate is parallel to water surface and at a depth of 2 m from water surface. [**Ans.** 126.52 kN, 2.996 m]

18. Find the horizontal and vertical components of the total force acting on a curved surface AB, which is in the form of a quadrant of a circle of radius 2 m as shown in Fig. 3.57. Take the width of the gate 2 m. [**Ans.** F_x = 117.72 kN, F_y = 140.114 kN]

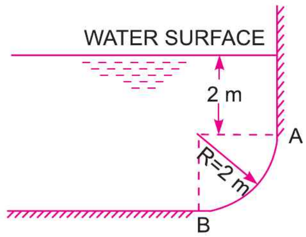

Fig. 3.57

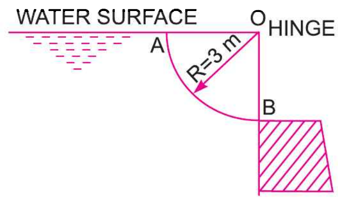

Fig. 3.58

19. Fig. 3.58 shows a gate having a quadrant shape of radius of 3 m. Find the resultant force due to water per metre length of the gate. Find also the angle at which the total force will act. [**Ans.** 82.201 kN, θ = 57° 31′]

20. A roller gate is shown in Fig. 3.59. It is cylindrical form of 6.0 m diameter. It is placed on the dam. Find the magnitude and direction of the resultant force due to water acting on the gate when the water is just going to spill. The length of the gate is given 10 m. [**Ans.** 2.245 MN, θ = 38° 8′]

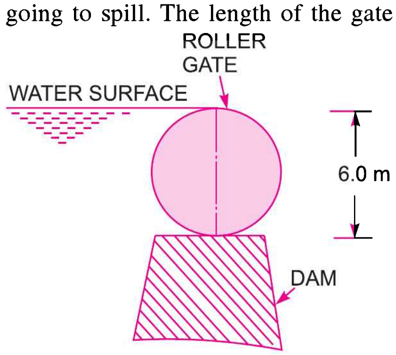

Fig. 3.59

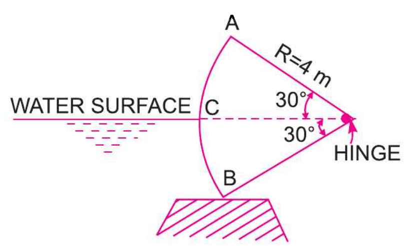

Fig. 3.60

21. Find the horizontal and vertical components of the water pressure exerted on a tainter gate of radius 4 m as shown in Fig. 3.60. Consider width of the gate unity. [**Ans.** F_x = 19.62 kN, F_y = 7102.44 N]

22. Find the magnitude and direction of the resultant water pressure acting on a curved face of a dam which is shaped according to the relation $y = \frac{x^2}{6}$ as shown in Fig. 3.61. The height of water retained by the dam is 12 m. Take the width of dam as unity. [**Ans.** 970.74 kN, θ = 43° 19′]

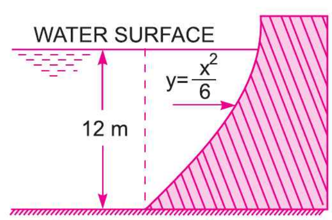

Fig. 3.61

23. Each gate of a lock is 5 m high and is supported by two hinges placed on the top and bottom of the gate. When the gates are closed, they make an angle of 120°. The width of the lock is 4 m. If the depths of water on the two sides of the gates are 4 m and 3 m respectively, determine : (*i*) the magnitude of resultant pressure on each gate, and (*ii*) magnitude of the hinge reactions. [**Ans.** (*i*) 79.279 kN, (*ii*) R_T = 27.924 kN, R_B = 51.355 kN]

24. The end gates *ABC* of a lock are 8 m high and when closed make an angle of 120°. The width of lock is 10 m. Each gate is supported by two hinges located at 1 m and 5 m above the bottom of the lock. The depth of water on the upstream and downstream sides of the lock are 6 m and 4 m respectively. Find :

(*i*) Resultant water force on each gate.

(*ii*) Reaction between the gates *AB* and *BC*, and

(*iii*) Force on each hinge, considering the reaction of the gate acting in the same horizontal plane as resultant water pressure. [**Ans.** 566.33 kN, (*ii*) 566.33 kN, and (*iii*) $R_T = 173.64$ kN, $R_B = 392.69$ kN]

25. A hollow circular plate of 2 m external and 1 m internal diameter is immersed vertically in water such that the centre of plate is 4 m deep from water surface. Find the total pressure and depth of centre of pressure. [**Ans.** 92.508 kN, 4.078 m]

26. A rectangular opening 2 m wide and 1 m deep in the vertical side of a tank is closed by a sluice gate of the same size. The gate can turn about the horizontal centroidal axis. Determine : (*i*) the total pressure on the sluice gate and (*ii*) the torque on the sluice gate. The head of water above the upper edge of the gate is 1.5 m. [**Ans.** (*i*) 39.24 kN, (*ii*) 1635 Nm]

27. Determine the total force and location of centre of pressure on one face of the plate shown in Fig. 3.62 immersed in a liquid of specific gravity 0.9. [**Ans.** 62.4 kN, 3.04 m]

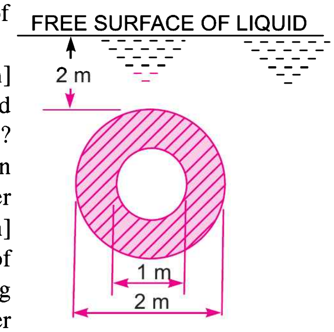

Fig. 3.62

28. A circular opening, 3 m diameter, in the vertical side of water tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter ? Calculate: (*i*) the force on the disc, and (*ii*) the torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 4 m. [**Ans.** (*i*) 270 kN, and (*ii*) 38 kN m]

29. A penstock made up by a pipe of 2 m diameter contains a circular disc of same diameter to act as a valve which controls the discharge passing through it. It can rotate about a horizontal diameter. If the head of water above its centre is 20 m, find the total force acting on the disc and the torque required to maintain it in the vertical position.

30. A circular drum 1.8 m diameter and 1.2 m height is submerged with its axis vertical and its upper end at a depth of 1.8 m below water level. Determine :

(*i*) total pressure on top, bottom and curved surfaces of the drum,

(*ii*) resultant pressure on the whole surface, and

(*iii*) depth of centre of pressure on curved surface.

31. A circular plate of diameter 3 m is immersed in water in such a way that its least and greatest depth from the free surface of water are 1 m and 3 m respectively. For the front side of the plate, find (*i*) total force exerted by water and (*ii*) the position of centre of pressure. [**Ans.** (*i*) 138684 N ; (*ii*) 2.125 m]

32. A tank contains water upto a height of 10 m. One of the sides of the tank is inclined. The angle between free surface of water and inclined side is 60°. The width of the tank is 5 m. Find : (*i*) the force exerted by water on inclined side and (*ii*) position of centre of pressure. [**Ans.** (*i*) 283.1901 kN, (*ii*) 6.67 m]

33. A circular plate of 3 m diameter is under water with its plane making an angle of 30° with the water surface. If the top edge of the plate is 1 m below the water surface, find the force on one side of the plate and its location. *(J.N.T.U., Hyderabad S 2002)*

[**Hint.** $d = 3$ m, $\theta = 30°$, height of top edge = 1 m, $\bar{h} = 1 + 1.5 \times \sin 30°$ $= 1.75$

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times \left(\frac{\pi}{4} \times 3^2\right) \times 1.75 = \mathbf{121.35\ kN.}$$

$$h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64}\left(3^4\right)^4 \times \frac{1}{4}}{\frac{\pi}{4}\left(3^2\right) \times 1.75} + 1.75 = 0.08 + 1.75 = \mathbf{1.83\ m.}]$$

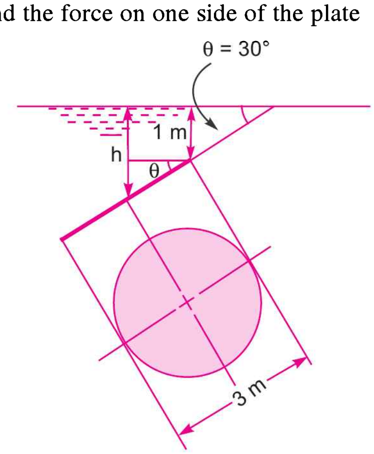

Fig. 3.63