2 Hydrostatics: water at rest

2.1 Introduction

Hydrostatics is the study of water which is not moving, that is, it is at rest. It is important to civil engineers for the design of water storage tanks and dams. What are the forces created by water and how strong must a tank or a dam be to resist them? It is also important to naval architects who design ships and submarines. How deep can a submarine go before the pressures become too great and damage it? The answers to these questions can be found from studying hydrostatics. The theory is quite simple both in concept and in use. It is also a well-established theory that was set down by Archimedes (287–212BC) over 2000 years ago and is still used in much the same way today.

2.2 Pressure

The term *pressure* is used to describe the force exerted by water on each square metre of some object submerged in water, that is, force per unit area. It may be the bottom of a tank, the side of a dam, a ship or a submerged submarine. It is calculated as follows:

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Introducing the units of measurement:

$$\text{pressure (kN/m}^2) = \frac{\text{force (kN)}}{\text{area (m}^2)}$$

Force is in kilo-Newtons (kN), area is in square metres (m^2) and so pressure is measured in kN/m^2. Sometimes pressure is measured in *Pascals* (Pa) in recognition of Blaise Pascal (1620–1662) who clarified much of modern-day thinking about pressure and barometers for measuring atmospheric pressure.

$$1\ \text{Pa} = 1\ \text{N/m}^2$$

One Pascal is a very small quantity and so kilo-Pascals are often used so that:

$$1 \text{ kPa} = 1 \text{ kN/m}^2$$

Although it is in order to use Pascals, kilo-Newtons per square metre is used throughout this text for the dimensions of pressure.

EXAMPLE: CALCULATING PRESSURE IN A TANK OF WATER

Calculate the pressure on a flat plate 3 m by 2 m when a mass of 50 kg rests on it. Calculate the pressure when the plate is reduced to 1.5 m by 2 m (Figure 2.1).

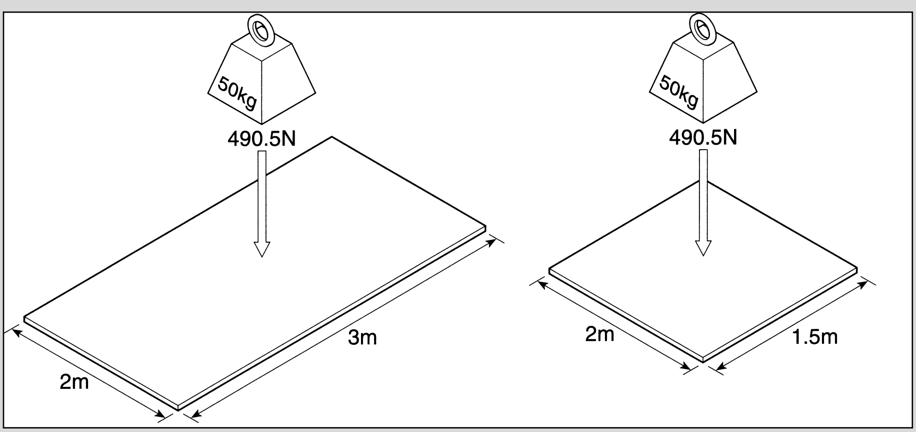
First calculate the weight on the plate. Remember weight is a force.

$$\begin{aligned} \text{mass on plate} &= 50 \text{ kg} \\ \text{weight on the plate} &= \text{mass} \times \text{gravity constant} \\ &= 50 \times 9.81 = 490.5 \text{ N} \\ \text{plate area} &= 3 \times 2 = 6 \text{ m}^2 \\ \text{pressure on plate} &= \frac{\text{force}}{\text{area}} = \frac{490.5}{6} \\ &= 81.75 \text{ N/m}^2 \end{aligned}$$

When the plate is reduced to 1.5 m by 2 m:

$$\begin{aligned} \text{plate area} &= 1.5 \times 2 = 3 \text{ m}^2 \\ \text{pressure on plate} &= \frac{490.5}{3} \\ &= 163.5 \text{ N/m}^2 \end{aligned}$$

Note that the mass and the weight remain the same in each case. But the areas of the plate are different and so the pressures are also different.



2.1 Different areas produce different pressures for the same force.

2.3 Force and pressure are different

Force and pressure are terms that are often confused. The difference between them is best illustrated by an example. If you had to choose between an elephant standing on your foot or a woman in a high-heel (stiletto) shoe, which would you choose? The sensible answer would be the elephant, as it is less likely to do damage to your foot than the high-heel shoe. To understand this is to appreciate the important difference between force and pressure.

The weight of the elephant is obviously greater than that of the woman but the pressure under the elephant's foot is much less than that under the high-heel shoe (see calculations in the box). The woman's weight (force) is small in comparison to that of the elephant, but the area of the shoe heel is very small and so the pressure is extremely high. So the high-heel shoe is likely to cause you more pain than the elephant. This is why high-heel shoes, particularly those with a very fine heel, are sometimes banned indoors as they can so easily punch holes in flooring and furniture!

There are many other examples which highlight the difference. Agricultural tractors often use wide (floatation) tyres to spread their load and reduce soil compaction. Military tanks use caterpillar tracks to spread the load to avoid getting bogged down in muddy conditions. Eskimos use shoes like tennis rackets to avoid sinking into the soft snow.

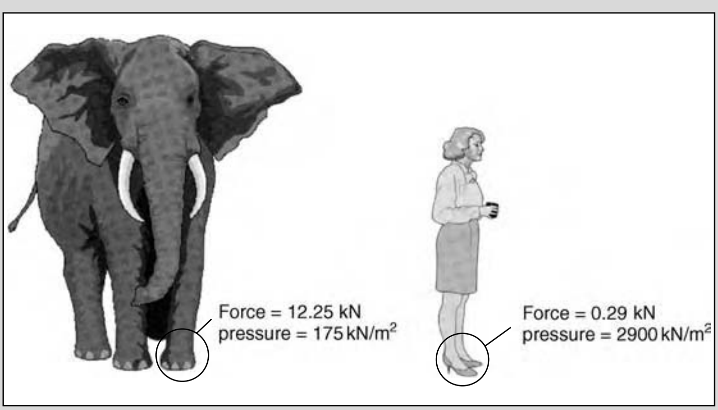
EXAMPLE: THE ELEPHANT'S FOOT AND THE WOMAN'S SHOE

An elephant has a mass of 5000 kg and its feet are 0.3 m in diameter. A woman has a mass of 60 kg and her shoe heel has a diameter of 0.01 m. Which produces the greater pressure – the elephant's foot or woman's shoe heel (Figure 2.2)?

First calculate the pressure under the elephant's foot:

$$\begin{aligned} \text{elephant's mass} &= 5000 \text{ kg} \\ \text{elephant's weight} &= 5000 \times 9.81 \\ &= 49\,050 \text{ N} = 49 \text{ kN} \end{aligned}$$

$$\text{weight on each foot} = \frac{49}{4} = 12.25 \text{ kN}$$



2.2 Which produces the greater pressure?

$$\text{foot area} = \frac{\pi d^2}{4} = \frac{\pi 0.3^2}{4}$$
$$= 0.07 \text{ m}^2$$
$$\text{pressure under foot} = \frac{\text{force}}{\text{area}} = \frac{12.25}{0.07}$$
$$= 175 \text{ kN/m}^2$$

Now calculate the pressure under the woman's shoe heel:

$$\text{woman's mass} = 60 \text{ kg}$$
$$\text{woman's weight} = 589 \text{ N} = 0.59 \text{ kN}$$
$$\text{weight on each foot} = \frac{0.59}{2} = 0.29 \text{ kN}$$
$$\text{area of shoe heel} = \frac{\pi d^2}{4} = \frac{\pi 0.01^2}{4}$$
$$= 0.0001 \text{ m}^2$$
$$\text{pressure under heel} = \frac{\text{force}}{\text{area}} = \frac{0.29}{0.0001}$$
$$= 2900 \text{ kN/m}^2$$

The pressure under the woman's heel is 16 times greater than under the elephant's foot. So which would you rather have standing on your foot?

2.4 Pressure and depth

The pressure on some object under water is determined by the depth of water above it. So the deeper the object is below the surface, the higher will be the pressure. The pressure can be calculated using the pressure-head equation:

$$p = \rho g h$$

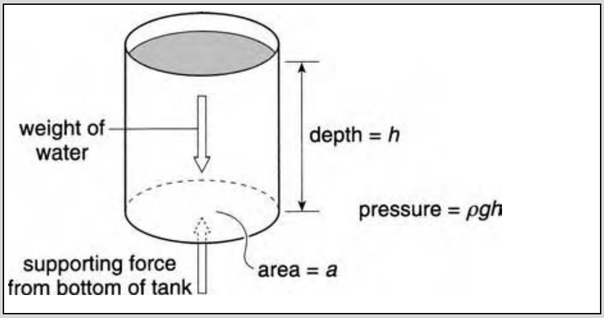
where p is pressure (kN/m^2); ρ is mass density of water (kN/m^3); g is gravity constant (m/s^2); h is depth of water (m).

This equation works for all fluids and not just water, provided of course that the correct value of density is used for the fluid concerned.

To see how the pressure-head equation is derived look in the box below.

DERIVATION: PRESSURE-HEAD EQUATION

Imagine a tank of water of depth h and a cross-sectional area of a. The weight of water on the bottom of the tank (remember that weight is a force and is acting downwards) is balanced by an upward force from the bottom of the tank supporting the water (Newton's third law). The pressure-head equation is derived by calculating these two forces and putting them equal to each other (Figure 2.3).



2.3 Calculating the forces on the bottom of a tank.

First calculate the downward force. This is the weight of water. To do this first calculate the volume and then the weight using the density:

$$\text{volume of water} = \text{cross-sectional area} \times \text{depth}$$
$$= a \times h$$

And so:

$$\text{weight of water in tank} = \text{volume} \times \text{density} \times \text{gravity constant}$$
$$= a \times h \times \rho \times g$$

This is the downward force of the water ↓. Next calculate the supporting (upward) force from the base:

$$\text{supporting force} = \text{pressure} \times \text{area}$$
$$= p \times a$$

Now put these two forces equal to each other:

$$p \times a = a \times h \times \rho \times g$$

The area *a* cancels out from both sides of the equation and so:

$$p = \rho g h$$
$$\text{pressure} = \text{mass density} \times \text{gravity constant} \times \text{depth of water}$$

This is the pressure-head equation and it links pressure with the depth of water. It shows that pressure increases directly as the depth increases. Note that it is completely independent of the shape of the tank or the area of base.

EXAMPLE: CALCULATING PRESSURE AND FORCE ON THE BASE OF A WATER TANK

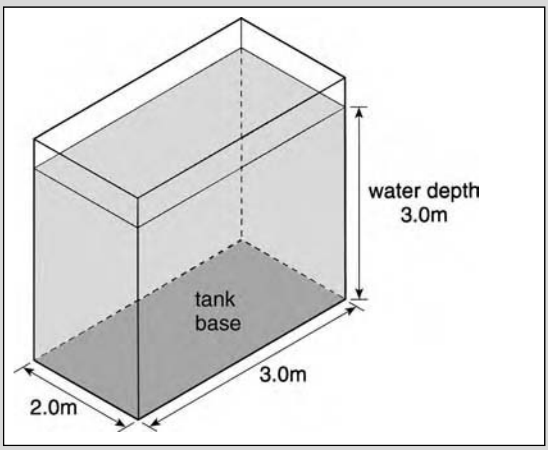
A rectangular tank of water is 3 m deep. If the base measures 3 m by 2 m, calculate the pressure and force on the base of the tank (Figure 2.4).

Use the pressure-head equation:

$$\begin{aligned} p &= \rho g h \\ &= 1000 \times 9.81 \times 3.0 \\ &= 29\,430 \text{ N/m}^2 \\ &= 29.43 \text{ kN/m}^2 \end{aligned}$$

Calculate the force on the tank base using the pressure and the area:

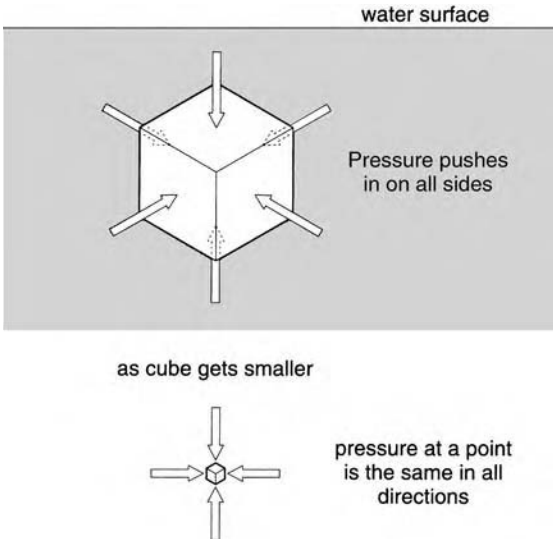
$$\begin{aligned} \text{force} &= \text{pressure} \times \text{area} \\ \text{base area} &= 3 \times 2 = 6 \text{ m}^2 \\ \text{force} &= 29.43 \times 6 \\ &= 176.6 \text{ kN} \end{aligned}$$



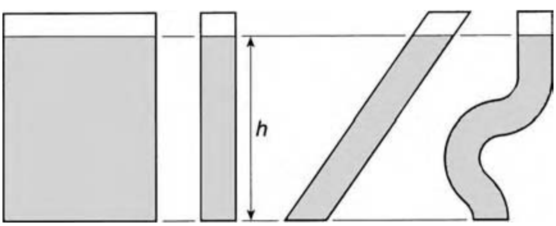
2.4 Calculating force and pressure on tank base.

2.5 Pressure is same in all directions

Although in the box example the pressure is used to calculate the downward force on the tank base, pressure does not in fact have a specific direction – it pushes in all directions. To understand this, imagine a cube immersed in water (Figure 2.5). The water pressure pushes on all sides of the cube and not just on the top. If the cube was very small then the pressure on all six faces would be almost the same. If the cube gets smaller and smaller until it almost disappears, it becomes clear that *the pressure at any point in the water is the same in all directions*. So the pressure pushes in all directions and not just vertically. This idea is important for designing dams because it is the horizontal action of pressure which pushes on a dam and which must



2.5 Pressure is the same in all directions.



2.6 Pressure is the same at the base of all the containers.

be resisted if the dam is not to fail. Note also that the 'pressures' in Figure 2.5 are drawn pushing inwards. But they could equally have been drawn pushing outwards to make the same argument – remember Newton's third law.

2.6 The hydrostatic paradox

It is often assumed that the size of a water tank or its shape influences pressure but this is not the case (Figure 2.6). It does not matter if the water is in a large tank or in a narrow tube. The pressure-head equation tells us that water depth is the only variable that determines the pressure. So the base area has no effect on the pressure nor does the amount of water in the tank. What is different of course is the force on the base of different containers. The *force* on the base of each tank is equal to weight of water in each of the containers. But if the depth of water in each is the same then the pressure will also be the same.

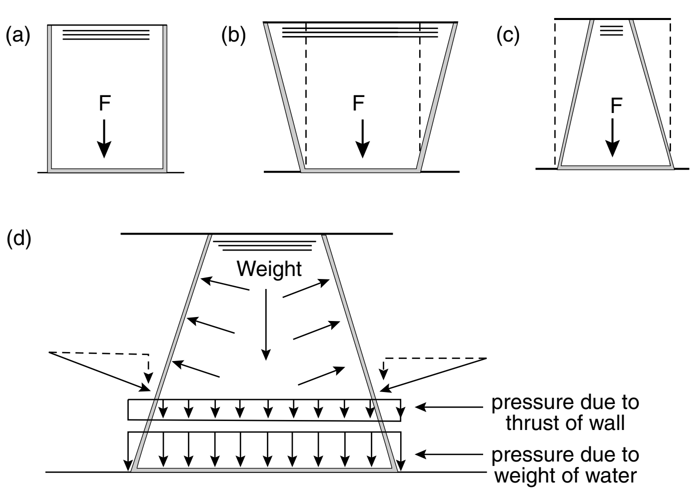
2.6.1 The bucket problem

The Dutch mathematician Simon Stevin (1548–1620) made a similar point by showing that the *force* on the base of a tank depended only on the area of the base and the vertical depth of water – and not the weight of water it contained. This is well demonstrated using three different-shaped buckets each with the same base area and the same depth of water in them (Figure 2.7).

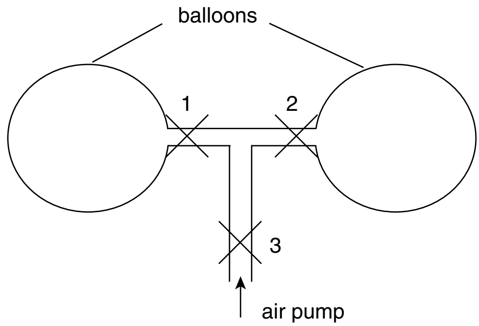
The weight of water in each bucket is clearly different and a casual observer might assume from this that the force on the base of each bucket will also be different: the force on the base of bucket *b* being greater than bucket *a* and the force on bucket *c* being less than bucket *a*. But thinking about this in hydraulic terms, the pressure-head equation tells us otherwise. In fact it predicts that the force on the base is the *same* in each case. It is equal to the area of the base multiplied by the pressure on the base, which is only a function of the water depth. So the force on the base is the same regardless of whether the sides are vertical or inclined inwards or outwards. In bucket *a*, with vertical sides, the force does in fact equal the weight of water in the container. But when the sides slope outwards, as in bucket *b*, the force is less than the weight of water and when the sides slope inwards, as in bucket *c*, the force on the base is greater than the weight of water. All this seems rather absurd but it is true.

The key to the paradox lies with the fact that the pressure at any point in the water is the same in all directions. The water not only pushes down onto the base but also pushes on the sides of the container as well. So when the sides slope inwards (bucket *c*) the water pushes outwards and also upwards. Newton's second law says that this produces a corresponding downward force on the water and this is transmitted to the base adding to the force due to the weight of the water (Figure 2.7d). In fact, the total vertical force on the walls and base (the force on the base less the upward force on the walls) is exactly equal to the weight of water in the bucket! The same argument can be applied to bucket *b*. The water pushes on the sides of the tank and in this case push outwards and downwards. Newton's second law says that this produces a corresponding upward force on the water and this is transmitted to the base reducing the force due to the weight of the water. So in this case the force on the base is less than the weight of water in the bucket.

Clear? If so then you are well on your way not only to understanding the important difference between force and pressure but also appreciating the significance of Newton's contribution to our understanding of the way in which our world works.



2.7 The bucket problem.



2.8 The balloon problem.

2.6.2 The balloon problem

One more 'absurdity' to test your understanding. Two identical balloons are connected to a manifold and blown up independently so that one is larger than the other (Figure 2.8). When valve 3 is closed and valves 1 and 2 are opened the air can flow between the balloons to equalise the air pressure. The question is – What happens to the balloons?

The normal expectation is that air moves from the larger balloon to the smaller one so they both become the same size, but this is not what happens. The larger balloon in fact gets larger and the smaller balloon gets smaller. The reason for this is again explained by the difference between pressure and force. The larger balloon has a much greater surface area than the small one and the force on the skin of the balloon will be greater as it approaches bursting point. But because of the large surface area the pressure inside is much smaller than it is in the smaller balloon. So when the two balloons are connected the higher air pressure in the small balloon flows into the larger balloon thus making the large balloon even larger and the small balloon smaller. So do not confuse size with pressure. If you are not convinced or you are still confused, try the balloon experiment by making up a small manifold using some plastic pipes and laboratory taps and see for yourself.

2.7 Pressure head

Engineers often refer to pressure in terms of metres of water rather than as a pressure in kN/m^2. So, referring to the pressure calculation in the box, instead of saying the pressure is 29.43 kN/m^2 they will say the pressure is 3 m head of water. They can do this because of the unique relationship between pressure and water depth ($p = \rho g h$). It is called the *pressure head* or just *head* and is measured in metres. It is the water depth h referred to in the pressure-head equation. Both ways of stating the pressure are correct and one can easily be converted to the other using the pressure-head equation.

Engineers prefer to use head measurements because, as will be seen later, differences in ground level can affect the pressure in a pipeline. It is then an easy matter to add (or subtract) changes in ground level to pressure values because they both have the same dimensions.

A word of warning though. When head is measured in metres it is important to say what the liquid is – 3 m head of water will be a very different pressure from 3 m head of mercury. This is because the density term ρ is different. So the rule is – say what liquid is being measured, for example, 3 metres head of water or 3 metres head of mercury etc. See the worked example in the box.

EXAMPLE: CALCULATING PRESSURE HEAD IN MERCURY

Building on the previous example, calculate the depth of mercury needed in the tank to produce the same pressure as 3 m depth of water (29.43 kN/m^2). Specific gravity (SG) of mercury is 13.6.

First calculate the density of mercury:

$$\rho \text{ (mercury)} = \rho \text{ (water)} \times \text{SG (mercury)}$$
$$= 1000 \times 13.6$$
$$= 13\,600 \text{ kg/m}^3$$

Use the pressure-head equation to calculate the head of mercury:

$$p = \rho g h$$

Where ρ is now the density and h is the depth of mercury:

$$29\,430 = 13\,600 \times 9.81 \times h$$
$$h = 0.22 \text{ m of mercury}$$

So the depth of mercury required to create the same pressure as 3 m of water is only 0.22 m. This is because mercury is much denser than water.

2.8 Atmospheric pressure

The pressure of the atmosphere is all around us pressing on our bodies. Although we often talk about things being 'as light as air' when there is a large depth of air, as on the earth's surface, it creates a very high pressure of approximately 100 kN/m^2. The average person has a skin area of 2 m^2 so the force acting on each of us from the air around us is approximately 200 kN (the equivalent of 200 000 apples or approximately 20 tons). A very large force indeed! Fortunately there is an equal and opposite pressure from within our bodies that balances the air pressure and so we feel no effect (Newton's third law).

At high altitudes where atmospheric pressure is less than at the earth's surface, some people suffer from nose bleeds due to their blood pressure being much higher than that of the surrounding atmosphere. We also notice slight, sudden changes in air pressure. For instance, when we fly in an aeroplane, even though the cabin is pressurised, our ears pop as our bodies adjust to changes in the cabin pressure. But if for some reason the cabin pressure system failed suddenly removing one side of this pressure balance then the result could be catastrophic. Inert gases such as nitrogen, which are normally dissolved in our body fluids and tissues, would rapidly start to form gas bubbles which can result in sensory failure, paralysis and death. Deep sea divers are well aware of this rapid pressure change problem and so make sure that they return to the surface slowly so that their bodies have enough time to adjust to the changing pressure. It is known as 'the bends'. A good practical demonstration of what happens can be seen when you open a fizzy drink bottle. When the cap is removed from the bottle, gas is heard escaping, and bubbles can be seen forming in the drink. This is carbon dioxide gas coming out of solution as a result of the sudden pressure drop inside the bottle as it equalises with the pressure of the atmosphere.

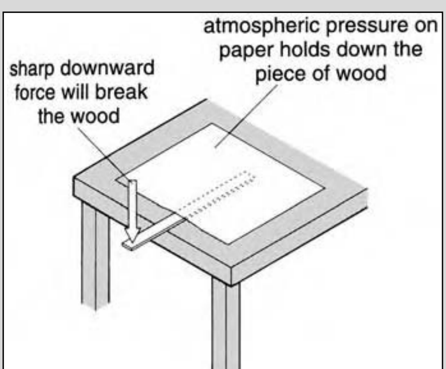
It was in the 17th century that scientists such as Evangelista Torricelli (1608–1647), a pupil of Galileo Galilei (1564–1642), began to understand about atmospheric pressure and to study the importance of vacuums – the empty space when all the air is removed. Scientists previously explained atmospheric effects by saying *that nature abhors a vacuum*. By this they meant that if the air is sucked out of a bottle it will immediately fill by sucking air back in again when it is opened to the atmosphere. But Galileo commented that a suction pump could not lift water more than 10 m so there appeared to be a limit to this abhorrence. Today we know that it is not the vacuum in the bottle that sucks in the air but the outside air pressure that pushes the air in. The end result is the same (i.e. the bottle is filled with air), but the mechanism is quite different.

Galileo realised that this had important consequences for suction pumps. Suction pumps do not 'suck' up water as was commonly thought. It is atmospheric pressure on the surface of the water that pushes water into the pump and to do this the air must first be removed from the pump to create a vacuum – a process known as 'priming'. The implication of this is that atmospheric pressure (10 m of water) puts an absolute limit on how high a pump can be located above the water surface and still work. In practice the limit is a lot lower than this but more about this in Section 8.4. Siphons too rely on atmospheric pressure in a similar way (Section 7.11).

Atmospheric pressure does vary over the surface of the earth. It is lower in mountainous regions and also varies as a result of the earth's rotation and temperature changes in the atmosphere which both cause large air movements. They create high and low pressure areas that create winds as air flows from high pressure to low pressure areas in an attempt to try and equalise the air pressure. This may be important in meteorology but in hydraulics such differences are relatively small and have little effect on solving problems – except of course if you happen to be building a pumping station for a community in the Andes or the Alps. So for all intents and purposes atmospheric pressure close to sea level can be assumed constant at 100 kN/m^2 – or approximately 10 m head of water.

EXAMPLE: EXPERIENCING ATMOSPHERIC PRESSURE

One way of experiencing atmospheric pressure is to place a large sheet of paper on a table over a thin piece of wood. If you hit the wood sharply it is possible to strike a considerable blow without disturbing the paper. You may even break the wood. This is



2.9 Experiencing atmospheric pressure.

because the paper is being held down by the pressure of the atmosphere.

If the paper is 1.0 m^2 then the force holding down the paper can be calculated as follows:

$$\text{force} = \text{pressure} \times \text{area}$$

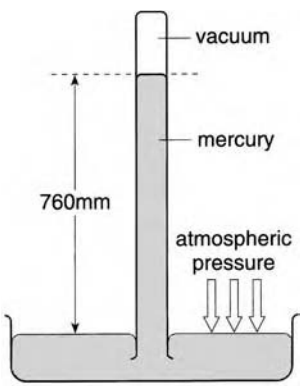
In this case:

$$\begin{aligned}\text{pressure} &= \text{atmospheric pressure}\\ &= 100\ \text{kN/m}^2\end{aligned}$$

And so:

$$\text{force} = 100 \times 1 = 100\ \text{kN}$$

In terms of apples this is about 100 000, which is a large force. It is little wonder that the wood breaks before the paper lifts.



2.10 Measuring atmospheric pressure.

2.8.1 Mercury barometer

One of the instruments used to measure atmospheric pressure is the mercury barometer. It was developed by Evangelista Torricelli in 1643, and has largely remained unchanged since except for the introduction of a vernier measuring scale to measure accurately the small changes in atmospheric pressure. This was done by Fortin in 1810 and so the instrument is now referred to as the *Fortin barometer*.

Torricelli's barometer consists of a vertical glass tube closed at one end, filled with mercury and inverted with the open end immersed in a cistern of mercury (Figure 2.10). The cistern surface is exposed to atmospheric pressure and this supports the mercury column, the height of which is a measure of atmospheric pressure. It is normally measured in mm and the long-term average value at sea level is 760 mm.

Torricelli could have used water for the barometer instead of mercury, but he would have needed a tube over 10 m high to do it – not a very practical proposition for the laboratory or for taking measurements.

EXAMPLE: MEASURING ATMOSPHERIC PRESSURE USING A MERCURY BAROMETER

Calculate atmospheric pressure when the reading on a mercury barometer is 760 mm of mercury. What would be the height of the column if the same air pressure was measured using water instead of mercury?

The pressure-head equation links together atmospheric pressure and the height of the mercury column, but remember the fluid is now mercury and not water:

$$\text{atmospheric pressure} = \rho g h$$

h is 760 mm and ρ for mercury is 13 600 kg/m^3 (13.6 times denser than water)

So:

$$\begin{aligned}\text{atmospheric pressure} &= 13\,600 \times 9.81 \times 0.76\\ &= 101\,400\ \text{N/m}^2 \text{ or } 101.4\ \text{kN/m}^2\end{aligned}$$

Calculate the height of the water column to measure atmospheric pressure using the pressure-head equation again:

$$\text{atmospheric pressure} = \rho g h$$

This time the fluid is water and so:

$$\begin{aligned}101\,400 &= 1000 \times 9.81 \times h\\ h &= 10.32\ \text{m}\end{aligned}$$

This is a very tall water column and there would be practical difficulties if it was used for routine measurement of atmospheric pressure. Hence the reason why a very dense liquid like mercury is used to make measurement more manageable.

Atmospheric pressure is also used as a unit of measurement for pressure both for meteorological purposes and in hydraulics. This unit is known as the *bar*. For convenience I bar pressure is rounded off to 100 kN/m^2.

A more commonly used term in meteorology is the *millibar*.

So:

$$1\ \text{millibar} = 0.1\ \text{kN/m}^2 = 100\ \text{N/m}^2$$

To summarise – there are several ways of expressing atmospheric pressure:

$$\begin{aligned}\text{atmospheric pressure} &= 1\ \text{bar}\\ \text{or} &= 100\ \text{kN/m}^2\\ \text{or} &= 10\ \text{m of water}\\ \text{or} &= 760\ \text{mm of mercury}\end{aligned}$$

EXAMPLE: CALCULATING PRESSURE HEAD

A pipeline is operating at a pressure of 3.5 bar. Calculate the pressure in metres head of water.

$$1 \text{ bar} = 100 \text{ kN/m}^2 = 100\,000 \text{ N/m}^2$$

And so:

$$3.5 \text{ bar} = 350 \text{ kN/m}^2 = 350\,000 \text{ N/m}^2$$

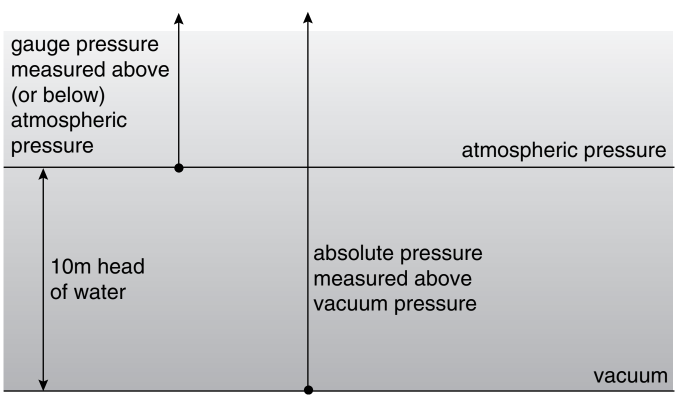
Use the pressure-head equation:

$$p = \rho g h$$
$$350\,000 = 1000 \times 9.81 \times h$$

Calculate head h:

$$h = 35.67 \text{ m}$$

Round this off: 3.5 bar = 36 m of water (approximately)



2.11 Gauge and absolute pressures.

2.9 Measuring pressure

2.9.1 Gauge and absolute pressures

Pressure measuring devices work in the atmosphere with normal atmospheric pressure all around them. Rather than add atmospheric pressure each time a measurement is made it is common practice to assume that atmospheric pressure is equal to zero and so it becomes the base line (or zero point) from which all pressure measurements are made. It is rather like setting sea level as the zero from which all ground elevations are measured (Figure 2.11). Pressures measured in this way are called *gauge pressures*. They can either be positive (above atmospheric pressure) or negative (below atmospheric pressure).

Most pressure measurements in hydraulics are gauge pressures but some mechanical engineers, working with gas systems occasionally measure pressure using a vacuum as the datum. In such cases the pressures are referred to as *absolute pressures*. It is not possible to have a pressure lower than vacuum pressure and so all absolute pressures have positive values.

To summarise:
Gauge pressures are pressures measured above or below atmospheric pressure. Absolute pressures are pressures measured above a vacuum.
To change from one to the other:

absolute pressure = gauge pressure + atmospheric pressure

Note, if only the word pressure is used, it is reasonable to assume that this means gauge pressure.

2.9.2 Bourdon gauges

Pressure can be measured in several ways. The most common instrument used is the *Bourdon Gauge* (Figure 2.12a). This is located at some convenient point on a pipeline or pump to record pressure, usually in kN/m^2 or bar. It is a simple device and works on the same principle as a party toy. When you blow into it, the coil of paper unfolds and the feather rotates. Inside a Bourdon gauge there is a similar curved tube which tries to straighten out under pressure and causes a pointer to move through a gearing system across a scale of pressure values.

2.9.3 Piezometers

This is another device for measuring pressure. A vertical tube is connected to a pipe so that water can rise up the tube because of the pressure in the pipe (Figure 2.12a). This is called a *piezometer* or *standpipe.* The height of the water column in the tube is a measure of the pressure in the pipe, that is, the pressure head. The pressure in kN/m^2 can be calculated using the pressure-head equation.

EXAMPLE: MEASURING PRESSURE USING A STANDPIPE

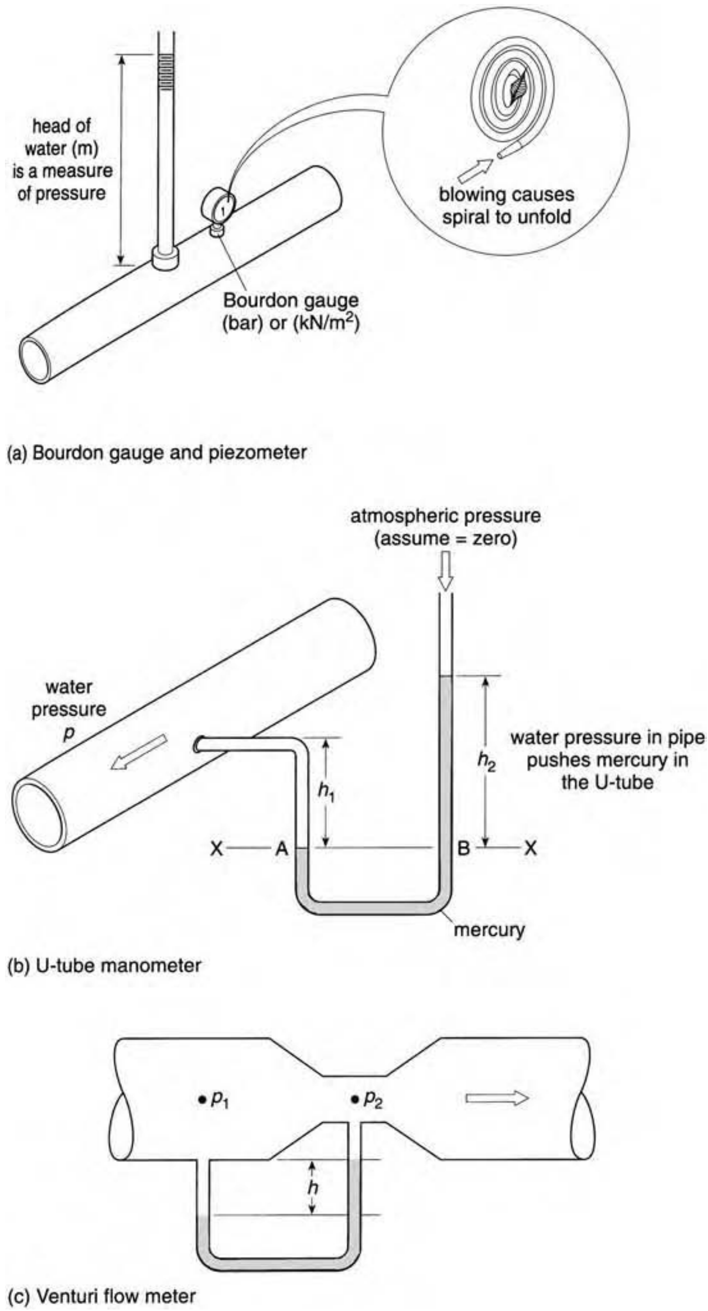
Calculate the height of a standpipe needed to measure a pressure of 200 kN/m^2 in a water pipe.
Using the pressure-head equation:

$$p = \rho g h$$
$$200\,000 = 1000 \times 9.81 \times h$$

Note in the equation pressure and density are both in N – not kN.

$$h = 20.4 \text{ m}$$

A very high tube would be needed to measure this pressure and it would be a rather impracticable measuring device! For this reason high pressures are normally measured using a Bourdon gauge or a manometer.



2.12 Measuring pressure.

2.9.4 Manometers

Vertical standpipes are not very practical for measuring high pressures (see example in box). An alternative is to use a *U-tube manometer* (Figure 2.12b).

The bottom of the U-tube is filled with a different liquid which does not mix with that in the pipe. When measuring pressures in a water system, oil or mercury is used. Mercury is very useful because high pressures can be measured with a relatively small tube (see atmospheric pressure).

To measure pressure, a manometer is connected to a pipeline and mercury is placed in the bottom of the U-bend. The basic assumption is that as the mercury in the manometer is not moving the pressures in the two limbs must be the same. If a horizontal line X–X is drawn through the mercury surface in the first limb and extended to the second limb then it can be assumed that:

pressure at point A = pressure at point B

This is the fundamental assumption on which all manometer calculations are based. It is then a matter of adding up all the components which make up the pressures at A and B to work out a value for the pressure in the pipe.

First calculate the pressure at A:

$$\begin{aligned}\text{pressure at A} &= \text{water pressure at centre of pipe } (p) \\ &\quad + \text{pressure due to water column } h_1 \\ &= p + \rho_{(water)} g\, h_1 \\ &= p + (1000 \times 9.81 \times h_1) \\ &= p + 9810 \times h_1\end{aligned}$$

Now calculate the pressure at B:

$$\begin{aligned}\text{pressure at B} &= \text{pressure due to mercury column } h_2 \\ &\quad + \text{atmospheric pressure}\end{aligned}$$

Normally atmospheric pressure is assumed to be zero. So:

$$\begin{aligned}\text{pressure at B} &= \rho_{(mercury)}\, g\, h_2 + 0 \\ &= 1000 \times 13.6 \times 9.81 \times h_2 \\ &= 133\,430\, h_2\end{aligned}$$

Putting the pressure at A equal to the pressure at B:

$$p + 9810\, h_1 = 133\,430\, h_2$$

Rearrange this to determine the pressure in the pipe p:

$$p = 133\,430\, h_2 - 9810\, h_1$$

Note that p is in N/m^2.

So the pressure in this pipeline can be calculated by measuring h_1 and h_2 and using the above equation.

Some manometers are used to measure pressure differences rather than actual values of pressure. One example of this is the measurement of the pressure difference in a venturi meter used for measuring water flow in pipes (Figure 2.12c). In this case it is the drop (difference) in pressure as water passes through a narrow section of pipe that is important. By connecting one limb of the manometer to the main pipe and the other limb to the narrow section, the difference in pressure can be determined. Note that the pressure difference is not just the difference in the mercury readings on the two columns as is often thought. The pressure difference must be calculated using the principle described above for the simple manometer. More about venture meters and using manometers in Section 4.10.

The best way to deal with manometer measurements is to remember the principle on which all manometer calculations are based and not the formula for p. There are many different ways of arranging manometers with different fluids in them and so there will be too many formulae to remember. So just remember and apply the principle – pressure on each side of the manometer is the same across a horizontal line AB – then the pressure can be easily determined. See the worked example in the box.

EXAMPLE: MEASURING PRESSURE USING A MANOMETER

A mercury manometer is used to measure the pressure in a water pipe (Figure 2.12c). Calculate the pressure in the pipe when $h_1 = 1.5$ m and $h_2 = 0.8$ m.

To solve this problem start with the principle on which all manometers are based:

pressure at A = pressure at B

Calculate the pressures at A and B:

$$\begin{aligned}\text{pressure at A} &= \text{water pressure in pipe } (p) \\ &\quad + \text{pressure due to water column } h_1 \\ &= p + \rho_{(\text{water})} g h_1 \\ &= p + 1000 \times 9.81 \times 1.5 \\ \text{pressure at B} &= \text{pressure due to mercury column } h_2 \\ &\quad + \text{atmospheric pressure} \\ &= \rho_{(\text{mercury})} g h_2 + 0 \\ &= 1000 \times 13.6 \times 9.81 \times 0.8\end{aligned}$$

Note that as all the pressures are gauge pressures, atmospheric pressure is assumed to be zero.

Putting the pressure at A equal to the pressure at B:

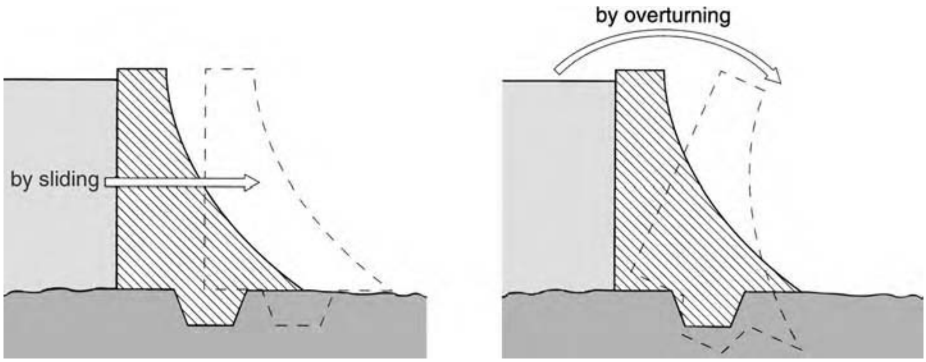
$$p + 1000 \times 9.81 \times 1.5 = 1000 \times 13.6 \times 9.81 \times 0.8$$

Rearrange this to determine p:

$$\begin{aligned}p &= (1000 \times 13.6 \times 9.81 \times 0.8) - (1000 \times 9.81 \times 1.5) \\ &= 106\,732 - 14\,715 \\ &= 92\,017 \text{ N/m}^2 \\ p &= 92 \text{ kN/m}^2\end{aligned}$$

2.10 Designing dams

Engineers are always interested in the ways in which things fall down or collapse so they can devise design and construction procedures that produce safe reliable structures. Dams in



2.13 Dams can fail by sliding and overturning.

particular are critical structures because failure can cause a great deal of damage and loss of life. Hydraulically a dam structure can fail in two ways – the pressure of water can cause the dam to slide forward and it can also cause it to overturn (Figure 2.13). The engineer must design a structure that is strong enough to resist both these possible modes of failure. This is where the principles of hydrostatics play a key role – the same principles apply to small dams only a few metres high as they do for major dams 40 m or more in height.

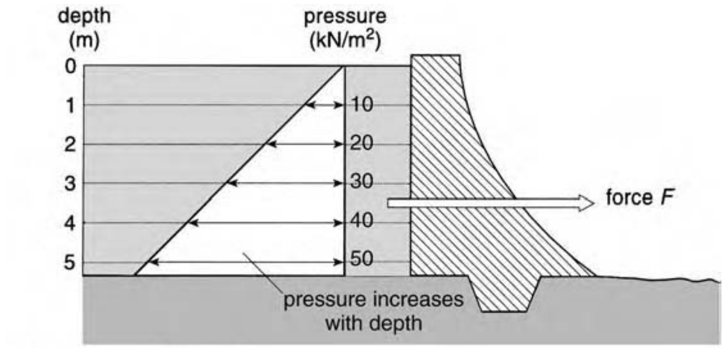
The pressure of water stored behind a dam produces a horizontal force which could cause it to slide forward if the dam was not strong enough to resist. So the total force resulting from the water pressure must first be calculated. The location of this force is also important. If it is near the top of the dam then it may cause the dam to overturn. If it is near the base then it may fail by sliding.

The force on a dam is calculated from the water pressure (Figure 2.14a). Remember that pressure pushes in all directions; in this case it is the horizontal push on the dam which is important. At the water surface the pressure is zero, but 1.0 m below the surface the pressure rises to 10 kN/m^2 (approximately), at 2.0 m it reaches 20 kN/m^2 and so on (remember the pressure-head equation $p = \rho gh$). A graph of the changes in pressure with depth is a straight line; together with the axes of the graph it forms a triangle. The pressure at the top of the triangle (the water surface) is zero and increases uniformly with depth. This triangle is called the pressure diagram and shows how the pressure varies with depth on the upstream face of a dam.

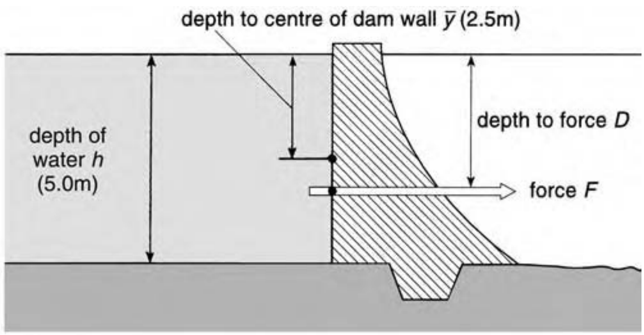
The force on the dam can be calculated from the pressure and the area of the dam face using the equation $F = pa$. But the pressure is not constant – it varies down the face of the dam and so the question is: which value of pressure should be used? One approach is to divide the dam face into lots of small areas and use the average pressure for each area. The force on each area is then calculated using the equation $F = pa$. But this results in lots of small forces to deal with. A much simpler method is to use a formula derived from combining all the small forces mathematically into a single larger force (F) (Figure 2.14a). This single force has the same effect as the sum of all the smaller forces and is much easier to deal with.

force (F) = $\rho ga\bar{y}$

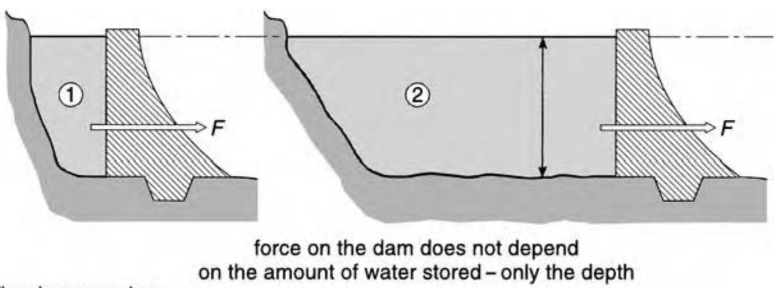
where ρ is density of water (kg/m^3)
g is gravity constant (9.81 m/s^2)
a is area of the face of the dam (m^2)
$\bar{y}$ is the depth from the water surface to the centre of the area of the dam (m).



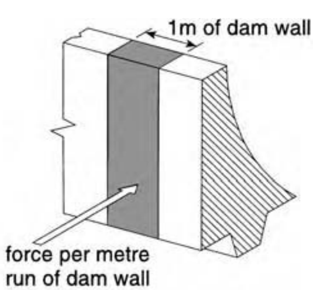
(a) The pressure diagram



(b) Location of force



(c) The dam paradox



(d) Assume dam is 1.0m long.

(e) Typical concrete dam

2.14 Designing a dam.

For the mathematically minded, a derivation of this formula can be found in most engineering hydraulics text books.

Returning to the pressure diagram, this can also be used to determine the resultant force. It is in fact equal to the area of the diagram, that is, the area of the triangle. To see how this, and the formula for force, works look at the example of how to calculate the force on a dam in the box.

The position of this force is also important. To determine the depth D from the water surface to the resultant force F (Figure 2.14b) on the dam the following formula can be used:

$$D = \frac{h^2}{12\bar{y}} + \bar{y}$$

where h is height of the dam face in contact with the water (m) and $\bar{y}$ is the depth from the water surface to the centre of the area of the dam (m).

As with the force formula, this one can also be derived from the principles of hydrostatics. The pressure diagram can also be used to determine D. The force is located at the centre of the diagram; as this is a triangle it is located two-thirds down from the apex (i.e. from the water surface).

Note that these formulae only work for simple vertical dams. When more complex shapes are involved, such as earth dams with sloping sides, then the formulae do not work. But solving the problem is not so difficult – it relies on applying the same hydrostatic principles. Most standard civil engineering texts will show you how.

EXAMPLE: CALCULATING THE FORCE ON A DAM

A farm dam is to be constructed to contain water up to 5 m deep. Calculate the force on the dam and the position of the force in relation to the water surface (Figure 2.14b).
Calculate the force F:

$$F = \rho g a \bar{y}$$

where $\rho = 1000$ kg/m^3, $g = 9.81$ m/s^2 and $a = b \times h = 1 \times 5 = 5$ m.
Note: when the length of the dam is not given assume that $b = 1$ m. The force is then the force per metre length of the dam (Figure 2.14d).

$$\bar{y} = \frac{h}{2} = 2.5 \text{ m}$$

Put all these values into the formula for F:

$$F = 1000 \times 9.81 \times 5 \times 2.5$$
$$= 122\,625 \text{ N}$$
$$F = 122.6 \text{ kNm length of the dam}$$

Using the alternative method of calculating the area of the pressure diagram:

$$\text{area of pressure diagram (triangle)} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times \rho g h \times h$$
$$= \frac{1}{2} \times 1000 \times 9.81 \times 5 \times 5$$
$$F = 122.6 \text{ kNm length of the dam}$$

This produces the same answer as the formula.
To locate the force use:

$$D = \frac{h^2}{12\bar{y}} + \bar{y}$$
$$= \frac{5^2}{12 \times 2.5} + 2.5$$
$$D = 3.33 \text{ m below the water surface}$$

Using the pressure diagram method, the force is located at the centre of the triangle, which is two-thirds the depth from the apex (or the water surface):

$$D = \frac{2}{3} \times h = \frac{2}{3} \times 5$$
$$D = 3.33 \text{ m below the water surface}$$

This produces the same answer as the formula.

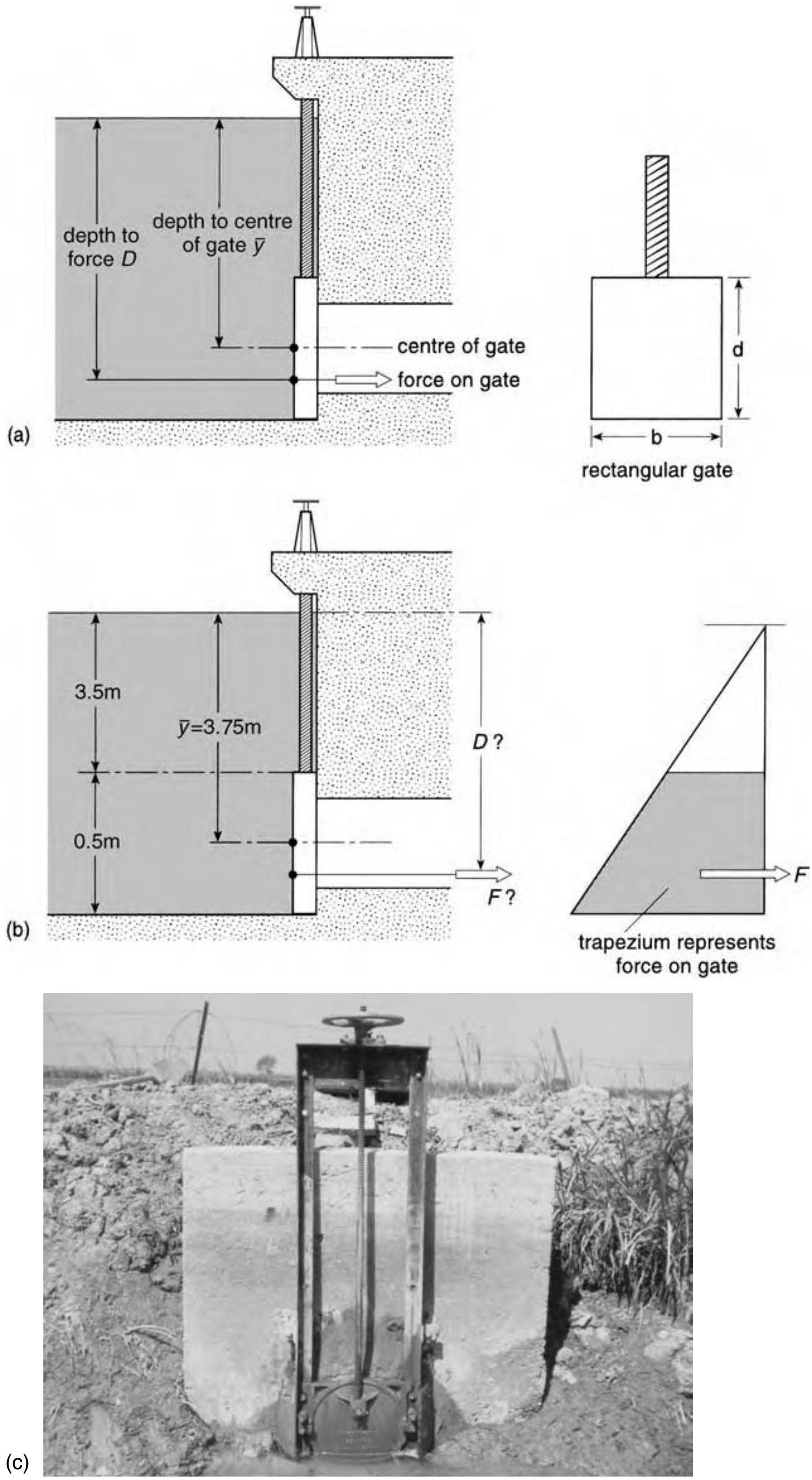
2.10.1 The dam paradox

Dams raise an interesting paradox. If two dams are built and are the same height but hold back very different amounts of water how will they differ in their hydraulic design (Figure 2.14c)? Many people would say that dam 2 would need to be much stronger than dam 1 because it is holding more water. But this is not the case. Hydraulically the design of the two dams will be the same. The force on dam 1 is the same as on dam 2 because the force depends only on the depth of water and not the amount stored. The effects of failure would obviously be more serious with dam 2 as the potential for damage and loss of life from all that extra water could be immense. The designer may introduce extra factors of safety against failure. So if you thought the forces would be different, place your trust in the well-established principles of hydrostatics and not your intuition.

2.11 Forces on sluice gates

Sluice gates are used to control the flow of water from dams into pipes and channels (see Section 7.2). They may be circular or rectangular in shape and are raised and lowered by turning a wheel on a threaded shaft (Figure 2.15a).

Gates must be made strong enough to withstand the forces created by hydrostatic pressure. The pressure also forces the gate against the face of the dam which can make it difficult to lift easily because of the friction it creates. So the greater the pressure the greater will be the force required to lift the gate. This is the reason why some gates have gears and hand-wheels fitted to make lifting easier.



2.15 Forces on sluice gates.

The force on a gate and its location can be calculated in the same way as for a dam. The force on any gate can be calculated using the same formula as was used for the dam:

$$F = \rho g a \bar{y}$$

In this case a is the area of the gate and $\bar{y}$ is the depth from the water surface to the centre of the gate. The formula for calculating D, the depth to the force, depends on the shape of the gate.
For rectangular gates:

$$D = \frac{d^2}{12\bar{y}} + \bar{y}$$

where d is depth of gate (m), $\bar{y}$ is depth from water surface to centre of the gate (m). Note: in this case d is the depth of the gate (m) and *not* the depth of water behind the dam.
For circular gates:

$$D = \frac{r^2}{4\bar{y}} + \bar{y}$$

where r is radius of the gate (m).

The depth D from the water surface to the force F must not be confused with $\bar{y}$. D is the depth to the point where the force acts on the gate. It is always greater than $\bar{y}$.

The force and its location can also be obtained from the pressure diagram but in this case it is only that part of the diagram in line with the gate that is of interest. The force on the gate can be calculated from the area of the trapezium and its location is at the centre of the trapezium. This can be found by using the principle of moments. But if you are not so familiar with moments, the centre can be found by cutting out a paper shape of the trapezium and freely suspending it from each corner in turn and drawing a vertical line across the shape. The point where all the lines cross is the centre. A common mistake is to assume that depth D is two-thirds of the depth from the water surface. It is true for a simple dam but not for a sluice gate.

The above equations cover most hydraulic sluice gate problems, but occasionally gates of different shapes may be encountered and they may also be at an angle rather than vertical. It is possible to work out the forces on such gates, but more difficult. Other hydraulic text books will show you how if you are curious enough. An example of calculating the force and its location on a hydraulic gate is shown in the box.

EXAMPLE: CALCULATING THE FORCE ON A SLUICE GATE

A rectangular sluice gate controls the release of water from a reservoir. If the gate is 0.5 m × 0.5 m and located 3.5 m below the water surface calculate the force on the gate and its location below the water surface (Figure 2.15b).

First calculate the force F on the gate

$$F = \rho g a \bar{y}$$

where:

a = area of the gate = 0.5 × 0.5 = 0.25 m^2

$\bar{y}$ = depth from water surface to the centre of the gate

= 3.5 + 0.25 = 3.75 m

$F = 1000 \times 9.81 \times 0.25 \times 3.75$
$F = 8580$ N or 8.58 kN

Next calculate depth from water surface to where force F is acting:

$$D = \frac{d^2}{12\bar{y}} + \bar{y}$$

$$= \frac{0.25}{12 \times 3.75} + 3.75$$

$$D = 3.76 \text{ m}$$

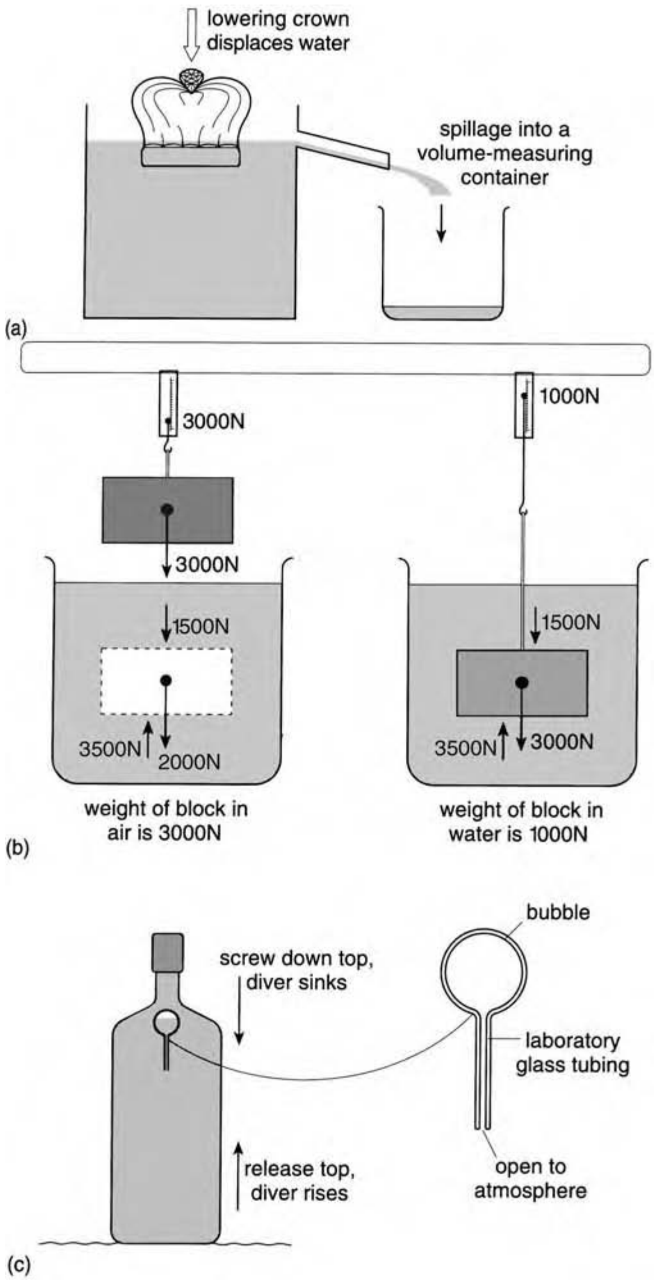
2.12 Archimedes' principle

Returning now to Archimedes who first set down the basic rules of hydrostatics. His most famous venture seems to have been in the public baths in Greece around the year 250BC. He allegedly ran naked into the street shouting 'Eureka!' – he had discovered an experimental method of detecting the gold content of the crown of the King of Syracuse. He realised that when he got into his bath, the water level rose around him because his body was displacing the water and that this was linked to the feeling of weight loss – that uplifting feeling everyone experiences in the bath. As the baths were usually public places he probably noticed as well that smaller people displaced less water than larger ones. It is at this point that many people draw the wrong conclusion. They assume that this has something to do with a person's weight. This is quite wrong – it is all about their volume. To explain this, let us return to the king's crown.

Perhaps the king had two crowns that looked the same in every way but one was made of gold and he suspected that someone had short-changed him by making the other of a mixture of gold and some cheaper metal. The problem that he set Archimedes was to tell him which was the gold one. Weighing them on a normal balance in air would not have provided the answer because a clever forger would make sure that both crowns were the same weight. If, however, he could measure their densities he would then know which was gold because the density of gold has a fixed value (19 300 kg/m^3) and this would be different from that of the crown of mixed metals. But to determine their densities their volumes must first be known. If the crowns were simple shapes such as cubes then it would be easy to calculate their volume. But crowns are not simple shapes and it would have been almost impossible to measure them accurately enough for calculation purposes. This is where immersing them in water helps.

The crowns may have weighed the same in air but when Archimedes weighed the crowns immersed in water he observed that they had different weights. Putting this another way, each crown experienced a different loss in weight due to the buoyancy effect of the water. It is this *loss in weight* that was the key to solving the mystery. By measuring the loss in weight of the crowns, Archimedes was indirectly measuring their volumes.

To understand this, imagine a crown is immersed in a container full of water up to the overflow pipe (Figure 2.16a). The crown displaces the water, spilling it down the overflow where it is caught in another container. The volume of the spillage water can easily be measured and it has exactly the same volume as the crown. But the most interesting point is that the weight of the spillage water (water displaced) is equal to the loss in weight of the crown. So by measuring the loss in



2.16 Archimedes' principle.

weight Archimedes was in fact measuring the weight of displaced water, that is, the weight of an equal volume of water. As the density of water is a fixed value (9810 N/m^3) it is a simple matter to convert this weight of water into a volume and so determine the density of the crown.

This is the principle that Archimedes discovered: when an object is immersed in water it experiences a loss in weight and this is equal to the weight of water it displaces.

What Archimedes measured was not actually the density of gold but its relative density or specific gravity as it is more commonly known. This is the density of gold relative to that of water and he calculated this using the formula:

$$\text{specific gravity} = \frac{\text{weight of crown}}{\text{weight loss when immersed in water}}$$

This may not look like the formula for specific gravity in Section 1.13.2 but it is the same. From Section 1.13.2:

$$\text{specific gravity} = \frac{\text{weight of an object}}{\text{weight of an equal volume of water}}$$

But Archimedes' principle states that:

weight loss when immersed in water = weight of an equal volume of water

So the two formulae are in fact identical and Archimedes was able to tell whether the crown was made of gold or not by some ingenious thinking and some simple calculations. The method works for all materials and not just gold, also for all fluids and not just water. Indeed, this immersion technique is now a standard laboratory method for measuring the volume of irregular-shaped objects and for determining their specific gravity.

Still not convinced? Try this example with numbers. A block of material has a volume of 0.2 m^3 and is suspended on a spring balance (Figure 2.16b) and weighs 3000 N. When the block is lowered into the water it displaces 0.2 m^3 of water. As water weighs 10 000 N/m^3 (approximately) the displaced water weighs 2000 N (i.e. 0.2 $m^3 \times$ 10 000 N/m^3). Now according to Archimedes the weight of this water should be equal to the weight loss by the block and so the spring balance should now be reading only 1000 N (i.e. 3000 N−2000 N).

To explain this, think about the space that the block (0.2 m^3) will occupy when it is lowered into the water (Figure 2.16b). The 'space' is currently occupied by 0.2 m^3 of water weighing 2000 N. Suppose that the water directly above the block weighs 1500 N (note that any number will do for this argument). These two weights of water added together are 3500 N and this is supported by the underlying water and so there is an upward balancing force of 3500 N. The block is now lowered into the water and it displaces 0.2 m^3 of water. The water under the block takes no account of this change and continues to push upwards with a force of 3500 N and the downward force of the water above it continues to exert a downward force of 1500 N. The block thus experiences a net upward force or a loss in weight of 2000 N (i.e. 3500 N−1500 N). This is exactly the same value as the weight of water that was displaced by the block. The reading on the spring balance is reduced by this amount from 3000 N down to 1000 N.

A simple but striking example of this apparent weight loss is to tie a length of cotton thread around a brick and try to suspend it first in air and then in water. If you try to lift the brick in air the thread will very likely break. But the uplift force when the brick is in water means that the brick can now be lifted easily. It is this same apparent loss in weight that enables rivers to move great boulders during floods and the sea to move shingle along the beach.

2.12.1 Floating objects

When an object such as a cork floats on water it appears that the object has *lost* all of its weight. If the cork was held below the water surface and then released it rises to the surface. This is because the weight of the water displaced by the cork is greater than the weight of the cork itself and so the cork rises under the unbalanced force. Once at the surface the weight of the cork is balanced by the lifting effect of the water. In this case the water displaced by the cork *is not a measure of its volume but a measure of its weight*.

Another way of determining if an object will float is to measure its density. When the density is less than that of water it will float. When it is greater it will sink. A block of wood, for example, is half the density of water and so it floats half submerged. Icebergs, which have a density close to that of water, float with only one tenth of their mass above the surface. The same principle also applies to other fluids. Hydrogen balloons for example, rise in air because hydrogen is 14 times less dense than air.

Steel is six times denser than water and so it will sink. People laughed when it was first proposed that ships could be made of steel and would float. But today we just take such things for granted. Ships float because even when loaded, much of their volume is filled with relatively light cargo and a lot of air space and so their average density is less than that of sea water.

Buoyancy is also affected by the density of seawater, which varies considerably around the world. In Bombay the sea is more salty than it is near Britain so ships ride higher in the water in Bombay. If a ship is loaded with cargo in Bombay and is bound for London, as it nears the UK it will lie much lower in the water and this could be dangerous if it is overloaded.

The Cartesian diver is an interesting example of an object, which can either sink or float by slightly varying its density a little above or below that of water (Figure 2.16c). The diver is really a small length of glass tubing, sealed and blown into a bubble at one end and open at the other. You can make one easily in a laboratory using a bunsen burner and a short length of glass tube. Next find a bottle with a screw top, fill it to the top with water and put the diver into the water. The diver will float because the air bubble ensures the average density is less than that of water. Now screw down the top and the diver will sink. This is because this action increases the water pressure, which compresses the air in the diver and increases its average density above that of water. Releasing the screw top allows the diver to rise to the surface again. This same principle is used to control submarines. When a submarine dives, its tanks are allowed to fill with water so that its average density is greater than that of water. The depth of submergence is determined by the extent to which its tanks are flooded. To make the submarine rise, water is blown out of its tanks using compressed air.

To summarise:

An object floats when it is less dense than water but sinks when it is denser than water. When an object floats it displaces water equal to its weight.

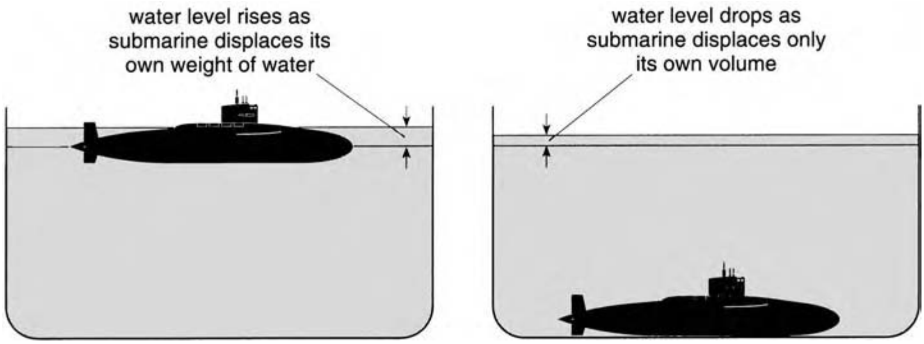
2.12.2 Applying the principle

Here are two problems to test your understanding of Archimedes' principle.

A submarine is floating in a lock (Figure 2.17). *It then submerges and sinks to the bottom. What happens to the water level in the lock? Does it rise or fall?*

Archimedes' principle says that when an object floats it displaces its own weight of water and when it sinks it displaces its own volume.

Applying this to the submarine – when it is floating on the water surface, the submarine displaces its own weight of water which will be substantial because submarines are heavy. But when the submarine sinks to the bottom it only displaces water equivalent to its volume. So the amount of water displaced by the floating submarine will be much greater than the volume of water displaced when it is submerged. This means that when the submarine dives the water level in the lock will drop (very slightly!).



2.17 Applying Archimedes' principle.

When ice is added to a tank of water, the water level rises (Figure 2.7b). When the ice melts what happens to the water level? Does it rise, fall or stay the same?

Ice is a solid object that floats and so it should behave in the same way as any other solid object. When it melts, however, it becomes part of the water and, in effect, it sinks.

To see what happens let's take 1.0 litre of water with a mass of 1 kg and weight 10 N and freeze it. Water expands as it freezes and so as it turns into ice its volume will increase by approximately 8% to 1.08 litres of ice. But remember it is still only 1.0 litre of water and so its weight has not changed – just its volume. If the block of ice is now put into the tank it will float on the water because the density of the ice is slightly lower than that of the water. The water level in the tank will also rise as a result of the displacement by the ice – like any solid object that floats the ice displaces its own weight of water, which is still 10 N. Now 10 N of water has a volume of 1.0 litre and so 1.0 litre of water will be displaced. It has nothing to do with the volume of the ice – only its weight. However, when the ice melts it 'sinks' into the tank and like any other object that sinks it displaces its own volume of water. But the volume of the melted ice (now water again) is 1.0 litre. So the displacement in each case is the same – 1.0 litre – which means the water level in the tank remains unchanged when the ice melts.

There is a lot of discussion about what happens to sea levels when polar ice melts as a result of global warming. Clearly if the ice is floating, which is the case with a lot of ice at the north pole, then melting will not change the sea level. If the ice melts off the land then of course it will cause the sea level to rise.

It is the volume of the ice that can mislead your thinking because it changes significantly when the water freezes. Do not be misled by this, just follow the principle of Archimedes and everything will work out right.

2.12.3 Drowning in quicksand: myth or reality?

A common scene in many adventure films is of someone stumbling into a patch of quicksand and getting sucked under. Great drama but is this what really happens? Recent research at the Ecole Normal Supérieure in Paris, based partly on Archimedes' principle, suggests otherwise. Apparently the work was inspired by a holiday trip to the legendary quicksands at Daryacheh -ye Namak salt lake, near Qom in Iran, where local shepherds speak of complete camels disappearing without trace.

Quicksand is a mixture of fine sand, clay and salt water, in which the grains are delicately balanced and very unstable. This makes the mixture appear solid but once it is disturbed it starts to behave like a liquid and so if you stand on it you will very easily start to sink into it. But just like any liquid there is a buoyancy effect (Archimedes' principle) and so how far you will sink depends on your density. So if you float in water you will also float in quicksand.

But the problem is not just a hydraulic one. The research showed that when the mixture liquefies the sand and clay fall to the bottom and create thick sediment that also helps to prevent you sinking further. So the good news is that the two combined mean that you are unlikely to sink much beyond your waist – not such good news if you fell in head first. Struggling and kicking will not make you sink further – it just makes the mixture more unstable and so you will sink faster. The bad news comes when you try to get out because the mixture will hold you fast. It can take as much force to pull you out of quicksand as it does to lift a typical family car. So you are more likely to have your limbs pulled off than get out of the mess! So how do you get out? One suggestion by the researchers is to gently wriggle your feet to liquefy the mixture and then slowly pull yourself up a few millimetres at a time. The myth surrounding quicksand probably originates from people falling in head first and in such circumstances you are most likely to drown. Science also spoils a good story – it means that all those film dramas about quicksand such as *The Hound of the Baskervilles* are pure fantasy!

2.13 Some examples to test your understanding

1. Determine the pressure in kN/m^2 for a head of a) 14 m of water and b) 1.7 m of oil. Assume the mass density of water is 1000 kg/m^3 and oil is 785 kg/m^3 (137.34 kN/m^2, 13.09 kN/m^2).
2. A storage tank, 2.3 m long by 1.2 m wide and 0.8 m deep is full of water. Calculate (a) the mass of water in the tank, (b) the pressure on the bottom of the tank, (c) the force on the end of the tank and (d) the position of this force below the water surface (2210 kg, 7848 N/m^2, 3767 N, 0.53 m below the water surface).
3. Calculate atmospheric pressure in kN/m^2 when the barometer reading is 750 mm of mercury. Calculate the height of a water barometer needed to measure atmospheric pressure (100.06 kN/m^2, 10.2 m).
4. Calculate the pressure in kN/m^2 and in m head of water in a pipeline carrying water using a mercury manometer when $h_1 = 0.5$ m and $h_2 = 1.2$ m. Assume the specific gravity of mercury is 13.6 (155 kN/m^2, 15.82 m).
5. A vertical rectangular sluice gate 1.0 m high by 0.5 m wide is used to control the discharge from a storage reservoir. Calculate the horizontal force on the gate and it location in relation to the water surface when the top of the gate is located 2.3 m below the water surface (13.73 kN, 2.83 m).
6. Calculate the force and its location below the water surface on a 0.75 m diameter circular sluice gate located when the top of the gate is located 2.3 m below the water surface (11.57 kN, 2.69 m).