“TIIAME” NRU
"Tashkent Institute of Irrigation & Agricultural Mechanization Engineers" National Research University

Lecture #2: Basic law of laminar filtration. Darcy’s equation

Lecture Outline

1. Introduction
2. Basic law of filtration
3. Darcy’s experiment
4. Darcy’s equation
5. Limitations to use Darcy’s filtration Law

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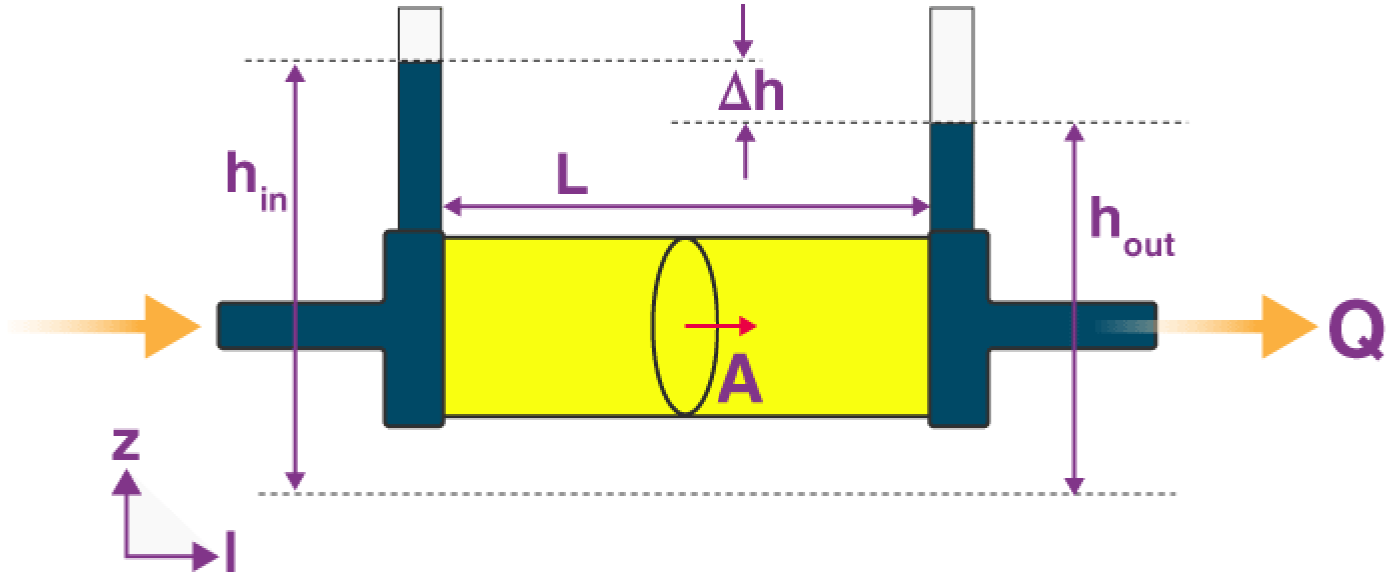
INTRODUCTION

- The basic law of filtration in a porous body was formulated by the French scientist Henry Darcy in 1856 (see next slide), as a result of experiments on the flow of water through beds of sand. It also forms the scientific basis of fluid permeability used in the earth sciences, particularly in hydrogeology:

- $$\vartheta = Q/A = K * dh/dl = K * I \quad (1)$$



the rate of filtration is directly proportional to the pressure gradient or hydraulic head

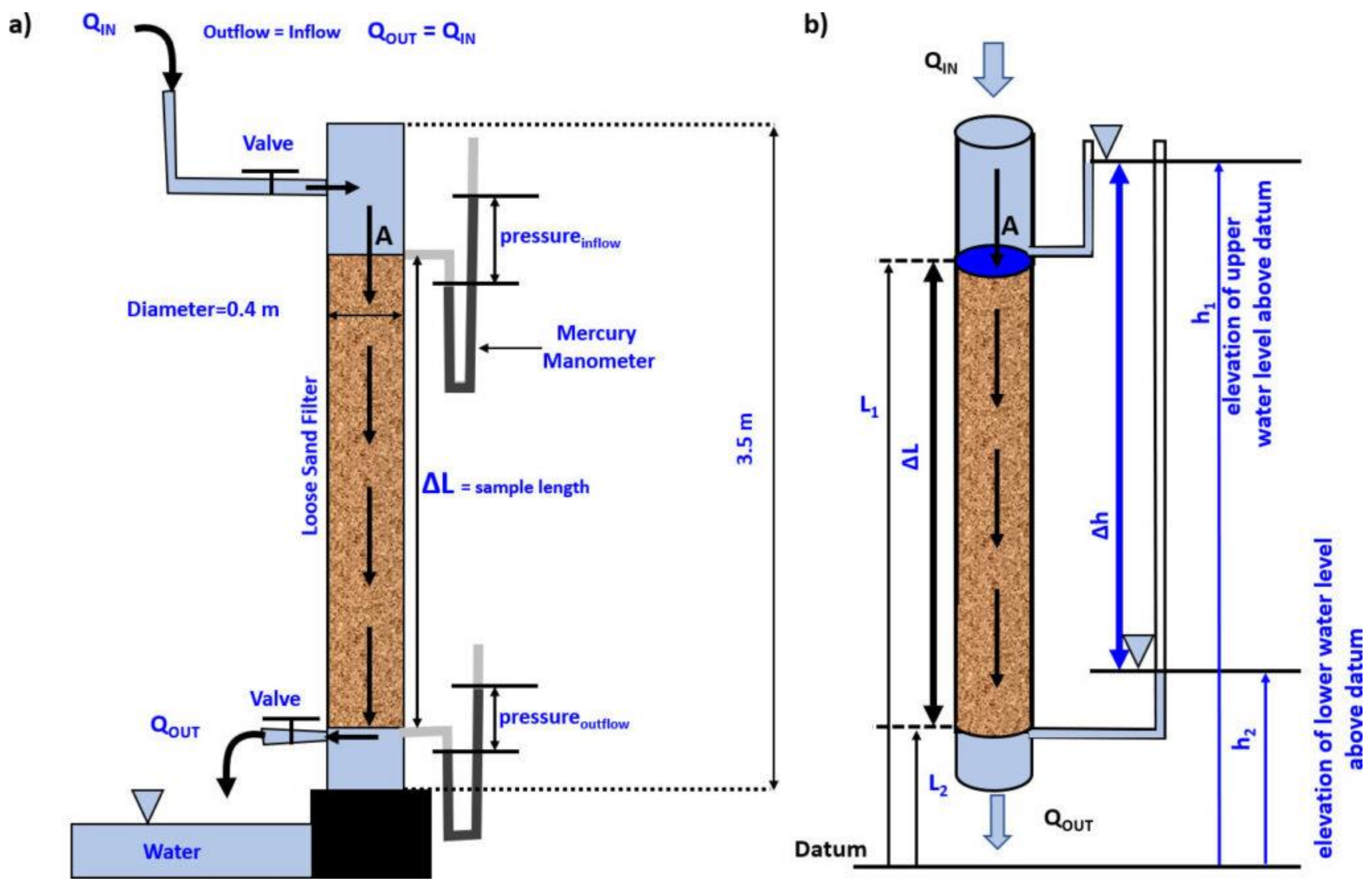
where, *K* is the filtration coefficient, *I* is the pressure gradient, and ϑ is the filtration rate (specific discharge/groundwater flux)

BASIC LAW OF FILTRATION

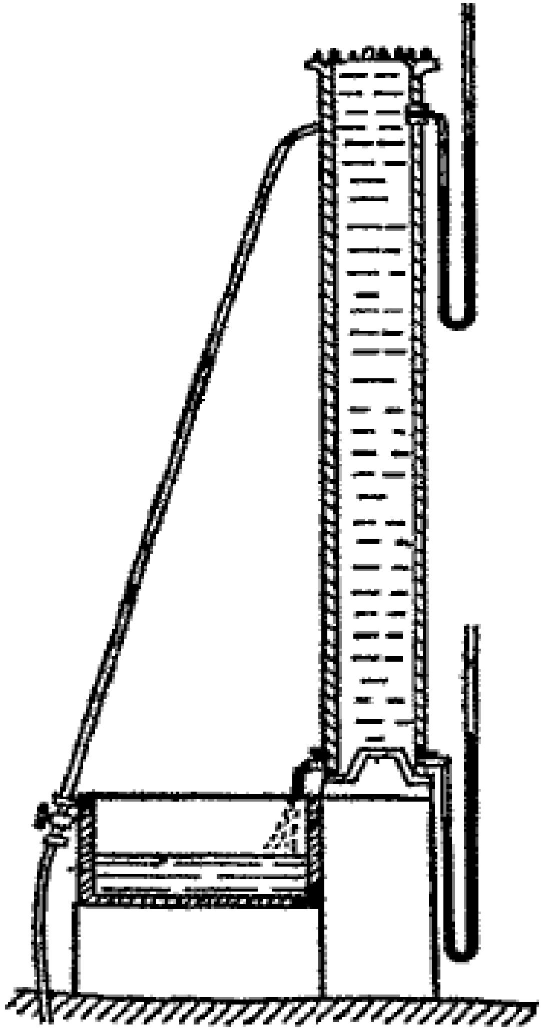
- Darcy's law is a simple mathematical statement which neatly summarizes several familiar properties that groundwater flowing in aquifers exhibits, including:
  - ✓ if there is no pressure gradient over a distance, no flow occurs, these are hydrostatic conditions;
  - ✓ if there is a pressure gradient, flow will occur from high pressure towards low pressure opposite the direction of increasing gradient, hence the negative sign in Darcy's law;
  - ✓ the greater the pressure gradient through the same formation material, the greater the discharge rate; and
  - ✓ the discharge rate of fluid will often be different, through different formation materials, or even through the same material in different directions, even if the same pressure gradient exists in both cases.
- Groundwater flows from high to low elevations, or more precise from high potential energy (=hydraulic head) to low potential energy. The hydraulic head is measured by determination of the vertical position of the water table in a well relative to a reference surface.
- Darcy's law is only valid for slow, viscous flow; fortunately, most groundwater flow cases fall in this category. Typically any flow with a Reynolds number less than unity (up to 10) is clearly laminar, and it would be valid to apply Darcy's law.

DARCY'S EXPERIMENT

- In 1856, Henry Darcy reported results of experiments used to enhance the water flow through sand filter beds used by the city of Dijon, France for water treatment (Darcy, 1856). As an engineer, he wanted to design sand beds that would efficiently and effectively filter the daily volume of water needed by the city. To evaluate the volume of water that could be filtered in a given period of time, Darcy experimented with changing: the type of sand; the area of the filter bed (diameter of the tube in his experiments); the thickness of the sand (length of the sample in his experiments); and, the force driving water through the filter bed:

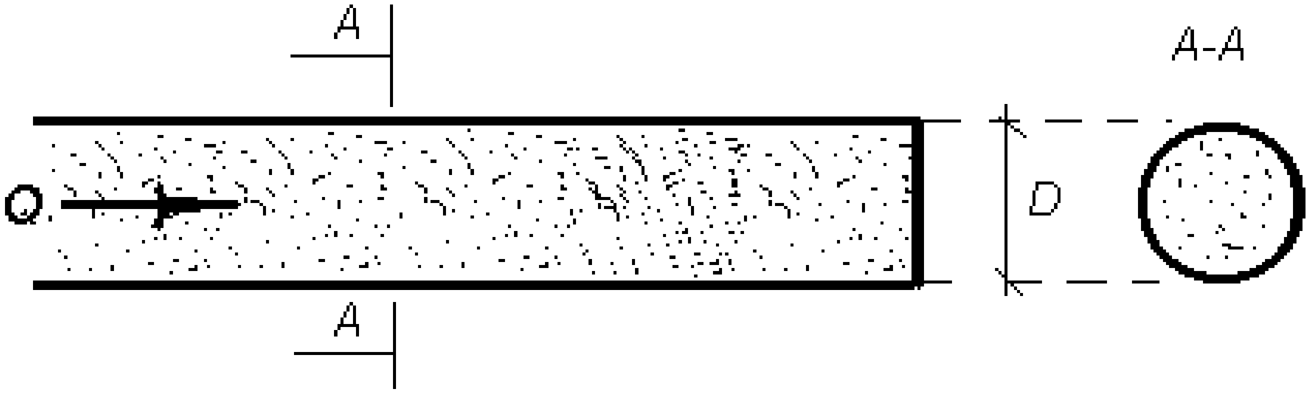


Darcy's experiment. a) In the original apparatus using mercury manometers to measure water pressures, water flowed at a constant rate into the top of the column and exited at the bottom. b) Commonly Darcy columns are equipped with water filled tubes referred to as piezometers to measure water pressures instead of mercury manometers (after Darcy, 1856).



- A_{por} – pores area of ground
- A_{par} – particles area of ground

- Simply, water flows through entire tube area $$A = \frac{\pi D^2}{4}$$



- Where $A = A_{por*} K * A_{par}$

- Hence velocity is: $$\vartheta = \frac{Q}{A}$$

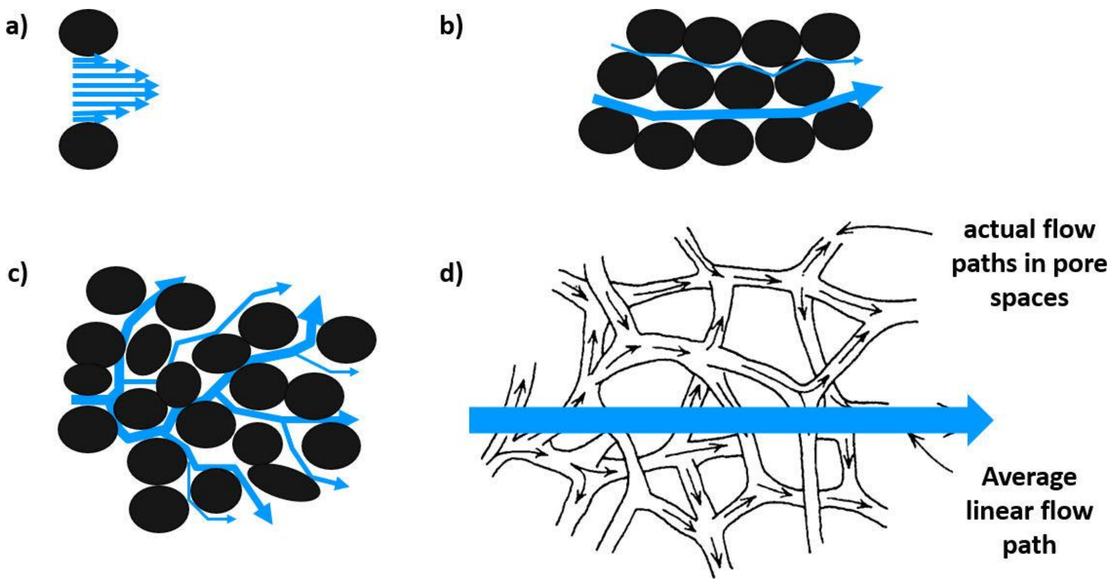
BASIC LAW OF FILTRATION

- Using the concept of filtration rate, we replace the discontinuities flow liquid through pores of soil particles with a hypothetical flow moving through a complete medium without any discontinuities. In this case, the movement of the flow is considered as the normal movement of water (we do not take into account the volumes of pores and soil and particles in the geometric sense at all) and the speed of movement of this hypothetical water flow is assumed to be equal to the "filtration rate".

- It can be seen that the filtration rate refers to some fictitious speed, the speed at which it is assumed that the water moves not only through the pores, but along the entire surface occupied by the sand. We denote the volumetric porosity of the soil by ***m*** and the coefficient of porosity on the surface by ***n***:

$$m = \frac{porous.volume}{ground.volume} \langle 1 \qquad n = \frac{A_{por}}{A} \langle 1$$

- The liquid flow within the rock is governed by the permeability of the rock. Permeability has to be determined in horizontal and vertical directions. For instance, shale consists of improbabilities which are less vertically. This indicates that it is not easy for liquid to flow up and down via shale bed but easier to flow side to side.



Conceptual model of pore-scale micro-velocities (a, b, c) represented as an average linear velocity (d): a) velocity distribution as water passes through a single pore where drag along grain surfaces slows flow as compared with higher velocities near the center of the pore; b) velocity varies because pore openings have different cross-sectional areas; c) velocity distribution caused by pore channel branching; and d) conceptual model of the average linear velocity (large straight arrow)

It is known that for homogeneous grounds ***m*=*n***, hence:

$$\frac{\vartheta}{\vartheta_d} = \frac{A_{por}}{A} = n$$

Or $\vartheta = m\vartheta_\partial$ and in most cases n=(0,35÷0,45) therefore filtration rate ϑ always smaller than actual rate

The Darcy's equation can be written as an another form: $Q = K \cdot A \cdot I$ (2)

The size of the filtration coefficient is the same as the size of the velocity, which is the speed when the gradient is *I*=1

Here, the Darcy's Law describes how head, hydraulic gradients and hydraulic conductivity are linked to quantify and describe groundwater flow. For example, to compute the discharge of groundwater (*Q*) through a cross-sectional area of sand below the water table that is 100 m by 30 m (*A*) with a hydraulic conductivity of 15 m/d (*K*), and with a head change (Δ*h*) of -2 m over a flow path length (Δ*L*) of 1000 m, above equation is applied and the discharge is calculated as follows:

$$Q = -K\frac{\Delta h}{\Delta L}\,A = -\left(15\frac{\mathrm{m}}{\mathrm{d}}\right)\frac{-2\ \mathrm{m}}{1000\ \mathrm{m}}\ (100\ \mathrm{m})\ (30\ \mathrm{m}) = 90\ \frac{\mathrm{m}^3}{\mathrm{d}}$$

- Formula (1) or (2) applies to laminar filtration motion and has a certain limit of application.
- Homogeneous soils (sand, gravel) can be applied only when Darcy's law fulfills Reynolds' following condition:

$$Re = \frac{\vartheta \cdot d}{\nu} \leq A \qquad A = 0{,}34 \div 10$$

if we consider ν = 0,018 см2/с for water, than

$$\vartheta_d \leq 0.07 \div 0.075 \qquad (3)$$

Where ϑ_d – filtration rate, m/s;

d- diameter of ground particles, cm.

If the condition (3) is not fulfilled, turbulent filtration will occur and Darcy's law will be violated.

In turbulent filtration, another expression is used instead of formula (1):

$$\vartheta = K \cdot I^{1/m} \quad \text{or} \quad I = \frac{1}{K^m} \cdot \vartheta^m \qquad (4)$$

where: m is defined through experiment and vary within $1 \leq m \leq 2$

or $$I = a\vartheta + b\vartheta^2 \qquad (5)$$

Where a and b are coefficients obtained based on experiment (for given water and ground temperature).

BASIC LAW OF FILTRATION

At small velocities (the second part $b\vartheta^2$ can be skipped), then expression (5) turns into Darcy's formula.

At the high speeds (the first part can be skipped): $I = b\vartheta^2$

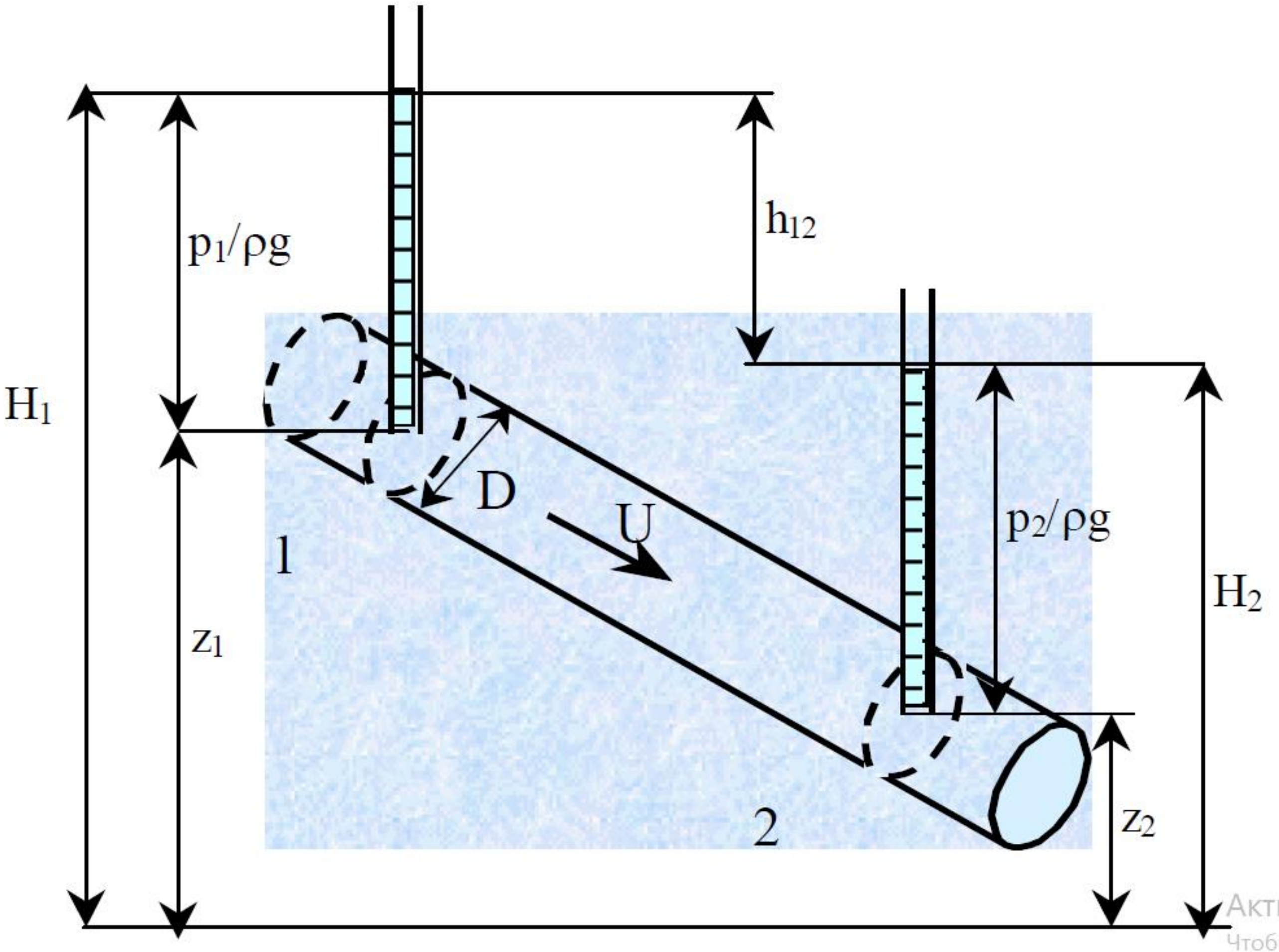
Which corresponds to the quadratic resistivity domain.

DARCY’S EQUATION

- In a porous medium, a homogeneous fluid moves under the influence of pressure and gravity.
- Darcy's law expresses the relationship between the rate of filtration and the forces present in the fluid.

- Let's look at Darcy's law in an experimental setup.

A fluid is rapidly filtered with velocity of ϑ through a pipe of diameter ***D*** and length ***L*** filled with porous soil. We select cross sections ***1*** and ***2***. The heights of the centers of gravity of these sections are ***z1*** and ***z2***. We determine the pressure in the sections ***p1*** and ***p2*** on the basis of piezometers.



DARCY'S EQUATION

- We write the Bernoulli equation for these sections:

$$\boldsymbol{H_1 = H_2 + h_{1\text{-}2}}, \qquad (6)$$

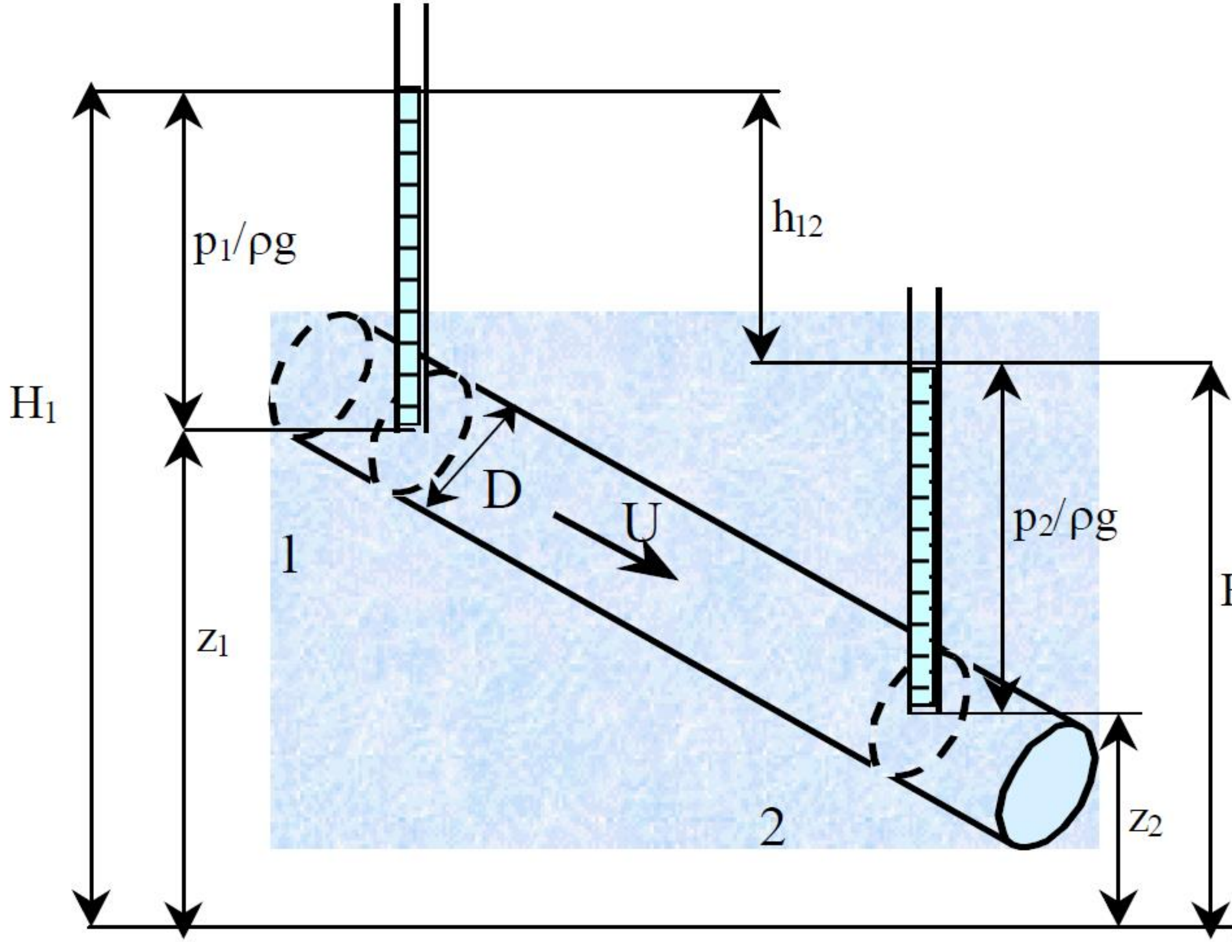
Where: $H_1 = z_1 + \dfrac{p_1}{\rho g} + \dfrac{\alpha_1 u_1^2}{2g}$

is hydrodynamic head;

$\boldsymbol{h_{1\text{-}2} = h(u)}$ is the lost pressure between sections, which is related to the filtration rate and cannot be determined by hydraulic formulas.

Since filtration velocities are of such small magnitude, the velocity gradient can be ignored. If we solve the equation (6) with respect to the filtration rate, we can determine:

$$\boldsymbol{u = f(H_1 - H_2)} \qquad (7)$$



DARCY'S EQUATION

Here the filtration rate does not depend on the cross-sectional area.
In addition, the rate of filtration is related to the properties of the fluid and the properties of the porous soil.
These properties are calculated by the filtration coefficient $\boldsymbol{k_f}$.

Then we can write the following equation (8):

$$u = k_{\phi} \frac{(H_1 - H_2)}{L}. \tag{8}$$

This equation was determined experimentally by the Darcy and is valid for many liquids and gases. But when determining the value of $\boldsymbol{k_f}$ for some liquids, the properties of liquids and soils should be taken into account separately.

The properties of liquids are taken into account by the dynamic viscosity coefficient $\boldsymbol{\mu}$ and density $\boldsymbol{\rho}$. In that case, the filtration coefficient can be determined by the following formula:

$$k_f = \frac{k}{\mu} \rho g. \tag{9}$$

Here, the $\boldsymbol{k}$-permeability coefficient is related to the property of the soil and refers to the ability to pass liquids and gases through it. The unit of measurement of this coefficient (in SI [k]=m^2= 1012 μm^2).

LIMITATIONS TO USE DARCY'S FILTRATION LAW

- Experiments conducted by different researchers have shown the correctness of the filtration law in some cases, while another group of researches has shown that the results of the experiments differ sharply from its results.

- It was clearly confirmed by the experiments conducted with sand soils with a particularly large diameter. This situation encouraged researchers to search for other laws and discover expressions that differ from the expression of the filtration law and fully express the law of filtration. As a result of the research, a number of empirical results were obtained and they expressed the law of filtration correctly, but this does not give the basis that the basic law of filtration is wrong. Perhaps it clarifies the limit of its application.

- Darcy's law can be applied to many situations but does not correspond to these assumptions.

✓ Unsaturated and Saturated flow.

✓ Flow in fractured rocks and granular media.

✓ Transient flow and steady-state flow.

✓ Flow in aquitards and aquifers.

✓ Flow in Homogeneous and heterogeneous systems.

- As we mentioned above, the motion of water subject to Darcy's law is laminar. Of course, this law does not obey when the flow pattern is turbulent. In addition, the Reynolds number must have a very small value in the laminar movement of the filtration flow, which obeys the Law of Filtration.

SOME SELF DEFINING AND CONTROL QUESTIONS

1. Define non-fluid filtration flow.
2. Where the filtration model is used for?
3. What is filtration rate?
4. What is water transfer coefficient?

LITERATURES

1. Manning, J.C. (1997) Applied Principles of Hydrology. Prentice Hall, third edition, 276p.
2. Freeze, R.A. and Cherry, J.A. (1979) Groundwater. Prentice Hall, 604p.
3. Агроскин И.И., Дмитреев Г.Т., Пикалов Ф.И. Гидравлика.- М.-Л.:Госэнергоиздат,.483 с.,1954.
4. Агроскин И.И., Дмитриев Г.Т., Пикалов Ф.И. Гидравлика -М.: Госэнергоиздат,.352с.,. 1964.
5. Бозоров Д.Р. ва бошқ.Гидравлика (амалий ва тажриба маш.).Т.ТИМИ.2009.
6. Красников Н.Д.Динамические свойства грунтов и методы их определения-М.Госстройиздат,1970.
7. Киселёв П.Г. Гидравлика.-М.-Л.: Госэнергоиздат,1963.
8. Рассказов Л.Н.и др. Гидротехнические сооружения,Часть 1,2.
9. Издательство Ассосации строительных ВУЗов,2008
10. Рассказов Л.Н. и др. Фильтрация в грунтовых плотинахв плоской и в пространственной постановке//Гидротехническое строительство.-1989,-№11.-С.26-32.
11. Справочник по гидравлике./Под ред. В.А.Большакова.-Киев: Высшая школа, 1977.
12. Угинчус А.А., Чугаева Е.А. Гидравлика.-Л.: Стройиздат, 1971.
13. Чугаев Р.Р. Гидравлика -Л.: Энергоатомиздат, 1982.
14. Чертоусов М.Д. Гидравлика(специальный курс).Л.1964
15. Штеренлихт Д.В. Гидравлика. I, II, III, IV т. -М.: Энергоатомиздат, 1991.
16. Штеренлихт Д.В. Очерки истории гидравлики, водных и строительных искусств. I, II, III,IV,V,VI т. -М.:Геос, 1999-2006.
17. Тейлор Д. Основы механики грунтов.-М.,ГСИ.,1960.С.597.

QUESTIONS?