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REVIEW OF MATHEMATICAL MODELS FOR THE REPRESENTATION OF A CONTINUOUS, TWO-COMPONENT, TWO-PHASE MEDIUM

Abstract. The paper analyzes various approaches to modeling the motion of a continuous two-component two-phase medium, reviewing the history of development and the problems of calculation and prediction of aerodynamic and hydrodynamic phenomena. It is revealed that the existing models cannot be used for small-scale phenomena. A search for finite-difference approximations to dynamic models is needed for such phenomena.

Keywords: general circulation, scale, instability, hydrostatics, Navier-Stokes equations.

Introduction. Modelling of solid, two-component, two-phase medium is of interest in the study of circulation processes in water and air, for pollution dispersion analysis, etc. At present, there are a number of different approaches to modeling, many mathematical models have been created, but the development of this direction continues.

The aim of this article is to analyse existing directions of mathematical modelling of continuous, two-component, two-phase environment and to evaluate possibilities of using existing models for calculation of local climatic comfort zones (microclimate systems).

1. Classification of mathematical models of air mass movement

The hierarchy of mathematical models according to their degree of detail can be represented as follows:

Atmospheric general circulation models with computational cell size of 10–100km.

Regional atmospheric models with a grid cell size of 1–10 km.

Atmosphere models with vortex calculations cell size 10–100m.

Turbulence models in the gas medium, computational cell size 1–10 mm.

A separate consideration should be given to integrated ocean-atmosphere models of general circulation models, which are much more complex and comprehensive.

At present, there are no computer facilities capable of calculating all listed model types in such a way as to evaluate both turbulent phenomena and general atmospheric circulation. Accounting for the interaction between the atmosphere and ocean complicates the entire computational process considerably. This is due to the huge difference in velocities of water and air masses. The time step limitations generate such a significant schematic (computational) viscosity that it becomes the dominant factor, masking the adequate reality of water and air masses movements.

2. Historical analysis of the development of mathematical models of air and water mass movement.

Let's take a closer look at the history of the development and emergence of computational aerodynamics.

The basis of theoretical and computational aerohydrodynamics are Navier-Stokes equations. The process of constructing systems of equations describing the motion of coupled media and named Navier-Stokes equations was completed at the beginning of the 19th century (about 1810–1820).

For more than 100 years, many scientists made many efforts to solve the derived equations of hydrodynamics. The hydrodynamic equations were so complex that there was not even an attempt to solve them in their entirety. Only the simplest problems, such as the flow of a cylindrical and spherical body by a laminar incoming flow, could be solved [7].

At the beginning of 1900, the development of similarity theory began. It was the time when physical analogues of processes and phenomena under study were reduced in size. Large replicas of reservoirs, large numbers of wind tunnels and the like were built. The similarity theory clearly stated that it was impossible in principle to build a reduced physical copy of the phenomenon under study in such a way that a complete correspondence between the phenomenon and its reduced model could be achieved. It is impossible to provide equal scaling of the Reynolds, Mach, Struhal, etc. parameters at the same time. The difference between the behaviour of a fluid or gas in models and reality is particularly strong when there is a significant difference in the size of the phenomenon and the model of the phenomenon. Huge wind tunnels and huge models of dams and river sections were built. The physical model of a section of the Amu Darya River at the new SANIIRI site was up to 15 metres wide and over 100 metres long. Even such huge physical models could not meet the demands of construction and design.

In 1956, Rakhmatullin published a system of equations of motion for multiphase media [12]. This event became an impetus for the emergence of a whole group of scientific schools of multiphase flow motion theory.

Especially rapidly, the multiphase flow motion theory began to develop in the Uzbek SSR. The School of Computational Aerodynamics of F. B. Abutaliev, Corresponding Member of the Academy of Sciences of the Uzbek SSR, studied only the atmospheric phenomena and climatic phenomena prediction using numerical methods.

The second school under the leadership of T. D. Dzhuraev conducted academic research of multiphase flow equation systems, their stability and uniqueness of solutions. Particular attention was paid to problems with unknown boundary of solution domain. That is, those hydrodynamic problems in which the position of the free surface limiting the fluid flow depends on the motion of the fluid itself were solved.

Also the hydrodynamics was considerably developed at the Central Asian Hydrometeorological Institute (CAHMI), which was one of the first recipients of high-performance computers of the IBM-22 and IBM-32 generation. This made it possible to build mathematical models of the atmosphere and to solve complex systems of equations. The first significant works on the construction of mathematical models of the atmosphere began to appear from 1970. One of the first significant works was published at the CAHMI in 1978 [5]. This work is interesting only in the depth of theoretical material and in writing the equations of motion for multiphase media in the most general, but also sufficiently complete form.

At that time outside the USSR there was no progress in creating mathematical models of atmospheric circulation. Computational fluid mechanics in Europe and the USA concentrated mainly on highly specialized problems related to the flow around bodies with complex configurations using the finite element method. The main customer of such research was the military-technical complex, so very often one could find in the literature descriptions of mathematical models adapted to calculate the aerodynamics of fast-moving objects – aircraft, machines and ships [1; 2].

Interestingly, this time coincides with the degradation of analogue modelling – the study of hydrodynamic phenomena on scaled-down physical models. By 1990, analogue modelling had disappeared as a class of research in its own right.

Computational fluid mechanics around the world are trying to solve the Navier-Stokes equations with finite difference methods. The finite difference method makes it easy to switch the generated solution algorithm for one problem to another problem. However, when solving full (or sufficiently full) Navier-Stokes equations and applying explicit schemes, the so-called "dynamic instability" occurs. All the characteristics of the solution begin to change sinusoidally, but the amplitude of these harmonic oscillations increases exponentially. The calculation terminates with an emergency stoppage of the computational process and nothing has been done to deal with this type of instability. Attempts to introduce artificial viscosity into the calculation to stabilize the dynamical instability failed. Tridiagonal matrices of algebraic equations were formed only for one-dimensional problems solved by Navier-Stokes equations, which is of little interest.

At that time in Novosibirsk branch of the USSR Academy of Sciences G. I. Marchuk finds and substantiates the method of subordinate and then factor splitting of multidimensional equations of motion of coupled medium (Navier-Stokes equations). Three-dimensional problems are decomposed into a series of sequentially solved one-dimensional problems. First everything related to motion along one axis, then another axis and then a third axis is computed. Each solution is the initial state for finding the next solution. The result is the ability to apply implicit calculation schemes using the economical 'forward and backward run' algorithm.

The solution obtained, of course, has some fluctuation, but it is stable, and this is the main thing. G. I. Marchuk proves the singleness theorem and existence of a solution for subordinately split hydrodynamic problems with a certain small given accuracy. G. I. Marchuk formed the Novosibirsk school of computational hydrodynamics, which published many articles dedicated to the calculation of circulations in the seas and oceans [8].

A number of scientists begin to actively apply the achievements of Novosibirsk hydrodynamicists in aerodynamics. Models of water circulation are created first for separate seas [11], parts of oceans [4], then oceans as a whole, and then global mathematical models of water circulation in the World Ocean appear. At the same time, G. I. Marchuk is engaged in the development of climatic mathematical models and the development of methods for their solution [6; 8; 9].

The development of the Novosibirsk scientific school coincided with the development of run-time methods for solving algebraic linear equations of a special kind. It turned out that for the case when algebraic equations can be written in the form of tridiagonal matrix with predominance of terms located on main diagonal, then they are solved by simple algorithm called "forward and backward run" [3, 13].

One of the most important conditions for solvability of the Navier-Stokes equations to describe the circulation in the seas and oceans was the so-called "truncation" of the vertical component of the equation of motion to the hydrostatic state. As a result of this action it was possible to create easily a calculation algorithm calculating the vertical component of velocity through the continuity equation for an incompressible fluid.

However, in this case it is impossible to take into account the hydrodynamic pressure and its influence on the motion of a cohesive medium. It was considered that the motion of a cohesive medium provides only hydrostatic pressure. However, the main achievement of this calculation was that the incompressibility of the fluid no longer created the danger of sound waves moving at infinite velocities.

3. Current problems in solving aerohydrodynamics problems

More than 40 years have passed but the practical approach to solving aerohydrodynamics problems has remained practically unchanged [10]. Most of the problems on circulation of water and atmospheric masses were solved by the method of establishment, which was theoretically developed in Novosibirsk by a group of scientists headed by G.I. Marchuk. But during the method of establishment it was thought without proof that even under changing boundary conditions the system is always in a stable equilibrium state. At the same time, inertia in the processes of mixing water and atmosphere was never calculated, most likely because any artificial numerical filters were used to exclude wave processes on the free surface of water or the upper bounding layer of the atmosphere. The filters have always been subjective and have in fact determined the ability of the current system to be driven entirely by changing boundary conditions. Calculations took a long time as computers of that time could carry out one calculation cycle in 5 seconds and it was not possible to speed up the process.

It means that the advance of the real process on a coarse grid could be tenfold. This means that calculation of a day would take 2.5 hours.

But even such a calculation necessarily required some heuristic filter to cut off the oscillations, because no one was able to reach the maximum allowable time step. Most characteristic was that the duration of the calculation was almost the same time as the real process.

Nowadays, with the advent of super-fast computers, the situation has improved, but not dramatically. Weather forecasts for at least a week ahead are still not achievable without the introduction of some artificial heuristic oscillation filters.

Parallel to the development of computational methods for solving the equations of aero-hydrodynamics, in-depth research on the development of the theory of interpenetrating motion of multiphase media continues. The equations of multiphase media were first written in phase space. This is a four-dimensional space in which there are three spatial coordinates and one additional coordinate – the characteristic dimensions of one of the phases represented by discrete entities of different sizes. For gas-liquid medium such parameter may be different diameters of spherical water droplets. The equations are written in a non-stationary form, specifically in a kinematic form (so-called Boltzmann equations). This model was developed by A.A. Dzhuraev and Y.M. Denisov [5], who completely uncovered the mechanism of motion of water mass between discrete droplets and the surrounding air mixture of dry air and water vapor.

By the end of 2021, there were more than a dozen atmospheric mathematical models being used for short-term weather forecasting – no more than 10 days ahead – and not everywhere is possible. The best mathematical models are used in Israel (7–10 days lead time) and relatively good ones are used in Japan and USA.

The website https://hmong.ru/wiki/Climate_model provides a list of mathematical models currently in active use to study atmospheric circulation.

All these models contain general achievements of Novosibirsk scientists of G. I. Marchuk school and in case of application of implicit calculation schemes – achievements of A. A. Samarsky: "cutting down" of vertical equation of motion to hydrostatic equation, incompressibility of continuous medium (including air in climate models) and possible application of economical solution of algebraic equation systems which can be written in form of tridiagonal matrix (application of implicit finite-difference schemes).

Conclusion. None of the models considered can be used as a basis for the investigation of microcirculation of water droplet-gas systems in microclimate control areas for the following reasons:

1. The scale of the phenomena is incommensurable.

2. The scales of existing models are hundreds of kilometres, and for scales of tens of metres "downsizing" to hydrostatics is unacceptable.

3. Gas is assumed to be a compressible medium.

4. The computational viscosity of implicit schemes in solving our problem is unacceptable.

Therefore, the objectives of further research are:

– the construction of a complete mathematical model of the dynamics of a multicomponent and multiphase medium;

– investigation of finite-difference approximations to solve the obtained algorithms;

– development of the algorithm for the solution of the stated tasks.

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