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Methods for Solving the Problem of Filter Flow with Furrow Irrigation

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Abstract. The article deals with the construction of hydraulic models of water systems included in the general model of water exchange. At the same time, the methods adopted for natural and field conditions, the theory of unsteady water filtration in soils and grounds, and modeling the dynamics and direction of hydrological, hydrogeological, and soil reclamation processes using modern technical means of observation and mathematical methods were used.

When considering the problems associated with mass transfer and interacting flows, it becomes necessary to study several issues on creating hydraulic models and their numerical implementation. This is the creation of the principles of conjugation of mathematical models corresponding to different branches of the hydrological cycle, analysis of mathematical problems of correctness, and study of the qualitative properties of solutions of the corresponding initial-boundary value problems. In this case, a special place is occupied by the development of numerical methods and effective algorithms for calculation, the establishment of boundary value problems for systems of equations of a non-standard form, taking into account their various features (nonlinearity, degeneration, etc.).

In the case of free-flow filtration of a liquid through a furrow in the soil, caused by the difference in the hydrostatic pressure of the liquid in a heterogeneous medium, the area of fluid movement is limited from above by a free surface, called the depression surface, at each point of which constant pressure acts. The section of the depression surface along the movement of the fluid through the porous medium is a depression curve. The resulting gap between the outlet of the depression curve and the groundwater is a pinch-out surface.

The subject of research in this article is the determination of the depression curve for unsteady filtration flow through the furrow during cotton irrigation by solving the Boussinesq equation.

INTRODUCTION

Surface irrigation technology, especially furrow irrigation, is the main irrigation method in the arid zone. However, traditional furrow irrigation methods have significant drawbacks, primarily in the uniformity of soil moisture along the length of the furrow. Despite improvements in the scientific foundations of irrigation technology, many other aspects of this scientific problem remain unresolved. One of them is to determine the infiltration characteristics of irrigation under various irrigation regimes and soil properties.

Usually, in land reclamation science, the modeling of the water-salt regime of ground and surface waters was carried out based on simplified models for individual components of the water runoff. Models of mass transfer by interconnected flows of ground and surface waters, as well as moisture in the humidification zone, taking into account the mass transfer between various components of the water runoff, as well as the problems of controlling the change in the state of the humidification zone, have not been considered so far. This significantly limited the possibility of using joint runoff models in solving many applied problems to assess the ecological and reclamation state of irrigated lands and the quality of ground and surface waters.

When considering the problem of coupled mass transfer by interacting flows, studying several issues on the hydraulic substantiation of models and their numerical implementation is necessary. Based on these conditions, we have made the definitions of the Bussensk limit problem of the nonstationary filtration theory, formulated for a scalar porous medium, into the Blasius problem, and then into the limit problem of smaller parameters.

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MATERIALS AND METHODS

In the research process, methods were adopted for natural and field conditions, the theory of unsteady water filtration in soils and grounds, and modeling the dynamics and direction of hydrological, hydrogeological, and soil reclamation processes using modern technical means of observation and mathematical methods. Experimental studies of water consumption of plants against a natural background using a lysimeter.

RESULTS AND DISCUSSIONS

The proposed reduction is performed to obtain a second-order ODE, the solution of which must be found on the interval (0,1), which is a compact set, which simplifies calculations concerning the equivalent Blasius problem, the solution of which must be sought on the semi-infinite interval $(0, \infty)$. The limiting problem of smaller parameters for a heterogeneous environment due to filtration of irrigation water along cotton furrows is formulated as follows:

$$2\varphi\varphi'' + u = 0, u_e \leq u_0 \leq u \leq 1,$$

$$\varphi := \int_u^1 \zeta dv, \varphi(1) = \varphi'(u_0) = 0, \zeta := \frac{x'}{2\sqrt{t'}}$$

An analytical solution to the equation of smaller parameters is made for its generalized formulation, a particular case of which is the formulation for a heterogeneous environment. A typical limiting problem of smaller parameters is formulated as follows [3,7,8]:

$$yy'' + \gamma x = 0, D(y) = (x: x_0 < x < 1),$$

$$y'(x_0) = y(1) = 0, \mathrm{Im}(y) = (y: y_0 > y > 0), \tag{1}$$

where $y_0 := y(x_0) > 0$. In the classical case of a typical limit problem $\gamma = \frac{1}{2}, x_0 = 0, y_0 := y(0)$. This case is considered below.

It can be proved that the two-point limit conditions (1) are equivalent to the Cauchy condition $y(0) - a = y'(0) = 0$. Let $a = 0$. Then $y(x) = \pm\sqrt{2/3(-x)^3}$ is the solution to the homogeneous one-point problem for the equation of smaller parameters on the negative semiaxis.

In hydrodynamic applications, $y(x)$ is the (dimensionless) friction, x is the (dimensionless) longitudinal component of the velocity in a heterogeneous medium propped up by a plane flow in the longitudinal direction. Then y_0 represents the shear stress of friction on the pore wall (Blasius constant) [4]. In the hydraulic theory of filtration, x is the (dimensionless) depth of the filtration flow through a scalar (homogeneous and isotropic) porous medium, y is the potential, defined as:

$$y(x) = \int_x^1 sdx', \ \ y(1) = y'(0) = 0,$$

where s is the longitudinal coordinate measured along the filtration flow. In filtration problems, the constant y_0 is proportional to the filtration flow rate in the cross-section of the flow exit to the medium boundary [4].

Stationary solutions for gravity filtration in a scalar medium are performed in terms of the analytic theory of ODEs [1]. Modern results are presented in [2,17].

The following statements are true:

Statement 1. The equation of smaller parameters has 2 branches of the solution, positive y_+(x), and negative, y_-(x). The negative branch is defined as the solution to the limiting problem:

$$2y_-y_-'' + \gamma x = 0, D(y_-) = (x: x_0 < x < 1),$$

$$y'_{-}(0) = y(1) = 0, \mathrm{Im}(y_{-}) = (y_{-}: -y_0 > y_{-} > 0),$$

In this case, $y_{+}(\mathrm{x}) + y_{-}(\mathrm{x}) = 0, 0 < x < 1$. The proof is obvious.
Further, only the positive branch of the solution to the equation of smaller parameters is considered, i.e., $\mathrm{y} := y_{+}$.
Statement 2. The solution of a typical limiting problem of smaller parameters (2) has the following properties:

$$y'(x) < 0, y''(x) < 0;$$

$$y'(x) \xrightarrow[x \to 1-0]{} -\infty.$$

Therefore, $y_0 > y(x), 0 < x < 1$. For the proof, we formally lower the order of the equation and reduce it to an integral equation:

$$2y' = -\int_0^x \frac{tdt}{y(t)} \longrightarrow y' \leq 0, 0 \leq x < 1,$$

as required.
The integral on the right-hand side can be calculated using Bonnet's mean value theorem. We get:

$$2yy' = -1/2(1 - \theta^2)x^2, \tag{2}$$

where θ is a regular fraction, it remains to pass to the limit as $x \to 1 - 0$.
The solution to equation (2.2) such that $y(1) = 0$ is:

$$y^2(x, \theta) = 1/6(1 - \theta^2)(1 - x^3). \tag{3}$$

This solution continuously depends on the value of the fraction θ. The value of the solution (2.3) averaged over θ represents the so-called weak solution of a typical limiting problem of smaller parameters, interpreted as a distribution over θ with a distribution density $y(x; \theta)$ [7,12].
By virtue of (3), a weak solution to a typical limiting problem of smaller parameters has the form:

$$y(x) = 1/3\sqrt{1 - x^3}, \tag{4}$$

and then $y_0 = y(0) = 1/3$, which is a good rational approximation for the Blasius constant. The exact value of the Blasius constant was calculated in [4]. As can be seen from formula (4), the solution can be continued to negative values of x while maintaining the continuity and smoothness of the solution at the point x = 0.
The solution of a typical limiting problem of smaller parameters is associated with the solution of a nonlinear integral equation:

$$y(x) = 1/2\left\{\int_0^1 \frac{(1 - s)sds}{y(s)} - \int_0^x \frac{(x - s)sds}{y(s)}\right\}. \tag{5}$$

Equation (5) yields the following expression for the Blasius constant:

$$y_0 := y(o) = 1/2\int_0^1 \frac{(1 - s)sds}{y(s)},$$

The solution to equation (5) can be obtained in the form of a Lagrange series [8]. It is proved that the radius of convergence of the Lagrange series is less than 1, and the series diverges as $x \to 1 - 0$. An alternative to the Lagrange series solution method can be an iterative process:

$$y_k(x) = 1/2\left\{\int_0^1 \frac{(1-s)sds}{y_{k-1}(s)} - \int_0^x \frac{(x-s)sds}{y_{k-1}(s)}\right\}, k = 1(1)\infty,$$

where the bottom icon identifies the iteration number. The iterated values of the Blasius constant are determined from the sequence:

$$y_k(0) = 1/2\int_0^1 \frac{(1-s)sds}{y_{k-1}(s)} \, .$$

We consistently find:

$$k = 1: y_0(x) = y_0 = \sqrt{1/12} = 0.2887;$$

$$k = 2: y_1(x) \cdot y_0 = 1/12(1-x^3), y_1(x) = (1-x^3)/\sqrt{12}\,, y_1(0) = 1/\sqrt{12}$$

$$k = 3: y_2(x) = \sqrt{3}\left(\int_0^1 \frac{(1-s)sds}{1-s^3} - \int_0^1 \frac{(x-s)sds}{1-s^3}\right) - \sqrt{3}\left\{\begin{array}{c} \ln\sqrt{3} - \dfrac{x+2}{3}\ln\sqrt{1+x+x^2} + \\ +1/3(1-x)\ln\left(\dfrac{1}{1-x}\right) - \dfrac{\pi}{6\sqrt{3}} + \\ +\dfrac{1}{\sqrt{3}}\left(arctg\,\dfrac{2x+1}{\sqrt{3}} - \dfrac{\pi}{6}\right) \end{array}\right\},$$

$$y_2(0) = \sqrt{3}\left(\ln\sqrt{3} - \frac{\pi}{6\sqrt{3}}\right),$$

etc. Accordingly, the first three iterated values of the Blasius constant form a sequence of values:

$$y_0(0) \;=\; 1/\;12 \;=\; 0.2887\ldots, y_1(0) \;=\; 0.2887\ldots, y_3(0) \;=\; 0.4278\ldots,$$

and, on average, for the first three iterations, $0{,}3299 < y(0) < 0{,}3344$. The iterative process leads to trivial and lengthy calculations, which is already seen at the third iteration. Obviously, any iterated solution possesses all the basic properties of solving the limiting problem (1):

$$\forall x \in (0,1), \forall k = 1(1)\infty, y_k'(x) < 0, y_k''(x) \xrightarrow[x \to 1-0]{} -\infty.$$

The inconvenience of the iterative process lies in the cumbersomeness of expressions for iterated solutions and in the absence of proof of the convergence of the iterative process. Both of these obstacles can be circumvented using the difference approximation of the limiting problem (5).

Interest in numerical solutions of the Blasius equation appeared immediately after the publication of [6] in the method of integration using power series, see [6] and the preprint of [2] containing the history of the issue. Modern works [5,7] are mainly devoted to improving the convergence of predictive correction methods for solving boundary layer ODEs. The work [6] is an exception, which is interpreted in terms of homotopy mappings of the interval of integration onto a compactum. In the case of limit problem (2), the mappings are compact. Let the linear homotopy $F(t,x)$: $((0 < t < 1) \times (0 < x < 1)) \to (0,a)$ represent the solution of the limit problem (5). Then $F(0,x)$ depicts the solution in the vicinity of the point x $= 0, F(1,x)$ depicts the solution in the vicinity of the point x = 1. For example, for a weak solution

$$F(0,x) = 1/3(1 - x^3/2 - x^6/8), F(1,x) = 1/\sqrt{3}\sqrt{1-x}\,.$$

Linear mapping has the form: $y(x) = F(t,x) = (1-t)F(0,x) + tF(1,x) = (1-t)/3(1 - x^3/3 - x^6/8) + t\sqrt{\frac{1-x}{3}}$.

A weak solution also represents a certain family of continuous mappings with parameter $\theta \in (0,1)$. Really:

$$y^2(x,\theta) = 1/6(1-\theta^2)(1-x^3), y^2(x,1) = 0, y^2(x,0) = 1/6(1-x^3).$$

Finally, work [1] is devoted to the reconstruction of the method of power expansions. But its results overlap with the results of work [6].

For the limiting problem of smaller parameters formulated for the filtration of irrigation water through furrows, it is proved that the obtained weak solution has all the properties of an exact solution. The value of the Blasius constant in the weak solution, $\gamma = 1/2,\ y(0) = 1/3,\ \gamma = 1,\ y(0) = 0{,}4714$, and differs from the exact value by less than 0,4%. The weak solution to the limiting problem of smaller parameters has the form:

$$\phi(u) = 1/3\sqrt{1-u^3}\,.$$

Limit Problem of Smaller Parameters and Depression Curve Configuration. The Boussinesq limiting problem, considered in the previous chapter, is equivalent to the limiting problem of smaller parameters for the "filtration potential" $\varphi(u)$:

$$\varphi(u) = \int_u^1 z(\nu)d\nu, z = s/(2\sqrt{\tau}),\ \ \varphi(1) = \varphi(u_0) = 0, 0 < u_0 < 1.$$

From the Boussinesq equation follows the differential equation for $\varphi(u)$:

$$2\varphi\varphi'' + u = 0, D(\varphi) = (u\colon u_0 < u < 1), \qquad (6)$$

together with the limiting conditions $\varphi(1) = \varphi'(u_0) = 0$ forms the so-called typical limiting problem of smaller parameters [5-13].

The domain $D(\varphi)$ of the solution to equation (6) can be extended to the left of the point u_0 while maintaining continuity at this point. The continuation of solutions to equation (6) plays an important role in constructing the instantaneous depression curve. It is nontrivial: on the negative semiaxis $u < 0$, equation (7) has an exact solution $\varphi(u) = \sqrt{-2/3u^3})$, which can be easily verified by substitution.

A weak positive solution of equation (2.6) for a typical limiting problem has the form (negative, weak solutions are not considered, they have no physical meaning for the problem being solved) [14-23]:

$$\varphi(u;u_0 = 1/6\sqrt{4(1-u^3) - 3u_0(1-u^2)6u_0^2(1-u)}\,, u_0 < u < 1, \qquad (7)$$

or extracting the root u = 1:

$$\varphi(u;u_0 = 1/6\sqrt{(1-u) - (4(1+u+u^2) - 3u_0(1+u) - 6u_0^2)}. \qquad (8)$$

Real values φ (u; u_0) are possible if the conditions

$$u \le 1;\ u^2 + (1 - 3/4u_0)u + 1 - 3/4u_0 - 3/2u_0^2 \ge 0,$$

$$u \le 1;\ u'' \le u, u \ge u',$$

$$u' := -1/2 + 3/8u_0 + \sqrt{105/64u_0^2 + 3/8u_0 - 3/4},\ \varphi(u'; u_0) = 0,$$

$$u'' := -1/2 + 3/8u_0 - \sqrt{105/64u_0^2 + 3/8u_0 - 3/4},\ \varphi(u''; u_0) = 0,$$

$$u_0^2 + 8/35u_0 - 16/35 \geq 0; 1 \geq u_0 \geq -4/35 + 4 \cdot 6/35 = 20/35 = 4/7.$$

Finally, if $u_0 < 4/7$, then the function $u = 1$, has no real roots, except for $\varphi(u; u_0)$.
So, if $u_0 < 4/7$, then the potential of smaller parameters has no zeros on the interval (0,1); if $4/7 < u_0 < 1$, then there is a real zero u '. If $1 > u_0 \geq -\frac{1}{4} + \sqrt{\frac{1}{16} + \frac{2}{3}} = 0{,}6 \ldots$ then there are positive, different from 1, zeros of the function φ (u; u_0), 0≤u '<1; if 4 / 7≤u_0≤-1/4 + √ (1/16 + 2/3), then u <0.

The graph of the function $\varphi(u; u_0)$, built by formula (1), is shown in Figure 1. As can be seen from (7), the solution can be extended to the interval $u < u_0$ while preserving the continuity and smoothness of the solution at the point $u = u_0$. Moreover, if $u_0 < 4/7$, then the extended solution has no zeros, except for u = 1. If $4/7 < u_0 < 1$, the extended solution has a root located to the left of the point $u = 1$. For example, let $\varphi(0, u_0) = 0$. Then $u_0 = -1/4 + \sqrt{1/16 + 2/3} > 4/7$. For such a value of u_0, the function φ(u,u0) vanishes for u = 0, u = 1. Equation with the limiting conditions $\varphi(0, u_0) = 0$ form a homogeneous limiting problem of smaller parameters.

Consequently, the solution of the homogeneous limit problem is obtained by sewing together at the point u_0 two typical limit problems for the intervals $0 < u < u_0 - 0, u_0 + 0 < u < 1$while maintaining continuity at the point $u = u_0$

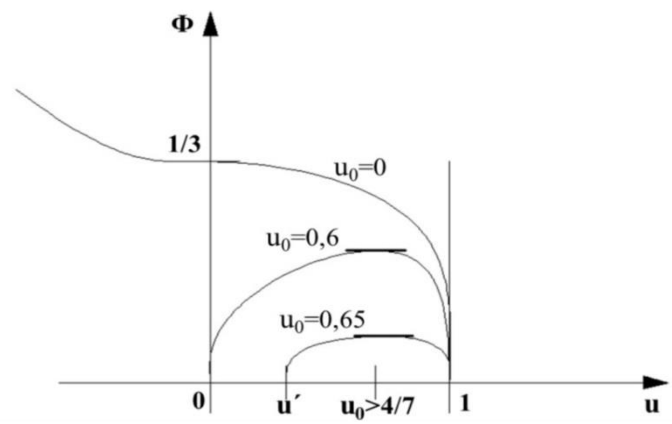

FIGURE 1. Potential $\varphi(u; u_0)$ for different u_0.

The value $a(u_0) := \varphi(u_0; u_0)$, by virtue of (8) is calculated as follows:

$$\varphi(u_0; u_0) := a(u_0) = 1/6(1 - u_0)\sqrt{4 + 5u_0}\,. \tag{9}$$

From formula (9), it follows that $a(u_0)$ decreases monotonically in the interval $0 < u_0 < 1$: $a(0) = 1/3 > a(u_0) > a(1) = 0$, figure 2. Otherwise, $u_0 \to +0$, then the value $a(0) = 1/3$practically coincides with the Blasius constant.

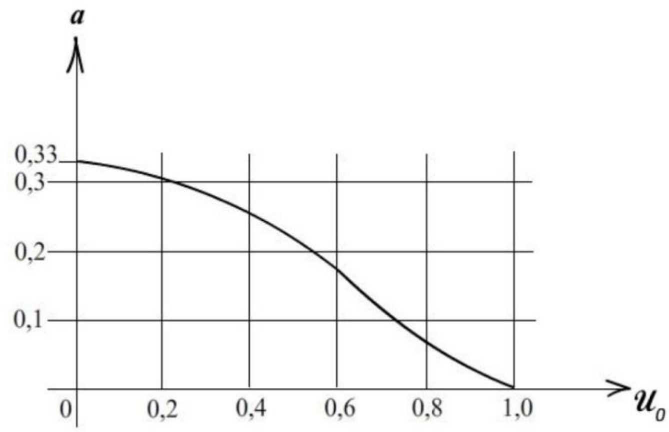

FIGURE 2. The value of the potential $\varphi(u0; u0)$.

The explicit form of the depression curve is: $\zeta(u, u_0) = -d\varphi/du$, or, using the weak solution (9):

$$\zeta(u, u_0) = \frac{u^2 - uu_0/2 - u_0^2/2}{\sqrt{4(1-u^3) - 3u_0(1-u^2) - 6u_0^2(1-u)}}. \tag{10}$$

As follows from (10), $u \to 1 - 0, \zeta \to \infty$. The depression curve crosses the $\zeta = 0$ axis at two points: at the point, u = u_0 and at the point $u = -u_0/2$. $4/7 < u_0 < 1$. In the range of values -$-u_0/2 < u < u_0$, the function $\zeta(u, u_0)$ has a discontinuity at the point $u = u'$, $u' -$ is a root of equation (8) different from u = 1. Exactly,

$$u' = -1/2 + 3u_0/8 \pm \sqrt{105/64u_0^2 + 3u_0/8 - 3/4}.$$

If $u_0 < 4/7$, then the continuation of solution (7) to the left of the point $u = u_0$ has no real zeros and on the interval $-\frac{u_0}{2} < u < u_0$ a continuous ambiguous continuation of the depression curve is formed, and the minimum point $\zeta(u, u_0)$ corresponds to the point where the depression exits onto the downstream slope, touching the downstream slope.

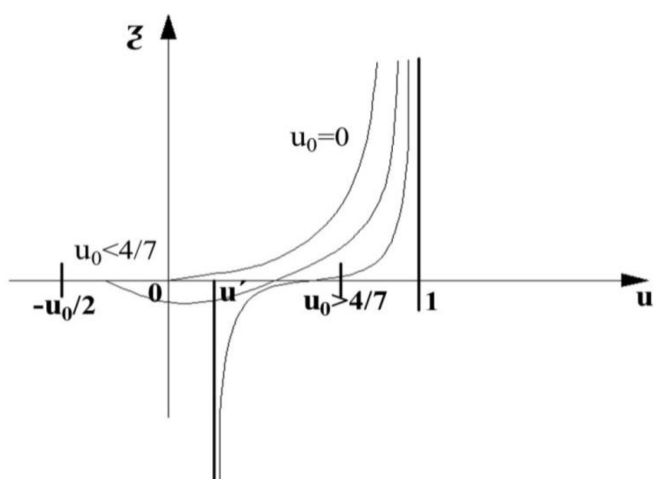

FIGURE 3. Formation of a depression curve.

We emphasize that this depression curve exists only for $u_0 < 4/7$, i.e., in long bridges with relatively small intervals of the protrusion (with a height of Δ not more than $\frac{4}{7} - u_e$ If $1 > u_0 > 4/7$, then this depression curve is not formed. In this case, the solution (8) of equation (6) extended to the left of the point u_0 has no zeros, except for the «obligatory zero» u = 1, Figure 3.

The potential of smaller parameters is related to the length of the active filtration zone, which is directly obtained from the identity:

$$\lambda := \int_0^\infty (1-u)dx = 2\sqrt{t}a(u_0). \tag{11}$$

Really:

$$\lambda(t) := \int_0^\infty (1-u)dx = 2\sqrt{t}\int_0^\infty (1-u)d\zeta = 2\sqrt{t}\int_{u_0}^\infty (1-u)\big(-\varphi''(u)\big)du\,,$$

and further integration by parts taking into account the limiting conditions for $\varphi(u)$. An easily verifiable chain of identities is valid $\theta_0(t)$:

$$\theta_0(t) := (u\partial u/\partial x)_{x=0} = \frac{1}{2\sqrt{t}}u_0\left(\frac{du}{d\zeta}\right)_{\zeta=0} = \frac{a(u_0)}{\sqrt{t}}. \tag{12}$$

So, from formula (12), it follows that the solution of the differential equation (1.14), such that $\lambda(0) = 0$, is: $\lambda(t) = 2\sqrt{t}a(u_0)$ which coincides with formula (11). Consequently, at the same time τ, the length of the active filtration zone through the cotton furrow increases with decreasing $(t) \leq 2/3t, t \geq 9/4\lambda^2$.

CONCLUSION

The solution of the equation of parameters with all the properties of the exact solution is obtained analytically. The obtained numerical solution of the parameter problem for the infiltration flow during furrow irrigation of cotton agrees with the analytical one. The depression curves obtained using the settling method are consistent with the analytical solution. The moisture suction gap tends to zero on the lysimetric complex. For L / H, more than eight dehumidification intervals are zero.

The inconsistent obtained solutions show that the created method of numerical modeling of saturated-unsaturated infiltration flow during furrow irrigation of cotton reliably describes the process under consideration.

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