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Critical point analysis of an irregular surface model

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Abstract. The article discusses approaches to modeling a surface by its discrete values. Possible passage positions are shown when modeling the surface according to the amount of information. Existing classification schemes for models according to the structure of input data are considered. The difference in the concepts of mathematical modeling in a narrow and broad sense is shown. The concept of an irregular critical point of a surface is considered, which is associated with the concept of the rank of a map. there are two types of metric points metrizing a given space. The definition of the curvature of the vertex and the degree of curvature is given. Types of points on the surface are defined. a method is proposed for determining irregular (critical) points of a surface by differentiating a matrix of initial data. Here the second partial and mixed derivatives are calculated. Possible cases of analysis of each vertex of the surface are considered. For each vertex, its degree of curvature and the type of vertex are determined. The original matrix with the source data is converted to a new matrix, where the elements show the type of vertex for each point. The task of identifying the desired model is reduced to constructing it in each cell.

1.   Introduction

The variety of approaches to the problem of surface construction on its discrete values reflects the multivariability of situations arising at its solution in various subject areas. Multiple variabilities of situations occur due to specificity of source data collection, their quantity, level of random observation errors, properties of the modeled surface.

The following initial positions are possible when modeling the surface by the volume of information:

1) Input data for building a surface model are sufficient. Thus a priori knowledge of properties of the modeled surface guarantees to a certain extent unambiguousness and adequacy of the model.

2) Input data for building a surface model are not enough. At the same time, the subjectivity of approach to the choice of modeling methods, to the estimation of model properties - "this is to a large extent the area of extrapolation, forecast" - is particularly affected. [4]

In existing schemes of classification of surface models, the following are offered as classification characteristics:

1) Data structures (composition and organization of interconnections of surface information), mathematical methods (surface restoration device), algorithms for transformation and movement of information;

2) Computer memory capacity at software implementation of the models, the cost of obtaining it, processor time;

3) Evaluation of modeling accuracy, resistance to change errors (possibility of their effective filtration), manufacturability of surface restoration algorithm. [5]

The method of mathematical modeling is understood as a display of input information $\{J_1\}$ utilizing some operator (conversion) P into output information $\{J_2\}$, as a result of which the given point set M_1 of power N_1 is converted into another point set M_2 of power N_1. [3]

By the nature of the input data (ID) - the finite number of points selected from the set of points of the displayed surface, they can be divided into three subclasses: irregular (ID incidents to characteristic surface points), partially regular (ID incidents to characteristic surface lines), regular (ID incidents to the nodes of the regular network), the latter is divided into uniform and uneven. Uniform ID structures belong to a network of squares, identical rectangles, equilateral triangles, etc., while non-uniform structures belong to networks of arbitrary rectangles, parallel profiles, etc.

A distinction is made between mathematical modeling in a narrow and wide sense

Mathematical modeling in the narrow sense of a specific object is understood as a set of equations, inequalities and other restrictions and conditions of their resolution, which allow the created coordinate system to determine the object solely. However, in modeling we compare the object not only with mathematically similar object - model but also with the theoretical concept, as well as with other objects suitable for this concept. Mathematical modeling (in the broad sense) is understood as mathematical formalization of the problem being solved, "embodiment of formal theory". Formal theory is defined as a motorcade T=<Ω, {F}, {p}>, where Ω is a set of relationship names, {F} is a set of formulas without free variables, {p}. - a set of exit rules. In the presence of a basic set, each character from {F} acquires a real content, turning into a real attitude on a given set, and defines a specific model as an implementation of formal theory. The number of formal theory implementations can be unlimited, which makes it possible to distinguish classes of isomorphic (quasiisomorphic) models with this structure among all mathematical models, regardless of the nature of the base set. [2]

The solution of many applied problems requires the creation of a model of some surface having both smooth areas and areas with different features.

Such surfaces can be considered as local - not smooth, and local irregular, having a set of both regular and irregular points.

The concept of irregular points is associated with the concept of the display rank $f(rank_{x_0}f)$ in point x_0. It is equal to the dimension of the subspace in R^m - the image R^n in the linear representation $D_{x_0}f$ because the matrix rank cannot exceed the number of rows and columns, then $rank_{x_0}f \leq min(n,m)$. The points at which $rank_{x_0}f = min(n,m)$ are regular. The points where $rank_{x_0}f < min(n,m)$ are irregular (critical, special). [1]

Let us select two types of points – vertices of some metric that metricize the given space. Let us define the curvature of each point $\{x_i\} \in X$ as $W^{\pm} = 2\pi - \theta$ where the θ sum of angles converging to the vertex.

Definition 1. Vertices with positive or negative curvature are called essential, vertices with zero curvature are called insignificant.

Definition 2. Let's introduce the concept of point curvature degree. An essential vertex has the first degree of curvature if $\frac{3}{2}\pi < W_1 < 2\pi$ the second degree of curvature at $\pi < W_2 < \frac{3}{2}\pi$, the third degree of curvature at $\frac{\pi}{2} < W_3 < \pi$, the fourth degree of curvature at $0 < W_4 < \frac{\pi}{2}$. The definition is also true for points with negative curvature. There are 6 types of vertices selected, depending on the sign of the point curvature and orientation relative to the bearing surface.

Definition 3. Let's name the point the type 1 vertex if the point of positive curvature $W^+ = 2\pi - \theta > O$ together with the neighborhood is located under the bearing surface, the type α_2 vertex if the point of positive curvature $W^+ = 2\pi - \theta > O$ together with the neighborhood is located above the bearing surface, a vertex of type γ – if the point has negative curvature $W = 2\pi - \theta < 0$, a vertex of type β_1 – if the point has zero curvature $W = 2\pi - \theta \approx O$ and the bearing surface, β_2 – is located under

the bearing surface together with the neighborhood, 3 – is located above the bearing surface together with the neighbourhood.

Regulating the source data network enables the user to differentiate the matrix:

$$z = \begin{bmatrix} z_{11}z_{12}.....z_{1n} \\ z_{21}z_{22}.....z_{2n} \\ .... \\ z_{m1}z_{m2}.....z_{mn} \end{bmatrix} \tag{1}$$

where z_{ij} – the values of heights of the incident starting points in some cell, make a preliminary analysis of the ID. The second private and mixed derivatives are calculated for each matrix element z_{ij} (the right and left extreme columns remain unchanged, the top and bottom rows are the area boundary).

$$\left[\frac{\partial^2 z}{\partial x^2}\right]_{ij} = \frac{z_{i+1,j} - 2z_{i,j} + z_{i-1,j}}{\Delta x_1 \Delta x_2} ; \left[\frac{\partial^2 z}{\partial y^2}\right]_{ij} = \frac{z_{i,j+1} - 2z_{i,j} + z_{i,j-1}}{\Delta y_1 \Delta y_2} ;$$

$$\left[\frac{\partial^2 z}{\partial x \partial y}\right]_{ij} = \frac{z_{i+1,j+1} - z_{i+1,j-1} - z_{i-1,j+1} + z_{i-1,j-1}}{4(\Delta x_1 + \Delta x_2)(\Delta y_1 + \Delta y_2)} ; \tag{2}$$

Hessian A and square form (Ax,y) are defined

$$A = \begin{vmatrix} \left[\dfrac{\partial^2 z}{\partial x^2}\right]_{ij} & \left[\dfrac{\partial^2 z}{\partial x \partial y}\right]_{ij} \\ \left[\dfrac{\partial^2 z}{\partial x \partial y}\right]_{ij} & \left[\dfrac{\partial^2 z}{\partial y^2}\right]_{ij} \end{vmatrix} \tag{3}$$

2. The character of which reveals the type of the vertex under study. The following cases are possible:

$$A = \begin{vmatrix} -\left[\dfrac{\partial^2 z}{\partial x^2}\right]_{ij} & 0 \\ 0 & -\left[\dfrac{\partial^2 z}{\partial y^2}\right]_{ij} \end{vmatrix} \neq 0 \quad (Ax, y) = -\lambda' x^2 - \lambda'' y^2 \tag{4}$$

The vertex z_{ij} is an unborn critical point - the vertex of type α_1 (Fig. 1) of the first order:

$$A = \begin{vmatrix} +\left[\dfrac{\partial^2 z}{\partial x^2}\right]_{ij} & 0 \\ 0 & +\left[\dfrac{\partial^2 z}{\partial y^2}\right]_{ij} \end{vmatrix} \neq 0 \quad (Ax, y) = \lambda' x^2 + \lambda'' y^2 \qquad (5)$$
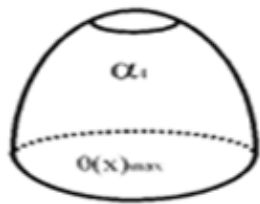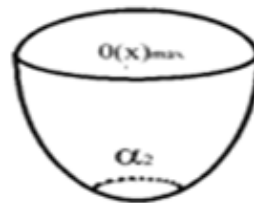

Figure 1. Vertex of type α_1

Figure 2. Vertex of type α_2

The apex z_{ij} is an unborn critical point of type (Fig. 2) of the first order;

$$A = \begin{vmatrix} +\left[\dfrac{\partial^2 z}{\partial x^2}\right]_{ij} & 0 \\ 0 & -\left[\dfrac{\partial^2 z}{\partial y^2}\right]_{ij} \end{vmatrix} \neq 0 \quad (Ax, y) = \lambda' x^2 - \lambda'' y^2 \qquad (6)$$

The vertex z_{ij} is an unborn critical point - the vertex of type γ (Fig. 3) of the first order.

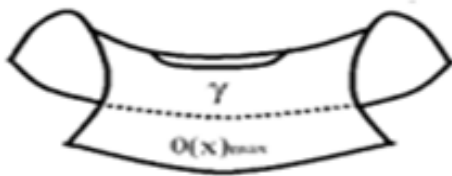$$A = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0 \quad (Ax, y) = 0 \qquad (7)$$
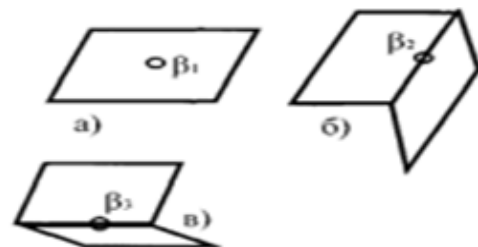

Figure 3. The vertex of type γ

Figure 4. The apex of type β, β_2, β_3

The apex z_{ij} is a degenerate critical point - the apex of type β, β_2, β_3 (Fig. 4).

$$A = \begin{vmatrix} \pm\left[\dfrac{\partial^2 z}{\partial x^2}\right]_{ij} & \pm\left[\dfrac{\partial^2 z}{\partial x \partial y}\right]_{ij} \\ \pm\left[\dfrac{\partial^2 z}{\partial x \partial y}\right]_{ij} & \pm\left[\dfrac{\partial^2 z}{\partial y}\right]_{ij} \end{vmatrix} \neq 0, (Ax, y) = \pm\lambda' x^2 \pm \lambda'' y^2 \pm \lambda''' yx \qquad (8)$$

The vertex z_{ij} is a non-critical point of type $\alpha_1, \alpha_2, \gamma$ with the complex of points of the second order, included in the maximal vicinity of the vertex z_{ij} (fig. 5).

Figure 5. The maximal vicinity of the vertex z_{ij}

Under real-world conditions of working with real variables on the computer, small derivative values $\left[\dfrac{\partial^2 z}{\partial x_i \partial y_i}\right] i = 1,2\, j = 1,2$ close to zero are identified as zeros.

The angles converging to each vertex are calculated and its degree of curvature is determined. As a result of the calculations, the matrix (1) is transformed into a matrix:

$$\begin{bmatrix} z_{11} & z_{12} & \dots z_{1n} \\ z_{21} & \alpha_i(w_i^{j\pm}) & \dots z_{2n} \\ \dots\dots & \dots\dots & \dots\dots \\ z_{m-1,1} & \beta_i(w_i^{j\pm}) & \dots z_{m-1,n} \\ z_{m1} & z_{m2} & \dots z_{mn} \end{bmatrix} \qquad (9)$$

This matrix allows you to visually identify the most characteristic (structural lines) surface areas.

To solve identification tasks the regularity model is suggested:

$$\Delta^2(B) = \frac{\sum_{j=1}^{N_B}(q_{table} - q_M)^2}{\sum_{i=1}^{N} q_{table}^2} \rightarrow \min \qquad (10)$$

Where $\Delta^2(B)$ is the middle square mistake, counted on new points, that had not been used to get model coefficient estimation. N_B – is the number of points of separated checking data choosing, q(table) – is the table initial data values, qm are values counted for this model.

Criterion is based on ID dividing into two parts: educating part N_A and checking part N_B. All identification data are ranged into a row according to values of their dispersion starting from the middle value, and this row is divided into two indicated parts (N=N_A+N_B)

To solve model identification tasks of surfaces, identification data are divided into three parts: N_A-mobile comprising points incident to structural ones, N_B – educating parts, N_C – checking ones. The following operation sequence is performed:

1) algorithms are realized to seek structural lines and triangulation of the model task field.

2) points, incident to structural lines , are selected into mobile choosing N_A

3) the remained points of ID table are divided into two parts depending on dispersion value starting from the middle value; new selections are being formed N_B and N_C so $N_A \supset N_B, N_A \supset N_C$

4) at points of educating selection N_B the model is being created
through the method chosen by a user.

5) exactness of created model is being cleared up by using points of checking selection N_c, on the base of regularity criterion Δ^2 [6]

For geometrical models the regularity criterion accepts values lying in the range from "0" till 1($0<\Delta^2<1$) under successful modeling method choosing; obviously, the case Δ^2=0 under geometrical modeling is not reachable. At the same time regularity $\Delta^2(B)$ criterion and its modifications Δ^2 qm, Δ^2(A, B) give the opportunity to determine homomorphism degree between relief site and its model: 1) $0,8 \leq \Delta^2 <1$ is weak homomorphism; 2) $0,5 \leq \Delta^2 \leq 0,8$ is middle homomorphism; 3) $0<\Delta^2 \leq 0,5$ is strong homomorphism.

References
[1]   Dubrovin B A, Novikov S P Fomenko A T 1998 *Modern Geometry, Methods and Applications vol. 1* (Moscow: Science) p 760
[2]   Shreyder Yu A, Sharov A A 1982 *Systems and models* (Moscow: Radio and connection) p 152
[3]   Stoyan Yu G, Yakovlev S V 1986 *Mathematical models and geometric design optimization methods* (*Kiyev: Nauk.Dumka*) p 268
[4]   Ivakhnenko A G 1989 *Reordered methods of model self-organization and clustering,* №4 (Automathics) p 15-17
[5]   Kuchkarova D F 2018 O*n the theory of topographic surfaces,* (Riga:LAP Lambert Academic Publishing/ Monography) p 119
[6]   Kuchkarova D F, Achilova D A *2019, Morphisms general properties of geometrical models,* Journal of Physics: Conference Series, (IOP Conf. Series: Journal of Physics: Conf. Series 1260 (2019) 072011)