Bayesian methods for combining climate forecasts

David B. Stephenson, Sergio Pezzulli, Caio Coelho (*)
Francisco J. Doblas-Reyes, Magdalena Balmaseda

1. Introduction
2. Conditioning and Bayes' theorem
3. Results

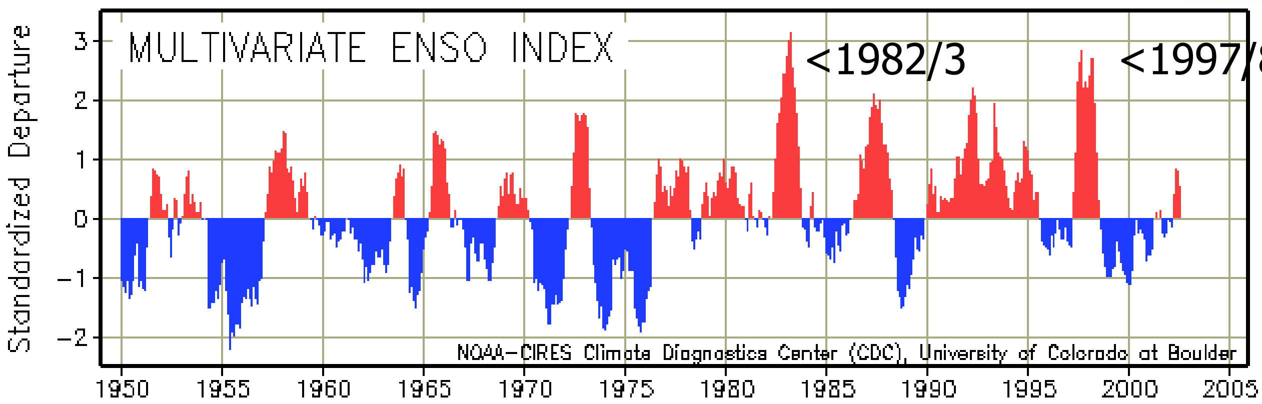
(*): Department of Meteorology, The University of Reading

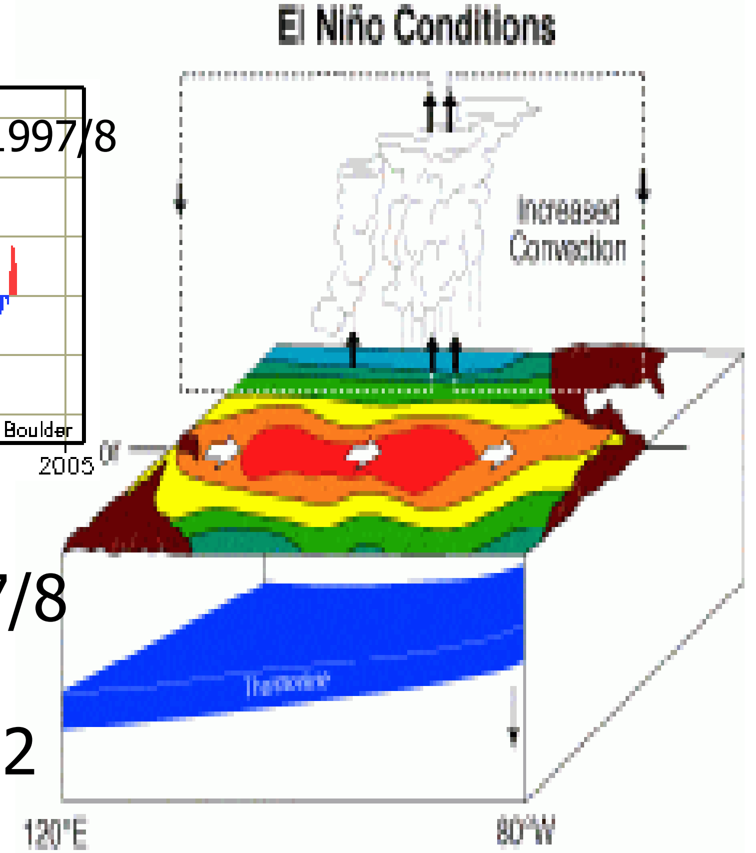
1. Introduction

Motivation

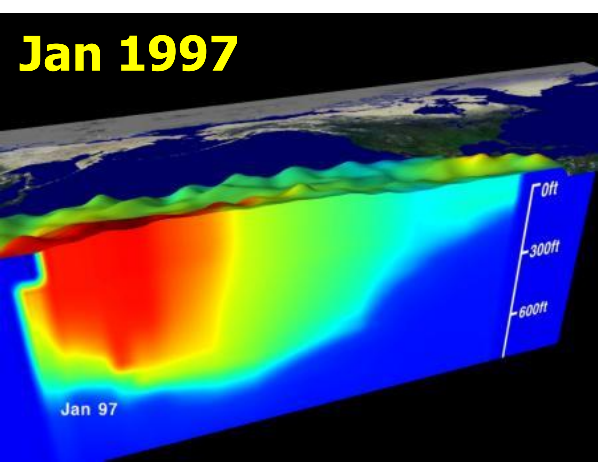
- Empirical versus dynamical forecasts?
- Why not combine both types of forecast in order to use ALL possible information?
- Ensemble forecasts + probability model → probability forecasts
- Use sample of ensemble forecasts to update historical (prior) probability information (post-forecast assimilation)

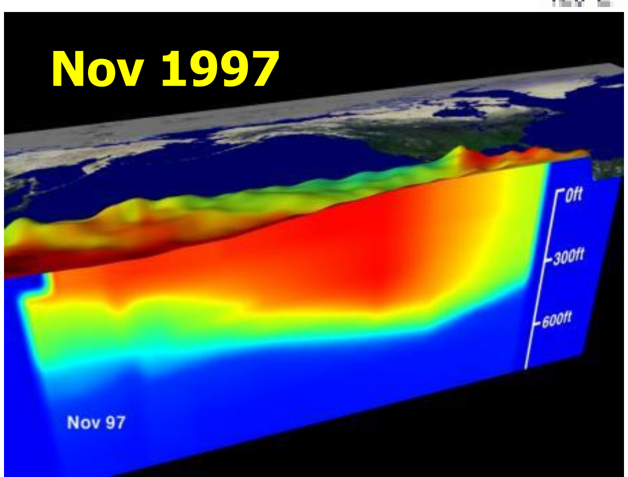
El Nino – Southern Oscillation

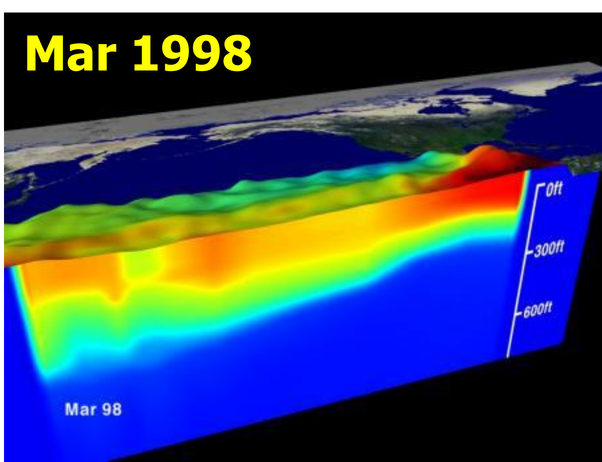




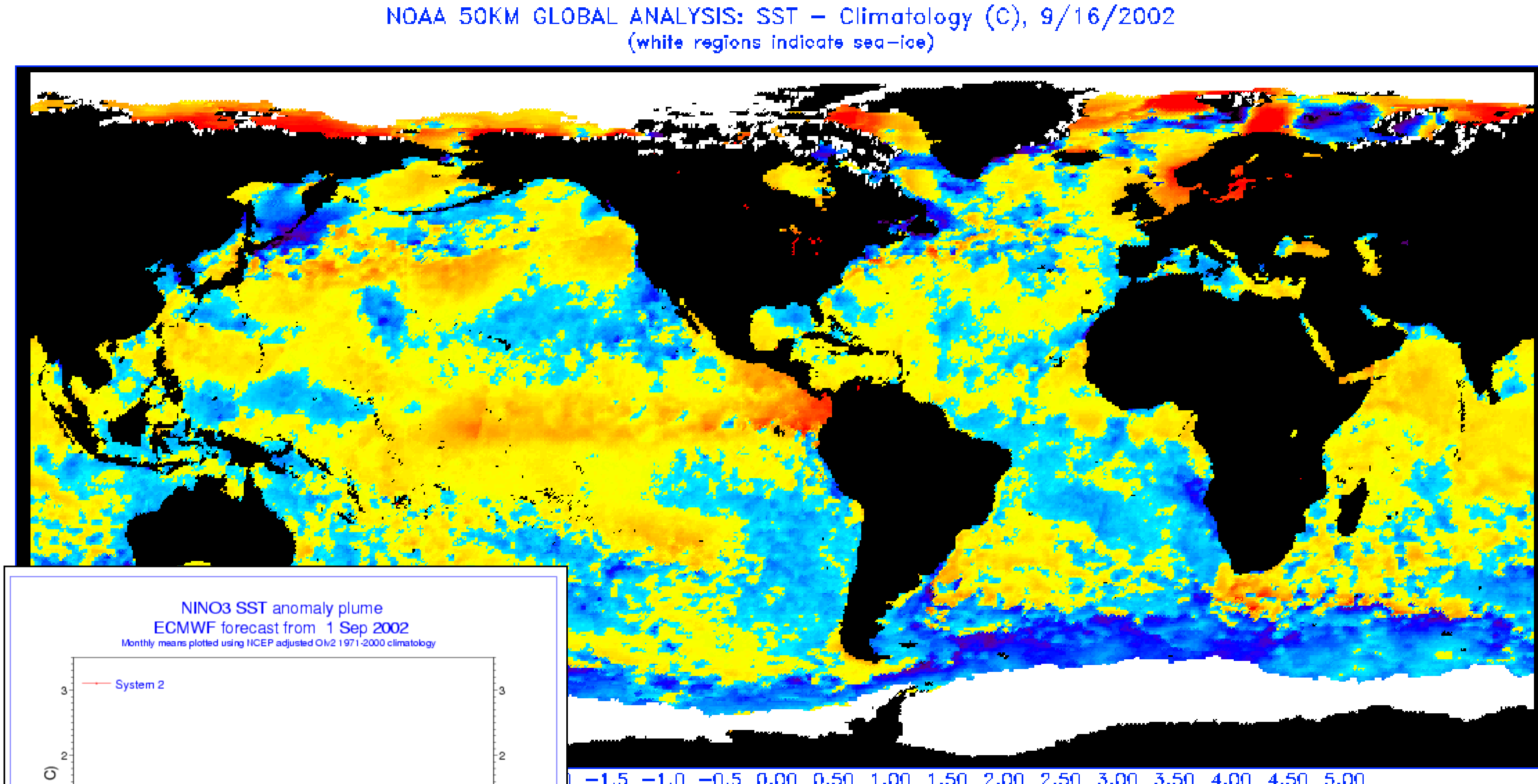
- Big El Nino events in 1982/3 and 1997/8
- La Nina/normal conditions since 1998
- El Nino event predicted for end of 2002

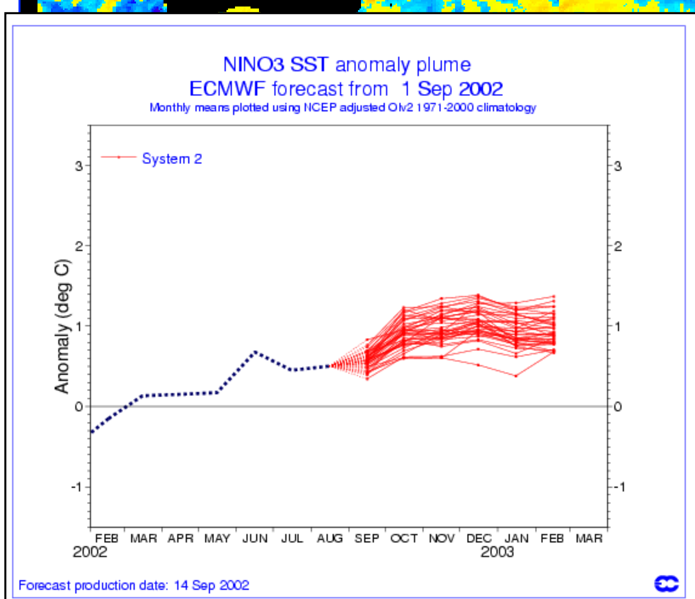






Recent sea temperature anomalies 16 Sep 2002





←ENSO forecasts from ECMWF, Reading
Sep 2002-Feb 2003

DATA

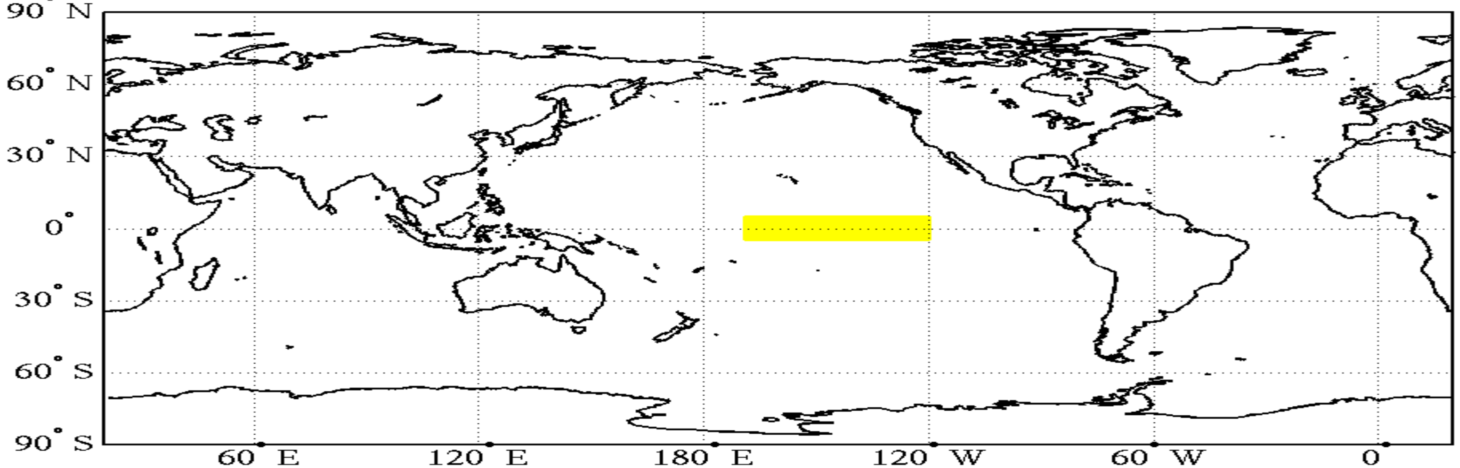
Sea Surface Temperatures (SST)

"at" location Nino 3.4

(5S - 5N , 170W - 120W)

December means of Nino 3.4:

- Reynolds SST : 1950-2001
- ECMWF DEMETER ensemble forecasts: 1987-1999



Some notation …

- Observed Dec Nino-3.4 θ_t
- Ensemble mean forecast $\overline{X}_t$
- Ensemble standard deviation s_X
- Normal (Gaussian) probability forecasts:

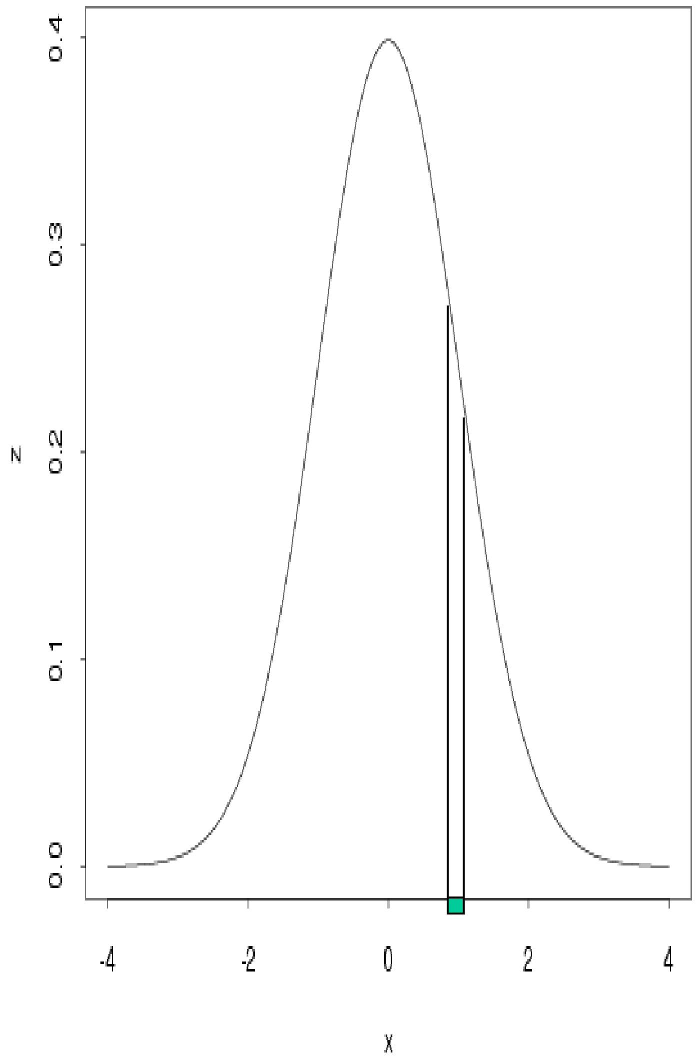
$$\hat{\theta}_t \sim N(\hat{\mu}_t, \hat{\sigma}_t)$$

$$\hat{\mu}_t = \text{forecast mean value}$$

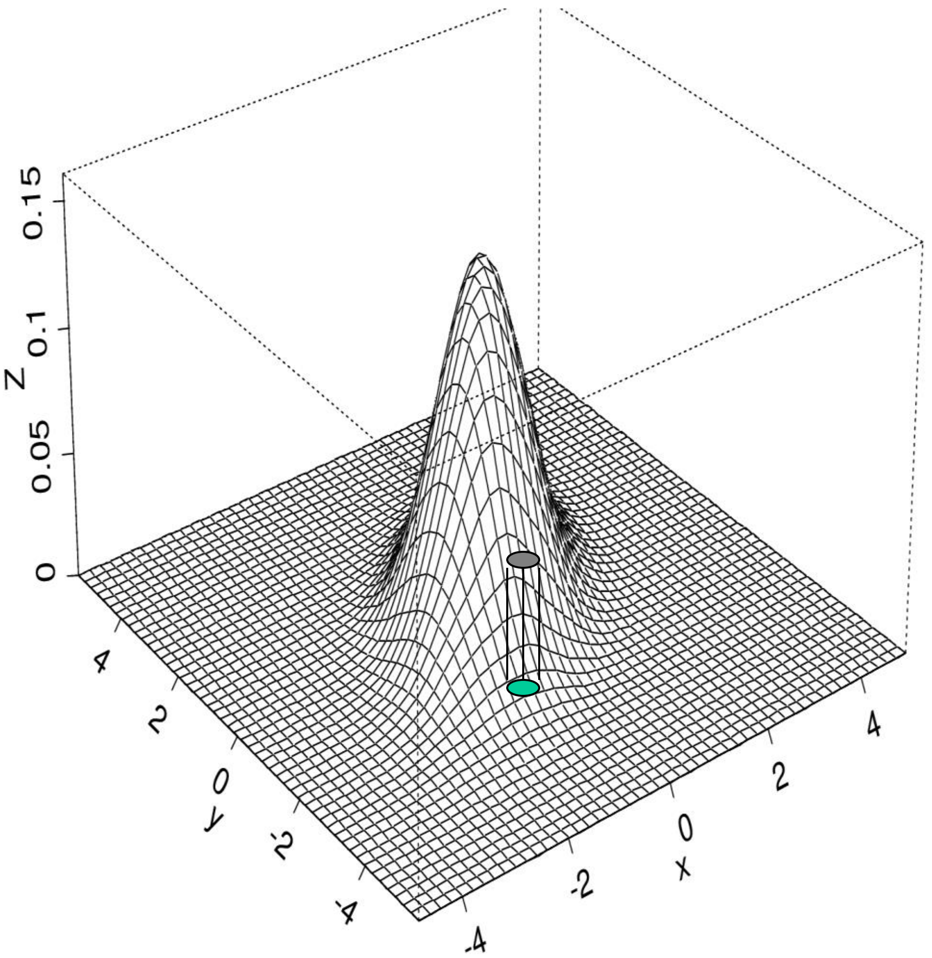
$$\hat{\sigma}_t = \text{forecast uncertainty}$$

2. Conditioning and Bayes theorem

Probability density functions (distributions)



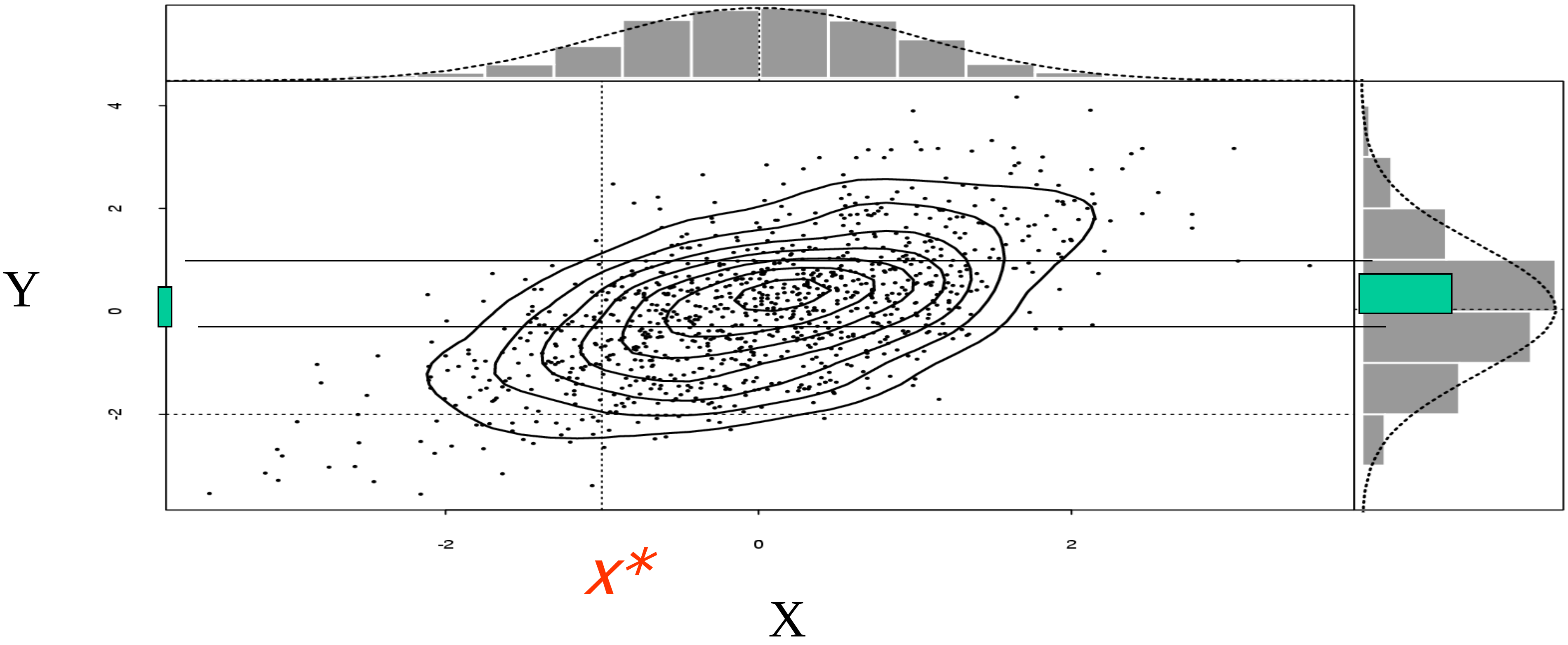
Uni-dimensional



Bi-dimensional
or Joint
distribution of X & Y

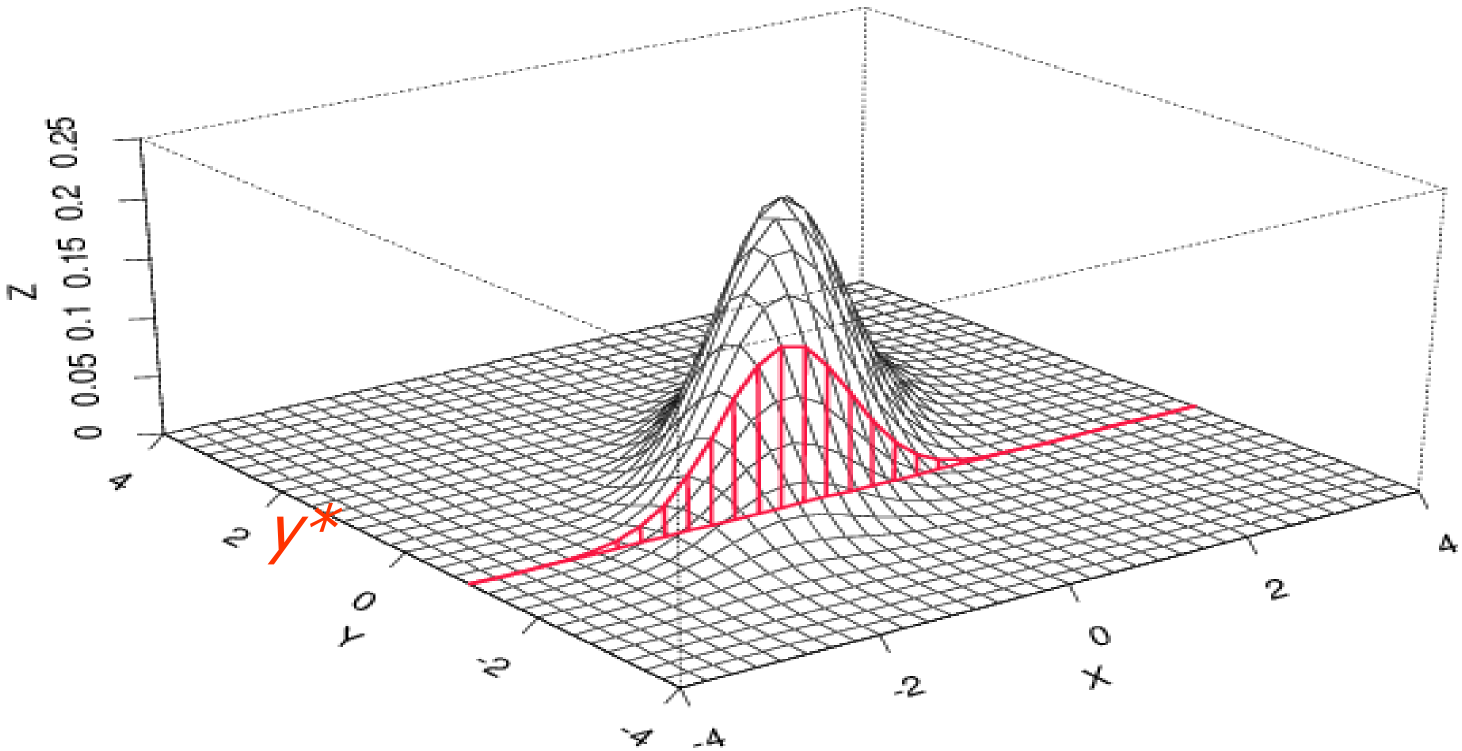
Marginal distributions

$$p(x^*) = \int p(x^*, y)\, dy$$



Conditional distributions

$$p(x \mid y^*) = p(x, y^*) / p(y^*)$$



Conditional-chain Rule

$$p(y)\,p(x|y) = p(x, y) = p(x)\,p(y|x)$$

Bayes Theorem

$$p(x|y) = p(x, y) / p(y)$$

$$\propto p(x, y)$$

$$= p(x)\,p(y|x)$$

Thomas Bayes
1701-1761

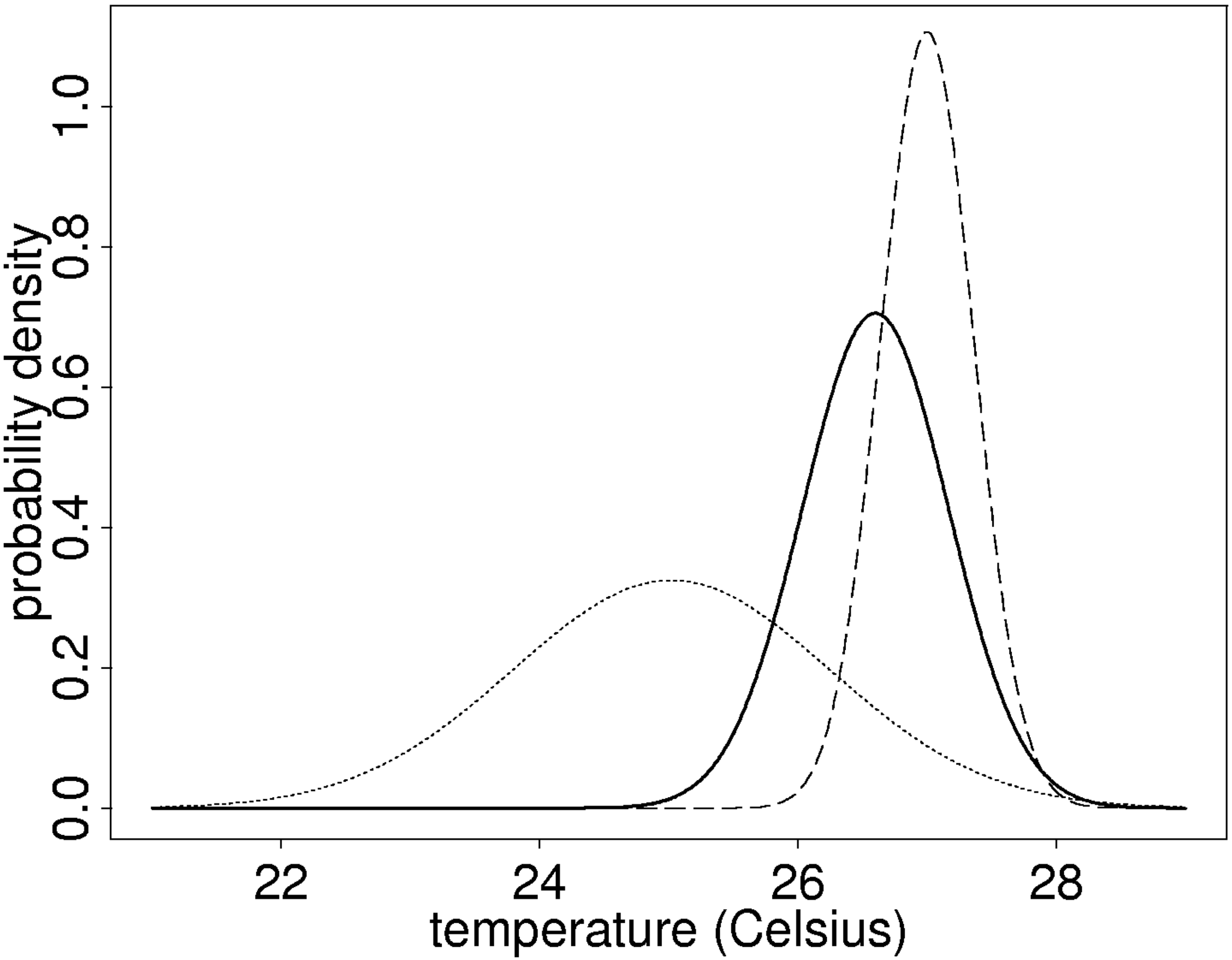
An Essay towards Solving a Problem
In the Doctrine of Chances.
Philosophical Transactions
of the Royal Society, 1763

The process of belief revision on any event
W (the weather)
consists in updating the probability of W when new information
F (the forecast)
becomes available

$$p(W \mid F) \propto p(W)\ p(F \mid W)$$

$$p(W) = N(\mu, \sigma^2)$$

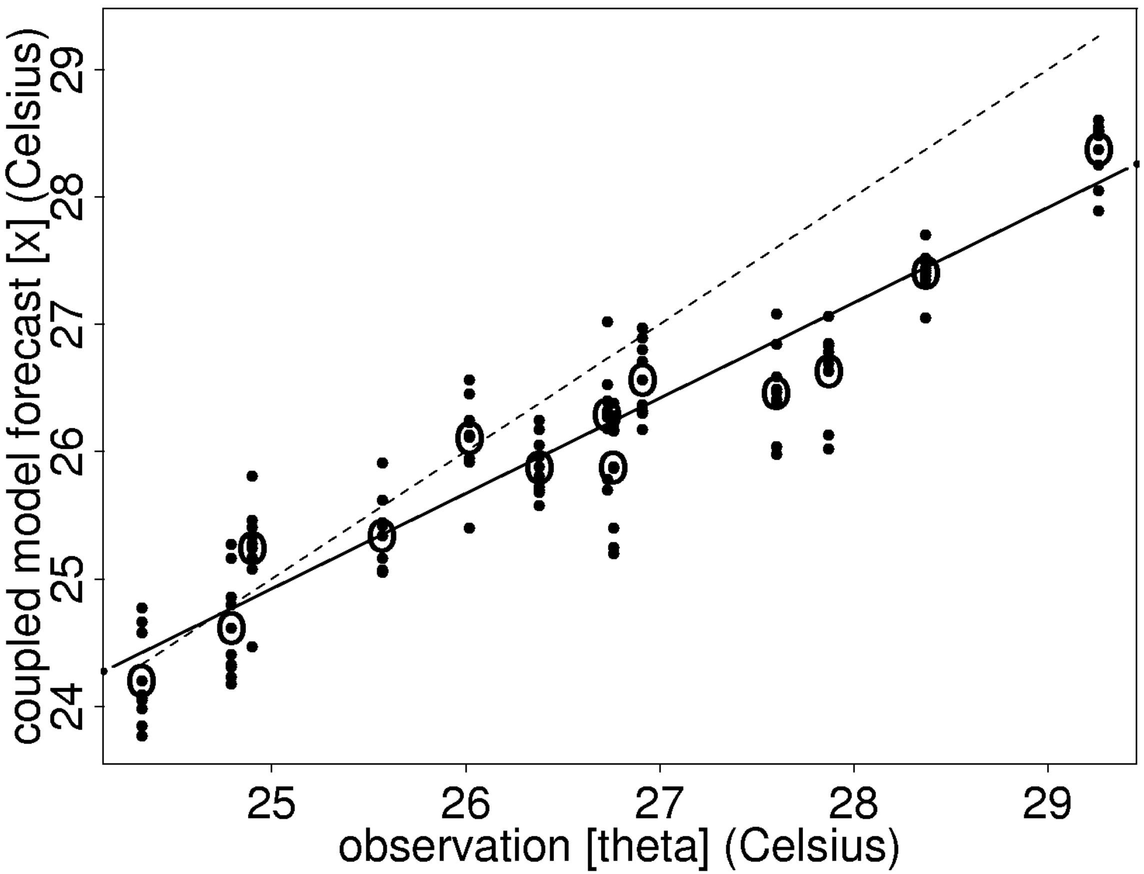
$$p(F \mid W) = N(\alpha + \beta W, \gamma V)$$



probability density
0.0
0.2
0.4
0.6
0.8
1.0
22
24
26
28
temperature (Celsius)

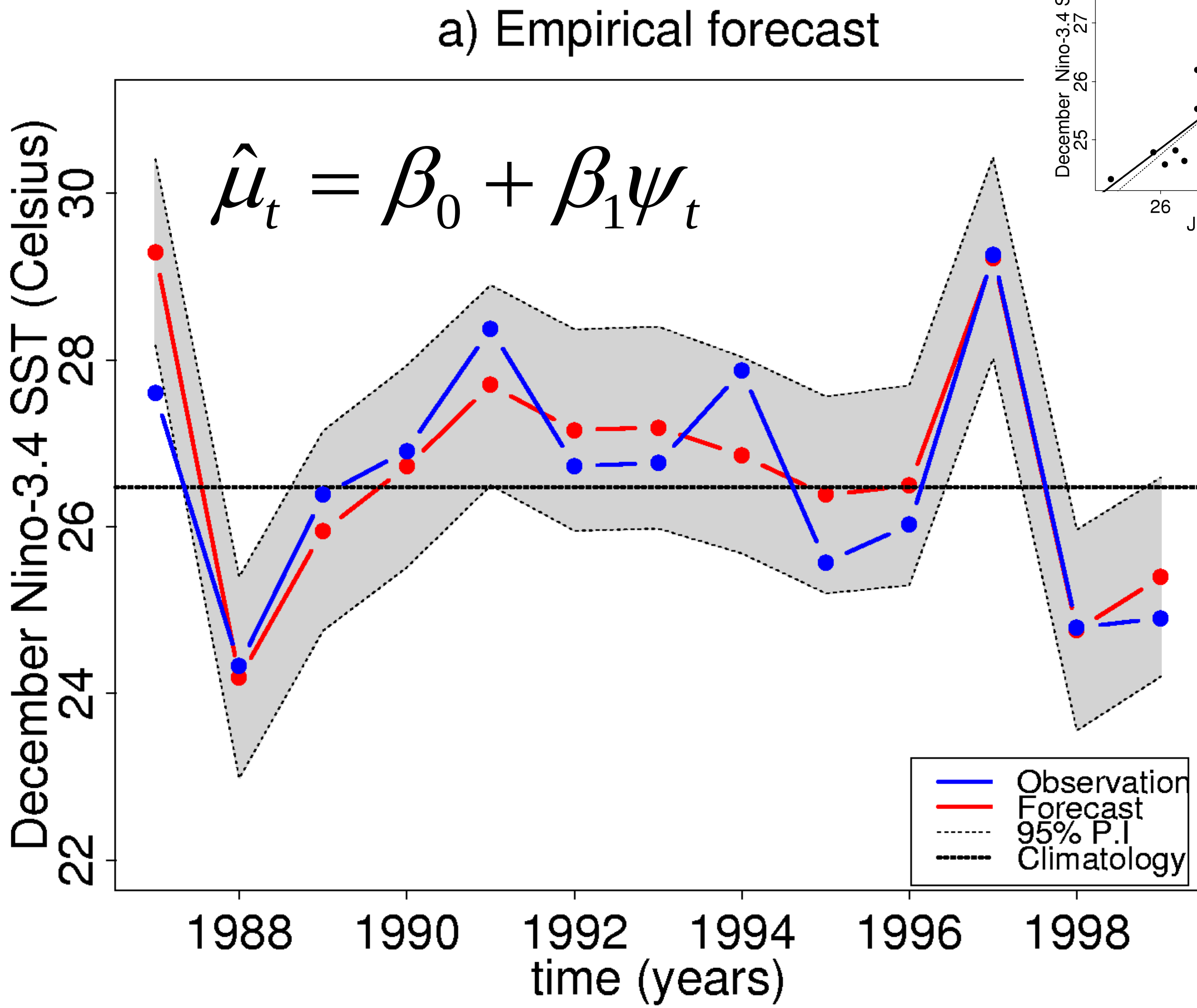
The Likelihood Model

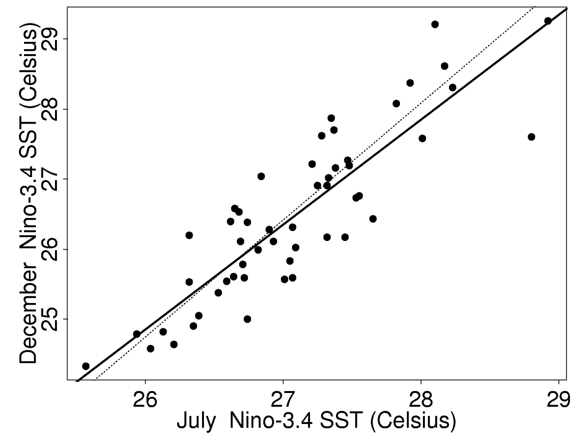
$$\bar{X}_t \mid \theta_t \sim N(\alpha + \beta\theta_t, \gamma V_t)$$



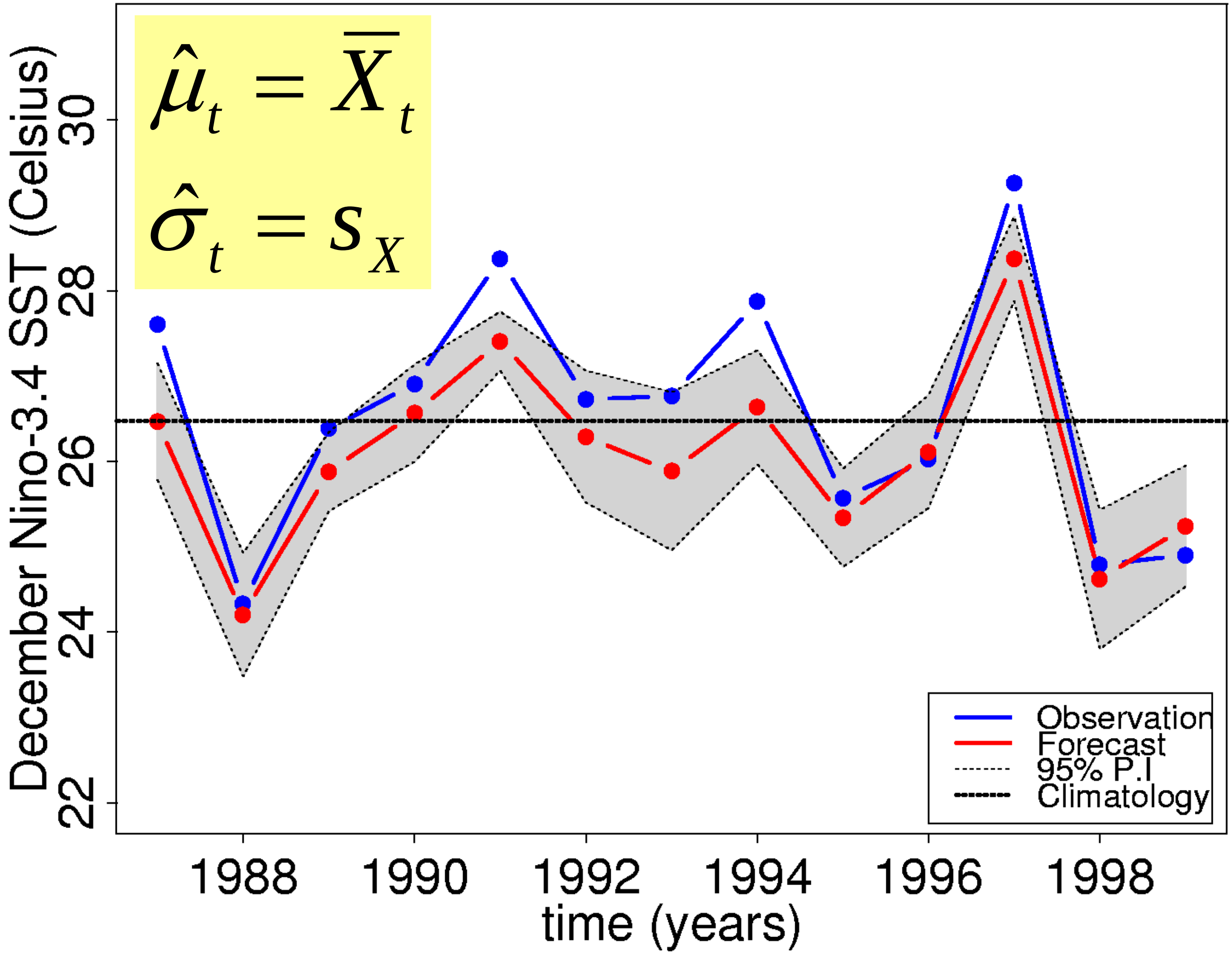
3. Forecast results

Empirical forecasts





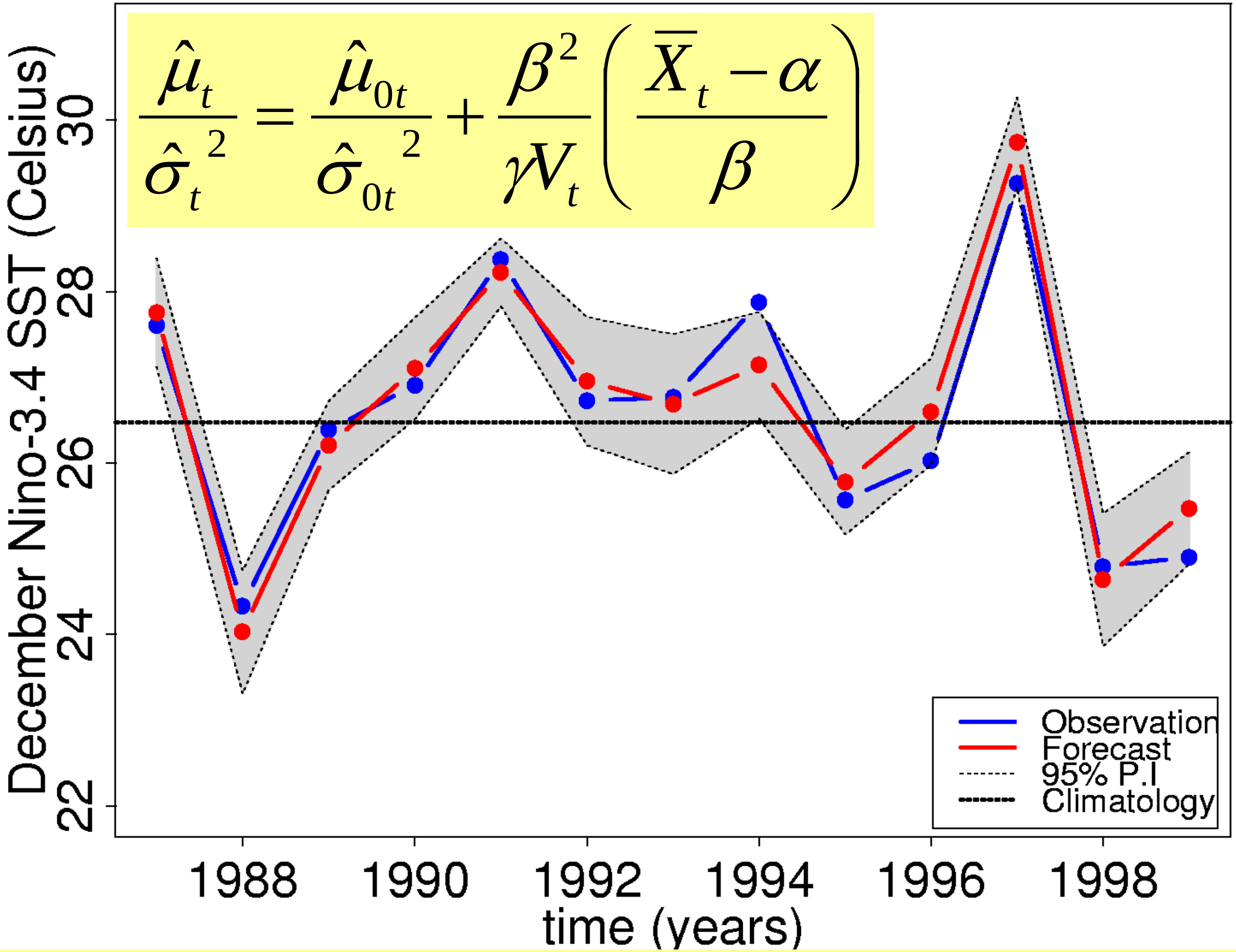
Coupled model forecasts



→ Note: many forecasts outside the 95% prediction interval!

Combined forecast

$$\frac{\hat{\mu}_t}{\hat{\sigma}_t^{\,2}} = \frac{\hat{\mu}_{0t}}{\hat{\sigma}_{0t}^{\,2}} + \frac{\beta^2}{\gamma V_t}\left(\frac{\overline{X}_t - \alpha}{\beta}\right)$$



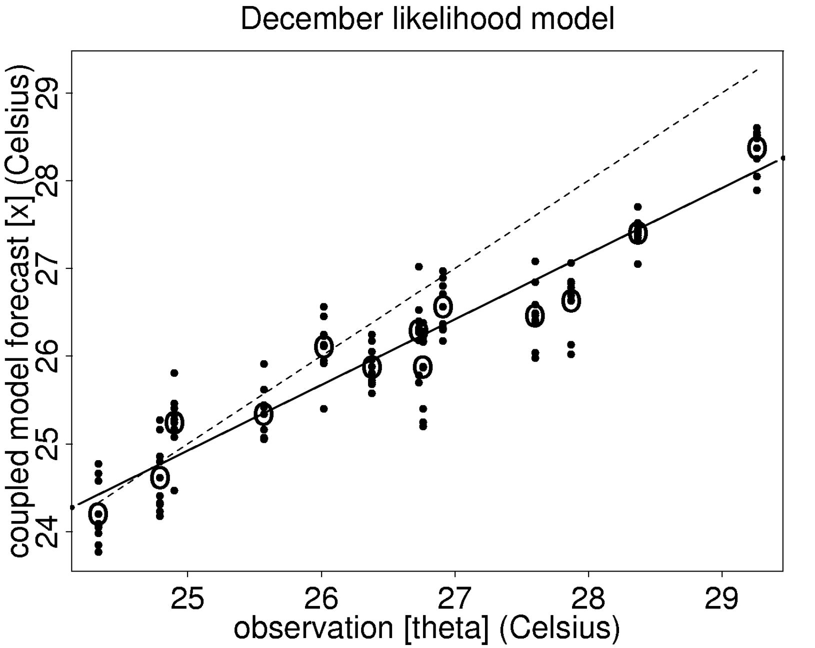
→ Note: more forecasts within the 95% prediction interval!

Mean likelihood model estimates

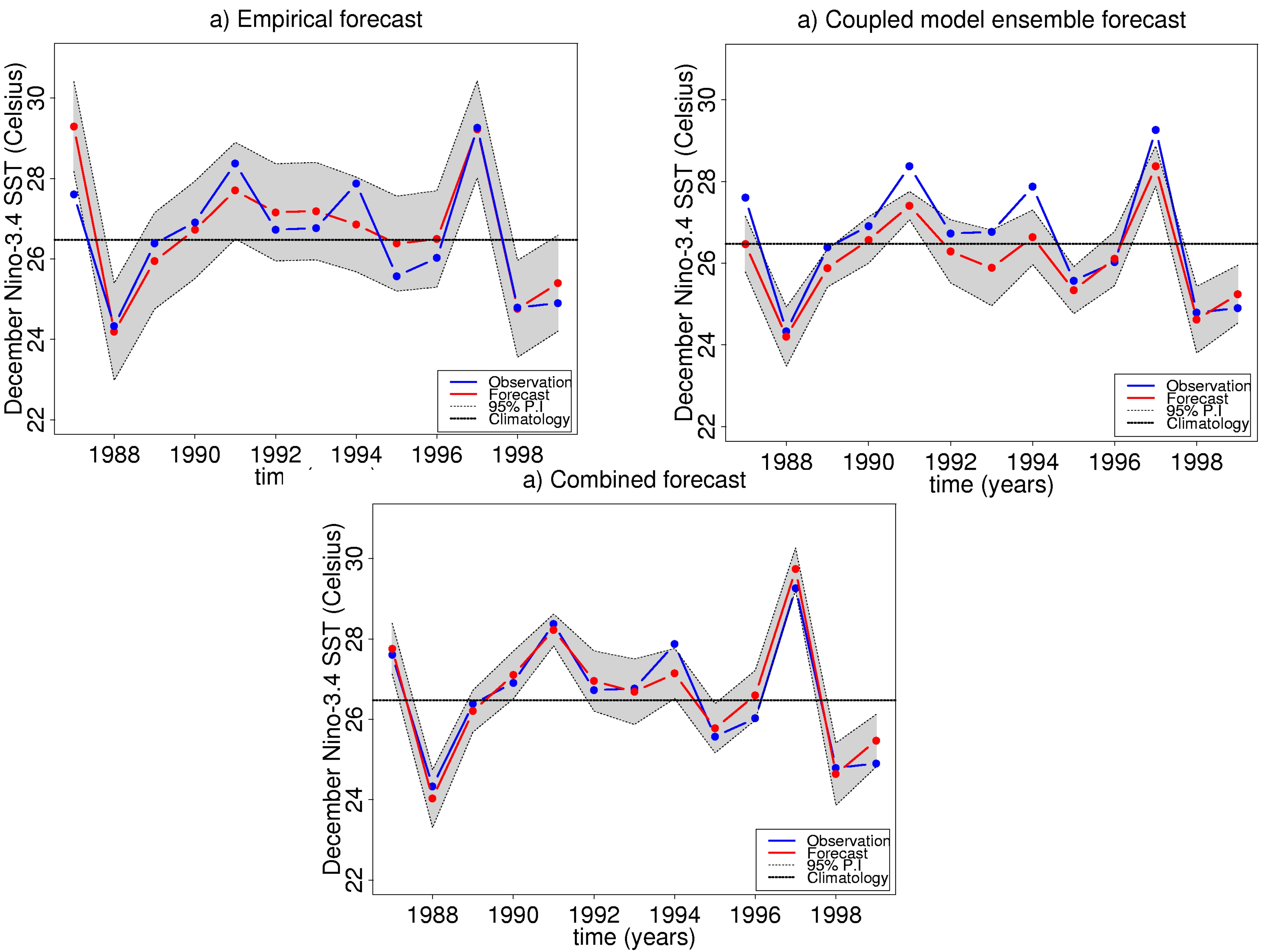
$$\hat{\alpha} = 6.27 \pm 1.44^0 C$$

$$\hat{\beta} = 0.75 \pm 0.05$$

$$\hat{\gamma} = 7.05 = m / m'$$



- ensemble forecasts too cold on average (alpha>0)
- ensemble forecast anomalies too small (beta<1)
- ensemble forecast spread underestimates forecast uncertainty


a) Empirical forecast
December Nino-3.4 SST (Celsius)
22
24
26
28
30
1988
1990
1992
1994
1996
1998
tim
Observation
Forecast
95% P.I
Climatology
a) Coupled model ensemble forecast
December Nino-3.4 SST (Celsius)
22
24
26
28
30
1988
1990
1992
1994
1996
1998
time (years)
Observation
Forecast
95% P.I
Climatology
a) Combined forecast
December Nino-3.4 SST (Celsius)
22
24
26
28
30
1988
1990
1992
1994
1996
1998
time (years)
Observation
Forecast
95% P.I
Climatology

Forecast statistics and skill scores

Forecast	MAE (deg C)	Skill Score	Uncertainty
Climatology	1.16	0%	1.19 deg C
Empirical	0.53	55%	0.61
Ensemble	0.57	51%	0.33
Combined	0.31	74%	0.32
Uniform prior	0.37	68%	0.39

Note that the combined forecast has:
→A large increase in MAE (and MSE) forecast skill
→A realistic uncertainty estimate

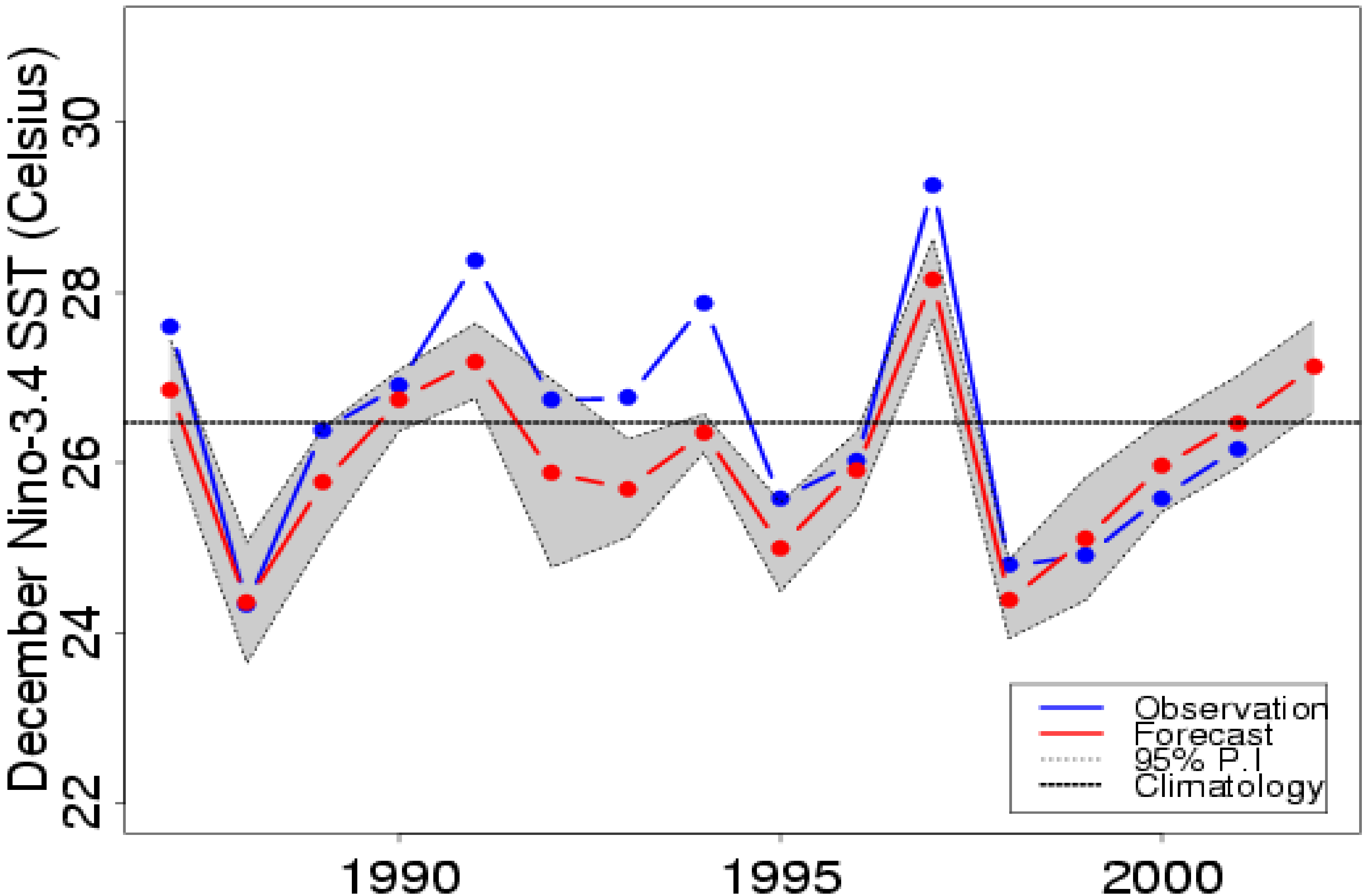
Conclusions and future directions

- Bayesian combination can substantially improve the skill and uncertainty estimates of ENSO probability forecasts

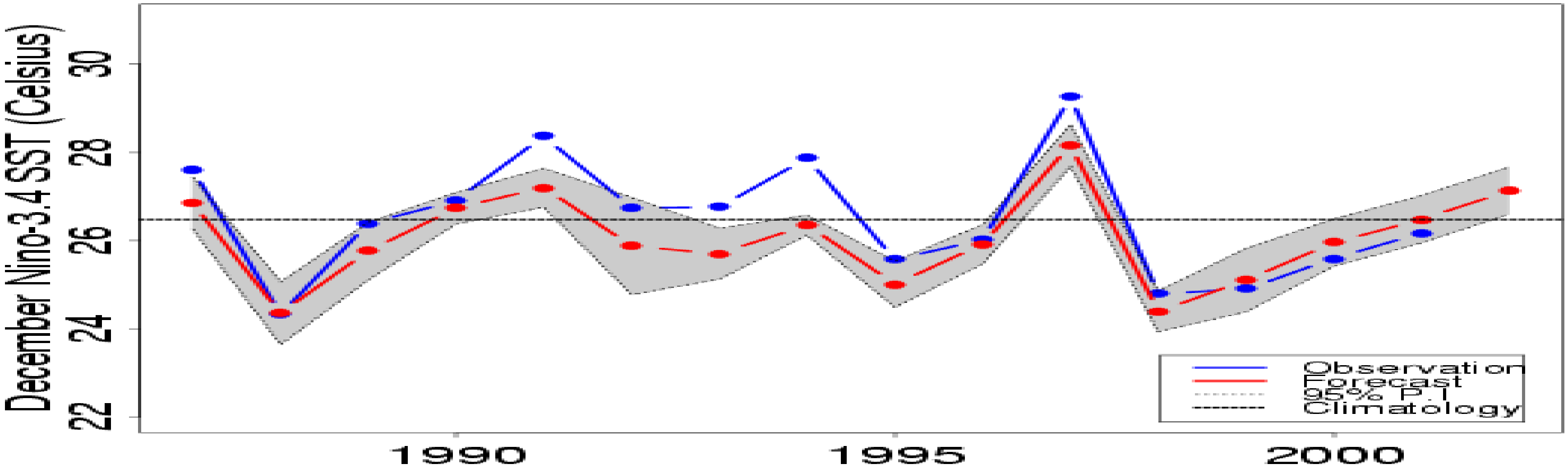
- Methodology will now be extended to deal with multi-model DEMETER forecasts

- Similar approach could be developed to provide better probability forecasts at medium-range (Issues: non-normality, more forecasts, lagged priors, etc.).

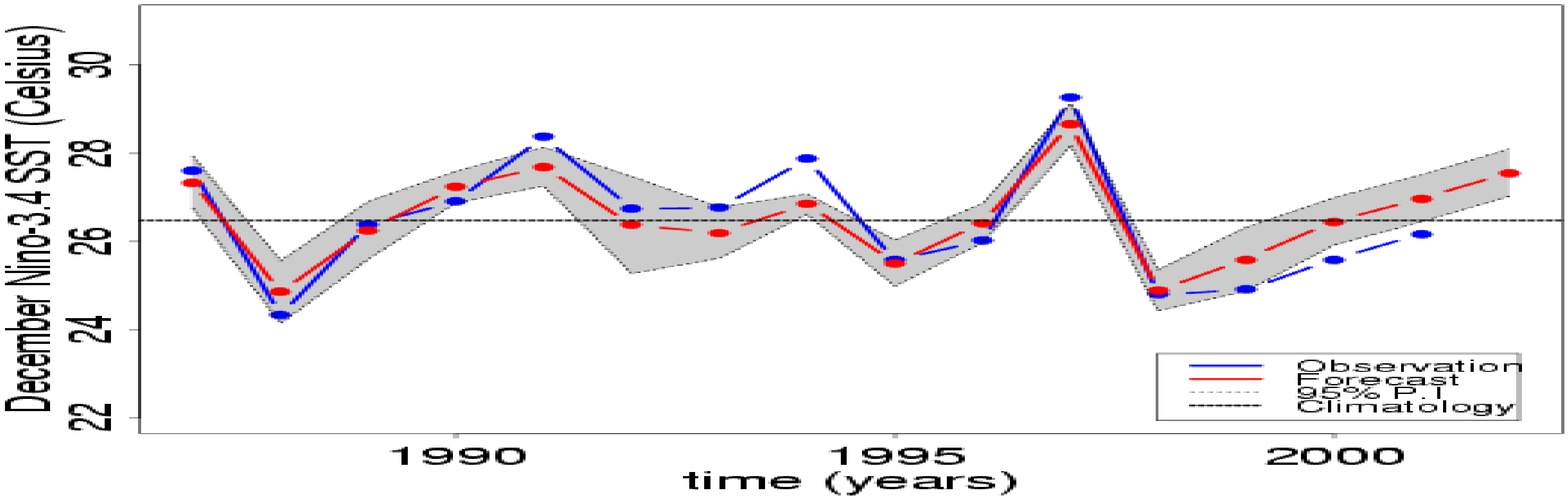
Coupled Model Ensemble Forecast



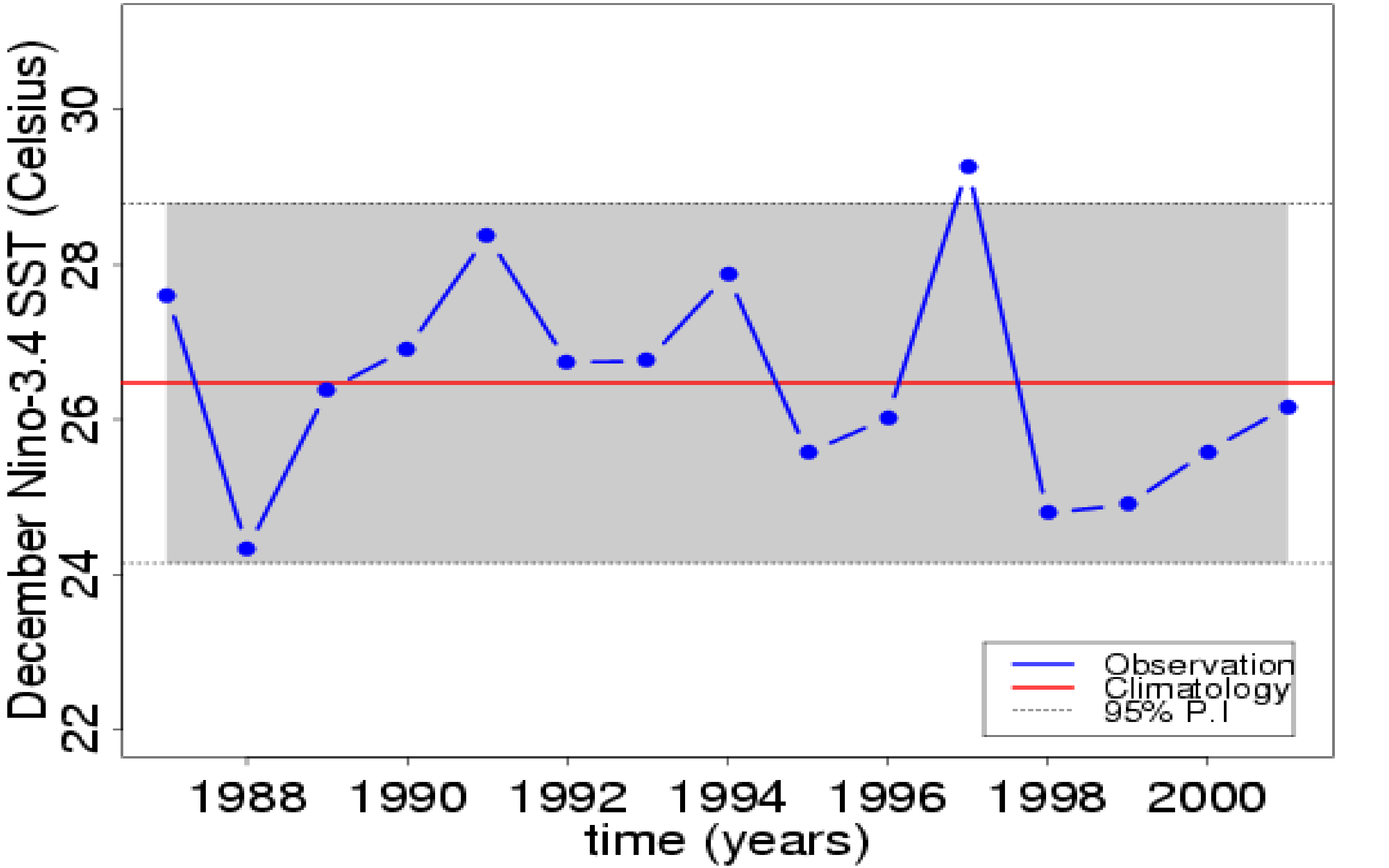
Ensemble Forecast and Bias Correction



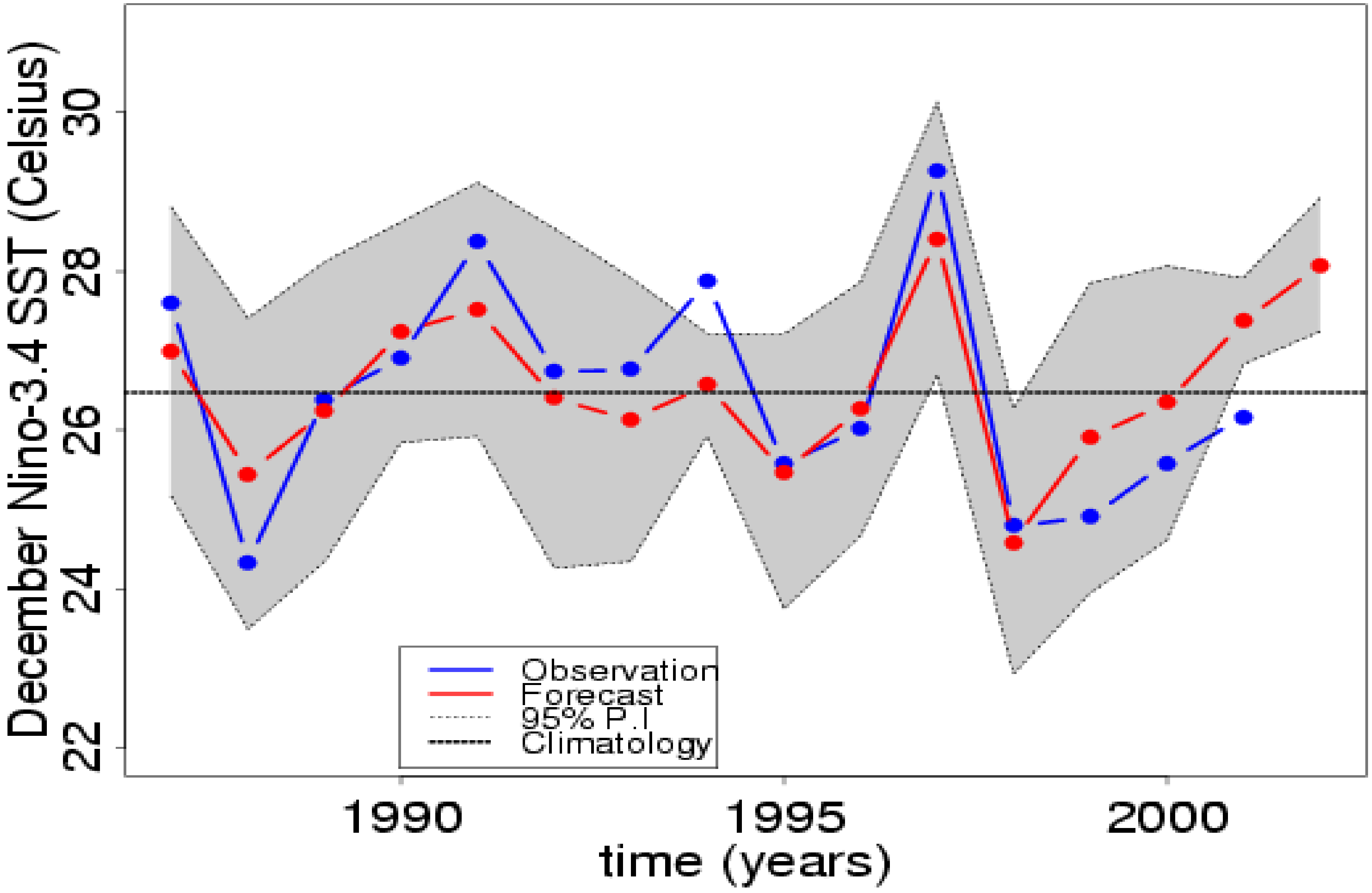
Bias Corrected



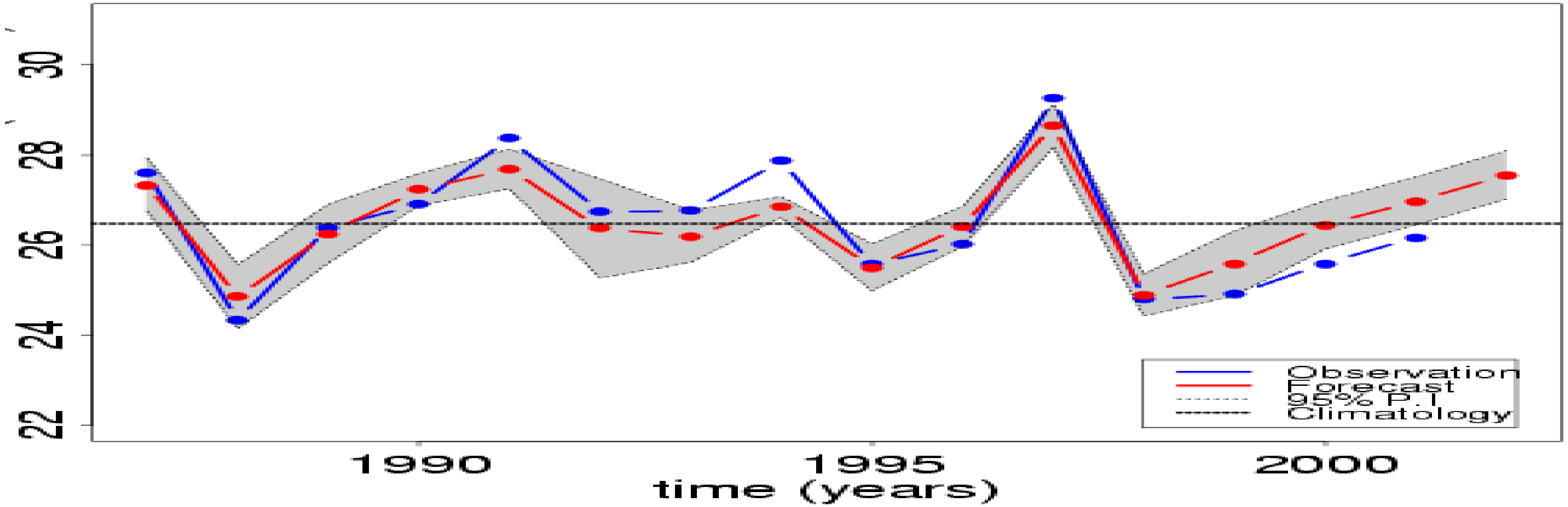
Climatology



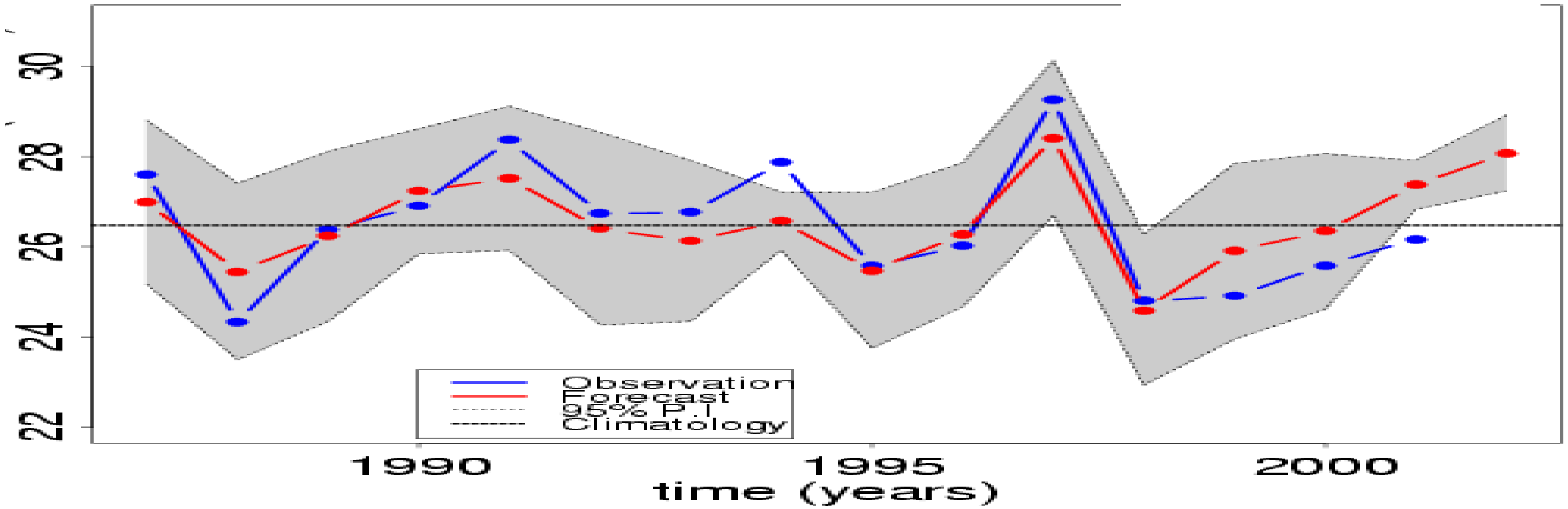
Climatology + Ensemble



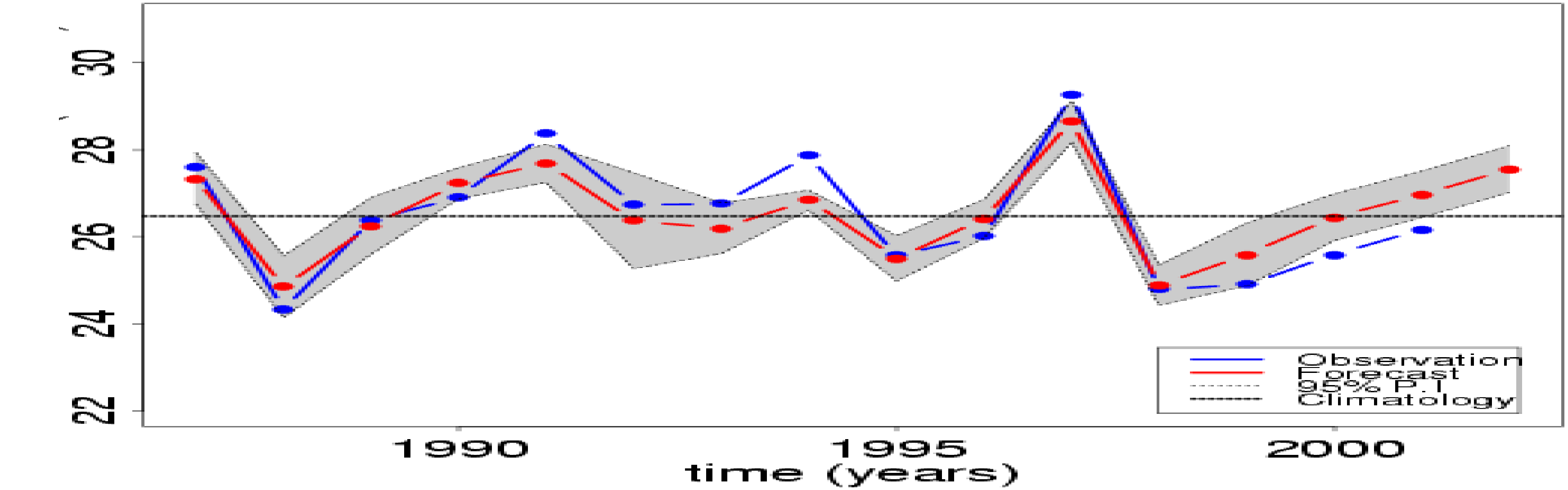
Coupled-Model Bias-Corrected Ensemble Forecast



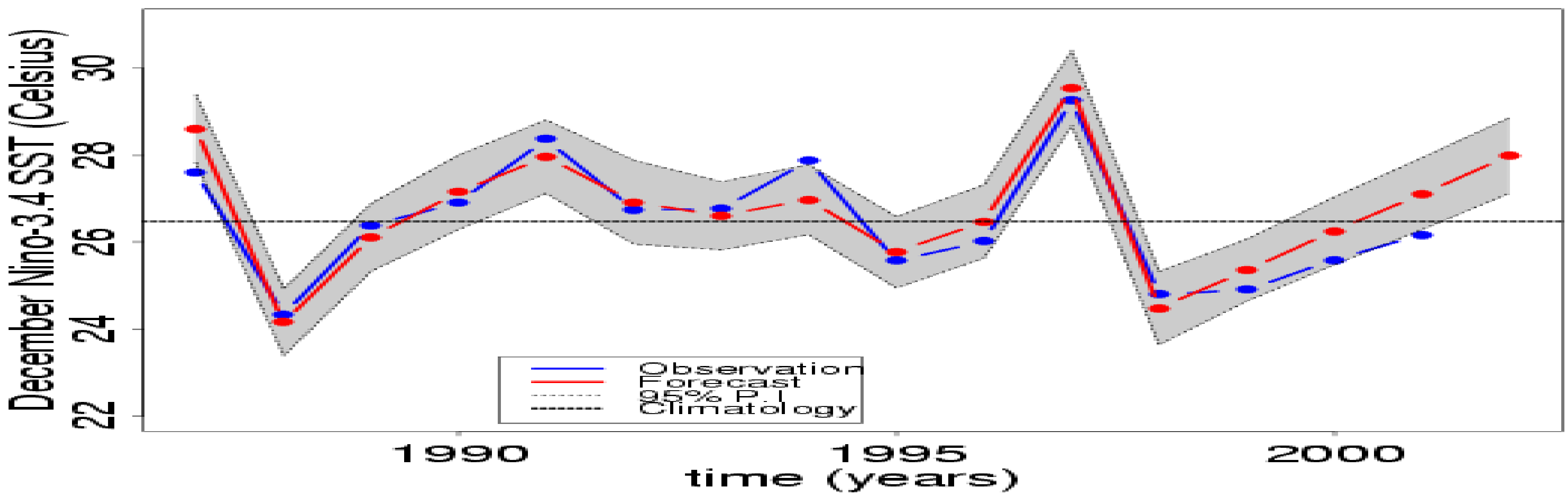
Climatology + Ensemble

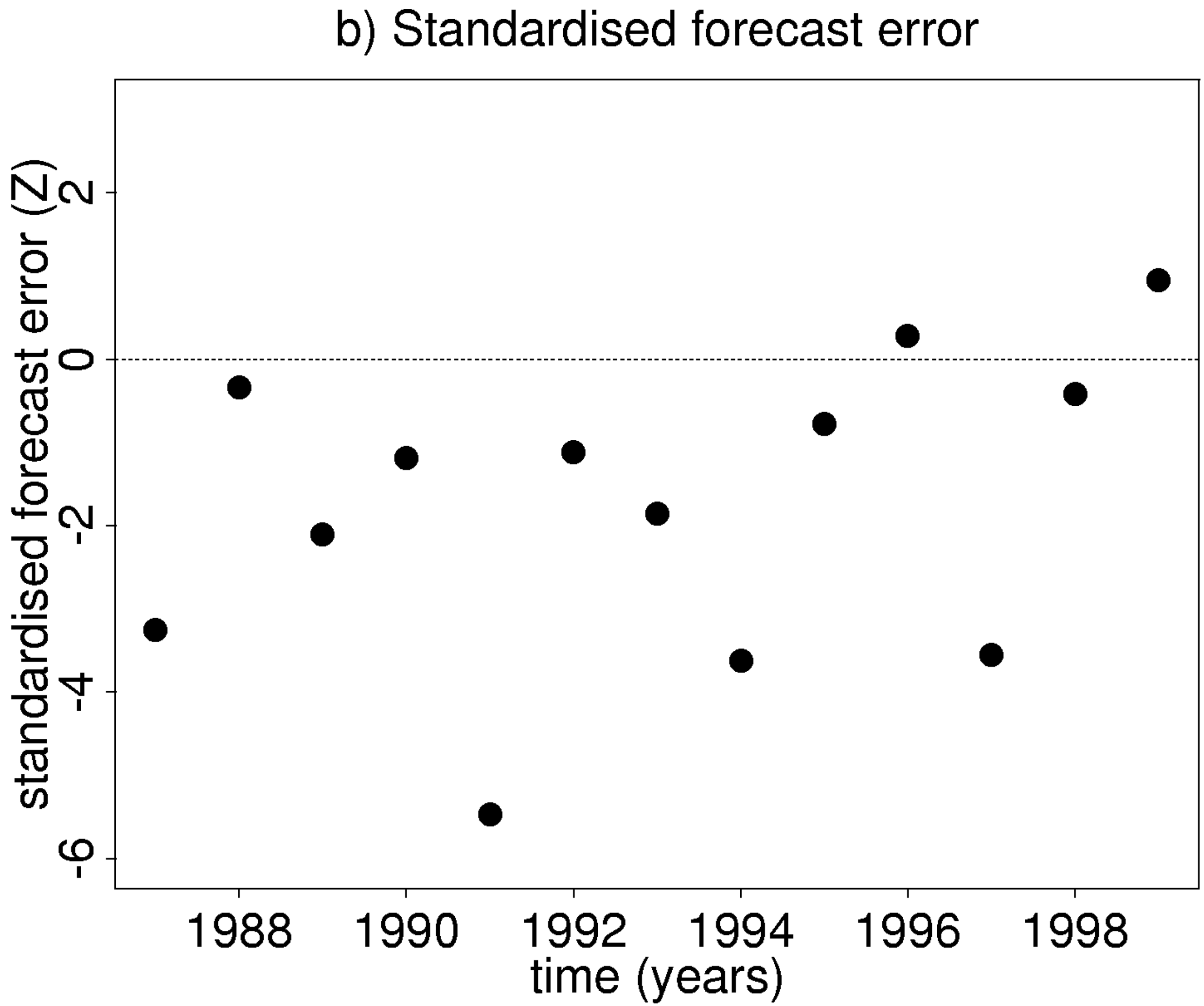


Coupled-Model Bias-Corrected Ensemble Forecast



Empirical Regression Model + Ensemble




b) Standardised forecast error
standardised forecast error (Z)
2
0
-2
-4
-6
1988
1990
1992
1994
1996
1998
time (years)