Mavzu : To'la orttirma va to'la differensial. To'la differensilaning taqribiy hisobga tatbiqlari. Murakkab va oshkormas funksiyaning hosilasi

Reja :

1.To'la orttirma va to'la differensial.

2.To'la differensilaning taqribiy hisobga tatbiqlari.

3.Murakkab va oshkormas funksiyaning hosilasi

To'la orttirma va to'la differensial

Ma'lumki, x va y o'zgaruvchilar mos ravishda orttirmalar olsa, funksiya to'la orttirma oladi. Bu to'la orttirmaning larga nisbatan chiziqli bo'lgan bosh qismi funksiyaning *to'la differensiali* deyiladi va dz bilan belgilanadi. funksiyaning to'la differensiali

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

formula bilan hisoblanadi, bu erda

$$dx = \Delta x, \; dy = \Delta y.$$

FUNKSIYANING TO'LA ORTTIRMASI VA TO'LA DIFFERENSIYALI

$\boldsymbol{z} = f(x, y)$ funksiya uzluksiz $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ xususiy hosilalarga $\Delta z = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y + \alpha_1 \Delta x + \alpha_2 \Delta y$ va $dz = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$ bo'lib, cheksiz kichik Δx, Δy lar uchun $\Delta z \approx dz$ bo'ladi, shuningdek

$f(x + \Delta x, y + \Delta y) \approx f(x, y) + \frac{\partial f(x,y)}{\partial x}\Delta x + \frac{\partial f(x,y)}{\partial y}\Delta x$ bo'ladi

Funksiyalarning to'la difirensiali topilsin

1) $z = x^2 y$; 2) $u = e^{\frac{s}{t}}$ 3) $z = \sqrt{x^2 + y^2}$

2) $\frac{\partial z}{\partial x} = 2xy$; $\frac{\partial z}{\partial x} = x^2$ shunda $dz = 2xydx + x^2 dy$

3) $\frac{\partial u}{\partial s} = e^{\frac{s}{t}} \cdot \frac{1}{t}$; $\frac{\partial u}{\partial t} = e^{\frac{s}{t}} \left\langle -\frac{s}{t^2} \right\rangle$ shunda

$du = e^{\frac{s}{t}} \left\langle \frac{1}{t} ds - \frac{s}{t^2} dt \right\rangle$ yoki $du = e^{\frac{s}{t}} \left\langle ds - \frac{s}{t} dt \right\rangle$

4) $\frac{dz}{ds} = \frac{2x}{\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+y^2}} = \frac{y}{\sqrt{x^2+y^2}}$

shunda $dz = \frac{xdx+ydx}{\sqrt{x^2+y^2}}$

Misol: $z = xy \cdot e^{5x^2}$ ni to'la ortirmasi topilsin.

Yechim: $\frac{\partial z}{\partial x} = ye^{5x^2} + xy \cdot 10x \cdot e^{5x^2} =$

$$= ye^{5x^2}(1 + 10x^2),$$

$\frac{\partial z}{\partial y} = xe^{5x^2}$

Bunda: $\Delta z = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y = y\,(1 + 10x^2)$

$$e^{5x^2}\Delta x + xe^{5x^2}\Delta y.$$

To'la differensilaning taqribiy hisobga tatbiqlari

To'la differensialdan funksiyaning taqribiy qiymatlarini

hisoblashda foydalanish mumkin, ya'ni $\Delta z \approx dz$ yoki

$$f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0) \approx dz,$$

bundan

$$f(x_0 + \Delta x,\ y_0 + \Delta y) \approx f(x_0, y_0) + z'_x dx + z'_y dy. \quad (2)$$

Uch argumentli $u = F(x, y, z)$ funksiyaning to'la differensiali

$$du = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz \quad (3)$$

formula bilan hisoblanadi.

Misol: $z = arctg\frac{y}{x}\ funksiyaning\ x = 1,\ y = 3,$

$$dx = 0{,}01,\ dy = -0{,}05$$

$$qiymatlaridagi\ to'la\ differensialini\ toping.$$

Yechish: 1- tartibli hususiy hosilalarni topamiz:

$$\frac{\partial z}{\partial x}=\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(-\frac{y}{x^2}\right)=-\frac{y}{x^2+y^2},\quad \frac{\partial z}{\partial y}=\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x}=\frac{x}{x^2+y^2}$$

Bu funksiyaning 1-tartibli to'la differensiali quyidagicha bo'ladi:

$dz=-\frac{ydx}{e^2+y^2}+\frac{xdy}{x^2+y^2}=\frac{xdy-ydx}{x^2+y^2}$,

$dz=\frac{1\cdot(-0,05)-3\cdot 0,01}{1^2+3^2}=-\frac{0,08}{10}$=-0,08

Ko'p o'zgaruvchili murakkab funksiyalarning

differensiallanuvchiligi. Murakkab funksiyaning hosilasi

$f(x_1,x_2,\ldots,x_m)$ funksiya M $(M\subset R^m)$ to'plamda berilgan bo'lib, $x_1,x_2,\ldots,x_m$ o'zgaruvchilarning har biri o'z navbatida $t_1,t_2,\ldots t_k$ o'zgaruvchilarning T $(T\subset R^k)$ to'plamda berilgan funksiya bo'lsin:

$$\begin{array}{l} x_1=\varphi_1(t_1,t_2,\ldots t_k),\\ x_2=\varphi_2(t_1,t_2,\ldots t_k),\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots,\\ x_m=\varphi_m(t_1,t_2,\ldots t_k), \end{array} \qquad (1)$$

Bunda $(t_1,t_2,\ldots t_k)\in T$ bo'lganda unga mos $(x_1,x_2,\ldots,x_m)\in M$ bo'lsin. Natijada ushbu

$$f(\varphi_1(t_1,t_2,\ldots t_k),\varphi_2(t_1,t_2,\ldots t_k),\ldots\varphi_m(t_1,t_2,\ldots t_k))=F(t_1,t_2,\ldots t_k)$$

murakkab funksiyaga ega bo'lamiz.

1^0. Murakkab funksiyaning differensiallanuvchanligi.

5-teorema. Agar (1) funksiyalarning har biri $(t_1^0,t_2^0,\ldots t_k^0)\in T$ nuqtada differensiallanuvchi bo'lib, $f(x_1,x_2,\ldots,x_m)$ funksiya esa mos $(x_1^0,x_2^0,\ldots x_k^0)\in M$ nuqtada $\left(x_1^0=\varphi_1(t_1^0,t_2^0,\ldots t_k^0),\quad x_2^0=\varphi_2(t_1^0,t_2^0,\ldots t_k^0),\ldots x_m^0=\varphi_m(t_1^0,t_2^0,\ldots t_k^0)\right)$ differensiallanuvchi bo'lsa, u holda murakkab funksiya $F(t_1,t_2,\ldots t_k)$ ham $(t_1^0,t_2^0,\ldots t_k^0)$ nuqtada differensiallanuvchi bo'ladi.

◄ $\left(t_1^0, t_2^0, \ldots, t_k^0\right) \in T$ nuqtani olib, uning koordinatalariga mos ravishda shunday $\left(\Delta t_1, \Delta t_2, \ldots, \Delta t_k\right)$ orttiruvchilar beraylikki, $\left(t_1^0 + \Delta t_1, t_2^0 + \Delta t_2, \ldots, t_k^0 + \Delta t_k\right) \in T$ bo'lsin. U holda (1) dagi har bir funksiya ham $\left(\Delta x_1, \Delta x_2, \ldots, \Delta x_m\right)$ orttirmalarga va nihoyat $f\left(x_1, x_2, \ldots, x_m\right)$ funksiya Δf orttirmaga ega bo'ladi.

Shartga ko'ra (1) dagi funksiyalarning har biri $\left(t_1^0, t_2^0, \ldots, t_k^0\right)$ nuqtada differensiallanuvchi. Demak,

$$\begin{aligned}
\Delta x_1 &= \frac{\partial x_1}{\partial t_1}\Delta t_1 + \frac{\partial x_1}{\partial t_2}\Delta t_2 + \ldots + \frac{\partial x_1}{\partial t_k}\Delta t_k + o(\rho), \\
\Delta x_2 &= \frac{\partial x_2}{\partial t_1}\Delta t_1 + \frac{\partial x_2}{\partial t_2}\Delta t_2 + \ldots + \frac{\partial x_2}{\partial t_k}\Delta t_k + o(\rho), \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\
\Delta x_m &= \frac{\partial x_m}{\partial t_1}\Delta t_1 + \frac{\partial x_m}{\partial t_2}\Delta t_2 + \ldots + \frac{\partial x_m}{\partial t_k}\Delta t_k + o(\rho)
\end{aligned} \qquad (2)$$

bo'ladi, bunda $\frac{\partial x_i}{\partial t_j}$ $\left(i = 1,2,\ldots,m;\ j = 1,2,\ldots,k\right)$ hususiy hosilalarning $\left(t_1^0, t_2^0, \ldots, t_k^0\right)$ nuqtadagi qiymatlari olingan, va

$$\rho = \sqrt{\Delta t_1^2 + \Delta t_2^2 + \ldots + \Delta t_k^2}$$

Shartga asosan, $f\left(x_1, x_2, \ldots, x_m\right)$ funksiya $\left(x_1^0, x_2^0, \ldots, x_m^0\right)$ nuqtada differensiallanuvchi. Demak,

$$\Delta f = \frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 + \ldots + \frac{\partial f}{\partial x_m}\Delta x_m + \alpha_1\Delta x_1 + \alpha_2\Delta x_2 + \ldots + \alpha_m\Delta x_m \qquad (3)$$

bo'ladi, bunda $\frac{\partial f}{\partial x_i}$ $\left(i = 1,2,\ldots,m\right)$ hususiy hosilalarning $\left(x_1^0, x_2^0, \ldots, x_m^0\right)$ nuqtadagi qiymatlari olingan va $\Delta x_1 \to 0, \Delta x_2 \to 0, \ldots, \Delta x_m \to 0$ da $\alpha_1 \to 0, \alpha_2 \to 0, \ldots, \alpha_m \to 0$ bo'ladi.

(2) va (3) munosabatlardan topamiz:

$$\Delta f = \frac{\partial f}{\partial x_1}\left[\frac{\partial x_1}{\partial t_1}\Delta t_1 + \frac{\partial x_1}{\partial t_2}\Delta t_2 + \ldots + \frac{\partial x_1}{\partial t_k}\Delta t_k + o(\rho),\right] +$$
$$+ \frac{\partial f}{\partial x_2}\left[\frac{\partial x_2}{\partial t_1}\Delta t_1 + \frac{\partial x_2}{\partial t_2}\Delta t_2 + \ldots + \frac{\partial x_2}{\partial t_k}\Delta t_k + o(\rho),\right] +$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$
$$+ \frac{\partial f}{\partial x_m}\left[\frac{\partial x_m}{\partial t_1}\Delta t_1 + \frac{\partial x_m}{\partial t_2}\Delta t_2 + \ldots + \frac{\partial x_m}{\partial t_k}\Delta t_k + o(\rho),\right] +$$
$$+ \alpha_1 \Delta x_1 + \alpha_2 \Delta x_2 + \ldots + \alpha_m \Delta x_m =$$

$$= \left[\frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_1} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_1} + \ldots + \frac{\partial f}{\partial x_m}\frac{\partial x_m}{\partial t_1}\right]\Delta t_1 +$$
$$+ \left[\frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_2} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_2} + \ldots + \frac{\partial f}{\partial x_m}\frac{\partial x_m}{\partial t_2}\right]\Delta t_2 +$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$
$$+ \left[\frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_k} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_k} + \ldots + \frac{\partial f}{\partial x_m}\frac{\partial x_m}{\partial t_k}\right]\Delta t_k +$$
$$+ \left[\frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2} + \ldots + \frac{\partial f}{\partial x_m}\right] o(\rho) + \tag{4}$$
$$+ \alpha_1 \Delta x_1 + \alpha_2 \Delta x_2 + \ldots + \alpha_m \Delta x_m.$$

Bu tenglikdagi $\left[\frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2} + \ldots + \frac{\partial f}{\partial x_m}\right]$ yig’indi o’zgarmas (ρ ga bog’liq emas) bo’lganligi sababli

$$\left[\frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2} + \ldots + \frac{\partial f}{\partial x_m}\right] o(\rho) = o(\rho) \tag{5}$$

bo’ladi.

Madomiki, $x_i = \varphi_i(t_1, t_2, \ldots, t_k)$ $(i = 1, 2, \ldots, m)$ funksiyalar $(t_1^0, t_2^0, \ldots, t_k^0)$ nuqtada differensiallanuvchi ekan, ular shu nuqtada uzluksiz bo’ladi. Unda uzluksizlik ta’rifiga ko’ra $\Delta t_1 \to 0, \Delta t_2 \to 0, \ldots, \Delta t_k \to 0$ da, ya’ni $\rho \to 0$ da $\Delta x_1 \to 0, \Delta x_2 \to 0, \ldots, \Delta x_m \to 0$ bo’ladi. Yana ham aniqroq aytsak, (2) formuladan

$\rho \to 0$ da $\Delta x_1 = o(\rho), \Delta x_2 = o(\rho), \ldots, \Delta x_m = o(\rho)$ ekanligi kelib chiqadi. $\Delta x_1 \to 0, \Delta x_2 \to 0, \ldots, \Delta x_m \to 0$ da esa $\alpha_1 \to 0, \alpha_2 \to 0, \ldots, \alpha_m \to 0$.

Demak,

$\rho \to 0 \Rightarrow$ barcha $\Delta x_i \to 0 \Rightarrow$ barcha $\alpha_i \to 0 \Rightarrow \alpha_1 \Delta x_1, \alpha_2 \Delta x_2, \ldots, \alpha_m \Delta x_m = o(\rho)$ *(6)*

Agar

$$A_j = \frac{\partial f}{\partial x_1} \cdot \frac{\partial x_1}{\partial t_j} + \frac{\partial f}{\partial x_2} \cdot \frac{\partial x_2}{\partial t_j} + \ldots + \frac{\partial f}{\partial x_m} \cdot \frac{\partial x_m}{\partial t_j}$$

$(j = 1, 2, 3, \ldots k)$ deyilsa, u holda (4), (5) va (6) munosabatlardan

$$\Delta f = A_1 \Delta t_1 + A_2 \Delta t_2 + \ldots + A_k \Delta t_k + o(\rho)$$

kelib chiqadi. ▶

2^0. Murakkab funksiyaning hosilasi. Endi

$$f(\varphi_1(t_1, t_2, \ldots, t_k), \varphi_2(t_1, t_2, \ldots, t_k), \ldots, \varphi_m(t_1, t_2, \ldots, t_k)) = F(t_1, t_2, \ldots, t_k)$$

murakkab funksiyaning $t_1, t_2, \ldots, t_k$ o'zgaruvchilar bo'yicha xususiy hosilalarini topamiz. Aytaylik $f(x_1, x_2, \ldots, x_m)$ va $x_1 = \varphi_1(t_1, t_2, \ldots, t_k)$, $x_2 = \varphi_2(t_1, t_2, \ldots, t_k), \ldots,$ $x_m = \varphi_m(t_1, t_2, \ldots, t_k)$ funksiyalar yuqoridagi 5- teoremaning shartlarini bajarsin. U holda 5-teoremaga ko'ra murakkab funksiya $(t_1^0, t_2^0, \ldots, t_k^0)$ nuqtada differensiallanuvchi bo'ladi.

Demak, bir tomondan

$$\Delta f = A_1 \Delta t_1 + A_2 \Delta t_2 + \ldots + A_k \Delta t_k + o(\rho) \qquad (7)$$

bo'lib, bunda

$$A_j = \frac{\partial f}{\partial x_1} \frac{\partial x_1}{\partial t_j} + \frac{\partial f}{\partial x_2} \frac{\partial x_2}{\partial t_j} + \ldots + \frac{\partial f}{\partial x_m} \frac{\partial x_m}{\partial t_j} \qquad (\mathrm{j} = 1, 2, \ldots, \mathrm{k}) \qquad (8)$$

(qaralsin 5-teorema) ikkinchi tamondan 1- natijaga asosan

$$\Delta f = \frac{\partial f}{\partial t_1}\Delta t_1 + \frac{\partial f}{\partial t_2}\Delta t_2 + \ldots + \frac{\partial f}{\partial t_k}\Delta t_k + o(\rho) \qquad (9)$$

bo'ladi. (7),(8) va (9) va munosabatlardan

$$\frac{\partial f}{\partial t_1} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_1} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_1} + \ldots + \frac{\partial f}{\partial x_m}\frac{\partial x_m}{\partial t_1},$$

$$\frac{\partial f}{\partial t_2} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_2} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_2} + \ldots + \frac{\partial f}{\partial x_m}\frac{\partial x_m}{\partial t_2}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\frac{\partial f}{\partial t_k} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_k} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_k} + \ldots + \frac{\partial f}{\partial x_m}\frac{\partial x_m}{\partial t_k}$$

bo'lishini topamiz.

Misol 1: Agar $z = \sqrt{x^2 + y^2}$ funksiyada $x = sint$ va $y = cost$ bo'lganda $\frac{dz}{dt}$ topilsin.

Yechim: $\frac{dz}{dt} = \frac{2x}{2\sqrt{x^2+y^2}} \cdot cost += \frac{2y}{2\sqrt{x^2+y^2}} \cdot (-\sin t),$

yoki

$\frac{dz}{dt} = \frac{\sin t \cos t - \sin t \cos t}{\sqrt{sin^2\, t + cos^2 t}} = \frac{0}{\sqrt{1}} = 0$

Oshkormas funksiyalar

1⁰. Oshkormas funksiya tushunchasi. Ma'lumki, $x \subset R$ to'plamdagi har bir x songa biror qoidaga ko'ra $Y \subset R$ to'plamdan bitta y son mos qo'yilgan bo'lsa, X to'plamda funksiya berilgan deb atalar va u

$$f : x \to y \text{ yoki } y = f(x)$$

kabi belgilanar edi.

Ikki x va y argumentlarning $F(x,y)$ funksiyasi

$$M = \{(x,y) \in R^2 : a < x < в,\ c < y < d\}$$

to'plamda berilgan bo'lsin. Ushbu

$$F(x,y)=0 \qquad (1)$$

tenglamani qaraylik. Biror x_0 sonni $(x_0 \in (a,в))$ olib, uni yuqoridagi tenglamadagi x ning o'rniga qo'yamiz. Natijada y ni topish uchun quyidagi

$$F(x_0,y)=0$$

tenglamaga kelamiz. Bu tenglamaning echimi haqida ushbu hollar bo'lishi mumkin:

1). (1) tenglama yagona haqiqiy y_0 echimga ega,

2). (1) tenglama bitta ham haqiqiy echimga ega emas,

3). (1) tenglama bir nechta, hatto cheksiz ko'p haqiqiy echimga ega.

Masalan,

$$F(x,y)=\begin{cases} y-x^2, \text{ agar } x\geq 0 \text{ bo'lsa,} \\ y^2+x, \text{ agar } x<0 \text{ bo'lsa} \end{cases}$$

u holda

$$F(x,y)=0$$

tenglama $x_0 \geq 0$ bo'lganda, yagona $y=x_0^2$ echimga, $x_0<0$ bo'lganda ikkita

$$y=\sqrt{-x_0}, \quad y=-\sqrt{-x_0}$$

echimga ega bo'ladi.

Agar biror $F(x,y)=0$ tenglama uchun 1)- hol o'rinli bo'lsa bunday tenglama e'tiborga loyiq. Uning yordamida funksiya aniqlanishi mumkin.

Endi x o'zgaruvchining qiymatlaridan iborat shunday X to'plamni qaraylikki, bu to'plamdan olingan har bir qiymatda $F(x,y)=0$ tenglama yagona echimga ega bo'lsin.

X to'plamdan ixtiyoriy x sonni olib, bu songa $F(x,y)=0$ tenglamaning yagona echimi bo'lgan y sonni mos qo'yamiz. Natijada X to'plamdan olingan har bir x ga yuqoridagi ko'rsatilgan qoidaga ko'ra bitta y mos qo'yilib, funksiya hosil bo'ladi. Bunda x va y o'zgaruvchilar orasidagi bog'lanish $F(x,y)=0$ tenglama yordamida bo'ladi. Odatda bunday berilgan (aniqlangan) funksiya oshkormas ko'rinishda berilgan funksiya (yoki oshkormas funksiya) deb ataladi va

$$x \to y: \ \ F(x,y)=0.$$

kabi belgilanadi.

3^0. Oshkormas funksiyaning hosilasi. Endi oshkormas funksiyaning hosilasini topish bilan shug'ullanamiz.

1-teorema. $F(x,y)$ funksiya $(x_0,y_0)\in R^2$ nuqtaning biror

$U_{h,k}((x_0,y_0))=\{(x,y)\in R^2: \ x_0-h<x<x_0+h; \ y_0-k<y<y_0+k\}$ atrofida $(h>0, k>0)$ berilgan va u quyidagi shartlarni bajarsin:

1) $U_{h,k}((x_0,y_0))$ da uzluksiz;

2) $U_{h,k}((x_0,y_0))$ da uzluksiz $F`_x(x,y)$, $F`_y(x,y)$ xususiy hosilalarga ega va $F`_y(x_0,y_0)\neq 0$;

3) $F`_y(x_0,y_0)=0$.

U holda (x_0,y_0) nuqtaning shunday

$U_{\delta,\varepsilon}((x_0,y_0))=\{(x,y)\in R^2: \ x_0-\delta<x<x_0+\delta; \ y_0-\varepsilon<y<y_0+\varepsilon\}$ atrofi $(0<\delta<h, \ 0<\varepsilon<\mathrm{k})$ topiladiki,

1^1) $\forall x\in(x_0-\delta, x_0+\delta)$ uchun

$$F(x,y)=0$$

tenglama yagona y echimga $y \in (y_0 - \varepsilon,\ y_0 + \varepsilon)$ ega, ya'ni $F(x,y) = 0$ tenglama yordamida

$$x \to y:\ F(x,y) = 0$$

oshkormas ko'rinishdagi funksiya aniqlanadi;

2[1]) $x = x_0$ bo'lganda unga mos keladigan y uchun $y = y_0$ bo'ladi;

3[1]) oshkormas ko'rinishda aniqlangan

$$x \to y:\ F(x,y) = 0$$

funksiya $(x_0 - \delta, x_0 + \delta)$ oraliqda uzluksiz bo'ladi;

4[1]) Bu oshkormas ko'rinishdagi funksiya $(x_0 - \delta, x_0 + \delta)$ oraliqda uzluksiz hosilaga ega bo'ladi.

◄Shartga ko'ra $F`_y(x,y)$ funksiya $U_{h,k}((x_0,y_0))$ da uzluksiz va $F`_y(x_0,y_0) \neq 0$. Aniqlik uchun $F`_y(x_0,y_0) > 0$ deylik. U holda uzluksiz funksiyaning xossasiga ko'ra (x_0,y_0) nuqtaning shunday

$$U_{\delta,\varepsilon}((x_0,y_0)) = \{(x,y) \in R^2:\ x_0 - \delta < x < x_0 + \delta;\ y_0 - \varepsilon < y < y_0 + \varepsilon\}$$

atrofi $(0 < \delta < h,\ 0 < \varepsilon < \mathrm{k})$ topiladiki, $\forall (x,y) \in U_{\delta,\varepsilon}((x_0,y_0))$ uchun $F'_y(x,y) > 0$ bo'ladi. Demak, $F(x,y)$ funksiya x o'zgaruvchining $(x_0 - \delta, x_0 + \delta)$ oraliqdan olingan har bir tayin qiymatida y o'zgaruvchining funksiyasi sifatida o'suvchi. Yuqorida isbot etilgan 11-teoremaga ko'ra

$$F(x,y) = 0$$

tenglama $(x_0 - \delta, x_0 + \delta)$ da

$$x \to y:\ F(x,y) = 0$$

oshkormas ko'rinishdagi funksiyani aniqlaydi, $x = x_0$ bo'lganda unga mos kelgan $y = y_0$ bo'ladi va oshkormas funksiya $(x_0 - \delta, x_0 + \delta)$ da uzluksiz bo'ladi.

Endi oshkormas funksiyaning hosilasini topamiz, x_0 nuqtaga shunday Δx orttirma beraylikki, $x_0 + \Delta x \in (x_0 - \delta, x_0 + \delta)$ bo'lsin. Natijada

$$x \to y:\ F(x,y) = 0$$

oshkormas funksiya ham orttirmaga ega bo'lib,

$$F(x_0 + \Delta x, y_0 + \Delta y) = 0$$

bo'ladi. Demak,

$$\Delta F(x_0, y_0) = F(x_0 + \Delta x, y_0 + \Delta y) - F(x_0, y_0) = 0 \qquad (2)$$

Shartga ko'ra $F`_x(x,y)$ va $F`_y(x,y)$ xususiy hosilalar $U_{\delta,\varepsilon}((x_0, y_0))$ da uzluksiz. Binobarin $F(x,y)$ funksiya (x_0, y_0) nuqtada differensiallanuvchi:

$$\Delta F(x_0, y_0) = F`_x(x_0, y_0)\Delta x + F`_y(x_0, y_0)\Delta y + \alpha\Delta x + \beta\Delta y \qquad (3)$$

Bu munosabatdagi α va β lar Δx va Δy larga bog'liq va $\Delta x \to 0$, $\Delta y \to 0$ da $\alpha \to 0$, $\beta \to 0$.

(2) va (3) munosabatlardan

$$\frac{\Delta y}{\Delta x} = -\frac{F`_x(x_0, y_0) + \alpha}{F`_y(x_0, y_0) + \beta}$$

ekanligi kelib chiqadi.

Oshkormas funksiyaning x_0 nuqtada uzluksizligini e'tiborga olib, keyingi tenglikda $\Delta x \to 0$ da limitga o'tib quyidagini topamiz:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\left(-\frac{F`_x(x_0, y_0) + \alpha}{F`_y(x_0, y_0) + \beta} \right) = -\frac{F`_x(x_0, y_0)}{F`_y(x_0, y_0)}.$$

Demak,

$$y`_{x=x_0} = -\frac{F`_x(x_0, y_0)}{F`_y(x_0, y_0)}$$

$U_{\delta,\varepsilon}((x_0,y_0))$ da $F'_x(x,y)$, $F'_y(x,y)$ xususiy hosilalar uzluksiz va $F'_y(x,y)\neq 0$ bo'lishidan oshkormas funksiyaning hosilasi

$$y`_x=-\frac{F`_x(x,y)}{F`_y(x,y)}$$

ning $(x_0-\delta,x_0+\delta)$ oraliqda uzluksiz bo'lishi kelib chiqadi. ►

Misol 1: Ushbu

$$F(x,y)=xe^y+ye^x-2=0 \qquad (4)$$

tenglama bilan aniqlanadigan oshkormas funksiyaning hosilasi topilsin.

◄Ravshanki, $F(x,y)=xe^y+ye^x-2$ funksiya $\{(x,y)\in R^2: -\infty<x<+\infty$ $-\infty<y<+\infty\}$ to'plamda yuqoridagi teoremaning barcha shartlarini qanoatlantiradi. Demak, $\forall(x_0,y_0)\in R^2$ nuqtaning $U_{\delta,\varepsilon}((x_0,y_0))$ atrofida (4) tenglama oshkormas ko'rinishdagi funksiyani aniqlaydi va bu oshkormas funksiyaning hosilasi

$$y`=-\frac{F`_x(x,y)}{F`_y(x,y)}=-\frac{e^y+ye^x}{xe^y+e^x}$$

bo'ladi. ►

Oshkormas ko'rinishdagi funksiyaning hosilasini quyidagicha ham hisoblasa bo'ladi. y ning x ga bog'liq ekanini e'tiborga olib, $F(x,y)=0$ dan topamiz:

$$F`_x(x,y)+F`_y(x,y)\cdot y`=0$$

Bundan esa

$$y`=-\frac{F`_x(x,y)}{F`_y(x,y)}$$

bo'lishi kelib chiqadi.

Yuqorida keltirilgan (4) tenglama yordamida aniqlangan oshkormas ko’rinishdagi funksiyaning hosilasini hisoblaylik:

$$F`_x(x,y)+F`_y(x,y)\cdot y`=e^y+ye^x+\left(xe^y+e^x\right)y`=0$$

$$y`=-\frac{e^y+ye^x}{xe^y+e^x}.$$

Misol 2: Agar $xe^{2y}-ye^{2x}=0$ bo’lsa $\frac{dy}{dx}$ topilsin.

Yechim: $(xe^{2y}-ye^{2x})'=0$

Oshkormas funksiyadan kelib chiqib:

$1\cdot e^{2y}+x\cdot e^{2y}\cdot 2y'-y'^{e^{2x}}-2ye^{2s}=0$

$y'(2xe^{2y}-e^{2x})=2ye^{2x}-e^{2y}$ bunda

$\frac{dy}{dx}=y'=\frac{2ye^{2x}-e^{2y}}{2xe^{2y}-e^{2x}}$