Funksiya hosilasi ta’riflari

$y = f(x)$ funksiya (a,b) intervalda aniqlangan bo‘lsin.

$$\Delta y = f(x+\Delta x) - f(x). \qquad \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

1-ta’rif. Agar $\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ limit mavjud bo‘lsa, bu limitga

$y = f(x)$ funksiyaning x nuqtagi hosilasi deyiladi va quyidagi

$f'(x)$ yoki y'

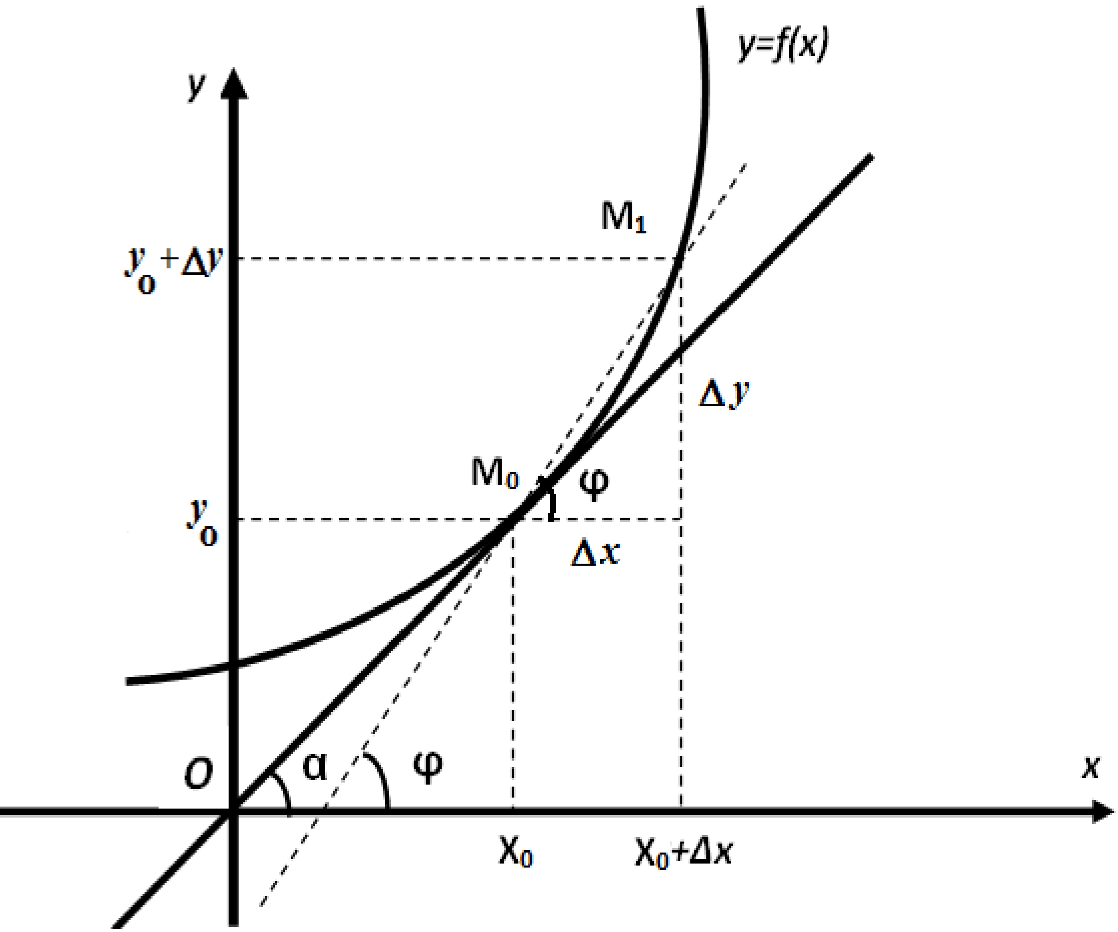
kabi belgilanadi.

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f\left(x+\Delta x\right) - f(x)}{\Delta x}.$$

Hosilaning geometrik ma'nosi

$$f(x) \qquad M_0(x_0; y_0) \qquad y - y_0 = k(x - x_0)$$

$$\alpha = \lim_{\Delta x \to 0} \varphi(\Delta x) = \lim_{\Delta x \to 0} arctg \frac{\Delta y}{\Delta x} = arctg\left(\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}\right) = arctg\left(f'(x_0)\right).$$

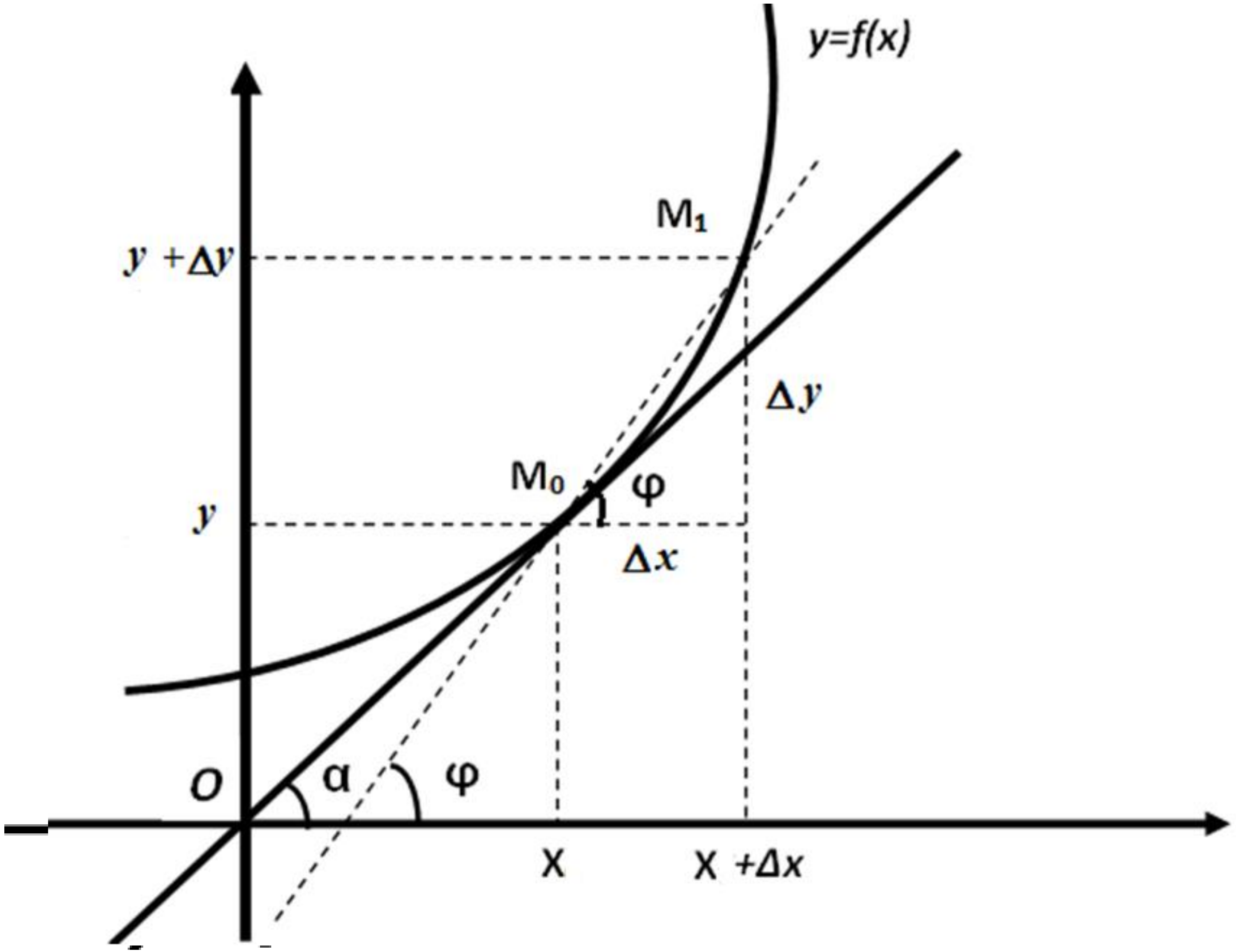


$$\alpha = arctg\left(f'(x_0)\right)$$

$$f'(x_0) = tg\alpha = k.$$

$$y - y_0 = f'(x_0)(x - x_0)$$

$$y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$$



y=f(x)
M1
y +Δy
Δy
M0
φ
y
Δx
O
α
φ
X
X +Δx

Hosilaning mexanik ma'nosi

$$S = f(t) \qquad \Delta t$$

$$\Delta S = f(t_0 + \Delta t) - f(t_0).$$

$$\frac{\Delta S}{\Delta t} = v_{o'r}$$

$$v(t_0) = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = \lim_{\Delta t \to 0} \frac{f(t_0 + \Delta t) - f(t_0)}{\Delta t} = f'(t_0).$$

1. $C' = 0; \quad C = const$

2. $\left(x^n\right)' = nx^{n-1};$

3. $\left(a^x\right)' = a^x \ln a;$

4. $\left(e^x\right)' = e^x;$

5. $\left(\log_a x\right)' = \frac{1}{x \ln a};$

6. $\left(\ln x\right)' = \frac{1}{x};$

7. $\left(\sin x\right)' = \cos x;$

8. $\left(\cos x\right)' = -\sin x;$

9. $\left(tgx\right)' = \frac{1}{\cos^2 x};$

10. $\left(ctgx\right)' = -\frac{1}{\sin^2 x};$

11. $\left(\arcsin x\right)' = \frac{1}{\sqrt{1 - x^2}};$

12. $\left(\arccos x\right)' = -\frac{1}{\sqrt{1 - x^2}};$

13. $\left(\mathrm{arc}\,tgx\right)' = \frac{1}{1 + x^2};$

14. $\left(\mathrm{arc}\,ctgx\right)' = -\frac{1}{1 + x^2};$

Hosila hisoblashning asosiy qoidalari

1. $(u+v)' = u' + v'$ $\qquad (u+v+...+w)' = u' + v' + ... + w'.$

2. $(u \cdot v)' = u'v + uv'.$ $\qquad (u \cdot v \cdot w)' = u'vw + u \cdot v' \cdot w + uvw'.$

3. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}.$

$$\triangle y = \frac{u + \triangle u}{v + \triangle v} - \frac{u}{v} = \frac{\triangle u \cdot v - u \cdot \triangle v}{v(v + \triangle v)}.$$

$$y' = \lim_{\triangle x \to 0} \frac{\triangle y}{\triangle x} = \lim_{\triangle x \to 0} \frac{\triangle u \cdot v - u \cdot \triangle v}{v(v + \triangle v) \cdot \triangle x} = \lim_{\triangle x \to 0} \frac{\frac{\triangle u}{\triangle x} \cdot v - u \cdot \frac{\triangle v}{\triangle x}}{v(v + \triangle v)} = \frac{u'v - uv'}{v^2}$$

Murakkab funksiyaning hosilasi

$y = f(u)$ va $u = \varphi(x)$ $y = f[\varphi(x)],$

$$y'_x = y'_u \cdot u'_x.$$

$$\frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta u} \cdot \frac{\Delta u}{\Delta x}. \qquad \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta u} \cdot \frac{\Delta u}{\Delta x} \right) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u} \cdot \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}.$$

$$y = [f(x)]^{\varphi(x)} \quad (f(x) > 0) \qquad \ln y = \ln[f(x)]^{\varphi(x)} = \varphi(x) \cdot \ln f(x)$$

$$\frac{1}{y} \cdot y' = \varphi'(x) \cdot \ln f(x) + \varphi(x) \cdot \frac{1}{f(x)} \cdot f'(x)$$

$$\left([f(x)]^{\varphi(x)} \right)' = \left[\varphi'(x) \ln f(x) + \frac{\varphi(x)}{f(x)} \cdot f'(x) \right] \cdot [f(x)]^{\varphi(x)}$$

Funksiyaning differensiali

$$f'(x) = \lim_{\triangle x \to 0} \frac{\triangle f(x)}{\triangle x}. \qquad \triangle x \neq 0$$

$$\triangle f(x) = f'(x)\triangle x + \alpha(\triangle x) \cdot \triangle x$$

$\triangle x \to 0$ $\qquad \alpha(\triangle x) \to 0.$

Ta'rif. Agar $\triangle f(x)$ ni quyidagicha

$$\triangle f(x) = A\triangle x + \alpha(\triangle x) \cdot \triangle x$$

ifodalash mumkin bo'lsa, $f(x)$ funksiya x nuqtada differensiallanuvchi deyiladi.

$$f'(x) = \lim_{\triangle x \to 0} \frac{\triangle f(x)}{\triangle x} = \lim_{\triangle x \to 0} \left(A + \alpha(\triangle x)\right) = A.$$

$$\Delta y = f'(x)\triangle x + \alpha(\triangle x)\cdot\triangle x$$

Ta'rif.

$$dy = df(x) = f'(x)\triangle x.$$

$$dx = \triangle x.$$

$$dy = f'(x)dx$$

$$\frac{dy}{dx} = f'(x).$$

$$\lim_{\triangle x\to 0}\frac{\triangle y}{dy}=\lim_{\triangle x\to 0}\frac{f'(x)\triangle x+\alpha(\triangle x)\cdot\triangle x}{f'(x)\triangle x}=$$

$$=\lim_{\triangle x\to 0}\left(1+\frac{1}{f'(x)}\alpha(\triangle x)\right)=1+\frac{1}{f'(x)}\lim_{\triangle x\to 0}\alpha(\triangle x)=1.$$

$$\frac{\triangle y}{dy}\approx 1$$

$$f(x+\triangle x)-f(x)\approx f'(x)\triangle x,$$

$$f(x+\triangle x)\approx f(x)+f'(x)\triangle x.$$

Misol. $arctg1,02$ miqdorni taqribiy hisoblang.

$$x+\triangle x=1,02 \qquad x=1 \qquad \triangle x=1,02-1=0,02.$$

$$y=arctgx$$

$$y'=\left(arctgx\right)'=\frac{1}{1+x^2},$$

$$y'(1)=\frac{1}{1+1^2}=\frac{1}{2}$$

$$arctg1,02\approx arctg1+y'(1)\cdot\triangle x=$$

$$=\frac{\pi}{4}+\frac{1}{2}\cdot 0,02\approx 0,7852+0,01=0,7952.$$

Yuqori tartibli hosilalar

$$(u \pm v)^{(n)} = u^{(n)} \pm v^{(n)},$$

$$(uv)^{(n)} = u^{(n)} \cdot v + C_n^1 u^{(n-1)} \cdot v^1 + C_n^2 u^{(n-2)} \cdot v^{(2)} + \ldots + C_n^k u^{(n-k)} \cdot v^{(k)} + \ldots$$

$$\ldots + u \cdot v^{(n)} = \sum_{k=0}^{n} C_n^k u^{(n-k)} \cdot v^{(k)},$$

$$u^{(0)} = u, C_n^k = \frac{n!}{k!(n-k)!}; n! = 1 \cdot 2 \cdot \ldots \cdot n; 0! = 1.$$

$$y = x^2 \cdot e^{3x}$$

$$y^{(10)} = (e^{3x})^{(10)} \cdot x^2 + C_{10}^1 (e^{3x})^{(9)} \cdot (x^2)' + C_{10}^2 (e^{3x})^{(8)} \cdot (x^2)'' + \ldots =$$

$$= 3^{10} \cdot e^{3x} \cdot x^2 + 10 \cdot 3^9 \cdot e^{3x} \cdot 2x + \frac{10 \cdot 9}{2} \cdot e^{3x} \cdot 3^8 \cdot 2 = 3^9 \cdot e^{3x}(3x^2 + 20x + 30).$$

Parametrik ko‘rinishda berilgan funksiyalarning hosilasi

$$\begin{cases} x = \varphi(t) \\ y = g(t) \end{cases} \qquad t_0 \le t \le T. \qquad y = g(t), t = \Phi(x),$$

$$y'_x = y_t{}' \cdot t_x{}' = g_t{}'(t) \cdot \Phi_x{}'. \qquad y'_x = g_t{}'(t) \cdot \frac{1}{\varphi_t{}'(t)}$$

$$\Phi_x{}'(x) = \frac{1}{\varphi_t{}'(t)}.$$

$$y'_x = \frac{y_t{}'}{x_t{}'} \qquad \frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}.$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{\dot{y}}{\dot{x}}\right) = \frac{d}{dt}\left(\frac{\dot{y}}{\dot{x}}\right)\frac{dt}{dx} = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{\dot{x}^2} \cdot t_x{}' = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{\dot{x}^2} \cdot \Phi_x{}' =$$

$$= \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{\dot{x}^2} \cdot \frac{1}{\varphi_t{}'} = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{\dot{x}^2} \cdot \frac{1}{\dot{x}}$$

$$\frac{d^2y}{dx^2} = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{(\dot{x})^3}.$$