FUNKSIYA TUSHUNCHASI.

- O‘zgaruvchi va o‘zgarmas miqdorlar
- Funksiya ta’rifi.
- Funksiyaning berilish usullari
- Elementar funksiyalar
- Chegaralangan funksiyalar

O‘zgaruvchi va o‘zgarmas miqdorlar

Funksiya ta’rifi

$$y = f(x)$$

$f: X \to Y$ $X \xrightarrow{f} Y$ $f: x \to y.$

Ta’rif. X to‘plamga funksiyaning aniqlanish sohasi deyiladi.

Aniqlanish sohasi D_f yoki D_y bilan belgilanadi.

E_f yoki E_y -funksiyaning qiymatlar sohasi

$f(x) = \sqrt[4]{x}$ $D_f = E_f = [0; +\infty),$

$y = e^x$ $D_y = (-\infty; +\infty),$ $E_y = (0; +\infty).$

Funksiyaning berilish usullari

Funksiya quyidagi usullarda berilishi mumkin: jadval, grafik va analitik.

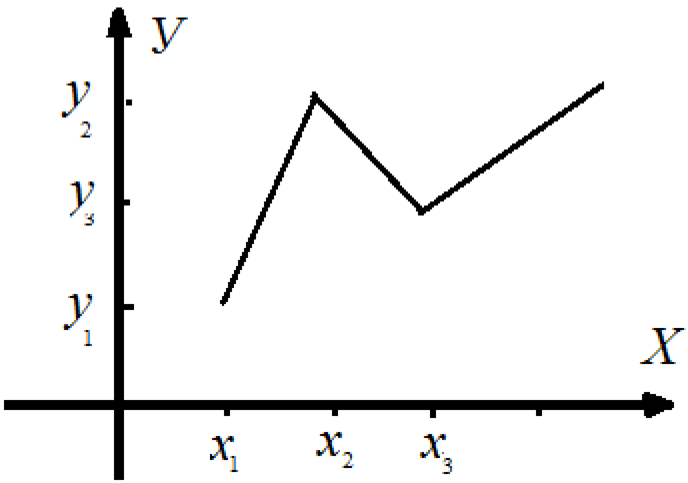
Funksiyaning jadval usulida berilishi.

x	x_1	x_2	x_3	..	x_n
y	y_1	y_2	y_3	..	y_n

t - haftalar	3	4	...	10
Y - g‘o‘za o‘sishi (mm)	10	30	...	400

t - kunlar	1	2	3	4	5
h - to‘g‘onda suvning balandligi	40	40,2	40,3	40,1	40,3

Funksiyaning grafik usulda berilishi.



Funksiyaning analitik usulda berilishi.

1. $f(x) = x^3.$ 2. $f(x) = \begin{cases} e^x, & x \leq 0, \\ 3x, & x > 0. \end{cases}$ 3. $y = -x^2.$

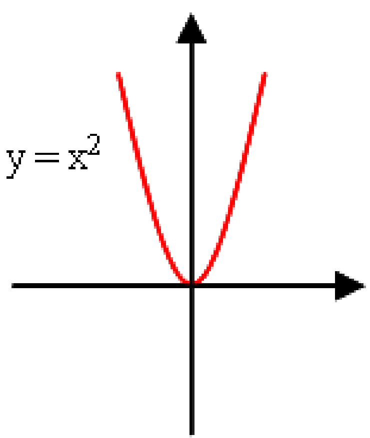
Masalan, $y = \dfrac{\sqrt{x+2}}{x-5}.$

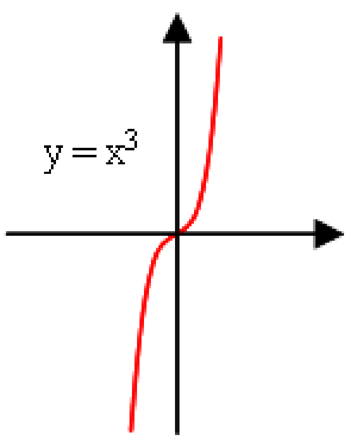
Elementar funksiyalar

Ta'rif. Quyidagi funksiyalarga asosiy elementar funksiyalar deyiladi:
•Darajali funksiya
•Ko'rsatkichli funksiya
•Logarifmik funksiya
•Trigonometrik funksiyalar
•Teskari trigonometrik funksiyalar.

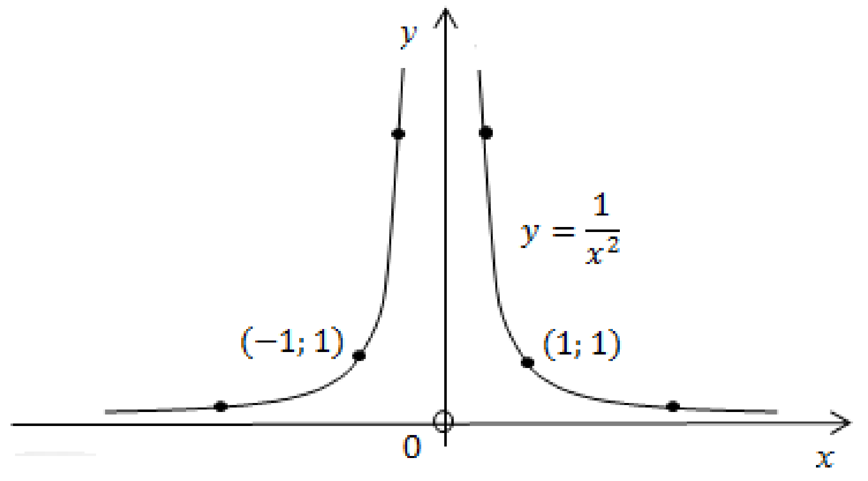
Darajali funksiya, $y = x^{\alpha}$

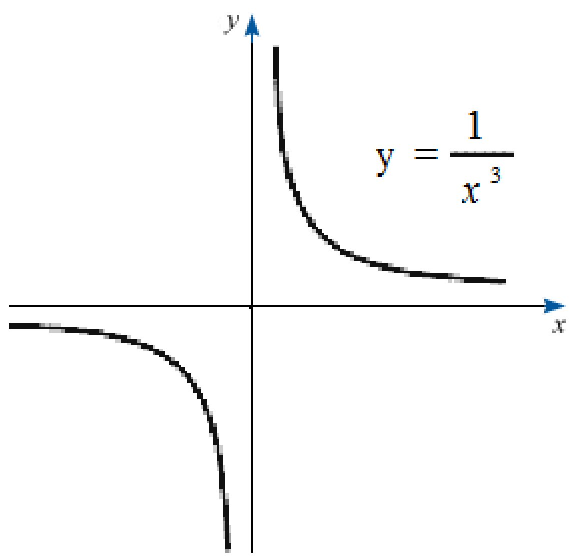
1. $\alpha = n,\quad n$ - natural son.



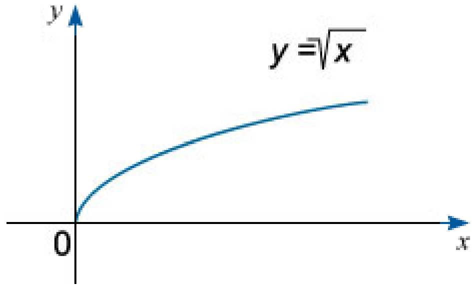


2. $\alpha=-n \quad (n\in N)$.

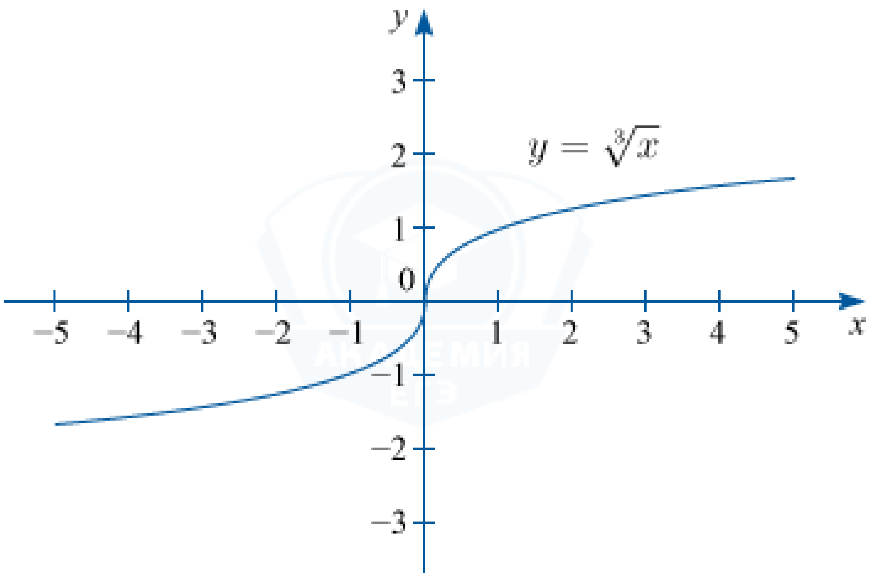




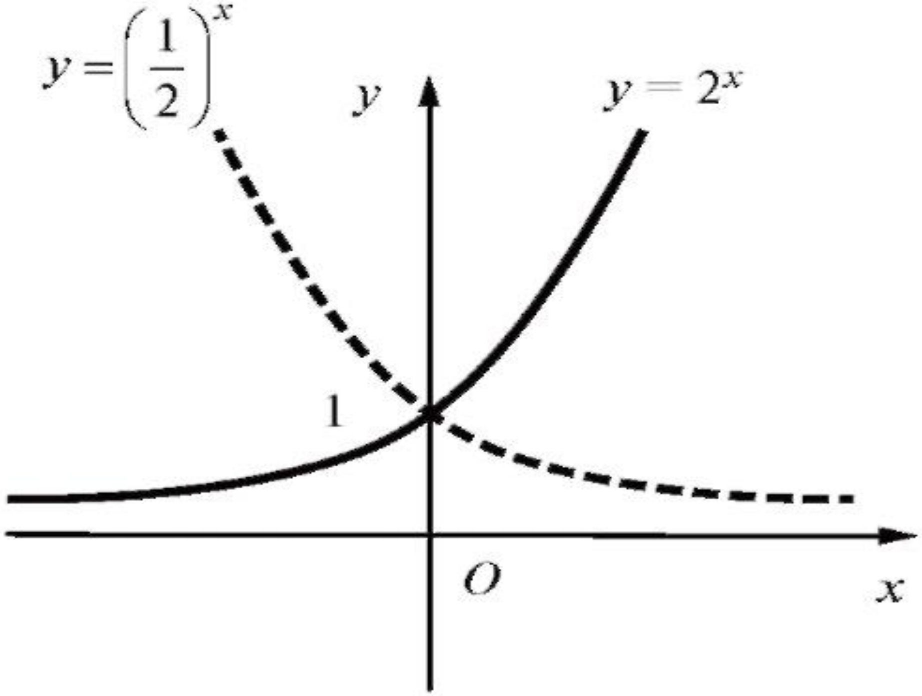
3. $\alpha = \frac{m}{n} \quad (m, n \in N, m < n, n = 2k).$



4. $\alpha = \frac{m}{n} \quad (m, n \in N, n = 2k + 1).$

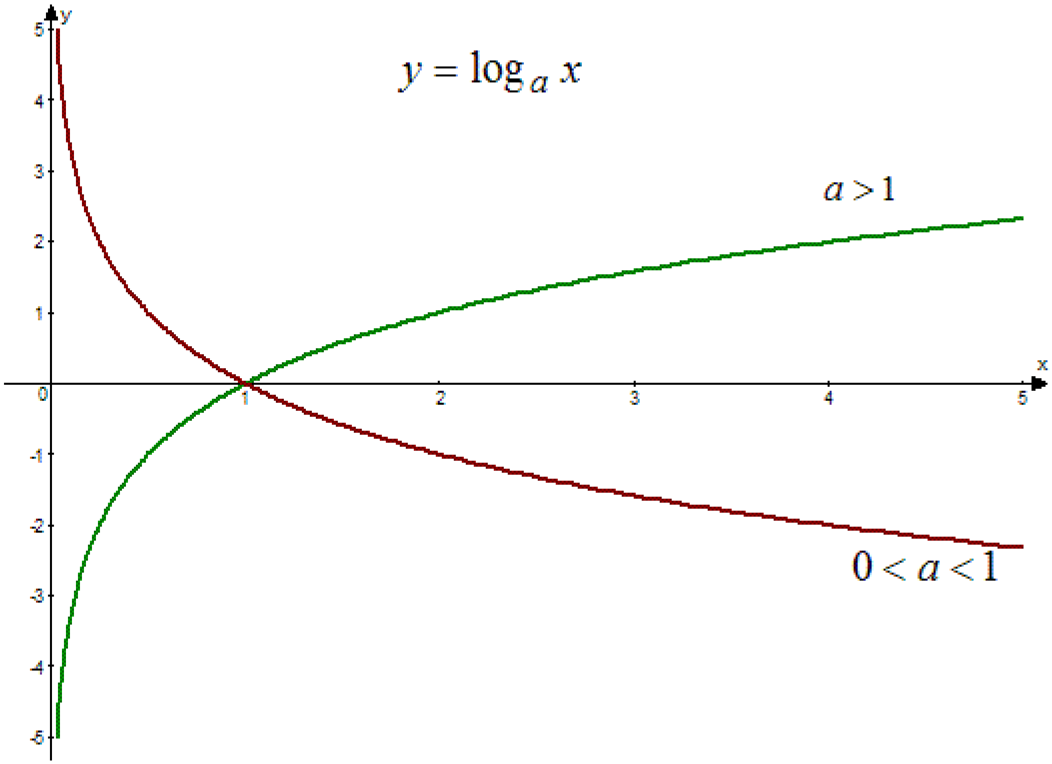


Koʻrsatkichli funksiya, $y = a^x, a > 0 \quad a \neq 1$



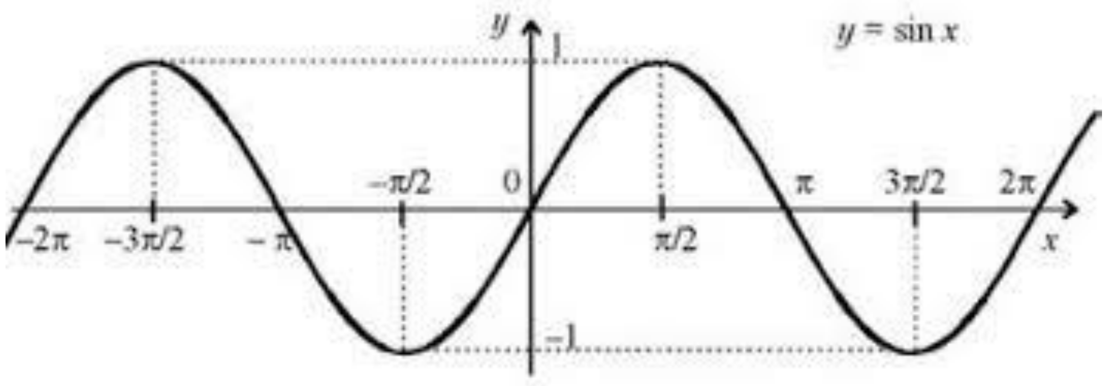
e^x

Logarifmik funksiya, $y = \log_a x, \quad a > 0 \quad a \neq 1 \quad x > 0$

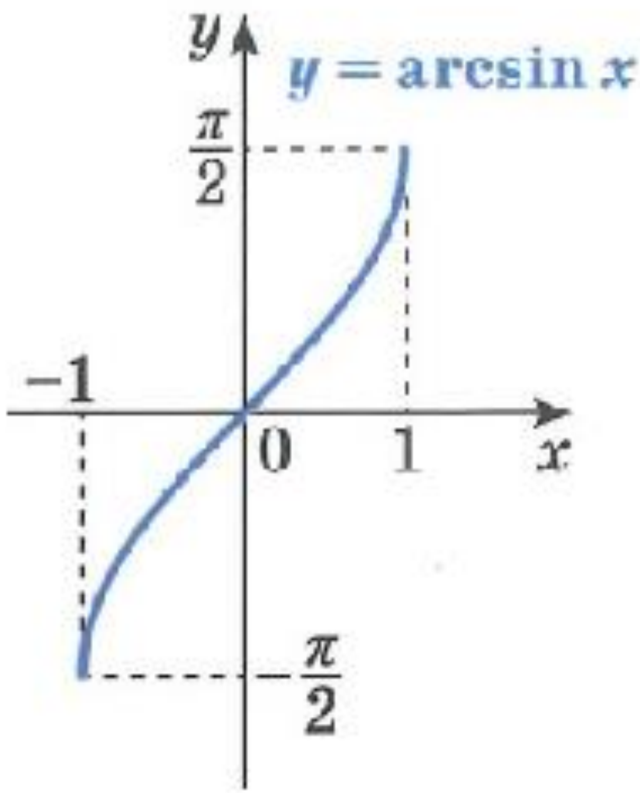


Trigonometrik funksiyalar, $\sin x, \quad \cos x, \quad tgx, \quad ctgx.$

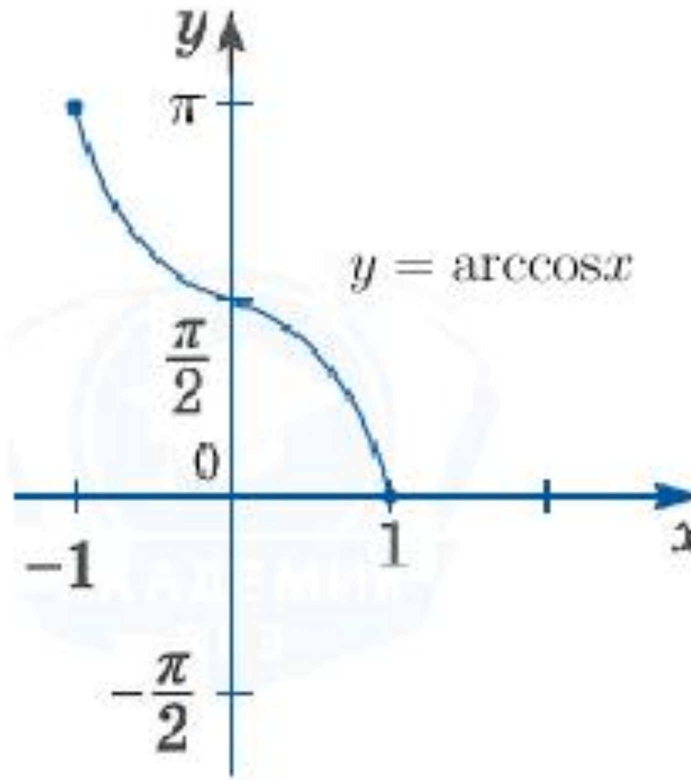
$$y = \sin x$$



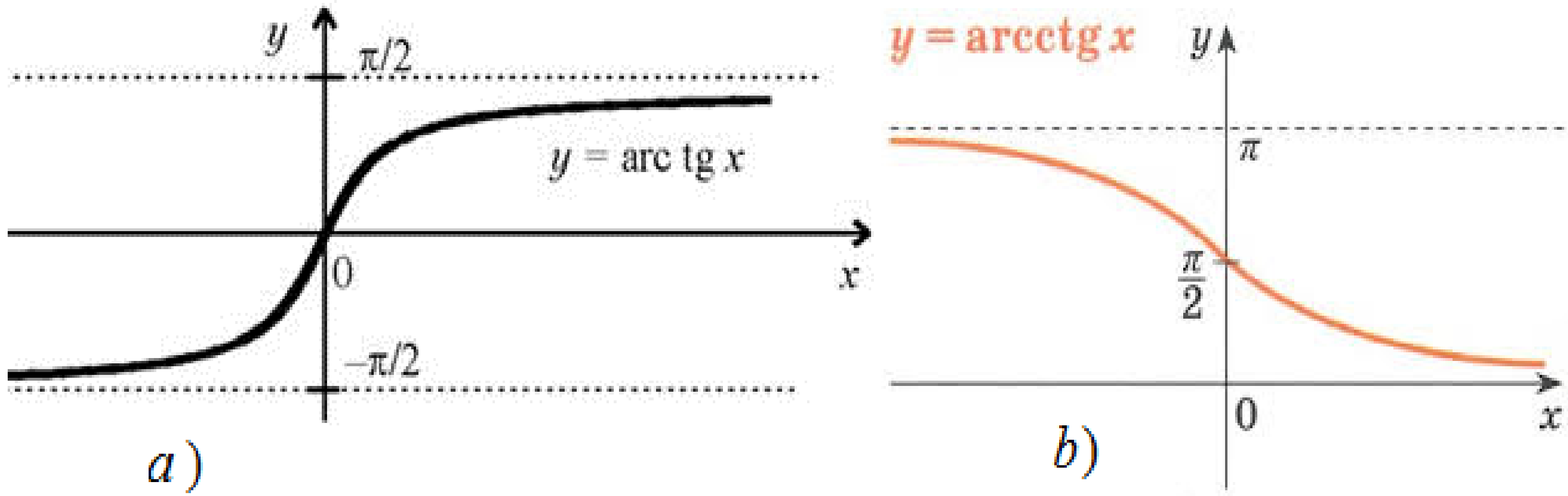
Teskari trigonometrik funksiyalar, $\arcsin x,\ \arccos x,\ \mathrm{arc}\,tgx,\ \ \mathrm{arc}\,ctgx.$



a)



b)


y
π/2
y = arc tg x
0
x
−π/2
a)
y = arcctg x
y
π
π/2
0
x
b)

Ta'rif. Chekli sondagi arifmetik amallar yordamida asosiy elementar funksiyalarning kompozitsiyasidan tuzilgan funksiyalarga elementar funksiyalar deyiladi.

Elementar funksiyalarni quyidagi sinflarga ajratiladi:

I. Ko'phadlar.

$$P_n(x) = \sum_{k=0}^{n} a_k x^k.$$

II. Ratsional funksiyalar (ratsional kasrlar).

$$\frac{P(x)}{Q(x)}, \qquad P(x), Q(x) \text{ - ko'phadlar,} \qquad Q(x) \neq 0.$$

III. Irratsional funksiyalar.

Misol. $y = \sqrt[3]{ax+b}.$

IV. Transsendent funksiyalar.

Chegaralangan funksiyalar

Biror X sonlar to‘plami berilgan bo‘lsin.

Ta’rif. Agar shunday o‘zgarmas M (m) soni topilsaki, $\forall x \in X$ uchun

$$f(x) \le M \qquad \left(f(x) \ge m \right)$$

tengsizlik bajarilsa, u holda $f(x)$ funksiya X to‘plamda yuqoridan (quyidan) chegaralangan funksiya deyiladi.

Masalan, $f(x) = ax^2 + bx + c$

$$a < 0 \qquad M = f\left(-\frac{b}{2a}\right).$$

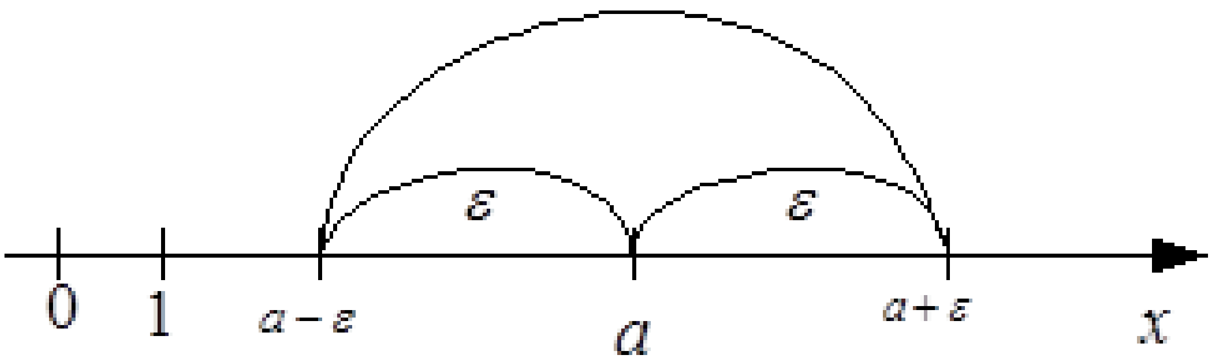
$$a > 0 \qquad m = f\left(-\frac{b}{2a}\right).$$

$$m \le f(x) \le M$$

$$-1 \le \sin x \le 1, \qquad -1 \le \cos x \le 1.$$

Sonlar ketma-ketligi

Ta’rif. $(a-\varepsilon, \quad a+\varepsilon)$ interval a ning ε atrofi deyiladi.



Biror X to‘plam berilgan bo‘lsin. Har bir $n \in N$ natural songa qandaydir bir $x_n \in X$ elementni mos qo‘yamiz. Natijada quyidagi funksiya hosil bo‘ladi:

$$x_n = f(n): N \to X.$$

Hosil bo‘lgan funksiya qiymatlarini X to‘plam elementlaridan iborat cheksiz ketma-ketlik deyiladi va $\{x_n\}$ bilan belgilanadi.

$$x_1, \quad x_2, \quad x_3, \ldots, x_n, \ldots$$

x_n - ga ketma-ketlikning umumiy hadi

$M.1. \quad 1, \frac{1}{4}, \frac{1}{9}, \ldots$

$-1, 1, -1, 1, \ldots$

$2, \frac{3}{2}, \frac{4}{3}, \ldots$

Ta'rif. Agar $\forall \varepsilon > 0$ uchun shunday $n_\varepsilon \in N$ topilsaki, $\forall n > n_\varepsilon$ uchun $|x_n - a| < \varepsilon$ tengsizlik o'rinli bo'lsa, u holda $a \in R$ soni $\{x_n\}$ ketma-ketlik limiti deyiladi va quyidagicha yoziladi:

$$\lim_{n\to\infty} x_n = a.$$

Masalan, $\lim_{n\to\infty} \frac{1}{n} = 0.$

Funksiya limiti

Ta'rif. Agar $\forall \varepsilon > 0$ son uchun shunday $\delta > 0$ son topilsaki, argument x ning $0 < |x-a| < \delta$ tengsizlikni qanoatlantiruvchi barcha qiymatlarida $|f(x) - b| < \varepsilon$ tengsizlik bajarilsa, b son $f(x)$ funksiyaning a nuqtadagi ($x \to a$ dagi) limiti deyiladi va

$$\lim_{x \to a} f(x) = b$$

kabi belgilanadi.

Ta'rif. Agar $\forall E > 0$ son uchun shunday $\delta > 0$ son topilsaki, argument x ning ($a-\delta,\ a+\delta$) intervaldan olingan $(x \neq a)$ barcha qiymatlarida

$$|f(x)| > E \qquad (\ f(x) > E; \quad -f(x) > E\ )$$

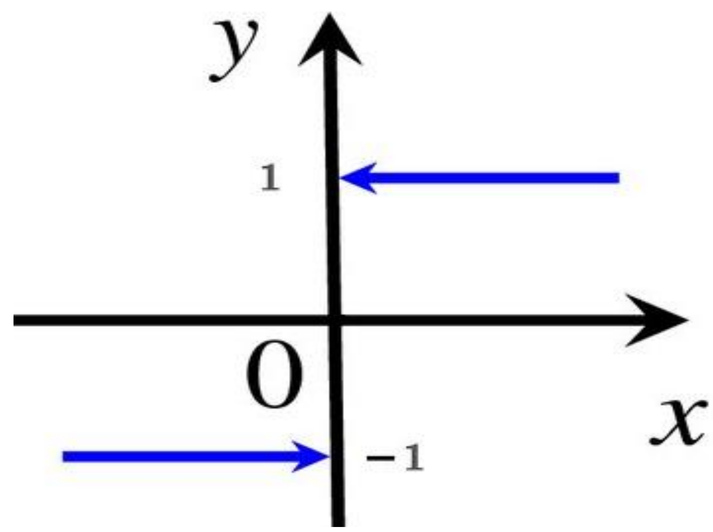
tengsizlik bajarilsa, $f(x)$ funksiyaning a nuqtadagi limiti ∞ ($+\infty,\ -\infty$) deyiladi va

$$\lim_{x \to a} f(x) = \infty \qquad \left(\lim_{x \to a} f(x) = +\infty, \quad \lim_{x \to a} f(x) = -\infty \right)$$

kabi belgilanadi.

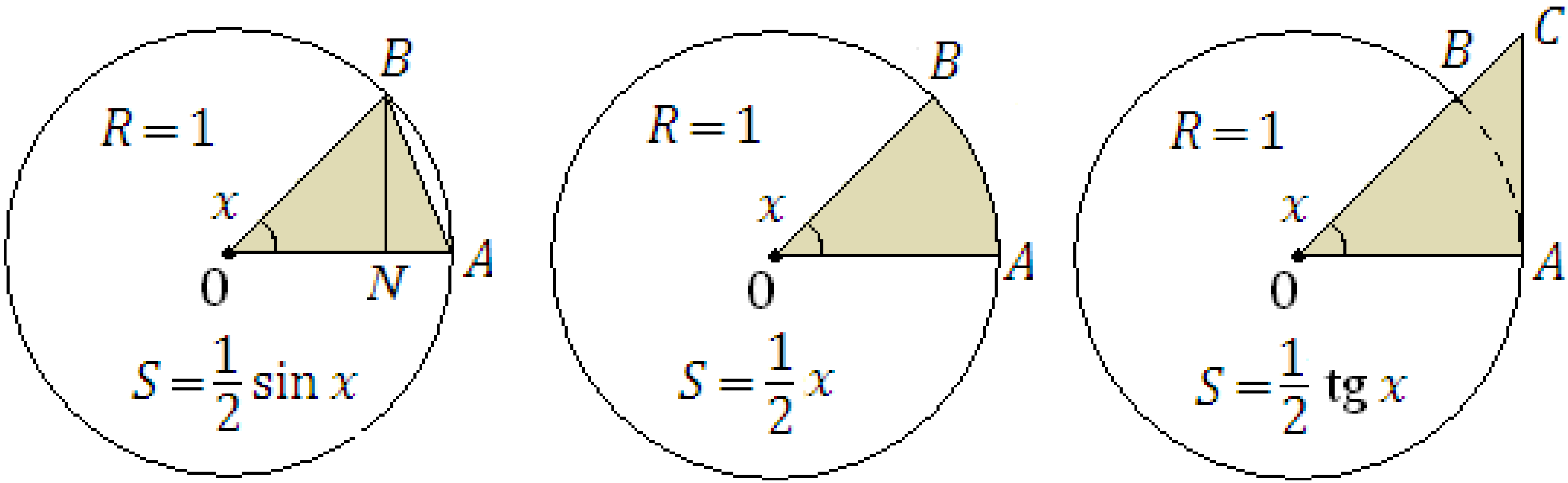
Bir tomonli limitlar.

$$f(x) = sign(x) = \begin{cases} -1, \text{ агар } x > 0, \\ 0, \text{ агар } x = 0. \\ -1, \text{ агар } x < 0. \end{cases}$$



1. Birinchi ajoyib limit:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1.$$



Ikkinchi ajoyib limit

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e.$$

e soni irratsional son.

Uning verguldan keyingi o‘nta ishonchli raqamli qiymati

$$e = 2,7182818284...$$

$$\frac{1}{1\cdot 2}\cdot\left(1-\frac{1}{n}\right)<\frac{1}{1\cdot 2}\cdot\left(1-\frac{1}{n+1}\right)$$

$$\left(1-\frac{1}{n}\right)<1; \qquad \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)<1$$

$$\left(1+\frac{1}{n}\right)^n<1+1+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{n!}.$$

$$\frac{1}{3!}<\frac{1}{2^2}; \qquad \frac{1}{4!}<\frac{1}{2^3}; \qquad \ldots, \qquad \frac{1}{n!}<\frac{1}{2^{n-1}},$$

$$\left(1+\frac{1}{n}\right)^n<1+\underbrace{1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^{n-1}}}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+7}=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\cdot\left(1+\frac{1}{n}\right)^{7}=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\cdot\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{7}=$$

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{x}=\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{\frac{x}{3}\cdot 3}=\left[\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{\frac{x}{3}}\right]^{3}=$$

$$\lim_{x\to\infty}\left(\frac{x+2}{x-1}\right)^{x}=\lim_{x\to\infty}\left(1+\frac{x+2}{x-1}-1\right)^{x}=\lim_{x\to\infty}\left(1+\frac{3}{x-1}\right)^{x}=$$

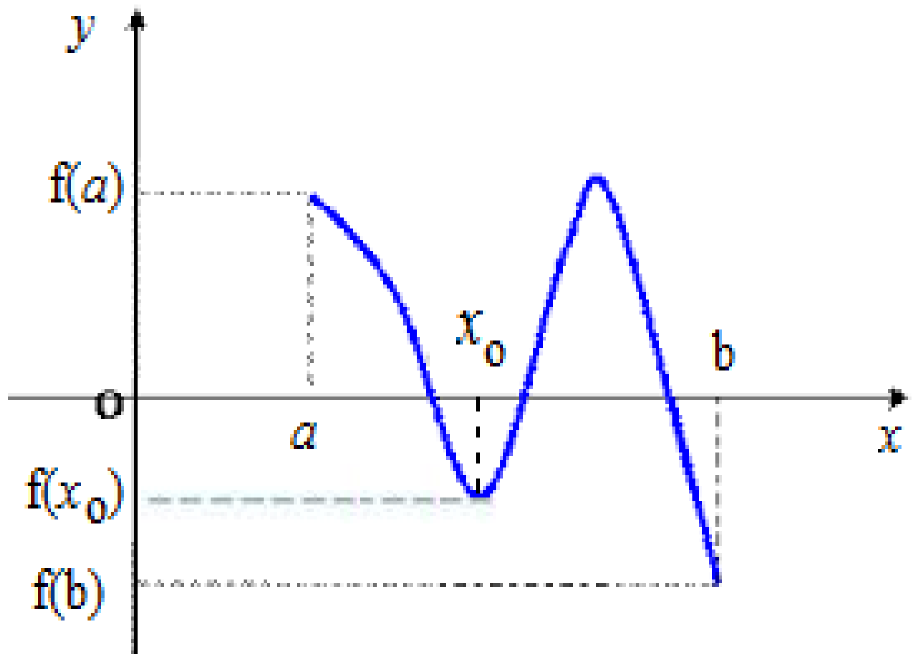
$$\left|\begin{matrix} y=\dfrac{3}{x-1},\, x=\dfrac{3}{y}+1 \\ x\to\infty,\, y\to 0\end{matrix}\right|=\lim_{y\to 0}(1+y)^{\frac{3}{y}+1}=\lim_{y\to 0}(1+y)^{\frac{1}{y}\cdot 3}\cdot\lim_{y\to 0}(1+y)=$$

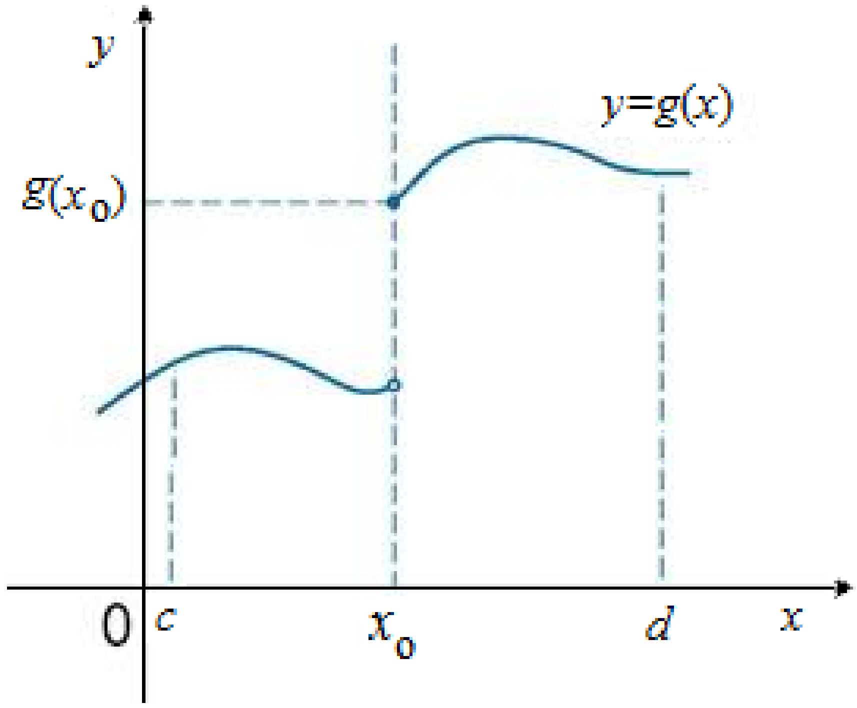
$$S_t = S_0\left(1+\frac{p}{100}\right)^t.$$

$$\frac{1}{m} \qquad\qquad \frac{p\%}{m}$$

$$S_t = \lim_{m\to\infty}\left(S_0\left(1+\frac{p}{100m}\right)^{mt}\right) = S_0\lim_{m\to\infty}\left(\left(1+\frac{p}{100m}\right)^{\frac{100m}{p}}\right)^{\frac{pt}{100}}$$

Funksiyaning uzluksizligi





Misol.

$$f(x)=\begin{cases}-x^3, \text{ агар } x\le 1 \text{ бўлса,}\\ x, \text{ агар } x>1 \text{ бўлса}\end{cases}$$

funksiyani uzluksizlikka tekshiring.

Funksiyaning uzilishi

$f(x)$ funksiya x_0 nuqtada uzluksiz bo‘lishi uchun:

1º. x_0 nuqtada va uning biror atrofida aniqlangan bo‘lishi;

2º. $x \to x_0$ da o‘ng va chap limitlarga ega bo‘lib,

$$\lim_{x \to x_0 + 0} f(x) = \lim_{x \to x_0 + 0} f(x) = f(x_0)$$

bo‘lishi zarur va etarli.

1 . $$f(x) = \frac{1}{x-2}$$

2 . $$f(x) = \begin{cases} -\frac{1}{4}x^2, \text{ агар } x \le 4 \text{ бўлса,} \\ x, \text{ агар } x > 4 \text{ бўлса} \end{cases}$$

1. *Bartaraf etiladigan uzilish*.

2. *I-tur uzilish.*

3. *II-tur uzilish*.