OSHKORMAS FUNKSIYA HOSILASI.

1. Oshkormas funksiya tushunchasi. Ma`lumki, $x \subset R$ to`plamdagi har bir x songa biror qoidaga ko`ra $Y \subset R$ to`plamdan bitta y son mos qo`yilgan bo`lsa, X to`plamda funksiya berilgan deb atalar va u $f : x \to y$ yoki $y = f(x)$ kabi belgilanar edi.

Endi ikki x va y argumentlarning $F(x, y)$ funksiyasi

$$M = \{(x, y) \in R^2 : a < x < в,\ \ c < y < d\}$$

to`plamda berilgan bo`lsin. Ushbu

$$F(x, y) = 0$$

tenglamani qaraylik. Biror x_0 sonni $(x_0 \in (a, в))$ olib, uni yuqoridagi tenglamadagi x ning o`rniga qo`yamiz. Natijada y ni topish uchun quyidagi

$$F(x_0, y) = 0 \qquad \textbf{(1.1)}$$

tenglamaga kelamiz. Bu tenglamaning yechimi haqida ushbu hollar bo`lishi mumkin:

1. (1.1) tenglama yagona haqiqiy y_0 yechimga ega,

2. (1.1) tenglama bitta ham haqiqiy yechimga ega emas,

3. (1.1) tenglama bir nechta, hatto cheksiz ko`p haqiqiy yechimga ega.

Masalan,

$$F(x,y)=\begin{cases} y-x^2, \text{ agar } x\geq 0 \text{ bo'lsa,} \\ y^2+x, \text{ agar } x<0 \text{ bo'lsa} \end{cases}.$$

U holda

$$F(x,y)=0 \qquad \textbf{(1.2)}$$

tenglama $x_0 \geq 0$ bo`lganda, yagona $y=x_0^2$ yechimga, $x_0<0$ bo`lganda ikkita

$$y=\sqrt{-x_0}, \quad y=-\sqrt{-x_0}$$

echimga ega bo`ladi.

Agar biror $F(x,y)=0$ tenglama uchun 1.-hol o`rinli bo`lsa bunday tenglama e'tiborga loyiq. Uning yordamida funksiya aniqlanishi mumkin.

Endi x o`zgaruvchining qiymatlaridan iborat shunday $X$ to`plamni qaraylikki, bu to`plamdan olingan har bir qiymatda $F(x,y)=0$ tenglama yagona echimga ega bo`lsin.

X to`plamdan ixtiyoriy $x$ sonni olib, bu songa $F(x,y)=0$ tenglamaning yagona echimi bo'lgan $y$ sonni mos qo`yamiz. Natijada X to`plamdan olingan har bir $x$ ga yuqoridagi ko`rsatilgan qoidaga ko`ra bitta $y$ mos qo`yilib, funksiya hosil bo`ladi. Bunda $x$ va $y$ o`zgaruvchilar orasidagi bog`lanish $F(x,y)=0$ tenglama yordamida bo`ladi. Odatda bunday berilgan (aniqlangan) funksiya ***oshkormas ko`rinishda berilgan funksiya*** (yoki ***oshkormas funksiya)*** deb ataladi va

$$x \to y : F(x,y)=0 .$$

kabi belgilanadi.

2. Oshkormas funksiyaning hosilasi. Endi oshkormas funksiyaning hosilasini topish bilan shug'ullanamiz.

1-teorema. $F(x,y)$ funksiya $(x_0,y_0)\in R^2$ nuqtaning biror

$$U_{h,k}((x_0,y_0))=\{(x,y)\in R^2 :\ x_0-h<x<x_0+h;\ \ y_0-k<y<y_0+k\}$$

atrofida $(h>0,k>0)$ berilgan va u quyidagi shartlarni bajarsin:

1) $U_{h,k}((x_0,y_0))$ da uzluksiz,

2) $U_{h,k}((x_0,y_0))$ da uzluksiz $F`_x(x,y)$, $F`_y(x,y)$ xususiy hosilalarga ega va $F`_y(x_0,y_0)\neq 0$;

β) $F'_y(x_0,y_0)=0$.

U holda (x_0,y_0) nuqtaning shunday

$$U_{\delta,\varepsilon}((x_0,y_0))=\{(x,y)\in R^2:\ x_0-\delta<x<x_0+\delta;\ y_0-\varepsilon<y<y_0+\varepsilon\}$$

atrofi $(0<\delta<h,\ 0<\varepsilon<\mathrm{k})$ topiladiki,

a) $\forall x\in(x_0-\delta,x_0+\delta)$ uchun

$$F(x,y)=0$$

tenglama yagona y yechimga $y\in(y_0-\varepsilon,\ y_0+\varepsilon)$ ega, ya'ni $F(x,y)=0$ tenglama yordamida

$$x\to y:\ F(x,y)=0$$

oshkormas ko`rinishdagi funksiya aniqlanadi;

b) $x=x_0$ bo`lganda unga mos keladigan $y$ uchun $y=y_0$ bo`ladi;

c) oshkormas ko`rinishda aniqlangan

$$x\to y:\ F(x,y)=0$$

funksiya $(x_0-\delta,x_0+\delta)$ oraliqda uzluksiz bo`ladi;

d) Bu oshkormas ko`rinishdagi funksiya $(x_0-\delta,x_0+\delta)$ oraliqda uzluksiz hosilaga ega bo`ladi.

◄Shartga ko`ra $F'_y(x,y)$ funksiya $U_{h,k}((x_0,y_0))$ da uzluksiz va $F'_y(x_0,y_0)\neq 0$.

Aniqlik uchun $F`_y(x_0, y_0) > 0$ deylik. U holda uzluksiz funksiyaning xossasiga ko'ra (x_0, y_0) nuqtaning shunday

$$U_{\delta,\varepsilon}((x_0, y_0)) = \{(x, y) \in R^2 : \; x_0 - \delta < x < x_0 + \delta; \; y_0 - \varepsilon < y < y_0 + \varepsilon\}$$

atrofi $(0 < \delta < h, \; 0 < \varepsilon < \mathrm{k})$ topiladiki, $\forall (x, y) \in U_{\delta,\varepsilon}((x_0, y_0))$ uchun $F'_y(x, y) > 0$ bo'ladi. Demak, $F(x, y)$ funksiya x o'zgaruvchining $(x_0 - \delta, x_0 + \delta)$ oraliqdan olingan har bir tayin qiymatida y o'zgaruvchining funksiyasi sifatida o'suvchi.

$$F(x, y) = 0$$

tenglama $(x_0 - \delta, x_0 + \delta)$ da

$$x \to y : \; F(x, y) = 0$$

oshkormas ko`rinishdagi funksiyani aniqlaydi, $x = x_0$ bo`lganda unga mos kelgan $y = y_0$ bo`ladi va oshkormas funksiya $(x_0 - \delta, x_0 + \delta)$ da uzluksiz bo`ladi.

Endi oshkormas funksiyaning hosilasini topamiz. x_0 nuqtaga shunday Δx orttirma beraylikki, $x_0 + \Delta x \in (x_0 - \delta, x_0 + \delta)$ bo`lsin. Natijada

$$x \to y : \; F(x, y) = 0$$

oshkormas funksiya ham orttirmaga ega bo`lib,

$$F(x_0 + \Delta x, y_0 + \Delta y) = 0$$

bo'ladi. Demak,

$$\Delta F(x_0, y_0) = F(x_0 + \Delta x, y_0 + \Delta y) - F(x_0, y_0) = 0 \qquad \textbf{(1.3)}$$

Shartga ko`ra $F`_x(x,y)$ va $F`_y(x,y)$ xususiy hosilalar $U_{\delta,\varepsilon}((x_0, y_0))$ da uzluksiz. Binobarin $F(x,y)$ funksiya (x_0, y_0) nuqtada differensiallanuvchi:

$$\Delta F(x_0, y_0) = F`_x(x_0, y_0)\Delta x + F`_y(x_0, y_0)\Delta y + \alpha\Delta x + \beta\Delta y \qquad \textbf{(1.4)}$$

Bu munosabatdagi α va β lar Δx va Δy larga bog'liq va $\Delta x \to 0$, $\Delta y \to 0$ da $\alpha \to 0$, $\beta \to 0$.

(1.3) va **(1.4)** munosabatlardan

$$\frac{\Delta y}{\Delta x} = -\frac{F`_x(x_0, y_0) + \alpha}{F`_y(x_0, y_0) + \beta}$$

ekanligi kelib chiqadi.

Oshkormas funksiyaning x_0 nuqtada uzluksizligini e'tiborga olib, keyingi tenglikda $\Delta x \to 0$ da limitga o'tib quyidagini topamiz:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\left(-\frac{F`_x(x_0, y_0) + \alpha}{F`_y(x_0, y_0) + \beta}\right) = -\frac{F`_x(x_0, y_0)}{F`_y(x_0, y_0)}.$$

Demak,

$$y`_{x=x_0} = -\frac{F`_x(x_0, y_0)}{F`_y(x_0, y_0)}$$

$U_{\delta,\varepsilon}((x_0, y_0))$ da $F'_x(x,y)$, $F'_y(x,y)$ xususiy hosilalar uzluksiz va $F'_y(x,y) \neq 0$ bo'lishidan oshkormas funksiyaning hosilasi

$$y`_x = -\frac{F`_x(x,y)}{F`_y(x,y)}$$

ning $(x_0 - \delta, x_0 + \delta)$ oraliqda uzluksiz bo'lishi kelib chiqadi. ▶

1-misol. Ushbu

$$F(x,y) = xe^y + ye^x - 2 = 0 \qquad \textbf{(1.5)}$$

tenglama bilan aniqlanadigan oshkormas funksiyaning hosilasi topilsin.

◀Ravshanki, $F(x,y) = xe^y + ye^x - 2$ funksiya $\{(x,y) \in R^2 : -\infty < x < +\infty$ $-\infty < y < +\infty\}$ to'plamda yuqoridagi teoremaning barcha shartlarini qanoatlantiradi. Demak, $\forall (x_0, y_0) \in R^2$ nuqtaning $U_{\delta,\varepsilon}((x_0, y_0))$ atrofida (1.4) tenglama oshkormas ko`rinishdagi funksiyani aniqlaydi va bu oshkormas funksiyaning hosilasi

$$y` = -\frac{F`_x(x,y)}{F`_y(x,y)} = -\frac{e^y + ye^x}{xe^y + e^x}$$

Bo`ladi. ▶

Oshkormas ko`rinishdagi funksiyaning hosilasini quyidagicha ham hisoblasa bo`ladi. y ning x ga bog`liq ekanini e'tiborga olib, $F(x,y)=0$ dan topamiz:

$$F`_x(x,y) + F`_y(x,y)\cdot y` = 0$$

Bundan esa

$$y` = -\frac{F`_x(x,y)}{F`_y(x,y)}$$

bo'lishi kelib chiqadi.

Yuqorida keltirilgan (1.4) tenglama yordamida aniqlangan oshkormas ko`rinishdagi funksiyaning hosilasini hisoblaylik:

$$F`_x(x,y) + F`_y(x,y)\cdot y` = e^y + ye^x + \left(xe^y + e^x\right)y` = 0$$

$$y` = -\frac{e^y + ye^x}{xe^y + e^x}.$$

YUQORI TARTIBLI XUSUSIY HOSILALAR

1. Funksiyaning yuqori tartibli xususiy hosilalari. $f(x_1, x_2, \ldots, x_m)$ funksiya ochiq $M \ (M \subset R^m)$ to`plamda berilgan bo`lib, uning har bir $(x_1, x_2, \ldots, x_m)$ nuqtasida $f'_{x_1}, f'_{x_2}, \ldots, f'_{x_m}$ xususiy hosilalarga ega bo`lsin. Ravshanki, bu xususiy hosilalar o`z navbatida $x_1, x_2, \ldots, x_m$ o`zgaruvchilarga bog`liq bo`lib, ularning funksiyalari bo`lib qolishi mumkin. Berilgan funksiya xususiy hosilalari $f'_{x_1}, f'_{x_2}, \ldots, f'_{x_m}$ larning ham xususiy hosilalarini qarash mumkin.

1-ta'rif. $f(x_1, x_2, \ldots, x_m)$ funksiya xususiy hosilalari $f'_{x_1}, f'_{x_2}, \ldots, f'_{x_m}$ larning $x_k \ (k = 1, 2, \ldots, m)$ o`zgaruvchi bo`yicha xususiy hosilalari berilgan funksiyaning ikkinchi tartibli xususiy hosilalari deb ataladi va

$$f''_{x_1 x_k}, f''_{x_2 x_k}, \ldots, f''_{x_m x_k} \qquad (k = 1,\ 2,\ \ldots,\ m)$$

yoki

$$\frac{\partial^2 f}{\partial x_1 \partial x_k}, \frac{\partial^2 f}{\partial x_2 \partial x_k}, \ldots, \frac{\partial^2 f}{\partial x_m \partial x_k} \qquad (k = 1,\ 2,\ \ldots,\ m)$$

kabi belgilanadi. Demak,

$$\frac{\partial^2 f}{\partial x_1 \partial x_k} = f''_{x_1 x_k} = \frac{\partial}{\partial x_k}\left(\frac{\partial f}{\partial x_1}\right),$$

$$\frac{\partial^2 f}{\partial x_2 \partial x_k} = f''_{x_2 x_k} = \frac{\partial}{\partial x_k}\left(\frac{\partial f}{\partial x_2}\right),$$

.................................,

$$\frac{\partial^2 f}{\partial x_m \partial x_k} = f''_{x_m x_k} = \frac{\partial}{\partial x_k}\left(\frac{\partial f}{\partial x_m}\right) \qquad (k = 1,\ 2,\ \ldots,\ m)$$

Bu ikkinchi tartibli xususiy hosilalarni umumiy holda

$$\frac{\partial^2 f}{\partial x_i \partial x_k} = f''_{x_i x_k} \qquad (i = 1, 2, \ldots, m;\ k = 1,\ 2,\ \ldots,\ m)$$

Ko`rinishda yozish mumkin, bunda $k = i$ bo`lganda

$$\frac{\partial^2 f}{\partial x_k \partial x_k} = f''_{x_k x_k}$$

deb yozish o`rniga

$$\frac{\partial^2 f}{\partial x_k^2} = f''_{x_k^2}$$

deb yoziladi.

Agar yuqoridagi ikkinchi tartibli xususiy hosilalar turli o`zgaruvchilar bo`yicha olingan bo`lsa, unda bu

$$\frac{\partial^2 f}{\partial x_i \partial x_k} = f''_{x_i x_k} \qquad (i \neq k)$$

2-tartibli xususiy hosilalar aralash hosilalar deb ataladi.

Xuddi shunga o`xshash, $f(x_1, x_2, \ldots, x_m)$ funksiyaning uchinchi, to`rtinchi va xokazo tartibdagi xususiy hosilalari ta'riflanadi. Umuman, $f(x_1, x_2, \ldots, x_m)$ funksiya $(n-1)$-tartibli xususiy hosilalarning xususiy hosilasi berilgan funksiyaning n -tartibli xususiy hosilasi deb ataladi.

1-misol. Ushbu

$$f(x_1, x_2) = arctg\,\frac{x_1}{x_2} \qquad (x_2 \neq 0)$$

funksiyaning 2-tartibli xususiy hosilasi topilsin.

◄Ravshanki, $\frac{\partial f}{\partial x_1} = \frac{x_2}{x_1^2 + x_2^2}$, $\quad \frac{\partial f}{\partial x_2} = -\frac{x_1}{x_1^2 + x_2^2}$,

$$\frac{\partial^2 f}{\partial x_1^2} = \frac{\partial}{\partial x_1}\left(\frac{\partial f}{\partial x_1}\right) = \frac{\partial}{\partial x_1}\left(\frac{x_2}{x_1^2 + x_2^2}\right) = -\frac{2x_1 x_2}{\left(x_1^2 + x_2^2\right)^2},$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial}{\partial x_2}\left(\frac{\partial f}{\partial x_1}\right) = \frac{\partial}{\partial x_2}\left(\frac{x_2}{x_1^2 + x_2^2}\right) = \frac{x_1^2 - x_2^2}{\left(x_1^2 + x_2^2\right)^2},$$

$$\frac{\partial^2 f}{\partial x_2 \partial x_1} = \frac{\partial}{\partial x_1}\left(\frac{\partial f}{\partial x_2}\right) = \frac{\partial}{\partial x_1}\left(-\frac{x_1}{x_1^2 + x_2^2}\right) = \frac{x_1^2 - x_2^2}{\left(x_1^2 + x_2^2\right)^2},$$

$$\frac{\partial^2 f}{\partial x_2^2} = \frac{\partial}{\partial x_2}\left(\frac{\partial f}{\partial x_2}\right) = \frac{\partial}{\partial x_2}\left(-\frac{x_1}{x_1^2 + x_2^2}\right) = \frac{2x_1 x_2}{\left(x_1^2 + x_2^2\right)^2}. \blacktriangleright$$

2-misol. Ushbu

$$f(x_1, x_2) = \begin{cases} x_1 x_2 \dfrac{x_1^2 - x_2^2}{x_1^2 + x_2^2}, & x_1^2 + x_2^2 \neq 0 \\ 0 & x_1^2 + x_2^2 = 0 \end{cases}$$

funksiyaning aralash hosilalari topilsin.

◀ Aytaylik $(x_1, x_2) \neq (0, 0)$ bo'lsin. U holda

$$\frac{\partial f}{\partial x_1} = x_2\left(\frac{x_1^2 - x_2^2}{x_1^2 + x_2^2} + \frac{4x_1^2 x_2^2}{\left(x_1^2 + x_2^2\right)^2}\right), \quad \frac{\partial f}{\partial x_2} = x_1\left(\frac{x_1^2 - x_2^2}{x_1^2 + x_2^2} - \frac{4x_1^2 x_2^2}{\left(x_1^2 + x_2^2\right)^2}\right),$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial}{\partial x_2}\left(\frac{\partial f}{\partial x_1}\right) = \frac{x_1^2 - x_2^2}{x_1^2 + x_2^2}\left(1 + \frac{8x_1^2 x_2^2}{\left(x_1^2 + x_2^2\right)^2}\right),$$

$$\frac{\partial^2 f}{\partial x_2 \partial x_1} = \frac{\partial}{\partial x_1}\left(\frac{\partial f}{\partial x_2}\right) = \frac{x_1^2 - x_2^2}{x_1^2 + x_2^2}\left(1 + \frac{8x_1^2 x_2^2}{\left(x_1^2 + x_2^2\right)^2}\right) \text{bo'ladi.}$$

Berilgan $f(x_1,x_2)$ funksiyaning $(0,0)$ nuqtadagi xususiy hosilalarini ta'rifga ko`ra topamiz:

$$\frac{\partial f(0,0)}{\partial x_1} = \lim_{\Delta x_1 \to 0} \frac{f(\Delta x_1,0) - f(0,0)}{\Delta x_1} = 0,$$

$$\frac{\partial f(0,0)}{\partial x_2} = \lim_{\Delta x_2 \to 0} \frac{f(0,\Delta x_2) - f(0,0)}{\Delta x_2} = 0,$$

$$\frac{\partial^2 f(0,0)}{\partial x_1 \partial x_2} = \lim_{\Delta x_1 \to 0} \frac{\frac{\partial f(0,\Delta x_2)}{\partial x_1} - \frac{\partial f(0,0)}{\partial x_1}}{\Delta x_2} = \lim_{\Delta x_2 \to 0} \frac{-\Delta x_2^3}{\Delta x_2^3} = -1,$$

$$\frac{\partial^2 f(0,0)}{\partial x_2 \partial x_1} = \lim_{\Delta x_2 \to 0} \frac{\frac{\partial f(\Delta x_1,0)}{\partial x_2} - \frac{\partial f(0,0)}{\partial x_2}}{\Delta x_1} = \lim_{\Delta x_1 \to 0} \frac{\Delta x_1^3}{\Delta x_1^3} = 1,$$

Bu keltirilgan misollar ko`rinadiki, funksiyaning $\frac{\partial^2 f}{\partial x_1 \partial x_2}$ va $\frac{\partial^2 f}{\partial x_2 \partial x_1}$ aralash hosilalari bir-biriga teng bo`lishi ham, teng bo`lmasligi ham mumkin ekan.

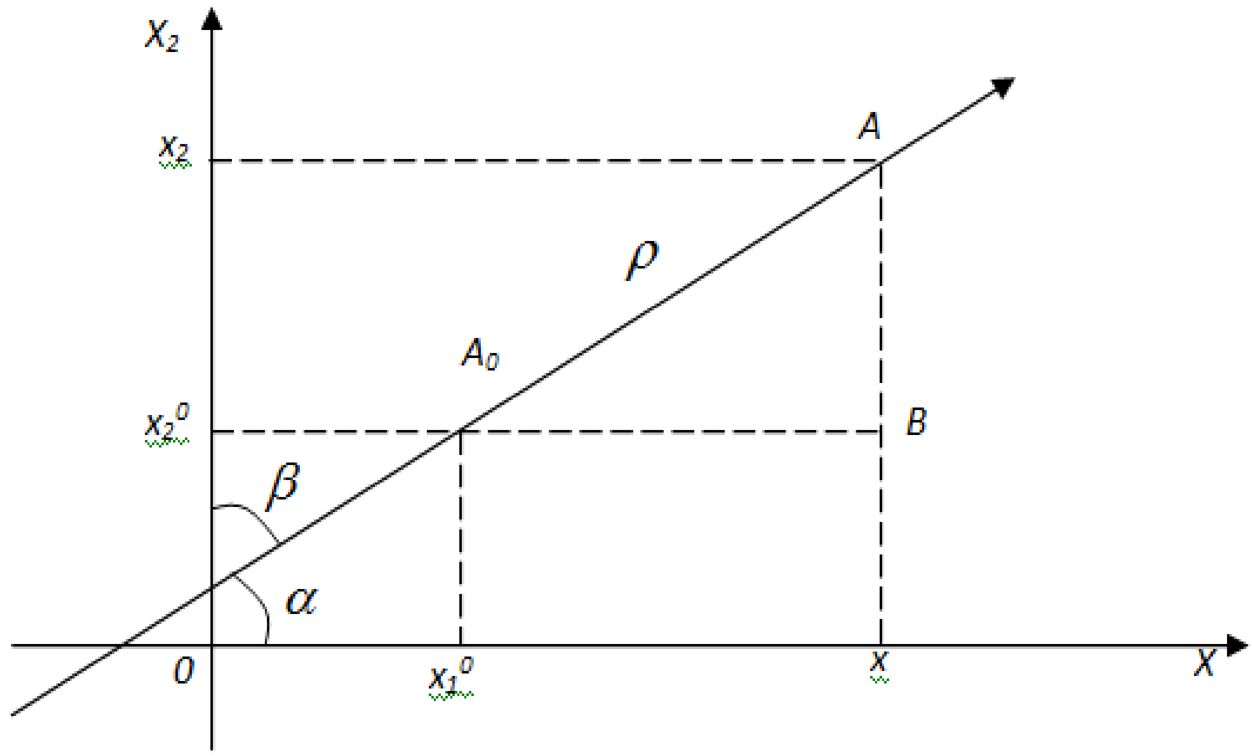
YO`NALISH BO`YICHA HOSILA

Ma'lumki, bir o`zgaruvchili $y = f(x)$ funksiyaning $(x \in R, y \in R)$ $\frac{df}{dx}$ hosilasi bu funksiyaning o`zgarish tezligini bildirar edi. Ko`p o`zgaruvchili $y = f(x_1, x_2, \ldots, x_m)$ funksiyaning xususiy hosilalari ham bir o`zgaruvchili funksiyaning hosilasi kabi ekanligini e'tiborga olib, bu $\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_m}$ xususiy hosilalar ham $y = f(x_1, x_2, \ldots, x_m)$ funksiyaning mos ravishda $ox_1, ox_2, \ldots, ox_m$ o`qlar bo`yicha o`zgarish tezligini ifodalaydi deb aytish mumkin.

Endi funksiyaning ixtiyoriy yo`nalish bo`yicha o`zgarish tezligini ifodalovchi tushuncha bilan tanishaylik. Soddalik uchun ikki o`zgaruvchili funksiyani qaraymiz.

$y = f(x_1, x_2) = f(A)$ funksiya ochiq M to`plamda $(M \subset R^2)$ berilgan bo`lsin. Bo` to`plamda ixtiyoriy $A_0 = (x_1^0, x_2^0)$ nuqtani olib, u orqali biror to`g`ri chiziq o`tkazaylik va undagi ikki yo`nalishdan birini musbat yo`nalish, ikkinchisini manfiy yo`nalish deb qabul qilaylik. Bu yo`nalgan to`g`ri chiziqni l deylik.

α va β deb l yo`nalgan to`g`ri chiziq musbat yo`nalishi bilan mos ravishda ox_1 va ox_2 koordinata o`qlarining musbat yo`nalishi orasidagi burchaklarni olaylik (49-chizma).



49-chizma

l to`g`ri chiziqda A_0 nuqtadan farqli va M to`plamga tegishli bo`lgan A nuqtani $(A=(x_1,x_2))$ olaylikki, A_0A kesma M to`plamga tegishli bo`lsin. Agarda A nuqta A_0 ga nisbatan l to`g`ri chiziqning musbat yo`nalishi tomonidan bo`lsa (shakldagidek), u holda A_0A kesma uzunligi $\rho(A_0,A)$ ni musbat ishora bilan olishga kelishaylik.

ΔA_0AB dan

$$\frac{x_1 - x_1^0}{\rho} = \cos\alpha, \qquad \frac{x_2 - x_2^0}{\rho} = \cos\beta \qquad \textbf{(1)}$$

Bo`lishi kelib chiqadi. Odatda $\cos\alpha$ va $\cos\beta$ lar $l$ to`g`ri chiziqning yo`naltiruvchi kosinuslari deyiladi.

1-ta'rif. A nuqta l yo`nalgan to`g`ri chiziq bo`ylab A_0 nuqtaga intilganda $(A \to A_0)$ ushbu nisbat

$$\frac{f(A) - f(A_0)}{\rho(A_0, A)} = \frac{f(x_1, x_2) - f(x_1^0, x_2^0)}{\rho((x_1^0, x_2^0),(x_1, x_2))}$$

ning limiti mavjud bo`lsa, bu limit $f(x_1, x_2) = f(A)$ funksiyaning $A_0 = (x_1^0, x_2^0)$ nuqtadagi $l$ ***yo'nalish bo`yicha hosilasi*** deb ataladi va

$$\frac{df(A_0)}{dl} \qquad \text{yoki} \qquad \frac{df(x_1^0, x_2^0)}{dl}$$

kabi belgilanadi. Demak,

$$\frac{df}{dl} = \lim_{A \to A_0} \frac{f(A) - f(A_0)}{\rho(A_0, A)}.$$

Endi $f(x_1, x_2)$ funksiyaning l yo`nalish bo`yicha hosilasining mavjudligi hamda uni topish masalasi bilan shug`ullanamiz.

1-teorema. $f(x_1,x_2)$ funksiya ochiq M to`plamda $(M \subset R^2)$ berilgan bo`lib, $A_0=(x_1^0,x_2^0)$ nuqtada $((x_1^0,x_2^0)\in M)$ differensiallanuvchi bo`lsa, funksiya shu nuqtada har qanday yo`nalish bo`yicha hosilaga ega va

$$\frac{df(x_1^0,x_2^0)}{dl}=\frac{\partial f(x_1^0,x_2^0)}{\partial x_1}\cos\alpha+\frac{\partial f(x_1^0,x_2^0)}{\partial x_2}\cos\beta \qquad \textbf{(2)}$$

bo`ladi.

◀ Shartga ko`ra $f(x_1,x_2)$ funksiya $A_0=(x_1^0,x_2^0)$ nuqtada differensiallanuvchi. Demak, funksiya orttirmasi

$$f(A)-f(A_0)=f(x_1,x_2)-f(x_1^0,x_2^0)$$

uchun

$$f(A)-f(A_0)=\frac{\partial f(x_1^0,x_2^0)}{\partial x_1}(x_1-x_1^0)+\frac{\partial f(x_1^0,x_2^0)}{\partial x_2}(x_2-x_2^0)+o(\rho) \qquad (13.8)$$

bo'ladi, bunda

$$\rho=\rho(A_0,A)=\sqrt{(x_1-x_1^0)^2+(x_2-x_2^0)^2}\,.$$

(2) tenglikning har ikki tomonini $\rho=\rho(A_0,A)$ ga bo`lib, so`ng topamiz.

$$\frac{f(A)-f(A_0)}{\rho(A_0,A)}=\frac{\partial f(x_1^0,x_2^0)}{\partial x_1}\cos\alpha+\frac{\partial f(x_1^0,x_2^0)}{\partial x_2}\cos\beta+\frac{o(\rho)}{\rho}.$$

Bu tenglikda $A \to A_0$ da (ya'ni $\rho \to o$ da) limitga o'tsak, unda

$$\lim_{A \to A_0} \frac{f(A) - f(A_0)}{\rho(A_0, A)} = \lim_{\rho \to o} \frac{f(A) - f(A_0)}{\rho} = \frac{df\left(x_1^0, x_2^0\right)}{dx_1} \cos\alpha + \frac{df\left(x_1^0, x_2^0\right)}{dx_2} \cos\beta$$

bo'ladi. Demak,

$$\frac{df\left(x_1^0, x_2^0\right)}{dl} = \frac{\partial f\left(x_1^0, x_2^0\right)}{\partial x_1} \cos\alpha + \frac{\partial f\left(x_1^0, x_2^0\right)}{\partial x_2} \cos\beta . \blacktriangleright$$

1-misol. Ushbu

$$f(x_1, x_2) = arctg \frac{x_1}{x_2}$$

funksiyaning l yo'nalish bo'yicha hosilasi topilsin, bunda l birinchi kvadrantning $(1,\ 1)$ nuqtadan o'tuvchi va $(0,\ 0)$ nuqtadan $(1,\ 1)$ nuqtaga qarab yo'nalgan bissektrisasidan iborat.

$\blacktriangleleft$ Berilgan funksiyaning $A_0 = (1,\ 1)$ nuqtadagi l yo'nalish bo'yicha hosilasini (13.7) formulaga ko'ra topamiz

Ravshanki,

$$f(x_1, x\ ) = arctg \frac{x_1}{x_2}$$

funksiya $A_0 = (1,1)$ nuqtada differensiallanuvchi. Unda formulaga ko`ra

$$\frac{df(1,\,1)}{dl}=\frac{\partial f(1,\,1)}{\partial x_1}cos\frac{\pi}{4}+\frac{\partial f(1,\,1)}{\partial x_2}cos\frac{\pi}{4}=$$

$$=\left(\frac{x_2}{x_1^2+x_2^2}-\frac{x_1}{x_1^2+x_2^2}\right)_{\substack{x_1=1\\x_2=1}}\frac{\sqrt{2}}{2}=0$$

bo`ladi. ▶

GRADIENT.

$u = F(x, y)$ tenglama biror sohaning har bir (x, y) nuqtasida u ni aniqlab beradi, o`sha soha ***skalyar*** u ***ning maydoni*** deyiladi.

$F(x, y) = u_1$, $F(x, y) = u_2, \ldots$ lardagi u_1, u_2, $\ldots$, lar o`zgarmas bo`lganda chiziqlarning har biri bo`yicha skalyar $u$ o`zgarmas bo`lib, u faqat $(x, y)$ nuqta bir chiziqdan ikkinchi chiziqqa o`tgandagina o`zgaradi. Bu chiziqlar ***yuksaklik chiziqlari*** yoki ***izochiziqlar*** (izotermalar, izobaralar) va shunga o`xshash deyiladi.

$u = F(x, y, z)$ tenglama uch o`lchovli fazoning biror qismida skalyar u ***ning maydonini*** aniqlaydi. U holda ***izosirtlar*** yoki ***yuksaklik sirtlarining*** tenglamalari

$$F(x, y, z) = u_1,\ F(x, y, z) = u_2, \ldots,$$

lardan iborat bo`ladi.

(x, y, z) nuqta $x = x_0 + l\cos\alpha$, $y = y_0 + l\cos\beta$, $z = z_0 + l\cos\gamma$ to`g`ri chiziq bo`yicha $\dfrac{dl}{dt} = 1$ tezlik bilan harakat qilsin.

U holda $F(x, y, z)$ skalyar

$$v = \frac{du}{dt} = \frac{du}{dl} = \frac{\partial F}{\partial x}\cos\alpha + \frac{\partial F}{\partial y}\cos\beta + \frac{\partial F}{\partial z}\cos\gamma = N \cdot l_0$$

tezlik bilan o`zgaradi, bundagi $N\left\{\dfrac{\partial F}{\partial x}; \dfrac{\partial F}{\partial y}; \dfrac{\partial F}{\partial z}\right\}$ -***izosirtning normal*** vektori bo`lib, $l_0\{\cos\alpha; \cos\beta; \cos\gamma\}$- l yo`nalishning birlik vektoridan iborat.

$l_0\{\cos\alpha; \cos\beta; \cos\gamma\}$- l yo`nalishning birlik vektoridan iborat.

Ushbu

$$\frac{du}{dl} = \frac{\partial F}{\partial x}\cos\alpha + \frac{\partial F}{\partial y}\cos\beta + \frac{\partial F}{\partial z}\cos\gamma = N \cdot l_0$$

hosila $u = F(x, y, z)$ ***funksiyaning berilgan*** $l_0\{\cos\alpha; \cos\beta; \cos\gamma\}$ ***yo`nalish bo`yicha hosilasi*** deyiladi.

$u = F(x, y, z)$ skalyarning ***gradienti*** deb

$$grad\, u = \frac{\partial u}{\partial x} i + \frac{\partial u}{\partial y} j + \frac{\partial u}{\partial z} k$$

vektorga aytiladi. Gradient skalyar u ning ***eng tez*** o`zgarishi tezligining vektoridan iborat.