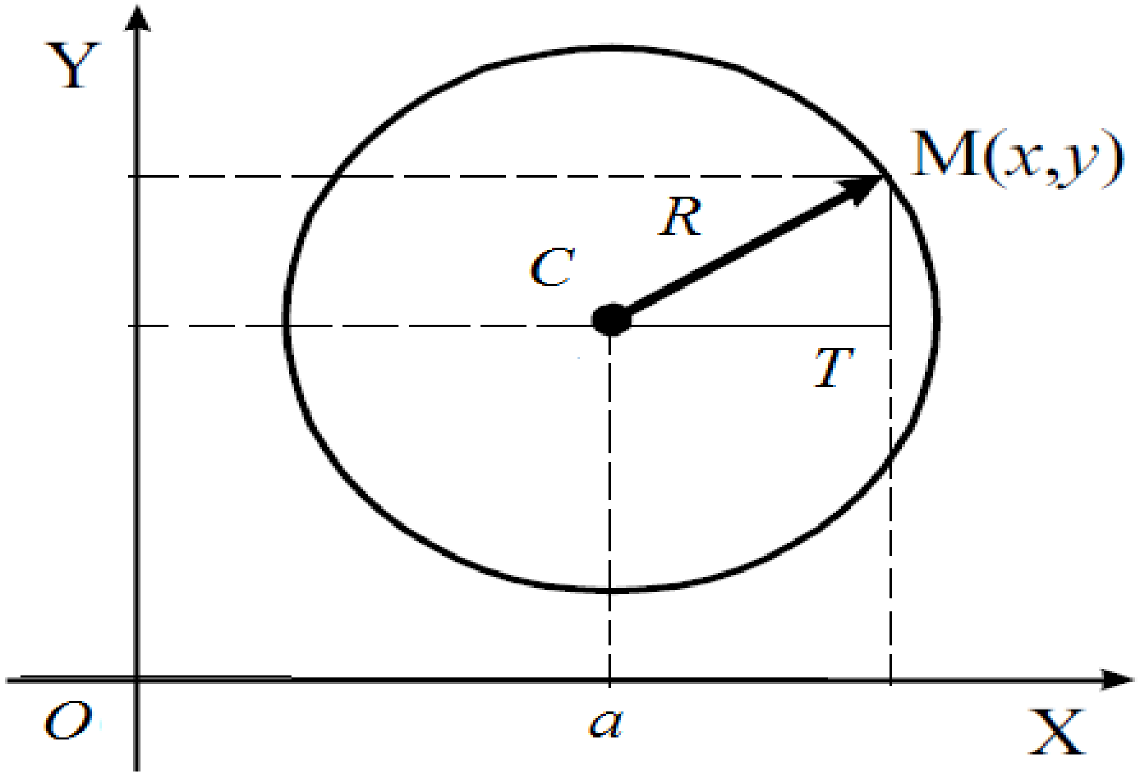
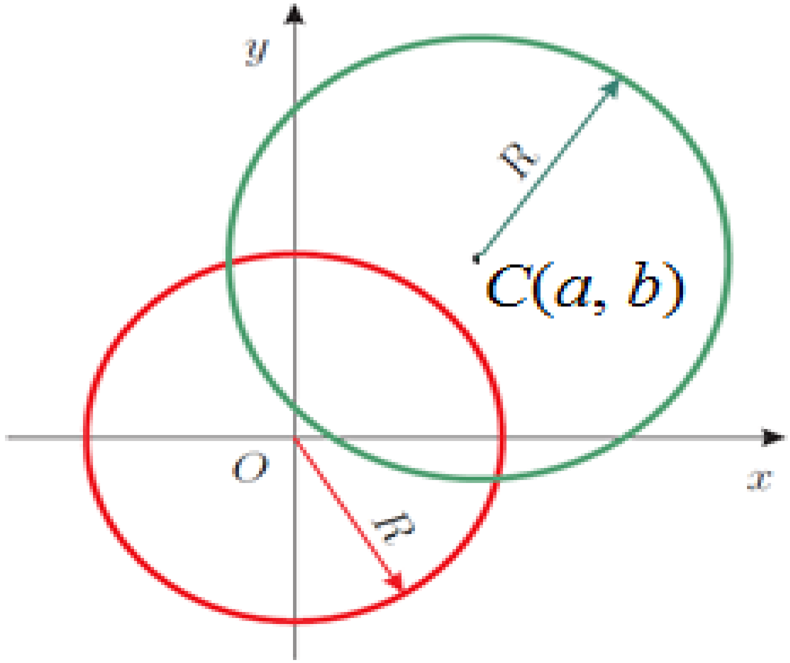
IKKINCHI TARTIBLI EGRI CHIZIQLAR

Aylana





$$(x-a)^2+(y-b)^2=R^2$$

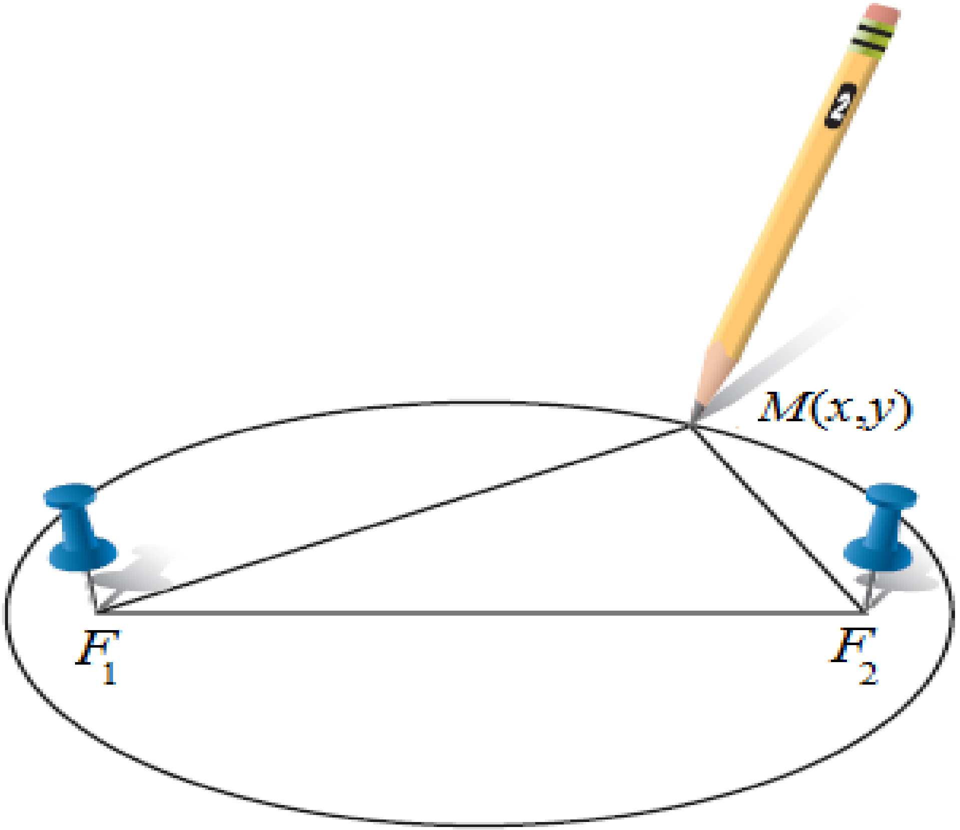
$$x^2+y^2=R^2$$

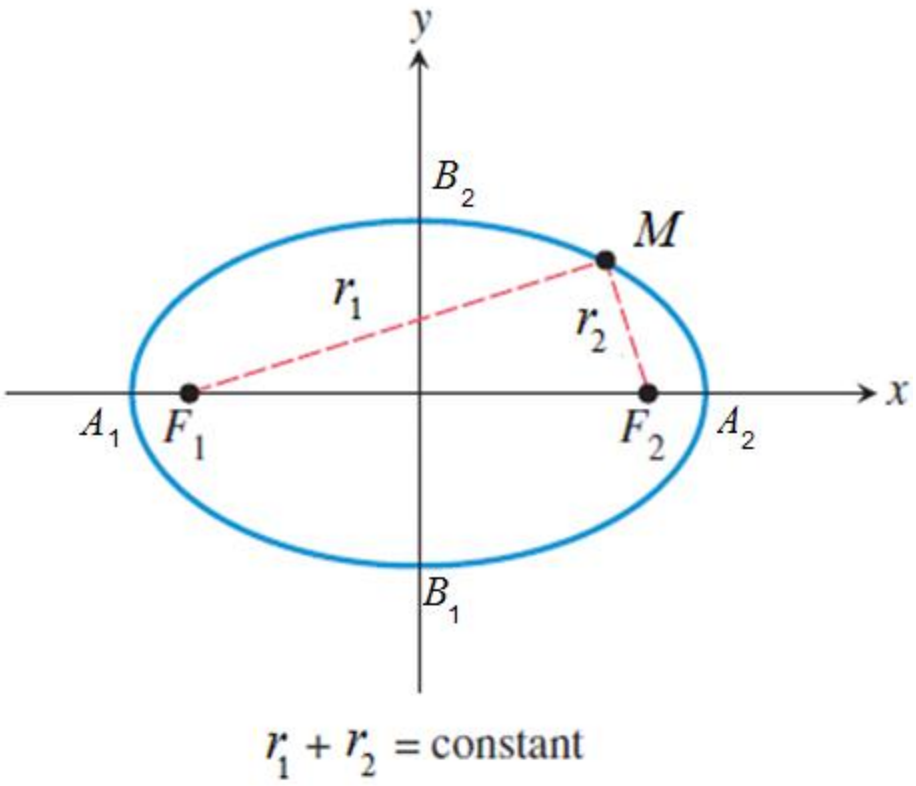
$$x^2+y^2+mx+ny+p=0$$

2-misol. $x^2+y^2-4x+6y-3=0$

aylana markazi va radiusini toping.

Ellips





$$|F_1M| + |F_2M| = 2a$$

$$|F_1M| = \sqrt{(x-c)^2 + y^2}, \quad |F_2M| = \sqrt{(x+c)^2 + y^2}.$$

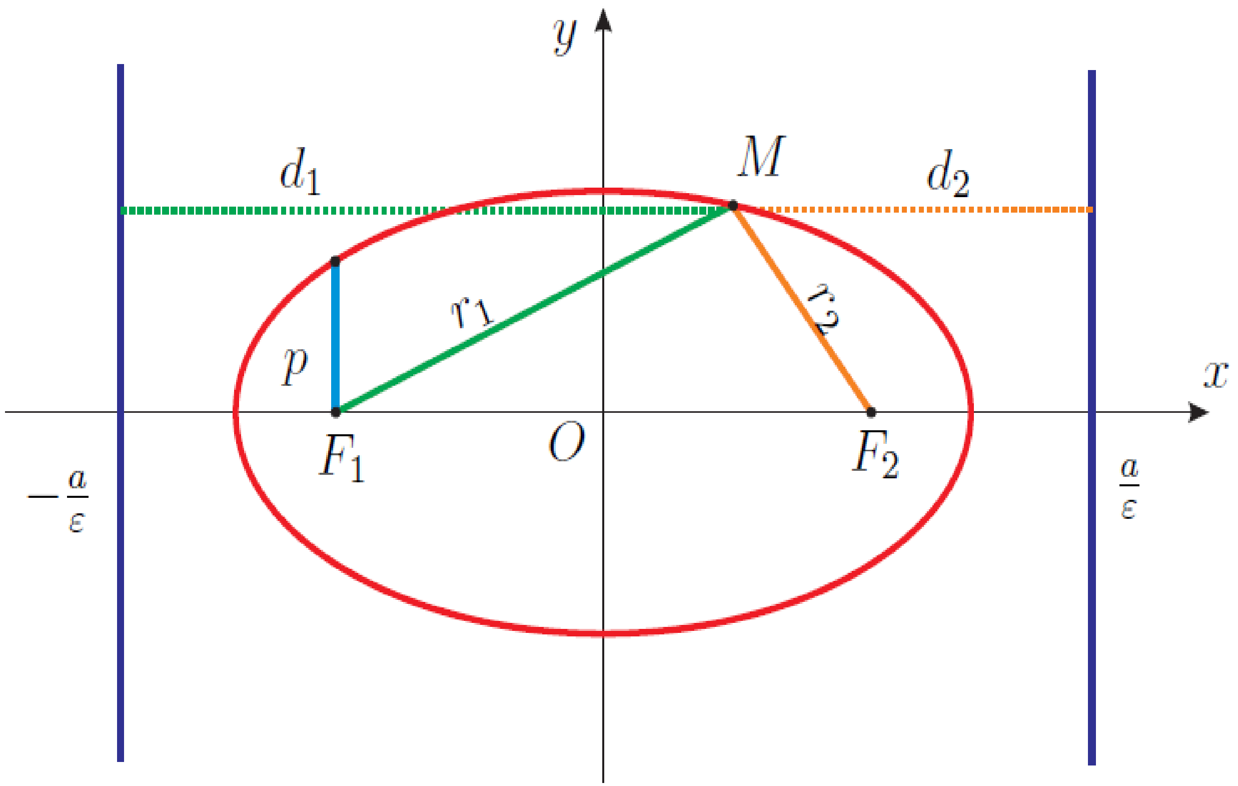
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

$$r_{1,2} = a \pm \varepsilon x,$$

$\varepsilon = \dfrac{c}{a}$ – miqdor ***ellipsning ekssentrisiteti*** deyiladi.

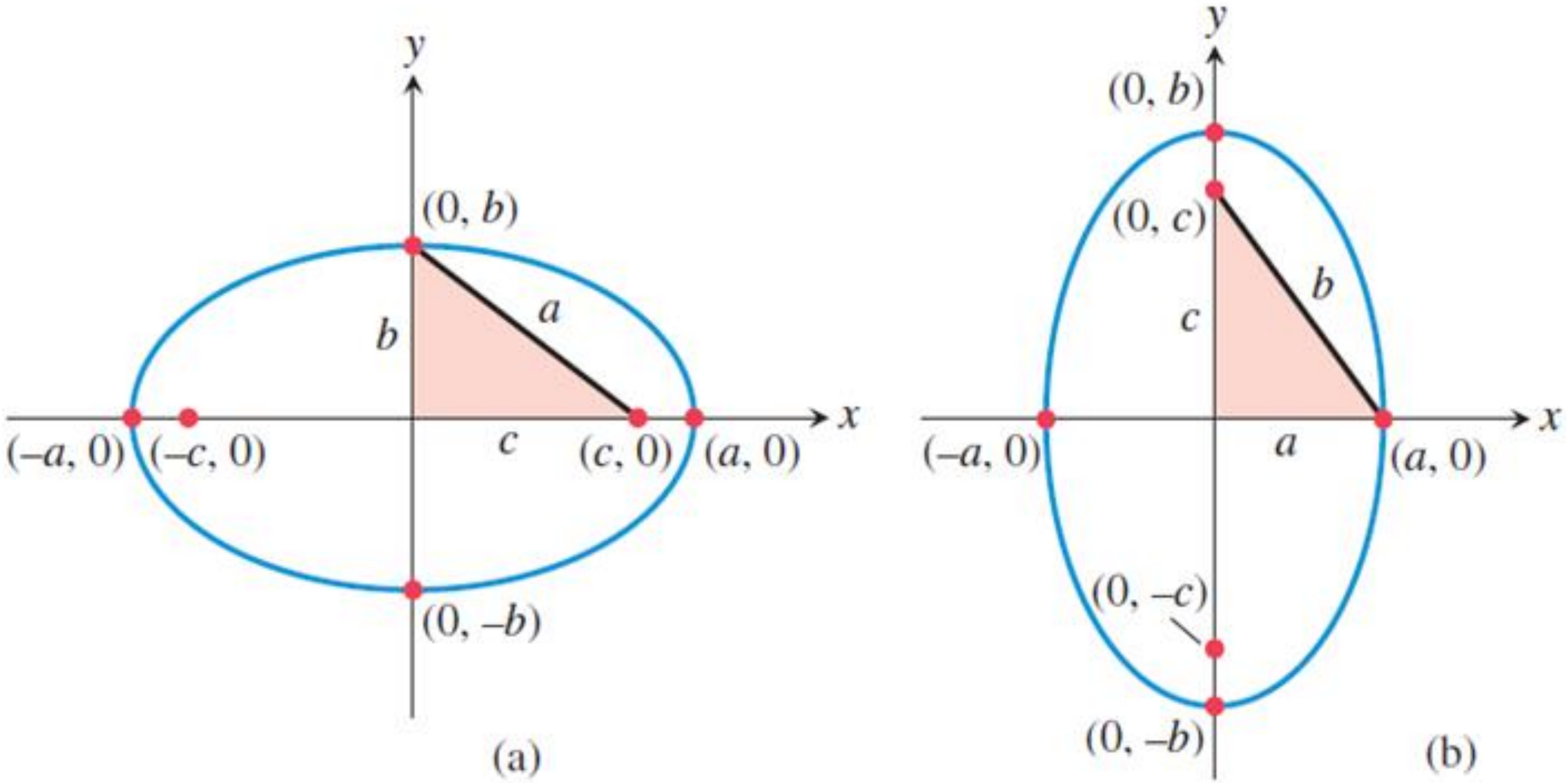
$x = \pm \dfrac{a}{\varepsilon}$ ***ellipsning direktrisalari*** deyiladi

p

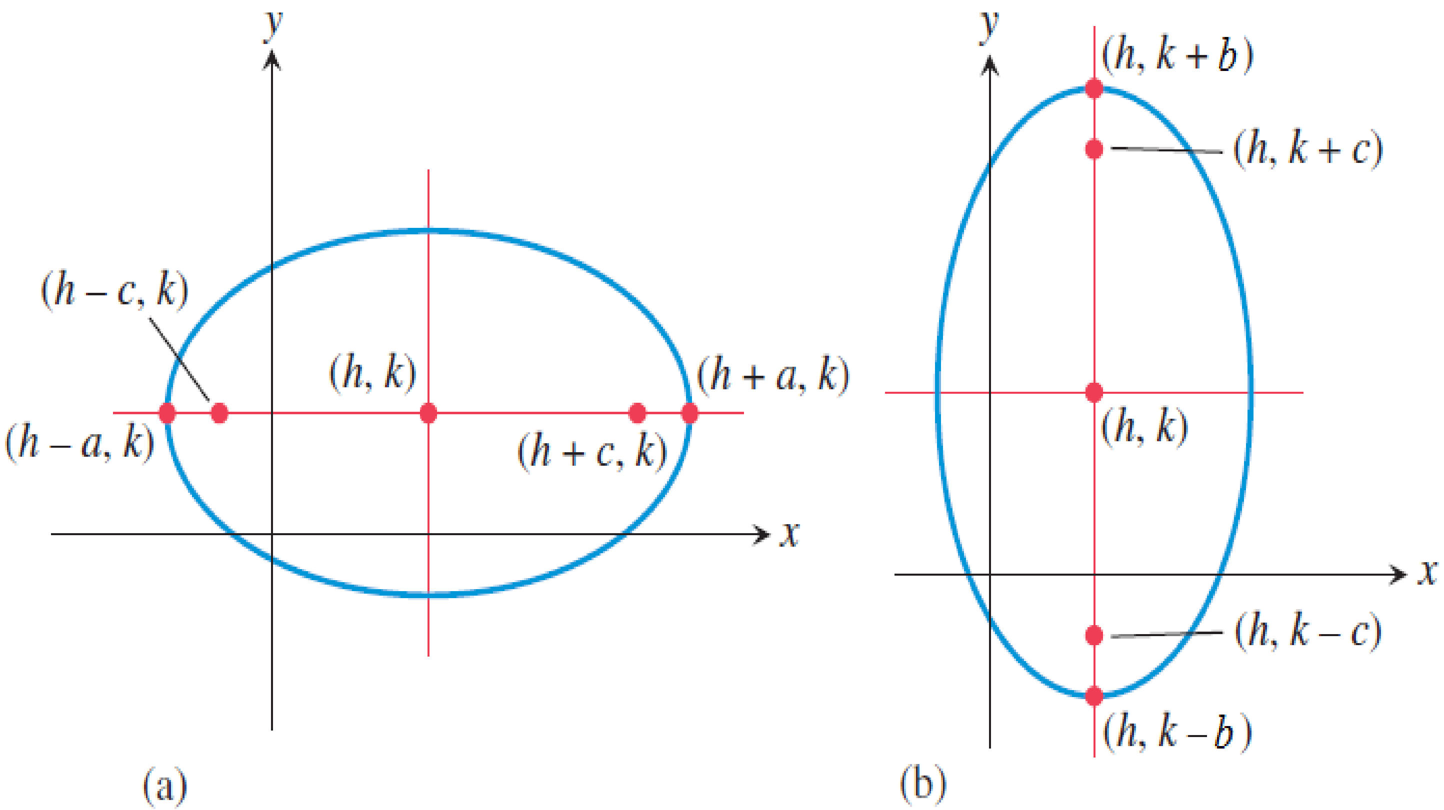


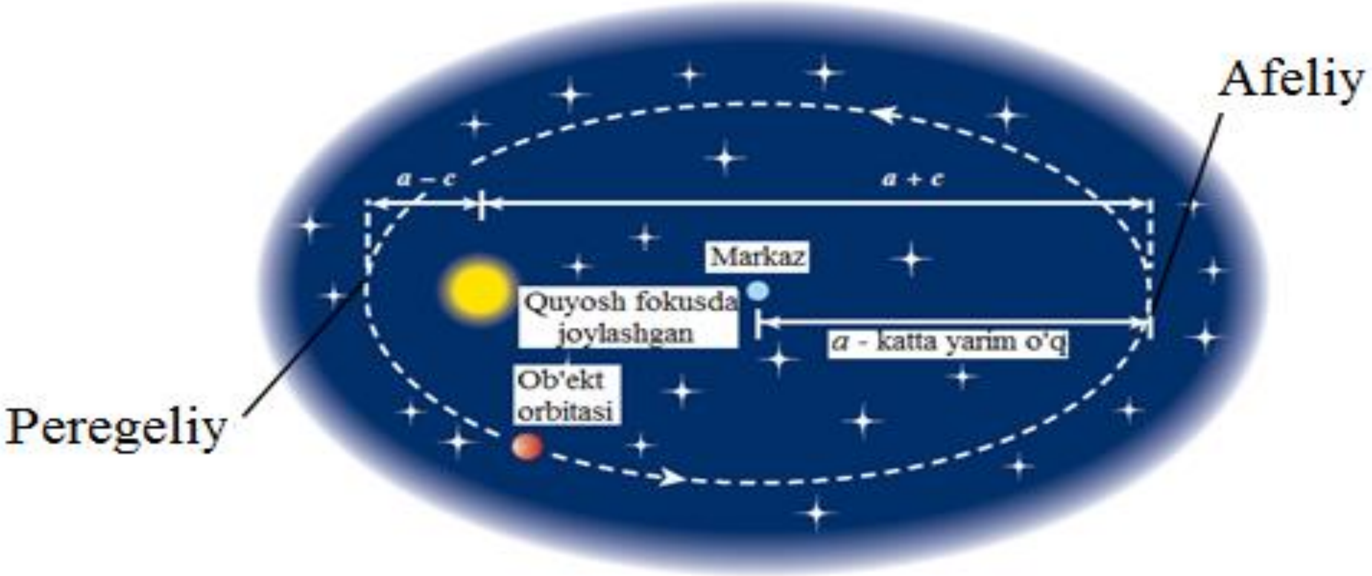
$$p = \frac{b^2}{a}$$ Ellipsning fokal parametri

Markazi O(0, 0) nuqtada bo‘lgan ellips		
Standart tenglamasi	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$ $a>b$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$ $a<b$
Fokuslari	$c=\sqrt{a^2-b^2}$	$c=\sqrt{b^2-a^2}$
Fokus koordinatasi	$F(\pm c,0)$	$F(0,\pm c)$
Ekssentrisiteti	$\varepsilon=\frac{c}{a}$	$\varepsilon=\frac{c}{b}$
Katta yarim o‘q uzunligi	a	b
Kichik yarim o‘q uzunligi	b	a
Direktrisasi	$x=\pm\frac{a}{\varepsilon}$	$y=\pm\frac{b}{\varepsilon}$
Fokal o‘qi	Ox	Oy
	10-rasm, (*a*)	10-rasm, (*b*)


y
x
(0, b)
(0, –b)
(–a, 0)
(–c, 0)
(c, 0)
(a, 0)
a
b
c
(a)
y
x
(0, b)
(0, c)
(0, –c)
(0, –b)
(–a, 0)
(a, 0)
a
b
c
(b)

Markazi O(h, k) nuqtada bo‘lgan ellips		
Tenglama ko‘rinishi	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,$ $a>b$	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,$ $a<b$
Fokuslari	$F(h\pm c,k)$	$F(h,k\pm c)$
Fokus koordinatasi	$c=\sqrt{a^2-b^2}$	$c=\sqrt{b^2-a^2}$
Ekssentrisiteti	$\varepsilon=\frac{c}{a}$	$\varepsilon=\frac{c}{b}$
Katta yarim o‘q uzunligi	a	b
Kichik yarim o‘q uzunligi	b	a
Direktrisasi	$x=\pm\frac{a}{\varepsilon}+h$	$y=\pm\frac{b}{\varepsilon}+k$
Fokal o‘qi	$y=k$	$x=h$
Fokal o‘qdagi uchlari	$(h-a,k)$; $(h+a,k)$	$(h,k+b)$; $(h,k-b)$
	11-rasm (a)	11-rasm (b)


y
(h − c, k)
(h, k)
(h + a, k)
(h − a, k)
(h + c, k)
x
(a)
y
(h, k + b)
(h, k + c)
(h, k)
x
(h, k − c)
(h, k − b)
(b)



Afeliy
a − c
a + c
Markaz
Quyosh fokusda joylashgan
a - katta yarim o'q
Ob'ekt orbitasi
Peregeliy

3-misol. $x^2 + 2y^2 = 2$

ellipsning ekssentrisiteti va direktrisasini toping.

4-misol. Ekssentrisiteti $\varepsilon = \frac{2}{3}$

ga teng bo‘lgan ellipsning fokuslaridan biri (6;0) nuqtada bo‘lsa, uning kanonik tenglamasini tuzing.

7-misol. (-2, -1) va (8, -1) nuqtalar ellipsning katta o‘qi uchlarini tashkil etadi. Uning kichik o‘qi uzunligi 8 ga teng. Ellipsning kanonik tenglamasini tuzing.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,$$

Ellips markazi (h, k) nuqta koordinatalarini aniqlaymiz:

$$h = \frac{8+(-2)}{2} = 3, \quad k = \frac{-1+(-1)}{2} = -1.$$

Ellipsning katta va kichik yarim o‘qlari uzunliklari:

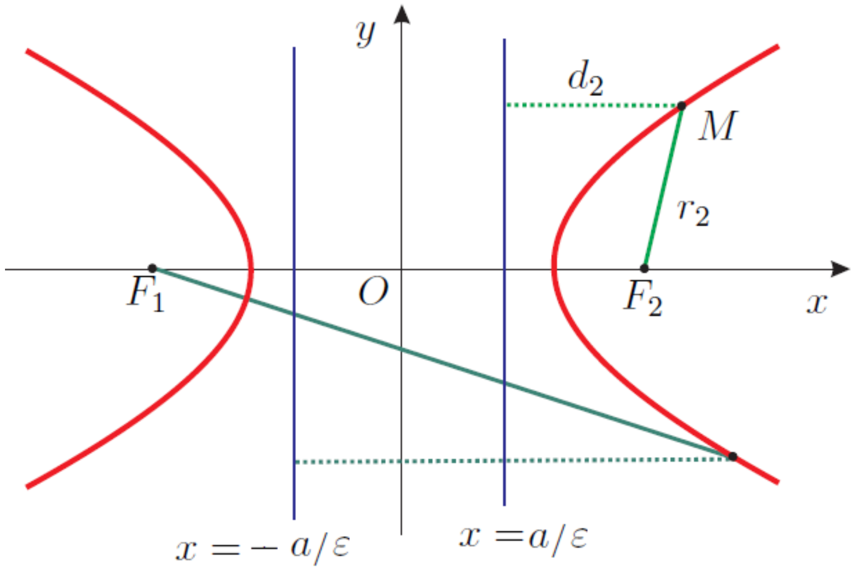
$$a = \frac{8-(-2)}{2} = 5, \quad b = \frac{8}{2} = 4.$$

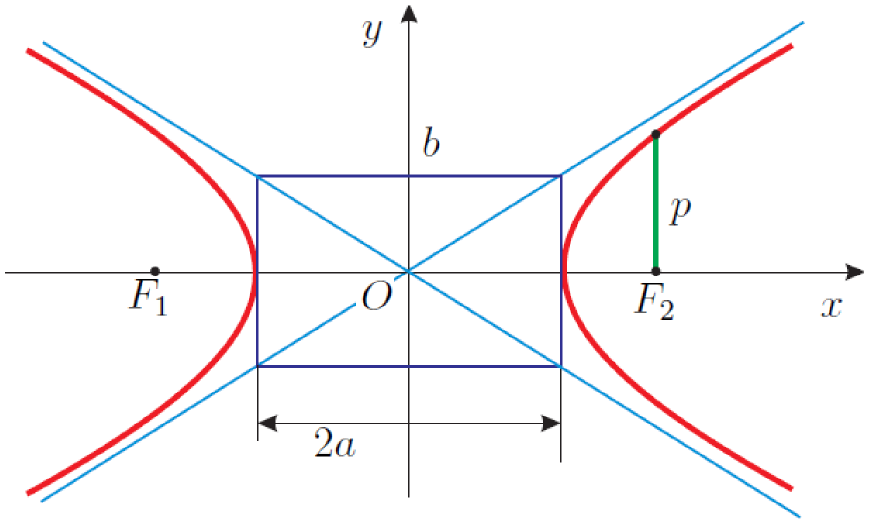
Giperbola

$$|F_1M| - |F_2M| = \pm 2a$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

$$r_{1,2} = |\varepsilon x \pm a|,$$





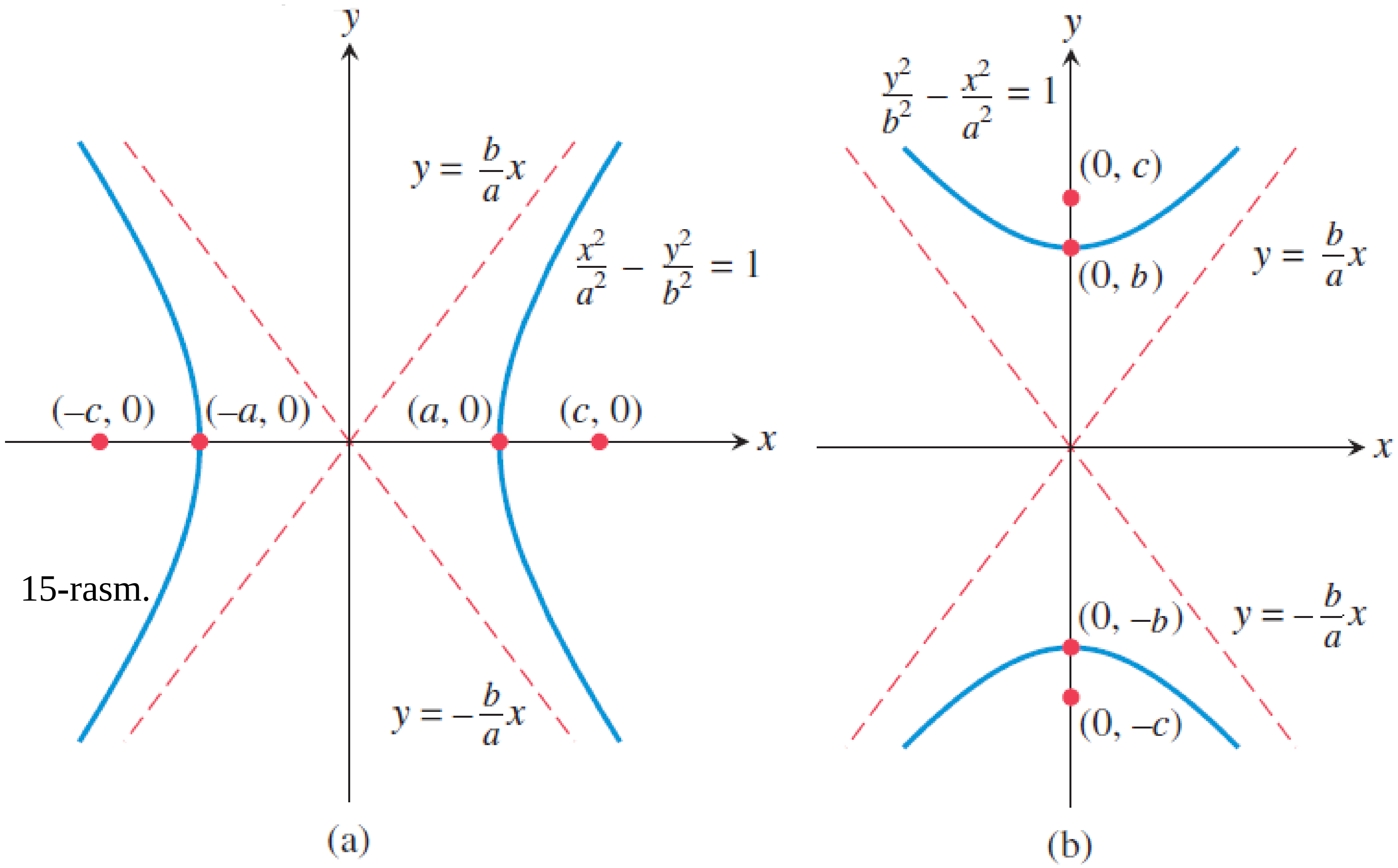
Quyidagilar *giperbola elementlarini* tashkil etadi:

- Onuqta – giperbola markazi;
- $A_1(-a;0)$va $A_2(a;0)$ nuqtalar – giperbola uchlari;
- $F_1(-c;0),\ \ F_2(c;0)$– giperbola fokuslari;
- $2c$– fokuslar orasidagi masofa, bu yerda $c=\sqrt{a^2+b^2}$;
- $A_1A_2=2a$– giperbolaning haqiqiy o‘qi, $B_1B_2=2b$ giperbolaning mavhum o‘qi;
- $a>0$ – giperbolaning haqiqiy yarim o‘qi, $b>0$ giperbolaning mavhum yarim o‘qi uzunliklari;
- $\varepsilon=\frac{c}{a}$– giperbola ekssentrisiteti ($\varepsilon>1$);
- $x=\pm\frac{a}{\varepsilon}$ – giperbola direktrisatenglamasi;
- $y=\pm\frac{b}{a}x$ – giperbola asimptotatenglamasi.

9-misol.$\frac{x^2}{25}-\frac{y^2}{16}=1$giperbolaning ekssentrisiteti, direktrisa va asimptotalarini toping.

Markazi O(0, 0) nuqtadabo‘lgangiperbola

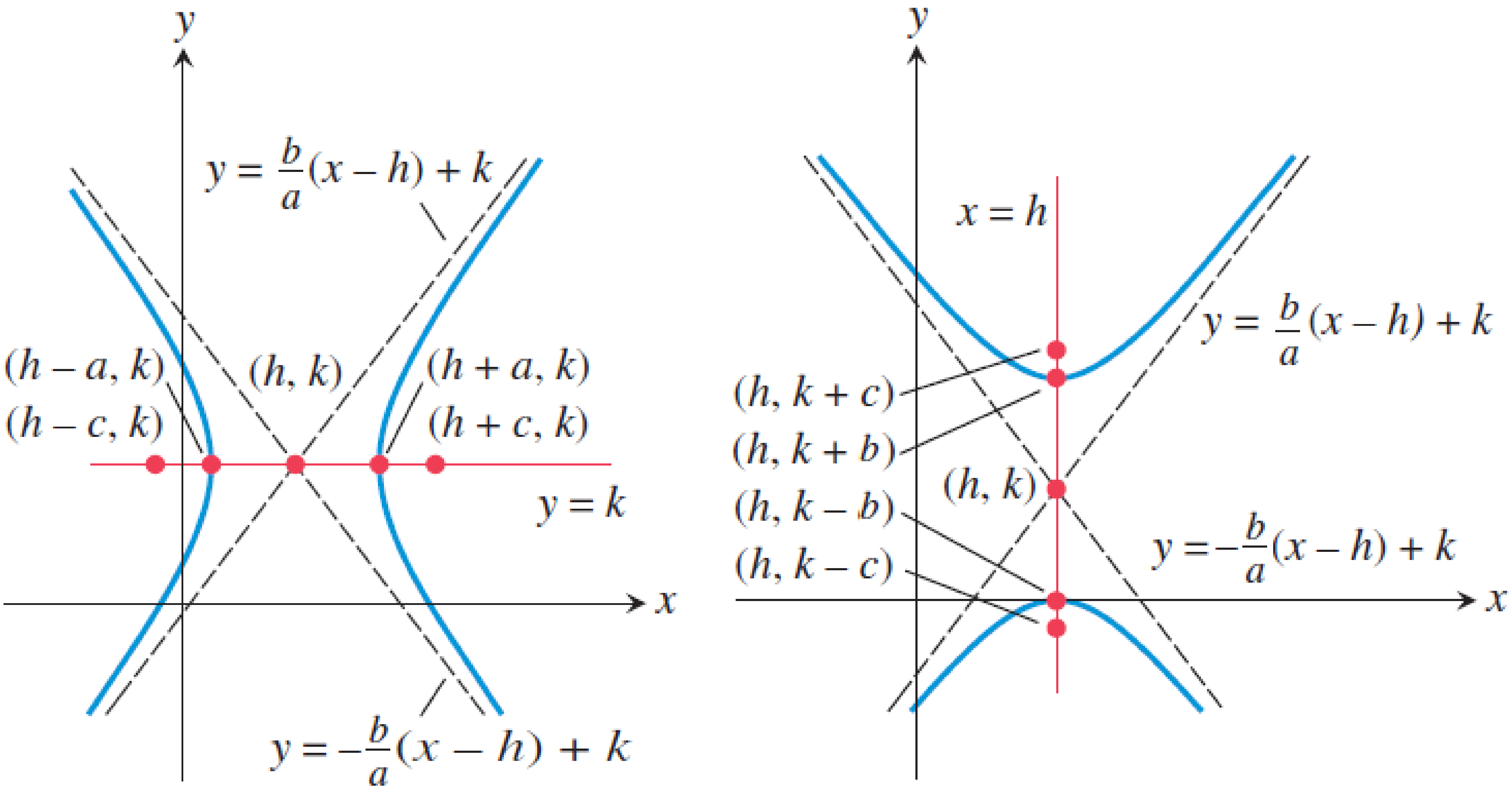
Standarttenglamasi	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$
Fokuslari	$F(\pm c, 0)$	$F(0, \pm c)$
Fokus koordinatasi	$c=\sqrt{a^2+b^2}$	$c=\sqrt{a^2+b^2}$
Ekssentrisiteti	$\varepsilon=\frac{c}{a}$	$\varepsilon=\frac{c}{b}$
Haqiqiy yarim o‘q uzunligi	a	b
Mavhum yarim o‘q uzunligi	b	a
Direktrisasi	$x=\pm\frac{a}{\varepsilon}$	$y=\pm\frac{b}{\varepsilon}$
Fokal o‘qi	Ox	Oy
Asimptota tenglamasi	$y=\pm\frac{b}{a}x$	$y=\pm\frac{b}{a}x$
	15-rasm, (a)	15-rasm, (b)



y
x
$y = \frac{b}{a}x$
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
$(-c, 0)$
$(-a, 0)$
$(a, 0)$
$(c, 0)$
$y = -\frac{b}{a}x$
(a)
y
x
$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
$(0, c)$
$(0, b)$
$y = \frac{b}{a}x$
$(0, -b)$
$y = -\frac{b}{a}x$
$(0, -c)$
(b)

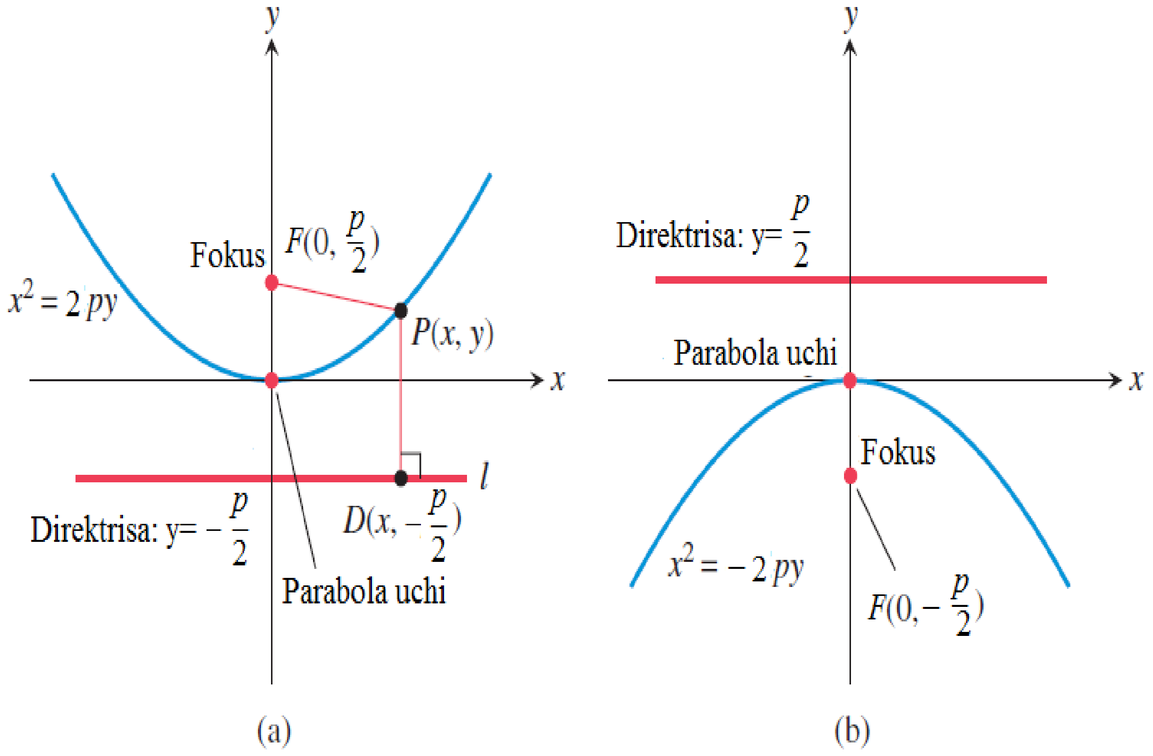
15-rasm.

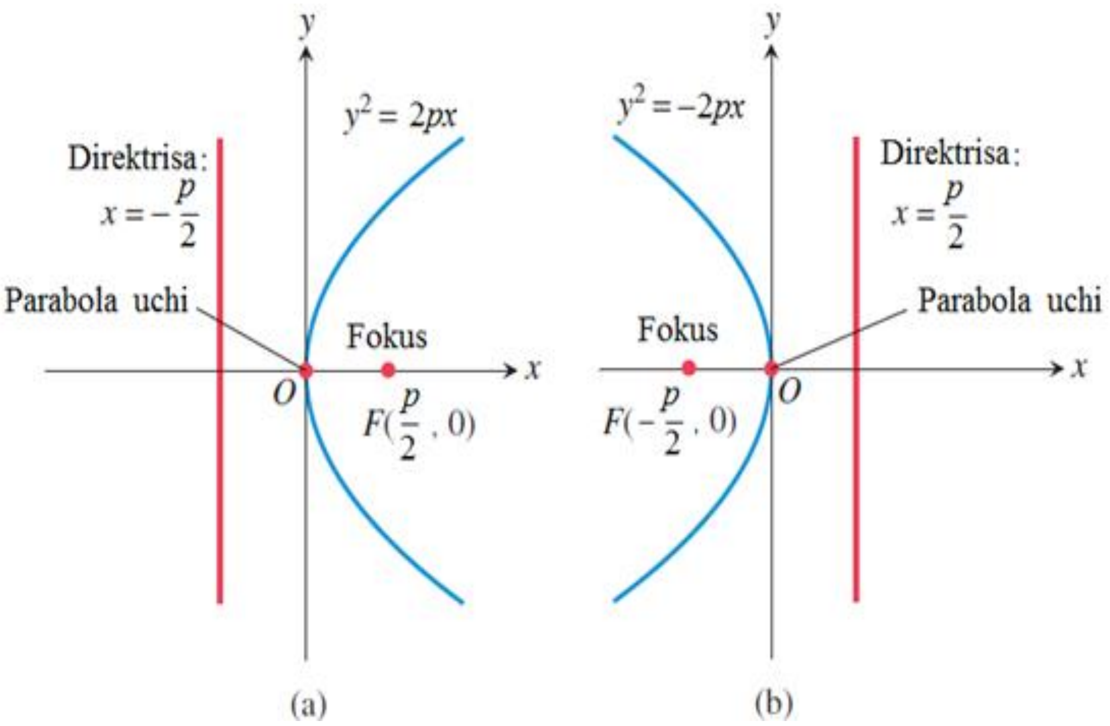
Markazi O(h, k) nuqtadabo‘lgangiperbola		
Tenglama ko‘rinishi	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1,$	$\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$
Fokuslari	$F(h \pm c, k)$	$F(h, k \pm c)$
Fokus koordinatasi	$c=\sqrt{a^2+b^2}$	$c=\sqrt{b^2+a^2}$
Ekssentrisiteti	$\varepsilon=\frac{c}{a}$	$\varepsilon=\frac{c}{b}$
Haqiqiy yarim o‘q uzunligi	a	b
Mavhum yarim o‘q uzunligi	b	a
Direktrisasi	$x=\pm\frac{a}{\varepsilon}+h$	$y=\pm\frac{b}{\varepsilon}+k$
Fokal o‘qi	$y=k$	$x=h$
Asimptota tenglamasi	$y=\pm\frac{b}{a}(x-h)+k$	$y=\pm\frac{b}{a}(x-h)+k$
	16-rasm, (a)	16-rasm, (b)


y
x
$y = \frac{b}{a}(x - h) + k$
$(h - a, k)$
$(h - c, k)$
(h, k)
$(h + a, k)$
$(h + c, k)$
$y = k$
$y = -\frac{b}{a}(x - h) + k$
y
x
$x = h$
$y = \frac{b}{a}(x - h) + k$
$(h, k + c)$
$(h, k + b)$
(h, k)
$(h, k - b)$
$(h, k - c)$
$y = -\frac{b}{a}(x - h) + k$

(a) (b)

12-misol. (-2, -1) va (8, -1) nuqtalar giperbolaning haqiqiy o‘qi uchlarini tashkil etadi. Uning mavhum o‘qi uzunligi 8 ga teng. Giperbolaning kanonik tenglamasini tuzing.

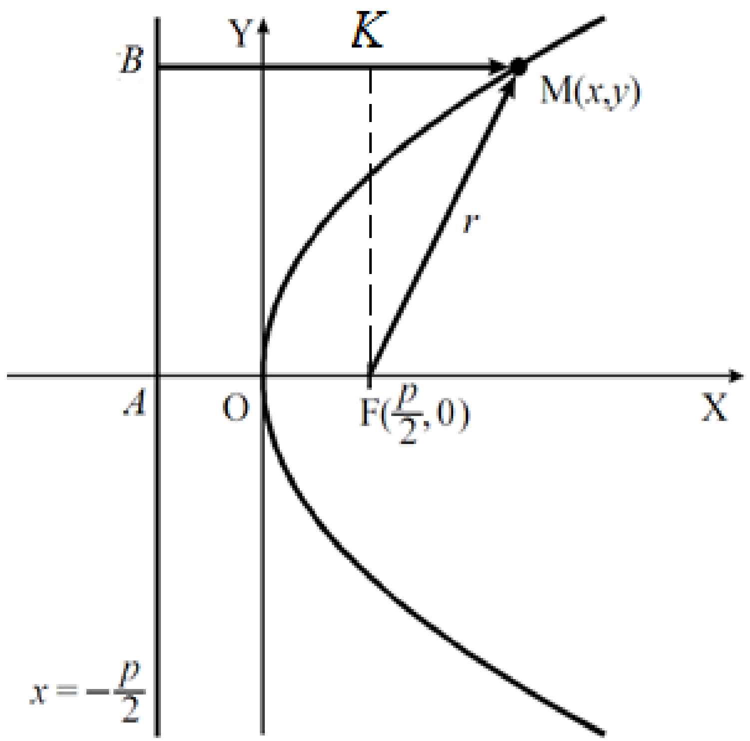




Parabola

$$y^2 = 2px$$

$$r = x + \frac{p}{2}.$$

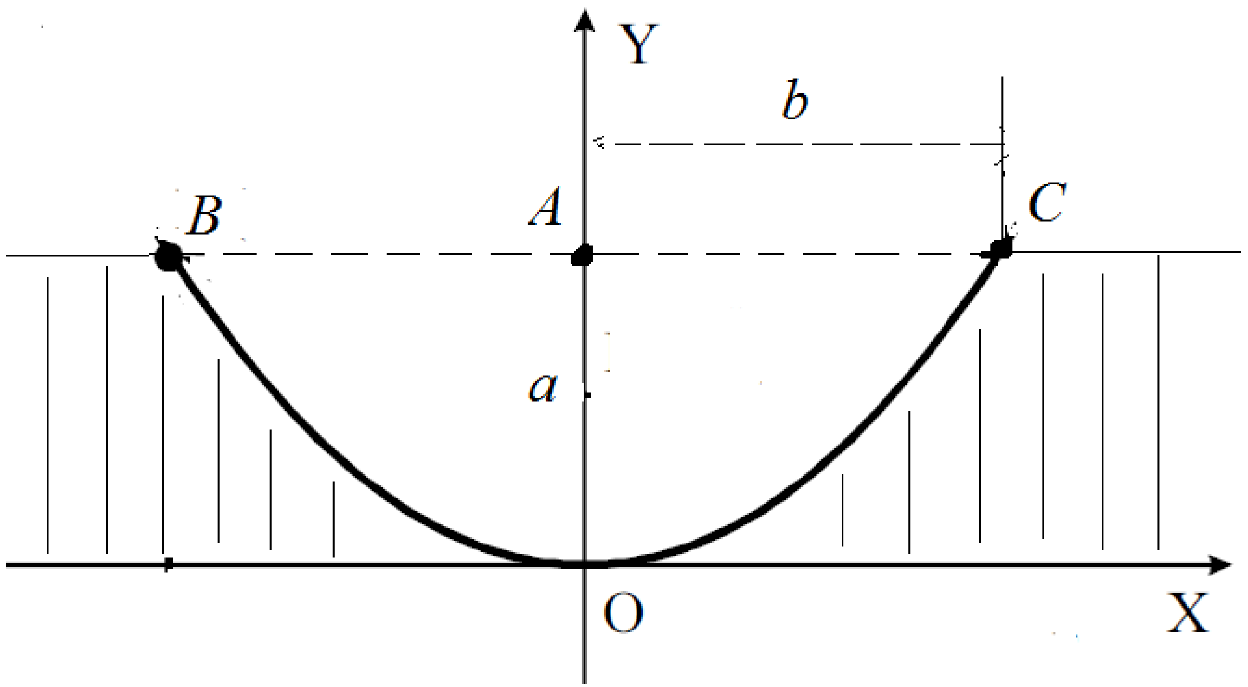


Quyidagilar *parabola elementlarini* tashkil etadi:

- Onuqta parabola uchi;
- $F\left(\frac{p}{2};0\right)$– parabola fokusi;
- $x = -\frac{p}{2}$ – parabola direktrisasi tenglamasi;
- $\varepsilon = 1$– parabola ekssentrisiteti;
- p fokal parametr (fokusdan direktrisagacha bo‘lgan masofa yoki Ox o‘qiga perpendikulyar ravishda fokusdan o‘tgan vatar uzunligining yarmi).

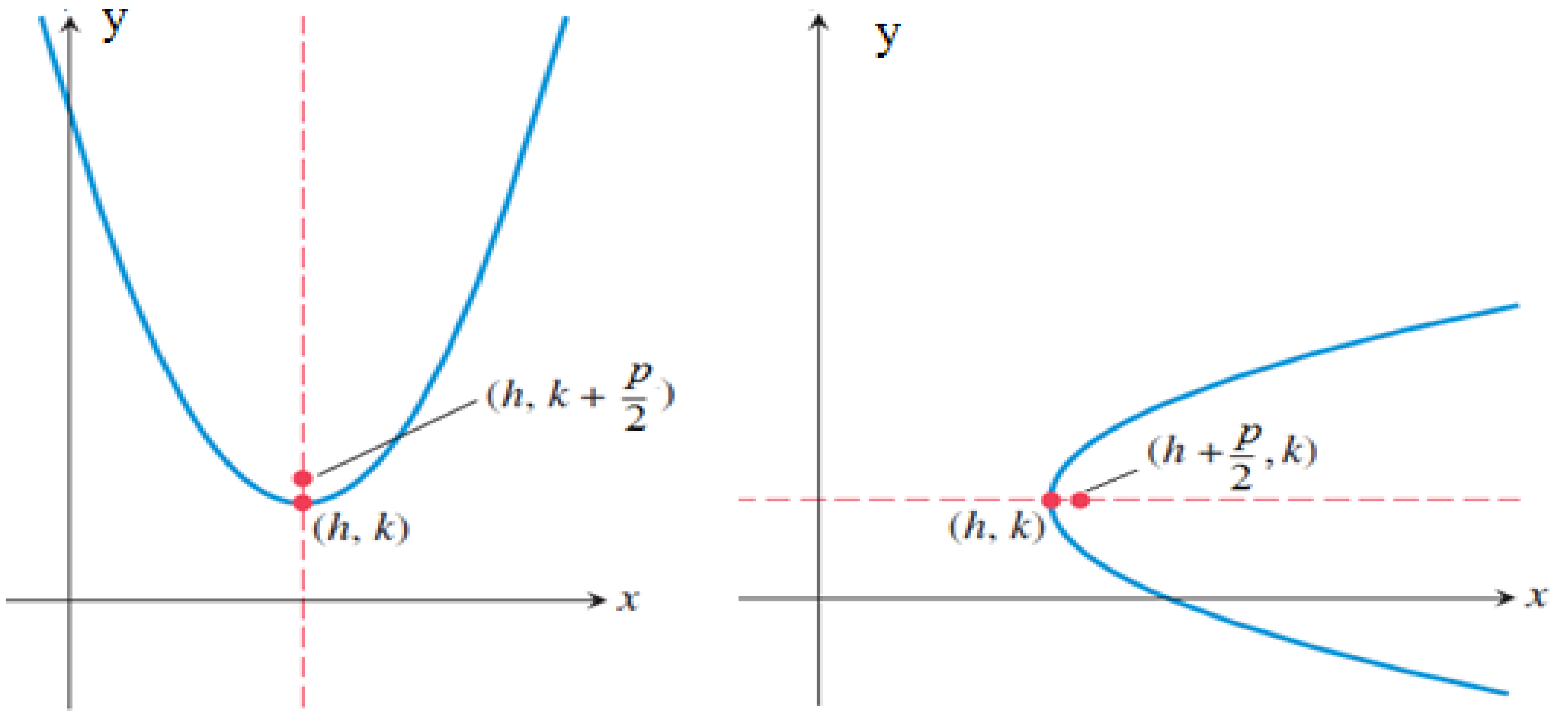
14-misol. Uchikoordinadaboshidava Ox o‘qqanisbatansimmetrikbo‘lgan parabola, A(9;6) nuqtadano‘tadi. Uningkanoniktenglamasiniyozing.

16-misol. Kanalning ko‘ndalang kesimi parabola shakliga ega (18-rasm).
Agar $OA=a$, $BC=2b$ bo‘lsa, rasmadagi ko‘rsatilgan o‘qlarga nisbatan uning tenglamasini yozing.

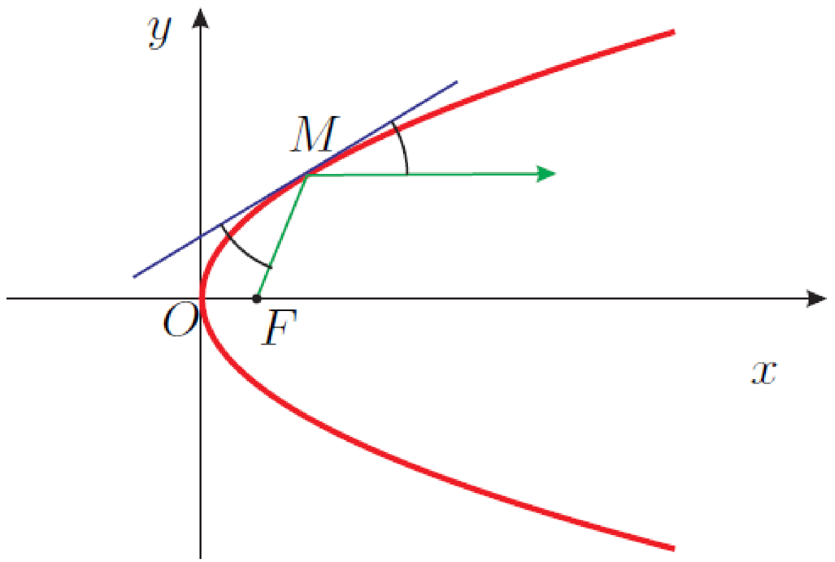


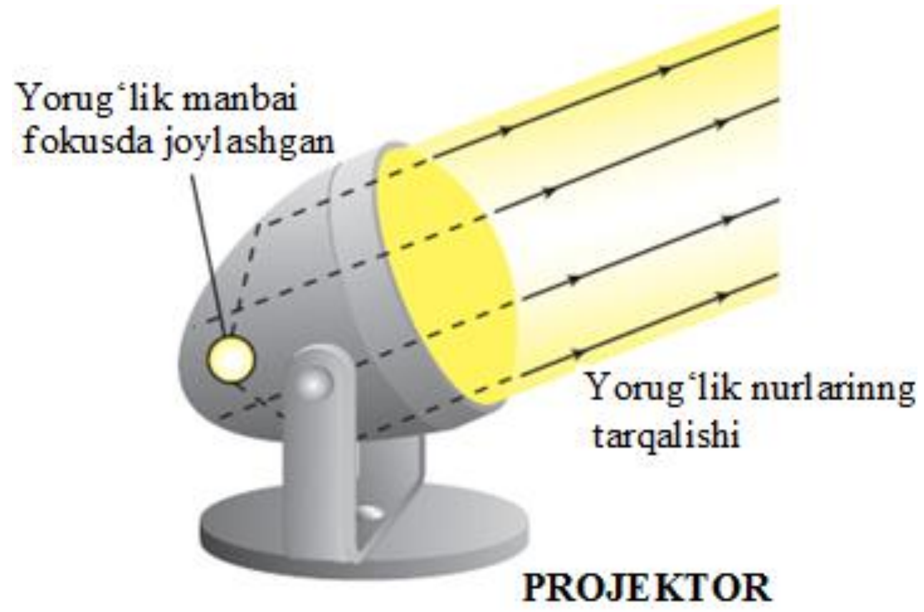
18-rasm.

Uchi O(h, k) nuqtadabo‘lgan parabola		
Tenglama ko‘rinishi	$(x-h)^2 = 2p(y-k)$	$(y-k)^2 = 2p(x-h)$
Fokusi	$F(h, k+\frac{p}{2})$	$F(h+\frac{p}{2}, k)$
Direktrisasi	$y = k - \frac{p}{2}$	$x = h - \frac{p}{2}$
Simmetriya o‘qi	$x = h$	$y = k$
	21-rasm, (a)	21-rasm, (b)

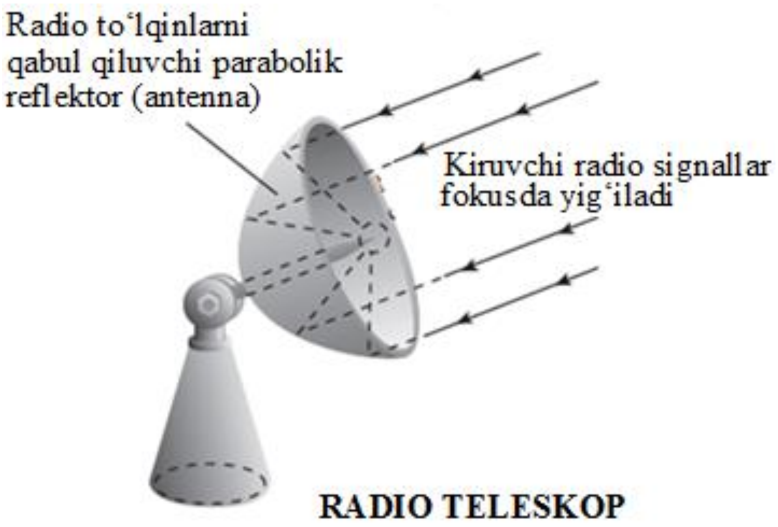


(a) (b)


y
M
O
F
x


Yorug‘lik manbai
fokusda joylashgan
Yorug‘lik nurlarinng
tarqalishi
PROJEKTOR

(a)


Radio to‘lqinlarni
qabul qiluvchi parabolik
reflektor (antenna)
Kiruvchi radio signallar
fokusda yig‘iladi
RADIO TELESKOP

(b)

Ikkinchi tartibli egri chiziqlarning umumiy tenglamasi

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$$

1. Agar $A = C$ bo‘lsa, aylana.
2. Agar $A \cdot C > 0$ va $A \neq C$ bo‘lsa, ellips.
3. Agar $A \cdot C < 0$ bo‘lsa, giperbola.
4. Agar $A \cdot C = 0, A^2 + C^2 \neq 0$ bo‘lsa, parabola.

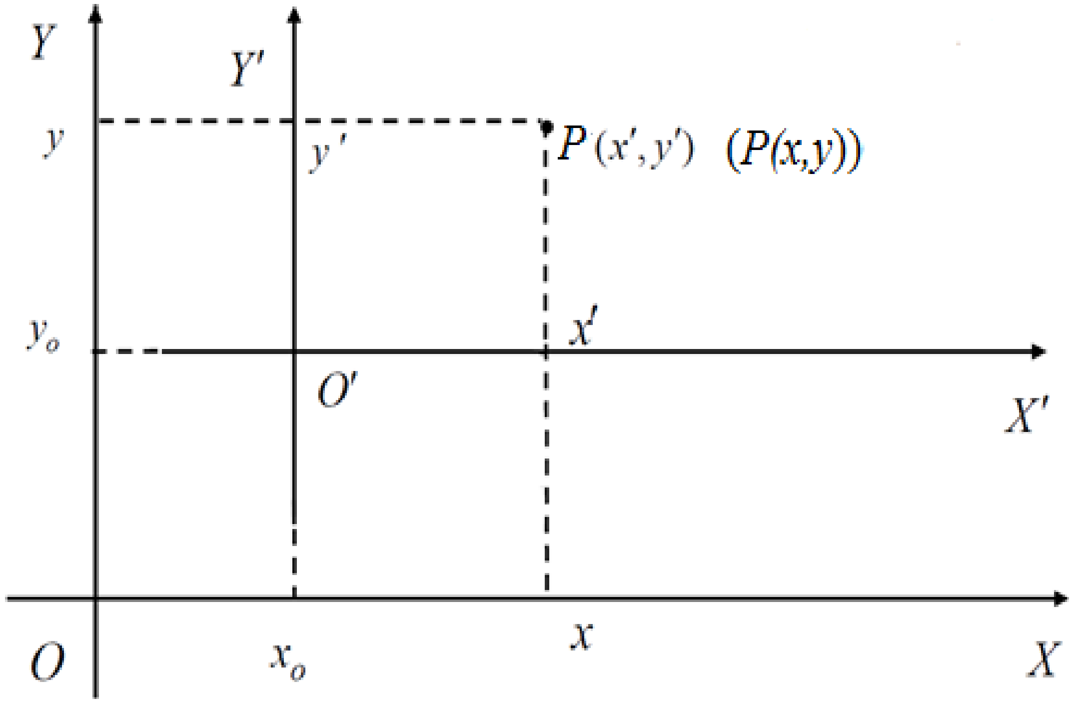
Masalan:

1. $2x^2 + 5y^2 - 3x + 7y - 5 = 0$ – ellips tenglamasi, chunki $A = 2, C = 5,$

$$A \cdot C = 2 \cdot 5 = 10 > 0.$$

2. $8x - 7y - 2x^2 - 2y^2 + 4 = 0$–aylana, chunki $A = -2$, $C = -2$.

Koordinata o‘qlarini parallel ko‘chirish.

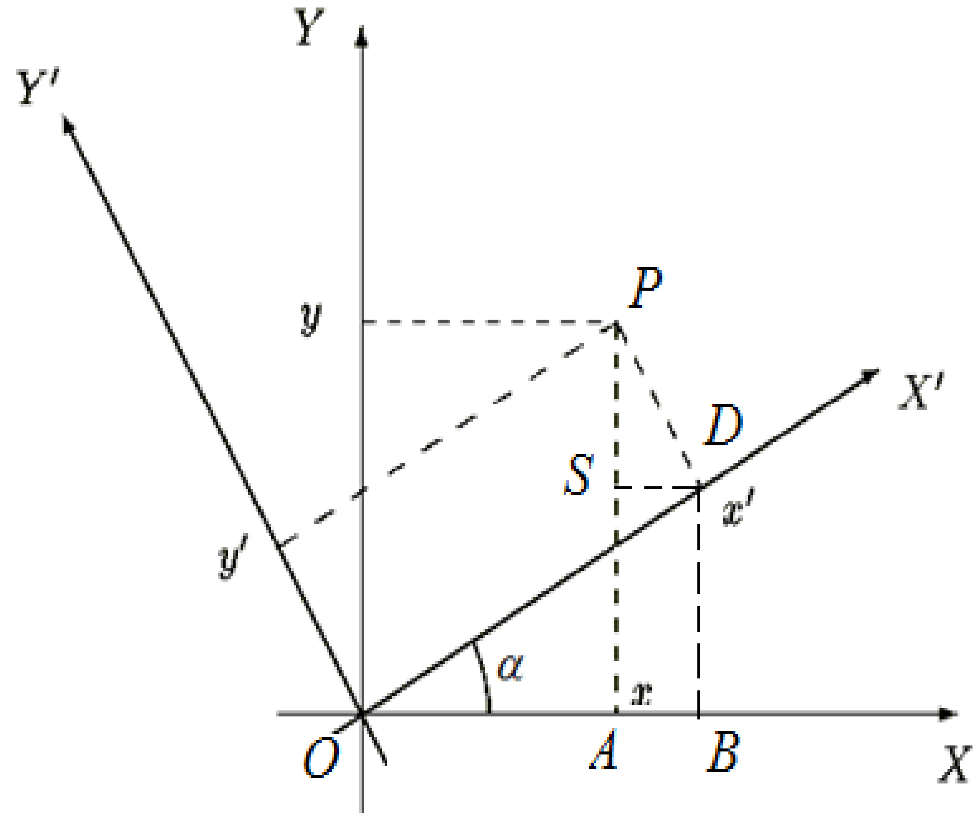


$$\begin{cases} x = x' + x_0, \\ y = y' + y_0, \end{cases} \tag{27}$$

yoki $\begin{cases} x' = x - x_0, \\ y' = y - y_0. \end{cases}$ (28)

18-misol. $2x^2 - 8x + 2y^2 + 4y - 62 = 0$ egri chiziq tupini aniqlang va tenglamani kanonik ko‘rinishga keltiring.

Koordinata o‘qlarini burish



ΔSPDuchburchakda $\angle SPD = \alpha, \;\; OD = x', \;\; PD = y'$.

Ravshanki,

$$x = OA = OB - AB = OB - SD,$$

$$y = PA = AS + SP = DB + SP.$$

Ma’lumki,

$$OB = x' \cos\alpha\,, \qquad SD = y' \sin\alpha\,,$$

$$SP = y' \cos\alpha\,, \qquad DB = x' \sin\alpha\,.$$

Natijada

$$\begin{cases} x = x' \cos\alpha - y' \sin\alpha\,, \\ y = x' \sin\alpha + y' \cos\alpha\,. \end{cases}$$

$$\begin{cases} x' = x \cos\alpha + y \sin\alpha \\ y' = -x \sin\alpha + y \cos\alpha \end{cases}$$

$\alpha = -45°$

22-misol. Teng tomonli $x^2 - y^2 = a^2$ giperbola grafigida koordinata o‘qlari $\alpha = 45°$burchakka burildi. Yangi koordinatalar sistemasida giperbola tenglamasini yozing.

Yechish. Eski koordinata o‘qlarini soat strelkasi yo‘nalishida 45° ga buramiz. (29) formulaga ko‘ra

$$x = x' \cos\alpha - y' \sin\alpha\,,$$

$$y = x' \sin\alpha + y' \cos\alpha\,,$$

bu yerda $\alpha = -45°$ ga teng, u holda $x = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y' = \frac{\sqrt{2}}{2}(x' + y'),$

$$y = -\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y' = \frac{\sqrt{2}}{2}(-x' + y').$$

$x$va y ning koordinatalarini $x^2 - y^2 = a^2$tenglamaga qo‘yamiz:

$$\frac{1}{2}(x' + y')^2 - \frac{1}{2}(-x' + y')^2 = a^2$$

Qavslarni ochib, o‘xshash hadlarni soddalashtiramiz:

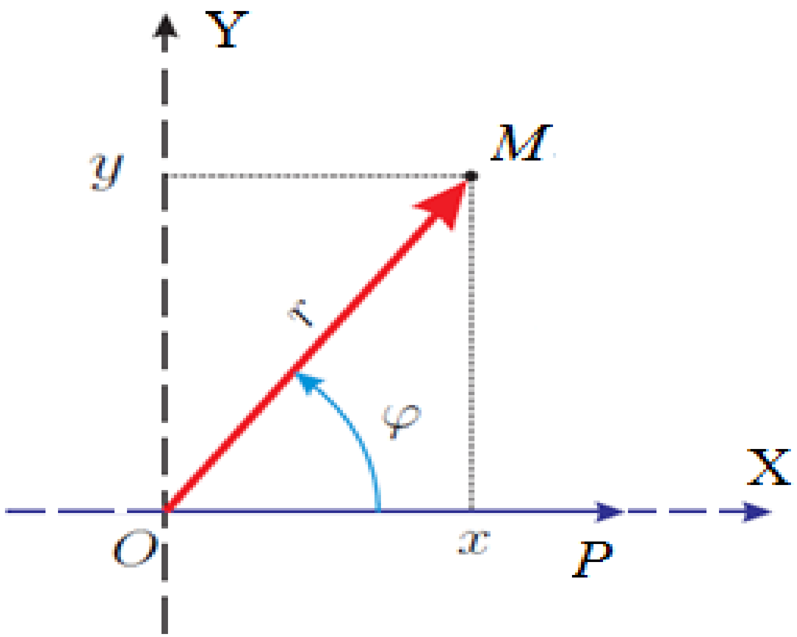
$$2x'y' = a^2,\text{yoki} x'y' = +\frac{a^2}{2}$$

Natijada $y' = +\frac{a^2}{2x'}$tenglama hosil bo‘ladi. Agar $a = \sqrt{2k}$ (k – parametr) belgilash kiritamiz, u holda maktab o‘quvchilariga tanish bo‘lgan $y' = \frac{k}{x'}$giperbola tenglamasi kelib chiqadi.

Qutb koordinatalari sistemasi. Ikkinchi tartibli egri chiziqlarning qutb koordinatalar sistemasidagi tenglamasi

Ta'rif. Tekislikda ***qutb koordinatalar sistemasi*** deb, qutb va qutb o'qi bilan aniqlanadigan koordinata sistemasiga aytiladi.

Де Сен-Венсан Грегуар (1584-1667) – бельгиялик математик.
Кавальери Бонавентура (1598-1647) – италиялик математик.



$$\begin{cases} x = r\cos\varphi, \\ y = r\sin\varphi, \end{cases} \qquad (37)$$

$$r = \sqrt{x^2 + y^2}, \quad \text{tg}\varphi = \frac{y}{x}.$$

28-misol. Dekart koordinatalar sistemasida berilgan $(x-R)^2+y^2=R^2$ aylana tenglamasini qutb koordinatalar sistemasida yozing.

Yechish. Tenglamadagi qavsni ochib chiqamiz:

$$x^2-2Rx+R^2+y^2-R^2=0 \iff x^2+y^2-2Rx=0.$$

(37) formulaga ko‘ra $x$va y ning qiymatini hosil bo‘lgan tenglamaga qo‘yamiz:

$$(r\cos\varphi)^2+(r\sin\varphi)^2-2Rr\cos\varphi=0\,,$$

$$r^2-2Rr\cos\varphi=0.$$

Tenglikni r $(r\neq 0)$ga bo‘lamiz va natijada aylananing qutb koordinatalar sistemasidagi tenglamasi kelib chiqadi:

$$r=2R\cos\varphi\,.$$

$$r=\frac{p}{1-\varepsilon\cos\varphi} \qquad (39)$$

bu yerda ε – ekssentrisitet, p– parametr. Ellips va giperbola uchun$p=\frac{b^2}{a}$ga teng bo‘ladi.

29-misol. Qutb koordinatalar sistemasida ikkinchi tartibli egri chiziq $r = \frac{9}{5-4\cos\varphi}$ tenglamabilan berilgan. Dekart koordinatalar sistemasidagi kanonik tenglamasini yozing.

Yechish. Tenglamaning chap tomonini (39) ko‘rinishga keltirish uchun surat va maxrajini 5 ga bo‘lamiz:

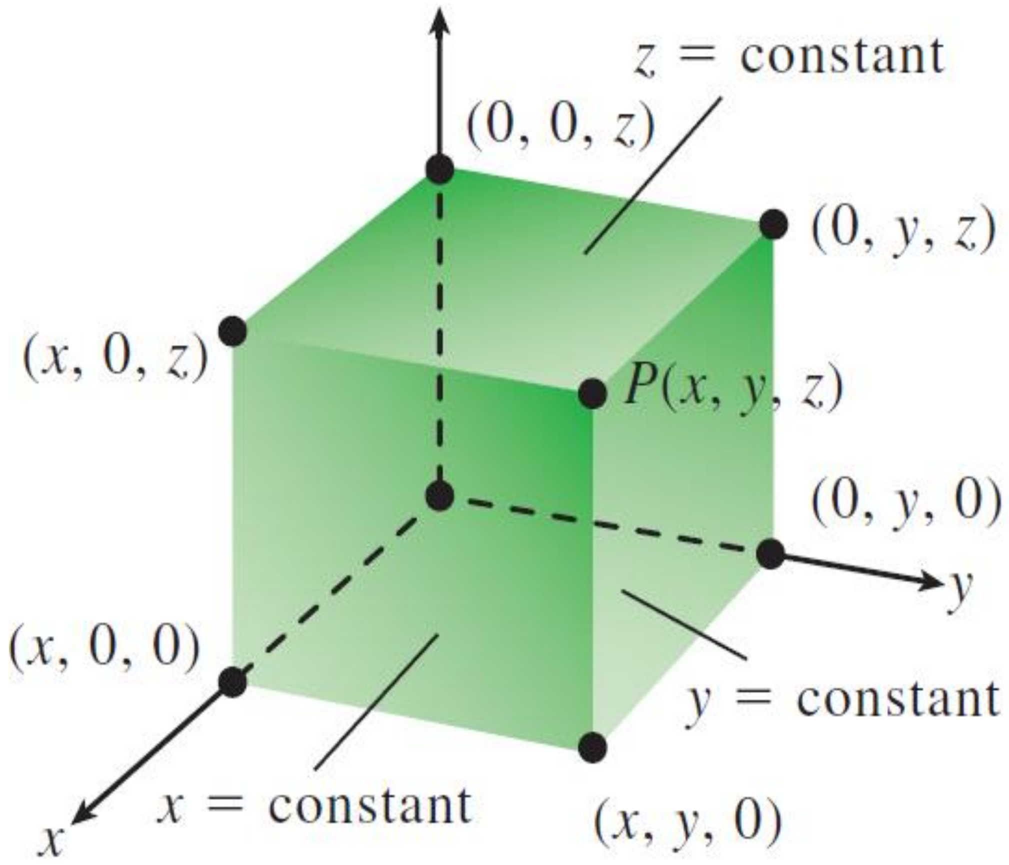
$$r = \frac{\frac{9}{5}}{1 - \frac{4}{5}\cos\varphi}.$$

Bu yerda $\varepsilon = \frac{4}{5} < 1$.Demak, egri chiziq ellipsdir.

$$p = \frac{b^2}{a} = \frac{9}{5}, \qquad \varepsilon = \frac{4}{5} = \frac{c}{a}, \qquad b^2 = \frac{9}{5}a, \qquad c = \frac{4}{5}a.$$

$$a = 5, \qquad b = 3$$

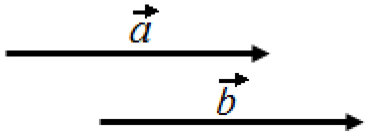
Fazoda koordinatalar sistemasi



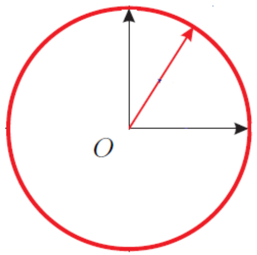
Vektor tushunchasi

Agar vektor uzunligi birga teng bo‘lsa, uni ***birlik vektor*** yoki ***ort*** deyiladi.

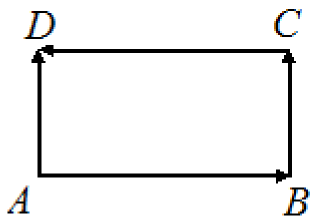
O‘zaro parallel, bir tomonga yo‘nalgan va uzunliklari teng bo‘lgan $\vec{a}$ va $\vec{b}$ vektorlar ***teng vektorlar*** deyiladi va $\vec{a} = \vec{b}$ kabi belgilanadi (1-rasm).



1-rasm.



2-rasm.



3-rasm

1-misol.A(-2; 4; 1), B(4; 2; 7) nuqtalar berilgan. $\overrightarrow{AB}$ vektor koordinatalarini aniqlang vavektormodulini toping.

Bitta to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqda yotgan $\vec{a}$ va $\vec{b}$ vektorlar ***kollinear vektorlar*** deyiladi:

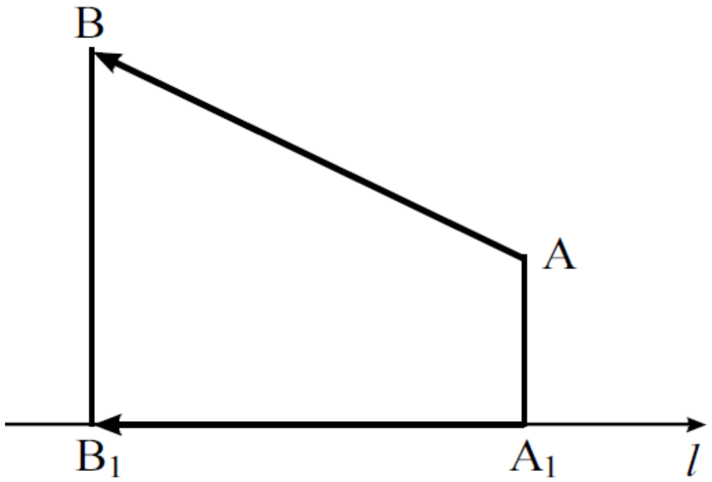
$\vec{a} \uparrow\uparrow \vec{b}$ – bir yo‘nalishli vektorlar;

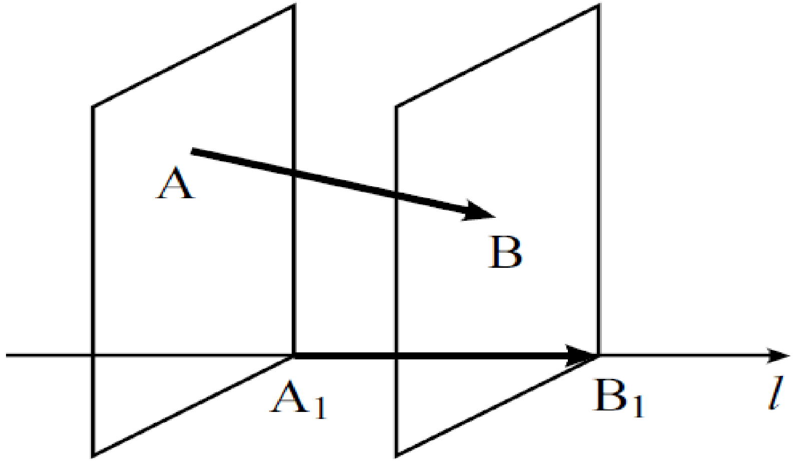
$\vec{a} \downarrow\uparrow \vec{b}$ – qarama-qarshi yo‘nalishli vektorlar;

$\vec{a} \parallel \vec{b}$ – umumiy holda, parallel vektorlar (o‘zaro yo‘nalishi ko‘rsatilmagan hol).

Bitta tekislikka parallel bo‘lganuchta va undan ortiq vektorlar to‘plami ***komplanar vektorlar*** deyiladi. Jumladan, bitta tekislikda yotgan vektorlar komplanardir.

Vektorning o‘qdagi proeksiyasi





$pr_{l}\vec{a} =$ - $|A_1B_1|$

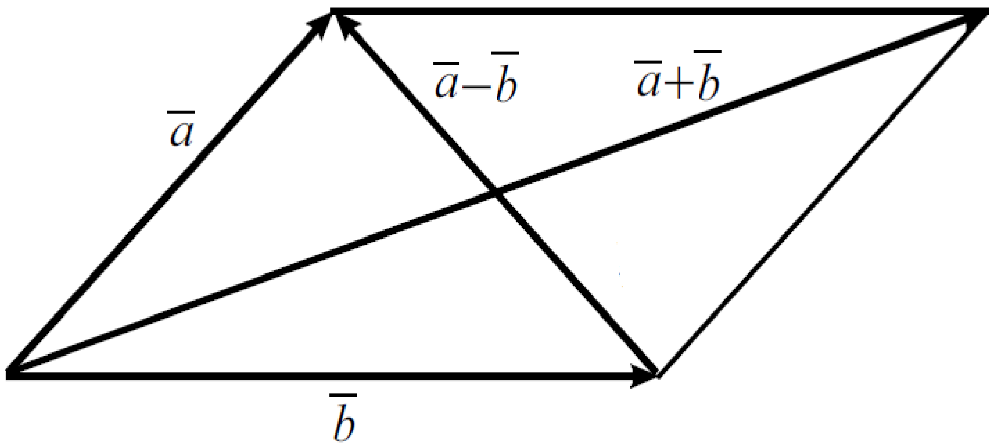
$pr_{l}\vec{a} = |A_1B_1|$

$pr_{l}\vec{a} = |\vec{a}| \cdot cos\alpha$

$\vec{a} = (a_x, a_y, a_z)$

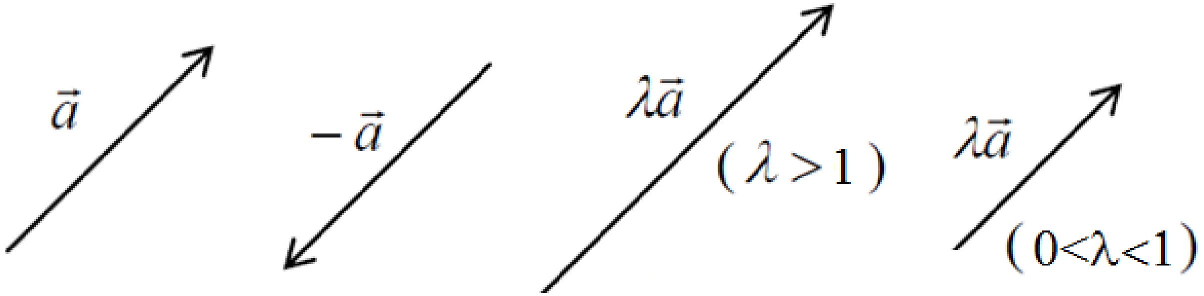
$a_x = |\vec{a}| \cdot cos\alpha,$ $a_y = |\vec{a}| \cdot cos\beta,$ $a_z = |\vec{a}| \cdot cos\gamma$

Vektorlar ustida arifmetik amallar



Faraz qilaylik, agrofirma tarkibida 2 ta fermer xo‘jaligi mavjud. 1-fermer xo‘jaligi joriy yilda 400 tonna paxta, 210 tonna g‘alla, 70 tonna sabzavot mahsulotlari va 30 tonna poliz mahsulotlari yetishtiradi. 2-fermer xo‘jaligi esa 320 tonna paxta, 230 tonna g‘alla, 80 tonna sabzavot mahsulotlari va 50 tonna poliz mahsulotlari yetishtiradi. 1-fermer xo‘jaligining joriy yildagi qishloq xo‘jaligi mahsulotlari ishlab chiqarish hajmini $\vec{a}$=(400; 210; 70; 30) vektor, 2-fermer xo‘jaligining qishloq xo‘jaligi mahsulotlari ishlab chiqarish hajmi $\vec{b}$=(320; 230; 80; 50) vektor yordamida ifodalash mumkin. U holda agrofirmaning yillik qishloq xo‘jalik mahsulotlari ishlab chiqarish hajmi $\vec{a}$ va $\vec{b}$ vektorlar yig‘indisidan iborat bo‘ladi:

$$\vec{a}+\vec{b}=(720;\ 440;\ 150;\ 80).$$



2-misol. Agar $\vec{a} = (1;\ -2;\ \ 3)$ va $\vec{b} = (-5;\ \ 3;\ -1)$ vektorlar berilgan bo‘lsa, $\vec{a} + 2\vec{b}$ vektor uzunligini toping.

Vektorlar ustida kiritilgan amallarga nisbatan quyidagi xossalar o‘rinli:

1°. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (kommutativlik xossasi).

2°. $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$ (assotsiativlik xossasi).

3°. $\vec{a}+0=\vec{a}$.

4°. $\alpha(\vec{a} + \vec{b}) = \alpha\vec{a}+\alpha\vec{b}$ $(\alpha+\beta)\vec{a}=\alpha\vec{a}+\beta\vec{a}$ (distributivlik xossasi).

5°. $\alpha\cdot(\beta\cdot \vec{a})=\alpha \cdot \beta \cdot \vec{a}$.

Uchta $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$vektorlaruchunquyidagishartlarbajarilsa, ular***koordinatabazislari***deyiladi:

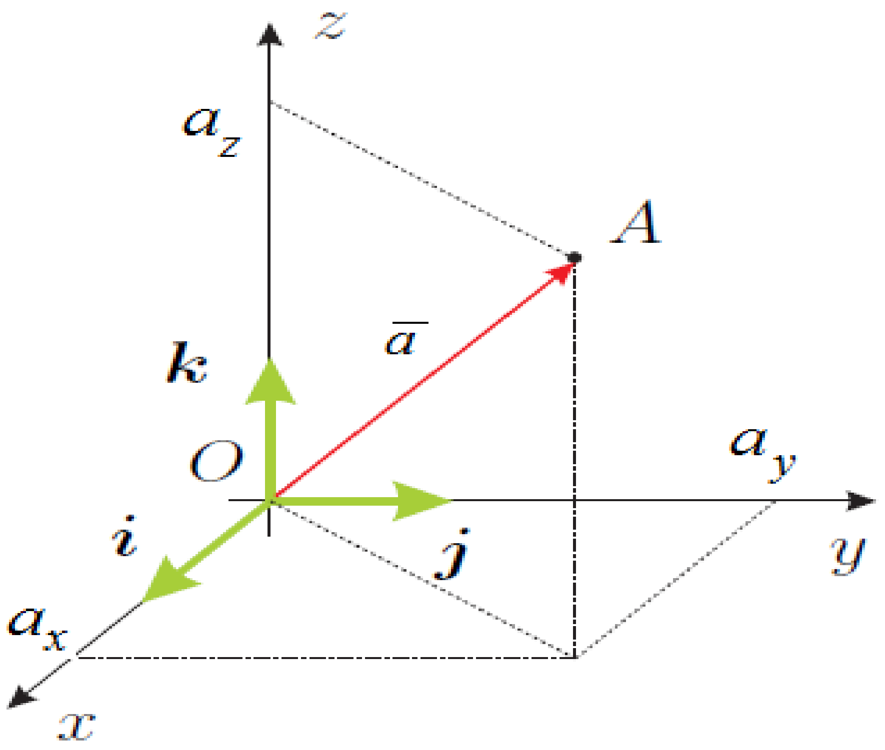
1) $\boldsymbol{i}$vektorOxo‘qida, $\boldsymbol{j}$vektorOyo‘qida, $\boldsymbol{k}$vektorOzo‘qidayotsa;

2) har bir$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$vektorlaro‘zo‘qidamusbattomongayo‘nalganbo‘lsa;

3) $|\vec{\imath}| = 1, \quad |\vec{\jmath}| = 1va|\vec{k}| = 1$, ya’niularbirlikvektorlarbo‘lsa.

Ixtiyoriy$\vec{a} = (a_x; a_y; a_z)$vektornii**i, j, k**bazisbo‘yichayoyishmumkin (9-rasm), ya’ni

$$\vec{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}. \quad (2)$$



Ta'rif. Kamida bittasi noldan farqli bo'lgan shunday $\lambda_1, \lambda_2, \ldots, \lambda_k$ sonlar mavjud bo'lib, ular uchun

$$\lambda_1 \boldsymbol{a_1} + \lambda_2 \boldsymbol{a_2} + \cdots + \lambda_k \boldsymbol{a_k} = 0$$

tenglik o'rinli bo'lsa, $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_k}$ vektorlar sistemasi ***chiziqli bog'liq*** deyiladi.

$$b = \lambda_1 \boldsymbol{a_1} + \lambda_2 \boldsymbol{a_2} + \cdots + \lambda_k \boldsymbol{a_k}$$

vektor $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_k}$ vektorlarning ***chiziqli kombinatsiyasi*** deyiladi.

Vektorlarning chiziqli kombinatsiyasi faqat $\lambda_1 = \lambda_2 = \cdots == \lambda_k = 0$ bo'lgandagina nolga teng bo'lsa, u holda $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_k}$ vektorlar sistemasi ***chiziqli bog'liqmas*** (yoki ***chiziqli erkli***) deyiladi.

4-misol. $\vec{a} = (3; -2;\ 5)$ va $\vec{b} = (4;\ \ 1;\ -2)$ vektorlarning $\lambda_1 = 2,\quad \lambda_2 = 3$ koeffitsiyentlar bilan chiziqli kombinatsiyasini toping.

Ikki vektorning skalyar ko'paytmasi

$$\vec{a}\cdot\vec{b}=|\vec{a}|\cdot\left|\vec{b}\right|\cdot\cos\varphi.$$

$$\vec{a}\cdot\vec{b}=(a_x\boldsymbol{i}+a_y\boldsymbol{j}+a_z\boldsymbol{k})\cdot(b_x\boldsymbol{i}+b_y\boldsymbol{j}+b_z\boldsymbol{k})=$$
$$=a_x\,b_x(\boldsymbol{i}\cdot\boldsymbol{i})+a_xb_y(\boldsymbol{i}\cdot\boldsymbol{j})+a_xb_z(\boldsymbol{i}\cdot\boldsymbol{k})+$$
$$+a_yb_x(\boldsymbol{j}\cdot\boldsymbol{i})+a_yb_y(\boldsymbol{j}\cdot\boldsymbol{j})+a_yb_z(\boldsymbol{j}\cdot\boldsymbol{k})+$$
$$+a_zb_x(\boldsymbol{k}\cdot\boldsymbol{i})+a_zb_y(\boldsymbol{k}\cdot\boldsymbol{j})+a_zb_z(\boldsymbol{k}\cdot\boldsymbol{k})=$$
$$=a_x\cdot b_x+a_y\cdot b_y+a_z\cdot b_z,$$

6-misol. Ushbu $\vec{a}$=(1; -3; 4), $\vec{b}$=(2; 1; -1) vektorlarning skalyar ko'paytmasini toping.

7-misol. $\vec{a}$=(6; -4; 3), $\vec{b}$=(3; -2; -4) vektorlarorasidagiburchaknihisoblang.

Skalyar ko'paytmaning xossalari:

1°. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$;

2°. $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$;

3°. $\lambda\vec{a} \cdot \vec{b} = \vec{a} \cdot \lambda\vec{b} = \lambda \cdot (\vec{a} \cdot \vec{b})$;

4°. $\vec{a} \parallel \vec{b}$ bo'lsa, $\vec{a} \cdot \vec{b} = \pm|\vec{a}| \cdot |\vec{b}|$.

Xususan, $\vec{a} \cdot \vec{a} = |\vec{a}|^2$;

5°. $\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$ $(\vec{a} \neq 0, \vec{b} \neq 0)$;

6°. $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot Pr_{\vec{a}}\vec{b} = |\vec{b}| \cdot Pr_{\vec{b}}\vec{a}$.

Skalyar ko'paytmaning fizik ma'nosi. Biror $\vec{F}$ kuch ta'siri ostida moddiy nuqta to'g'ri chiziq bo'yicha harakat qilsin, bunda kuchning yo'nalishi harakat yo'nalishi bilan bir xil bo'lsin. Moddiy nuqta M_1 nuqtadan M_2 nuqtagacha ko'chganda $\vec{F}$ kuchning bajargan ishi quyidagi formula (skalyar ko'paytma) yordamida aniqlanadi:

$$A=\vec{F}\cdot\vec{S}, \qquad (9)$$

bu erda $\vec{S}=\overrightarrow{M_1M_2}$.

13-misol. Bir nuqtaga $\vec{F}_1=3\vec{\imath}-4\vec{\jmath}+5\vec{k},\ \vec{F}_2=2\vec{\imath}+\vec{\jmath}-4\vec{k}$ va $\vec{F}_3=-\vec{\imath}+6\vec{\jmath}+2\vec{k}$ kuchlar qo'yilgan. Ularning teng ta'sir etuvchi $\vec{F}$ kuch qo'yilish nuqtasi to'g'ri chiziqli harakat qilib, $M_1(4,2,-3)$ nuqtadan $M_2(7,4,1)$ nuqtaga o'tganda, $\vec{F}$ kuch bajargan ishni hisoblang.

Yechish. Ravshanki, $\vec{F}=\vec{F}_1+\vec{F}_2+\vec{F}_3=(4;3;3).\ \vec{S}=\overrightarrow{M_1M_2}=(3;2;4).$ U holda $\vec{F}$ kuch yordamida bajarilgan A ish (9) formulaga ko'ra aniqlanadi:

$$A=\vec{F}\cdot\vec{S}=4\cdot3+3\cdot2+3\cdot4=30\ (J).$$

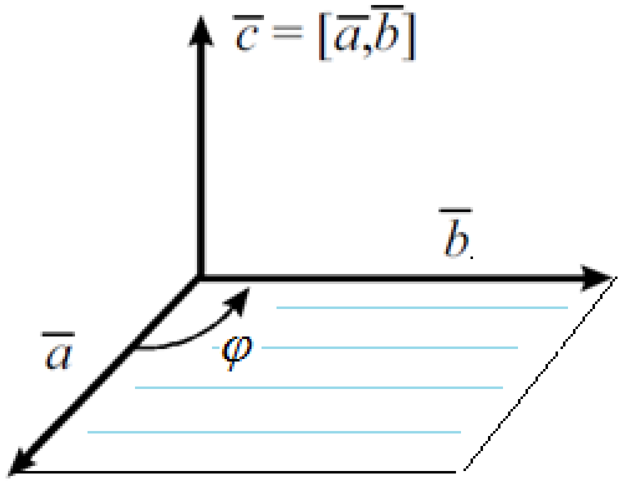
Vektor ko‘paytma

1) $|\vec{c}| = |\vec{a}| \cdot |\vec{b}| \cdot sin\varphi$

2) $\vec{c} \perp \vec{a}, \ \vec{c} \perp \vec{b}$

($\vec{c}$ vektor parallelogramm tekisligiga perpendikulyar); yoki .

3) $\vec{a}, \vec{b}, \vec{c}$ vektorlar o‘ng bog‘lam tashkil etadi.



Vektor ko‘paytma [$\vec{a}, \vec{b}$] yoki $\vec{a} \times \vec{b}$ kabi belgilanadi:

[$\vec{a}, \vec{b}$]= $\vec{c}$ $\vec{a} \times \vec{b} = \vec{c}$

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Vektor ko'paytmaning xossalari:

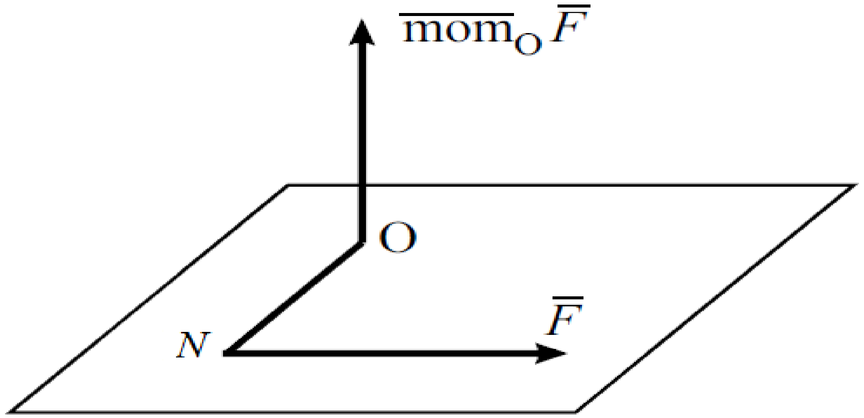
- 1. $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$
- 2. $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
- 3. $(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$
- 4. $\lambda\vec{a} \times \vec{b} = \vec{a} \times \lambda\vec{b} = \lambda \cdot (\vec{a} \times \vec{b})$
- 5. $\vec{a} \times \vec{a} = 0$
- 6. $\vec{a} \times \vec{b} = 0 \iff \vec{a} \parallel \vec{b}$

$S_{parallelogramm} = |\vec{a} \times \vec{b}|$ $S_{uchburchak} = \frac{1}{2}|\vec{a} \times \vec{b}|$

15-misol. $\vec{a}$ =(-2; 0; 1),
$\vec{b}$=(1; 4; 3) vektorlardan yasalgan parallelogramm yuzini toping.

Mexanika kursida qandaydir qattiq jismning qo‘zg‘almas *O* nuqtasiga nisbatan
.

$\vec{F}$ kuch momenti deb ataladigan vektor ushbu formuladan aniqlanadi



$$\overline{mom}_0 \overline{F} = \overline{ON} \times \overline{F}$$

Qo‘zg‘almas o‘q atrofida $\overline{\omega}$ burchak tezlik bilan aylanayotgan nuqtaning chiziqli $\overline{V}$ tezligi shu nuqtaning $\overline{R}$ radius vektori va $\overline{\omega}$ burchak tezlik vektorining vektor ko‘paytmasi bilan aniqlanadi:

$$\overline{V} = \overline{\omega} \times \overline{R}$$

18-misol.A(0; 2; -1) nuqtaga $\vec{F} = (2;\ 3; 1)$

qo‘yilgan. B(1; 1; 2) nuqtaga nisbatan

$\vec{F}$ kuch mometining qiymatini va yonalishini aniqlang.

Uch vektorning aralash ko‘paytmasi

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

$$V = |\vec{a}\vec{b}\vec{c}|$$

$$V_{pir} = \pm \frac{1}{6} \vec{a}\vec{b}\vec{c}$$

$$V_{prizma} = \pm \frac{1}{2} \vec{a}\vec{b}\vec{c}$$

Aralash ko‘paytmaning xossalari

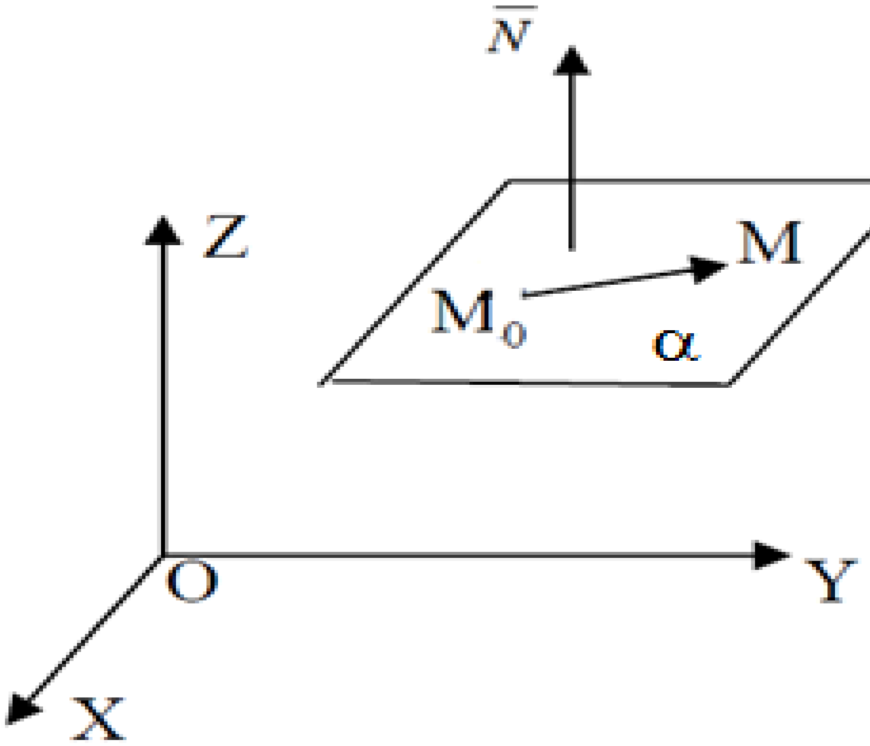
1) $\vec{a} \cdot (\vec{b} \times \vec{c}) = -\vec{a} \cdot (\vec{c} \times \vec{b})$

2) Agar uch vektordan ikkitasi teng yoki parallel bo‘lsa, u holda aralash ko‘paytma nolga teng bo‘ladi.

3) ($\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c})$

FAZODA ANALITIK GEOMETRIYA

1. Tekislikning umumiy tenglamasi



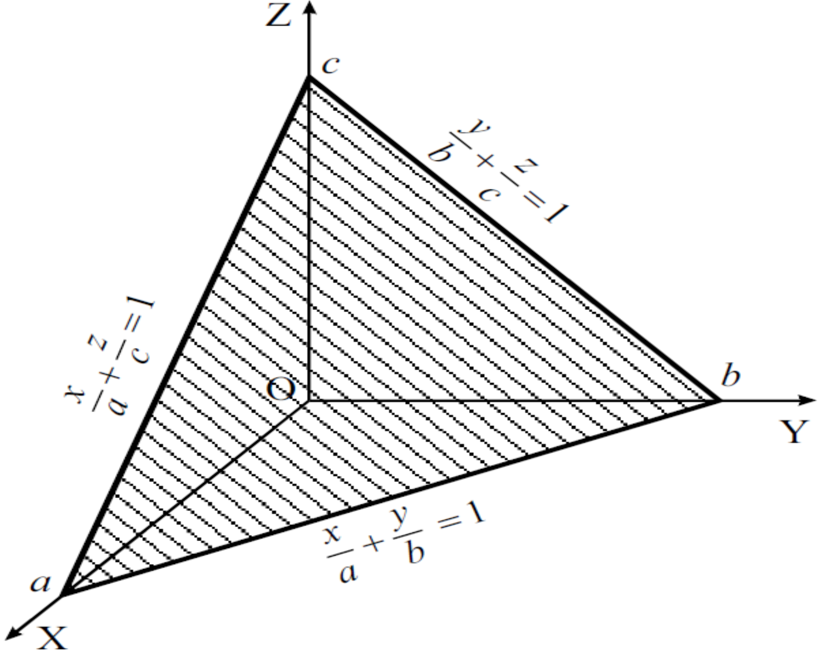
$$\vec{N} \cdot \overrightarrow{M_0M} = 0. \quad (1)$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0 \quad (2)$$

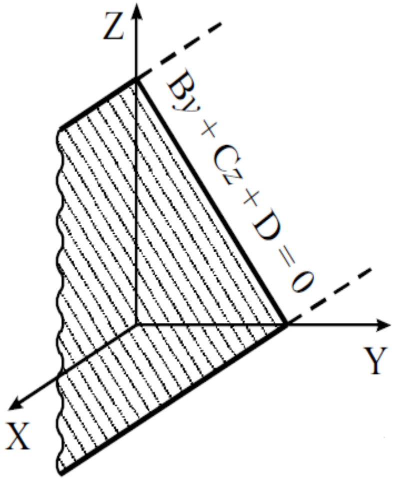
$$Ax + By + Cz + D = 0 \quad (3)$$

$\vec{N}(A, B, C)$ – tekislikning normal vektori

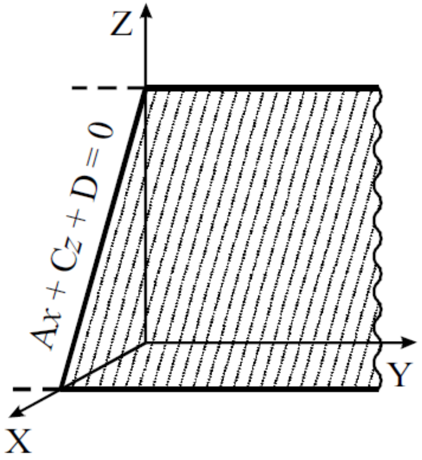
1°. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ ***Tekislikning o‘qlardan ajratgan kesmalar bo‘yicha tenglamasi***



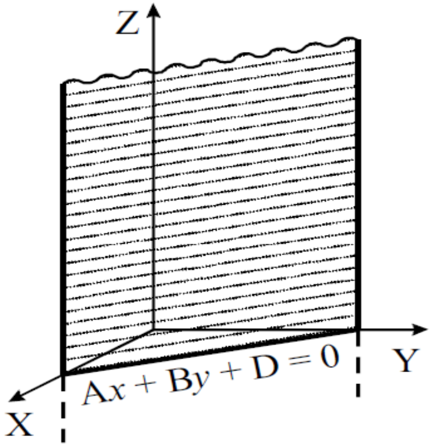
2°. $A=0 \Rightarrow By+Cz+D=0.$



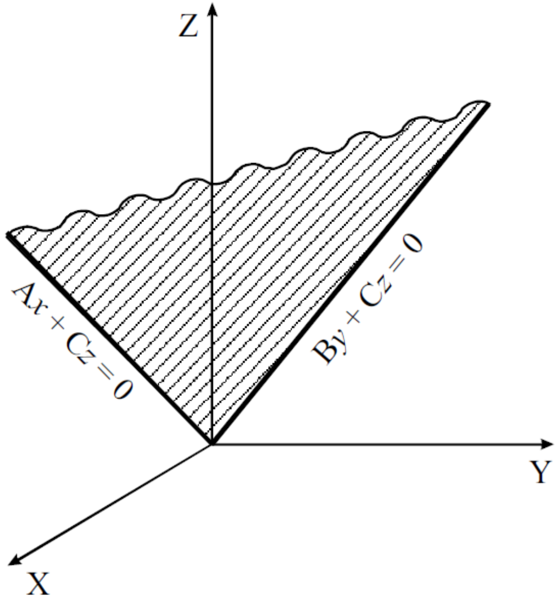
$Ax+Cz+D=0$



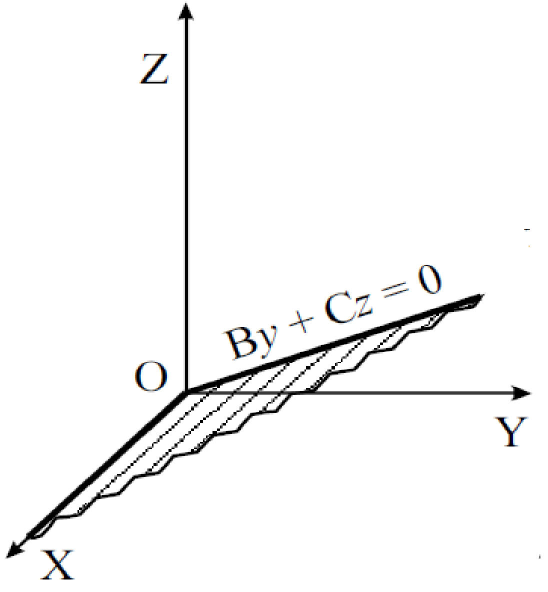
$Ax+By+D=0$



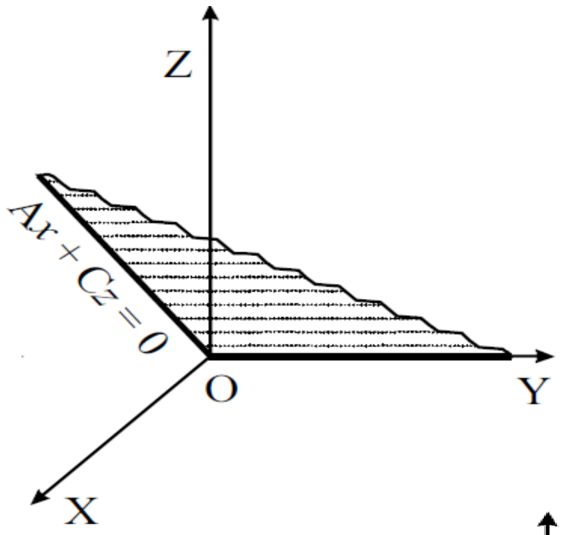
3°. $D = 0 \implies Ax + By + Cz = 0.$



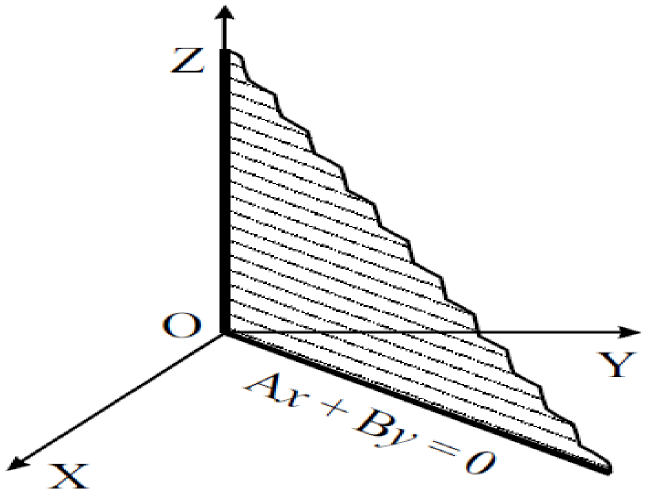
4°. $By + Cz = 0,\ A = 0,\ D = 0$ tekislik Ox o‘qi orqali o‘tadi.



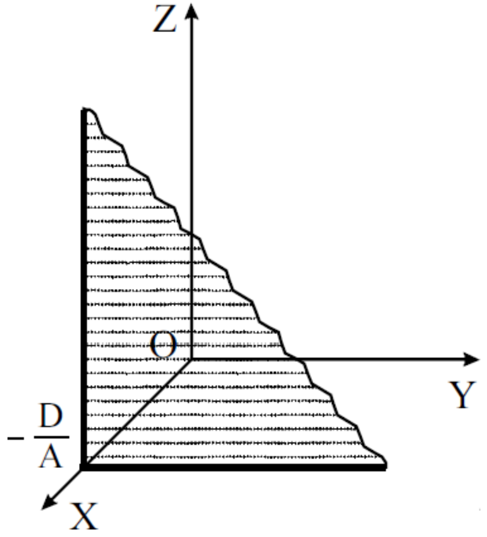
$Ax + Cz = 0$



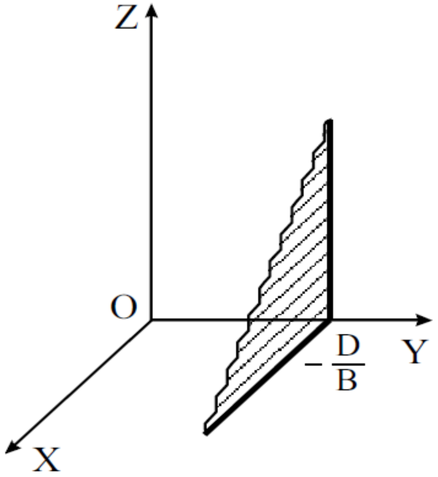
$Ax + By = 0$



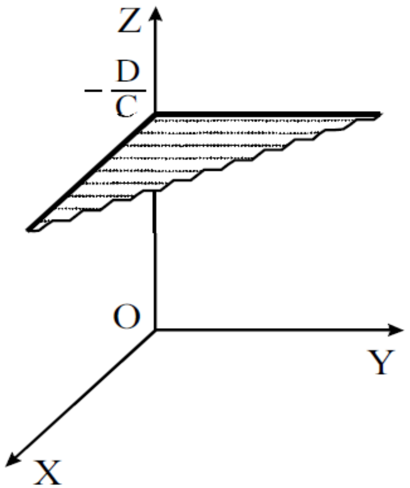
5°. $Ax + D = 0$ tekislik Oyz tekisligiga parallel boladi



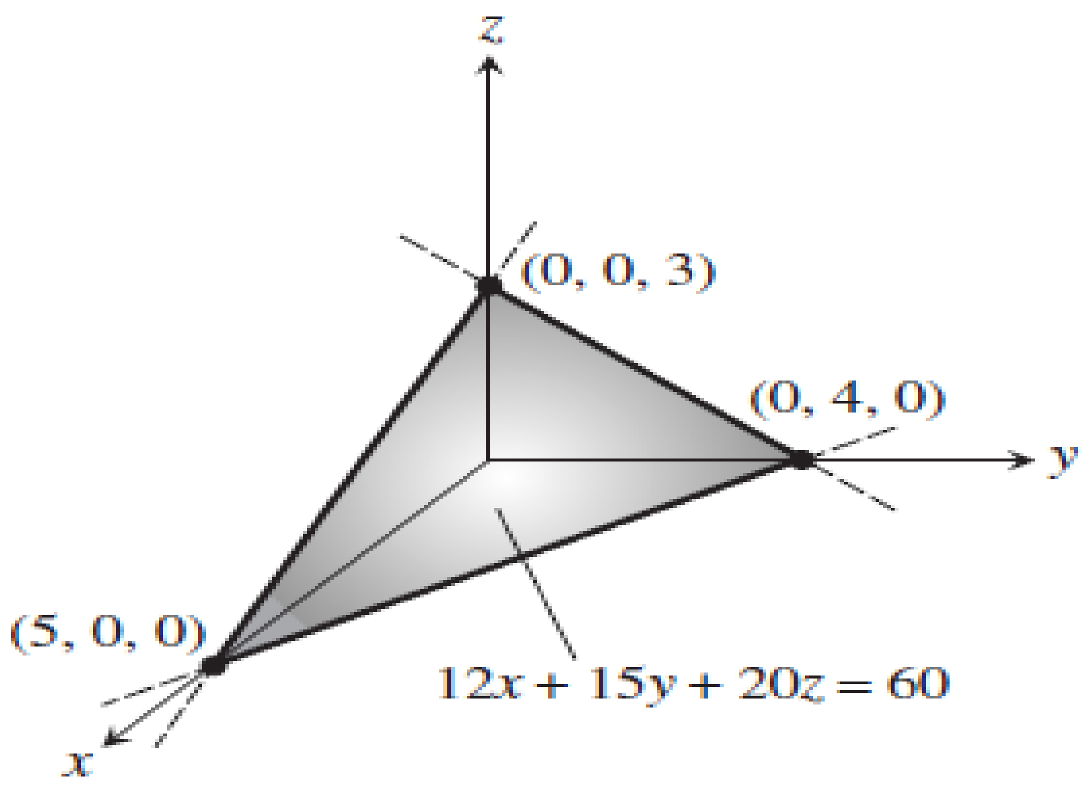
$By + D = 0$ tekislik Oxz tekisligiga parallel boladi



$Cz + D = 0$ tekislik Oxy tekisligiga parallel boladi



1-misol. $12x + 15y + 20z - 60 = 0$ tekislikning koordinata o‘qlaridan ajratgan kesmasini aniqlang.



2-misol. Tekislik $M(6;-10;1)$ nuqtadan o‘tib, abssissalar o‘qidan $a=-3$ va applikatalar o‘qidan $c=2$ kesmalar ajratadi. Bu tekislikning koordinata o‘qlaridan ajratgan kesmalar bo‘yicha tenglamasini tuzing.

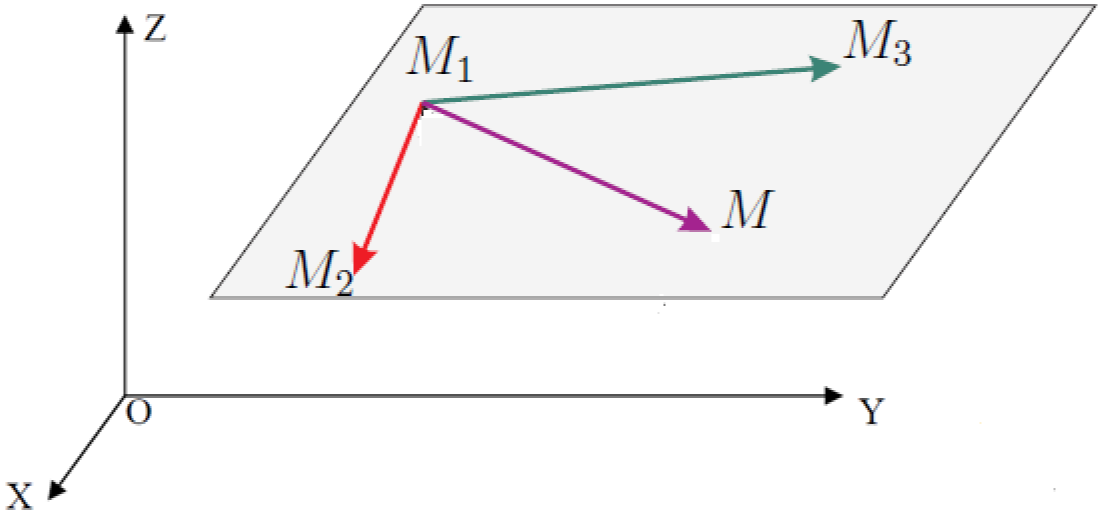
3-misol. *Oz* o‘qi va $M(3;-4;6)$ nuqtadan o‘tuvchi tekislik tenglamasini yozing.

Berilgan uchta nuqtadan o‘tuvchi tekislik tenglamasi

$M_1(x_1, y_1, z_1),\ M_2(x_2, y_2, z_2),\ M_3(x_3, y_3, z_3)$

$$\overrightarrow{M_1M} = (x - x_1,\ y - y_1,\ z - z_1),$$
$$\overrightarrow{M_2M_1} = (x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1),$$
$$\overrightarrow{M_3M_1} = (x_3 - x_1,\ y_3 - y_1,\ z_3 - z_1).$$

$$\overrightarrow{M_1M} \cdot \left(\overrightarrow{M_1M_2} \times \overrightarrow{M_1M_3}\right) = 0.$$



$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$$

5-misol. $M_1(0; 2; 1),\ M_2(4; -1; 1),\ M_3(3; 2; 4)$ nuqtalardan o‘tuvchi tekislik tenglamasi tuzing.

Ikki tekislik orasidagi burchak. Tekisliklarning parallellik va perpendikulyarlik shartlari

$\alpha_1: A_1x + B_1y + C_1z + D_1 = 0, \ \overrightarrow{N_1} = (A_1, \ B_1, \ C_1),$ $\alpha_2: A_2x + B_2y + C_2z + D_2 = 0, \ \overrightarrow{N_2} = (A_2, \ B_2, \ C_2)$

$$\cos\varphi = \pm\frac{\overrightarrow{N_1}\cdot\overrightarrow{N_2}}{|\overrightarrow{N_1}|\cdot|\overrightarrow{N_2}|} = \pm\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{{A_1}^2 + {B_1}^2 + {C_1}^2}\cdot\sqrt{{A_2}^2 + {B_2}^2 + {C_2}^2}}$$

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

$\overrightarrow{N_1} \perp \overrightarrow{N_2}$ $\overrightarrow{N_1}\cdot\overrightarrow{N_2} = 0$ $A_1A_2 + B_1B_2 + C_1C_2 = 0$

6-misol. $x + y + \sqrt{2}z + 7 = 0$ va $z = 0$ tekisliklar orasidagi burchakni toping.

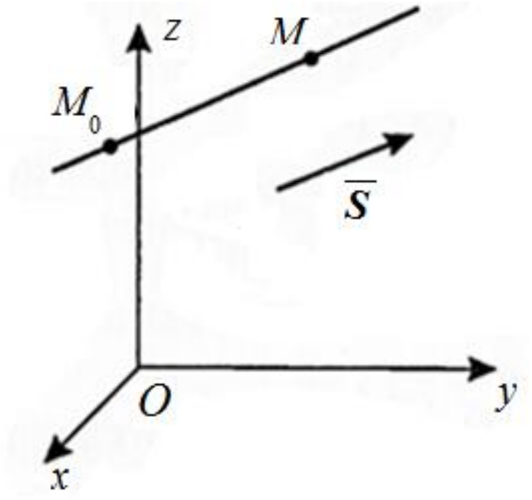
7-misol. $M(3; 2; 4)$ nuqtadan o‘tib, $2x - 6y - 3z + 5 = 0$ tekislikka parallel bo‘lgan tekislik tenglamasini toping.

Fazoda to‘g‘ri chiziq tenglamasi

1.To‘g‘ri chiziqning kanonik va parametrik tenglamalari

Faraz qilaylik, $M_0(x_0, y_0, z_0)$ nuqta $\vec{S}(n; m; p)$ vektorga parallel bo‘lgan l to‘g‘ri chiziqqa tegishli bo‘lsin.

$\overrightarrow{M_0M} \parallel \vec{S}$



$$\frac{x - x_0}{n} = \frac{y - y_0}{m} = \frac{z - z_0}{p}.$$

1-ta’rif. Ushbu ko‘rinishdagi $\frac{x-x_0}{n} = \frac{y-y_0}{m} = \frac{z-z_0}{p}$ tenglama fazoda l ***to‘g‘ri chiziqning kanonik tenglamasi*** deyiladi,

$\vec{S}(n, m, p)$ vektor ***to‘g‘ri chiziqning yo‘naltiruvchi vektori*** deyiladi.

2-ta’rif. Ushbu ko‘rinishdagi

$$\begin{cases} x = nt + x_0 \\ y = mt + y_0 \\ z = pt + z_0 \end{cases}$$

tenglamalar sistemasi l ***to‘g‘ri chiziqning parametrik tenglamasi*** deyiladi.

8-misol. $A(3;4;\ -2)$ va $B(7;5;\ -5)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning kanonik va parametrik tenglamalarini yozing.

9-misol. $\alpha:\ x - 2y - 2z + 4 = 0$ tekislikdan $M(5;\ 1;\ -1)$ nuqtagacha bo‘lgan masofani toping .

2.Fazoda berilgan ikki nuqta orqali o‘tuvchi to‘g‘ri chiziq tenglamasi

Fazoda $M_1(x_1, y_1, z_1)$ va $M_2(x_2, y_2, z_2)$ nuqtalar berilgan bo‘lsin.

$$\overrightarrow{M_1M_2} = (x_2 - x_1;\ \ y_2 - y_1;\ \ z_2 - z_1). \qquad \overrightarrow{M_1M} = (x - x_1;\ \ y - y_1;\ \ z - z_1).$$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \qquad (11)$$

10-misol. Fazoda $M_1(3;\ -2; 4),\ M_2(-7;\ -3; 6)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning kanonik tenglamasini yozing.

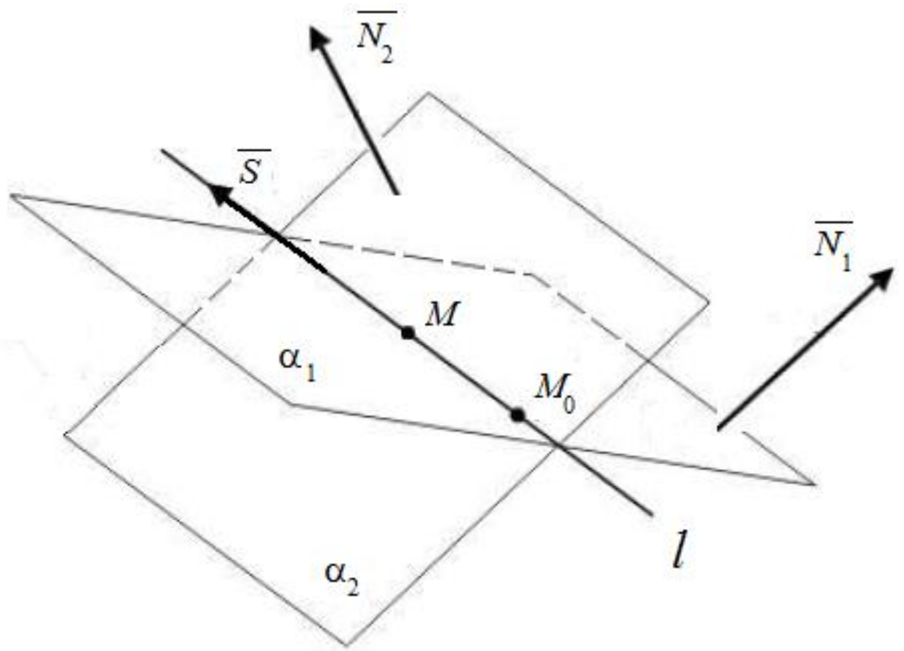
3.Fazoda to‘g‘ri chiziqning umumiy tenglamasi

$$\alpha_1:\ A_1x + B_1y + C_1z + D_1 = 0,$$

$$\alpha_2:\ A_2x + B_2y + C_2z + D_2 = 0.$$

Faraz qilaylik, $\frac{A_1}{A_2} \neq \frac{B_1}{B_2}$ bo‘lsin.

Ushbu $\begin{cases} A_1x + B_1y + C_1z + D_1 = 0, \\ A_2x + B_2y + C_2z + D_2 = 0 \end{cases}$ sistemaga ***fazoda to‘g‘ri chiziqning umumiy tenglamasi*** deyiladi.



$$\begin{cases} A_1x + B_1y = -D_1 - C_1z_0 \\ A_2x + B_2y = -D_2 - C_2z_0 \end{cases}$$

$$\frac{A_1}{A_2} \neq \frac{B_1}{B_2}$$

$$x = \frac{\begin{vmatrix} -D_1 - C_1z_0 & B_1 \\ -D_2 - C_2z_0 & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} A_1 & -D_1 - C_1z_0 \\ A_2 & -D_2 - C_2z_0 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}.$$

$$\vec{S} = \overrightarrow{N_1} \times \overrightarrow{N_2}.$$

$$\vec{S} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} = \vec{i}\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix} + \vec{j}\begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix} + \vec{k}\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}.$$

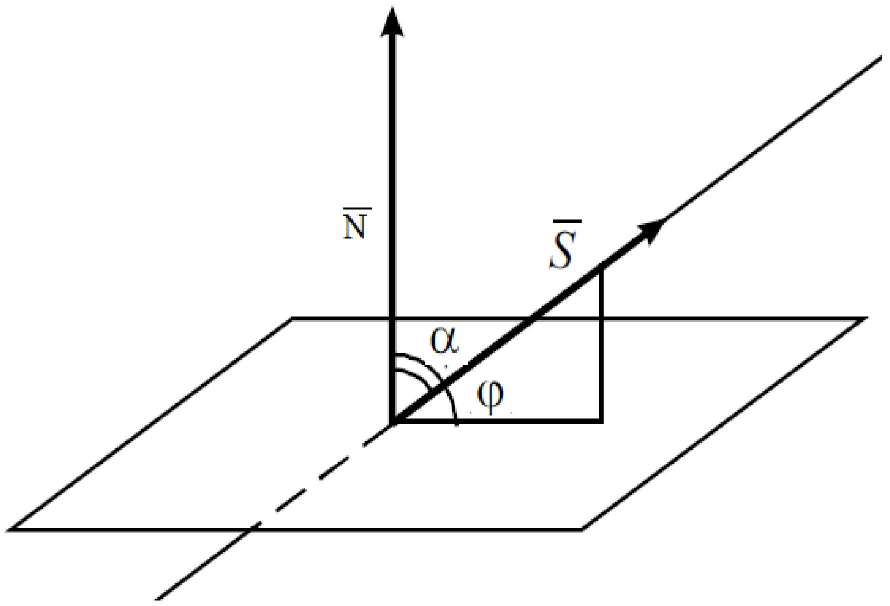
$$\vec{S} = n \cdot \vec{i} + m \cdot \vec{j} + p \cdot \vec{k},$$

$$\frac{x - x_0}{n} = \frac{y - y_0}{m} = \frac{z - z_0}{p}$$

11-misol. To‘g‘ri chiziqning $\begin{cases} 2x - 3y + z - 5 = 0, \\ 3x + y - 2z - 4 = 0 \end{cases}$

umumiy tenglamasini kanonik ko‘rinishda ifodalang.

4. To‘g‘ri chiziq va tekislik orasidagi burchak. To‘g‘ri chiziq va tekislikning parallellik va perpendikulyarlik shartlari



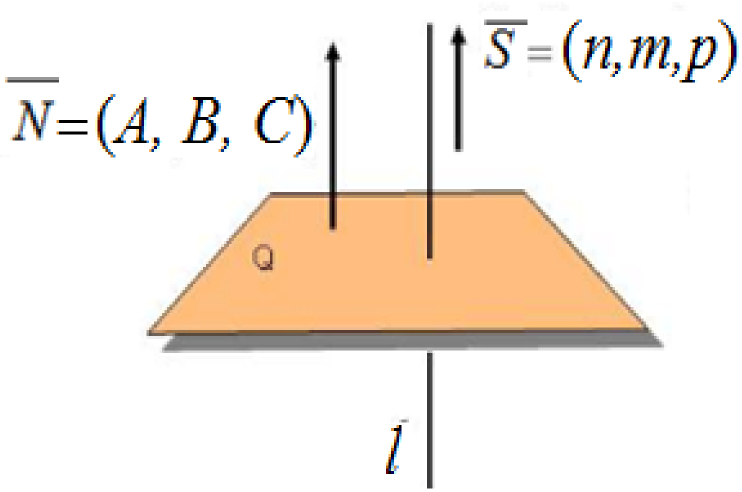
$$\frac{x-a}{n} = \frac{y-b}{m} = \frac{z-c}{p}$$

$$Ax + By + Cz + D = 0$$

$$\cos\alpha = \frac{\vec{N}\cdot\vec{S}}{|\vec{N}|\cdot|\vec{S}|} \qquad \cos(90^\circ - \varphi) = \frac{\vec{N}\cdot\vec{S}}{|\vec{N}|\cdot|\vec{S}|} \Rightarrow \sin\varphi = \frac{\vec{N}\cdot\vec{S}}{|\vec{N}|\cdot|\vec{S}|}.$$

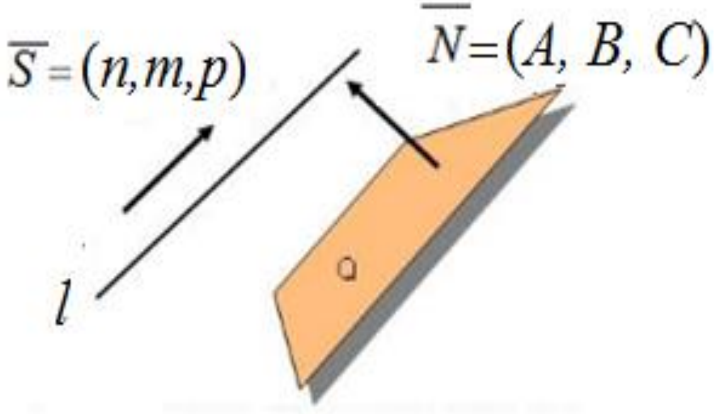
$$\sin\varphi = \frac{|A\cdot n + B\cdot m + C\cdot p|}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{n^2+m^2+p^2}}$$

$\vec{S} \parallel \vec{N}$



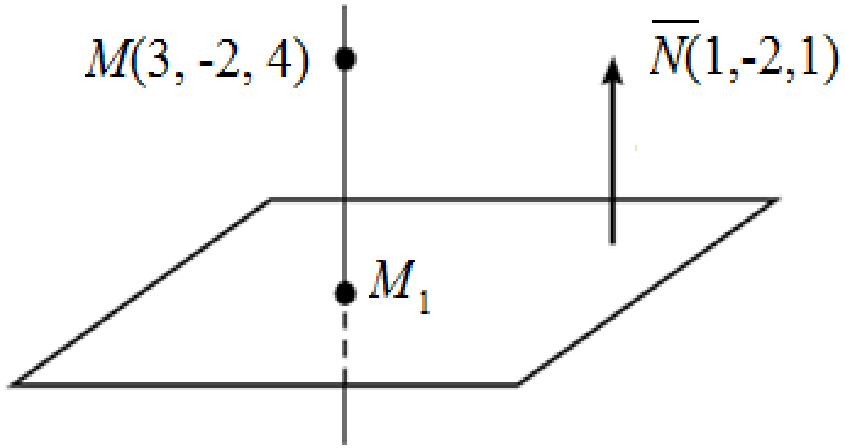
$\frac{A}{n} = \frac{B}{m} = \frac{C}{p}$ formula ***to‘g‘ri chiziq va tekislikning perpendikulyarlik shartidir***.

$\vec{N} \perp \vec{S}$



$\vec{N} \cdot \vec{S} = 0 \implies A \cdot n + B \cdot m + C \cdot p = 0$ formula ***to‘g‘ri chiziq va tekislikning parallellik shartidir***.

13-misol. $M(3,-2,4)$ nuqtaning $x-2y+z-5=0$ tekislikdagi proyeksiyasini toping.



5. Fazoda ikki to‘g‘ri chiziqning parallellik, perpendikulyarlik va bitta tekislikda yotish shartlari

.

$$l_1: \ \frac{x-a_1}{n_1}=\frac{y-b_1}{m_1}=\frac{z-c_1}{p_1}, \qquad \vec{S}_1(n_1,m_1,p_1), \quad M_1(a_1,b_1,c_1).$$

$$l_2: \ \frac{x-a_2}{n_2}=\frac{y-b_2}{m_2}=\frac{z-c_2}{p_2}, \qquad \vec{S}_2(n_2,m_2,p_2), \quad M_2(a_2,b_2,c_2)$$

l_1 va l_2 to‘g‘ri chiziqlarning $\frac{n_2}{n_1}=\frac{m_2}{m_1}=\frac{p_2}{p_1}$ parallellik sharti

$n_1n_2+m_1m_2+p_1p_2=0$ perpendikulyarlik sharti

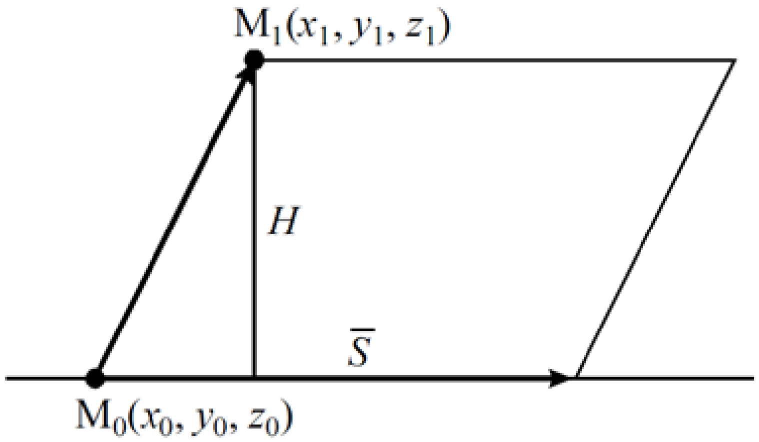
$\vec{S}_1 \nparallel \vec{S}_2$ $\quad \overrightarrow{M_1M_2}\cdot(\vec{S}_1\times\vec{S}_2)$ =0

l_1 va l_2 to‘g‘ri chiziqlarning bitta tekislikda yotish sharti

$$\begin{vmatrix} a_2-a_1 & b_2-b_1 & c_2-c_1 \\ n_1 & m_1 & p_1 \\ n_2 & m_2 & p_2 \end{vmatrix}=0$$

Tekislik va to‘g‘ri chiziqqa doir aralash masalalar

1. $M_1(x_1, x_2, x_3)$ **nuqtadan** $\frac{x - x_0}{n} = \frac{y - y_0}{m} = \frac{z - z_0}{p}$ **to‘g‘ri chiziqgacha bo‘lgan masofa**



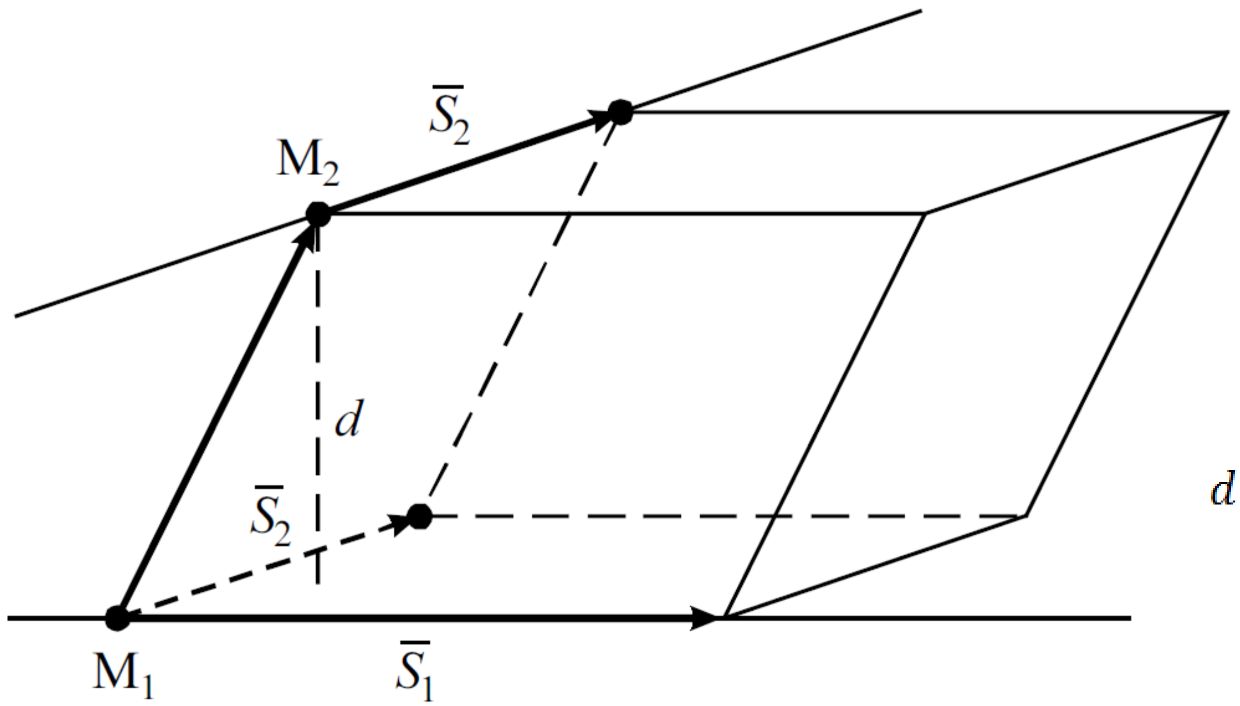
$M_0\ (x_0, y_0, z_0),$
$\overline{S} = (n, m, p)\ .$

$$d = H = \frac{|\vec{S} \times \overrightarrow{M_0M_1}|}{|\vec{S}|}. \qquad (22)$$

2.Ayqash to'g'ri chiziqlar orasidagi eng qisqa masofa.

$$l_1: \frac{x-a_1}{n_1}=\frac{y-b_1}{m_1}=\frac{z-c_1}{p_1}, \qquad \text{bu yerda} \vec{S}_1(n_1,m_1,p_1);\ M_1(a_1,b_1,c_1);$$

$$l_2: \frac{x-a_2}{n_2}=\frac{y-b_2}{m_2}=\frac{z-c_2}{p_2}, \qquad \text{bu yerda} \vec{S}_2(n_2,m_2,p_2);\ M_2(a_2,b_2,c_2);$$



$$d=H=\frac{V_{par-d}}{S_{asos}}=\frac{\left|\overrightarrow{M_1M_2}\cdot(\vec{S}_1\times\vec{S}_2)\right|}{\left|\vec{S}_1\times\vec{S}_2\right|}. \qquad (23)$$

3. Berilgan to‘g‘ri chiziqdan o‘tuvchi va berilgan tekislikka perpendikulyar tekislik tenglamasi.

$$\alpha: Ax + By + Cz + D = 0, \quad \vec{N}(A, B, C),$$

$$l: \ \frac{x-a}{n} = \frac{y-b}{m} = \frac{z-c}{p},$$ bu yerda $\vec{S}(n, m, p)$, $M_1(a, b, c)$.

$$\vec{N} \parallel \vec{S} \qquad \vec{S} \parallel \overrightarrow{MM_1}$$

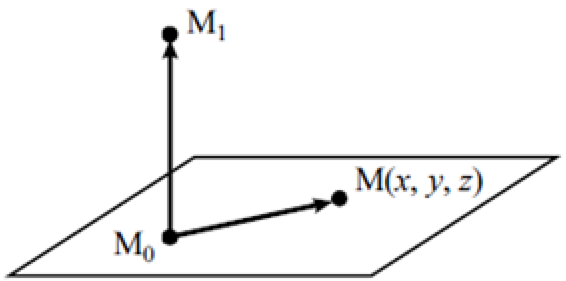
$$\overrightarrow{MM_1} \cdot (\vec{S} \times \vec{N}) = 0$$

$$\begin{vmatrix} x-a & y-b & z-c \\ n & m & p \\ A & B & C \end{vmatrix} = 0.$$

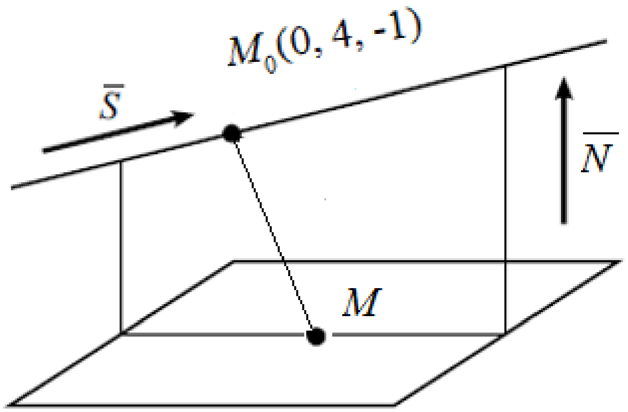
15-m isol. $\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-2}{2}$ va $\frac{x}{-1} = \frac{y-2}{3} = \frac{z}{3}$

to‘g‘ri chiziqlar orasidagi eng qisqa masofani toping.

16-misol. $M_1(5,3,-4)$ nuqtadan teki slikka perpendikulyar tushirilgan. Perpendikulyarning asosini shu tekislikdagi $M_0(2,4,-1)$ nuqta tashkil etadi. Tekislik tenglamasini tuzing.



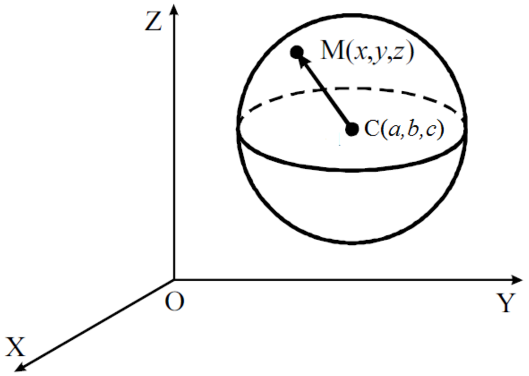
17- misol. $\frac{x}{4}=\frac{y-4}{3}=\frac{z+1}{-2}$ to‘g‘ri chiziqning $x-y+3z+8=0$ Tekislikdagi proyeksiyasini toping



Ikkinchi tartibli sirtlar

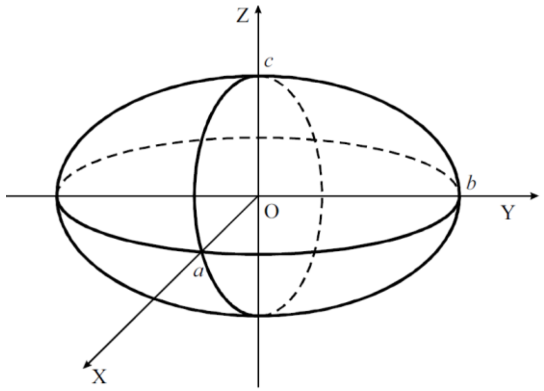
1. Sfera. Ellipsoid.

$$x^2 + y^2 + z^2 = R^2$$



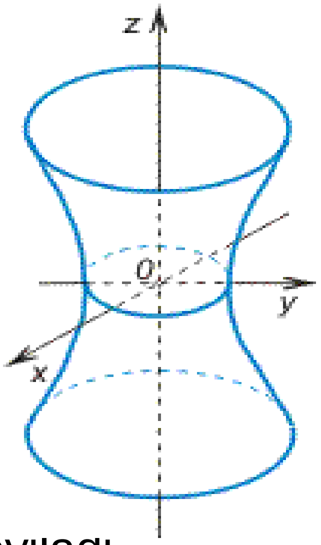
1-ta'rif. Kanonik tenglamasi

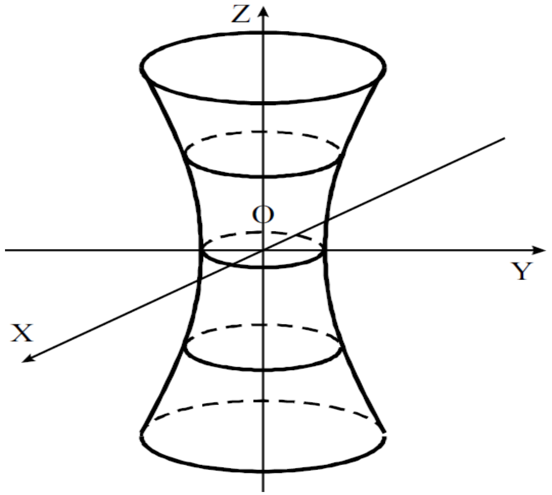
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



ko'rinishda bo'lgan sirtga ***ellipsoid*** deyiladi.

2. Giperboloidlar.

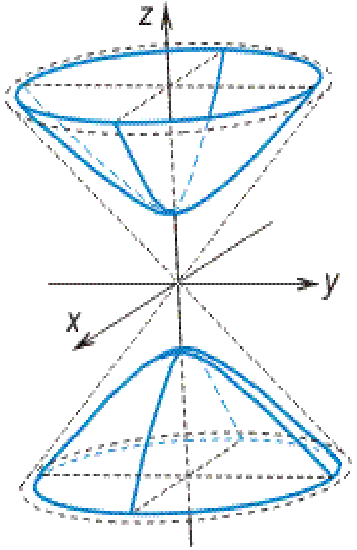




1-ta'rif. Kanonik tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1,$$

ko'rinishdagi sirtga ***bir pallali giperboloid*** deyiladi



2-ta'rif. Kanonik tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

ko'rinishdagi sirtga ***ikki pallali giperboloid*** deyiladi

3. Paraboloidlar.

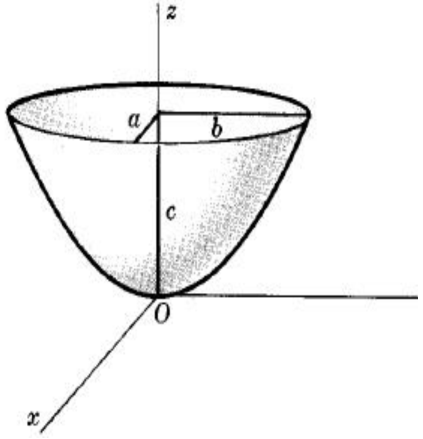
$$x^2 = 2pz, y = 0$$

Bu parabolani Oz o‘qi atrofida aylantirishdan hosil bo‘lga n sirt ***aylanma paraboloid*** deyiladi.

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 2z$$

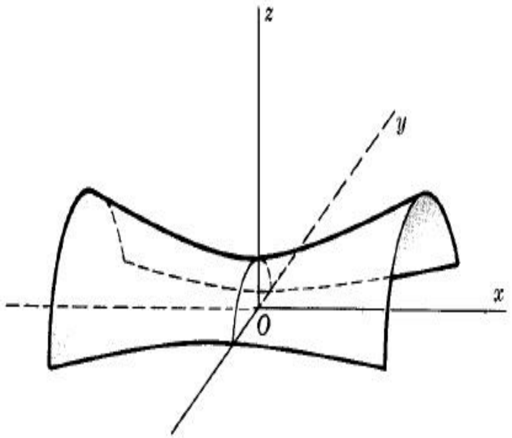
1-ta’rif. Kanonik tenglamasi $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2z,$

ko‘rinishdagi sirtga ***elliptik paraboloid*** deyiladi.



2-ta’rif. Kanonik tenglamasi $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z,$

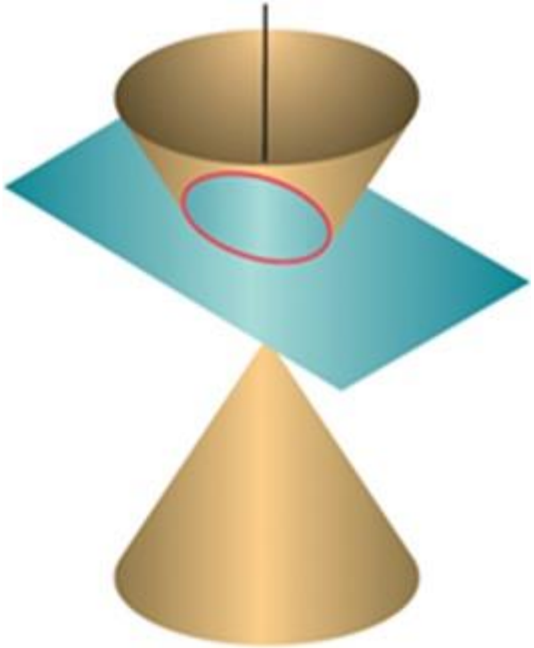
ko‘rinishdagi sirtga ***giperbolik paraboloid*** deyiladi.



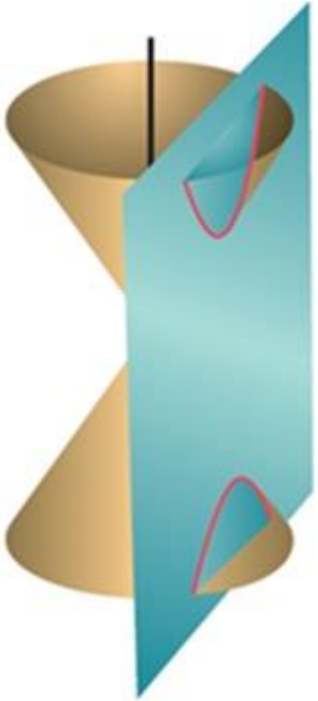
4. Konus.

Ta'rif. Kanonik tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$

ko'rinishdagi sirtga ***konus*** deb ataladi.



Kesimi ellips



Kesimi giperbola

Ikkinchi tartibli sirtlarning umumiy tenglamasi

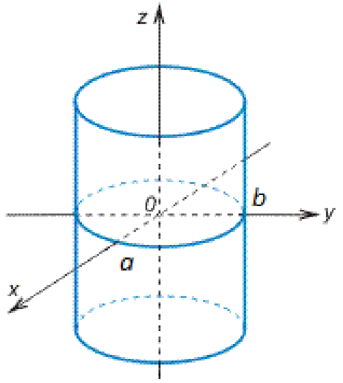
$$Ax^2 + By^2 + Cz^2 + 2Dxy + 2Exz + 2Fyz + Gx + Hy + Kz + L = 0$$

$$F(x, y, z) = 0$$

1-ta'rif. Kanonik tenglamasi

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

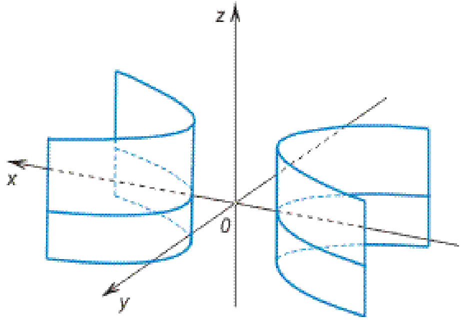
ko'rinishdagi sirtga ***elliptik silindr*** deyiladi.



2-ta'rif. Kanonik tenglamasi

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

ko'rinishdagi sirtga ***giperbolik silindr*** deyiladi.



3-ta'rif. Kanonik tenglamasi

$$y^2 = 2px$$

ko'rinishda bo'lgan sirt ***parabolik silindr*** deyiladi