Mavzu: Bo‘laklab integrallash usuli.

REJA:

1. **Bo‘laklab integrallash usuli**
2. **Bo‘laklab integrallash usuli orqali yechiladigan integrallar**

Bo‘laklab integrallash usuli. Bu usul ikki funksiya ko‘paytmasining differensiali formulasidan kelib chiqadi. Ma’lumki, agar *u(x)* va *v(x)* funksiyalar differensiallanuvchi funksiyalar bo‘lsa, u holda *d(uv)=udv+vdu* yoki *udv=d(uv)-vdu* bo‘ladi. Bu tenglikni ikkala tomonini integrallasak,

$\int udv=\int d(uv)- \int vdu$, yoki

$\int udv=uv - \int vdu$ (1)

formula hosil bo‘ladi. Bu formula bo‘laklab integrallash formulasi deyiladi. Bu formula yordamida $\int udv$ ni hisoblash boshqa, $\int vdu$ integralni, hisoblashga keltiriladi. Bu formuladan $\int udv$ ga nisbatan $\int vdu$ integralni hisoblash oson bo‘lganda foydalaniladi.

1-*misol*. $\int xcosxdx$ ni hisoblang.

Yechish. *u=x, du=x, v=sinx, dv=cosxdx* belgilashlarni kiritamiz. U holda

$$\int x\cdot\cos xdx=\int udv=u\cdot v-\int vdu=x\cdot\sin x-\int\sin xdx=x\sin x+\cos x+C$$

bo‘ladi.

2-*misol*. $\int lnxdx$ ni hisoblang.

Yechish. $u=lnx$, $du=\frac{dx}{x}$, $v=x$, $dv=dx$ almashtirishni kiritamiz. U holda,

$$\int\ln xdx=\int udv=x\cdot\ln x-\int x\cdot\frac{dx}{x}=x\cdot\ln x-x+C$$ bo‘ladi.

Endi amaliyotda tez-tez uchrab turadigan va bo‘laklab integrallash usuli bilan hisoblanadigan integrallar tiplarini keltiramiz.

1. $\int P_n(x)e^{kx}dx,\ \int P_n(x)\sin kxdx,\ \int P_n(x)\cos kxdx$ ko‘rinishdagi integrallar, bu yerda $P_n(x)$ - n – darajali ko‘phad, k – biror son. Bu integrallarni hisoblash uchun $u=P_n(x)$ deb olish va (1) formulani n marta qo‘llash yetarli.

2. $\int P_n(x)\ln xdx,\ \int P_n(x)\arcsin xdx,\ \int P_n(x)\arccos xdx,\ \int P_n(x)arctgxdx,$ $\int P_n(x)arcctgxdx$ ko‘rinishdagi integrallar, bu yerda $P_n(x)$ - n – darajali ko‘phad. Bu integrallarni bo‘laklab integrallash uchun $P_n(x)$ oldidagi ko‘payuvchi funksiyani u deb olish lozim.

3. $\int e^{ax}\cos bxdx,\ \int e^{ax}\cos bxdx$, bu yerda a va b lar haqiqiy sonlar. Bu integrallar ikki marta bo‘laklab integrallash usuli bilan hisoblanadi.

3-*misol*. $\int \arcsin xdx$ integralni hisoblang.

Yechish. Bu integral 2-tipga kiradi, bunda $P_0(x)=1$ va $u=arcsinx$ deb olamiz. U holda

$$\int \arcsin xdx = \left| \begin{matrix} u=\arcsin x, & du=\dfrac{dx}{\sqrt{1-x^2}} \\ dv=dx, & v=x \end{matrix} \right| = x\arcsin x - \int \frac{xdx}{\sqrt{1-x^2}} =$$

$$= x\arcsin x + \frac{1}{2}\int (1-x^2)^{-\frac{1}{2}}d(1-x^2) = x\arcsin x + \sqrt{1-x^2} + C \text{ bo‘ladi.}$$

4-*misol*. $\int e^{-x}\cos\frac{x}{2}dx$ integralni hisoblang.

Yechish. Bu integral 3-tipga mansub. u sifatida dx ning oldidagi ko‘paytuvchilardan ixtiyoriy birini olamiz va ikki marta bo‘laklab integrallashni bajaramiz. Ikkinchi marta integrallaganimizda avval berilgan integralni o‘z ichida saqlaydigan tenglikka ega bo‘lamiz. Bu tenglikdan berilgan integralni topamiz:

$$\int e^{-x}\cos\frac{x}{2}dx = \left| \begin{matrix} u=e^{-x}, & du=-e^{-x}dx \\ dv=\cos\frac{x}{2}dx, & v=2\int\cos\frac{x}{2}d(\frac{x}{2})=2\sin\frac{x}{2} \end{matrix} \right| = 2e^{-x}\sin\frac{x}{2} +$$

$$+2\int e^{-x}\sin\frac{x}{2}dx = \left| \begin{matrix} u=e^{-x}, & du=-e^{-x}dx \\ dv=\sin\frac{x}{2}dx, & v=-2\cos\frac{x}{2} \end{matrix} \right| = 2e^{-x}\sin\frac{x}{2} - 4e^{-x}\cos\frac{x}{2} -$$

$-4\int e^{-x}\cos\frac{x}{2}dx$, ya’ni $\int e^{-x}\cos\frac{x}{2}dx = 2e^{-x}\sin\frac{x}{2} - 4e^{-x}\cos\frac{x}{2}$ - $4\int e^{-x}\cos\frac{x}{2}dx$,

bundan $5\int e^{-x}\cos\frac{x}{2}dx = 2e^{-x}\sin\frac{x}{2} - 4e^{-x}\cos\frac{x}{2}$, yoki

$\int e^{-x}\cos\frac{x}{2}dx = \frac{2}{5}(e^{-x}\sin\frac{x}{2} - 2e^{-x}\cos\frac{x}{2})$.

5 – м и с о л . ***Ushbu***

$$\mathbf{I_n = \int \frac{dx}{\left(x^2 + a^2\right)^n} \qquad (n = 1,2,3...)}$$

Integral hisoblansin

◀Ravshanki,

$$\mathbf{I_1 = \int \frac{dx}{x^2 + a^2} = \frac{1}{a}arctg\frac{x}{a} + C}$$

Berilgan integralni bo’laklab integrallaymiz:

$$\mathbf{I_n = \int \frac{dx}{\left(x^2 + a^2\right)^n} = \begin{bmatrix} u = \frac{1}{\left(x^2 + a^2\right)^n}, & du = -\frac{2nxdx}{\left(x^2 + a^2\right)^{n+1}} \\ dv = dx, & v = x \end{bmatrix} = \frac{x}{\left(x^2 + a^2\right)^n} +}$$

$$\mathbf{+ 2n\int \frac{x^2}{\left(x^2 + a^2\right)^{n+1}}dx = \frac{x}{\left(x^2 + a^2\right)^n} + 2n\int \frac{dx}{\left(x^2 + a^2\right)^n}dx - 2na^2\int \frac{dx}{\left(x^2 + a^2\right)^{n+1}} =}$$

$$\mathbf{= \frac{x}{\left(x^2 + a^2\right)^n} + 2n \cdot I_n - 2na^2 \cdot I_{n+1}}$$

Keyingi tenglikdan topamiz:

$$\mathbf{I_{n+1} = \frac{1}{2na^2} \cdot \frac{x}{\left(x^2 + a^2\right)^n} + \frac{2n - 1}{2na^2} I_n} . ▶$$

O‘z- o‘zini tekshirish uchun savollar

1. $y=f(x)$ funksiyaning boshlang‘ich funksiyasi deb nimaga aytiladi?
2. berilgan funksiyaning ikkita turli boshlang‘ich funksiyalari x ga farq qilishi mumkinmi?
3. Aniqmas integral nima (son, funksiya, funksiyalar to‘plami)?

4. Asosiy integrallar jadvali integralning qaysi xossasiga asoslanib tuzilgan?

5. Aniqmas integralning xossalarini ayting.

6. Qanday integrallash qoidalarini bilasiz?

7. Qanday integrallash usullarini bilasiz?

8. Bo‘laklab integrallash usuli differensial hisobning qaysi formulasiga asoslangan?

9. Aniqmas integralda bo‘laklab integrallash qanday amalga oshiriladi?

10. Rekurrent formula bilan hisoblash nimaga asoslangan?

11. Ushbu integrallarda u bilan nimani belgilash kerak:

a) $\int x^3 arctgx dx$; b) $\int (x+1)^5 e^{3x} dx$? Javobingizni asoslang.

Mustaqil yechish uchun misol va masalalar

Bo‘laklab integrallash usulidan foydalanib, quyidagi integrallarni toping.

a) $\int x \sin 2x dx$; b) $\int x arctgx dx$; c) $\int \arccos x dx$;

d) $\int x^2 e^{-x} dx$; e) $\int \frac{x^2 dx}{(1+x^2)^2}$; f) $\int x^2 \ln(1+x) dx$.

Javoblar:

a) $0{,}25\sin 2x - 0{,}5x\cos 2x + C$; b) $\frac{x^2+1}{2} arctgx - \frac{x}{2} + C$;

c) $x\arccos x - \sqrt{1-x^2} + C$; d) $C - (x^2+2x+2)e^{-x}$;

e) $C - \frac{x}{2(1+x^2)} + 0{,}5 arctgx$; f) $\frac{1}{3}(x^2+1)\ln(1+x) - \frac{x^3}{9} + \frac{x^2}{6} - \frac{x}{3} + C$.

1. $\int x^n \ln x dx \quad (n \neq -1)$

2. $\int \arcsin x dx$

3. $\int x^3 e^{-x^2} dx$

4. $\int \frac{\arcsin x}{x^2} dx$

5. $\int arctgx dx$

6. $\int \ln\left(x + \sqrt{1+x^2}\right) dx$

7. $\int \sin x \ln(tgx) dx$

8. $\int \cos(\ln x) dx$

9. $\int e^{ax}\cos bx\,dx$

10. $\int e^{ax}\sin bx\,dx$

11. $\int \sqrt{x}\cdot \ln^2 x\,dx$

12. $\int e^{2x}\cdot \sin^2 x\,dx$

13. $\int x\cdot(\mathrm{arctg}x)^2\,dx$

14. $\int x\ln\frac{1+x}{1-x}\,dx$

15. $\int x^2 \mathrm{sh}x\,dx$

16. $\int \left(\frac{\ln x}{x}\right)^2 dx$

17. $\int \frac{x}{\cos^2 x}\,dx$

18. $\int \frac{\mathrm{arctg}(e^x)}{e^x}\,dx$

Қуйидаги I_n $(n\in N)$ интеграллар учун реккурент формулалар топилсин.

19. $I_n=\int x^n e^{ax}\,dx,\quad a\neq 0$

20. $I_n=\int \ln^n x\,dx$

21. $I_n=\int x^{\alpha}\ln^n x\,dx,\quad \alpha\neq -1$

22. $I_n=\int \frac{x^n dx}{\sqrt{x^2+a}},\quad n>2$

23. $I_n=\int \sin^n x\,dx,\quad n>2$

24. $I_n=\int \cos^n x\,dx,\quad n>2$

25. $I_n=\int \frac{dx}{\sin^n x},\quad n>2$

FOYDALANILGAN ADABIYOTLAR RO'YXATI:

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