

**MATRITSA VA ULAR USTIDA
AMALLAR. TESKARI MATRITSA**

Ta'rif. m ta yo'l va n ta ustundan iborat bo'lgan ushbu jadvalga

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \text{ yoki } \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{vmatrix} \quad (1)$$

$m \times n$ -o'lchamli *matritsa* deyiladi.

Masalan, $A = \begin{pmatrix} 2 & 1 & \pi \\ 1 & \sqrt{2} & -5 \end{pmatrix}$ - 2×3 -o'lchamli *matritsa*.

$B = \begin{pmatrix} e^t & 1 & -1 & \cos t \\ 0 & 4t & -7 & 1-t \end{pmatrix}$ - 2×4 -o'lchamli *matritsa*.

Matritsalarini A, B, C, \dots bosh harflar bilan belgilaymiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \text{yoki } A = \|a_{ij}\| = (a_{ij})$$

$$(i=1,2,\dots,m, \quad j=1,2,\dots,n)$$

a_{ij} -lar *matritsa elementlari* deyiladi.

Agar (1) matritsaning barcha elementlari nolga teng bo'lsa,

$$0 = \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

u *nol matritsa* deyiladi.

n – tartibli kvadrat matritsa berilgan bo'lsin:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} (2)$$

$$A_1 = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix} \text{yoki}$$

$$B = \begin{pmatrix} b_{11} & 0 & \cdots & 0 \\ b_{21} & b_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{pmatrix}.$$

A_1 – *yuqori uchburchakli*, B – *quyi uchburchakli* matritsadir.

$$\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix} - \textit{diagonal matritsa} \quad (3)$$

Xususiy holda, (3) matritsada

$$a_{11} = a_{22} = \dots = a_{nn} = 1$$

bo'lsa,

$$E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix} - \textit{birlik matritsa}$$

$$A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{pmatrix} - \textit{transponirlangan matritsa}$$

Masalan, agar $A = \begin{pmatrix} 3 & 4 \\ 5 & -2 \\ 2 & 0 \end{pmatrix}$ bo'lsa, u holda

$$A^T = \begin{pmatrix} 3 & 5 & 2 \\ 4 & -2 & 0 \end{pmatrix}$$

bo'ladi.

Matritsalar ustida amallar va ularning xossalari

Matritsalar ni qo'shish va ayirish.

$A = (a_{ij}) - m \times n$ – o'lchovli,

$B = (b_{ij}) - m \times n$ – o'lchovli.

$A + B = C - m \times n$ – o'lchovli,

$$c_{ij} = a_{ij} + b_{ij} \quad (4)$$

($i=1,2,\dots,n, j=1,2,\dots,m$)

1-misol. $A = \begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & 2 \end{pmatrix}$ va $B = \begin{pmatrix} -1 & 6 & 3 \\ 8 & 12 & 14 \end{pmatrix}$. $A + B$ yig'indini toping.

Yechish. $A + B = \begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & 2 \end{pmatrix} + \begin{pmatrix} -1 & 6 & 3 \\ 8 & 12 & 14 \end{pmatrix} =$

$$= \begin{pmatrix} 1 + (-1) & 2 + 6 & -3 + 3 \\ 4 + 8 & 0 + 12 & 2 + 14 \end{pmatrix} = \begin{pmatrix} 0 & 8 & 0 \\ 12 & 12 & 16 \end{pmatrix}$$

Matritsalar ni qo‘shish xossalari:

$$1^0. A + \mathbf{0} = \mathbf{0} + A = A.$$

$$2^0. A + B = B + A.$$

$$3^0. (A + B) + C = A + (B + C)$$

Matritsani songa ko‘paytirish.

$$\lambda A = \begin{pmatrix} \lambda a_{11} & \lambda a_{12} & \cdots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22} & \cdots & \lambda a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda a_{n1} & \lambda a_{n2} & \cdots & \lambda a_{nn} \end{pmatrix}$$

2-misol. $A = \begin{pmatrix} -3 \\ 4 \\ 2t \\ \sin(2t) \end{pmatrix}$ matritsani $\sqrt{2}$ ga ko'paytiring.

Yechish. $\sqrt{2} \cdot A = \sqrt{2} \cdot \begin{pmatrix} -3 \\ 4 \\ 2t \\ \sin(2t) \end{pmatrix} = \begin{pmatrix} -3\sqrt{2} \\ 4\sqrt{2} \\ 2t\sqrt{2} \\ \sqrt{2}\sin(2t) \end{pmatrix}.$

Matritsalar ni songa ko'paytirish xossalari:

$$4^0. (\lambda\mu)A = \lambda(\mu A), \quad (\lambda, \mu = \text{const}).$$

$$5^0. \lambda(A+B) = \lambda A + \lambda B.$$

$$6^0. (\lambda+\mu)A = \lambda A + \mu A.$$

4-misol. Agar

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \\ 5 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ 4 & 1 \\ 6 & 5 \end{pmatrix} \text{ bo'lsa, } 3A - 4B \text{ matritsalar ni toping.}$$

$$\begin{aligned} 3A - 4B &= 3 \cdot \begin{pmatrix} 2 & -1 \\ 1 & 3 \\ 5 & 3 \end{pmatrix} - 4 \cdot \begin{pmatrix} 2 & 0 \\ 4 & 1 \\ 6 & 5 \end{pmatrix} = \begin{pmatrix} 6 & -3 \\ 3 & 9 \\ 15 & 9 \end{pmatrix} - \begin{pmatrix} 8 & 0 \\ 16 & 4 \\ 24 & 20 \end{pmatrix} = \\ &= \begin{pmatrix} -2 & -3 \\ -13 & 5 \\ -9 & -11 \end{pmatrix} \end{aligned}$$

Matritsalarni ko'paytirish.

$A = (a_{ij})$ - $m \times n$ – o'lchovli, $B = (b_{ij})$ $n \times p$ – o'lchovli.

$A \cdot B = R = (r_{ij})$ - $m \times p$ – o'lchovli.

$$r_{ij} = \sum_{k=1}^n a_{ik} b_{kj} \quad (5)$$

($i=1,2,\dots,m; j=1,2,\dots,p$)

5-misol. Ushbu

$$A = \begin{pmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 & 4 \\ 0 & 3 & 1 \end{pmatrix}$$

Matritsalar ko'paytmasini toping.

$$R = A \cdot B = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

(5) Formulaga ko'ra

$$r_{11} = 1 \cdot 2 + 0 \cdot 0 = 2, r_{12} = 1 \cdot (-1) + 0 \cdot 3 = -1,$$

$$r_{13} = 1 \cdot 4 + 0 \cdot 1 = 4, \quad r_{21} = 2 \cdot 2 + (-1) \cdot 0 = 4,$$

$$r_{22} = 2 \cdot (-1) + (-1) \cdot 3 = -5, r_{23} = 2 \cdot 4 + (-1) \cdot 1 = 7,$$

$$r_{31} = 0 \cdot 2 + 3 \cdot 0 = 0, \quad r_{32} = 0 \cdot (-1) + 3 \cdot 3 = 9,$$

$$r_{33} = 0 \cdot 4 + 3 \cdot 1 = 3$$

bo'ladi. Demak,

$$R = A \cdot B = \begin{pmatrix} 2 & -1 & 4 \\ 4 & -5 & 7 \\ 0 & 9 & 3 \end{pmatrix}.$$

A - $n \times n$ - o'lchovli, E - $n \times n$ - o'lchovli birlik matritsa bo'lsa,

$$\mathbf{A \cdot E = E \cdot A = A}$$

$$\mathbf{A \cdot B \neq B \cdot A}$$

7-misol. Ushbu

$$A = \begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 6 \\ 1 & 3 \end{pmatrix}$$

Matritsalar uchun $A \cdot B \neq B \cdot A$ munosabatni tekshirib ko'ring.

Yechish. $A \cdot B = \begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix} \cdot \begin{pmatrix} -2 & 6 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 6 \\ -8 & 0 \end{pmatrix},$

$$B \cdot A = \begin{pmatrix} -2 & 6 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix} = \begin{pmatrix} 10 & -24 \\ 6 & -12 \end{pmatrix}.$$

A , B va C matritsalar berilgan bo'lsin. U holda

$$7^0 . (A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$8^0 . (A+B) \cdot C = A \cdot C + B \cdot C, \quad A \cdot (B+C) = A \cdot B + A \cdot C$$

Teskari matritsa

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

$$A \cdot B = B \cdot A = E, \quad A^{-1} = B$$

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T = \frac{1}{\det A} \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}.$$

16-misol. $A = \begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & 3 \\ 1 & -3 & 1 \end{pmatrix}$

A matritsaga teskari A^{-1} matritsani toping.

Yechish. $\det A = 2 \cdot 3 + 0 \cdot 2 + 18 - 0 = 15$. Demak, $\det A \neq 0 \implies A^{-1}$ – mavjud.

$$A^{-1} = \frac{1}{15} \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix},$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 3 \\ -3 & 1 \end{vmatrix} = 10, \quad A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = 3,$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 1 \\ 1 & -3 \end{vmatrix} = -1,$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ -3 & 1 \end{vmatrix} = -5,$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = 0, \quad A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} = 5,$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 1 & 3 \end{vmatrix} = -5, \quad A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 2 \\ 0 & 3 \end{vmatrix} = -6,$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -1 \\ 2 & -1 \end{vmatrix} = 2$$

$$A^{-1} = \frac{1}{15} \begin{pmatrix} 10 & -5 & -5 \\ 3 & 0 & -6 \\ -1 & 5 & 2 \end{pmatrix}$$

Tekshirib ko‘ramiz:

$$\begin{aligned} A^{-1} \cdot A &= \frac{1}{15} \begin{pmatrix} 10 & -5 & -5 \\ 3 & 0 & -6 \\ -1 & 5 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & -1 & 2 \\ 0 & 1 & 3 \\ 1 & -3 & 1 \end{pmatrix} = \\ &= \frac{1}{15} \begin{pmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E. \end{aligned}$$

Matritsalarining amaliy masalalarga tatbiqi

1-masala. “Ravot” va “Qahramon” fermer xo‘jaliklarida yetishtirilgan poliz mahsulotlari shahardagi N_1 , N_2 va N_3 supermarketlarga har kuni yetkasilib turiladi. Bu fermer xo‘jaliklaridan kundalik poliz mahsulotlarining bir tonnasini N_1 - supermarketga yetkasib berish - 20 ming, N_2 - supermarketga yetkasib berish - 30 ming va N_3 - supermarketga yetkasib berish esa - 50 ming pul birligiga to‘g‘ri keladi. Har bir fermer xo‘jaligining kundalik transport xarajatlarini hisoblang.

Fermer xo‘jaliklari	Supermarketlarga kundalik yetkasilib berilgan poliz mahsulotlari (tonna hisobida)		
	N_1	N_2	N_3
“Ravot”	2	3	1
“Qahramon”	3	1	4

Yechish. A – matritsa bilan har kuni fermer xo‘jaliklaridan supermarketlarga yetkasib berilgan polis mahsulotlari (tonna hisobida), B – matritsa esa fermer xo‘jaligidan bir tonna mahsulotni supermarketga etkasib berish uchun sarflanadigan transport xarajatlari (narxlari) bo‘lsin:

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 1 & 4 \end{pmatrix}, \quad B = (20 \quad 30 \quad 50).$$

$$\begin{aligned} C = A \cdot B^T &= \begin{pmatrix} 2 & 3 & 1 \\ 3 & 1 & 4 \end{pmatrix} \cdot \begin{pmatrix} 20 \\ 30 \\ 50 \end{pmatrix} = \\ &= \begin{pmatrix} 2 \cdot 20 + 3 \cdot 30 + 1 \cdot 50 \\ 3 \cdot 20 + 1 \cdot 30 + 4 \cdot 50 \end{pmatrix} = \begin{pmatrix} 180 \\ 290 \end{pmatrix}. \end{aligned}$$

2-masala. Fermer xo‘jaligida 10 tonna kartoshka, 3 tonna piyoz va 6 tonna pomidor yetishtirish rejalashtirilgan. $X = (10 \ 3 \ 6)$ – fermer xo‘jaligining rejasi; $S = (1 \ 1 \ 3)$ - resurslar narxi (har bir tonna uchun); $P = (0 \ 3 \ 7)$ – transport xarajati (har bir tonna uchun).

Qishloq xo‘jalik mahsulotlari	1 tonna mahsulotni yetishtirish uchun sarflanadigan resurslar miqdori		
	T_1 Suv (ming, litr)	T_2 Mahaliy o‘g‘itlar(tonna)	T_3 Mineral o‘g‘itlar(tonna)
Kartoshka	2	2	1
Piyoz	3	1	3
Pomidor	4	3	2

- 1) Fermer xo‘jaligi bo‘yicha rejadagi qishloq xo‘jalik mahsulotlarini yetishtirish uchun sarflangan har bir resurslarning miqdorini aniqlang?
- 2) Mahsulotlar turlari bo‘yicha bir tonna qishloq xo‘jalik mahsulotini yetishtirish uchun sarflangan resurs xarajatlarini aniqlang?
- 3) Rejani bajarish uchun sarflangan jami resurs xarajatlari miqdorini aniqlang?
- 4) Fermer xo‘jaligi bo‘yicha resurs va transport xarajatlari umumiy yig‘indisini toping?

Yechish. 1) 1 tonna mahsulotni yetishtirish uchun sarflanadigan resurslar miqdorini $A = \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & 3 \\ 4 & 3 & 2 \end{pmatrix}$ - matritsa bilan ifodalaylik.

$$\begin{aligned} T &= X \cdot A = (10 \ 3 \ 6) \cdot \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & 3 \\ 4 & 3 & 2 \end{pmatrix} = \\ &= (10 \cdot 2 + 3 \cdot 3 + 6 \cdot 4 \quad 10 \cdot 2 + 3 \cdot 1 + 6 \cdot 3 \\ &\quad 10 \cdot 1 + 3 \cdot 3 + 6 \cdot 2) = (53 \ 41 \ 31). \end{aligned}$$

T_1 – 53 ming litr suv; T_2 – 41 tonna; T_3 – 31 tonna.

2) Bir tonna qishloq xo‘jalik mahsulotini yetishtirish maqsadida foydalanilgan resurslar uchun ketgan sarf-xarajatlarni hisoblaymiz:

$$A \cdot S^T = \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & 3 \\ 4 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 + 2 \cdot 1 + 1 \cdot 3 \\ 3 \cdot 1 + 1 \cdot 1 + 3 \cdot 3 \\ 4 \cdot 1 + 3 \cdot 1 + 2 \cdot 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 13 \\ 13 \end{pmatrix}.$$

3) Fermer xo‘jaligining uch turdagi mahsulotlarni yetishtirish uchun resurslarga sarflagan harajatini aniqlaymiz:

$$X \cdot (A \cdot S^T) = (10 \quad 3 \quad 6) \cdot \begin{pmatrix} 7 \\ 13 \\ 13 \end{pmatrix} = 187.$$

4) Resurslarni tashish uchun ketgan transport xarajatini hisoblaymiz:

$$T \cdot P = (53 \quad 41 \quad 31) \cdot \begin{pmatrix} 0 \\ 3 \\ 7 \end{pmatrix} = 0 + 123 + 217 = 340.$$

Demak, fermer xo'jaligining resurslar va transport xarajatlari umumiy yig'indisini quyidagiga teng:

$$X \cdot (A \cdot S^T) + T \cdot P = 187 + 340 = 527.$$

Mustaqil ishlash uchun misollar

Matritsali tenglamalarni yeching:

$$1) \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix};$$

$$2) X \cdot \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix};$$

$$3) \begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix} \cdot X \cdot \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix}.$$

ETIBORINGIZ UCHUN RAHMAT!