МАТЕМАТИКА

Тошкент-2016

2-MA'RUZA

DETERMINANTLAR VA ULARNING XOSSALARI

REJA

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2-TARTIBLI DETERMINANTLARNI HISOBLASH

Ta'rif. 2 - tartibli kvadrat matritsaning

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

determinanati deb, ushbu

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = . \; a_{11}a_{22} - a_{12}a_{21}$$

songa aytiladi. Bu yerda a_{11}, a_{12}, a_{21}, a_{22} -

1-misol.

2-tartibli determinantni hisoblang.

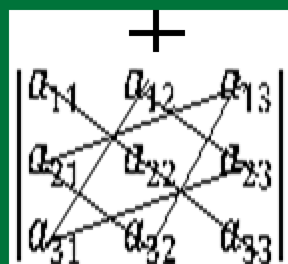
$$\begin{vmatrix} 2 & -4 \\ 3 & 5 \end{vmatrix} =$$

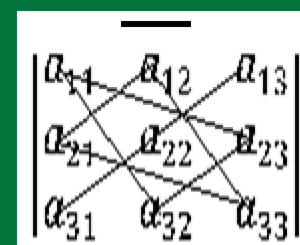
$$=2 \cdot 5 - (-4) \cdot 3 = 10 + 12 = 22.$$

$$detA = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

songa 3-tartibli determinant, $detA = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}\, a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - \ a_{12}\, a_{21}a_{33}$ unung qiymati deyiladi.

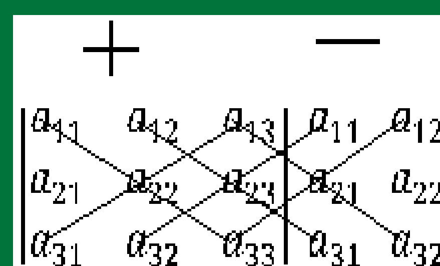
- Uchburchak qoidasi





Yoki

- Sarryus qoidasi



n - tartibli kvadrat matritsaning

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

determinanati deb, ushbu

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \qquad (3)$$

ifodaga aytiladi.

MINOR VA ALGEBRAIK TO'LDIRUVCHI

O‘zirilmay qolgan elementlardan ikkiinchi tartibli determinant hosil bo‘ladi. Unga a_{ik} elementning ***minori*** deyiladi va M_{ik} bilan belgilanadi. Masalan, uchinchi tartibli determinantda a_{23} element turgan yo‘l va ustunni o‘chirish

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

natijasida

$$M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

determinant hosil bo‘ladi. Bu berilgan determinant a_{23} elementining minoridir.

1-мisol. $\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix}$ determinantning minorlarini toping.

Ta'rif. (3) determinant a_{ij} elementining ***algebraik to'ldiruvchisi*** deb,

$$(-1)^{i+j} M_{ij}$$

miqdorga aytiladi va A_{ij} orqali belgilanadi:

$$A_{ij} = (-1)^{i+j} M_{ij}.$$

Kvadrat matritsa determinantining algebraik to'ldiruvchilari soni uning elementlari soniga teng bo'ladi.

Masalan,

$$\begin{vmatrix} 1 & 0 & -2 \\ 3 & 4 & 1 \\ 0 & -3 & 5 \end{vmatrix}$$

determinant elementlariga mos to'qqizta minorlar mavjud. $a_{32} = -3$ elementining algebraik to'ldiruvchisini topamiz:

$$A_{32} = (-1)^{3+2} M_{32} = (-1)^5 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = -\big(1 - (-6)\big) = -7.$$

DETERMINANTNI HISOBLASHGA DOIR MISOLLAR

2-misol. 3-tartibli determinantni hisoblang.

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & 3 & -1 \\ 4 & 5 & 2 \end{vmatrix} = 1 \cdot 3 \cdot 2 + (-2) \cdot (-1) \cdot 4 + 1 \cdot 2 \cdot 5 -$$

$$-1 \cdot 3 \cdot 4 - 1 \cdot (-1) \cdot 5 - (-2) \cdot 2 \cdot 2 =$$

$$= 6 + 8 + 10 - 12 + 5 + 8 = 25.$$

3-TARTIBLI DETERMINANTNI UCHBURCHAK USULIDA HISOBLASH

$$\Delta = \begin{pmatrix} a_{11} & - & - \\ - & a_{22} & - \\ - & - & a_{33} \end{pmatrix} + \begin{pmatrix} - & a_{12} & - \\ - & - & a_{23} \\ a_{31} & - & - \end{pmatrix} + \begin{pmatrix} - & - & a_{13} \\ a_{21} & - & - \\ - & a_{32} & - \end{pmatrix} -$$

$$- \begin{pmatrix} - & - & a_{13} \\ - & a_{22} & - \\ a_{31} & - & - \end{pmatrix} - \begin{pmatrix} - & a_{12} & - \\ a_{21} & - & - \\ - & - & a_{33} \end{pmatrix} - \begin{pmatrix} a_{11} & - & - \\ - & - & a_{23} \\ - & a_{32} & - \end{pmatrix}$$

UCHBURCHAK USULIGA MISOLLAR

$$\Delta = \begin{vmatrix} 2 & 3 & 4 \\ 5 & 1 & -3 \\ -3 & -2 & 1 \end{vmatrix} =$$

$$= 2 \cdot 1 \cdot 1 + 3 \cdot (-3) \cdot (-3) + 5 \cdot (-2) \cdot 4 -$$

$$- 4 \cdot 1 \cdot (-3) - 5 \cdot 3 \cdot 1 - (-2) \cdot (-3) \cdot 2 =$$

$$= 2 + 27 - 40 + 12 - 15 - 12 = -26$$

UCHBURCHAKLI DETERMINANT QIYMATI BOSH DIOGANOLDAGI ELEMENTLAR KO'PAYTMASIGA TENG

$$\Delta = \begin{vmatrix} 3 & -1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 3 \cdot 2 \cdot 1 = 6$$

Teskari matritsa

n - tartibli kvadrat matritsa $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$

berilgan bo‘lsin.

Agar A bilan n-tartibli A^{-1} - kvadrat matritsa ko‘paytmasi E - birlik matritsaga teng bo‘lsa

$$A \cdot A^{-1} = A^{-1} \cdot A = E$$

u holda A^{-1} matritsa A ga ***teskari matritsa*** deyiladi.

Teorema. A matritsaga A^{-1} teskari matritsa mavjud bo‘lishi uchun uning xosmas matritsa bo‘lishi zarur va etarlidir.

Teskari matritsani topish

$$A^{-1} = \frac{1}{detA}\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T = \frac{1}{detA}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} =$$

$$\begin{pmatrix} \frac{A_{11}}{detA} & \frac{A_{21}}{detA} & \frac{A_{31}}{detA} \\ \frac{A_{12}}{detA} & \frac{A_{22}}{detA} & \frac{A_{32}}{detA} \\ \frac{A_{13}}{detA} & \frac{A_{23}}{detA} & \frac{A_{33}}{detA} \end{pmatrix}.$$

Matritsaning rangi

Biror *mxn* - o‘lchamli *A matritsa*

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

berilgan bo‘lsin. *A* matritsaning ixtiyoriy *k* ta yo‘lini va *k* ta ustunini olib, ($k \leq \min(m,n)$) *k*-tartibli kvadrat matritsa tuzamiz. Bu kvadrat matritsaning determinanti *A* matritsaning ***k-tartibli minori*** deyiladi.

Ta’rif. *A* matritsaning noldan farqli bo‘lgan eng yuqori (katta) tartibli minoriga uning ***rangi*** deyiladi va rank *A* bilan belgilanadi.

Matritsaning rangi uning yo‘llari va ustunlari sonidan katta bo‘lmaydi, ya’ni rang $A \le \min(m,n)$.

ta’rifdan quyidagilar kelib chiqadi:

1) agar rang $A=k$ bo‘lsa, u holda *A* matritsa minorlari orasida noldan farqli k-tartibli kamida bitta minori mavjud bo‘ladi;

2) $(k+1)$ va undan yuqori tartibli minorlari (agar ular mavjud bo‘lsa) nolga teng.

MATRITSANING RANGINI TOPISH

1-misol. Ushbu $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ -4 & 1 & 1 \end{pmatrix}$

matritsaning rangini toping.

Berilgan matritsaning 2-tartibli minorlari bir nechta bo‘lib, ulardan biri $\begin{vmatrix} 3 & 1 \\ -4 & 1 \end{vmatrix} = 7$ bo‘ladi. A matritsaning 3-tartibli minori

$$\begin{vmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ -4 & 1 & 1 \end{vmatrix}$$

determinantlarning xossalariga ko‘ra nolga teng. rang A=2.

Foydalanilgan adabiyotlar

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