2. EQUATIONS OF LINES AND PLANES

I Main Topics

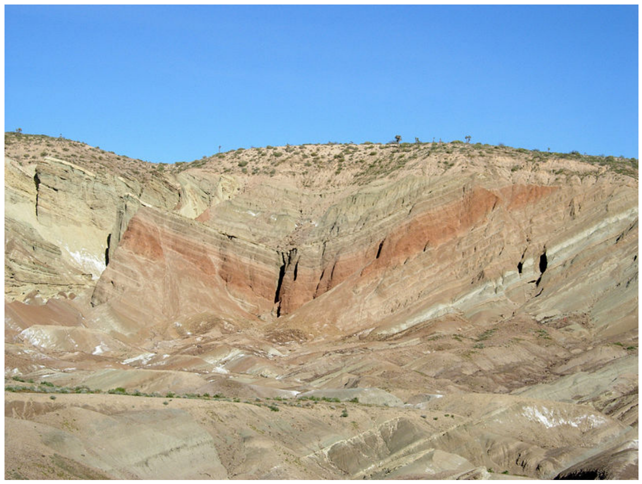
A Direction cosines

B Lines

C Planes

Lineations Along a Probable Fault Matterhorn Peak, California

Small Fold
Rainbow Basin, California



http://en.wikipedia.org/wiki/File:Rainbow_Basin.JPG

Deformation Bands
East Rim of Buckskin Gulch, Utah

Sheeting Joints
Yosemite National Park, California

Fractures
Austrian Alps

2. EQUATIONS OF LINES AND PLANES

II Direction cosines

A The cosines of the angles between a line and the coordinate axes

B The coordinates of the endpoint of a vector of unit length

C The ordered projection lengths of a line of unit length onto the x,y, and z axes

Direction Cosines from Geologic Angle Measurements (Spherical coordinates; z up)

Positive z-axis up
y = north; x = east

xy plane is horizontal plane

RIGHT-HANDED COORDINATES

Remember: Trends are azimuths and are measured in a horizontal plane. Plunges are inclinations and are measured in a vertical plane.

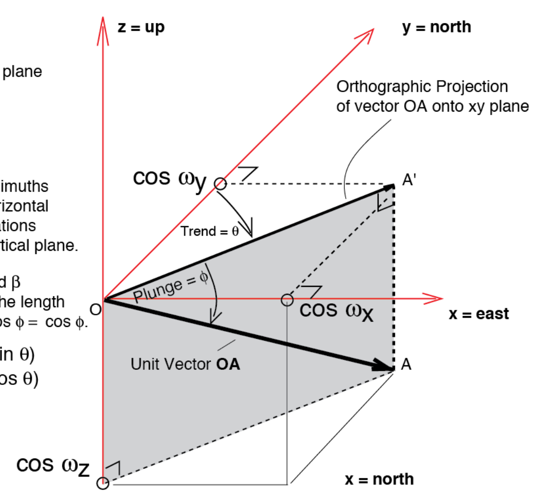
The direction cosines α and β are determined from OA', the length of OA' being $|OA'| = |OA| \cos\phi = \cos\phi$.

$\alpha = \cos\omega_x = (\cos\phi)(\sin\theta)$

$\beta = \cos\omega_y = (\cos\phi)(\cos\theta)$

$\gamma = \cos\omega_z = -(\sin\phi)$

$\alpha^2 + \beta^2 + \gamma^2 = 1$



Direction Cosines from Geologic Angle Measurements (Spherical coordinates; z down)

Positive z-axis down
x = north; y = east

xy plane is horizontal plane

RIGHT-HANDED COORDINATES

Remember: Trends are azimuths and are measured in a horizontal plane. Plunges are inclinations and are measured in a vertical plane.

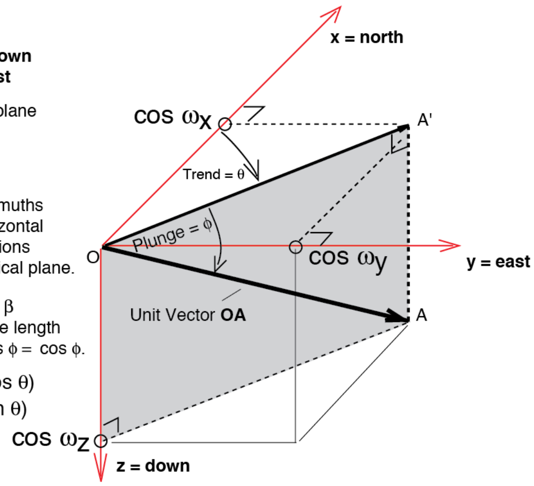
The direction cosines α and β are determined from OA', the length of OA' being |OA'| =|OA| cos ϕ = cos ϕ.

$\alpha = \cos \omega_x = (\cos \phi)(\cos \theta)$

$\beta = \cos \omega_y = (\cos \phi)(\sin \theta)$

$\gamma = \cos \omega_z = +(\sin \phi)$

$\alpha^2 + \beta^2 + \gamma^2 = 1$



2. EQUATIONS OF LINES AND PLANES MATLAB

Cartesian coordinates ➔ Spherical coordinates

```
>> [TH,PHI,R] = cart2sph(1,0,0)

TH =

   0

PHI =

   0

R =

   1
```

The θ and ϕ values are angles. R is a length.

Spherical coordinates ➔ Cartesian coordinates

```
>> [X,Y,Z] = sph2cart(0,0,1)

X =

   1

Y =

   0

Z =

   0
```

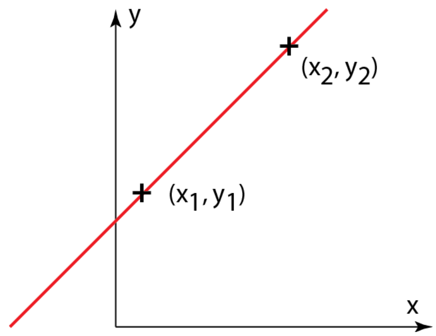
The x,y,z values here are direction cosines

2. EQUATIONS OF LINES AND PLANES

A Line defined by 2 points (two-point form)

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

where (x_1,y_1) and (x_2,y_2) are two known points on the line



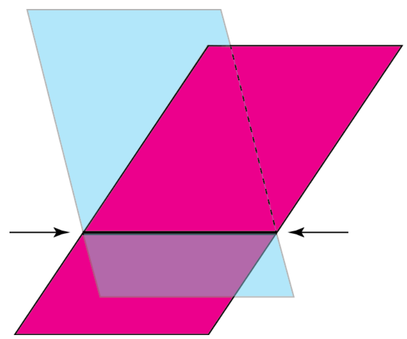
2. EQUATIONS OF LINES AND PLANES

B Line defined by 1 point (e.g., x_0,y_0,z_0) and a direction

1 <u>Slope-intercept form (2D)</u>
   $y = mx + b$
2 <u>General form (2D)</u>
   $Ax + By + C = 0$
3 <u>Parametric form (2D or 3D)</u>
   $x = x_0 + t\alpha$, $y = y_0 + t\beta$, $z = z_0 + t\gamma$,
   where α, β, and γ are direction cosines:
   $\alpha = \cos \omega_x$, $\beta = \cos \omega_y$, and (for 3D) $\gamma = \cos \omega_z$;
   In 2-D, $\cos \omega_x = \sin \omega_y$

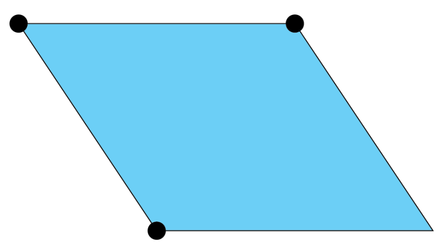
2. EQUATIONS OF LINES AND PLANES

C Line defined by the intersection of two planes



2. EQUATIONS OF LINES AND PLANES

IV Plane
- A Defined by three points (three non-colinear pts)
- B Defined by two intersecting lines
- C Defined by two parallel lines
- D Defined by a line and a point not on the line



2. EQUATIONS OF LINES AND PLANES

IV Plane

D Defined by a distance and a direction (or pole) from a point not on the plane

1 General form: Ax + By + Cz + D = 0

2 Normal form: αx + βy + γz = d

$$\alpha = \frac{A}{\pm\sqrt{A^2+B^2+C^2}} \quad \beta = \frac{B}{\pm\sqrt{A^2+B^2+C^2}} \quad \gamma = \frac{C}{\pm\sqrt{A^2+B^2+C^2}}$$

$$d = \frac{-D}{\pm\sqrt{A^2+B^2+C^2}}$$

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2. EQUATIONS OF LINES AND PLANES

3 **n• V** = d,

where

V is any vector from a given point O to plane P;

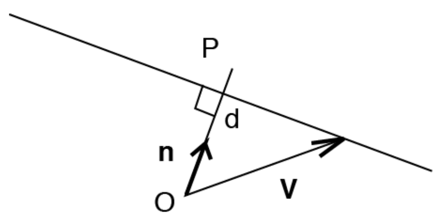
n is the unit normal vector to plane P with direction cosines α, β, and γ;

n also goes through point O;

d is the distance from O to plane along **n**;

• refers to the dot product:

$\langle x_1,y_1,z_1\rangle \bullet \langle x_2,y_2,z_2\rangle = x_1x_2 + y_1y_2 + z_1z_2$



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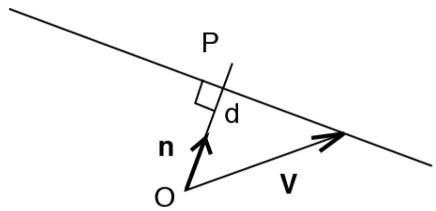
2. EQUATIONS OF LINES AND PLANES

3 **n• V** = d

a The distance from a reference point to a plane (as measured along a direction perpendicular to the plane) is d.

b The projection of **V** onto **n** has a length d



If **n** points from the reference point to the plane, then d>0. Otherwise, d<0.