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ГИДРОТЕХНИКА ИНШООТЛАРИ ВА НАСОС СТАНЦИЯЛАР

ҚИШЛОҚ ХЎЖАЛИГИНИ МЕХАНИЗАЦИЯЛАШ

ҚИШЛОҚ ХЎЖАЛИГИНИ ЭЛЕКТРЛАШТИРИШ ВА АВТОМАТЛАШТИРИШ

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AEROLASTIC VIBRATIONS AND STABILITY OF VISCOELASTIC PLATES TAKING INTO ACCOUNT THE SWEEP

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Abstract

It is shown in the article that when building mathematical models of the dynamics problem of the heredity theory of viscoelasticity, the Koltunov-Rzhanitsyn singular kernel of heredity adequately describes real mechanical processes and best approximates experimental data over a long period. A mathematical model of the problem of viscoelastic plate flutter is presented, taking into account the sweep angle moving in a gas at a high supersonic speed. Using the Bubnov-Galerkin method, discrete models of the flutter problem for viscoelastic plates in a supersonic gas flow are obtained. A numerical method for solving nonlinear integro-differential equations of the problem of the heredity theory of viscoelasticity with weakly singular kernels is developed. A general computational algorithm and a set of applied programs were developed that make it possible to study nonlinear dynamic problems of the heredity theory of viscoelasticity with weakly singular kernels. On the basis of numerical methods and algorithm proposed, nonlinear flutter problems for viscoelastic plates flowing about in a gas flow at an arbitrary angle are investigated. Flutter critical velocities are determined in a wide range of changes in various plate parameters. It is shown that the singularity parameter α affects not only oscillations of viscoelastic systems, but also the critical flutter velocity.

Key words: mathematical model, viscoelasticity, integro-differential equations, algorithm, flutter, plates, sweep angle

Introduction. Mathematical and computer modeling of the flutter of viscoelastic elements and structural units of an aircraft is an urgent scientific problem, the study of which is stimulated by the failure of aircraft structures, parts of space and jet engines.

Due to the complexity of the flutter phenomenon of aircraft elements, simplifying assumptions were used in numerous studies. However, these assumptions, as a rule, turn out to be so restrictive that the mathematical model ceases to reflect real conditions with sufficient accuracy. Therefore, many results of theoretical and experimental studies are still in poor agreement.

One of the characteristic features of the development of heredity theory is the wide possibilities for describing the dynamic processes of deformation of various materials. However, due to the lack of an adequate mathematical apparatus, the implementation of these possibilities in many cases is difficult, especially in the study of nonlinear dynamic processes. In recent years, the possibilities of computer technology have increased interest in nonlinear problems. Under these conditions, it is important to create and develop effective solution methods that could be applied to the widest possible class of problems.

The first mathematical models based on integral models for the study of aerodynamic problems were used by V.I. Matyash (1971) and G.S. Larionov (1974). In these works, to solve systems of Volterra integro-differential equations, the averaging method proposed by A.N. Filatov was used. In mentioned studies, an exponential kernel was used as the relaxation kernel.

A.A. Ilyushin, I.A. Kiyko (1994), V.D. Potapov (2011), S.D. Algazin (2021) devoted their studies to linear problems of aerodynamics, considering the heredity properties of the materials. In these studies, the method of Laplace integral transformations was used to solve systems of Volterra integro-differential equations. Note that this method is suitable for linear problems of aerodynamics.

The studies by G.S. Larionov (1974) are devoted to the nonlinear problems of aerodynamics, considering the heredity properties of the material. In his works, problems were considered in a geometrically nonlinear formulation based on the Kirchhoff-Love hypothesis. When solving these problems, the Bubnov-Galerkin variational method, based on the two-term approximation of deflections, was used in combination with the asymptotic averaging method. Calculations show that these methods do not give the expected result when solving aerodynamic problems.

Nonlinear problems of the dynamics of heredity theory under aerodynamic loading based on the Kirchhoff-Love hypothesis were studied by Badalov (1987), Badalov et al. (2007a, 2007b, 1987). The Kirchhoff-Love model used, makes it possible to obtain sufficiently accurate solutions to a number of practical problems, though, in most cases, they are incomplete. This primarily refers to thin-walled structures made of composite materials with heredity and non-homogeneous properties.

The noted properties of structural materials and the above factors increase the complexity of research and lead to the need to develop computational methods for studying the stability of viscoelastic elements of thin-walled structures. Therefore, the development of efficient computational algorithms for solving nonlinear integro-differential equations of dynamic problems of viscoelastic elements of thin-walled structures with weakly singular heredity kernels is relevant.

The issue of considering viscoelastic properties under dynamic deformation of plates and shells is currently one of the topical issues in the mechanics of deformable bodies. Its solution is an effective application of the theory of viscoelasticity to real processes. Therefore, the methods and problems of the theory of hereditary elasticity attract much attention from researchers. There are a number of publications devoted to solving problems of calculating the characteristics of viscoelastic thin-walled structures (Nguyen et al. (2014), Pouresmaeeli et al. (2013), Shokrollahi Saeed Shafaghat Salman (2017), Chung-Li Liao, Yee-Win Sun (1993), Kouchakzadeh et al (2010), Zhi-Guang Song, Feng-Ming Li (2012), Zhi-Guang Song et al (2018), Hai Zhao Dengqing Cao (2013), Pacheco et al (2017), M.K. Singha Mukul Mandal (2008)). At present, general theoretical foundations and methods for solving problems of determining the stress-strain state and analyzing the dynamic properties of loadbearing structures are developed, that take into account the characteristics of the rheological behavior of the materials.

Xiaochen Wang et al. (2017) investigated the nonlinear flutter of viscoelastic panels in a supersonic gas flow. To build a mathematical model of viscoelastic panels, the Karman theory was used. Aerodynamic pressure was determined according to piston theory. To describe the viscoelastic properties of materials, the model of a standard viscoelastic body (the Kelvin theory) was used.

L.Librescu et al. (1989) on the basis of Boltzmann's theory considered the dynamic stability of viscoelastic isotropic plates. Transverse shear deformation and rotational inertia were taken into account. To solve linear dynamic problems of viscoelasticity, the method of integral transformations was applied.

Mouafo Teifouet Armand et al. (2016) investigated viscoelastic rectangular plates under various boundary conditions. The Kelvin-Voigt theories were used to describe the deformation processes in viscoelastic materials. The numerical results obtained were compared with known results.

Sandwich (three-layered) shells have long been used in various fields of industry, aviation, and shipbuilding. S. Mahmoudkhani et al. (2016) investigated the flutter problem for viscoelastic sandwich cylindrical shells in a supersonic gas flow. A numerical study of the influence of geometric parameters, the parameter of viscoelastic damping, and temperature on the flutter boundaries of shells was conducted.

The flutter of plates and shallow shells, considering elastic and viscoelastic foundations, was considered by a number of authors (Bolotin (1961), Nguyen et al. (2014), Pouresmaeeli et al. (2013), Li et al. (2018)). Pouresmaeeli et al. (2013) investigated the natural frequency of orthotropic viscoelastic nanoplates lying on an elastic foundation employing the nonlocal classical plate theory. Bolotin (1961) considered an infinite plate lying on an elastic base and streamlined by a gas flow. Despite numerous publications, relatively few studies refer to the issue of nonlinear flutter of viscoelastic plates and panels.

One of the main difficulties in total understanding the phenomenon of supersonic panel flutter is that the critical velocity of the panel flutter depends on a large number of parameters. At present, the difficulty of isolating many of these factors in an experimental study does not make it possible to obtain a satisfactory agreement between the experimental and theoretical results. There are reviews of the investigated problems in the scientific literature; an extensive bibliography is given by Fung (1960), Eisley, Luessen (1963), Dowell and Ventres (1970). The development of problems on the flutter of a plate, taking into account the angle of flow, is reflected in the studies by Deman Tang and Dowell (2016), Andreea Koreanschi et al. (2016), Jiali Xiea et al. (2013), Kemal Yaman (2016), Shokrollahi Saeed Shafaghat Salman (2017), Yang et al. (2012), Sina Mirzaei Sefat et al. (2012, 2013), Bichiou et al. (2016), Attar et al. (2003) and others. It turns out that the sensitivity of the flutter velocity to such factors as the angle of flow is still incomplete.

Mathematical model

Let us consider a viscoelastic plate with sides a, b and thickness h, flown over in a supersonic gas flow. The edges of the plate are not oriented in the flow direction, so we take into account the sweep (Fig. 1). Despite the obvious importance of the problem under discussion, there is only one publication devoted to the study of the behavior of the plate considering the sweep (Eisley and Luessen (1963)).

Let us expand the velocity vector of the oncoming flow into two velocity components: perpendicular to the leading edge Vcosφ and along the leading edge Vsinφ. Then the flow around the swept-back plate (Fig. 1, a) is equivalent to the flow around the straight plate (Fig. 1, b) by a flow perpendicular to the leading edge at velocity Vcosφ and by a flow along the span of the plate at velocity Vsinφ.

When determining the aerodynamic forces acting on the plates, it can be assumed that only the normal component of velocity Vcosφ affects the pressure distribution.

In the flow of an inviscid medium, the velocity component Vsinφ does not affect the pattern of pressure distribution. Therefore, the flow around the plate (Fig. 1a) is equivalent to the flow around a straight plate (Fig. 1b).

Since a flow around a plate with sweep angle φ is equivalent to a flow around a flat plate at velocity V, we can use the following equations

Since a flow around a plate with sweep angle φ is equivalent to a flow around a flat plate at velocity *V*, we can use the following equations

$$\frac{D}{h}\left(1-R^*\right)\left[\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]-\frac{N_x}{h}\frac{\partial^2 w}{\partial x^2}-\frac{N_y}{h}\frac{\partial^2 w}{\partial y^2}=$$
$$=-\rho\frac{\partial^2 w}{\partial t^2}-\frac{B}{h}\frac{\partial w}{\partial t}+\frac{BV}{h}\frac{\partial w}{\partial x}-\frac{B_1V^2}{h}\left[\frac{\partial w}{\partial x}\right]^2 \quad (1)$$

for cases of absence of normal forces (N_x=N_y=0), obtained in the theory of a plate in a supersonic gas flow, by replacing *V* with *Vcos*φ (Dowell and Voss, (1965); Eisley and Luessen (1963), Dowell (1970)) :

$$\frac{D}{h}\left(1-R^*\right)\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^2 w\partial^2 x}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)= \quad (2)$$

$$=-\rho\frac{\partial^2 w}{\partial t^2}-\frac{B}{h}\frac{\partial w}{\partial t}+\frac{B}{h}V\cos\varphi\frac{\partial w}{\partial x}-\frac{B_1}{h}(V\cos\varphi)^2\left(\frac{\partial w}{\partial x}\right)^2.$$

Where $D=\frac{Eh^3}{12(1-\mu^2)}$; *h* is the plate thickness; *E* is the modulus of elasticity; μ is Poisson's ratio; ρ is the density of the material; *w* is the deflection of the plate; *V* is the flow rate; *R** is the integral operator with a weakly singular Abel type relaxation kernel *R(t)*:

$$R^*\varphi(t)=\int_0^t R(t-\tau)\varphi(\tau)d\tau,$$

$$\left(R(t)=A\cdot\exp(-\beta t)\cdot t^{\alpha-1},\ A>0,\beta>0,0<\alpha<1\right);$$

$B=\frac{\aleph p_\infty}{V_\infty}$, $B_1=\frac{\aleph(\aleph+1)p_\infty}{4V_\infty^2}$, $\aleph$ is the polytropic exponent for gas; p_∞, V_∞ – are the pressure and speed of sound in the undisturbed gas flow, respectively.

Discrete model

The solution of equation (2) is sought using the Bubnov-Galerkin method. Let $\{\varphi_{nm}(x,y)\}$ be a complete sequence of coordinate functions satisfying the boundary conditions. Introducing the following series into (2)

$$w(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}w_{nm}(t)\varphi_{nm}(x,y) \quad (3)$$

and performing the well-known procedure of the Bubnov-Galerkin method, we obtain systems of integro-differential equations. By introducing the following dimensionless quantities into integro-differential equations (IDE)

$$\frac{w}{h},\ \frac{V_\infty}{a}t,\ \frac{x}{a},\ \frac{y}{b}$$

and keeping the same notation, we obtain:

$$\sum_{n=1}^{N}\sum_{m=1}^{M}\left\{L_{k\ln m}\left(\ddot{w}_{nm}+a_1\dot{w}_{nm}\right)+\Omega^2F_{k\ln m}\left(1-R^*\right)w_{nm}\right.$$

$$\left.-\lambda_1\cos\varphi\sum_{n=1}^{N}\sum_{m=1}^{M}G_{k\ln m}w_{nm}\right\}+\lambda_2(\cos\varphi)^2\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}Q_{k\ln mir}w_{nm}w_{ir}=0, \quad (4)$$

$k=\overline{1,N}$; $l=\overline{1,M}$;

where

$L_{klnm}=\int_0^1\int_0^1\phi_{nm}\phi_{kl}dxdy;$ $F_{k\ln m}=\int_0^1\int_0^1\left(\phi^{IV}_{nm,xxxx}+2\lambda^2\phi^{IV}_{nm,xxyy}+\lambda^4\phi^{IV}_{nm,yyyy}\right)\phi_{kl}dxdy;$

$a_1=p_\infty\aleph a^4/(V_\infty Dt_1)$; $t_1=\sqrt{\rho ha^4/D}$;

$\Omega^2=\dfrac{\pi^4}{12(1-\mu^2)}M_E^2\left(\dfrac{h}{a}\right)^2;$ $\Psi=\aleph M_p^2\left(\dfrac{a}{h}\right);$ $\Psi_1=\aleph\ (\aleph+1)$

$\dfrac{M_p^2}{4}$; $\lambda=\dfrac{a}{b}$;

$M^*=\dfrac{V}{V_\infty}$ is the Mach number; $M_E=\sqrt{\dfrac{E}{\rho V_\infty^2}}$;

$M_p=\sqrt{\dfrac{P_\infty}{\rho V_\infty^2}}$; $\lambda_1=\Psi M^*$; $\lambda_2=\Psi_1(M^*)^2$; $G_{k\ln mir}$,

$Q_{k\ln mir}$ are the dimensionless coefficients.

The initial conditions for the system of equations (4) take the following form

$$W_{nm}(0)=\alpha_{nm},\quad \dot{W}_{nm}(0)=\beta_{nm}, \quad (5)$$

where α_{nm}, β_{nm} - are known constants.

Systems of (IDE) (4) are solved numerically using the method based on the quadrature formulas (Badalov et al. (1987, 2007a, 2007b)). We write this system in integral form and, using a rational transformation, we eliminate the weakly singular properties of integral operator R^*. Assuming that $t=t_i$, $t_i=i\Delta t$, $i=1,2,\ldots$ ($\Delta t=const$) and replacing the integrals with some quadrature formulas to calculate $w_{nm}=w_{nm}(t)$, we obtain the following recurrence relation:

$$\sum_{n=1}^{N}\sum_{m=1}^{M}L_{k\ln m}w_{pnm}=\frac{1}{1+A_pa_1}\left\{\sum_{n=1}^{N}\sum_{m=1}^{M}L_{k\ln m}\left\langle w_{0nm}+\left(\dot{w}_{0nm}+a_1w_{0nm}\right)t_p\right\rangle-\right.$$

$$-\sum_{j=0}^{p-1}C_j\left(a_1\sum_{n=1}^{N}\sum_{m=1}^{M}L_{k\ln m}w_{jnm}+\left(t_p-t_j\right)\left[\sum_{n=1}^{N}\sum_{m=1}^{M}\lambda_1\cos\varphi G_{k\ln m}w_{jnm}\right.\right.$$

$$-\Omega^2\sum_{n=1}^{N}\sum_{m=1}^{M}F_{k\ln m}\left(w_{jnm}-\frac{A}{\alpha}\sum_{s=0}^{j}B_s\exp(-\beta t_s)w_{j-s,nm}\right) \quad (6)$$

$$\left.\left.\left.-\sum_{n,i,j_1=1}^{N}\sum_{m,r,s_1=1}^{M}Q_{k\ln m}w_{jnm}\left(w_{jir}-\frac{A}{\alpha}\sum_{s=0}^{j}B_s\exp(-\beta t_s)w_{j-s,ir}\right)\right]\right)\right\}$$

$p=1,2,\ldots;$ $n=\overline{1,N};$ $m=\overline{1,M};$

where C_j, B_s are the numerical coefficients in relation to the quadrature formulas of the trapezium:

$$C_0=\frac{\Delta t}{2};\quad C_j=\Delta t,\quad j=\overline{1,i-1};\quad C_i=\frac{\Delta t}{2};$$

$$B_0=\frac{\Delta t^\alpha}{2};\quad s=j,\quad B_j=\frac{\Delta t^\alpha(j^\alpha-(j-1)^\alpha)}{2};\qquad B_s=\frac{\Delta t^\alpha((s+1)^\alpha-(s-1)^\alpha)}{2}.$$

Algorithm (6) is general and suitable for flutter problems both for ideally elastic and hereditarily deformable flexible plates, under different boundary conditions.

Numerical results

The calculation results are presented in Tables 1, 2 and are reflected in the graphs shown in Figs. 2-5.

The criterion of instability

Here the principal task is to determine the critical flutter velocity V_{cr}. Various criteria are used to find the critical flutter velocity. As a criterion that determines the critical flutter velocity, we take the condition that at this velocity the oscillation amplitude changes according to the harmonic law. At a velocity $V>V_{cr}$, an oscillatory motion occurs with rapidly increasing amplitudes, which can lead to the destruction of the structure. At $V<V_{cr}$ the flow velocity is less than the critical one, and the amplitude of viscoelastic plate oscillations is damping (Khudayarov (2010, 2005, 2008; Khudayarov et al. (2019, 2020)).

The procedure for finding the critical velocity

To determine $V=V_{cr}$, we consider the values of V_1 and V_2 located in interval (V_0, V_n) in such a way that $V_0<V_1<V_2<V_n$. Comparing the law of variation of w at $V=V_1$ and $V=V_2$, the following conclusions can be drawn:

a) if at $V<V_1$, the law of variation of function w is close to a harmonic law, then V_{cr} cannot be in interval (V_0, V_1); that is, V_{cr} lies in interval (V_1, V_n);

b) if at $V>V_1$, rapid growth of function w in time is observed, then V_{cr} lies in interval (V_0, V_1).

Processes a) and b), i.e. the processes of excluding the intervals that do not give rise to undesirable phenomena are repeated for (V_0, V_1) or (V_1, V_n), etc. The search ends when the remaining sub-interval is reduced to a sufficiently small size.

Table 1 shows the critical values of the flutter velocity depending on the physical-mechanical and geometric nature of the plate, taking into account the sweep.

From Table 1 it can be seen that critical value V_{cr} for the swept-back plate is obviously greater than for the plate at $\varphi=0$. The range of values of φ does not go beyond $\pi/4$ since otherwise λ_1 and λ_2 should be determined by the size of the panel in the spread direction, and not in the chord direction. For example, with sweep $\varphi=12^0$, the critical number V_{cr} of the swept-back plate increases by 3.7% compared with the corresponding values of V_{cr} of the non-swept-back plate, with $\varphi=18^0$ - by 8.3%, with $\varphi=22{,}5^0$- by 11.4%, with $\varphi=36^0$ - 25.7%, and with $\varphi=45^0$ - by 43.1%.

Table 1 gives a comparison of the effects of the viscoelastic properties of the plate material on the flutter velocity with the sweep angle. For an elastic plate (A=0) with sweep angle $\varphi=\pi/6$, the critical velocity is 1169, and for a viscoelastic plate (A=0.05) with the same sweep, the critical velocity is 597. The difference between them is 49%. It is interesting to note that the viscoelastic plate in the presence and absence of sweep shows the same decrease in relation to the elastic plate in the presence and absence of sweep.

Computational experiments showed that a slight increase in the singularity parameter α leads to a significant increase in the critical flutter velocity.

Table 1

Dependence of the flutter velocity on the physical-mechanical and geometrical parameters of the plate

A	α	β	λ	*a/h*	φ	M_{cr}	V_{cr}
0,0	0,25	0,05	3	350	0 π/6	2,98 3,44	1012 1169
0,05	0,25	0,05	3	350	0 π/6	1,50 1,76	510 597
0,05	0,25	0,05	1	180	0 π/15 π/10 π/8 π/5 π/4	1,60 1,66 1,74 1,79 2,01 2,29	545 565 590 607 685 780
0,05	0,2 0,5 0,75	0,05	1	180	π/6	1,76 2,13 2,28	600 725 775
0,05	0,25	0,01 0,1	1	180	π/6	1,90 1,88	645 640
0,05	0,25	0,05	0,8 1,0 1,1 1,5	180	π/6	1,38 1,87 2,19 4,04	472 635 745 1373
0,05	0,25	0,05	1	160 170 190	π/6	3,06 2,40 1,50	1050 815 510

Table 2

Influence of the aspect ratio parameter λ on the critical flutter velocity

A	α	β	*a/h*	φ	V_{cr}				
					λ=1,6	λ=1,8	λ=2	λ=2,2	λ=2,5
0,05	**0,25**	**0,05**	**250**	**0** **π/10** **π/6** **π/5**	**345** **366** **394** **425**	**455** **488** **535** **570**	**615** **650** **708** **760**	**795** **852** **920** **980**	**1180** **1238** **1365** **1474**

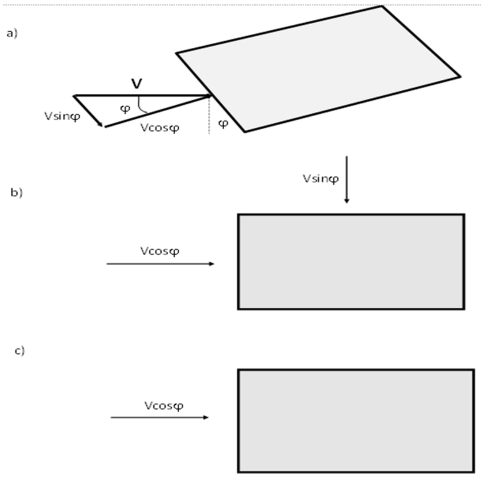

Fig. 1 Flow around a sweep-back plate

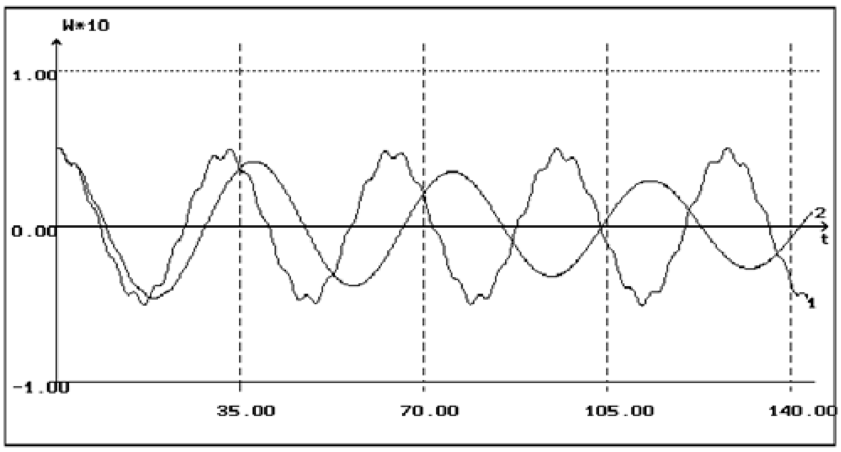

Fig. 2. A=0(1); A=0,05(2); α=0,25; a/h=350; β=0,05; λ=3; φ=π/6; V=500 m/sec.

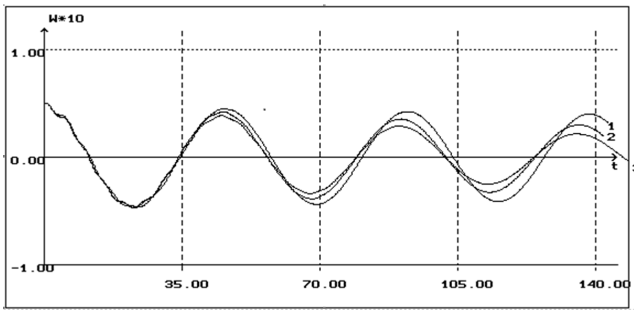

Fig. 3. α**=0(1);** $\pi/10$**(2);** $\pi/6$**(3); A=0,05;** α**=0,25; a/h=180;** β**=0,05;** λ**=1; V=400 m/sec.**

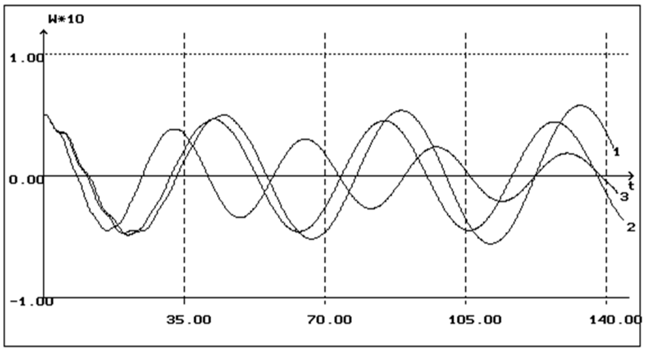

Fig. 4. α**=0,2(1); 0,5(2); 0,75(3); A=0,05; a/h=180;** β**=0,05;** λ**=1;** $\varphi=\pi/6$**; V=500 m/sec.**

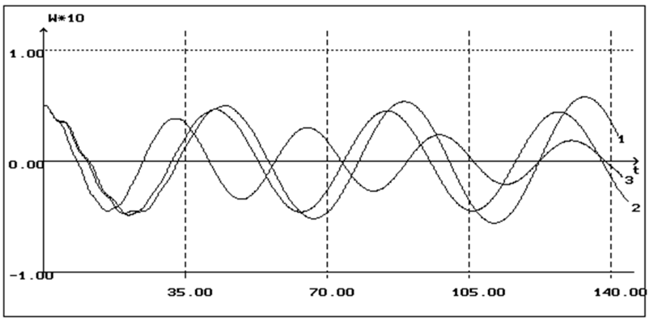

Fig. 5. λ**=0,8(1); 1(2); 1,5(3); A=0,05;** α**=0,25; a/h=180;**
β**=0,05;** $\varphi=\pi/6$**; V=600 m/sec.**

As seen from the table, the effect of damping parameter β of the heredity kernel on the plate flutter velocity is insignificant compared to the effect of viscosity parameter A and singularity α, which once again confirms the well-known conclusions that the exponential relaxation kernel is unable to fully describe the heredity properties of the structural material.

Table 2 shows the influence of the aspect ratio parameter λ at different sweep angles on the critical flutter velocities. With an increase in parameter λ and sweep angle φ, the critical flutter velocity increases. The critical flutter velocity for each λ increases by approximately 25-28% with sweep (φ=π/5) in relation to non-sweep.

Figure 2 shows the curves of dependence of *w* on time *t*, for the elastic (curve - 1, A=0) and viscoelastic (curve - 2, A=0.05) plates, taking into account the sweep. For a viscoelastic swept-back plate, the amplitude and frequency of oscillations decrease.

Further, the effect of the sweep on the oscillations of viscoelastic plate was studied. Figure 3 shows the dependence of the deflection of a plate flown about in a supersonic gas flow in time for various values of sweep angle φ. As parameter φ increases, the oscillations of the plate become damped. At that, the oscillation amplitude decreases, and the oscillation phase shifts to the right.

Figure 4 shows the influence of rheological parameter α on the behavior of a viscoelastic plate, taking into account the sweep φ=π/6. With an increase in the values of this parameter, the oscillation amplitude decreases, and the oscillation frequency increases.

The effect of the aspect ratio λ on the amplitude and frequency of oscillations of a viscoelastic plate at φ=π/6 (Fig. 5) was studied. For λ=0,8 (curve - 1), the amplitude of oscillations increases in time, and the movement of the plate is of a flutter nature. At values of λ=1 (curve - 2) and λ=1.5 (curve - 3) the amplitude of oscillations decays, and the frequency of oscillations increases.

5. Conclusions

It is shown in the article that in order to build mathematical models for the problem of dynamics of the heredity theory of viscoelasticity, the Koltunov-Rzhanitsyn singular kernel of heredity adequately describes real mechanical processes and best approximates experimental data over a long period of time. The critical flutter velocities are determined in a wide range of changes in various parameters of plates. It is shown that the singularity parameter α affects not only the oscillations of viscoelastic systems but also the critical flutter velocity. Therefore, it is important to take this influence into account when designing aircraft structures since the smaller the singularity parameter of the structure material, the more intense the dissipative processes in these structures. When modeling nonlinear flutter problems for viscoelastic plates, a number of new mechanical effects were obtained:

- it was stated that an account for the viscoelastic properties of the plate material leads to a decrease in the critical flutter velocity by 40 - 60%;
- it was found that the angle of flow around the plate contributes to a noticeable increase in the flutter velocity.

The developed models, algorithms and applied programs can be used in studying the dynamic behavior, designing and testing structural elements of aircraft made of composite viscoelastic materials, and other technical structures in various areas of aircraft and mechanical engineering.

References

1.Andreea Koreanschi, Mehdi ben Henia, Olivier Guillemette, Francois Michaud, Yvan Tondji, Oliviu Sugar Gabor, Ruxandra Mihaela Botez, Manuel Flores Salinas, "Flutter Analysis of a Morphing Wing Technology", INCAS BULLETIN, Vol. 8, Issue 1. (2016), pp. 99 – 124. doi: 10.13111/2066-8201.2016.8.1.10
2.Attar, P.J, Dowell, E.H, Tang, D.M, "A theoretical and experimental investigation of the effects of a steady angle of attack on the nonlinear flutter of a delta wing plate model", Journal of Fluids and Structures, Vol. 17, Issue 2, February (2003), pp. 243-259. https://doi.org/10.1016/S0889-9746(02)00123-8.
3.Badalov, F.B, "Methods for solving integral and integro-differential equations of the hereditary theory of viscoelasticity", Tashkent, (1987), 269 p.
4.Badalov, F.B, Abdukarimov, A, Khudayarov, B.A, "A numerical investigation of the influence of rheological parameters on the character of vibrations in heredity-defortable systems", Computational technologies, (2007), 12 (4), pp. 17-26.
5.Badalov, F.B, Eshmatov, Kh, Yusupov, M, "Some methods re¬sheniya systems of integral-differential equations encountered in problems of viscoelasticity", Applied Mathematics and Mechanics, (1987), Vol. 51, No. 5, pp. 867-871.
6.Badalov, F.B, Khudayarov, B.A, Abdukarimov A, "Effect of the hereditary kernel on the solution of linear and nonlinear dynamic problems of hereditary deformable systems", Journal of Machinery Manufacture and Reliability, (2007), 36 (4), pp.328-335.
7.Bichiou, Y, Nuhait, A.O, Abdelkefi, A, Hajj, M.R, "Unsteady aeroelastic behaviors of rigid airfoils with preset angles of attack", Journal of Vibration and Control, (2016) Vol. 22, Issue 4, pp.1010-1022. https://doi.org/10.1177/1077546314537106.
8.Bolotin V.V (1961). Non-conservative problems of the theory of elastic stability. – M.: Fizmatgiz, 340 p.
9.Chung-Li Liao, Yee-Win Sun, Flutter analysis of stiffened laminated composite plates and shells in supersonic flow, September 1993, AIAA Journal 31(10):1897-1905. DOI 10.2514/3.11865
10.D.R.Q. Pacheco, F.D. Marques, S. Natarajan, A.J.M. Ferreira, Nonlinear finite element post-flutter analysis of multibay composite panels in supersonic regime, Composite Structures, Volume 180, 15 November 2017, Pages 883-891. https://doi.org/10.1016/j.compstruct.2017.08.058
11.Deman Tang and Earl H. Dowell, "Experimental Aeroelastic Models Design and Wind Tunnel Testing for Correlation with New Theory", Aerospace (2016), 3, 12; doi:10.3390/aerospace3020012.
12.Dowell, E. H and Ventres, C. S, "Comparison of theory and experiment for nonlinear flutter of loaded plates", AIAA Journal, Vol. 8, No. 11 (1970), pp. 2022-2030.
13.EH Dowell (1970), Fatigue life estimation of fluttering panels, AIAA Journal 8 (10), 1879-1881.
14.EH Dowell, HM Voss, (1965) "Theoretical and experimental panel flutter studies in the Mach number range 1.0 to 5.0.", AIAA Journal 3 (12), 2292-2304.
15.Eisley J.G, Luessen G, "Flutter of Thin Plates under Combined Shear and Normal Edge Forces", AIAA Journal, Vol. 1, No. 3. (1963), pp. 347-356.
16.Fung, Y.C, "A summary of the theories and experiments on panel flutter", Air Force Office Sci. Research TN 60-224 (May 1960), 27 p.
17.Hai Zhao Dengqing Cao, A study on the aero-elastic flutter of stiffened laminated composite panel in the supersonic flow Journal of Sound and Vibration, Volume 332, Issue 19, 16 September 2013, Pages 4668-4679. https://doi.org/10.1016/j.jsv.2013.04.006
18.Ilyushin A.A., Kiyko I.A. A new statement of the problem of flutter of a shallow shell // Appl. mathematics and mechanics. - 1994. - V. 58. - Issue. 3. - pp. 167-171.
19.Jiali Xiea , Cunming Mab, Chunxu Leic and Qingsong Duand, "Research on Wind-resistant Capability of Two Common Section Forms for Long Span Bridges", Applied Mechanics and Materials, (2013), Vols. 438-439, pp. 930-934. doi:10.4028/www.scientific.net/AMM.438-439.930.
20.Kemal Yaman, "Subsonic Flutter of Cantilever Rectangular PC Plate Structure", International Journal of Aerospace Engineering Volume 2016 (2016), Article ID 9212364, http://dx.doi.org/10.1155/2016/9212364.
21.Khudayarov B.A, "Numerical study of the dependence of the critical flutter speed and time of a plate on rheological parameters", International Applied Mechanics, (2008), 44 (6), pp.676-682.
22.Khudayarov, B. A., Komilova, K. M. and Turaev, F. Z. (2019) 'The effect of two-parameter of Pasternak foundations on the oscillations of composite pipelines conveying gas-containing fluids', International Journal of Pressure Vessels and Piping, 176, p. 103946. doi: https://doi.org/10.1016/j.ijpvp.2019.103946.
23.Khudayarov, B.A, "Flutter of a viscoelastic plate in a supersonic gas flow", International Applied Mechanics, (2010), 46 (4), pp.455-460.
24.Khudayarov, B.A, "Mathematical modelling of nonlinear flutter of viscoelastic elements and units of the flying device", Mathematical Models and Computer Simulations, (2010), 22 (6), pp.111-131.
25.Khudayarov, B.A, "Numerical Analysis of the Nonlinear Flutter of Viscoelastic Plates", INTERNATIONAL J. APPLIED MECHANICS, (2005), Vol. 41, No. 5, pp. 538-542.
26.Khudayarov, B.A., Turaev, F.Zh. (2019), "Mathematical Simulation of Nonlinear Oscillations of Viscoelastic Pipelines Conveying Fluid", Applied Mathematical Modelling, Vol. 66, pp. 662-679. https://doi.org/10.1016/j.apm.2018.10.008
27.L. Librescu, N.K. Chandiramani, Dynamic stability of transversely isotropic viscoelastic plates, Journal of Sound and Vibration. Volume 130, Issue 3, 8 May 1989, Pages 467-486. https://doi.org/10.1016/0022-460X(89)90070-9
28.Larionov G.S. (1974), "Nonlinear flutter of an elastoviscous plate", Rigid Body Mechanics, 4, 95–100.
29.Li, XF., Jiang, SN. & Lee, K.Y (2018). Surface effect on dynamic stability of microcantilevers on an elastic foundation under a subtangential follower force. International Journal of Mechanics and Materials in Design. Volume 14, Issue 1, pp 91–104. https://doi.org/10.1007/s10999-016-9362-1.
30.M.A. Kouchakzadeh, M. Rasekh, H. Haddadpour, Panel flutter analysis of general laminated composite plates, Composite Structures Volume 92, Issue 12, November 2010, Pages 2906-2915. https://doi.org/10.1016/j.compstruct.2010.05.001
Volume 82, Issue 2, January 2008, Pages 295-301. https://doi.org/10.1016/j.compstruct.2007.01.007
cience China Technological Sciences, (2012) 55: 3370. https://doi.org/10.1007/s11431-012-4933-3