MAVZU:
Bir va ikki pallali giperboloid. Elliptik paraboloid.Konus, silindrlar.

Uch o‘lchovli $Oxyz$ Dekart koordinatalar sistemasida har qanday sirt biror $F(x,y,z)=0$ tenglama bilan yoziladi, bu erda $(x,y,z)-$ sirt ixtiyoriy nuqtasining koordinatasi. Agar $F(x,y,z)$ - x,y,z o‘zgaruvchilarga nisbatan ikkinchi darajali ko‘phad bo‘lsa, u holda $F(x,y,z)=0$ tenglama ikkinchi tartibli tenglama deyiladi, shu tenglama yordamida tasvirlanadigan sirt esa *ikkinchi tartibli sirt* deyiladi. Agar sirtning koordinatalar sistemasiga nisbatan joylashishi alohida xususiyatga ega bo‘lsa (masalan, ba’zi koordinatalar sistemalariga nisbatan simmetrik joylashgan bo‘lsa), u holda uning tenglamasi juda sodda ko‘rinishga ega bo‘ladi va u *kanonik tenglama* deyiladi.

Ikkinchi tartibli sirtning umumiy tenglamasi

$$a_{11}x^2+a_{22}y^2+a_{33}z^2+2a_{12}xy+2a_{13}xz+2a_{23}yz+$$

$$+2a_1x+2a_2y+2a_3z+a_0=0$$

ko‘rinishda bo‘ladi, bu erda a_{11}, a_{22}, a_{33}, a_{12}, a_{13}, a_{23}, a_1, a_2, a_3, a_0 – haqiqiy sonlar, bunda a_{11}, a_{22}, a_{33}, a_{12}, a_{13}, a_{23} koeffitsientlar bir vaqtda nolga teng emas.

Ikkinchi tartibli sirtlar nazariyasida sirtlar klassifikatsiya qilinadi va ularning turli ko‘rinishlari o‘rganiladi. Sirtlarni o‘rganishning usullaridan biri kesim usulidir. Bunda sirtlarning koordinata tekisliklariga parallel bo‘lgan yoki koordinata tekisliklarining o‘zi yordamidagi kesimlari o‘rganiladi. Hosil bo‘lgan kesimlarning ko‘rinishiga qarab sirt haqida xulosa chiqariladi.

Ikkinchi tartibli sirtlarning 17 ta ko‘rinishi bor. Sirtlarni klassifikatsiyalash g‘oyasi koordinatalar sistemasini kanonik sistemaga keltirish yo‘li bilan sirtlarning tenglamalarini kanonik ko‘rinishga keltirishga asoslangan.

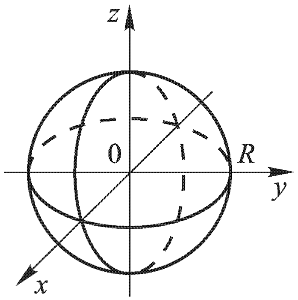
Ikkinchi tartibli sirtlarning 6 ta ko‘rinishini batafsil o‘rganamiz: ellipsoid, bir pallali giperboloid, ikki pallali giperboloid konus, elliptik paraboloid va giperbolik paraboloid.

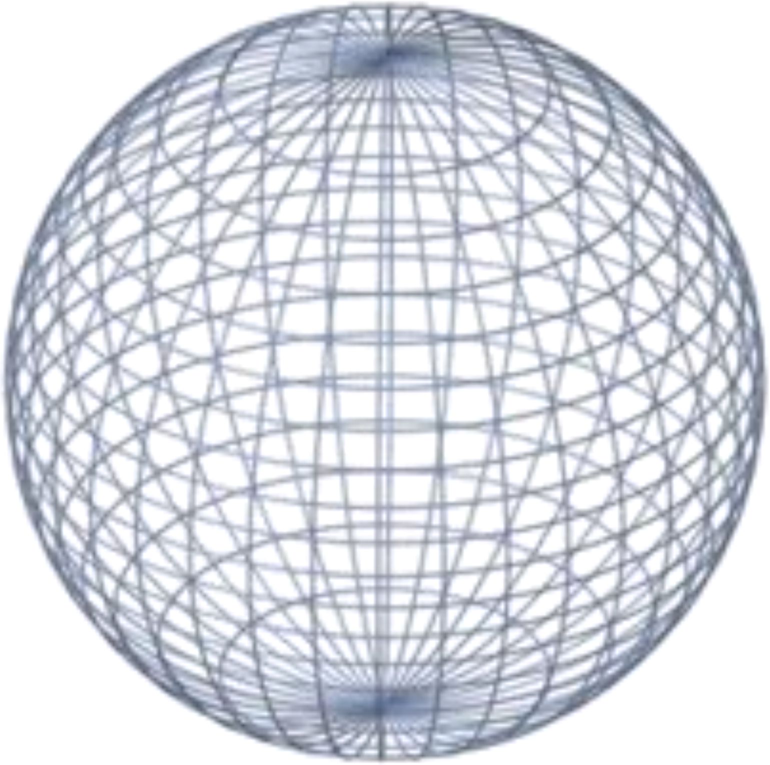
Ellipsoidlarning eng sodda ko‘rinishi sfera bo‘lib, bu sirt eng ko‘p o‘rganilgandir. SHuning uchun sferalarni ko‘rib chiqamiz.

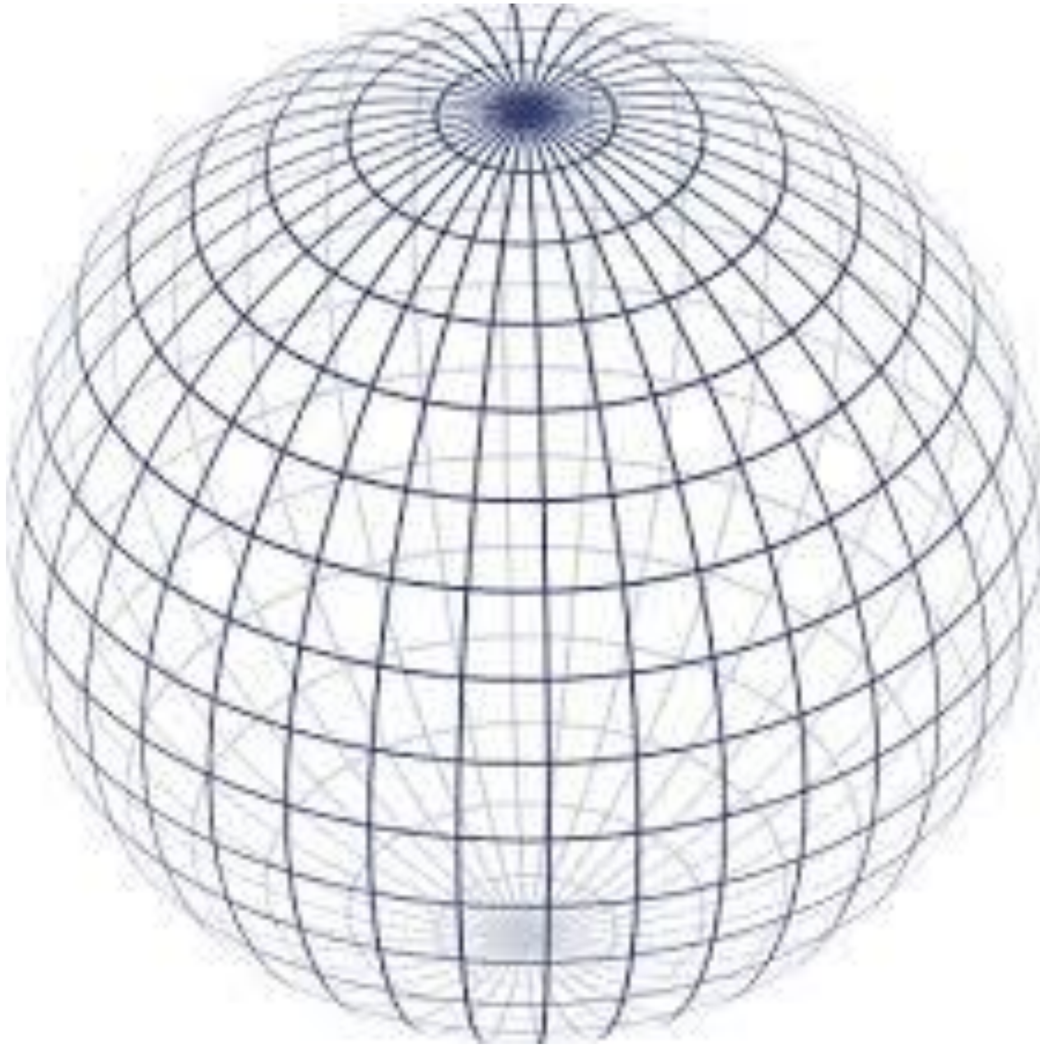
SFERA

Markazi koordinatalar boshida bo‘lgan *R* radiusli sfera tenglamasi

$$x^2 + y^2 + z^2 = R^2$$







Markazi $M(x_0, y_0, z_0)$ nuqtada bo‘lgan *R* radiusli sfera tenglamasi

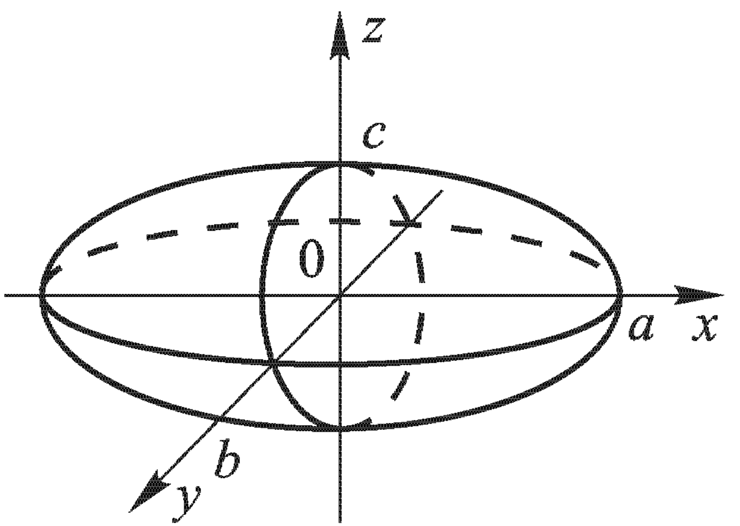
$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2$$

Ellipsoid

Ellipsoid deb, koordinatalarning kanonik sistemasidagi tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

ko‘rinishda bo‘lgan ikkinchi tartibli sirtga aytiladi. Xususan, agar $a=b=c=R$ bo‘lsa, markazi koordinatalar boshida bo‘lgan R radiusli sferani olamiz. a,b,c sonlar ellipsoidning yarim o‘qlari deyiladi. Agar yarim o‘qlar har xil bo‘lsa, ellipsoid uch o‘qli ellipsoid deyiladi. Ellipsoidning koordinata o‘qlari bilan kesishgan $A_1(-a,0,0)$, $A_2(a,0,0)$, $B_1(0,-a,0)$, $B_2(0,a,0)$, $C_1(0,0,-a)$, $C_2(0,0,a)$ nuqtalar ellipsoidning uchlari deyiladi.



Koordinatalar kanonik sistemasining o‘qlari ellipsoidning simmetriya o‘qlari, koordinatalar boshi – uning simmetriya markazi, koordinatalar tekisliklari esa simmetriya tekisliklari bo‘ladi.

Ellipsoidning $xOy: z=0$ tekislik bilan kesimini o‘rganamiz. Bu kesim

$$\begin{cases} \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1 \\ z=0 \end{cases}$$

sistema bilan beriladi va kanonik tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

bo‘lgan ellipsdan iborat bo‘ladi.

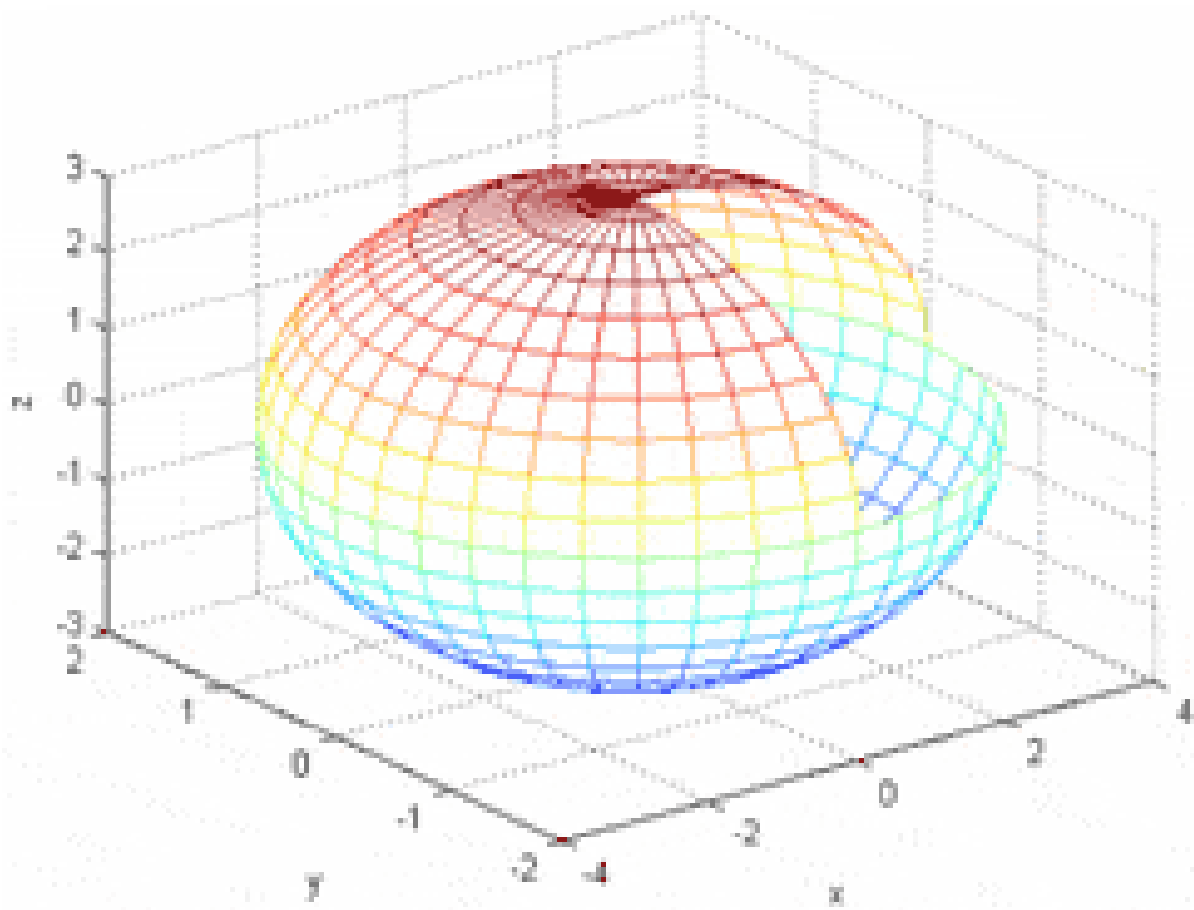
Ellipsoidning $xOz: y=0$ va $yOz: x=0$ koordinata tekisliklari hamda $x=h_1,\ y=h_2,\ z=h_3$ tekisliklar bilan kesimlarini olib, elliptik tipdagi ikkinchi tartibli egri chiziqlarga ega bo‘lamiz. Bu chiziqlar $h_1<a,\ h_2<b,\ h_3<c$ bo‘lganda bo‘lganda ellipsdan iborat bo‘ladi.

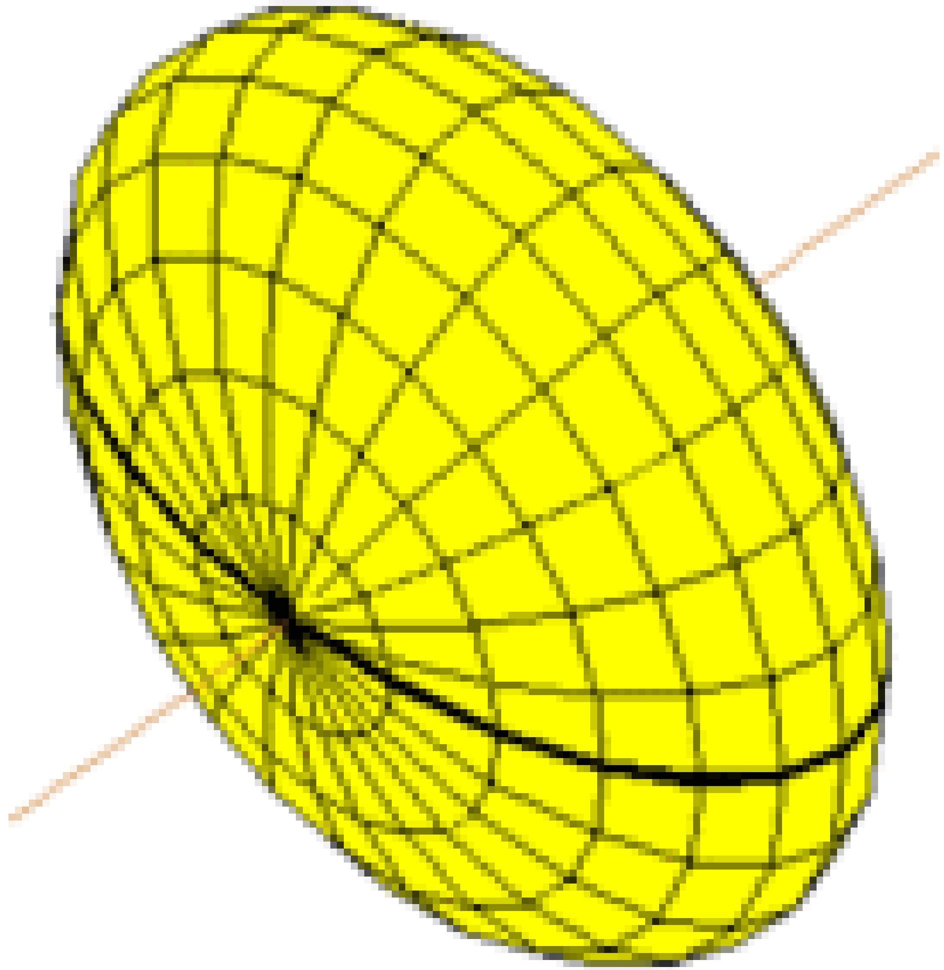
Markazi $M(x_0, y_0, z_0)$ nuqtada bo‘lgan ellipsoid tenglamasi

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1$$

ko‘rinishda bo‘ladi.

Sfera ham ellipsoiddir, chunki ellipsoid tenglamasida $a=b=c=R$ bo‘lsa, sfera tenglamasi hosil bo‘ladi.



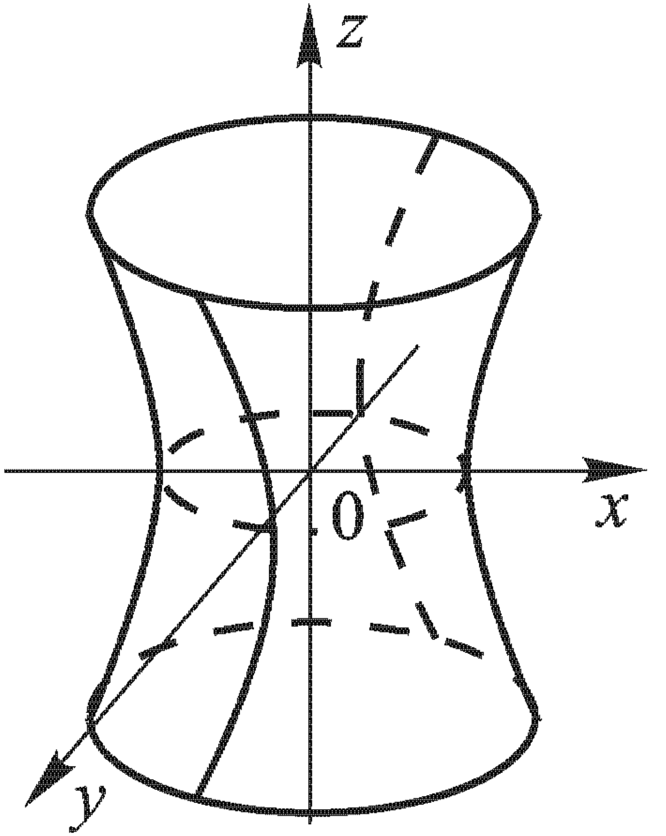


Bir pallali giperboloid

Bir pallali giperboloid deb, koordinatalarning kanonik sistemasidagi tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

ko‘rinishda bo‘lgan ikkinchi tartibli sirtga aytiladi.



Koordinatalar kanonik sistemasining o‘qlari bir pallali giperboloidning simmetriya o‘qlari, koordinatalar boshi – uning simmetriya markazi, koordinatalar tekisliklari esa simmetriya tekisliklari bo‘ladi.

Bir pallali giperboloidning koordinata o‘qlari bilan kesishgan $A_1(-a,0,0)$, $A_2(a,0,0)$, $B_1(0,-a,0)$, $B_2(0,a,0)$ nuqtalar bir pallali giperboloidning uchlari deyiladi. Bir pallali giperboloid bilan umumiy nuqtaga ega bo‘lmagan Oz o‘qi uning mavhum o‘qi deyiladi.

Bir pallali giperboloidning $xOy: z=0$ yoki $z=h_3$ tekisliklar bilan kesimini o‘rganamiz. Bu kesimda ellipslar hosil bo‘ladi. Endi bir pallali giperboloidning $xOz: y=0$ tekislik bilan kesimini olamiz. U

$$\begin{cases} \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-\dfrac{z^2}{c^2}=1 \\ y=0 \end{cases}$$

sistema bilan beriladi. Kesimda haqiqiy o‘qi Ox va kanonik tenglamasi

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}=1$$

bo‘lgan giperbola hosil bo‘ladi.

Xuddi shunga o‘xshash, bir pallali giperboloidning $yOz: x=0$, hamda $x=h_1$, $y=h_2$ tekisliklar bilan kesimlarini olsak, giperbolik tipdagi ikkinchi tartibli egri chiziqlarga ega bo‘lamiz. Masalan, $x=a$ tekislikning bir pallali giperboloid bilan kesimi

$$\begin{cases} \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \\ x=a \end{cases}$$

sistema bilan beriladi va u ikkita kesishuvchi to‘g‘ri chiziqlarning juftini beradi:

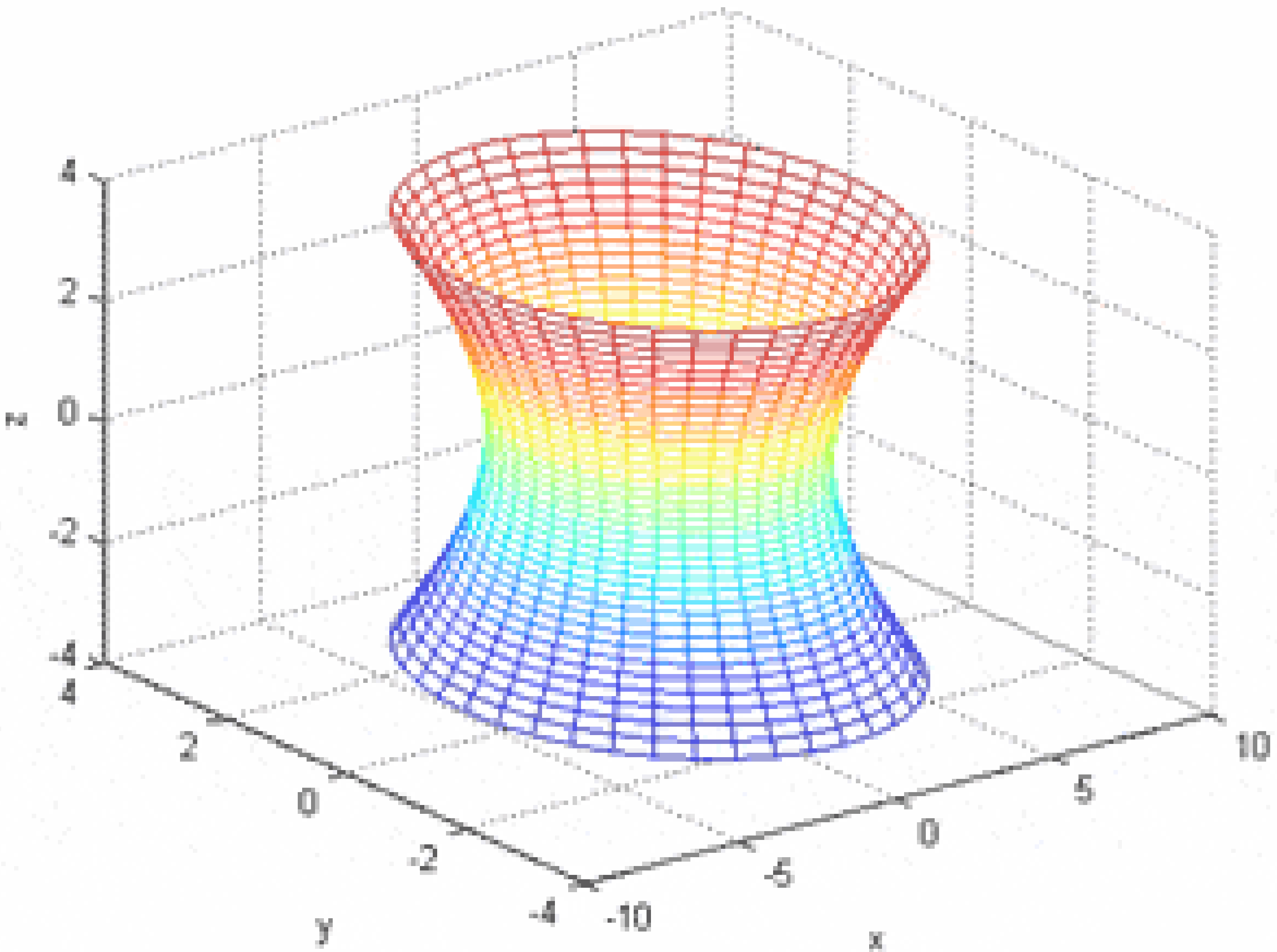
$$\frac{y^2}{b^2}-\frac{z^2}{c^2}=0 \cdot$$

Bir pallali giperboloidning boshqa ko‘rinishlari ham mavjud bo‘lib

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

ular ham yuqoridagi kabi o‘rganiladi.



4
2
0
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z
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-4
y
-10
-5
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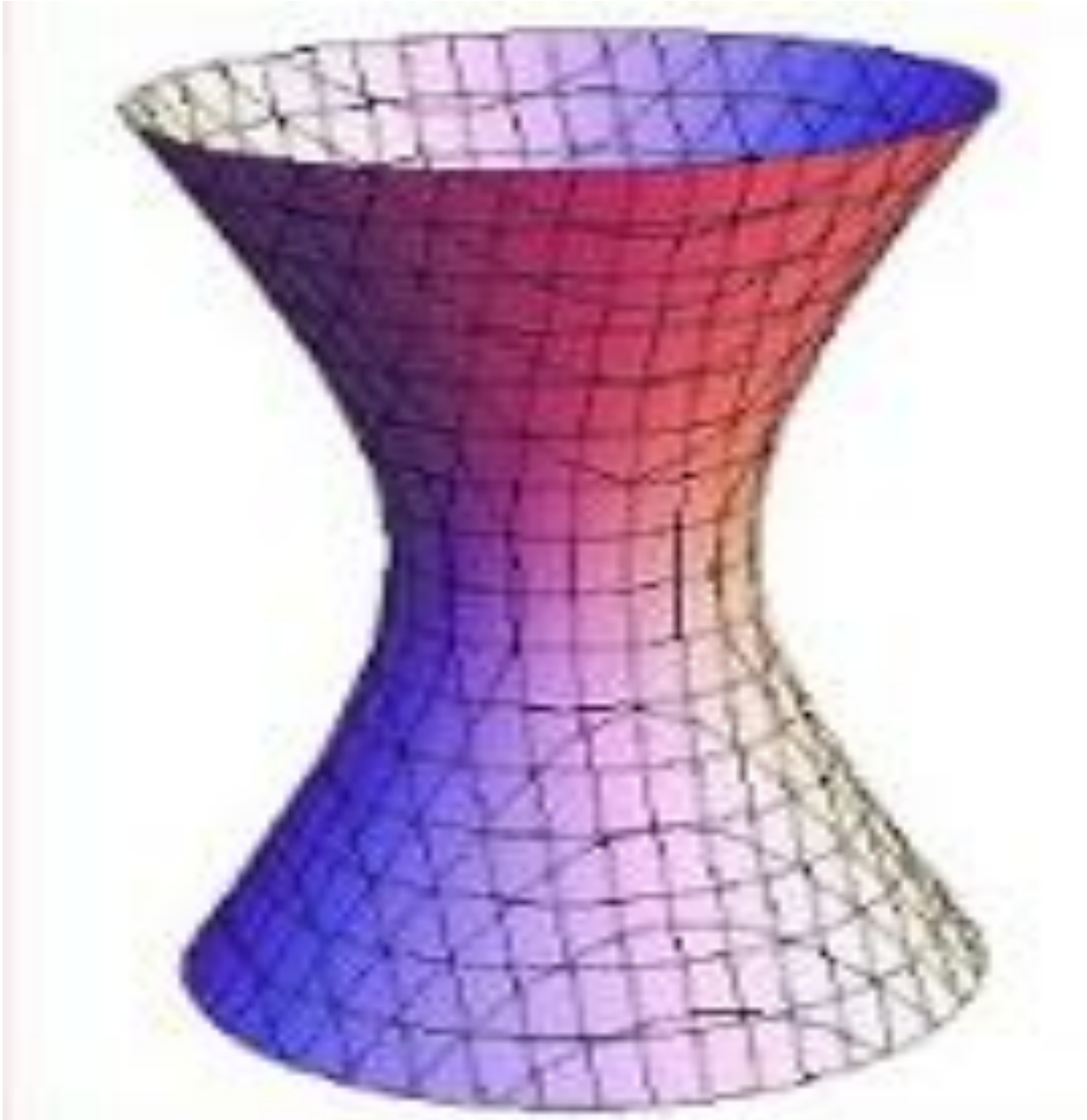
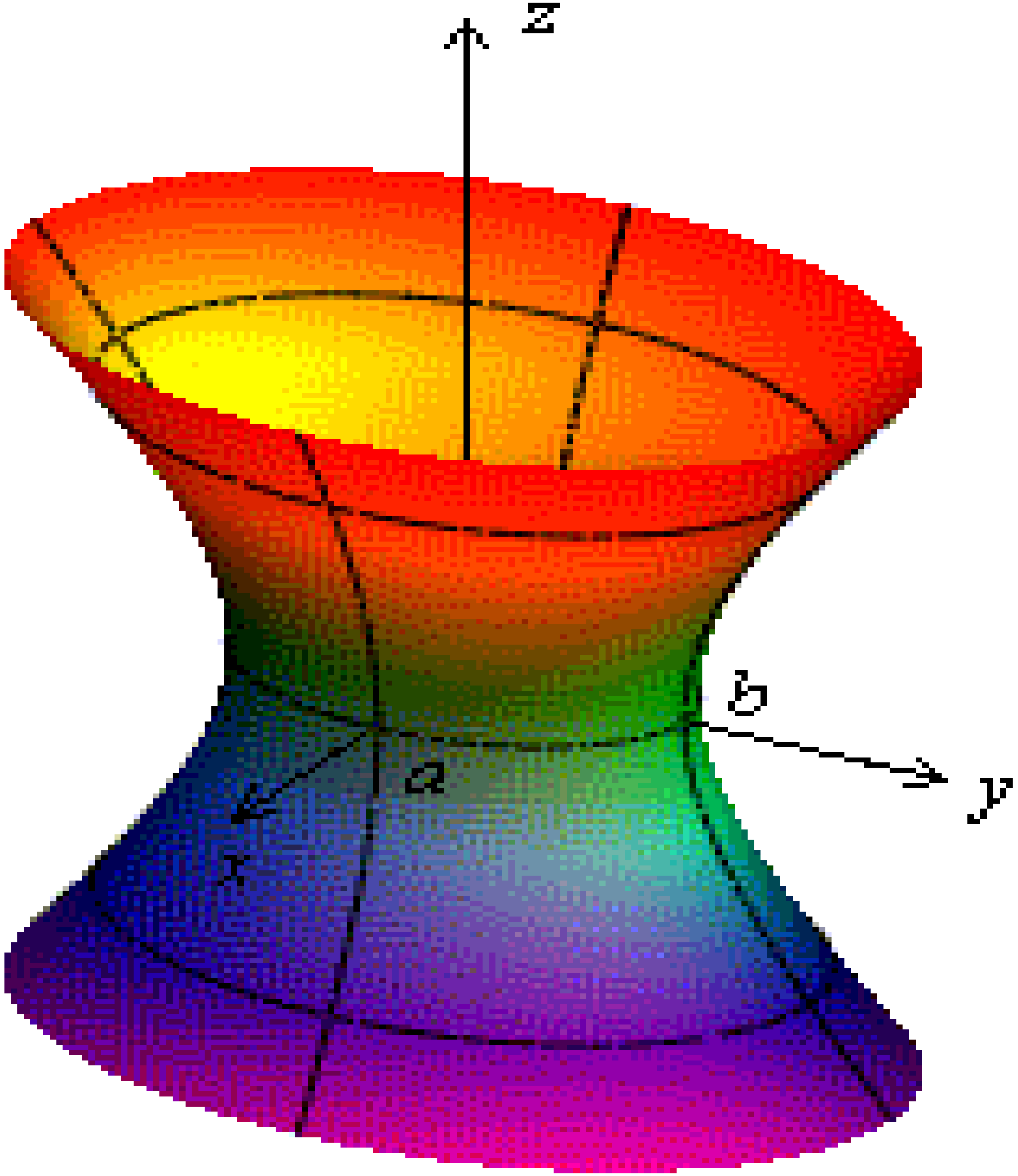
5
10
x





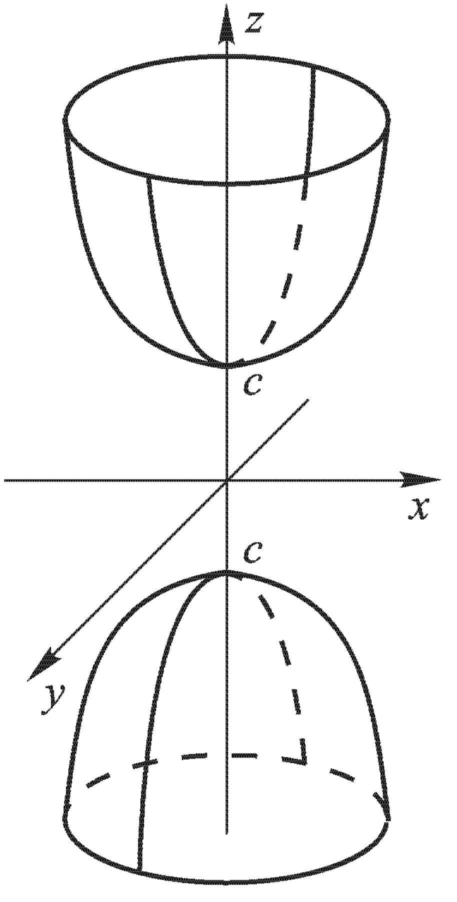
z
b
y
a
x

Ikki pallali giperboloid

Ikki pallali giperboloid deb, koordinatalarning kanonik sistemasidagi tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

ko‘rinishda bo‘lgan ikkinchi tartibli sirtga aytiladi.



Koordinatalar kanonik sistemasining o‘qlari ikki pallali giperboloidning simmetriya o‘qlari, koordinatalar boshi – uning simmetriya markazi, koordinatalar tekisliklari esa simmetriya tekisliklari bo‘ladi.

Ikki pallali giperboloidning koordinata o‘qlari bilan kesishgan $C_1(0,0,-c)$, $C_2(0,0,c)$ nuqtalar ikki pallali giperboloidning uchlari deyiladi. Ikki pallali giperboloid bilan umumiy nuqtaga ega bo‘lgan Oz o‘qi uning haqiqiy o‘qi deyiladi.

Ikki pallali giperboloidning $xOy: z=0$ yoki $z=h_3$ tekisliklar bilan kesimini o‘rganamiz. Bu kesimda ellipslar hosil bo‘ladi. Endi ikki pallali giperboloidning $z=h$ tekislik bilan kesimini olamiz. U

$$\begin{cases} \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-\dfrac{z^2}{c^2}=-1 \\ z=h \end{cases}$$

sistema bilan beriladi. Bu erdan ikkinchi tenglamani birinchi tenglamaga qo‘yib, ketma-ket

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{h^2}{c^2}-1$$

tenglamani, bundan

$$\frac{x^2}{\left(\frac{h^2}{c^2}-1\right)a^2}+\frac{y^2}{\left(\frac{h^2}{c^2}-1\right)b^2}=1$$

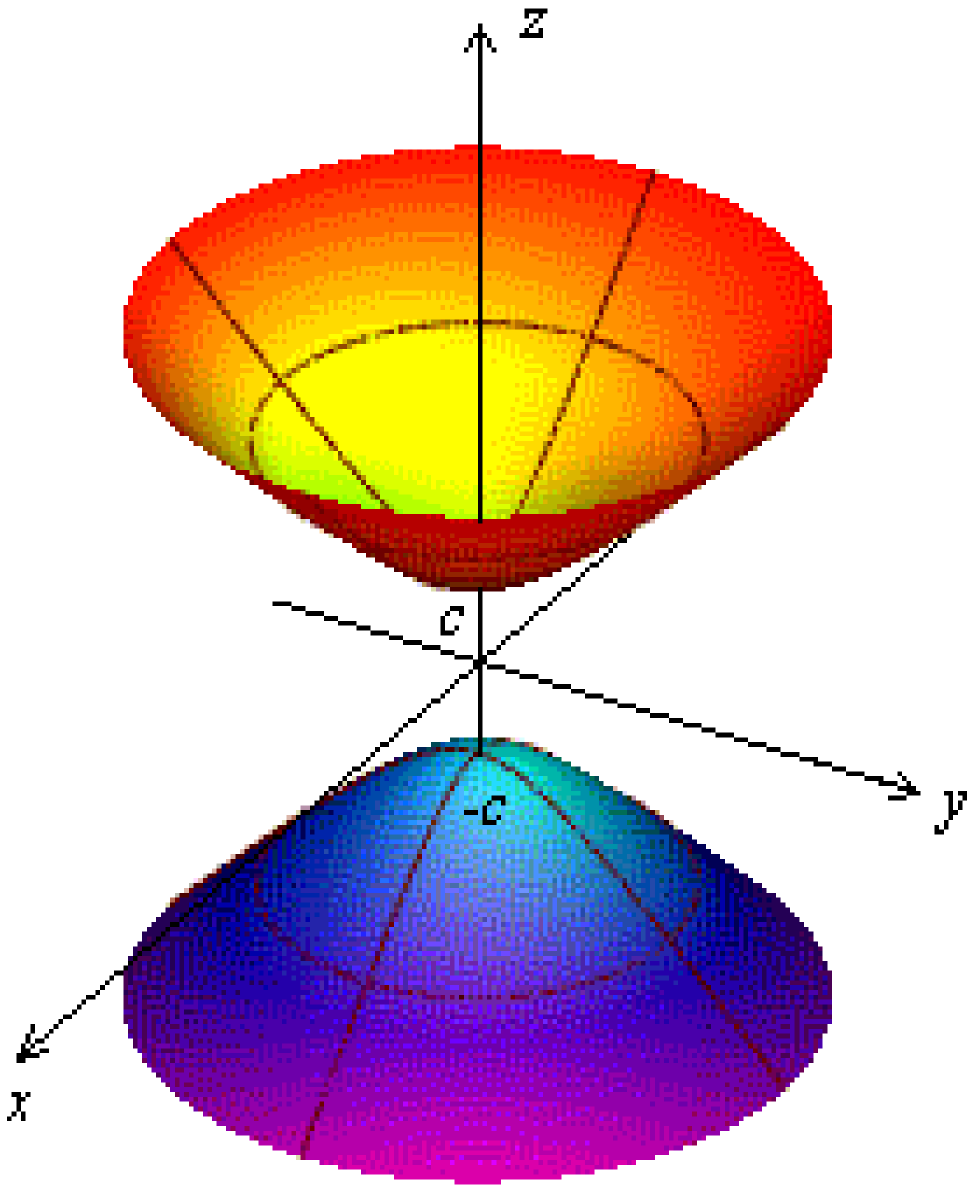
ellipsning kanonik tenglamasini olamiz.

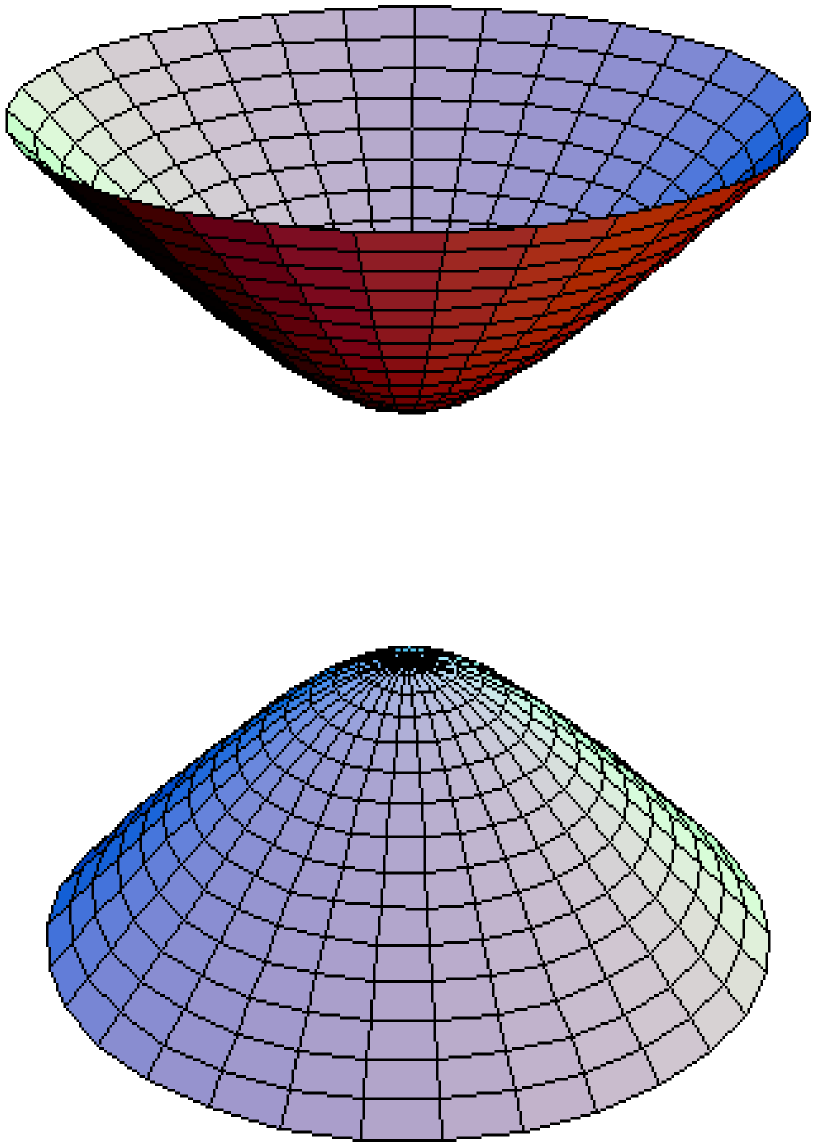
Ikki pallali giperboloidning boshqa ko‘rinishlari ham mavjud bo‘lib

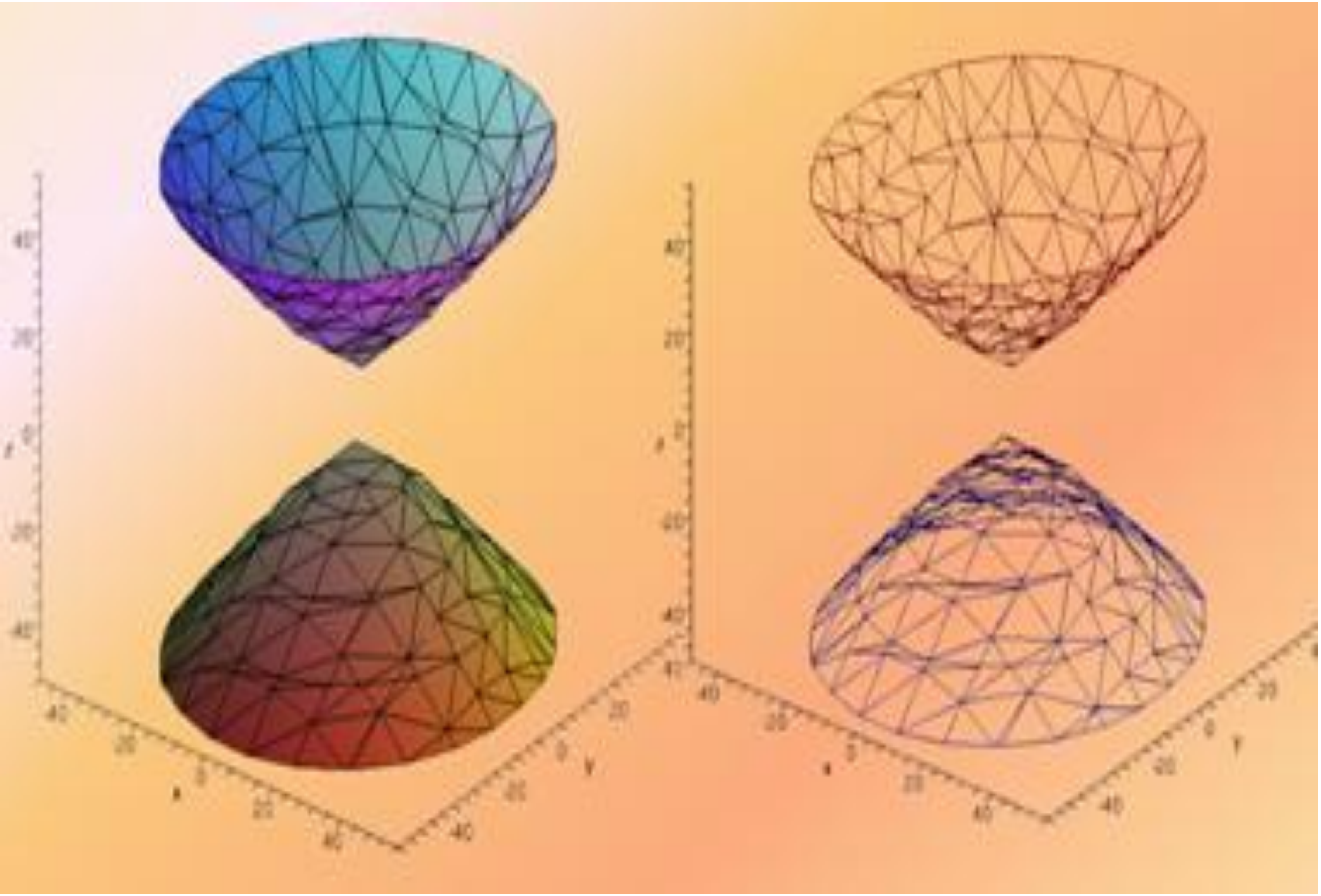
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$$

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$$

ular ham yuqoridagi kabi o‘rganiladi.


z
c
-c
y
x

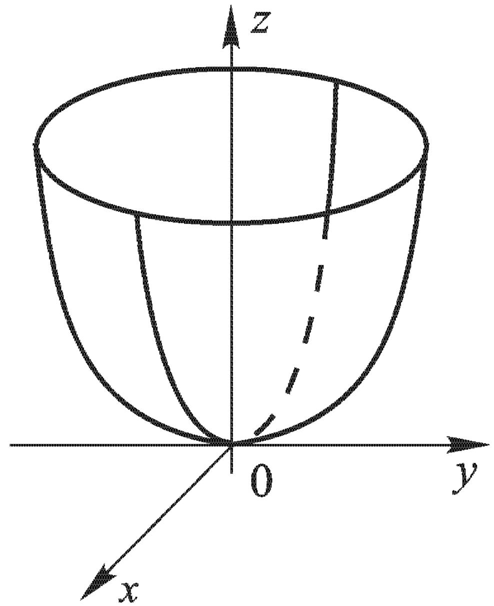


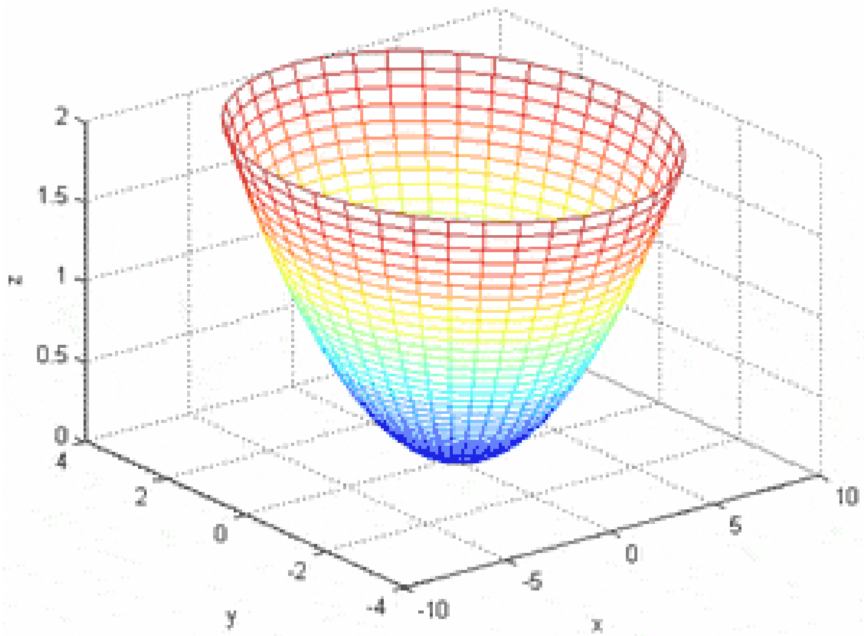


Silindrik va aylanma sirtlar

Elliptik paraboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2z$$





Elliptik paraboloidning $z = h$ gorizontal tekisliklar bilan kesimida ellipslar hosil bo‘ladi:

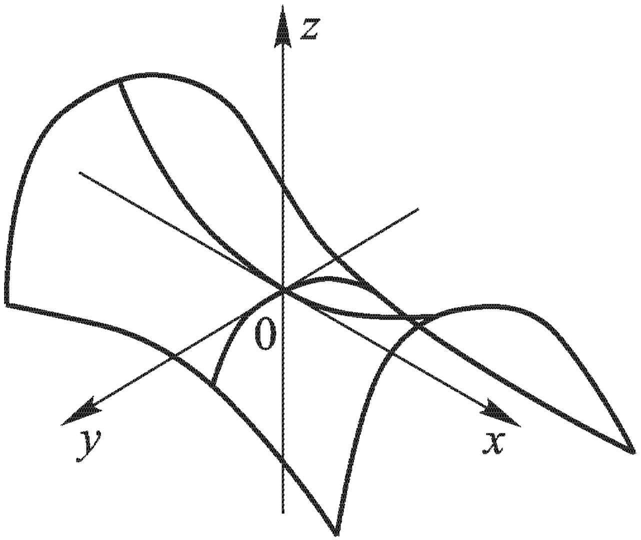
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2h$$

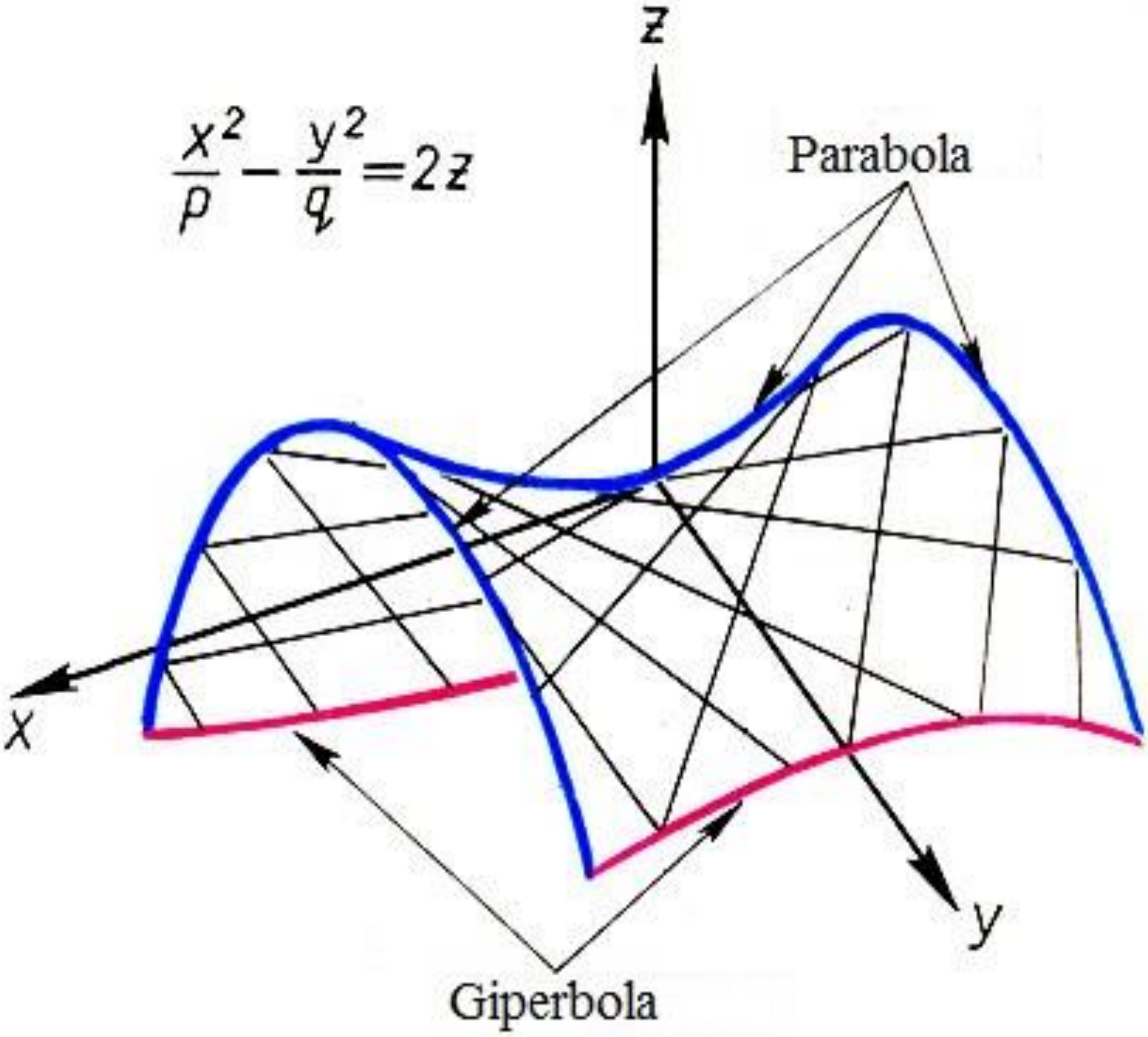
Elliptik paraboloidning $x = h$ va $y = h$ vertikal tekisliklar bilan kesimida parabolalar hosil bo‘ladi:

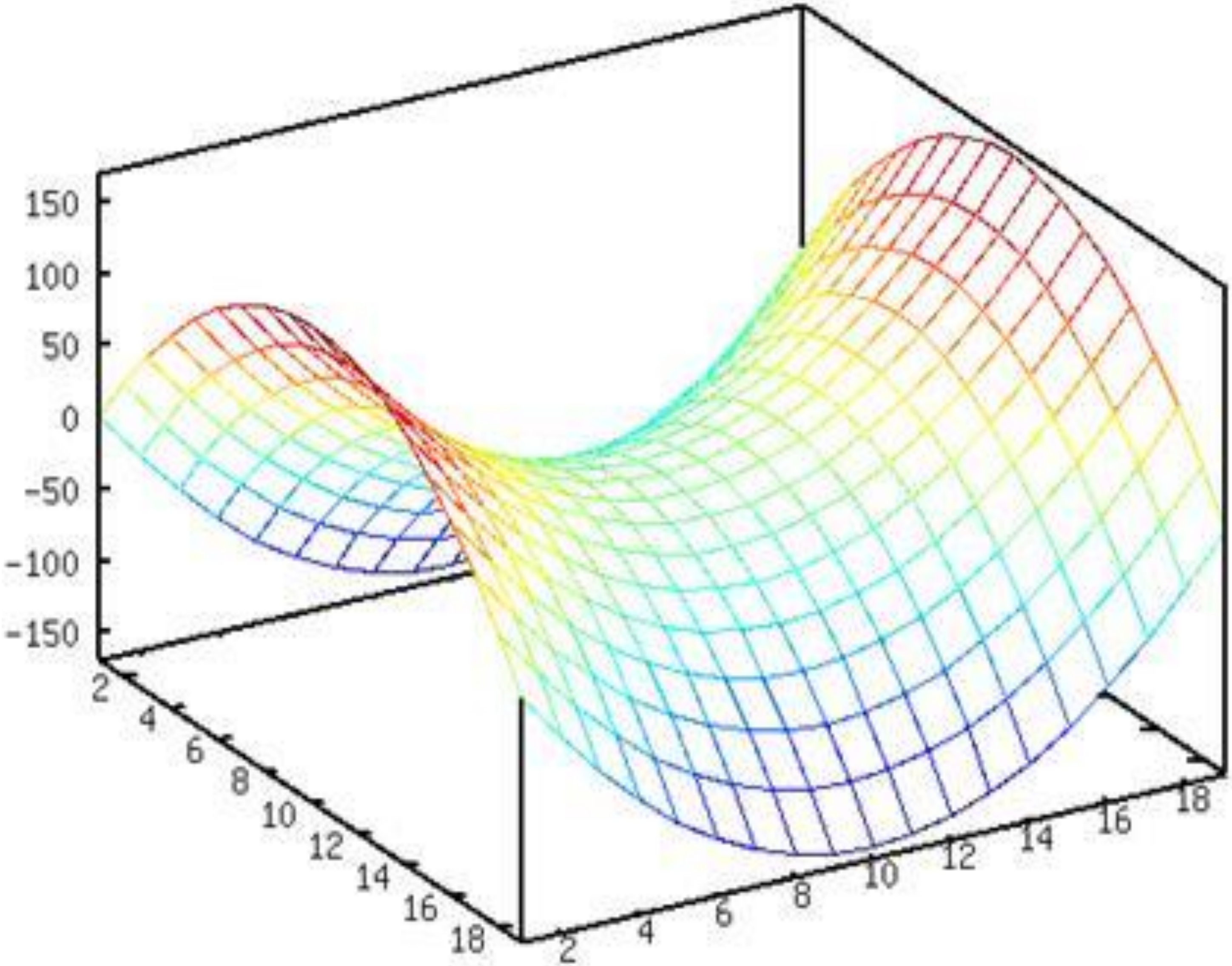
$$\frac{y^2}{b^2} = 2z - \frac{h^2}{a^2} \quad \text{yoki} \quad \frac{x^2}{a^2} = 2z - \frac{h^2}{b^2}.$$

Giperbolik paraboloid

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$$




z
$\frac{x^2}{p} - \frac{y^2}{q} = 2z$
Parabola
x
y
Giperbola


150
100
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-100
-150
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4
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16
18

Giperbolik paraboloidning $z = h$ gorizontal tekisliklar bilan kesimida giperbolalar hosil bo‘ladi:

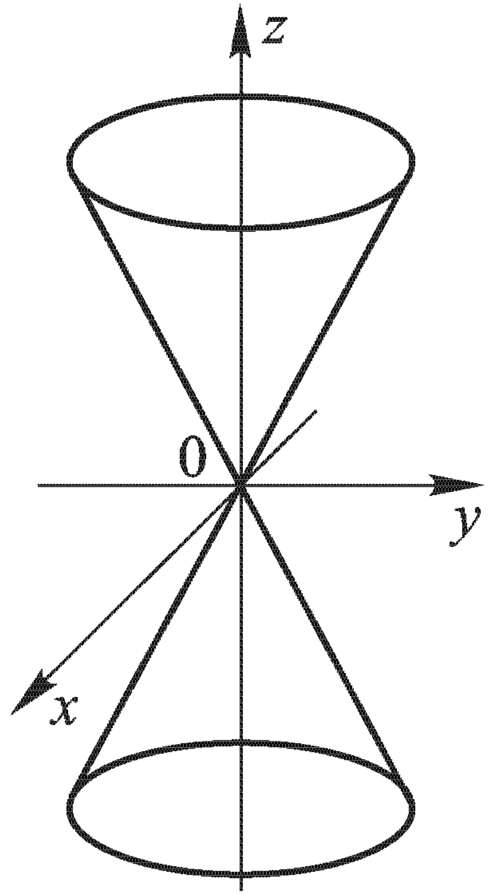
$$\frac{x^2}{2a^2h} - \frac{y^2}{2b^2h} = 1$$

Giperbolik paraboloidning $x = h$ va $y = h$ vertikal tekisliklar bilan kesimida parabolalar hosil bo‘ladi:

$$\frac{y^2}{b^2} = -2z + \frac{h^2}{a^2} \quad \text{va} \quad \frac{x^2}{a^2} = 2z + \frac{h^2}{b^2}.$$

Ikkinchi tartibli konus (elliptik konus)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$$

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$$

Agar $a = b$ bo‘lsa, u holda doiraviy konus hosil bo‘ladi.

Konusning $z = h$ gorizontal tekisliklar bilan kesimida ellipslar hosil bo‘ladi ($h = 0$ bo‘lganda konus nuqtaga aylanib qoladi):

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{h^2}{c^2}.$$

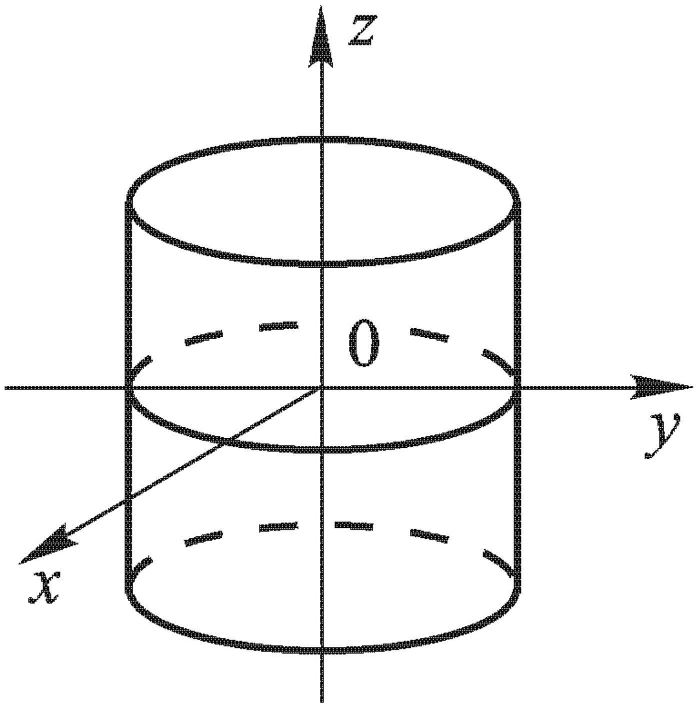
Konusningning $x = h$ va $y = h$ vertikal tekisliklar bilan kesimida giperbolalar hosil bo‘ladi:

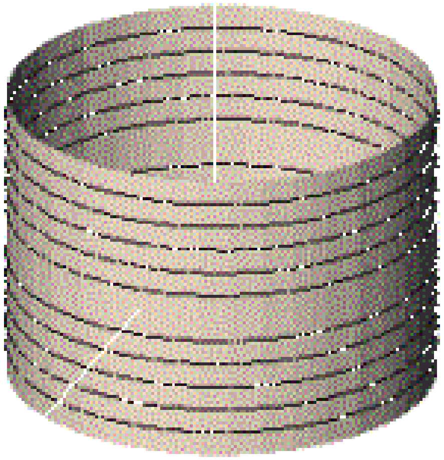
$$\frac{y^2}{b^2} - \frac{z^2}{c^2} = -\frac{h^2}{a^2} \qquad \text{yoki} \qquad \frac{x^2}{a^2} - \frac{z^2}{c^2} = -\frac{h^2}{b^2}$$

Silindrik sirtlar

Elliptik silindr

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

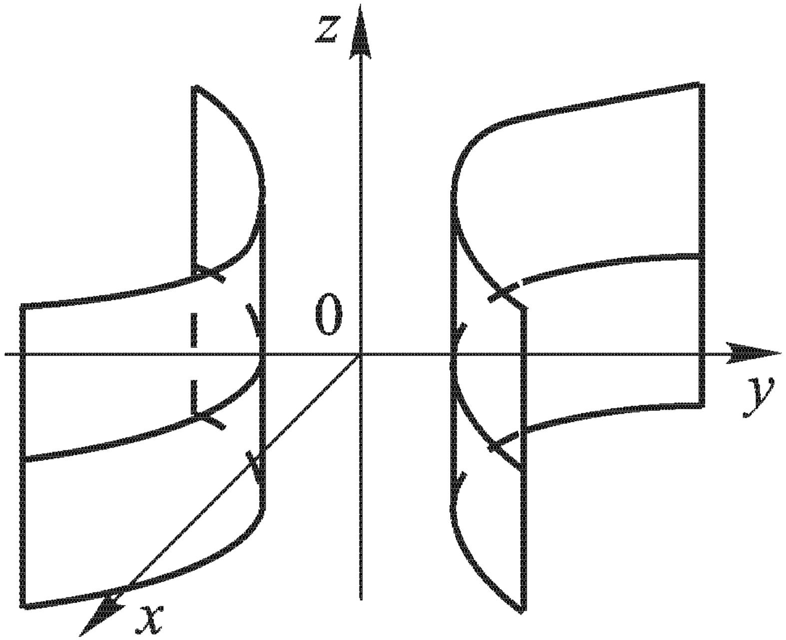


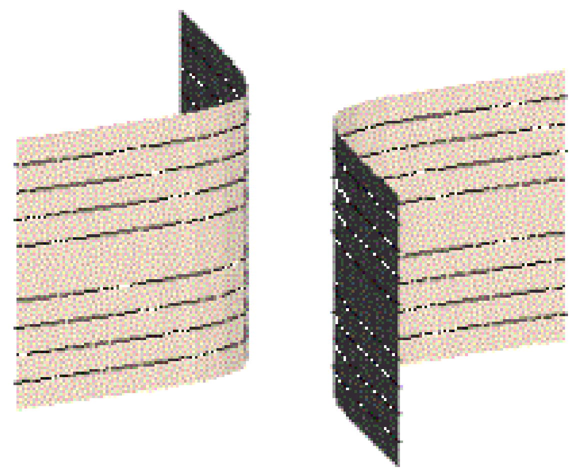


Agar $a = b = R$ bo‘lsa, $x^2 + y^2 = R^2$ doiraviy silindr hosil bo‘ladi.

Giperbolik silindr

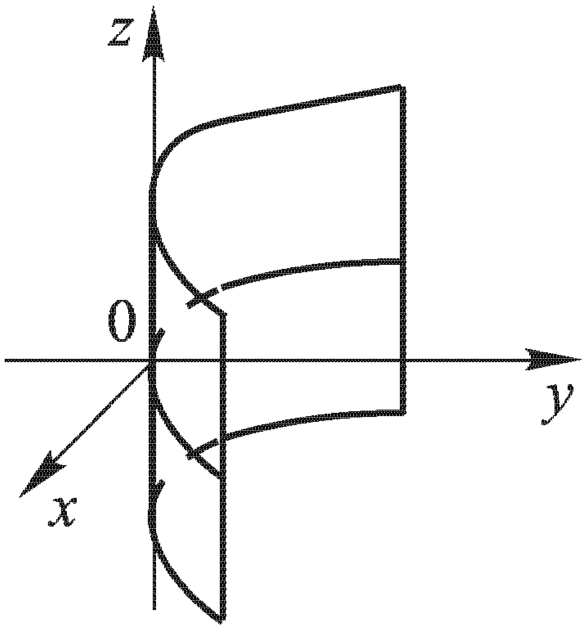
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

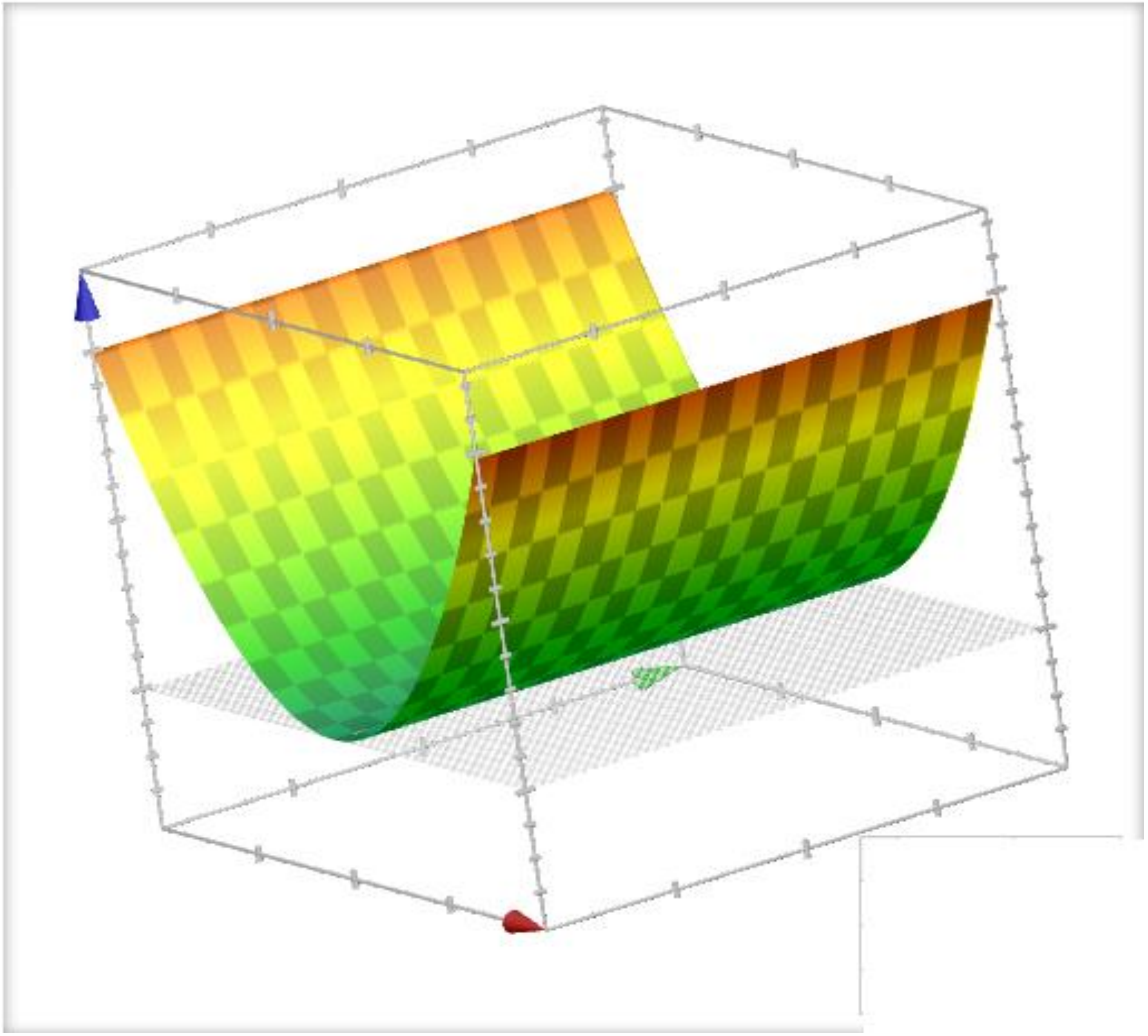

z
0
y
x

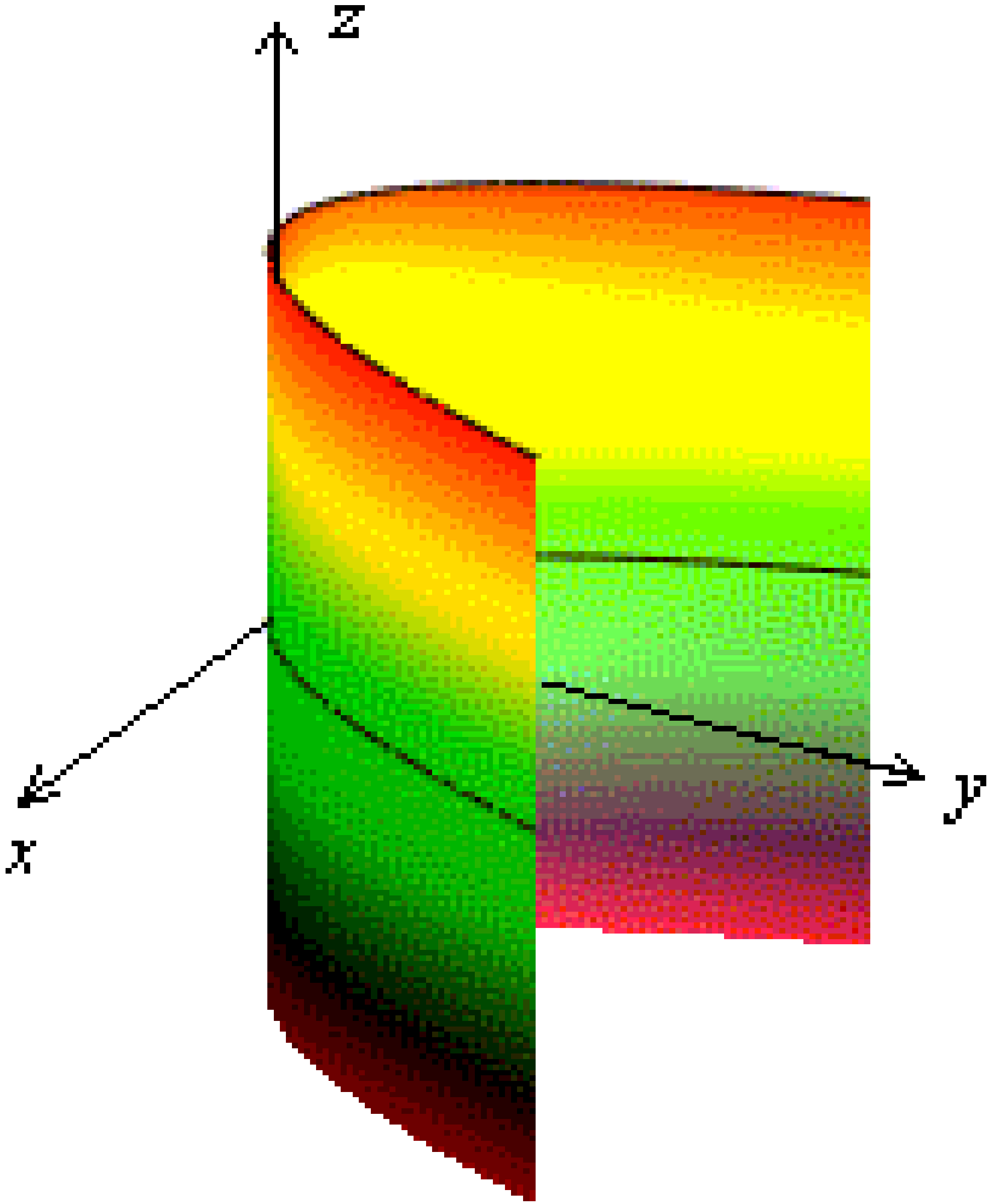


Parabolik silindr

$$x^2 = 2py$$






z
x
y

Aylanma sirtlar

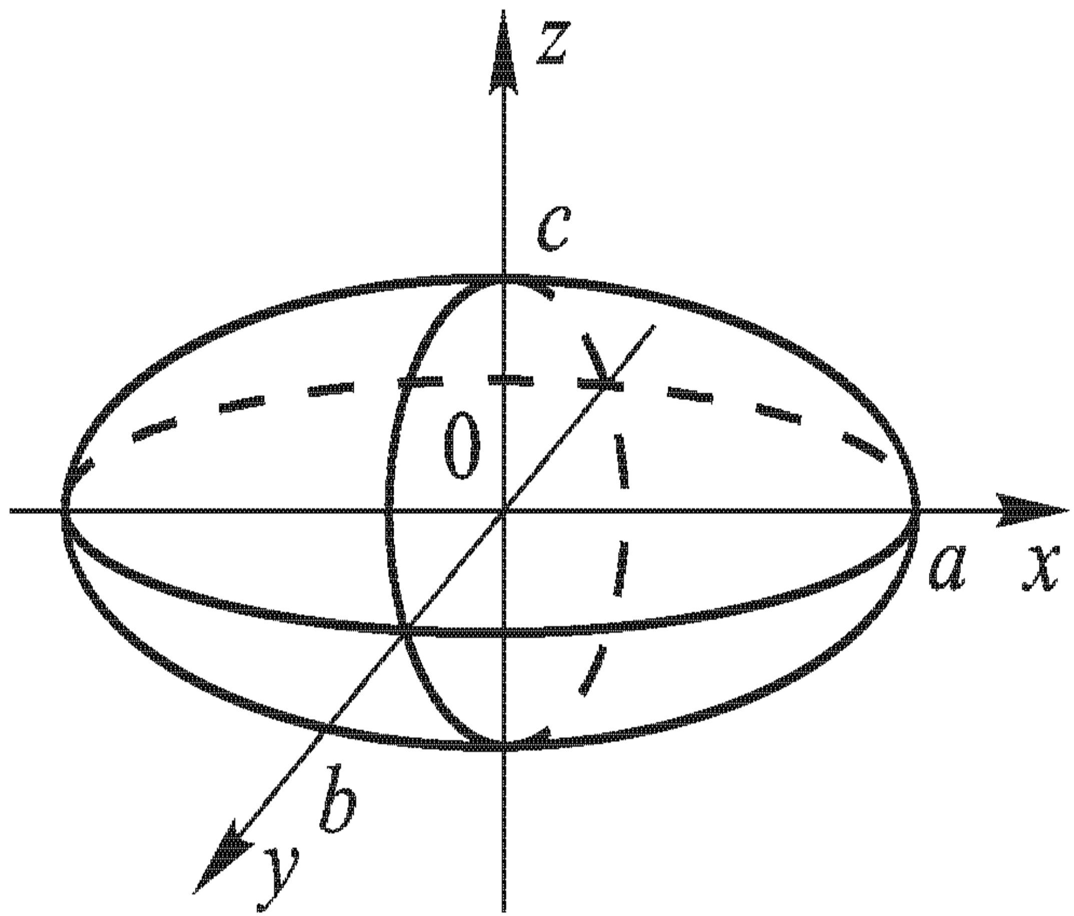
Aylanma ellipsoidlar

Ellipsoid tenglamasida $a = b$, $b = c$ yoki $a = c$ bo‘lganda aylanma ellipsoidlar paydo bo‘ladi.

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{c^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1$$

$$\frac{x^2 + z^2}{a^2} + \frac{y^2}{b^2} = 1$$



z
c
0
a
x
b
y

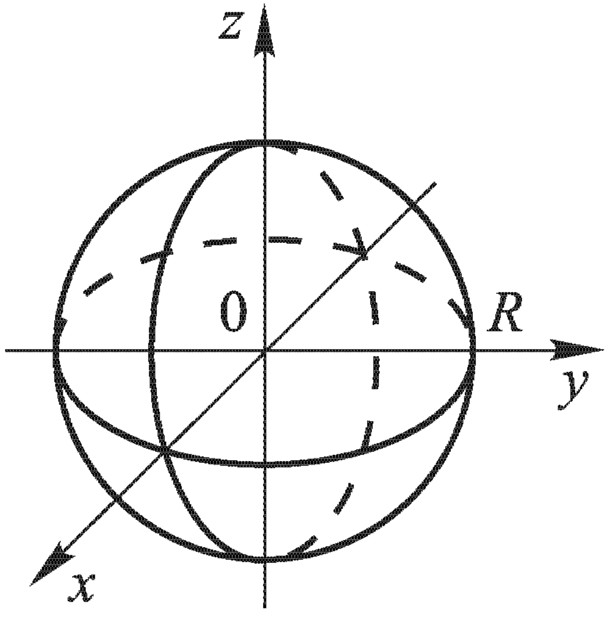
Ellipsoid tenglamasida $a = b = c = R$ sfera hosil bo‘ladi. Demak, sfera ham aylanma ellipsoid bo‘ladi, chunki sferani aylanma ellipsoid ko‘rinishida

yozish mumkin

$$\frac{x^2 + y^2}{R^2} + \frac{z^2}{R^2} = 1$$

$$\frac{x^2}{R^2} + \frac{y^2 + z^2}{R^2} = 1$$

$$\frac{x^2 + z^2}{R^2} + \frac{y^2}{R^2} = 1$$



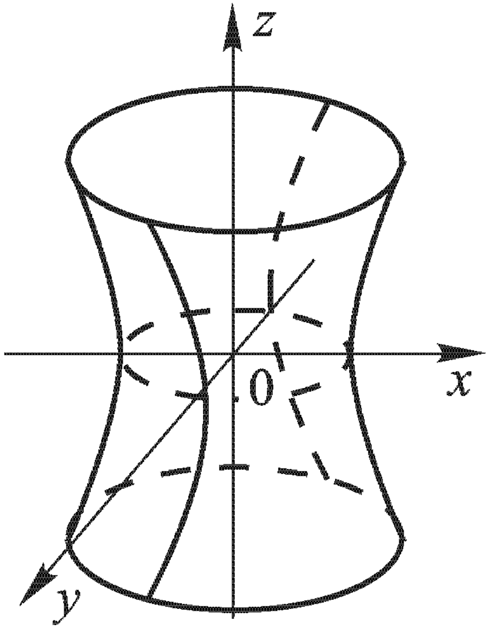
Bir pallali aylanma giperboloid

Bir pallali giperboloid tenglamasida $a=b$, $b=c$ yoki $a=c$ bo‘lganda bir pallali aylanma giperboloidlar paydo bo‘ladi.

$$\frac{x^2+y^2}{a^2}-\frac{z^2}{c^2}=1$$

$$\frac{x^2}{a^2}+\frac{y^2-z^2}{b^2}=1$$

$$\frac{x^2-z^2}{a^2}+\frac{y^2}{b^2}=1$$



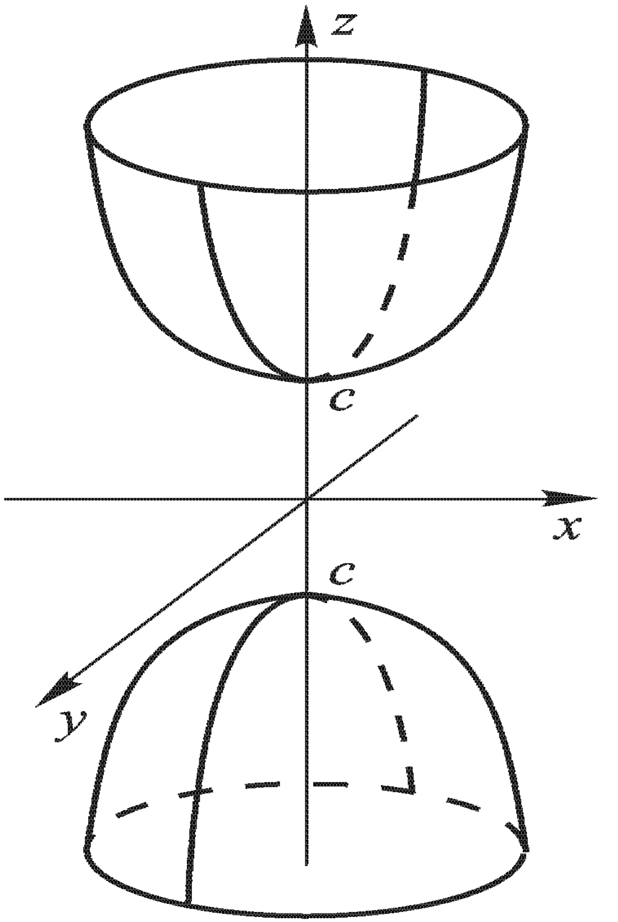
Ikki pallali aylanma giperboloid

Ikki pallali giperboloid tenglamasida $a = b$, $b = c$ yoki $a = c$ bo‘lganda ikki pallali aylanma giperboloidlar paydo bo‘ladi.

$$\frac{x^2 + y^2}{a^2} - \frac{z^2}{c^2} = -1$$

$$\frac{x^2}{a^2} + \frac{y^2 - z^2}{b^2} = -1$$

$$\frac{x^2 - z^2}{a^2} + \frac{y^2}{b^2} = -1$$



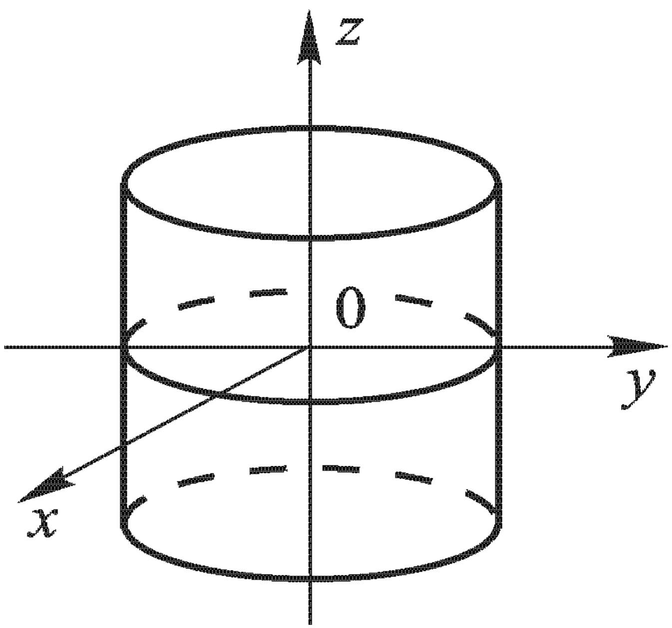
Aylanma elliptik paraboloid

Elliptik paraboloid tenglamasida $a=b$, $b=c$ yoki $a=c$ bo‘lganda aylanma elliptik paraboloidlar paydo bo‘ladi.

$$x^2+y^2=2pz$$

$$x^2+z^2=2py$$

$$y^2+z^2=2px$$



Aylanma giperbolik paraboloid

Giperbolik paraboloid tenglamasida $a = b$, $b = c$ yoki $a = c$ bo‘lganda aylanma giperbolik paraboloidlar paydo bo‘ladi.

$$x^2 - y^2 = 2pz$$

$$x^2 - z^2 = 2py$$

$$y^2 - z^2 = 2px$$