



TOSHKENT IRRIGATSIYA VA QISHLOQ
XO'JALIGINI MEXANIZATSIYALASH
MUHANDISLARI INSTITUTI



MAVZU: Chiziqli tenglamalar sistemasini yechish



REJA:

1. Kramer usuli.
2. Teskari matritsa usuli.
3. Gauss usuli.

Kramer usuli.

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}, \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0,$$

$$\Delta_x = \begin{vmatrix} b_{11} & a_{12} & a_{13} \\ b_{21} & a_{22} & a_{23} \\ b_{31} & a_{32} & a_{33} \end{vmatrix}, \Delta_y = \begin{vmatrix} a_{11} & b_{12} & a_{13} \\ a_{21} & b_{22} & a_{23} \\ a_{31} & b_{32} & a_{33} \end{vmatrix},$$

$$\Delta_z = \begin{vmatrix} a_{11} & a_{12} & b_{13} \\ a_{21} & a_{22} & b_{23} \\ a_{31} & a_{32} & b_{33} \end{vmatrix}. \quad x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta}.$$

$(x; y; z)$

1-misol. Chiziqli tenglamalar sistemasini Kramer usulida yeching.

$$\begin{cases} 2x - y + z = 2 \\ 3x + 2y + 2z = -2 \\ x - 2y + z = 1 \end{cases}$$

$$\text{Yechish. } \Delta = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & 2 \\ 1 & -2 & 1 \end{vmatrix} = 5, \quad \Delta_x = \begin{vmatrix} 2 & -1 & 1 \\ -2 & 2 & 2 \\ 1 & -2 & 1 \end{vmatrix} = 10,$$

$$\Delta_y = \begin{vmatrix} 2 & 2 & 1 \\ 3 & -2 & 2 \\ 1 & 1 & 1 \end{vmatrix} = -5, \quad \Delta_z = \begin{vmatrix} 2 & -1 & 2 \\ 3 & 2 & -2 \\ 1 & -2 & 1 \end{vmatrix} = -15$$

$$x = \frac{10}{5} = 2, \quad y = \frac{-5}{5} = -1, \quad z = \frac{-15}{5} = -3. \quad \mathbf{J:(2;-1;-3)}$$

2-misol. Teskari matritsa usulida yeching.

$$\begin{cases} 2x + y + 2z = 6 \\ x - 3y - z = -5 \\ 5x - 2y + z = -1 \end{cases}$$

Yechish. $A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & -3 & -1 \\ 5 & -2 & 1 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $B = \begin{pmatrix} 6 \\ -5 \\ -1 \end{pmatrix}$. $X = A^{-1} \cdot B$

$|A| = 10$, $A_{11} = -5$, $A_{12} = -6$, $A_{13} = 13$, $A_{21} = -5$,
 $A_{22} = -8$, $A_{23} = 9$, $A_{31} = 5$, $A_{32} = 4$, $A_{33} = -7$


$$A^{-1} = \frac{1}{10} \begin{pmatrix} -5 & -5 & 5 \\ -6 & -8 & 4 \\ 13 & 9 & -7 \end{pmatrix}$$

$$X = \frac{1}{10} \begin{pmatrix} -5 & -5 & 5 \\ -6 & -8 & 4 \\ 13 & 9 & -7 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -5 \\ -1 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} -30 + 25 - 5 \\ -36 + 40 - 4 \\ 78 - 45 + 7 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$$

$$(-1; 0; 4)$$



Uchburchakli matritsa.



Agar asosiy diogonaldan pastda (yoki yuqorida) turgan elementlar nollardan iborat bo'lsa, bunday matritsa ***uchburchakli*** matritsa deyiladi.

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \mathbf{0} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ \mathbf{0} & \mathbf{0} & \dots & a_{nn} \end{pmatrix}$$

yoki

$$\begin{pmatrix} a_{11} & \mathbf{0} & \dots & \mathbf{0} \\ a_{21} & a_{22} & \dots & \mathbf{0} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$



Asosiy va kengaytirilgan matritsa.



Asosiy matritsa – o'zgaruvchilarning oldidagi koeffitsiyenlardan iborat bo'ladi.

Kengaytirilgan matritsa =
asosiy matritsa + ozod hadlar ustuni.

Gauss usuli

3-misol.
$$\begin{cases} x - 2y + 3z = 6 \\ 2x + 3y - 4z = 16 \\ 3x - 2y - 5z = 12 \end{cases}$$
 sistemasni Gauss usulida yeching.

Yechish. Berilgan sistemaning kengaytirilgan matritsasini yozib olamiz va uni uchburchakli matritsa ko'rinishiga keltiramiz:

$$\begin{pmatrix} 1 & -2 & 3 & 6 \\ 2 & 3 & -4 & 16 \\ 3 & -2 & -5 & 12 \end{pmatrix}$$

1qatorni*(-2)+2 qatorga, 1qatorni*(-3)+3 qatorga

va natijada
$$\begin{pmatrix} 1 & -2 & 3 & 6 \\ 0 & 7 & -10 & 4 \\ 0 & 4 & -14 & -6 \end{pmatrix}$$
 ni hosil qilamiz.

Endi 4 ni nolga aylantiramiz: buning uchun

2qatorni $\cdot(-4)$ ga , 3qatorni $\cdot 7$ ga va ularni qo'shamiz

$$\begin{pmatrix} 1 & -2 & 3 & 6 \\ 0 & 7 & -10 & 4 \\ 0 & 0 & -58 & -58 \end{pmatrix}$$

Hosil bo'lgan matritsadan quyidagi sistemani yozamiz:

$$\begin{cases} x - 2y + 3z = 6 \\ 7y - 10z = 4 \\ -58z = -58 \end{cases} \Rightarrow \begin{cases} x - 2y + 3z = 6 \\ 7y - 10z = 4 \\ z = 1 \end{cases} \Rightarrow$$

$$\begin{cases} x - 2y + 3z = 6 \\ y = 2 \\ z = 1 \end{cases} \Rightarrow \begin{cases} x = 7 \\ y = 2 \\ z = 1 \end{cases} \Rightarrow \text{Javob:}(7; 2; 1)$$

Mustaqil yechish uchun misollar

Kramer usulida yeching.

$$1) \begin{cases} 2x - y + z = 2 \\ 3x + 2y + 2z = -2 \\ x - 2y + z = 1 \end{cases} \quad 2) \begin{cases} 2x - 4y + 3z = 1 \\ x - 2y + 4z = 3 \\ 3x - y + 5z = 2 \end{cases}$$

Teskari matritsa usulida yeching.

$$1) \begin{cases} 2x + y = 5 \\ x + 3z = 16 \\ 5y - z = 10 \end{cases} \quad 2) \begin{cases} 7x + 2y + 3z = 15 \\ 5x - 3y + 2z = 15 \\ 10x - 11y + 5z = 36 \end{cases}$$

Gauss usulida yeching.

$$1) \begin{cases} 2x + y + 4z = 20 \\ 2x - y - 3z = 3 \\ 3x + 4y - 5z = -8 \end{cases}$$

$$2) \begin{cases} 2x + y - 3z = 3 \\ 3x + 4y - 5z = 9 \\ 2y + 7z = 11 \end{cases}$$

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